Title: Black hole ringdown and quasinormal modes

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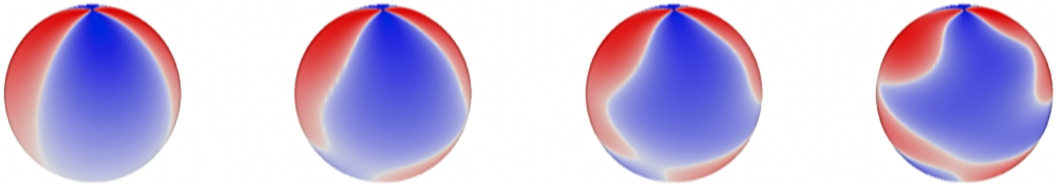
Abstract: The first detection of gravitational waves came with an unexpected windfall: a clear signal from the merger of two black holes into a final, spinning black hole. General Relativity predicts that following merger, the final black hole relaxes by emitting radiation in a characteristic spectrum of decaying modes. I will discuss these ``quasinormal modes" and what can be learned from them, as well as the black hole ringdown observed in GW150914. I will also explore the exotic side of ringdown, including the modes of nearly extremal black holes, and a tool for understanding the ringdown of black holes which differ from the standard Kerr solution.

Black hole ringdown and quasinormal modes



Aaron Zimmerman (CITA)

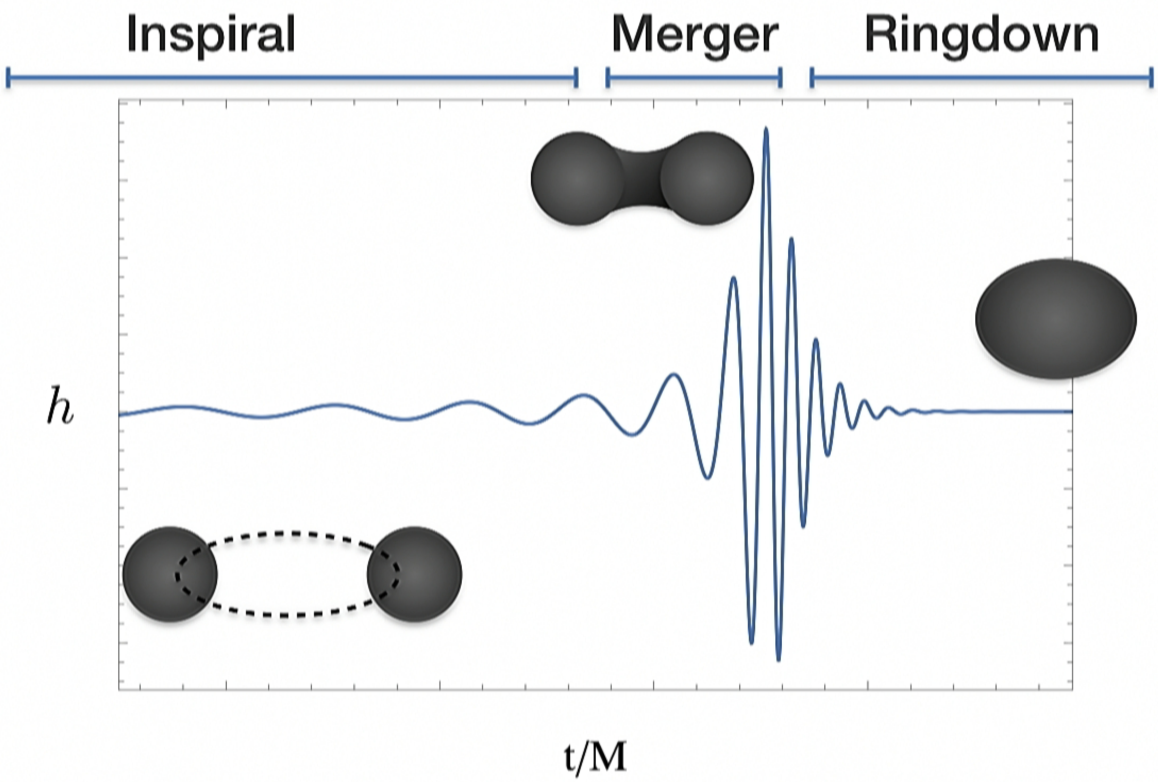
Collaborators: Huan Yang, Zachary Mark, David Nichols, Fan Zhang, Anil Zenginoglu, Yanbei Chen, Emanuele Berti, Luis Lehner

Cosmological Frontiers
June 15, 2016

Outline

- Introduction to ringdown and QNMs
- The ringdown of GW150914
- Modes of rapidly rotating BHs
- QNMs beyond Kerr
- Application: Parametric resonance and gravitational turbulence (That asymp. flat space case Stephen mentioned)

