Title: Turbulent gravity in asymptotically AdS spacetimes

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Abstract: Dynamics in asymptotically anti-de Sitter spacetimes with reflecting boundary conditions are characterized by reduced dissipation as compared to asymptotically flat spacetimes. Such spacetimes, thus, represent opportunities to study nonlinear gravitational interactions that would otherwise be quickly damped away. I will discuss two background spacetimes---large AdS black branes in d=4, and pure AdS---where small perturbations display turbulent behavior and energy cascades driven by nonlinear interactions. In each case, the presence of an unexpected conserved quantity---a gravitational "enstrophy" around the AdS black brane, and a "particle number" for pure AdS perturbations---significantly affects the energy flow direction throughout the cascade, and drives energy to longer distance scales. I will comment on implications for fundamental general relativity questions such as cosmic censorship, and potential for turbulence beyond AdS.

Turbulent gravity in asymptotically AdS spacetimes

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Perimeter Institute
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PRL **113**, 071601 (2014); arXiv:1403.6471 [hep-th]
PRD **91**, 064026 (2015); arXiv:1412.4761 [gr-qc]
PRD **92**, 084001 (2015); arXiv:1507.08261 [gr-qc]

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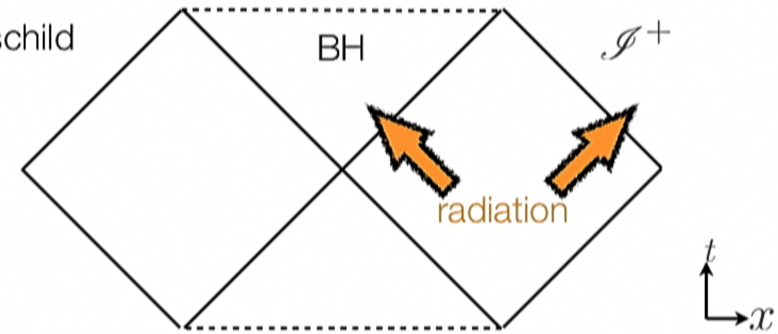
Introduction

- Asymptotically flat spacetimes, or spacetimes with black holes have dissipation.

E.g., Schwarzschild

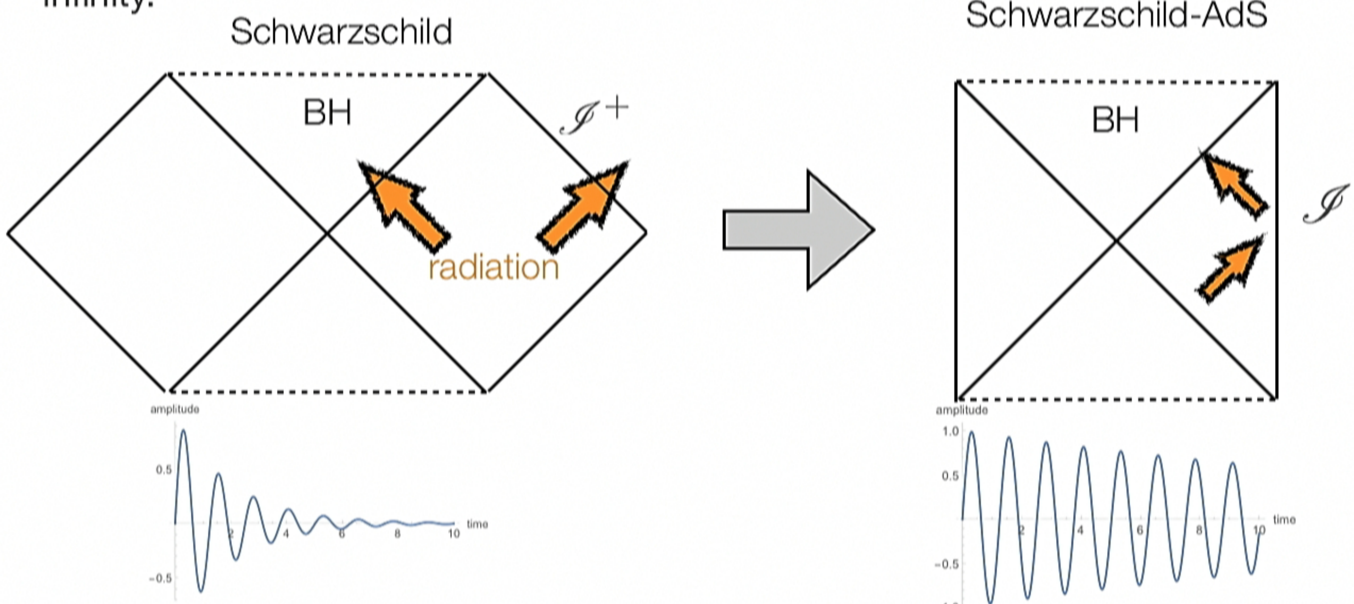


- For this reason, gravitational perturbations tend to decay rapidly, and nonlinear perturbations are typically rapidly damped away.

  E.g., quasinormal mode ringdown seen by LIGO.

Introduction

- Asymptotically AdS spacetimes have a reflective timelike infinity instead of null infinity.



- Large Schwarzschild-AdS black holes have very low dissipation / very long lived quasinormal modes.

Introduction

- Pure AdS spacetime (no black hole) has no source of dissipation.

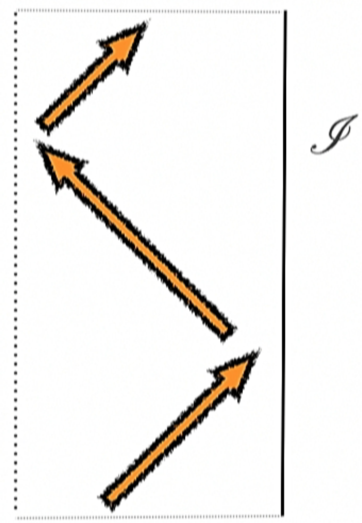
- Quasinormal modes are replaced by normal modes, with no decay.

- Reduced dissipation means that nonlinear gravitational interactions, including turbulence, can be studied in the following two backgrounds:

    1. Large AdS black holes

    2. Pure AdS

Global anti-de Sitter



Outline

1. Large AdS black branes (in 4 dimensions)
   - Gravity-fluid correspondence
   - Kelvin-Helmholtz instability in gravity
   - Turbulent decay of black hole perturbations, inverse energy cascades, conservation laws
2. Pure AdS
   - Instability to black hole formation for certain initial perturbations
   - Two-timescale analysis and new conservation law
   - Inverse energy cascades and islands of stability

1. Large anti-de Sitter black branes

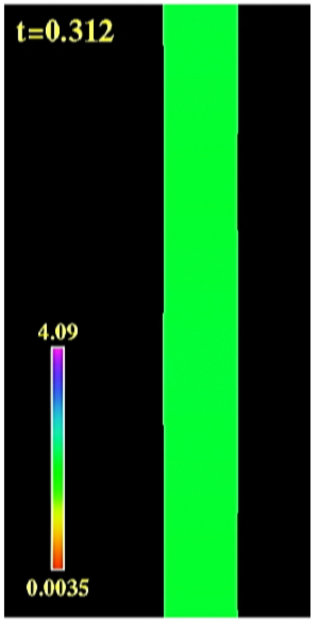
Gravity-fluid correspondence

- Many hints of a connection or analogy between fluid dynamics and gravity:
  - Black hole thermodynamics
  - Membrane paradigm for black holes
  - Fluid analogs of black hole spacetimes
  - Gregory-Laflamme instability of 5d black strings →

    vs

    Rayleigh-Plateau instability →

Video from Lehner and Pretorius (2010)



(Wikipedia)

Gravity-fluid correspondence

- AdS / CFT correspondence: Relates quantum gravity in (d+1) dimensions and quantum field theory on the d-dimensional boundary.

- There exists a purely classical limit, where

(d+1) quantum gravity ⟶ (d+1) perturbed black brane solutions to Einstein's equation in anti-de Sitter

(d+1) quantum gravity ↕ AdS/CFT ↕ d-dimensional CFT

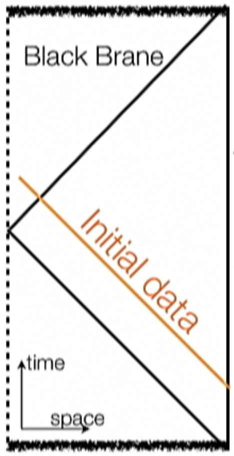
(d+1) perturbed black brane solutions to Einstein's equation in anti-de Sitter ↕ "gravity/fluid correspondence" ↕ d-dimensional fluid

d-dimensional CFT ⟶ d-dimensional fluid

- By studying dual fluid, we can obtain approximate general relativity solutions describing perturbed black brane.

