Title: Can inflation really begin with inhomogenous initial conditions?

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Abstract:

Can inflation really inflate?

Eugene A. Lim
Cosmological Frontiers in Fundamental Physics, PI.

with Katy Clough, Raphael Flaugher and Brendan DiNunno

Inflation

PHYSICAL REVIEW D VOLUME 23, NUMBER 2 15 JANUARY 1981

Inflationary universe: A possible solution to the horizon and flatness problems

Alan H. Guth*

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 11 August 1980)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.

If inflation needs homogenous initial conditions, then it defeats its own purpose.

Inhomogenous Inflation

Some older prior studies with numerical relativity

- 1+1D Spherical Scalar field collapse with inflationary potential (Goldwirth and Piran 1989)
- Inhomogenous scalar field in 3+1 numerical relativity (Laguna, Kurki-Susunio, Matzner, 1991)

"Canonical result" : spacetime needs to be homogenous around a patch of H_{act}^{-1}

$$H_{act}^2 = \frac{8\pi}{3}\left[V(\phi_0) + \frac{1}{2}(\nabla^2\phi)^2 + \frac{1}{2}\dot{\phi}^2\right]$$

Newer work!

- East, Kleban, Linde, Senatore (2015) : Previous talks by Senatore and East!

Result : Large field inflation is robust to large sub-Hubble gradient energies

$$\rho_{grad} = 1000\rho_{vacuum}$$

initially uniform expanding spacetime

This work

Simple Minded Low Brow Question :

Given an inflating potential, with the inflaton situated around the inflating plateau but with non-trivial scatter (gradients), can it still inflate?

What does it take to kill inflation?
Does robustness depend on models?

- Large vs Small field inflation
- Initially expanding/contracting/mixed spacetimes

"Successful inflation" : Some part of Hubble volume inflate

This work

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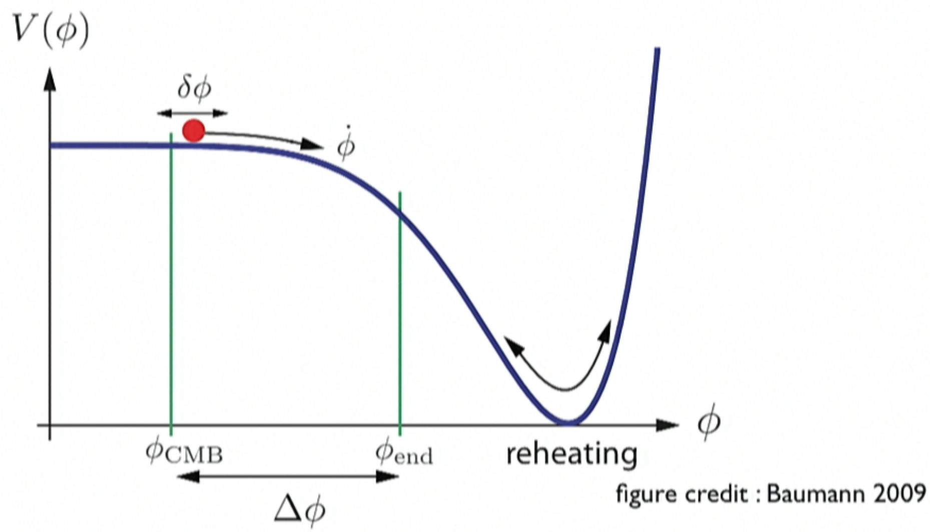
What can kill inflation?



$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$

kinetic gradient vacuum

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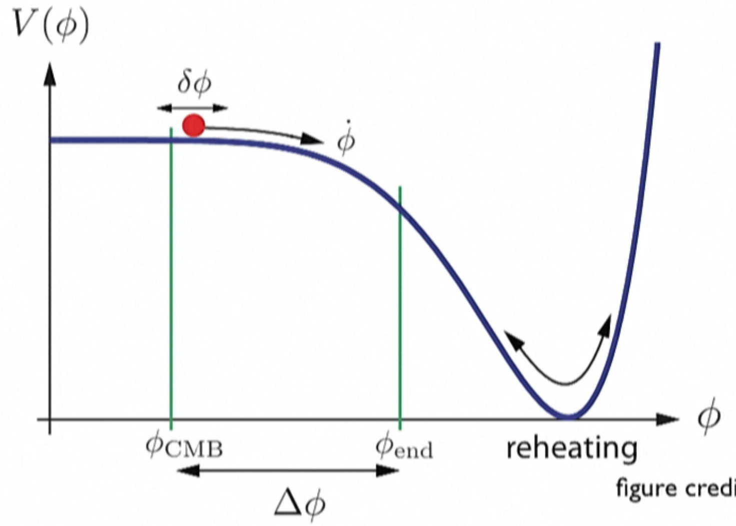

figure credit : Baumann 2009

$$\rho = \cancel{\frac{1}{2}\dot{\phi}^2} + \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$

kinetic ≈ 0 slow roll

gradient

vacuum

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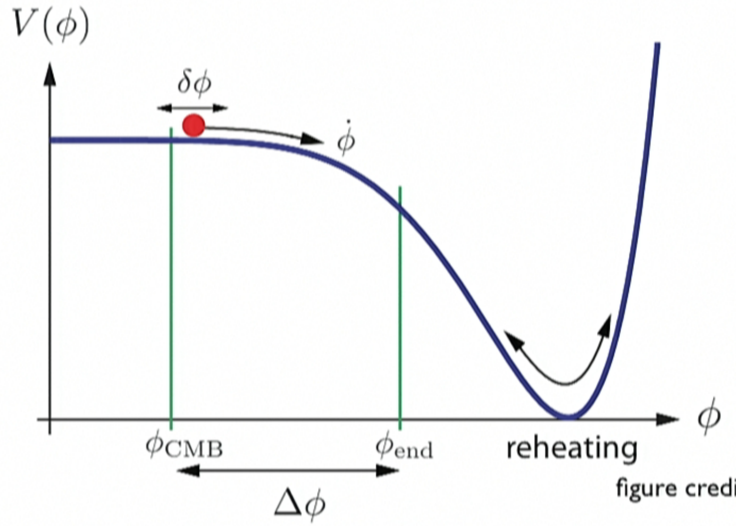

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$$\rho = \cancel{\frac{1}{2}\dot{\phi}^2} + \cancel{\frac{1}{2}(\nabla\phi)^2} + V(\phi)$$

kinetic	gradient	vacuum
≈ 0	≈ 0	
slow roll	homogeneity	

Inflation require *coherent slow roll* to work.

What can kill inflation?

Some hard and fast rules, in dS space

Vacuum energy $\rho_V = V(\phi) \propto \text{constant}$

Kinetic energy $\rho_K = \frac{1}{2}\dot{\phi}^2 \propto a^{-6}$

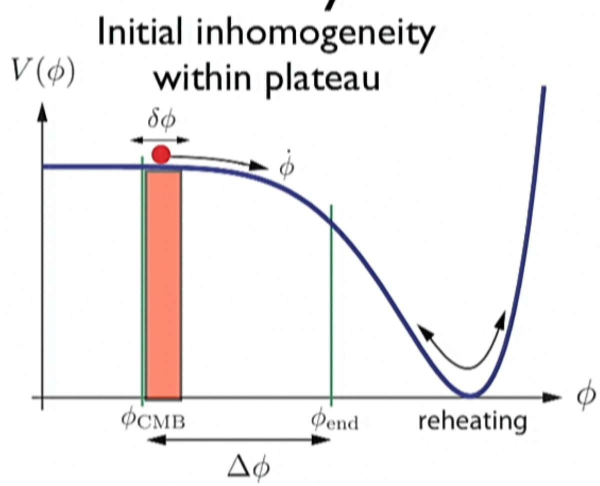
Gradient energy $\rho_{grad} = \frac{1}{2}\gamma^{ij}\partial_i\phi\partial_j\phi \propto a^{-4} \text{ to } a^{-5}$

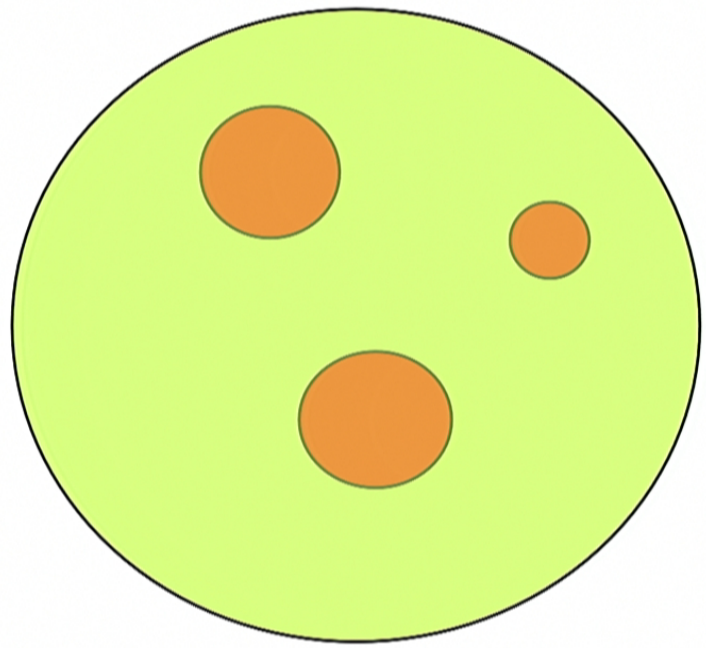
Since inflation wants to kill everything except vacuum energy, we have to work hard to kill it.

What can kill inflation?

Mechanism 1 : The Pit of Doom

Push some region of the inflating spacetime into a non-inflating part of the potential, then hope that scalar dynamics will "pull" in the rest.





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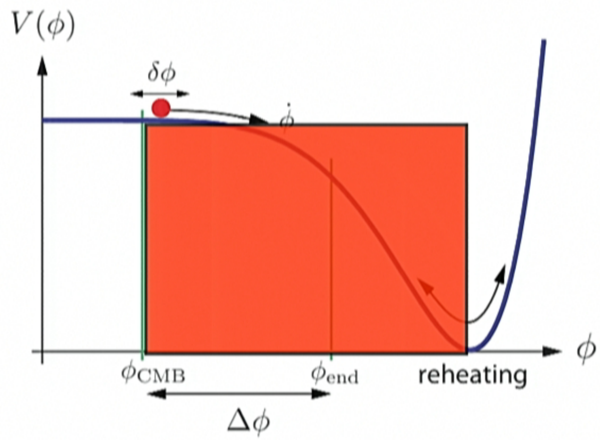
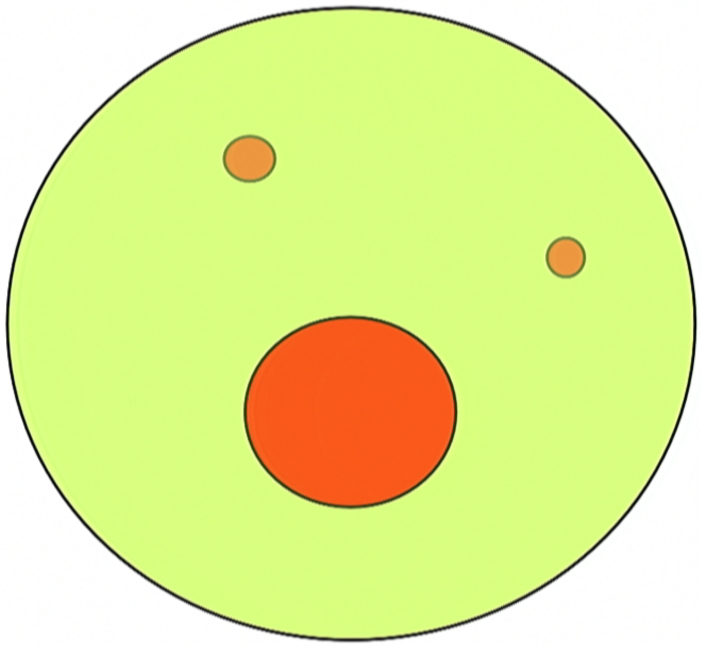
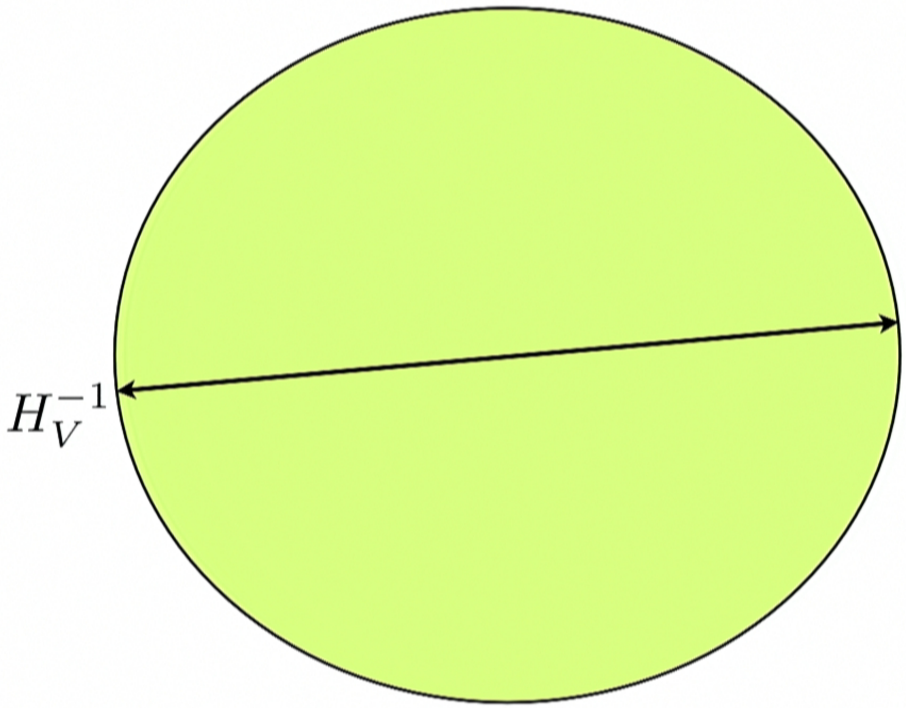




What can kill inflation?