GWs from compact binaries



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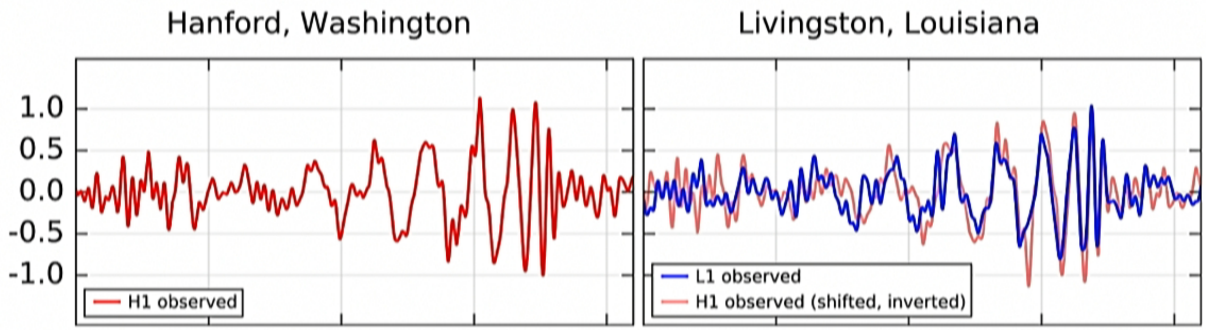
First detection of GWs

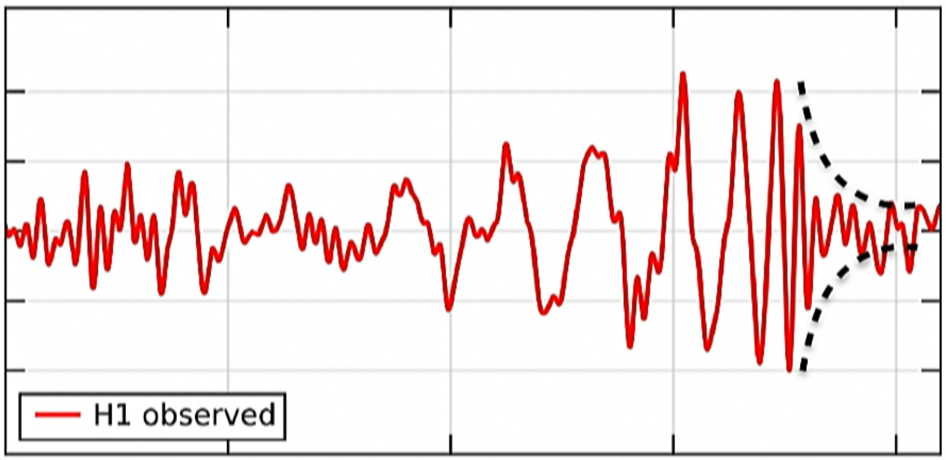


- GW150914: the merger of two BHs
- Masses $m_1 = 36^{+5}_{-4} M_\odot \quad m_2 = 29^{+4}_{-4} M_\odot$
- Final black hole $M_f = 62^{+4}_{-4} M_\odot \quad \chi_f = 0.67^{+0.05}_{-0.07}$
- Luminosity distance $D_L = 410^{+160}_{-180} \mathrm{Mpc}$
- Luminosity $L \sim 10^{23} L_\odot \sim 10^{13} L_{\mathrm{MWG}}$

LVC, PRL (2016)

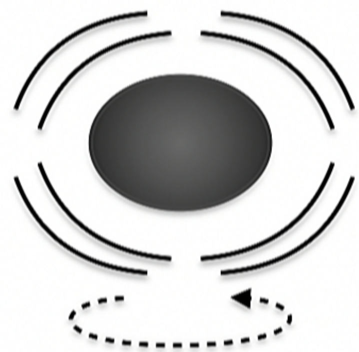
Observation of ringdown

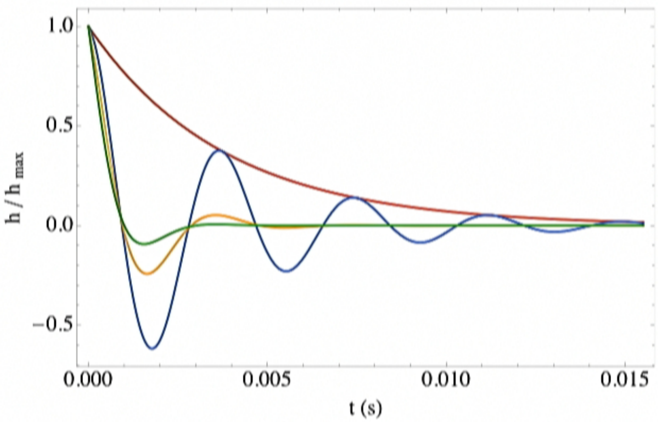
- Most exciting (to me): final black hole ringing!