Gravity-fluid correspondence

- Black Brane spacetime:



Bulk:
Asymptotically AdS black brane

Einstein Equation holds
$G_{AB} + \Lambda g_{AB} = 0$

Boundary:
Impose "boundary metric" = $\eta_{\mu\nu}$ (mirror)

Read off "boundary stress-energy"

$$T_{\mu\nu} \equiv \lim_{r\to\infty} \frac{r^{d-2}}{8\pi G_{d+1}} (K_{\mu\nu} - K\eta_{\mu\nu} - (d-1)\eta_{\mu\nu})$$

- *At late times, the boundary stress-energy takes the form of a relativistic, viscous conformal fluid.* This is derived in a derivative expansion; valid for long-wavelength perturbations. (Bhattacharyya et al, 2008)

- From the boundary stress-energy, can re-construct a bulk metric which solves the Einstein equation in the derivative expansion.

Boundary Fluid

- Resulting boundary stress-energy tensor (to 2nd order in derivatives):

$$T_{\mu\nu}^{[0+1+2]} = \frac{\rho}{d-1}\left(du_\mu u_\nu + \eta_{\mu\nu}\right) + \Pi_{\mu\nu}$$

where the viscous part is given by

1st order in derivatives

2nd order

$$\Pi_{\mu\nu} = -2\eta\sigma_{\mu\nu} + 2\eta\tau_\Pi\left(\langle u^\alpha\partial_\alpha\sigma_{\mu\nu}\rangle + \frac{1}{d-1}\sigma_{\mu\nu}\partial_\alpha u^\alpha\right) + \langle\lambda_1\sigma_{\mu\alpha}\sigma_\nu{}^\alpha + \lambda_2\sigma_{\mu\alpha}\omega_\nu{}^\alpha + \lambda_3\omega_{\mu\alpha}\omega_\nu{}^\alpha\rangle$$

shear

vorticity

Transport coefficients all functions of the density.
In particular, shear viscosity $\eta = \frac{s}{4\pi} \propto T^{d-1} \propto \rho^{(d-1)/d}$

- There is also a dual bulk metric, which is expressed in terms of $u_\mu,\ \rho$

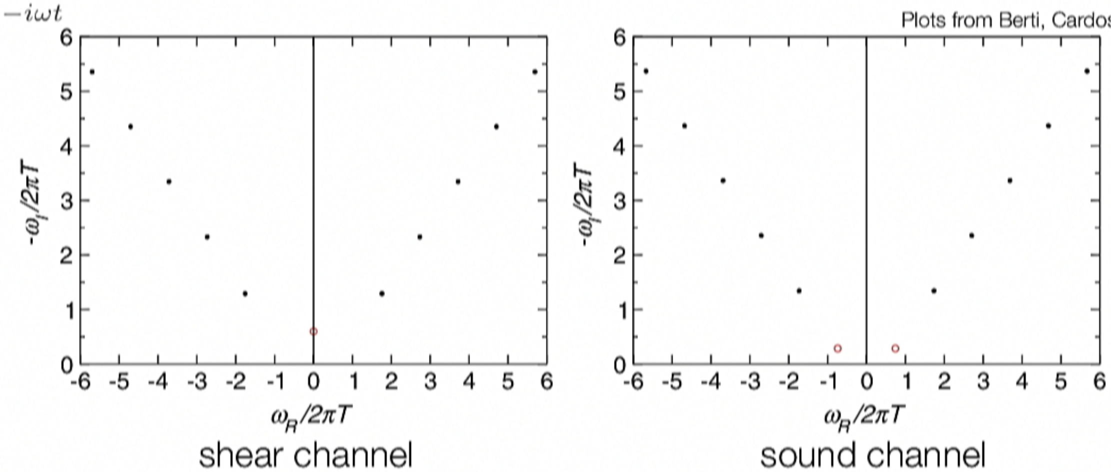
$$ds_{[0]} = -2u_\mu dx^\mu dr + r^2\left(\eta_{\mu\nu} + \frac{(d-1)\rho}{r^d}u_\mu u_\nu\right)dx^\mu dx^\nu$$

Black brane quasinormal modes

- Modes in red are the hydrodynamic modes, which are captured by the fluid.

$h \sim e^{-i\omega t}$

Plots from Berti, Cardoso, Starinets (2009)



- The shear mode has purely imaginary frequency, and for high T, becomes long lived.

- Higher modes not captured by dual fluid, but they decay more rapidly and are not relevant at long wavelengths.

Quasinormal modes from the fluid side

- Hydrodynamic shear quasinormal mode ⟷ fluid shear flow

- Consider a uniform fluid flow,
$$\rho_{(0)} = \text{constant}$$
$$u^{\mu}_{(0)} = (1,0,0)$$
$$\Pi^{(0)}_{\mu\nu} = 0,$$
linearly perturbed by shear flow,
$$u^{x}_{(1)} = u^{x}_{(1)}(t,y)$$
$$\Pi^{(1)}_{xy} = \Pi^{(1)}_{xy}(t,y)$$

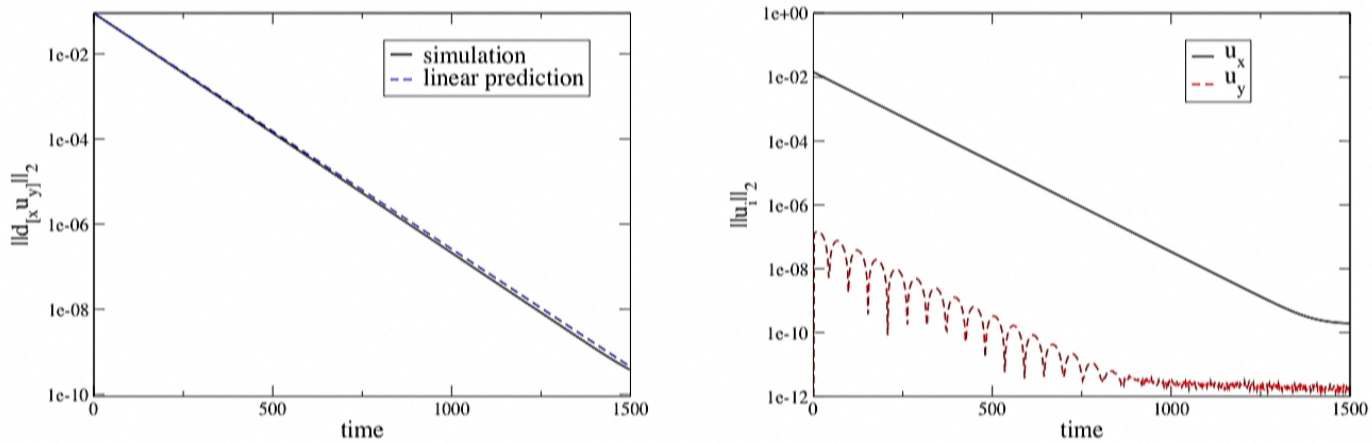
same as lowest shear QNM for black brane

- Find, for modes $\sim e^{-i\omega t + iky}$, a dispersion relation
$$\omega \approx -i\frac{2k^2\eta_{(0)}}{3\rho_{(0)}} = -i\frac{k^2}{4\pi T_{(0)}}$$

We use the fluid/gravity correspondence to study (3+1)-d black hole perturbations nonlinearly, via numerical simulations of the (2+1)-d dual fluid. We choose initial data corresponding to shear mode, and study its evolution under the full nonlinear fluid equations. We also include tiny random perturbations, in order to assess stability.

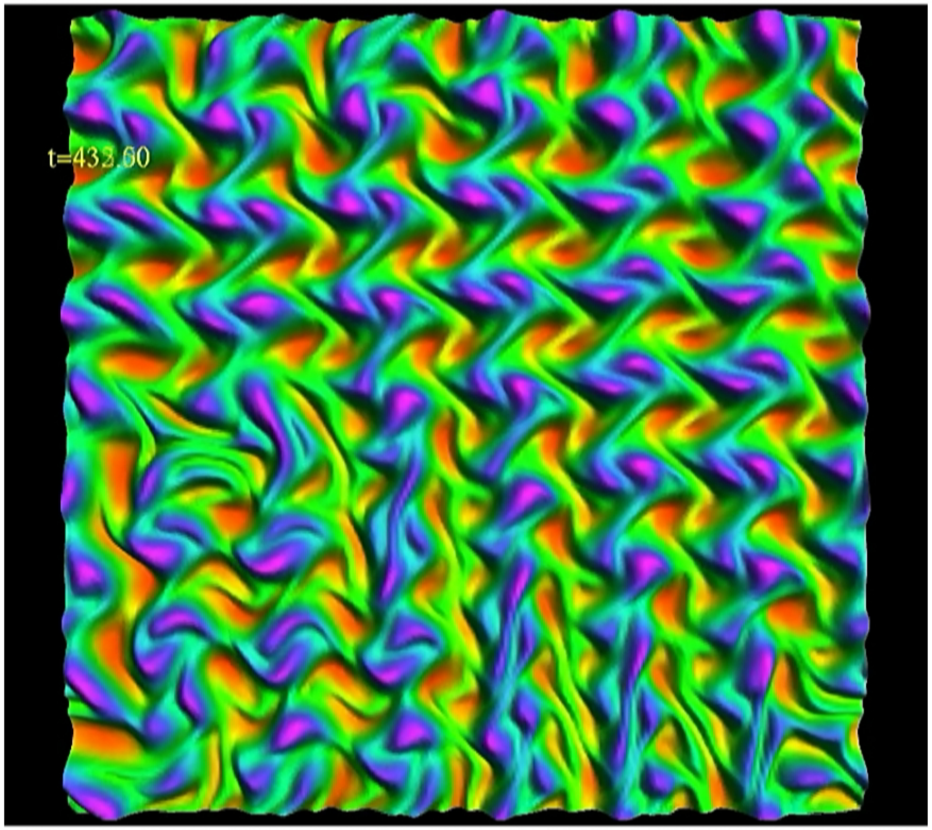
Shear flow at low Reynolds number

- The random seed perturbation does not grow. u_y remains small.
- Laminar flow matches quasinormal mode decay