Mechanism 2 : The Giant Death Black Hole

Make black holes, hope black hole drive local dynamics away from homogeneity



$$H_V = \sqrt{\frac{3}{8\pi V(\phi_0)}}$$

Vacuum Hubble length

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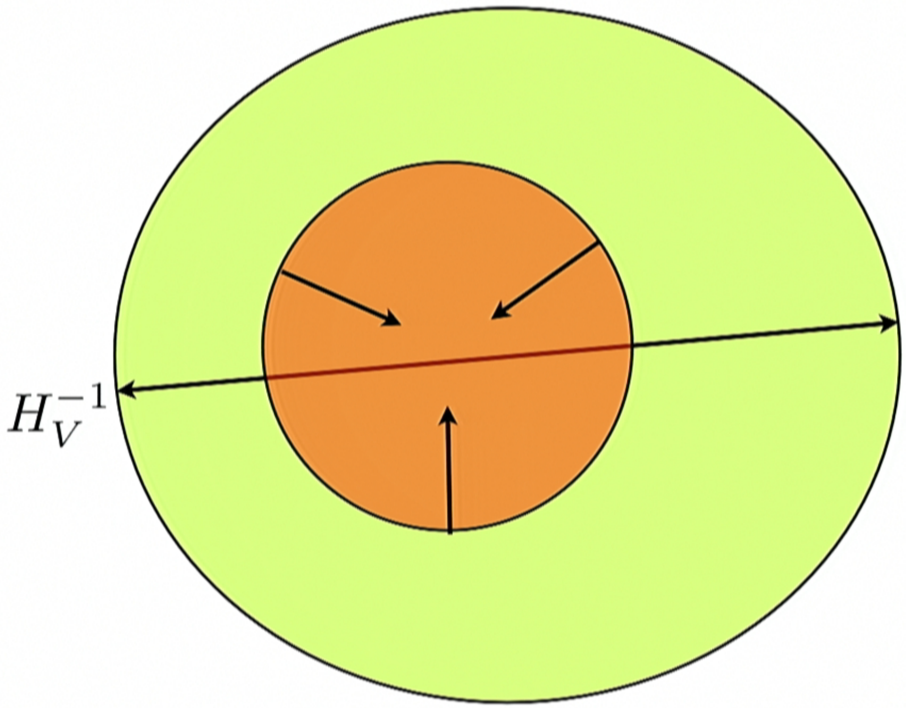

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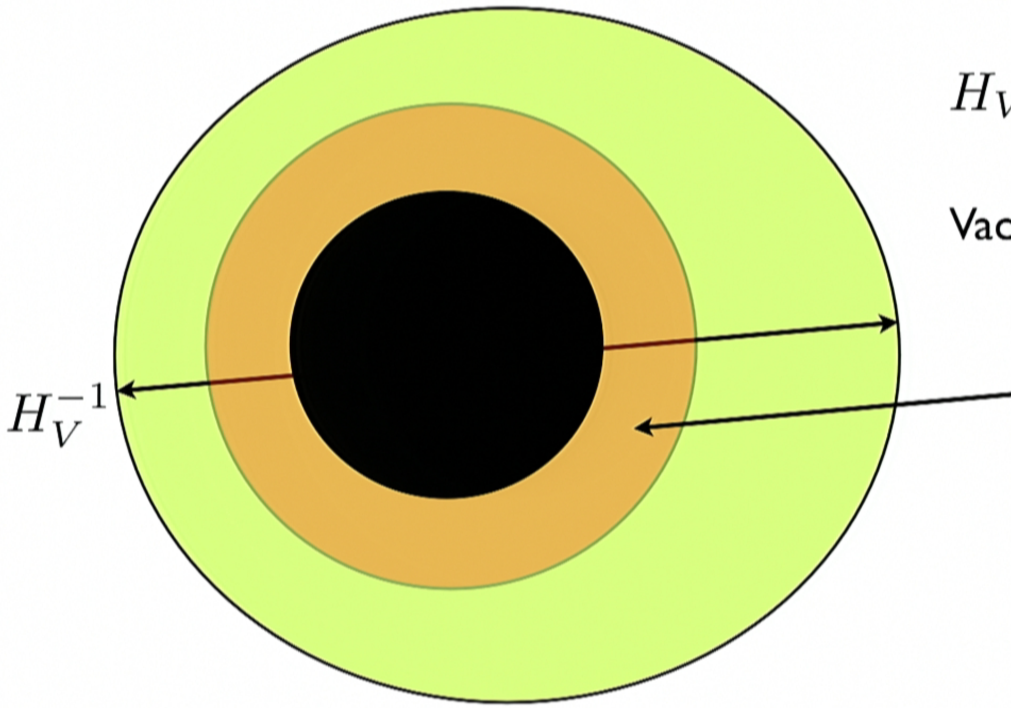


$$H_V = \sqrt{\frac{3}{8\pi V(\phi_0)}}$$

Vacuum Hubble length

Requires :

$$M_{BH} \approx H_V^{-1}$$

Plan of Attack

Robustness of Large vs Small Field Inflation

Robustness of Inflation to initial expansion/contraction/mixed spacetimes

Small vs Large field Inflation

Lyth Bound : amplitude of scalar perturbations and sufficient inflation imposes inflaton field traverse

Scalar amplitude $$\Delta_R^2 \approx \pi \frac{H^2}{M_p^2} \times \frac{1}{\epsilon} \approx 10^{-10}$$

Slow roll parameter

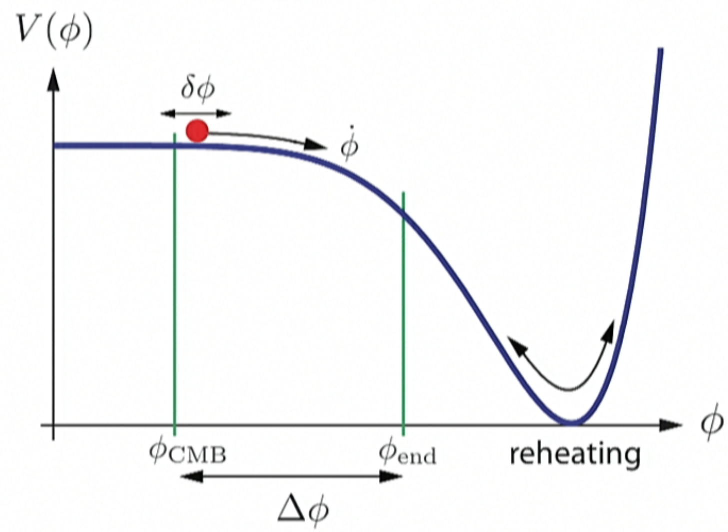
If H^2/M_p^2 is large/small, then the inflaton has to roll faster/slower.

Small vs Large field Inflation

Total Field Traverse

$$\Delta\phi \approx \frac{N}{2}\frac{H}{M_p}10^5 \times M_p$$

$N = \text{no. efolds}$



High scale inflation $\Rightarrow \Delta\phi \gg M_p$ "Large Field"

Low scale inflation $\Rightarrow \Delta\phi < M_p$ "Small Field"

GRChombo

Numerical GR with Adaptive Mesh Refinement

arXiv : 1503.03436
http://grchombo.github.io/

King's College London, University of London — University of Cambridge — Queen Mary University of London — Argonne National Laboratory


Katy Clough


Markus Kunesch


Pau Figueras


Hal Finkel


Saran Tunyasuvunakool


Eugene Lim

BSSN-CCZ4 Formalism   Built using *CHOMBO* libraries

Berger-Rigoutsos Block-structured AMR with sub-cycling

Public release late 2016 or soon™

Initial Set-up

Gravity sector:

$$ds^2 = -\alpha^2\, dt^2 + \gamma_{ij}(dx^i + \beta^i\, dt)(dx^j + \beta^j\, dt)$$

lapse 3-metric shift

Initially conformally flat $\gamma_{ij} = \chi^{-2}\delta_{ij}$

Extrinsic curvature K_{ij} initially trace-free.

BSSN with moving puncture gauge driver

Initial gauge conditions $\alpha = 1\ ,\ \beta_i = 0$

Gauge drivers

$$\partial_t\alpha = -\mu_{\alpha_1}\alpha^{\mu_{\alpha_2}}K + \mu_{\alpha_3}\beta^i\partial_i\alpha$$
$$\partial_t\beta^i = B^i$$
$$\partial_t B^i = \mu_{\beta_1}\alpha^{\mu_{\beta_2}}\partial_t\hat{\Gamma}^i - \eta B^i$$

Initial Set-up

Scalar sector:

Initial inhomogenous scalar field profile

$$\phi(t, \mathbf{x}) = \phi_0 + \frac{\delta\phi}{N} \sum_{k=1}^{N} \left(\sin \frac{2\pi k x}{L} + \sin \frac{2\pi k y}{L} + \sin \frac{2\pi k z}{L} \right)$$

Homogenous component (fixed at 100 efolds)

Inhomogenous component, L is some fixed length scale (not necessary the Hubble length)

kinetic term $\quad \eta \equiv \frac{1}{\alpha} \left(\dot{\phi} - \beta^k \partial_k \phi \right) = 0$

Initial metric configuration for χ solved using relaxation.

Initial Set-up

Numerical parameters:

Adaptive Mesh: 6-8 refinement levels, refinement ratio 2 per level, thresholded with K and ϕ .

Periodic Boundary Conditions

Simulation domain either H_V^{-1} or H_{act}^{-1}

$$H_V^2 = \frac{8\pi}{3}V(\phi_0) \qquad H_{act}^2 = \frac{8\pi}{3}\left(V(\phi_0) + \frac{1}{2}\gamma^{ij}\partial_i\phi\partial_j\phi + \frac{1}{2}\eta^2\right)$$

"Vacuum/Inflation scale" "Actual Hubble scale"

Note : in an inhomogenous initial state, these are just convenient length scales.

Large Field inflation

Model : $V(\phi) = m^2\phi^2$ and $H_{inf} = 1.25 \times 10^{-6} M_p$

Inhomogeneity wavelength and simulation domain — $L = H_V^{-1}$ (Because we want to make Death Black Hole)

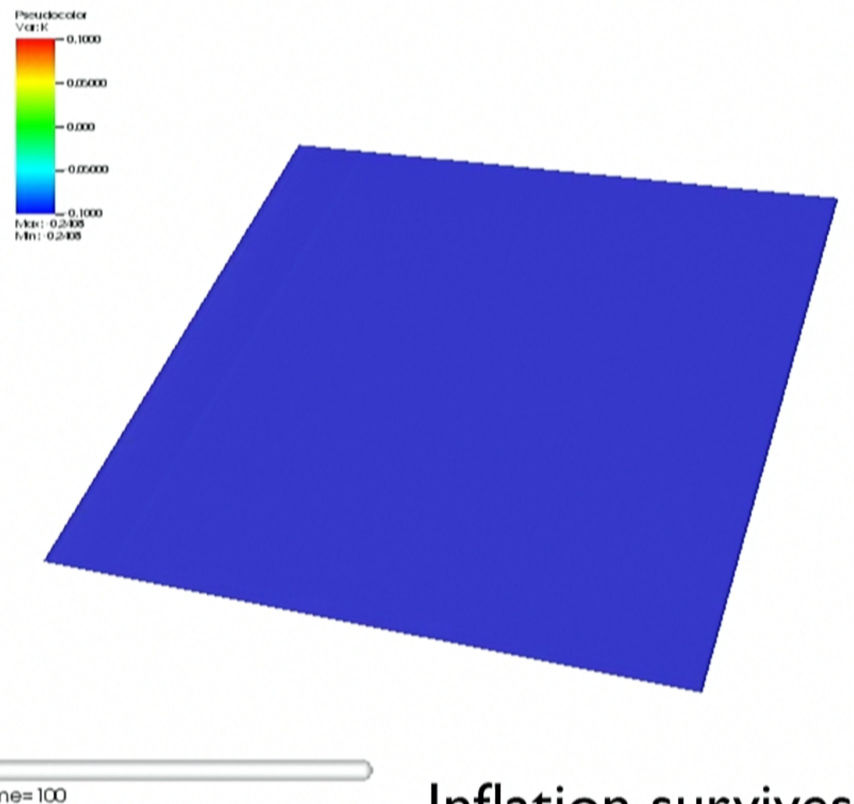
Constant initial expansion/contraction — $\mathrm{Tr}\ K_{ij} = K \approx -3H_{act}$

Scanning parameter — $\delta\phi = 0.01 M_p$ to $0.5 M_p$
or
$\rho_{grad} = 0.025\rho_V$ to $100\rho_V$

Large Field inflation

$\delta\phi = 0.15$ Initially expanding K

expansion K



Inflation survives, no BH

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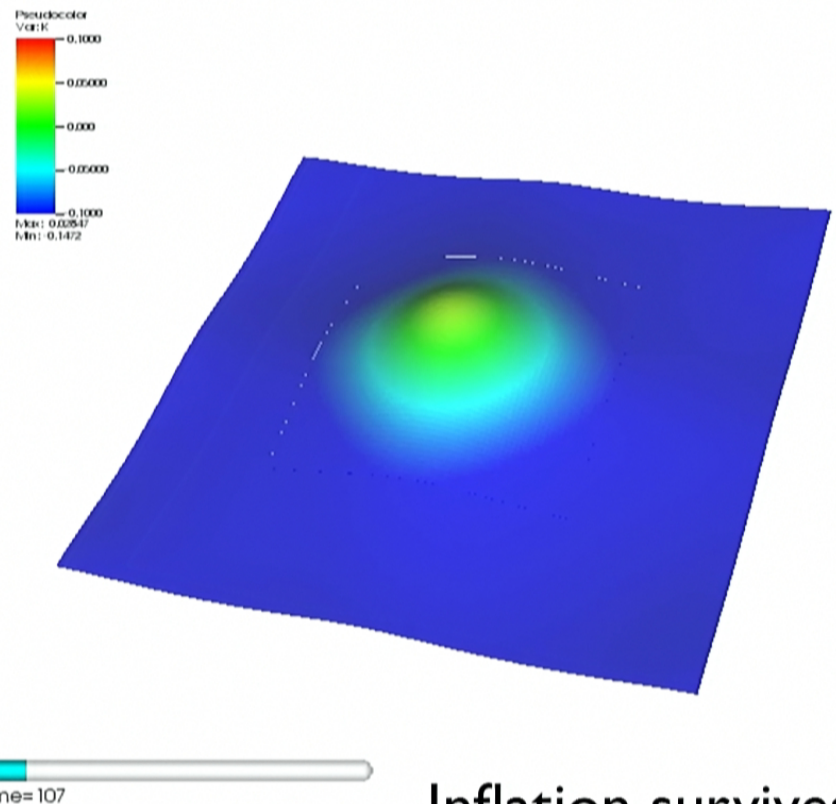