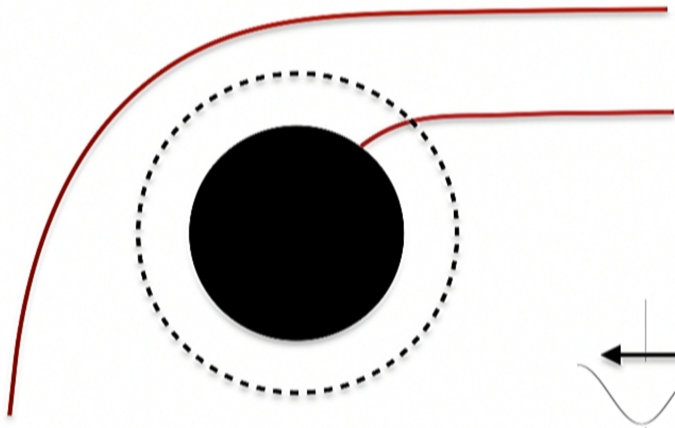
Black hole ringdown

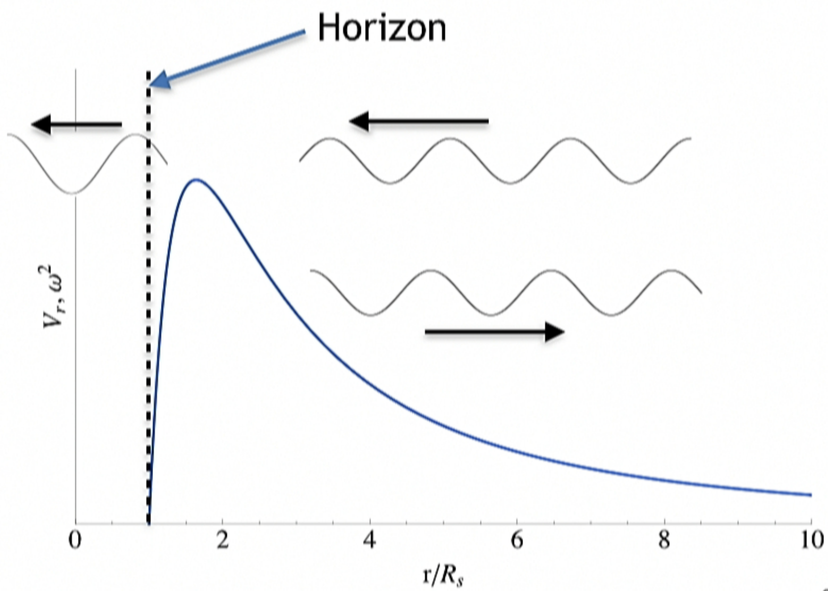
- After merger, the remnant BH is strongly deformed
- View as a perturbation about the final state
- Perts excite resonant modes of the BH
- These decay exponentially in a "ringdown"





Waves around black holes





Quasinormal modes: the math

- Scalar wave equation: $\Box_g \psi = 0$
- Separation of variables:

$$\psi_{\omega l m} \sim \frac{1}{r} e^{-i\omega t} u_{\omega l m}(r) Y_{lm}(\theta, \phi)$$

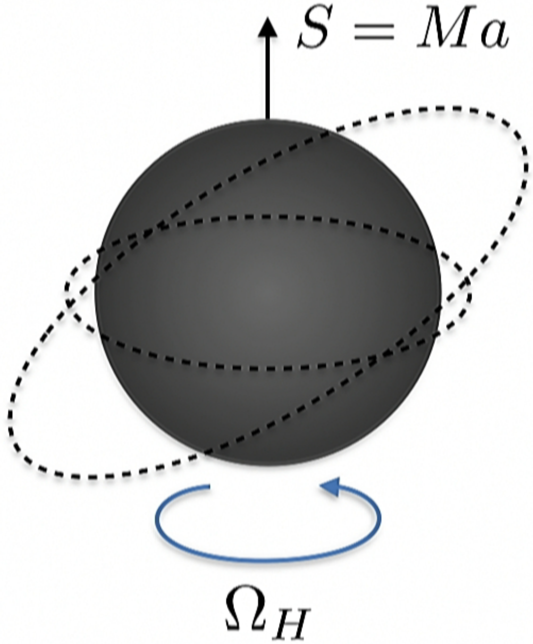
- Radial equation:

$$\frac{d^2 u_{\omega l m}}{dr_*^2} + \left(\omega^2 - V\right) u_{\omega l m} = 0$$

$$V = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right)$$

- Only certain freqs work for BCs: ω_{lm}, τ_{lm}
- Story same for GWs ${}_s\psi_{\omega l m} \to h$