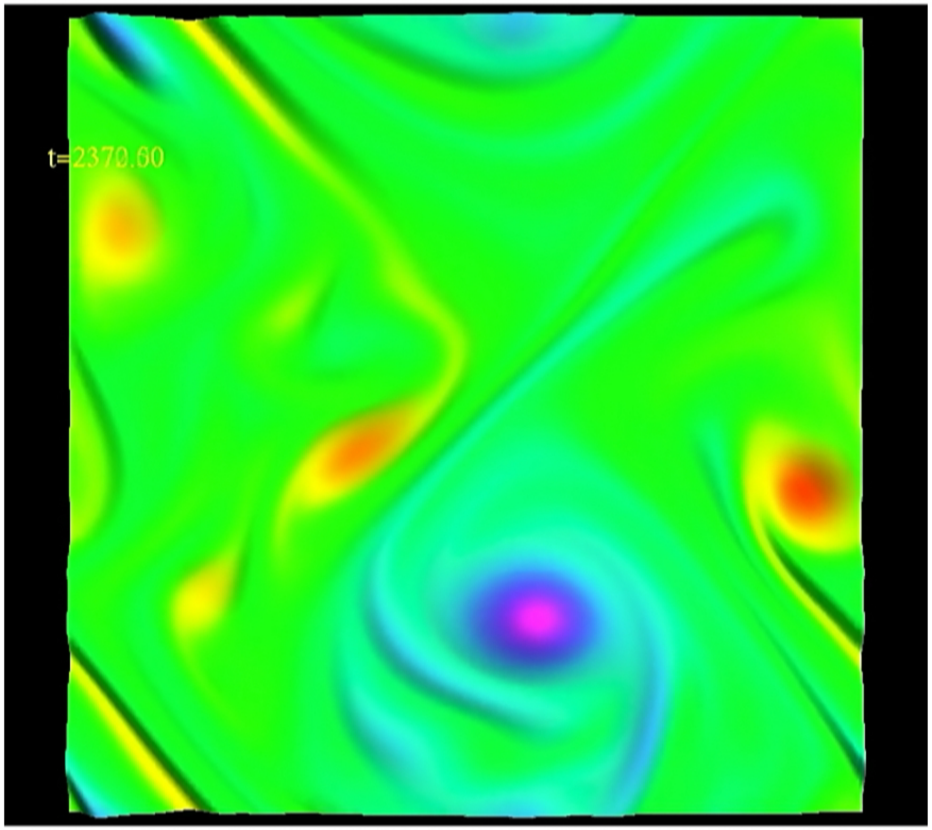
(a) Vorticity (b) Velocity

Shear flow at high Reynolds number



vorticity field

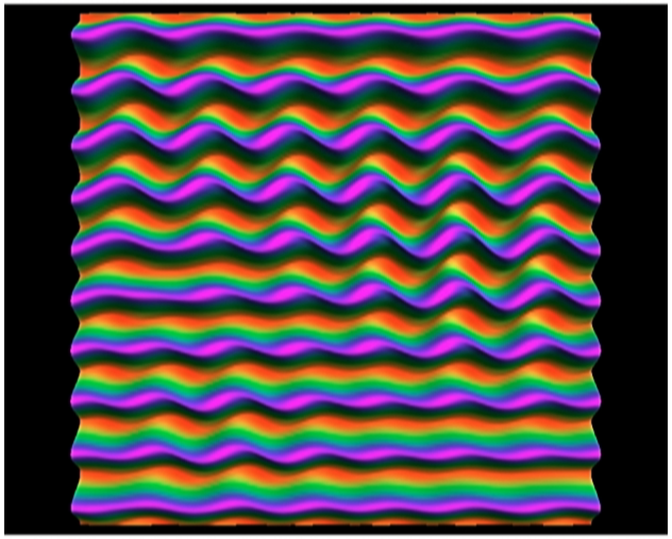
Shear flow at high Reynolds number



vorticity field

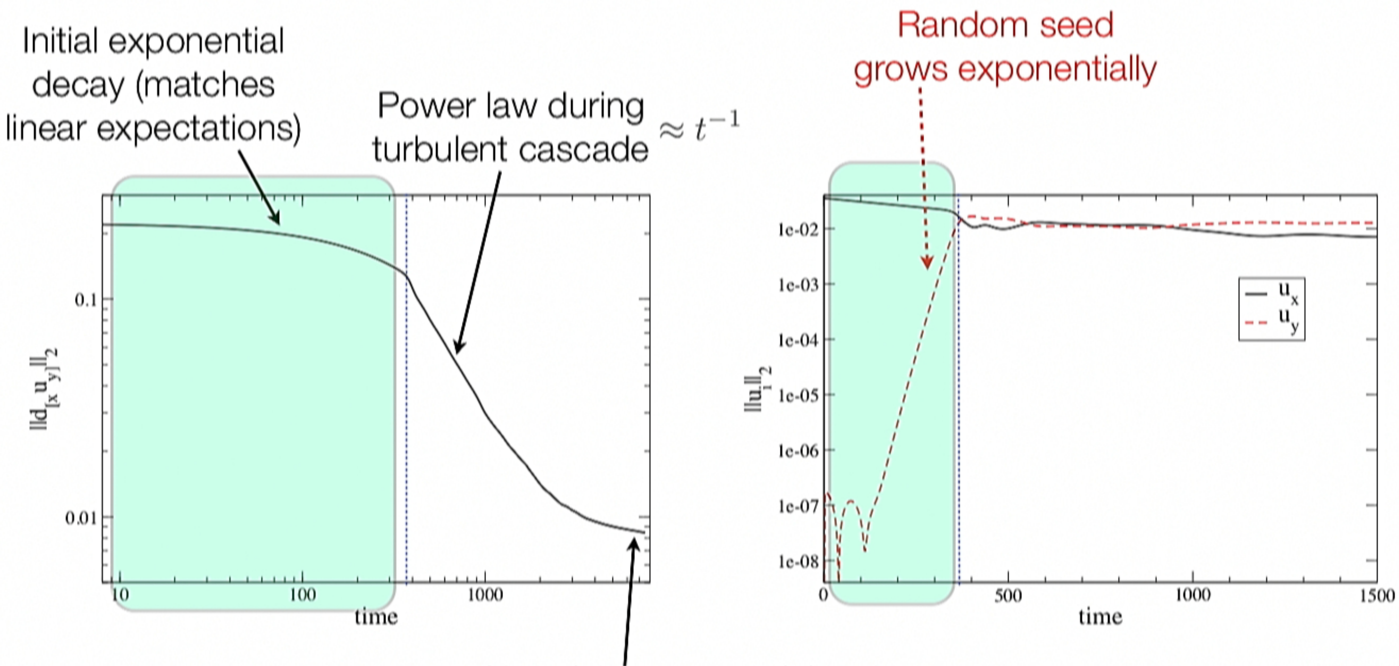
Initial instability

(a) $t = 0$



(b) $t = 350$

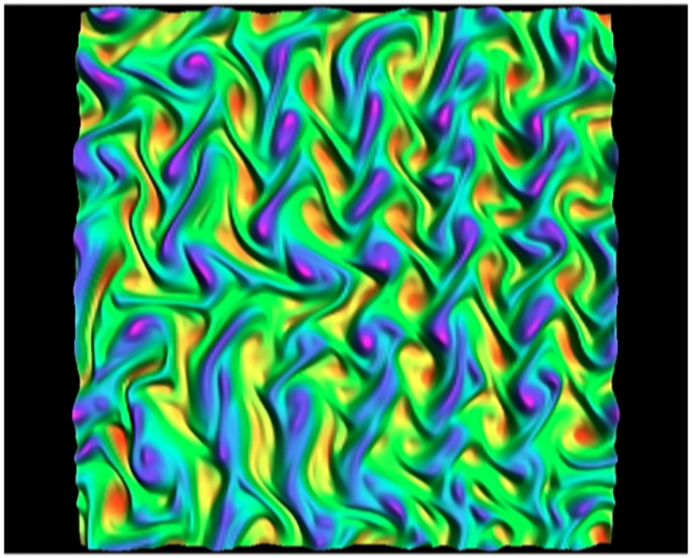
Shear flow at high Reynolds number



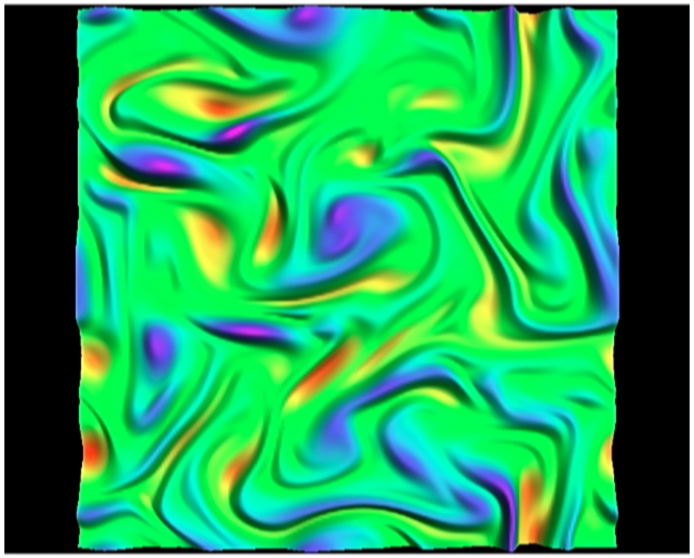
(a) Vorticity

(b) Velocity

Fully developed turbulence



(c) $t = 500$



(d) $t = 900$

- Turbulence characterized by inverse energy cascade, which transfers energy to larger scales.

Turbulence

- Turbulence in (2+1) dimensional Navier-Stokes fluids:

  E.g., for an incompressible fluid,

$$\partial_t v^i + v^j \partial_j v^i = \nu \partial^j \partial_j v^i - \partial^i P; \qquad i,j = x,y$$