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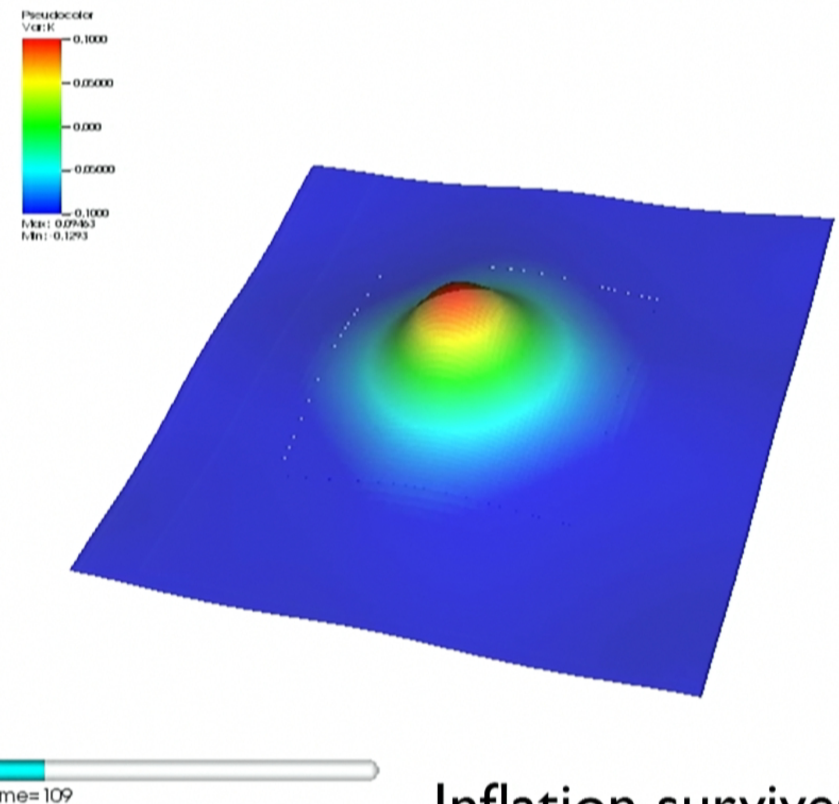

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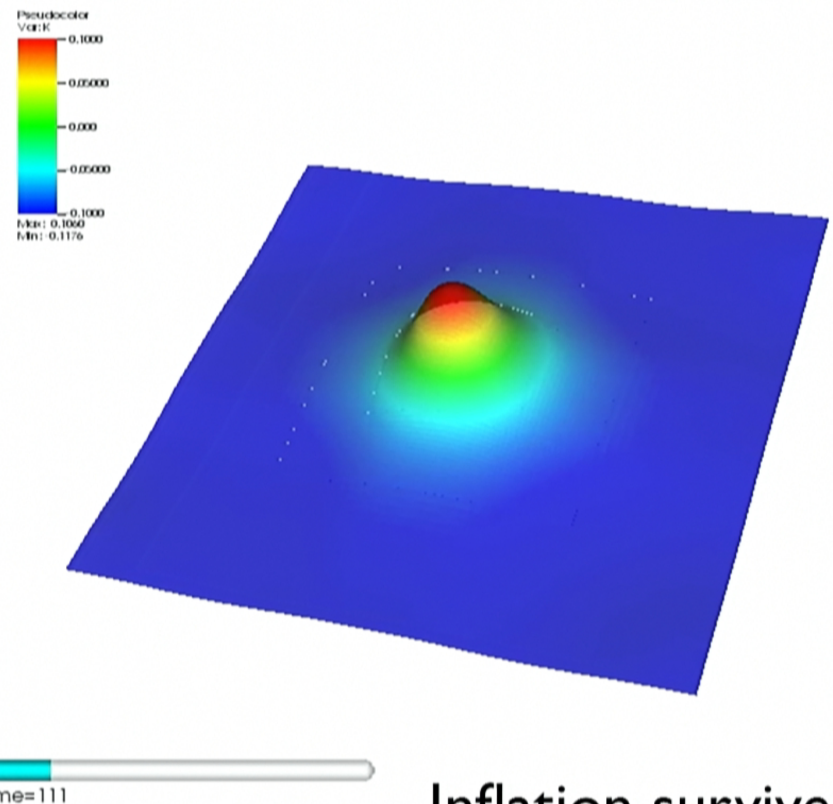

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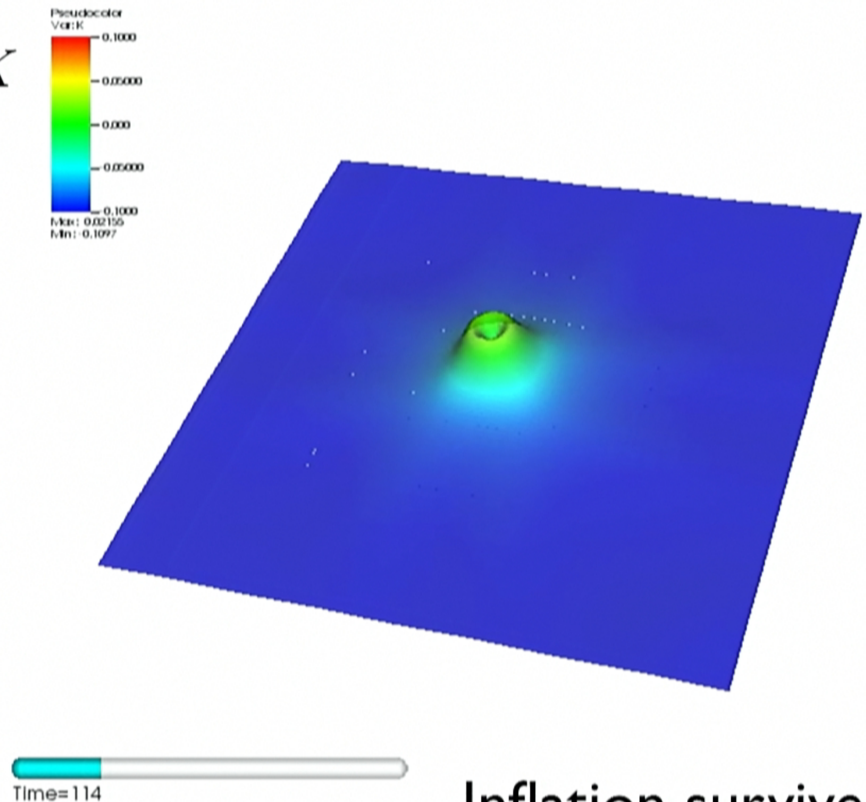

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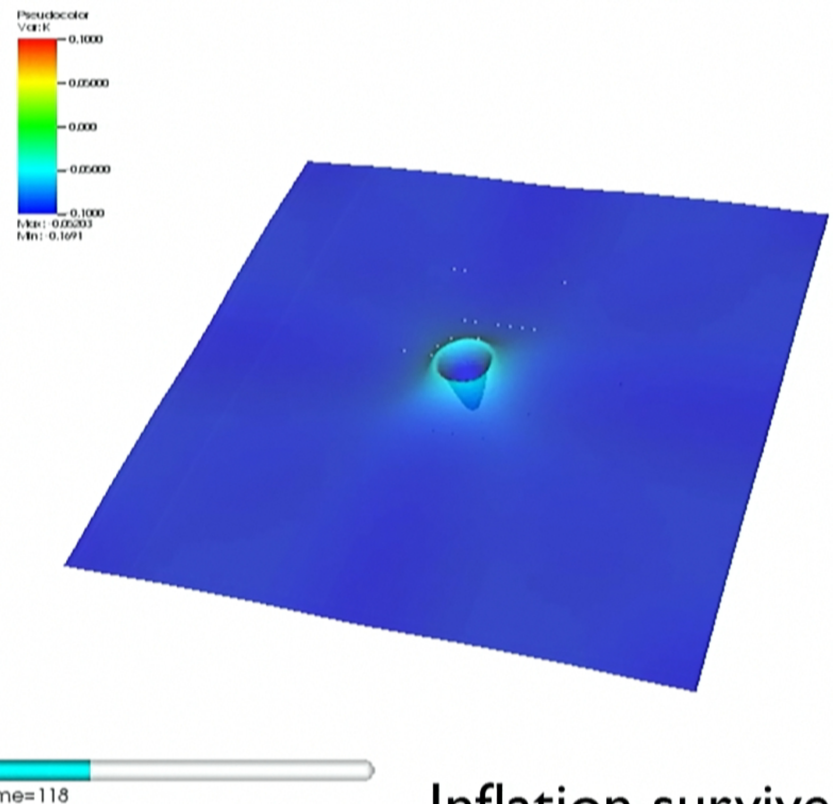

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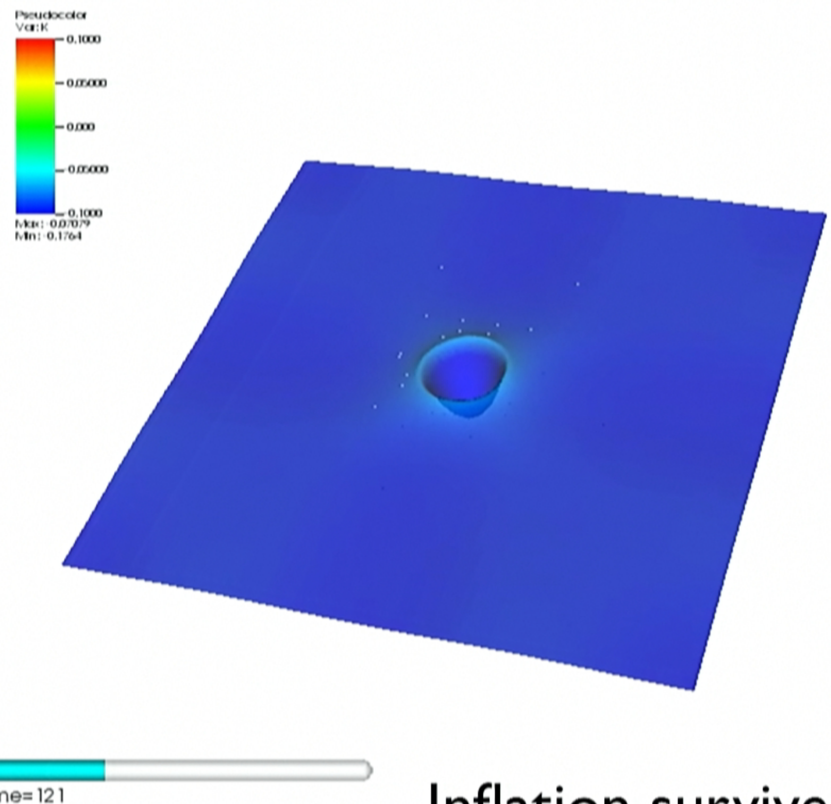

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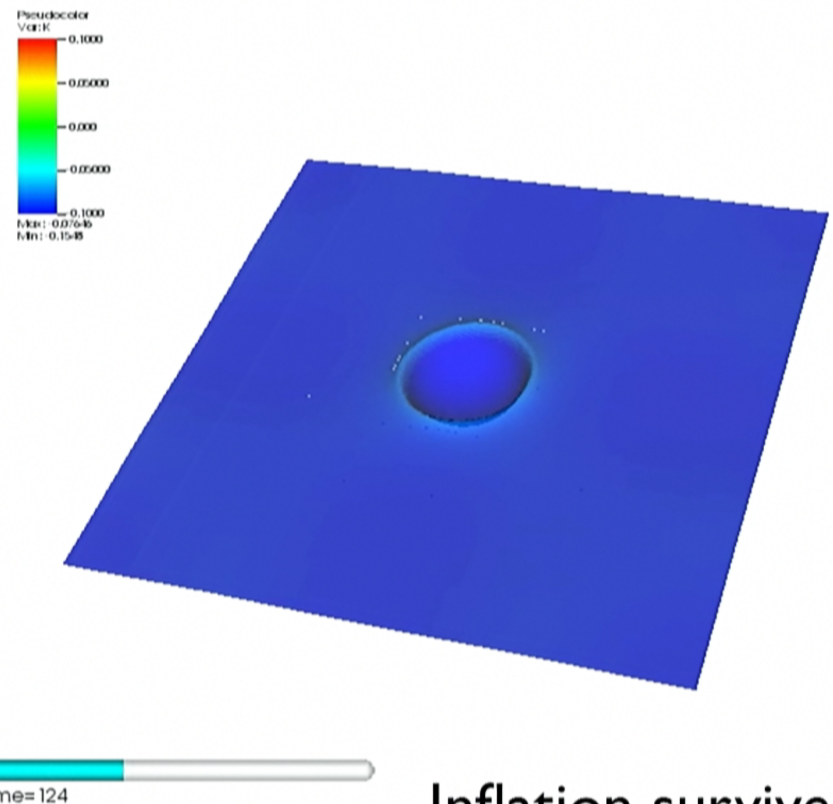

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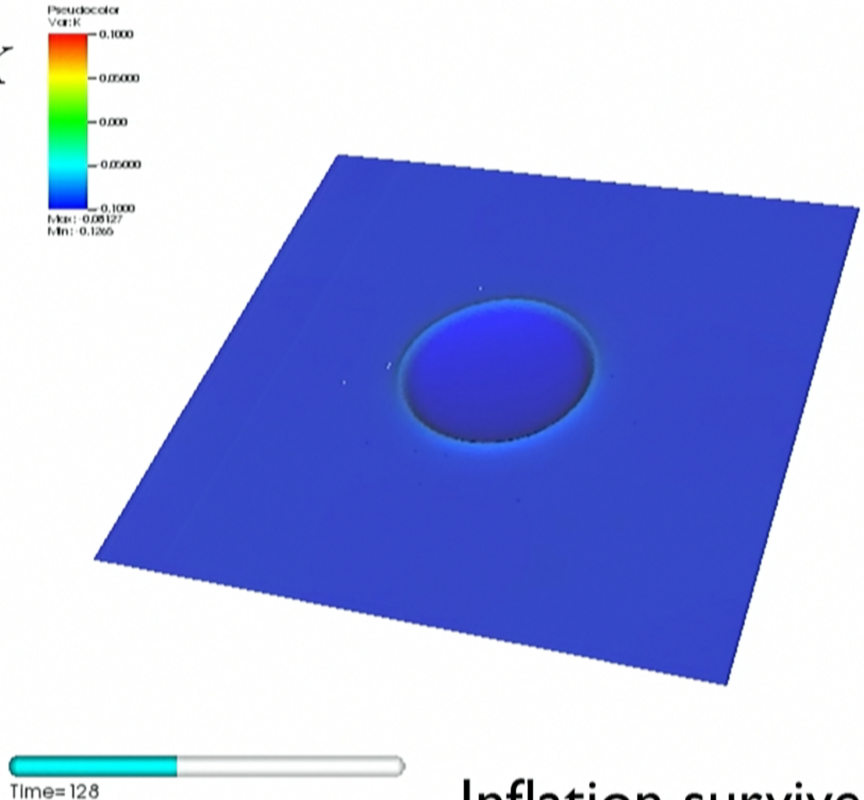