Modes of rotating black holes



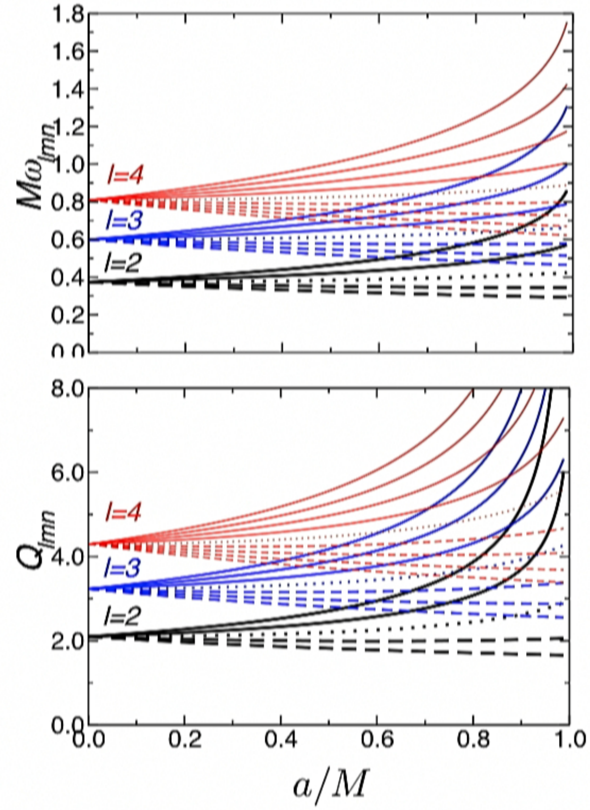
- Orbits of light split according to inclination
- Wave picture: everything holds, but freq and decay split with m
- Math story carries through ${}_s\psi_{\omega lm} \rightarrow h$
- As spin increases, freq increases, decay rate decreases

Measuring mass and spin

- Unique and clean measure of mass and spin
- Mass: overall scale

$$f \sim 32 \left(\frac{M_\odot}{M} \right) \mathrm{kHz}$$

- ω, τ vary with mode and spin
- Low $Q = \omega\tau/2$
- Two modes: test of GR



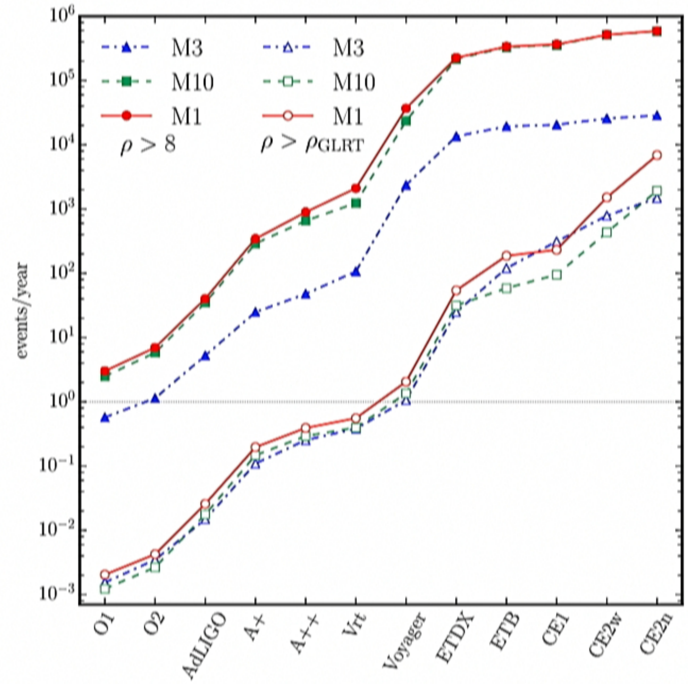
Berti,Cardoso, Starinets (2009)

Testing GR with ringdown

- The (2,2) mode dominates
- Large SNR is needed to detect additional modes (~100)

$$\rho_{\rm RD} \propto \frac{M_z^{3/2}}{S_n}$$

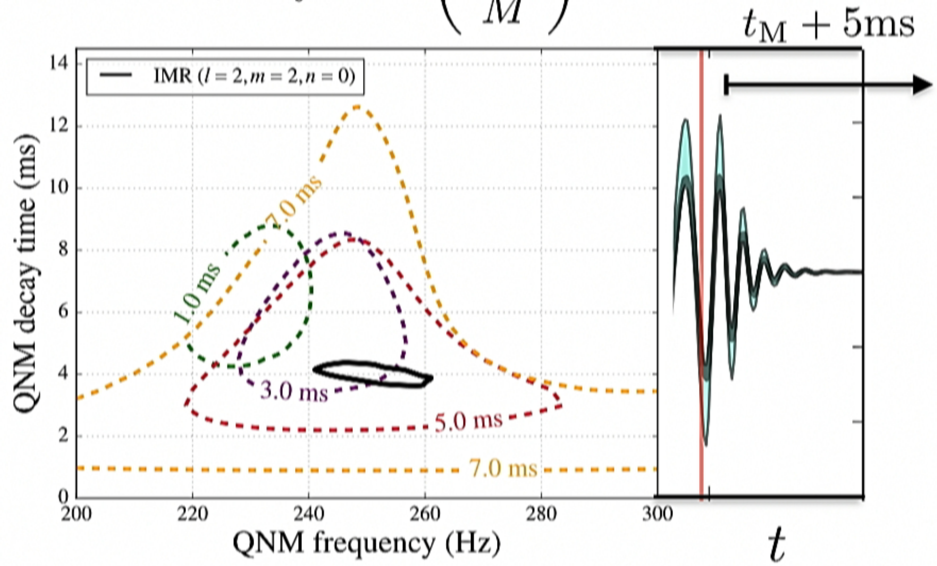
- Hard for ground-based detectors
- Easier for more massive binaries