$$\downarrow \text{curl}$$

$$\partial_t \omega + v^j \partial_j \omega = \nu \partial^j \partial_j \omega$$

In (2+1) dims, single vorticity component $\omega = \partial_x v_y - \partial_y v_x$

  - Inviscidly conserved quantity, *enstrophy*, $Z = \int \omega^2 d^2 x$
  - Leads to *direct cascade of enstrophy*, instead of energy.
  - *Inverse cascade of energy* takes energy to large scales, rather than small scales for higher dimensions.

Turbulence

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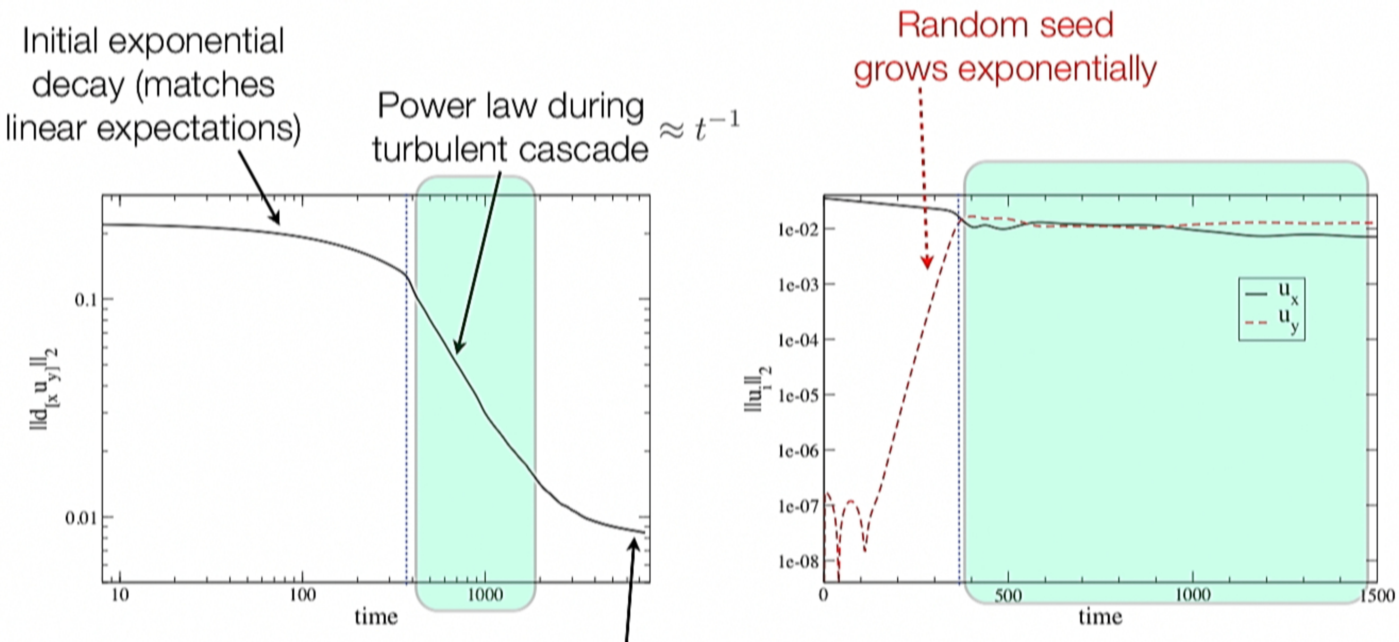
Shear flow at high Reynolds number

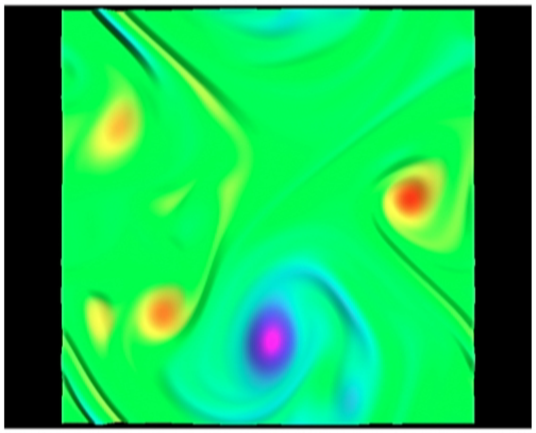


(a) Vorticity

(b) Velocity

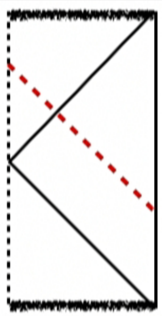
Mapping from boundary to bulk

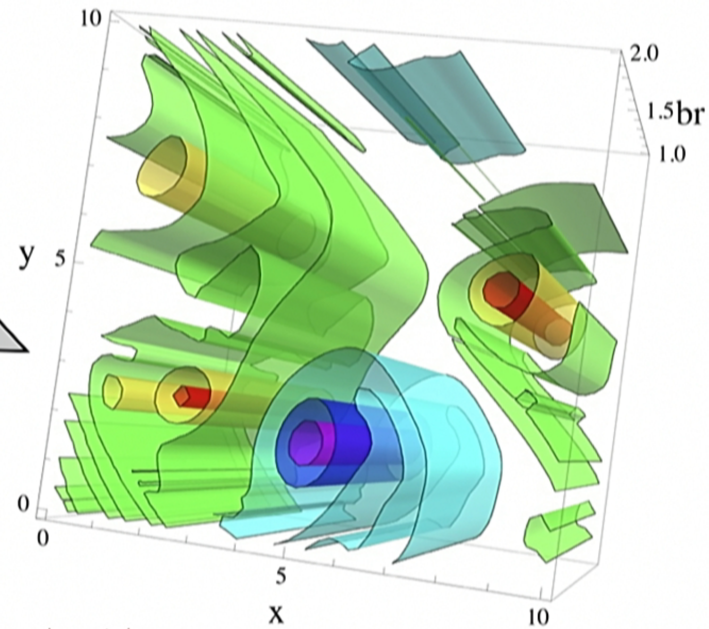
- Fluid quantities captured by bulk Weyl curvature invariants