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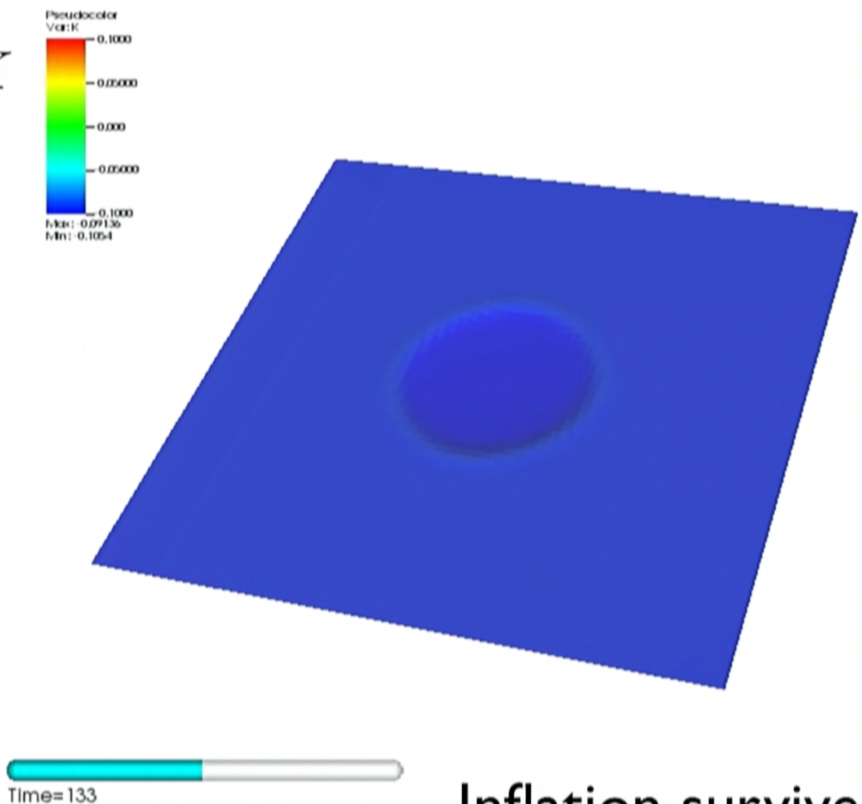

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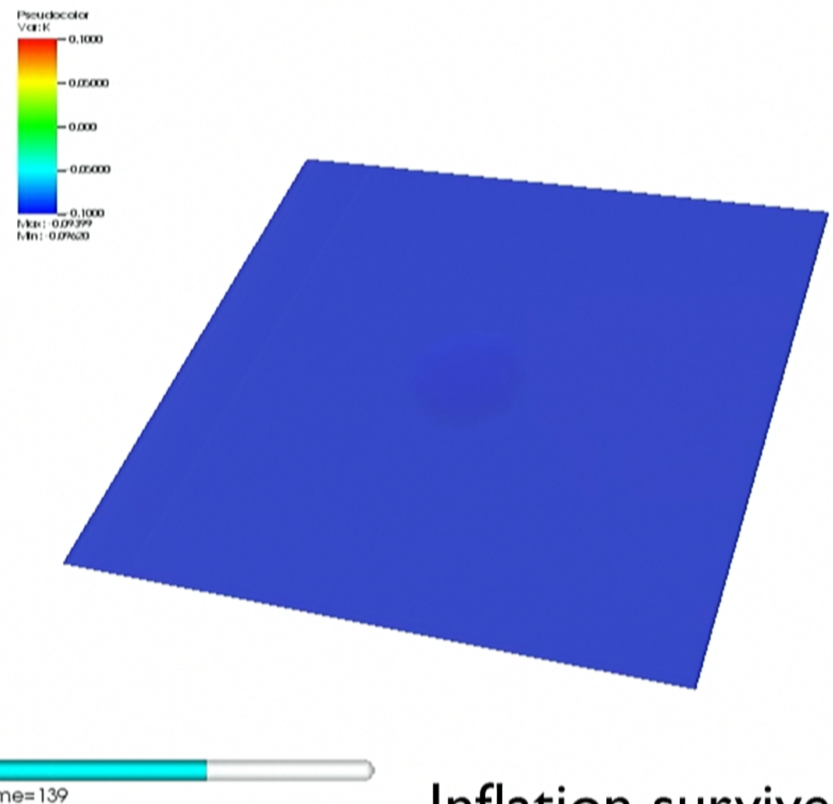

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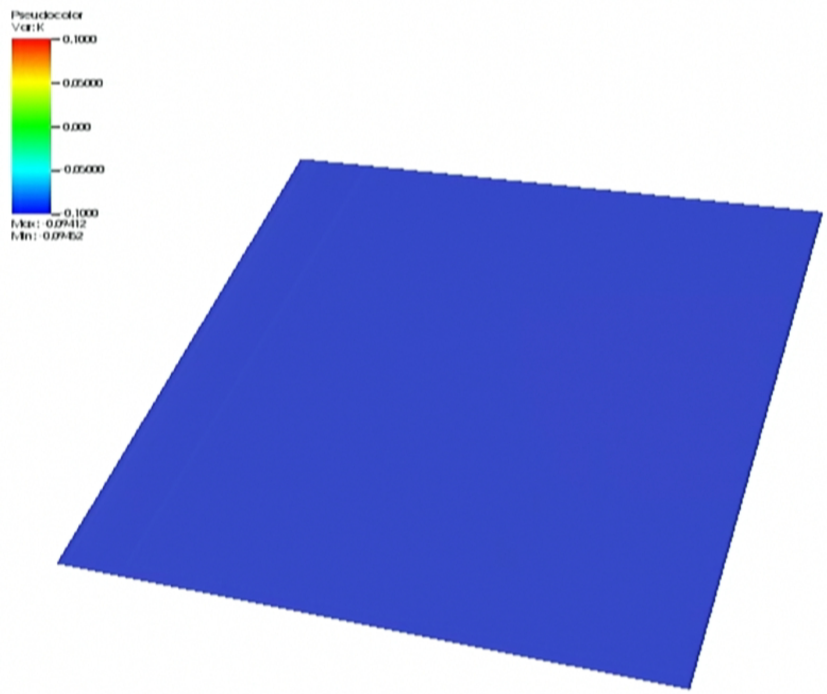


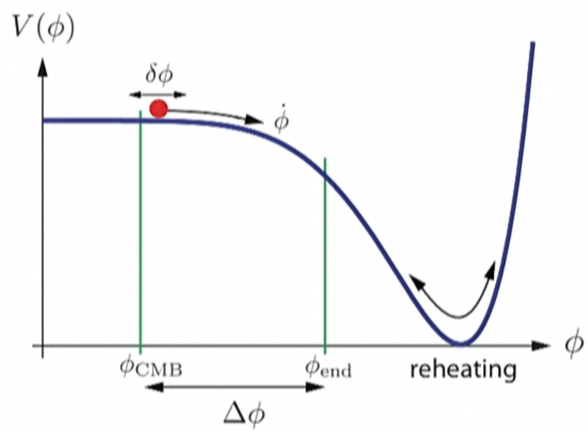
Time=144

Inflation survives, no BH

Large Field inflation

Broad conclusions:

1. Inflating domain $\Delta\phi \gg M_p$ but gradients $\delta\phi \ll \Delta\phi$ so never "fall into" reheating region.



2. Very robust to large gradients (agreeing with East et al).

3. To kill Large field, need Giant Death Black Hole.

Giant Death Black Hole?

A mode of wavelength L has gradient density:

$$\rho_{grad} \approx 3\pi^2 \frac{\delta\phi^2}{L^2}$$

Mass of a spherical blob of stuff of size L:

$$M(L) = \frac{4\pi}{3} L^3 \rho_{grad} = 4\pi^3 L \delta\phi^2$$

Hoop conjecture (assuming no initial expansion)

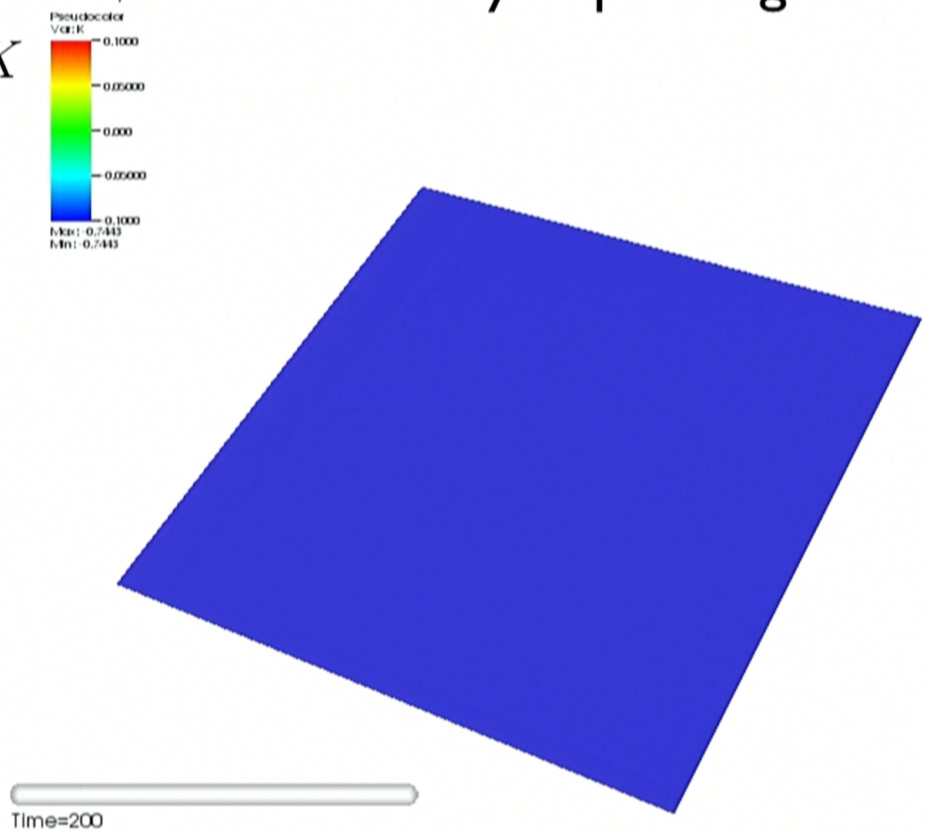
$$M(L) = 2L \Rightarrow \delta\phi_{crit} \geq \sqrt{2/\pi^3} \approx 0.25$$

Crank up $\delta\phi$ to see if we can make a GDBH.

Large Field inflation

$\delta\phi = 0.5$ Initially expanding K

expansion K



Inflation survives, not big enough BH

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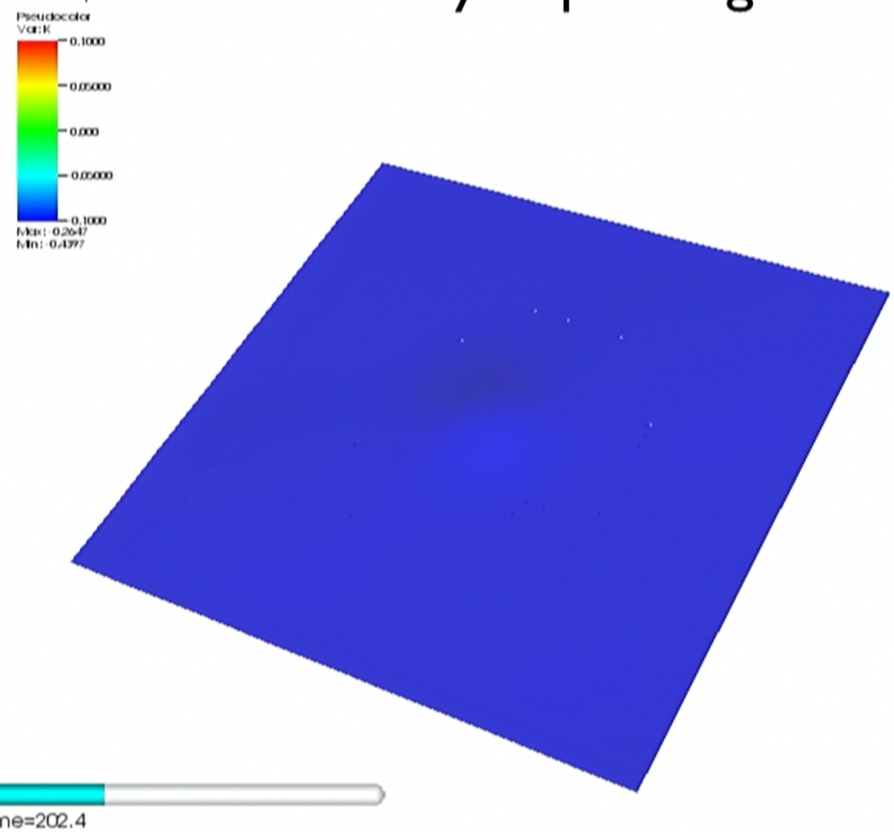

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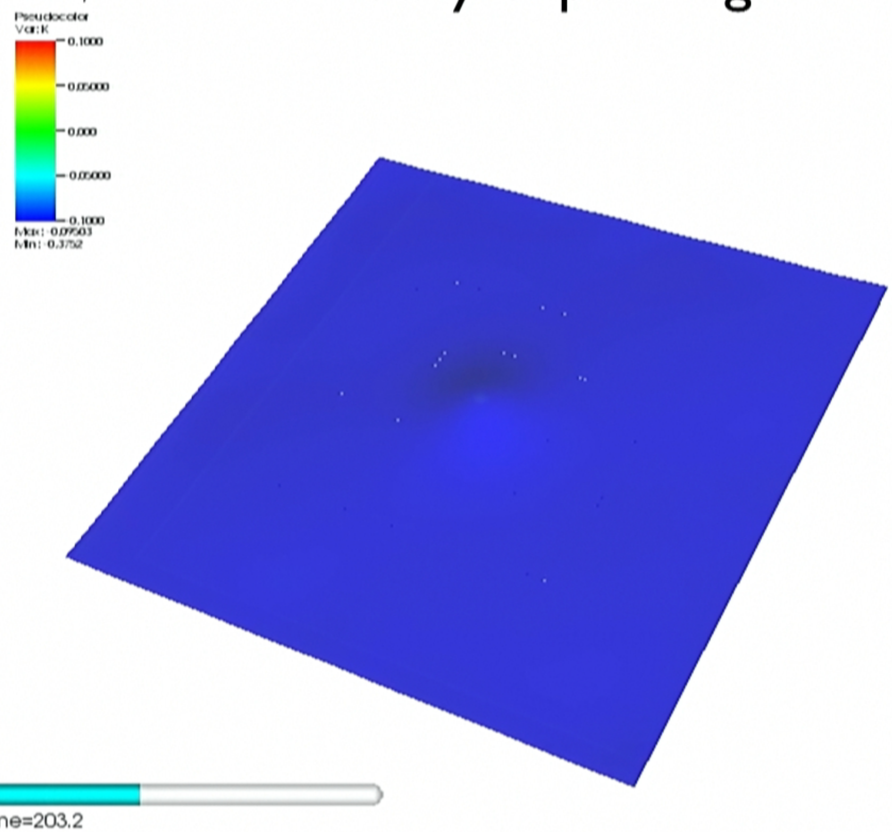