Berti et al. arXiv:1605.09286

The ringdown of GW150914

- First detection of a BH ringdown
- Freq and decay of lowest overtone for $\ell = 2,\, m = 2$
- Consistent with GR $f \sim 32 \left(\frac{M_\odot}{M} \right) \mathrm{kHz}$



LSC, arXiv:1602.03841

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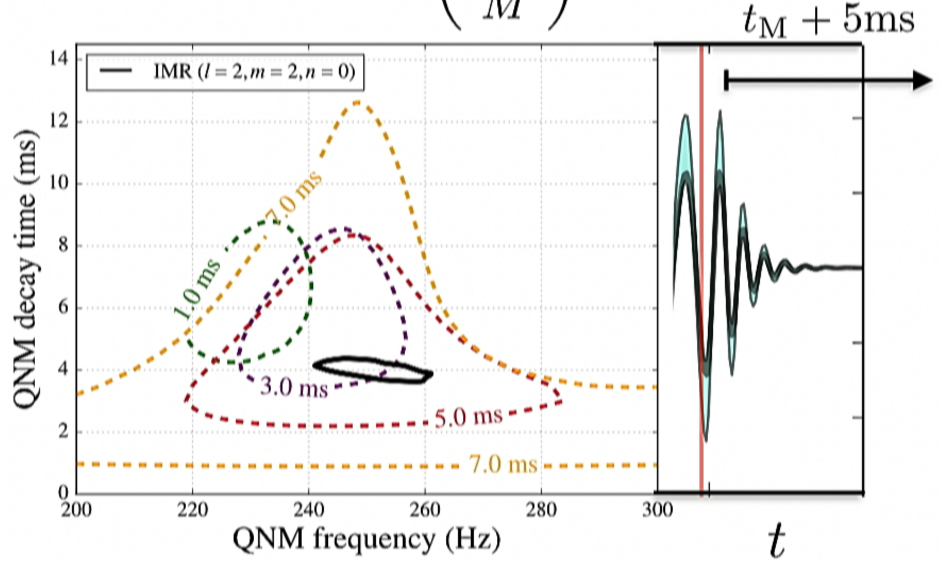
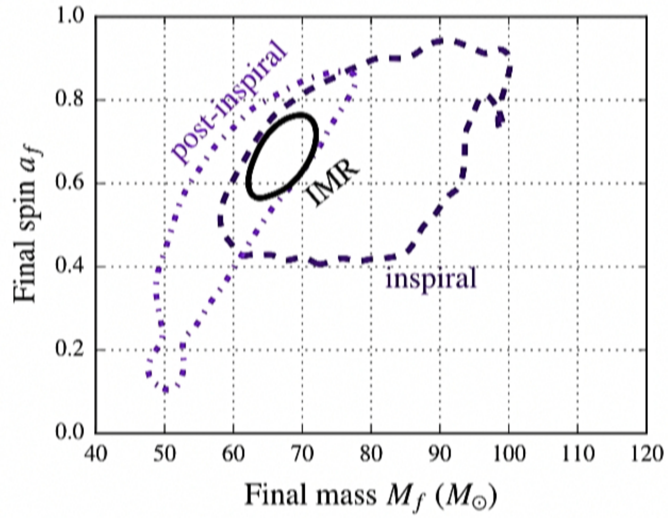
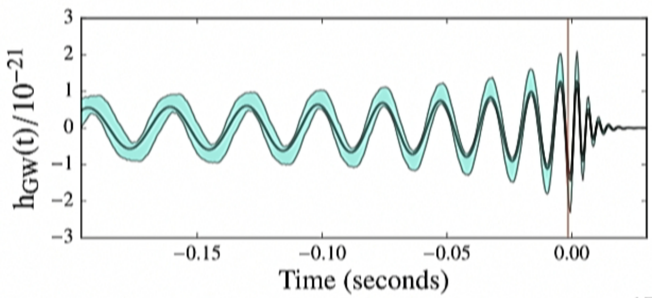


LSC, arXiv:1602.03841

Consistency tests of GR

- GW150914: Only 1 mode measured
- Consistency test still possible: split signal
- Compare inspiral with merger-ringdown
- Both consistent with IMR analysis