(e) $t = 2500$

vorticity

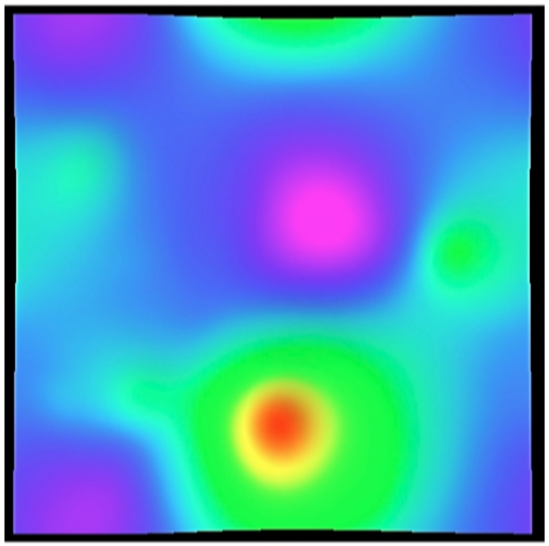




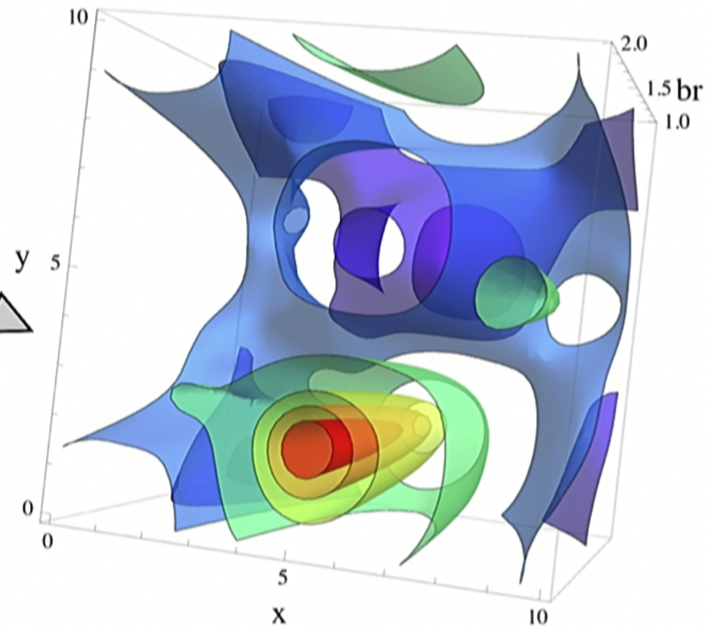
Funnels run along tubes connecting boundary and horizon

(b) $r^7 * C_{ABCD}C^{ABCD}$

Mapping from boundary to bulk



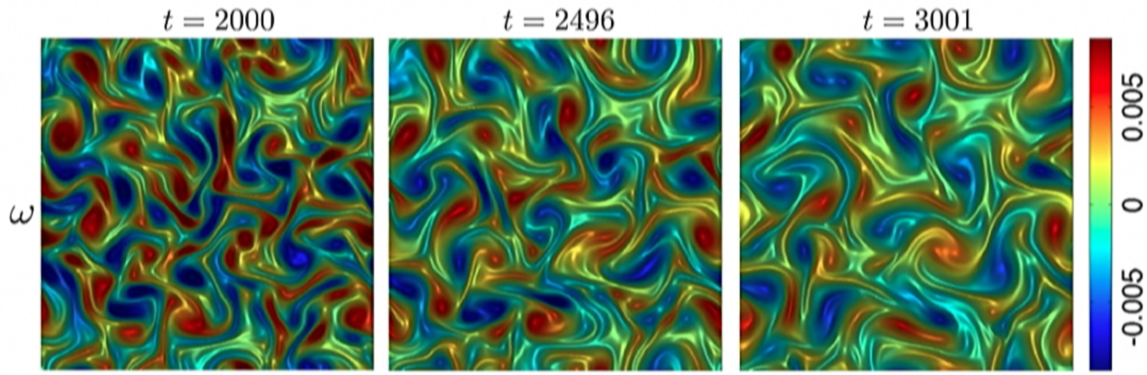
density



(a) $r^6\, C_{ABCD}C^{ABCD}$

Validity of correspondence

- Is turbulence really happening in gravity? Or is it an artifact of derivative expansion?
- Pure gravity simulations in (3+1) dimensions (Adams, Chesler and Liu, 2014):



New intuition for gravity

- Ordinary perturbation theory is not valid in low-dissipation regime.
- As with fluids, define a "gravitational Reynolds number", to measure the relative sizes of nonlinear and dissipative terms in the equations.

Long lived QNMs ⟷ High Reynolds number

In this fluid-gravity setup, $R_{GR} \propto T_{\text{Hawking}} \left\| h_{AB} \left(\frac{\partial}{\partial r} \right)^{B} \right\| L$

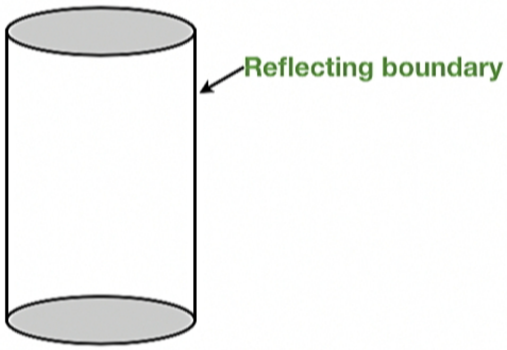
Fluid side		**Gravity side**
Laminar flow	⟶	Quasinormal mode regime
Turbulent flow	⟶	New, turbulent phase

Extensions / Open Questions

- Higher dimensions: Direct cascade to small scales.
- Black holes rather than black branes: Expect similar behavior.
- Beyond AdS: Other cases of long-lived quasinormal modes / slow dissipation / high Reynolds number might lead to turbulence. In particular, long lived modes of near-extremal Kerr black holes have been shown to exhibit the initial instability (Yang, Zimmerman and Lehner, 2014).
- Conserved enstrophy: Does conservation hold beyond gravity-fluid approximation? How generally does it hold? Does it act as a cosmic censor?

Pure anti-de Sitter spacetime

- Anti-de Sitter without the black hole = empty box with reflecting walls



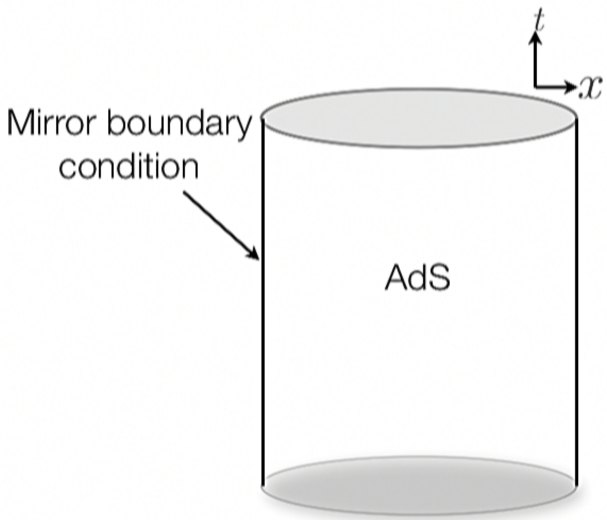
- Linear fields have normal modes with harmonic time-dependence. In contrast to black hole case, there is zero dissipation. Infinite Reynolds number.

- Nonlinearly, mode-couplings transfer energy between modes in a “turbulent” cascade.

- If sufficient energy transferred to high-frequency modes (short distance scales), black hole formation may result.

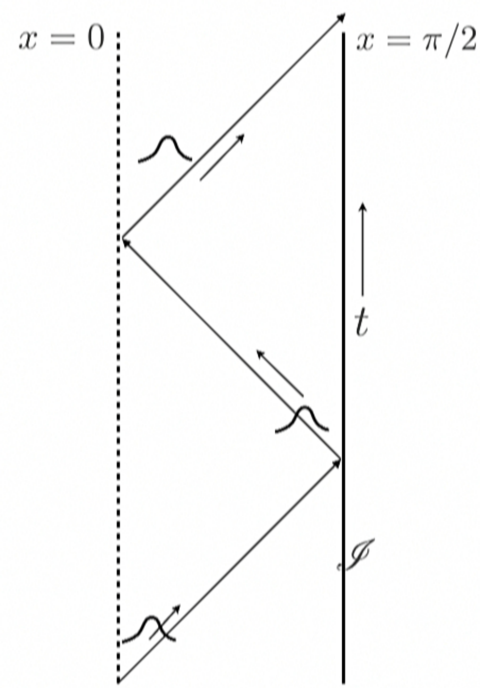
Stability of AdS

- **Question:** Is AdS stable to arbitrarily small perturbations? If not, which initial data collapse to black hole?