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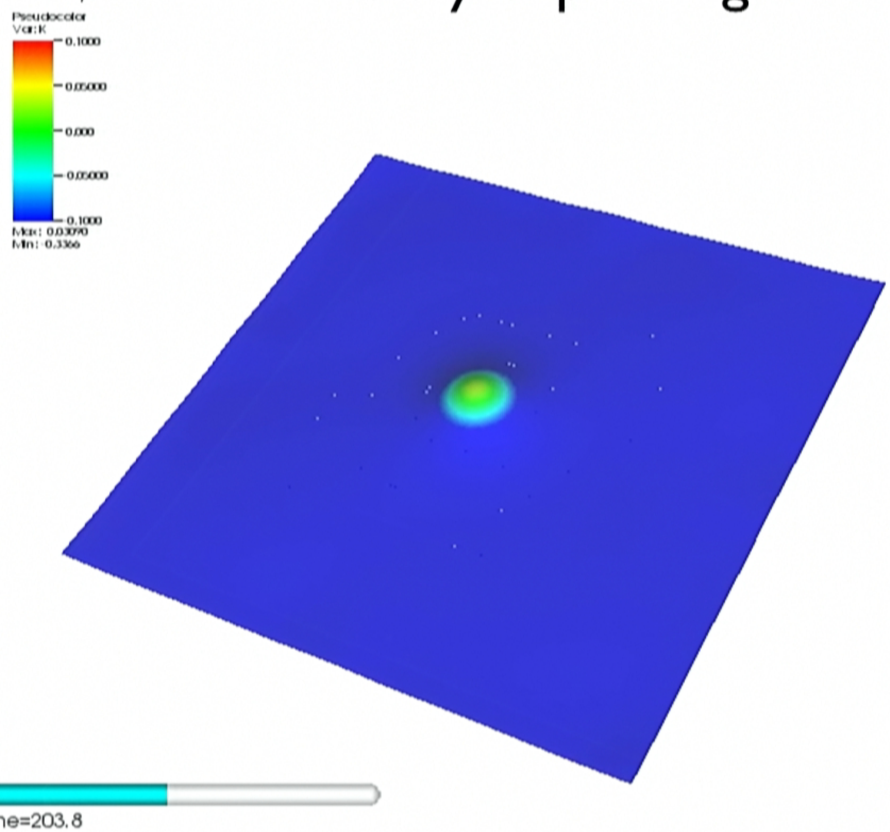

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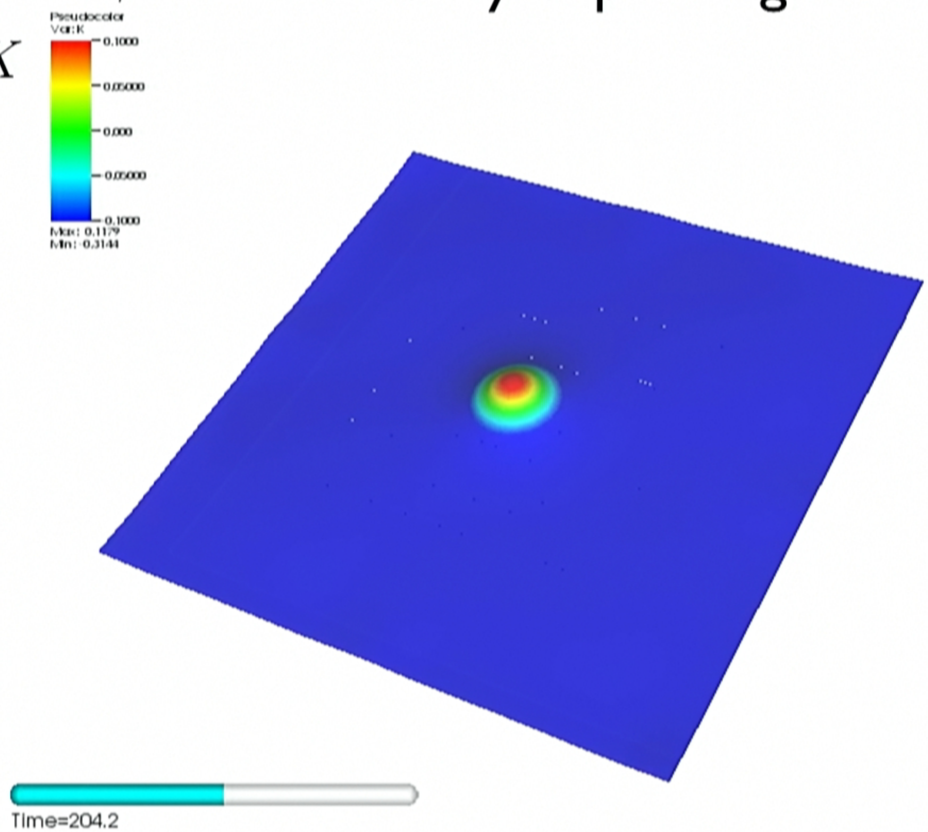

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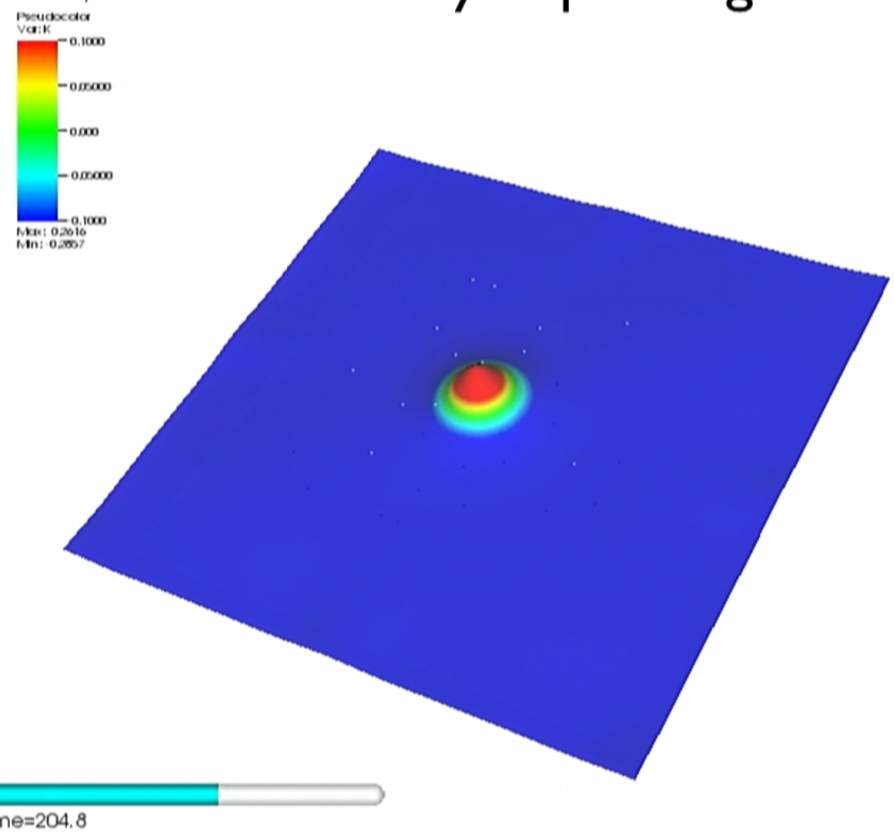

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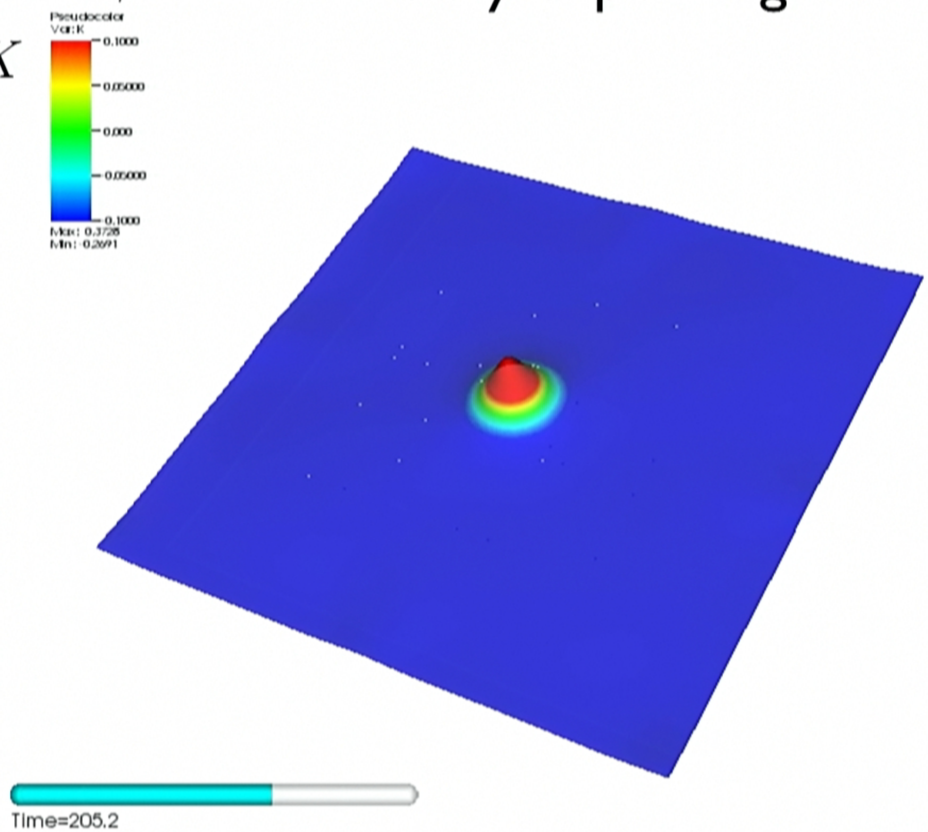

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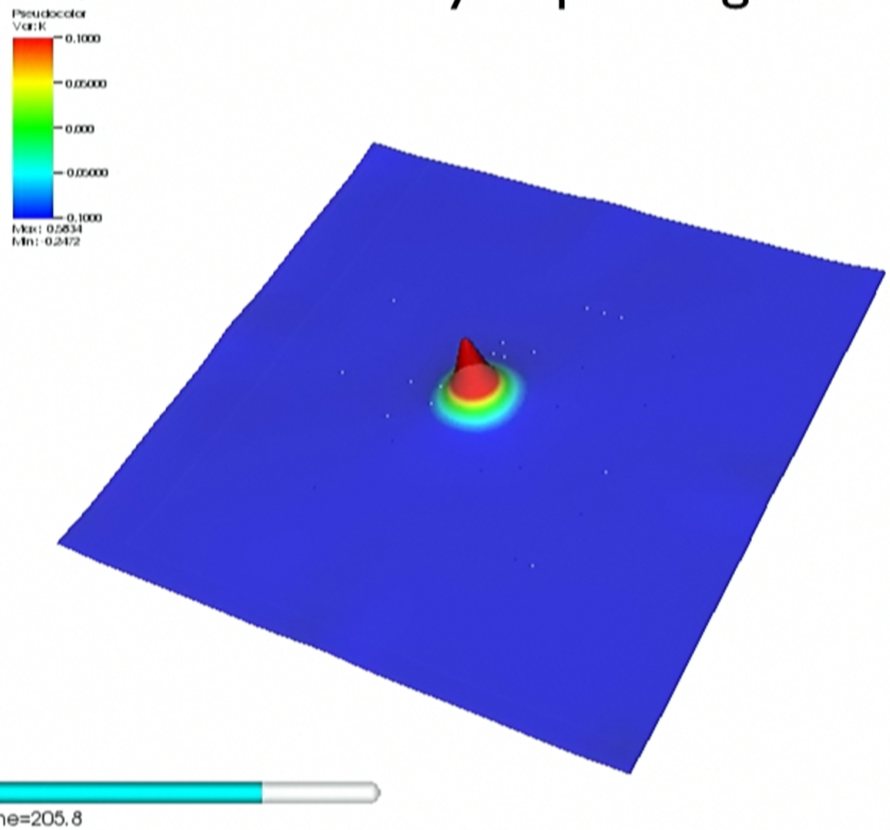

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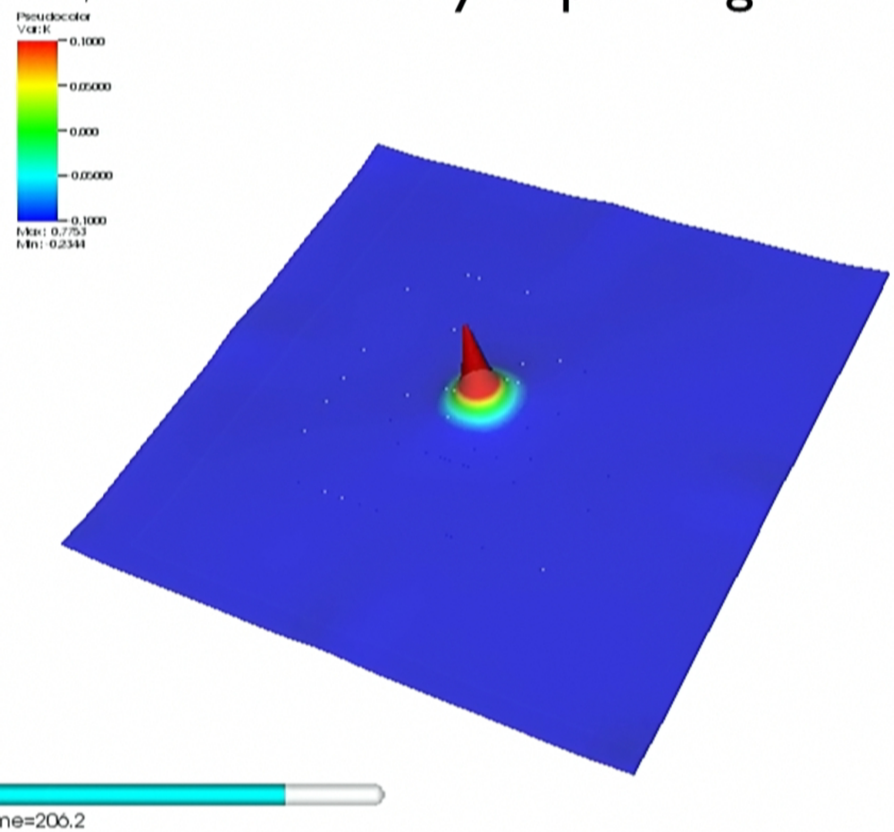