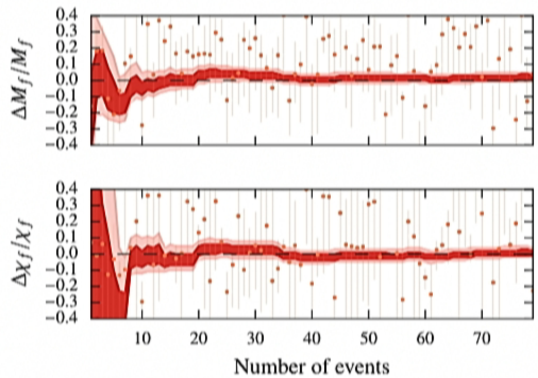


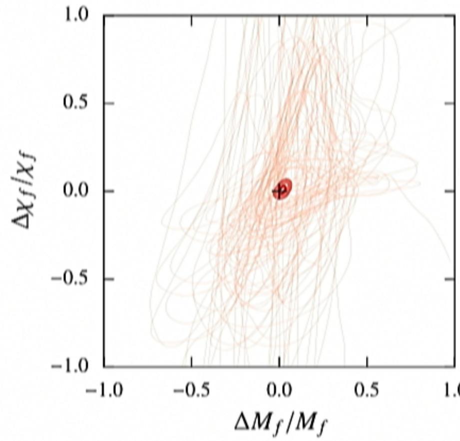


LSC, arXiv:1602.03841

Consistency tests of GR

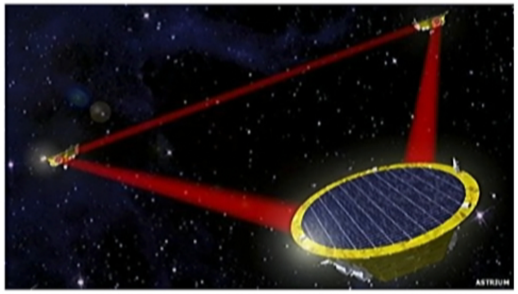
- Consistency tests can be stacked over many observations
- Ghosh et al. (2016): after ~100 observations at SNR 25, percent level test are achievable

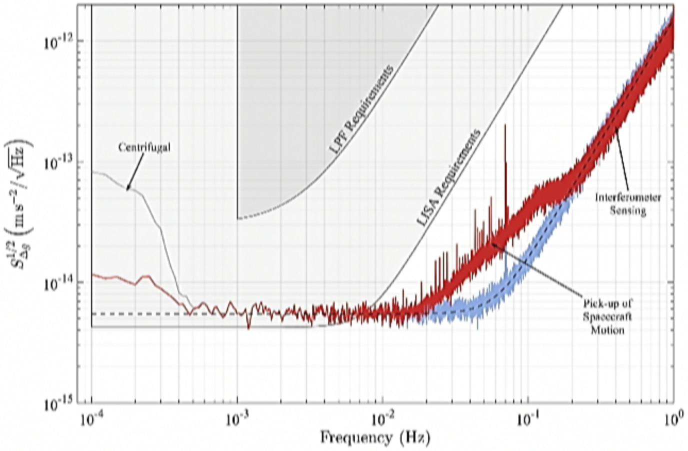




Ghosh et al arXiv:1602.02453

Space-based detectors

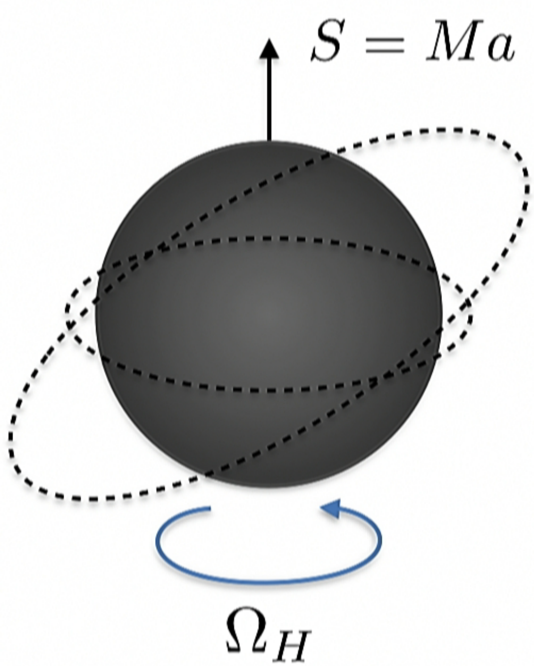




Armano et al. (2016)

- LISA Pathfinder outperformed expectations!
- Space-based missions open many frontiers: SMBH binaries, WD binaries
- Precision tests of GR with EMRIs