Stability of AdS

- **Study problem numerically.** Initial Gaussian pulse of field:



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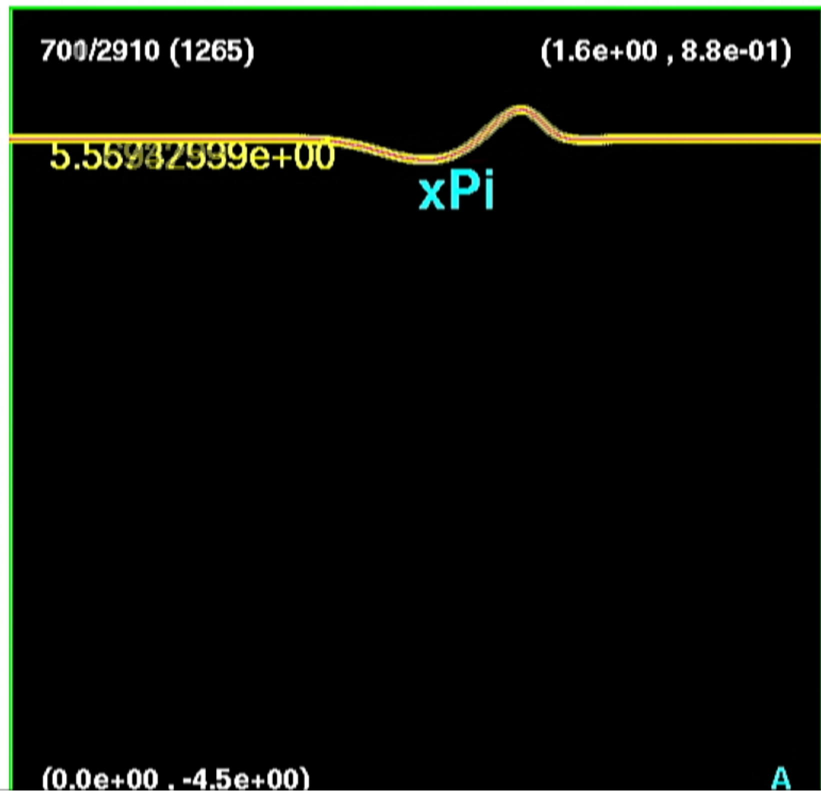
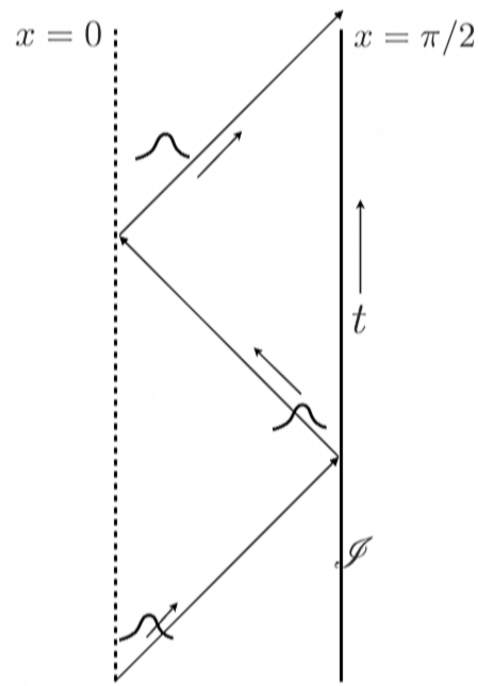
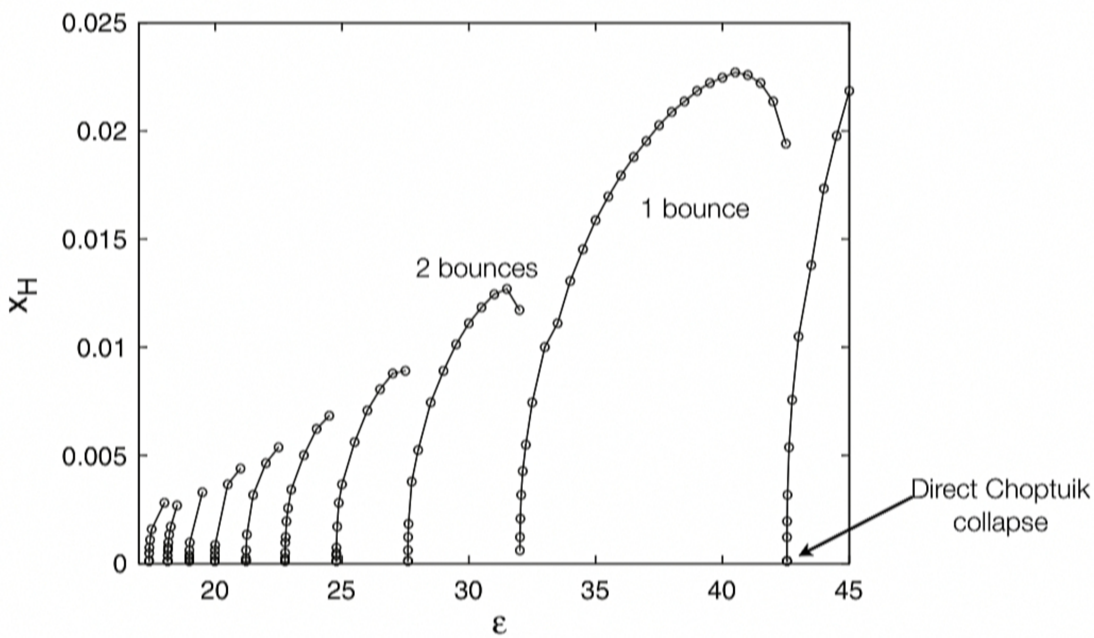




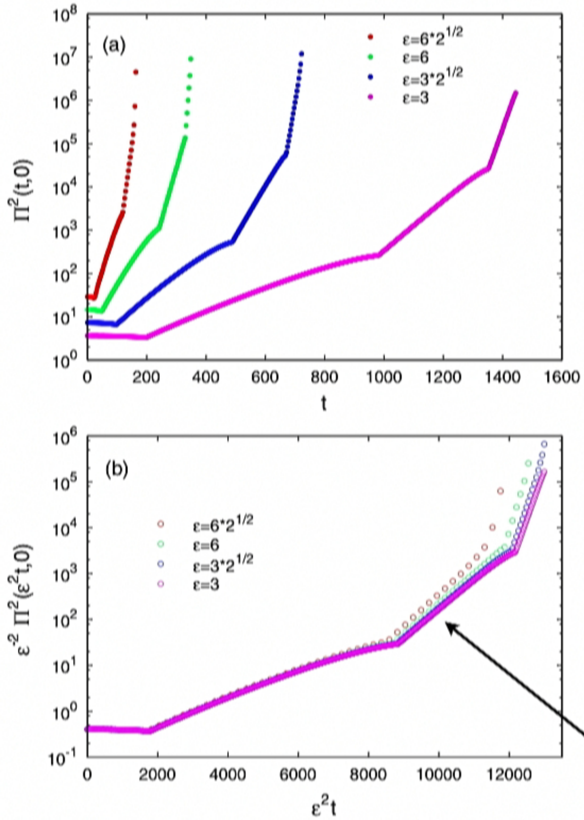
Introduction: Stability of AdS

- Bizon and Rostworowski (2011):

  For initial Gaussian field profile, numerical simulations showed collapse occurs even for arbitrarily small amplitude.



Take ϵ much smaller... (Bizon and Rostworowski, 2011)

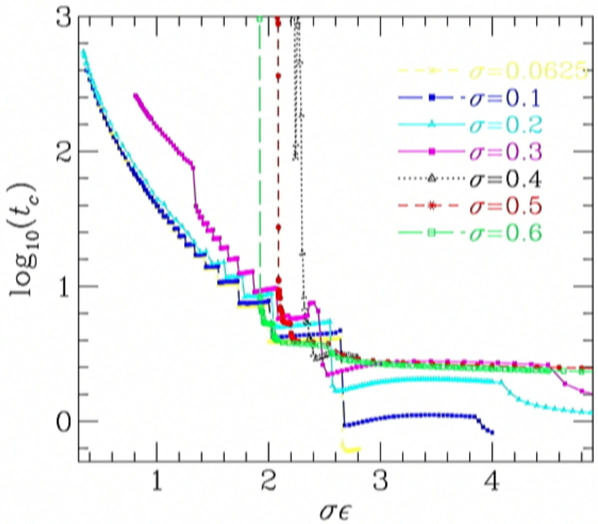


- AdS is effectively a confining mirrored box. So, a *non-gravitating* scalar field is characterized by normal modes.
- Gravitational focusing effects transfer energy *nonlinearly* to higher frequency modes in a direct turbulent cascade.
- High-frequency modes more highly peaked about the origin, leading to collapse.
- *"Gravitational turbulent instability"*

Scaling symmetry

But not all data collapses!

- Buchel, Lehner and Liebling (2013): Depending on *width* of Gaussian, collapse may not always occur.



- **Question:** What distinguishes these initial data?

- To make progress, must go beyond numerical simulations.

Ordinary perturbation theory

- 1st order: Free scalar field in AdS

$$\ddot{\phi}_{(1)} + L\phi_{(1)} = 0$$

General solution expressed in terms of eigenfunctions / eigenvalues of differential operator L:

$$\phi_{(1)}(t,x) = \sum_{j=0}^{\infty} \left(A_j e^{-i\omega_j t} + \bar{A}_j e^{i\omega_j t} \right) e_j(x)$$

$\omega_j = 2j + 3$

- 2nd order: Metric correction $g_{ab}^{(2)}(t,x)$ from Einstein constraint equations.