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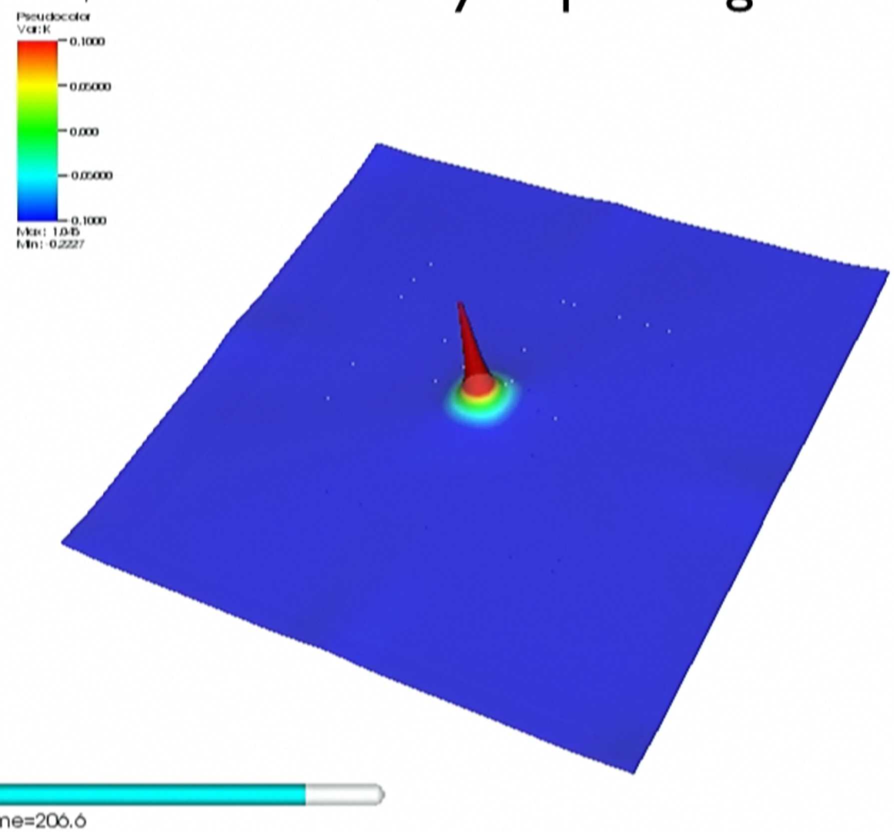

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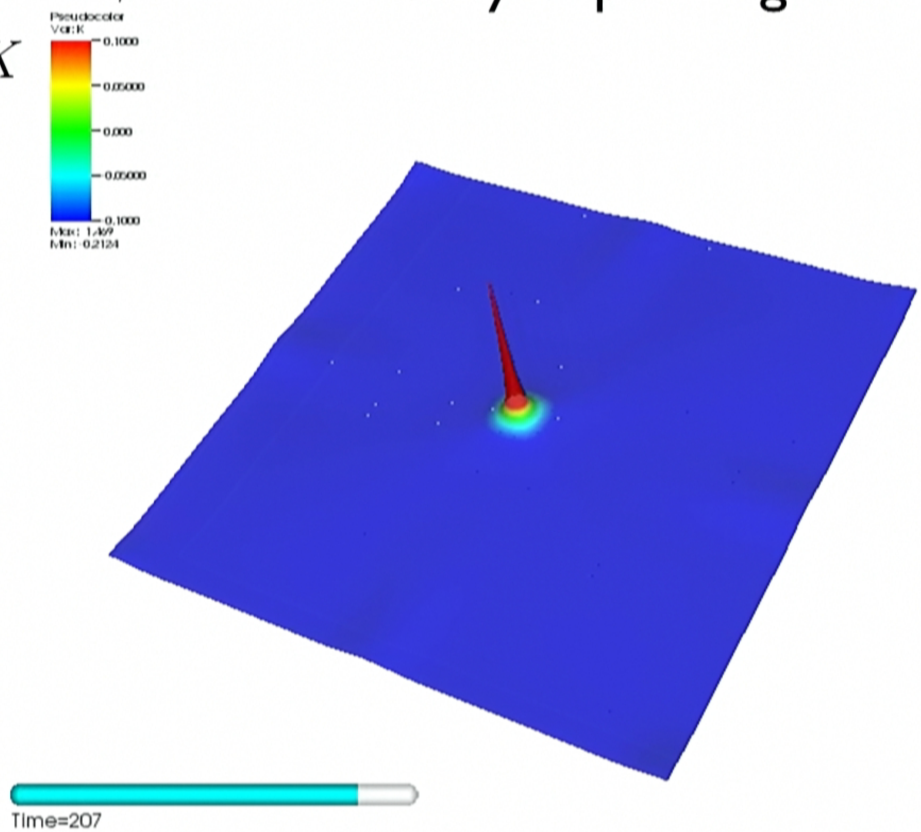

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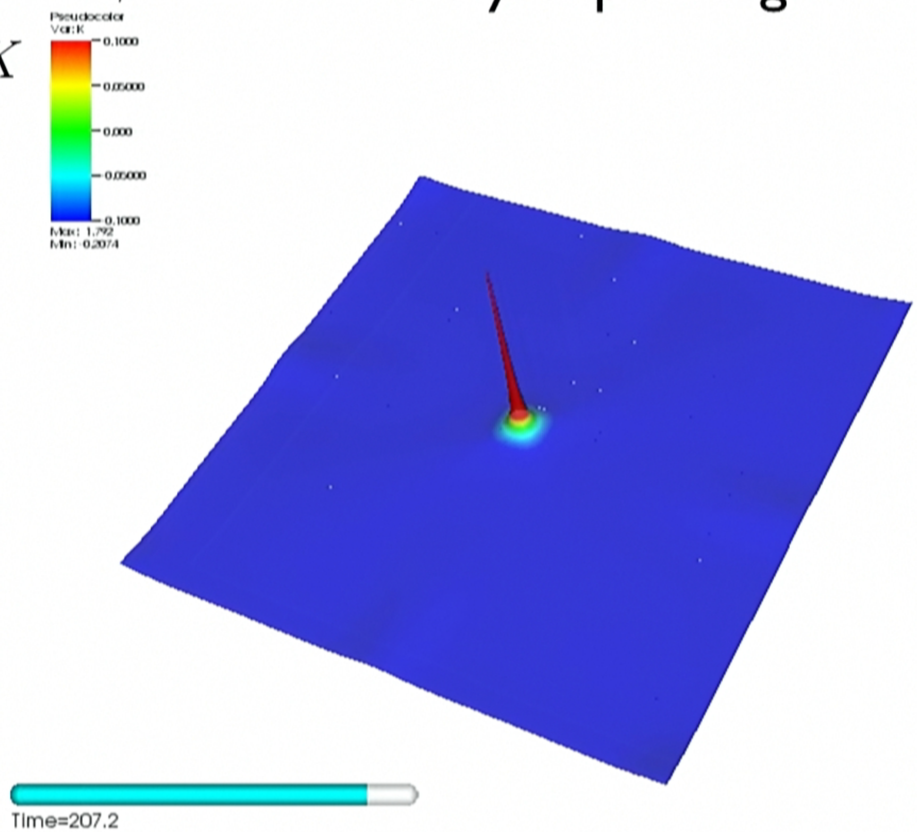

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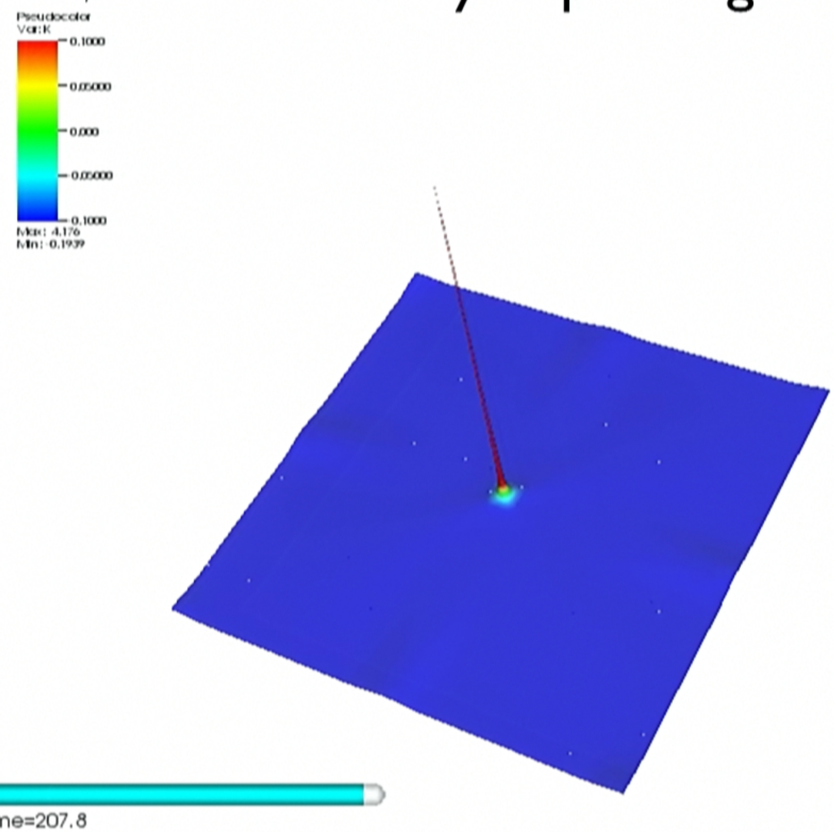

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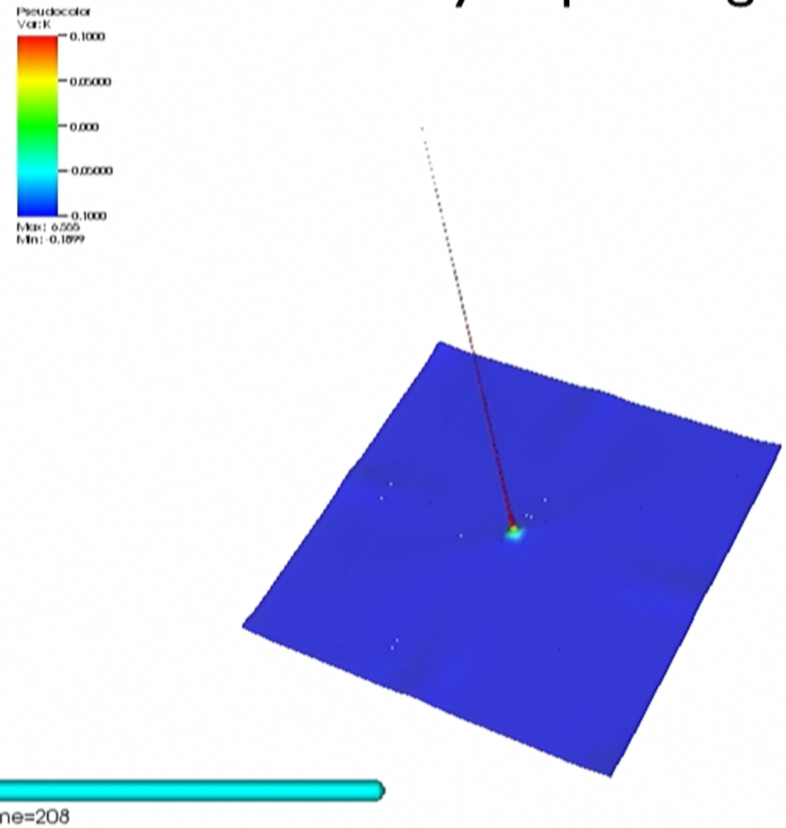


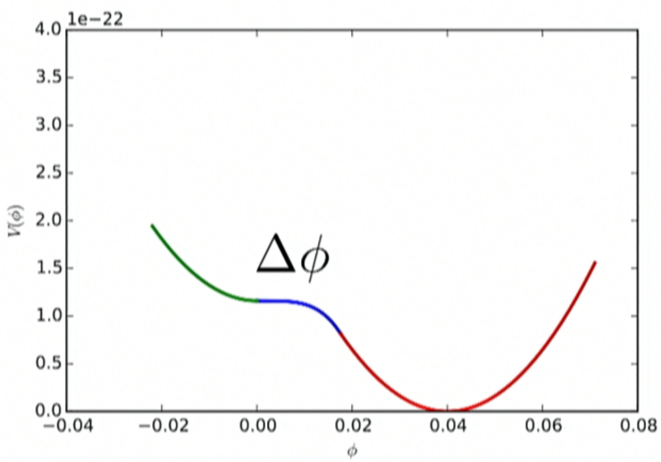
Inflation survives, not big enough BH

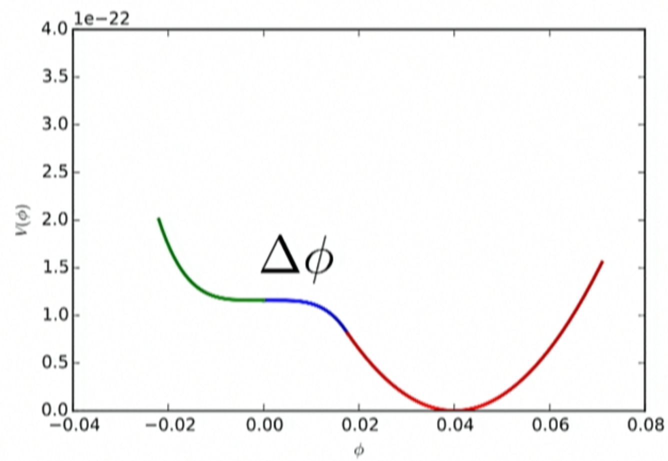
Giant Death Black Hole?

Not quite able to get GDBH, but it's mostly we haven't figure out the sweet spot for gauge driver conditions.

Small Field inflation

Model : $H_{inf} = 3.125 \times 10^{-11} M_p$





"Just enough model"

$$\Delta\phi \approx 10^{-3} M_p$$

Extended range models

$$\Delta\phi \gg 10^{-3} M_p$$

Due to large gradients, the left "cliff" will provide additional initial kinetic energy in "just enough" models.