The ringdown of rapidly rotating black holes



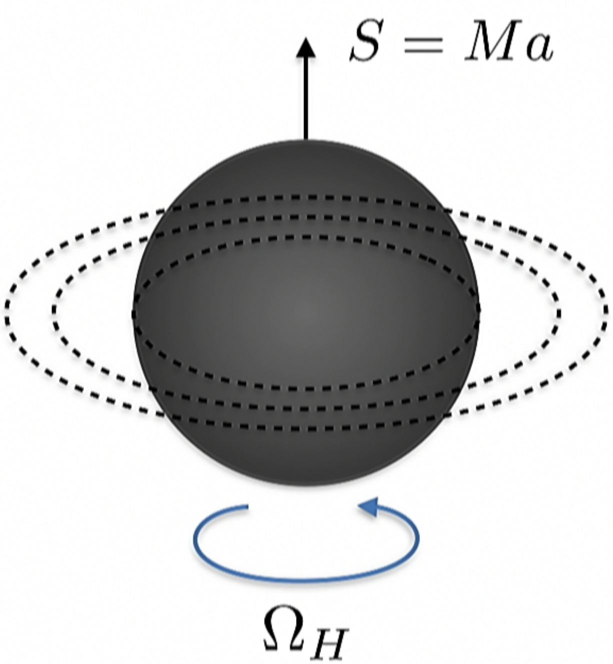
- At high spins, a lot changes
- Simple geometry, new appxs, Kerr/CFT duality

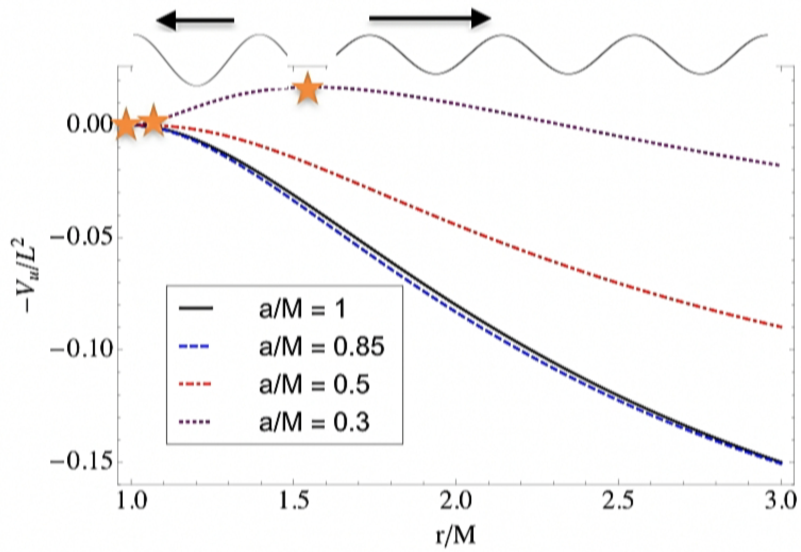
$$\epsilon = 1 - a/M \ll 1$$

- Censorship, instability...
- BHs spun up by accretion

Yang et al (2012)

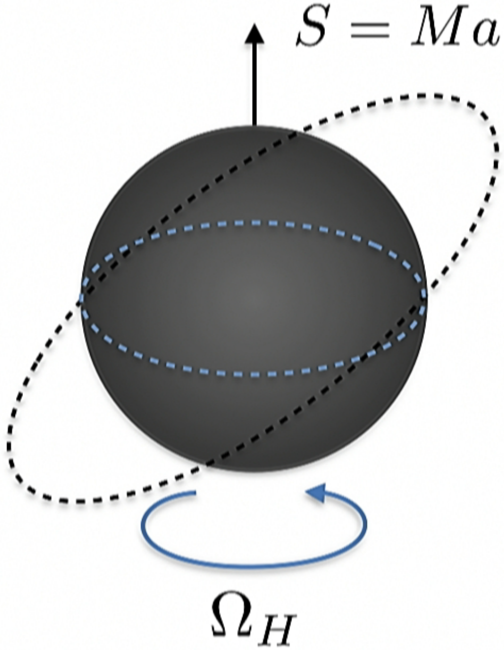
Modes of rapidly rotating BHs: WKB treatment





Yang et al (2012)

Modes of rapidly rotating BHs



- Analytically find ringdown freq, decay

$$\omega \approx m\Omega_H$$

$$\tau \approx \frac{\sqrt{2}}{\sqrt{\epsilon}\,(n + 1/2)}$$

Slow decay!