- 3rd order: Scalar wave equation with source term cubic in $\phi_{(1)}$

$$\ddot{\phi}_{(3)} + L\phi_{(3)} = S_{(3)}[\phi_{(1)}]$$

Commensurate frequency spectrum

Ordinary perturbation theory

- Example: 2 modes initially excited

$$\phi(0,x) = \epsilon[e_0(x) + e_1(x)]$$
$$\partial_t \phi(0,x) = 0$$

↓ 3rd order calculation

$$\phi(t,x) = \epsilon[e_0(x)\cos(3t) + e_1(x)\cos(5t)] + \epsilon^3 t \sin(7t) + \dots$$

- Perturbation theory breaks down after time $t \propto 1/\epsilon^2$

Two timescale framework (TTF)

- Our solution: Use a two-timescale framework to derive equations describing the interactions between modes.

- Allow all fields to depend on "slow time" $\tau = \epsilon^2 t$ and expand perturbatively as before:

$$\phi = \epsilon\phi_{(1)}(t,\tau,x) + \epsilon^3\phi_{(3)}(t,\tau,x) + O(\epsilon^5)$$
$$g_{ab} = g_{ab}^{AdS} + \epsilon^2 g_{ab}^{(2)}(t,\tau,x) + O(\epsilon^4)$$

Two timescale framework (TTF)

- At $O(\epsilon^3)$, where resonances are encountered in ordinary perturbation theory, use freedom in $A_j(\tau)$ to cancel them off.

- By requiring

$$-2i\omega_j \frac{dA_j}{d\tau} = \sum_{klm} \mathcal{S}^{(j)}_{klm} \bar{A}_k A_l A_m \quad \text{(TTF)}$$

resonant interactions are precisely accounted for in dynamics of $A_j(\tau)$.

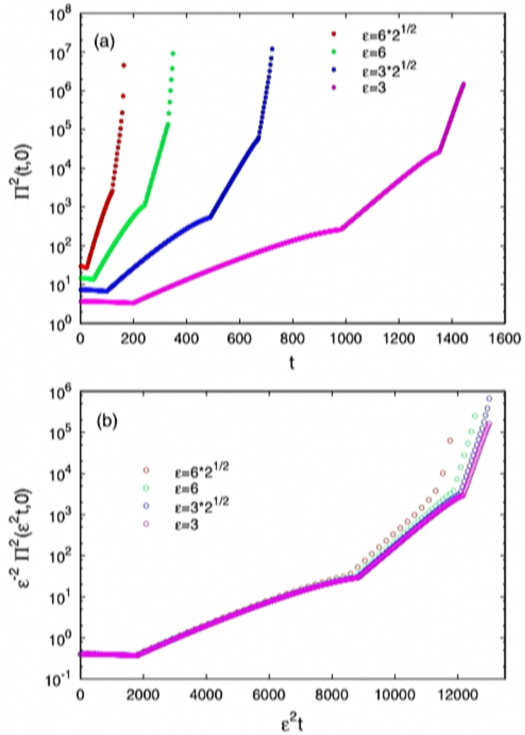
- Coefficients $\mathcal{S}^{(j)}_{klm}$ arise from overlap integrals of mode functions $e_j(x)$.

$$\mathcal{S}^{(j)}_{klm} = 0 \text{ unless } j + k = l + m \text{ (resonance condition)}$$

- *Rest of talk will focus on TTF equations, and how they explain collapse vs non-collapse.*

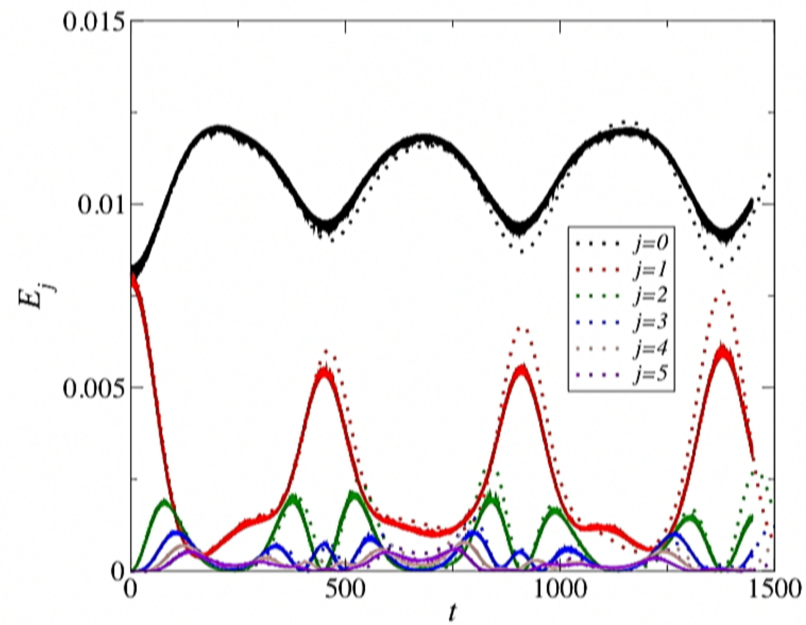
Collapsing initial data (full GR simulation)

- Initial data: Gaussian profile (original Bizon and Rostworowski data)



Non-collapsing initial data

- Initial data: Energy $E = \sum_j 4\omega_j^2 |A_j|^2$ evenly distributed between modes $j = 0, 1$
- Solid: Full numerics
  Dotted: TTF



Conserved quantities

- TTF equations conserve 3 quantities:

Symmetry

- Energy $E = \sum_j 4\omega_j^2 |A_j|^2$ — $A_j \to A_j e^{i\theta}$
- Particle number $N = \sum_j 4\omega_j |A_j|^2$ — $A_j \to A_j e^{i\omega_j \theta}$
- Hamiltonian $H = -\frac{1}{4} \sum_{jklm} S_{klm}^{(j)} \bar{A}_j \bar{A}_k A_l A_m + \frac{E}{8} \sum_j C_j \frac{d|A_j|^2}{d\tau}$

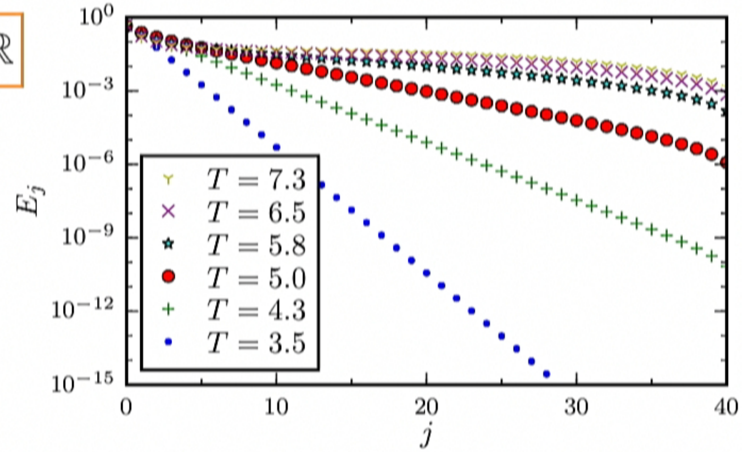
- *Conserved particle number is very unexpected for a real scalar field. It also holds beyond spherical symmetry.*

Quasi-periodic equilibrium solutions

- Ansatz $A_j^{\mathrm{QP}} = \alpha_j e^{-i\beta_j \tau},\ \beta_j \in \mathbb{R}$

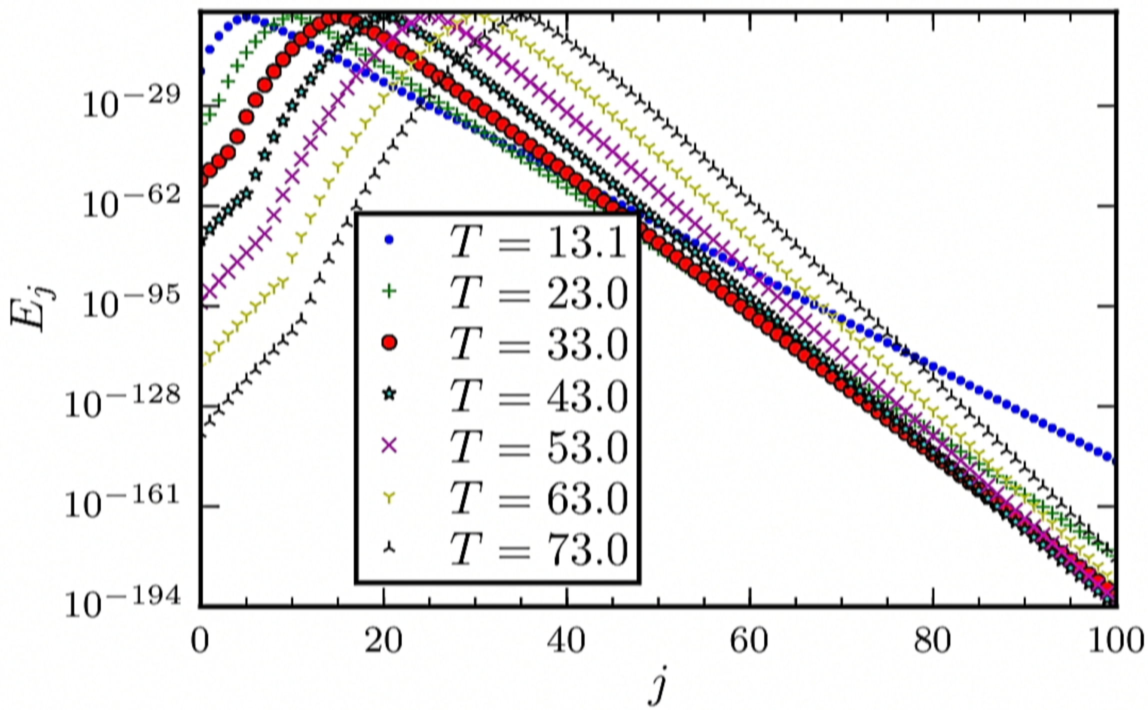
- 2-parameter family of solutions.