Small Field inflation

Model : $H_{inf} = 3.125 \times 10^{-11} M_p$

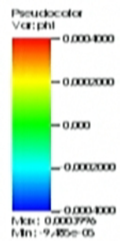
Inhomogeneity wavelength and simulation domain	$L = H_V^{-1} \approx H_{act}^{-1}$
Constant initial expansion/ contraction	$K \approx -3H_{act} \text{ or } +3H_{act}$
Scanning parameter	$\delta\phi = 0.0001 M_p \text{ to } 0.001 M_p$ **or** $\rho_{grad} = 2.5 \times 10^{-6} \rho_V \text{ to } 2.5 \times 10^{-4} \rho_V$

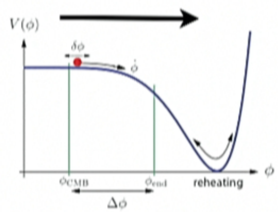
Small Field inflation

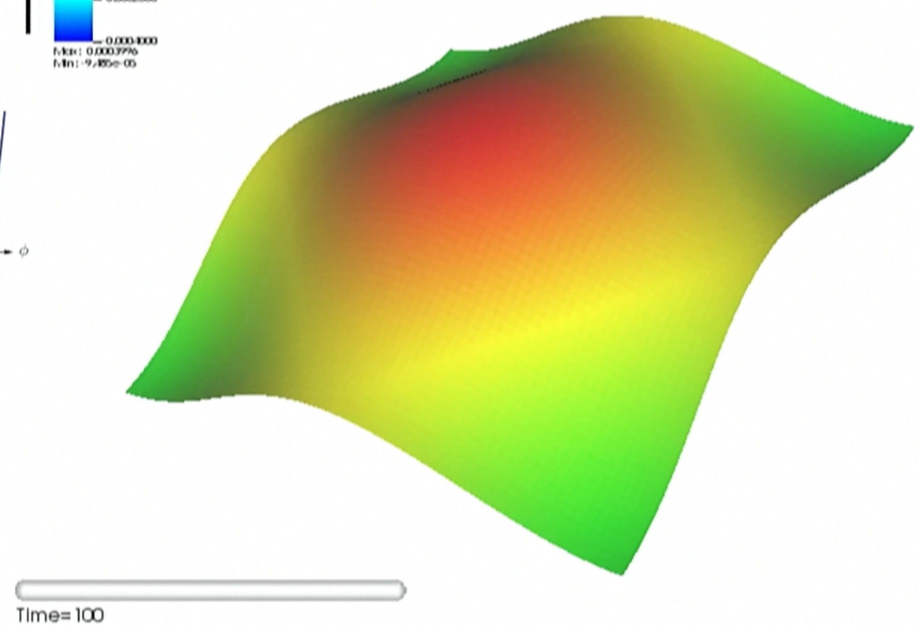
"Just enough model"

$\delta\phi = 0.0001 M_p = 0.1 \Delta\phi$ Initially expanding K

scalar ϕ







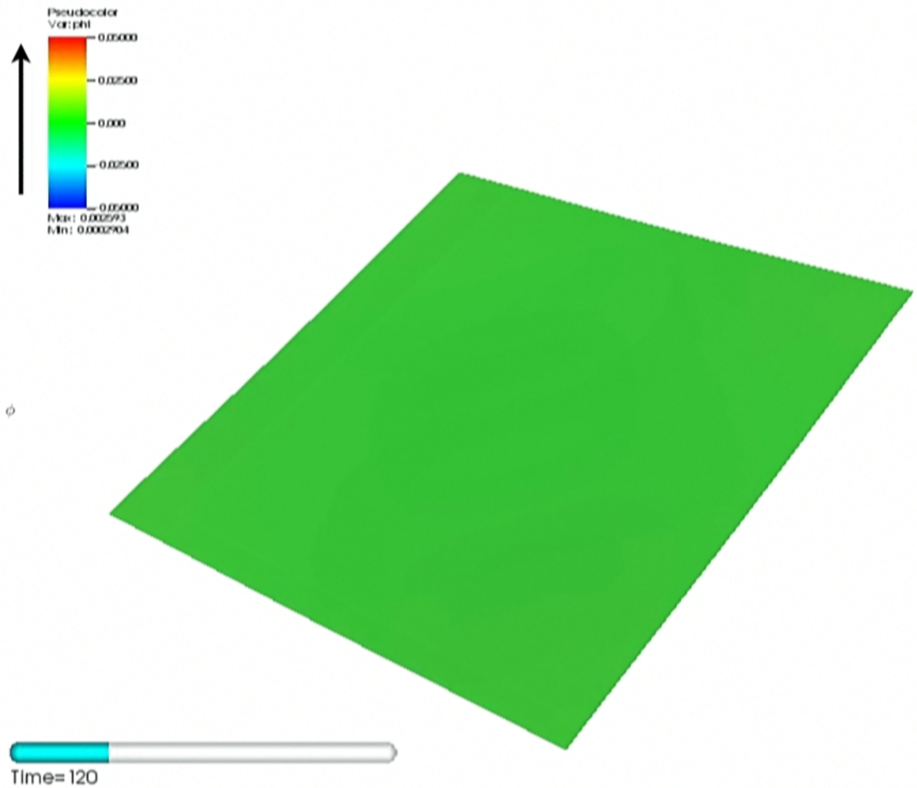
Inflation survives

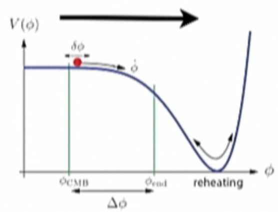
Small Field inflation

"Just enough model"

$\delta\phi = 0.0005 M_p = 0.5\Delta\phi$ Initially expanding K

scalar ϕ





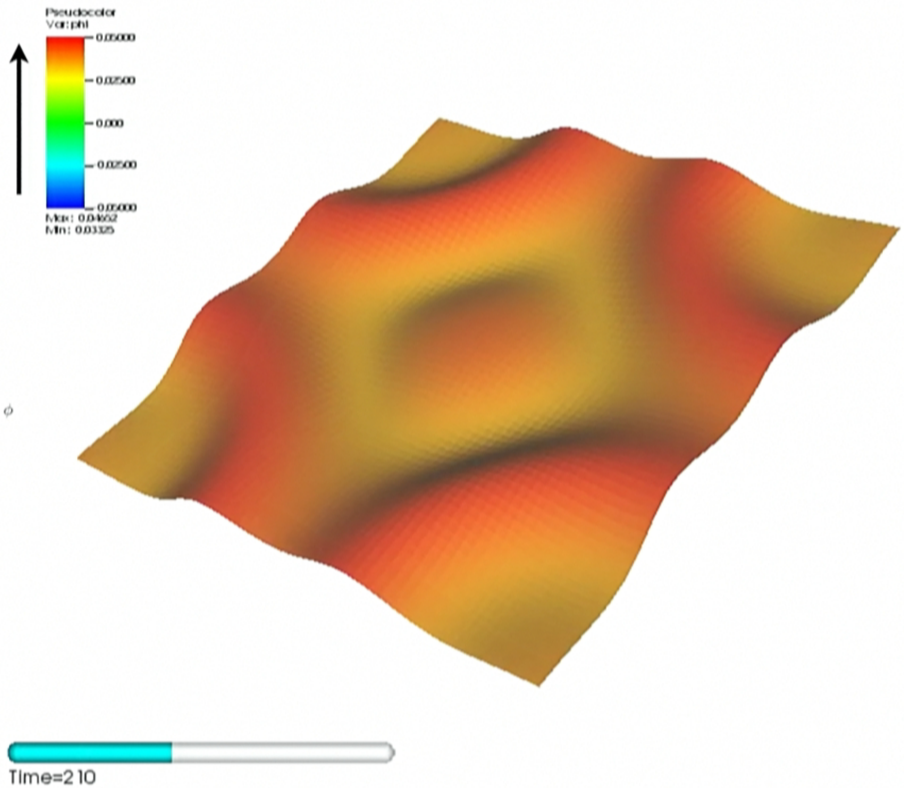
Inflation dies in the Pit of Doom

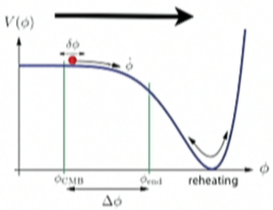
Small Field inflation

“Just enough model”

$\delta\phi = 0.0005 M_p = 0.5\Delta\phi$ Initially expanding K

scalar ϕ





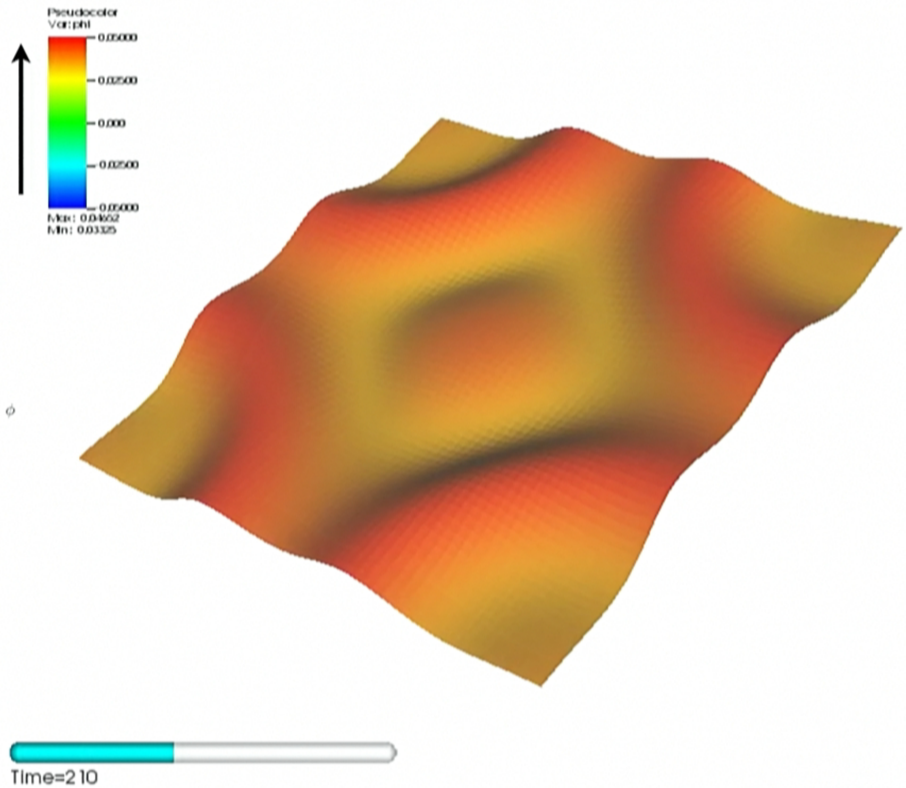
Inflation dies in the Pit of Doom

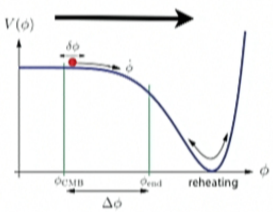
Small Field inflation

"Just enough model"

$\delta\phi = 0.0005 M_p = 0.5 \Delta\phi$ Initially expanding K

scalar ϕ





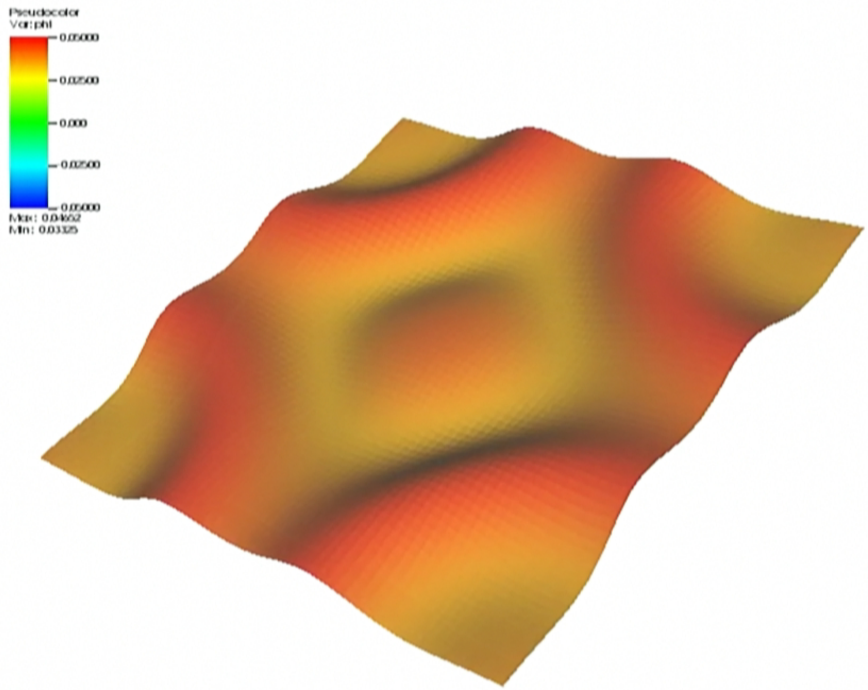
Inflation dies in the Pit of Doom

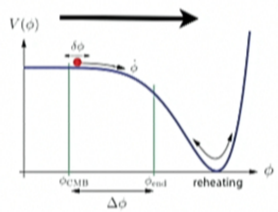
Small Field inflation

"Just enough model"

$\delta\phi = 0.0005 M_p = 0.5\Delta\phi$ Initially expanding K

scalar ϕ





Time=210

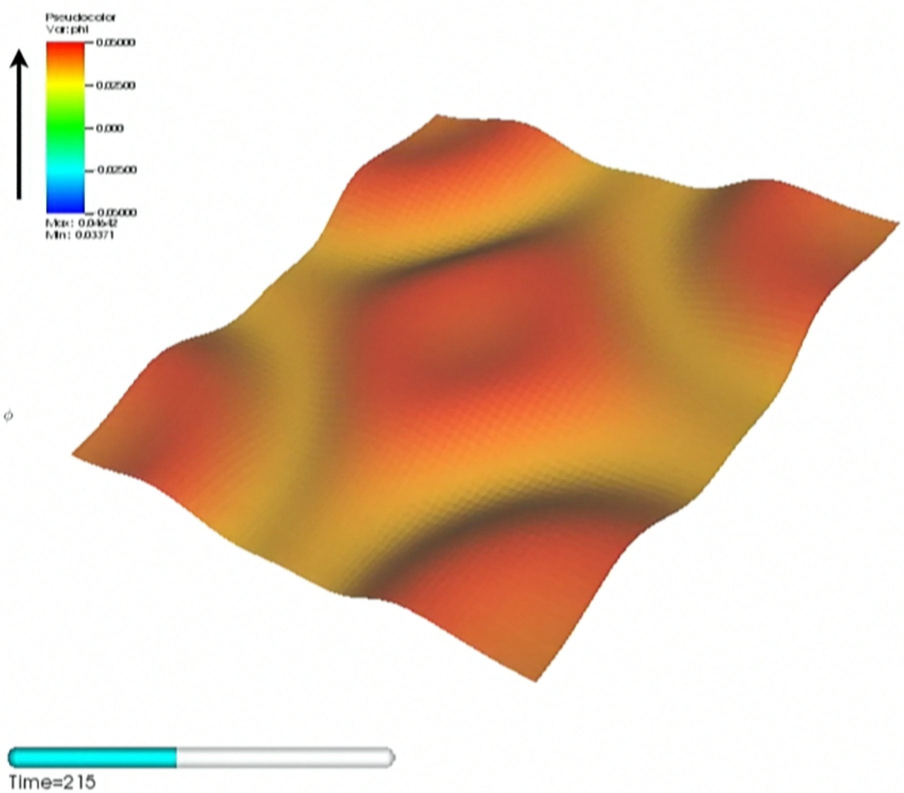
Inflation dies in the Pit of Doom

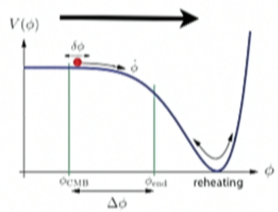
Small Field inflation

"Just enough model"

$\delta\phi = 0.0005 M_p = 0.5 \Delta\phi$ Initially expanding K

scalar ϕ

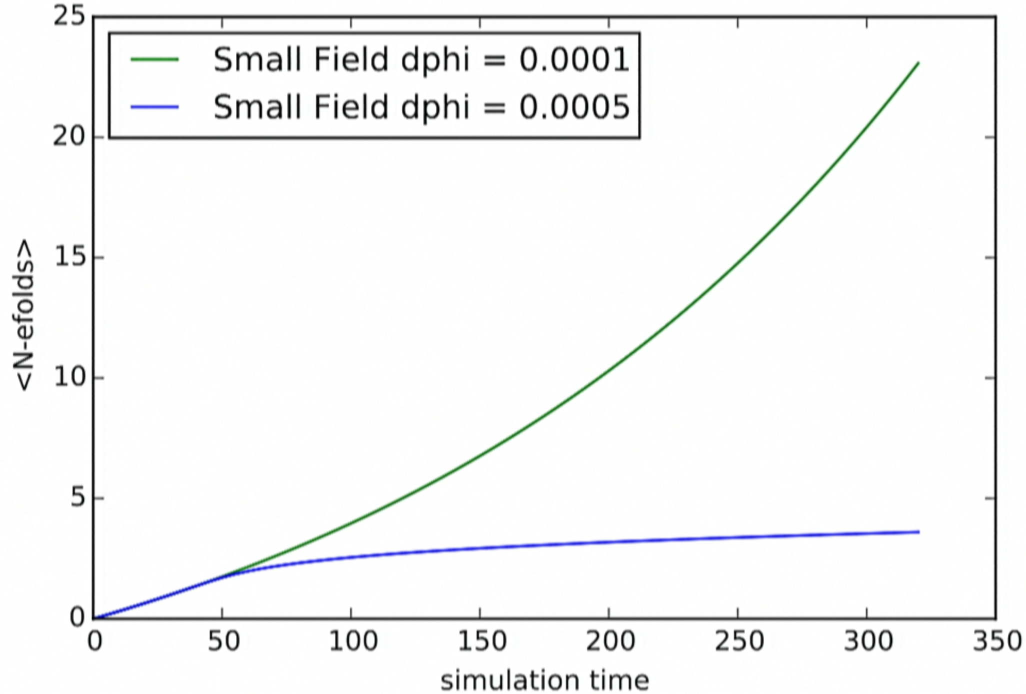




Inflation dies in the Pit of Doom

Small Field inflation

"Just enough model"   Initially contracting K

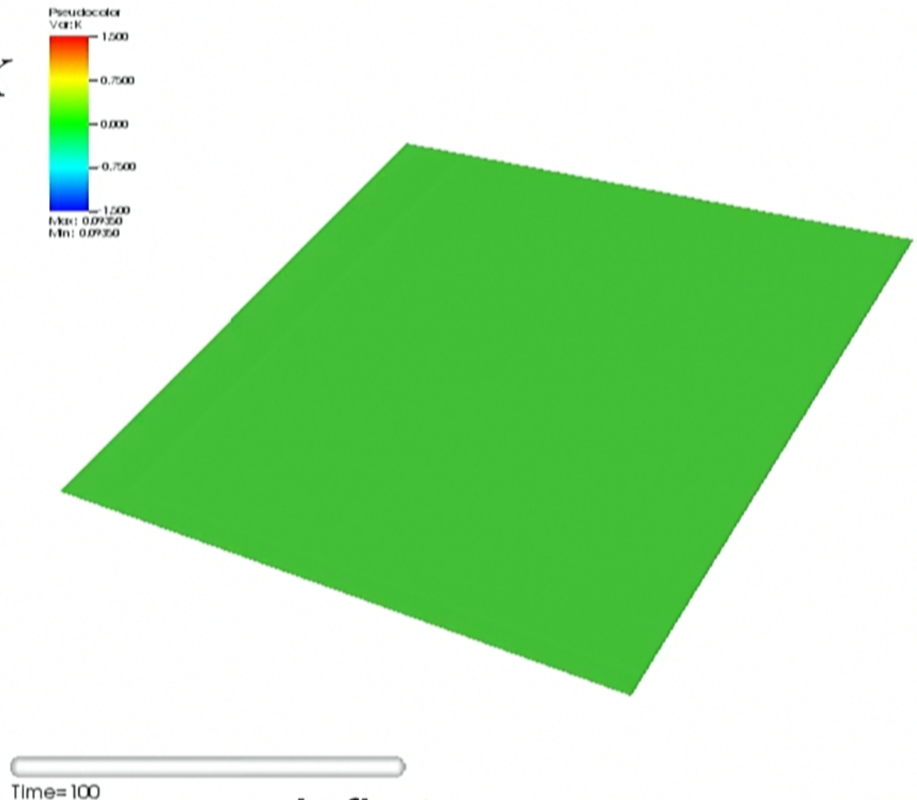


Small Field inflation

Extended range model

$\delta\phi = 0.0005 M_p$ Initially contracting K

expansion K



Inflation never got started

Small Field inflation

Broad conclusions:

1. Inflation gets killed by being pulled into the reheating regime (Pit of Doom), even if the initial gradient is well within inflating plateau.

2. "Just enough" model less robust than "extended field model".

3. Not very robust, critical ("Just enough model")

$$\delta\phi = 0.0005 M_p \Rightarrow \rho_{grad} \approx 10^{-4} \rho_V$$

Can we have a *local* quantum fluctuation that kills it instead?

Varying initial expansion/contraction

General initial conditions for K are hard to set up.

Special case trick : the *ansatz*

$$K(\mathbf{x}) = -C\phi(\mathbf{x}) + K_0 \quad \eta = -\frac{C}{12\pi} \quad C, K_0$$ free parameters

trivially solves the momentum constraint equation.

By choosing C, we can set up spacetimes with mixed initial expanding/contracting regions.

Simulation Domain and inhomogenous wavelength

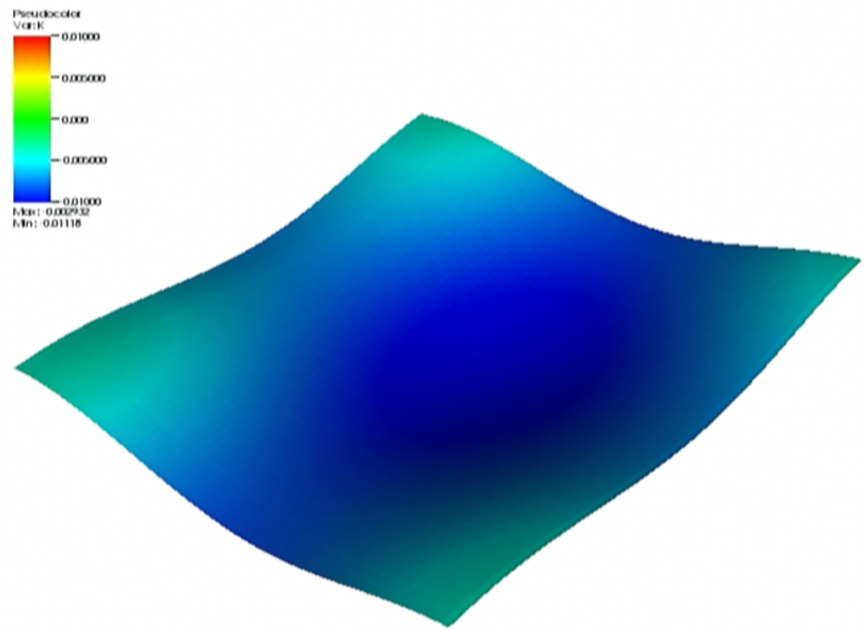
$$L = H_{act}^{-1} \qquad H_{act}^2 = \frac{8\pi}{3}(V(\phi_0) + \rho_{grad})$$

Varying initial expansion/contraction

Large field, cosmological constant, $\rho_{grad} = 1000\rho_V$

$\langle K \rangle = -H_{act} < 0$ Varying K, everywhere expanding

expansion K



Time=499883

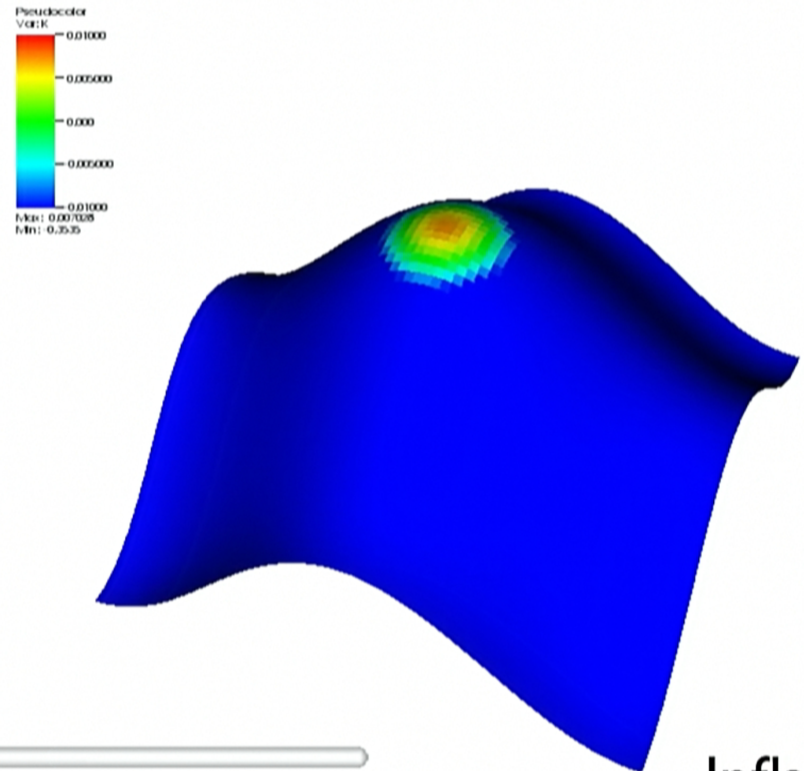
Inflation Survives

Varying initial expansion/contraction

Large field, cosmological constant, $\rho_{grad} = 1000\rho_V$

$\langle K \rangle = -H_{act} < 0$ Varying K, small region contracting

expansion K



Time=500

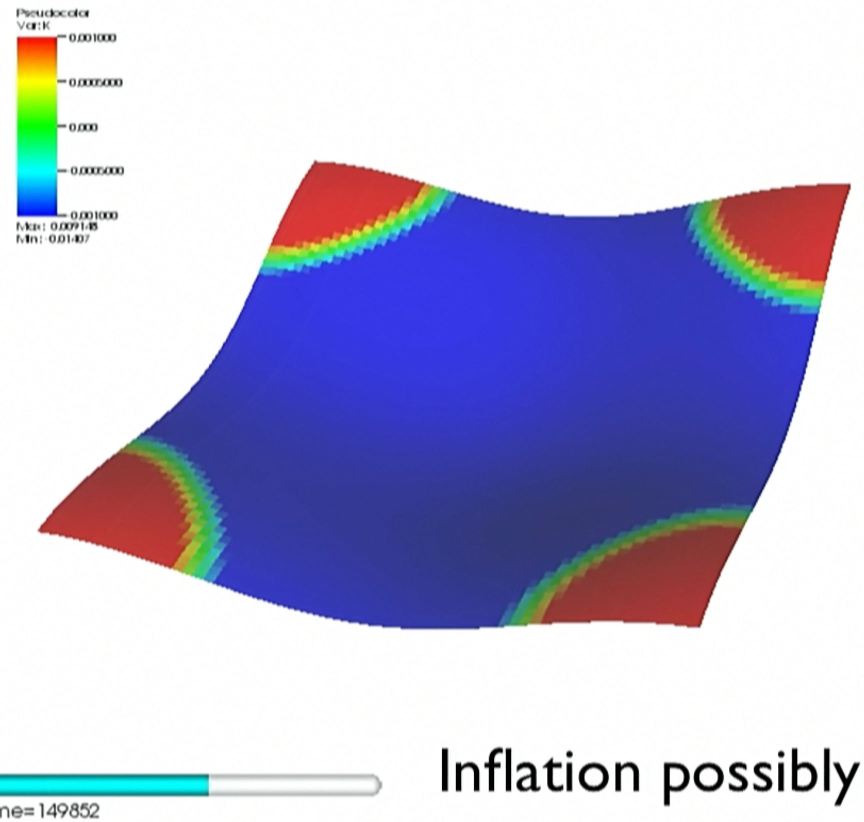
Inflation Survives

Varying initial expansion/contraction

Large field, cosmological constant, $\rho_{grad} = 1000\rho_V$

$\langle K \rangle = 0$ Varying K, equal expanding/contracting

expansion K

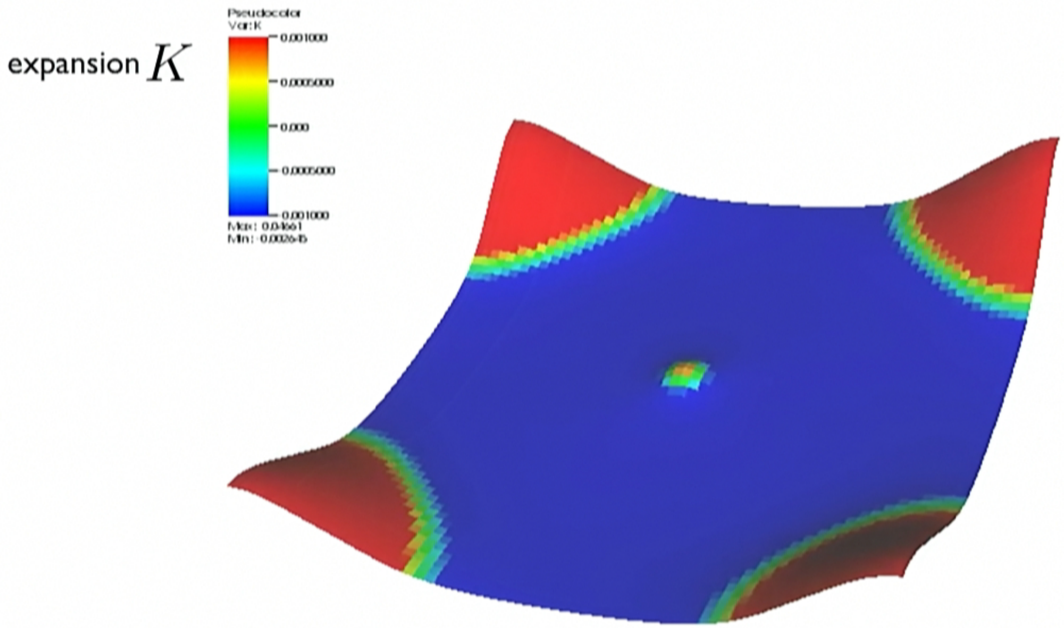


Inflation possibly dies

Varying initial expansion/contraction

Large field, cosmological constant, $\rho_{grad} = 1000\rho_V$

$\langle K \rangle = 0$ Varying K, equal expanding/contracting



Inflation possibly dies

Varying initial expansion/contraction

Small field varying K is really boring because ansatz has too large initial scalar kinetic velocity

$$\eta = -\frac{C}{12\pi}$$

so it walks right into the Pit of Doom on its own.

Need general initial prescription. 

Conclusions

- Large field inflation is robust to the Pit of Doom because gradients don't grow fast enough
- Small field inflation is not robust to the Pit of Doom because once gradient falls into the Pit, it drags the whole spacetime down.
- Does this mean we should expect high scale inflation and large primordial GW?
- Large field is robust to small amount of initially contracting spacetime.