Take parameters to be (E,N), or (T=E/N,overall scale).



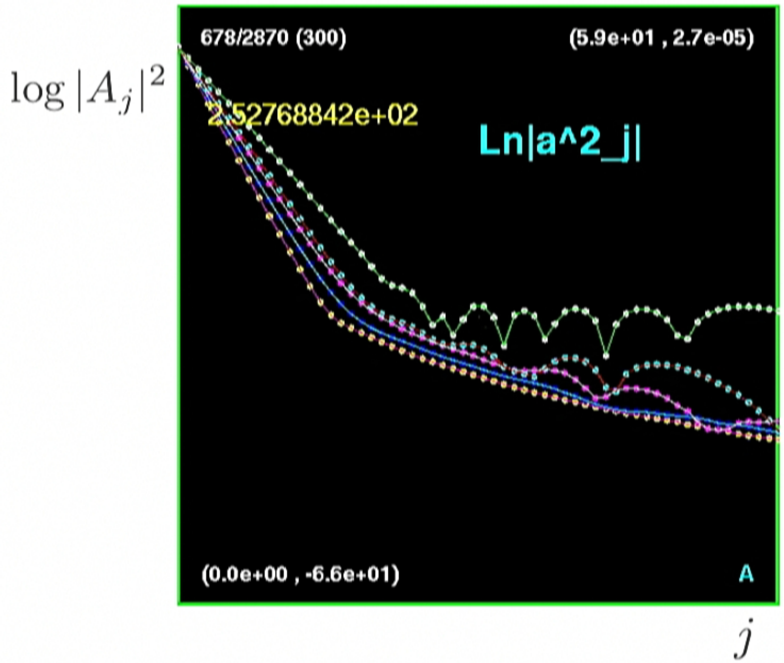
Other families of quasi-periodic solutions

- QP equations are nonlinear, so for given E and N there can be multiple solutions. Families of QP solutions, labelled by peak mode jr.



Stability of QP solutions

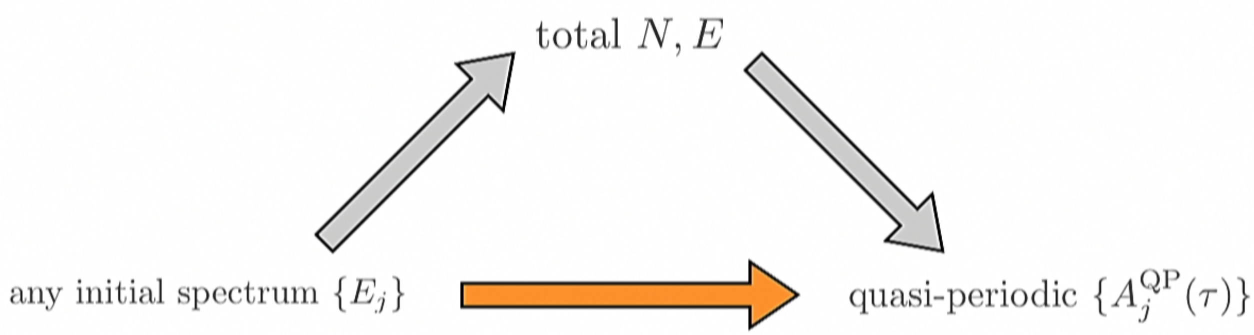
- QP solutions appear to be nonlinearly stable under full numerical simulations:



QP solutions as islands of stability

- **Claim:** Non-collapsing solutions are oscillating around QP solutions.

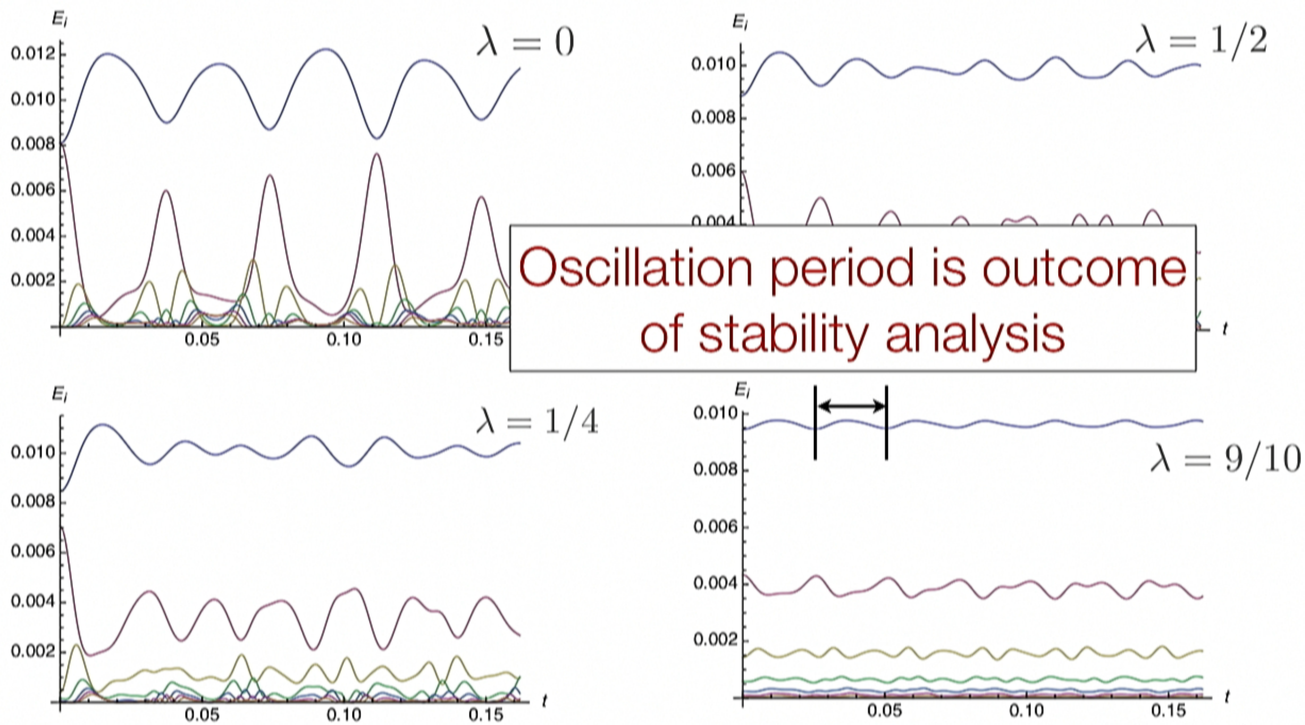
- Match initial data to QP solution by identifying total conserved N and E.



- Oscillation frequencies (recurrence times) arise from oscillations about QP solution.

QP solutions as islands of stability

- Interpolate initial data: $E_j^\lambda = (1-\lambda)E_j^{\text{2-mode}} + \lambda E_j^{\text{QP}}$



Conclusions for Part 2

- TTF effectively models the AdS-scalar system for small amplitude perturbations for time scales $t \approx 1/\epsilon^2$
- TTF has led us to uncover:
  - ★ Conserved particle number, energy and Hamiltonian
  - ★ Stable quasi-periodic solutions as islands of stability
  - ★ Direct calculation of recurrence times
- How much of this extends beyond spherical symmetry?

Overall conclusions and questions

- In the presence of long lived quasinormal or normal modes, standard approaches to gravitational perturbations break down. High gravitational Reynolds number regime.
    - Are there natural high gravitational Reynolds number regimes beyond AdS? Near-extreme Kerr? Cosmology?
- Turbulent cascades strongly influenced by presence of conserved quantities.
    - Are either particle number or enstrophy conserved beyond perturbative level?
    - Are there analogs of these AdS conserved quantities in dS spacetimes relevant to cosmology?
    - What role can these quantities play in fundamental gravity questions, including stability and cosmic censorship?

Thank you