- Match near horizon to far region

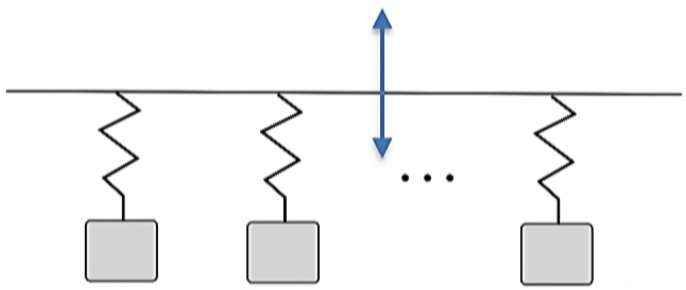
$$(r - r_+) \ll 1 \qquad (r - r_+) \gg \sqrt{\epsilon}$$

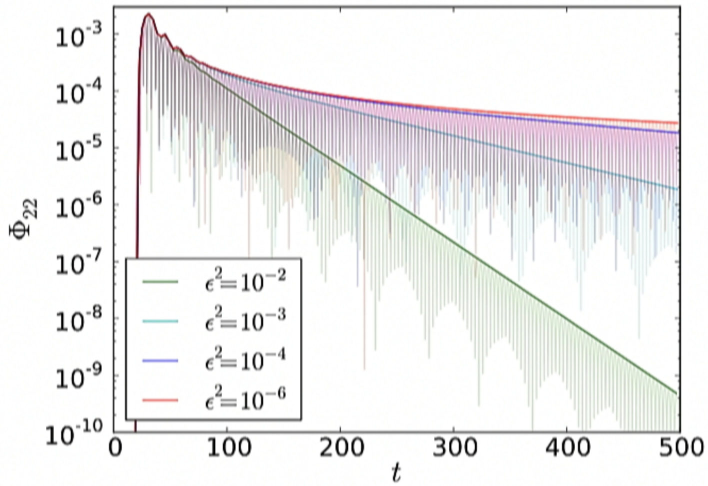
Yang et al (2012)

A power law ringdown

- Many modes with nearly equal frequencies: collective excitation
- Initially a power-law decay
- Slowest mode takes over at end

$$h \sim \begin{cases} t^{-1} & t \ll 1/\sqrt{\epsilon} \\ \sqrt{\epsilon} e^{-\sqrt{\epsilon/8}\, t} & t \gg 1/\sqrt{\epsilon} \end{cases}$$





Yang, AZ et al (2013)

QNMs in a nearly Kerr spacetime

$$g = g_K + \eta\, h \qquad \eta \ll 1$$

- Frequencies shift

$$\omega \to \omega + \eta\, \delta\omega + O(\eta^2)$$

- Teukolsky eq deformed

$$L[\psi] = 0 \to L[\psi] + \eta\, \delta L[\psi] = 0$$

- Harmonics: $\psi \sim \exp[-i(\omega + \eta\delta\omega)t]e^{im\phi}\psi_{m\omega}(r,\theta)$

Eigenvalue perturbation

- Just like undergrad quantum!

$$H|n\rangle = E_n|n\rangle \to (H + \eta\,\delta H)|n\rangle = (E_n + \eta\,\delta E_n)|n\rangle$$

$$\delta E_n = \frac{\langle n^{(0)}|\delta H|n^{(0)}\rangle}{\langle n^{(0)}|n^{(0)}\rangle}$$

- Need finite inner product: $\langle \psi|\chi\rangle = C$

$$\delta\omega = -\frac{\langle \psi_{m\omega}|\delta \tilde{L}|\psi_{m\omega}\rangle}{\langle \psi_{m\omega}|\partial_\omega \tilde{L}|\psi_{m\omega}\rangle}$$

Yang, AZ, Lehner (2015)
AZ, Yang et al (2015)

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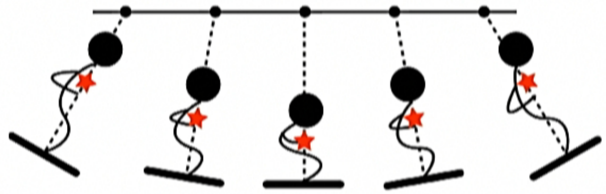
Yang, AZ, Lehner (2015)
AZ, Yang et al (2015)

Parametric resonance

- Forced oscillator

$$\ddot{x} + 2\gamma\dot{x} + \omega^2[1 + 2f(t)]x = 0$$

$$f(t) = f_0 \cos\omega' t$$



Nation et al. (2012)

- Resonance $\omega \approx \omega'/2$
- Weak forcing and damping:

$$x \propto \exp[(\alpha - \gamma)t]$$

$$\alpha \approx \omega' \sqrt{f_0^2 - \frac{4\Delta^2}{\omega^2}} \qquad \Delta = \omega - \omega'/2$$

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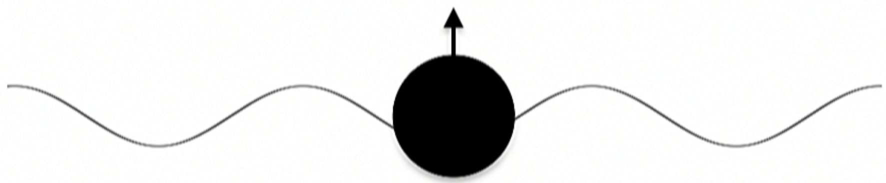
Parametric instability of BHs

- Toy model: use a rapidly spinning BH with a single mode active



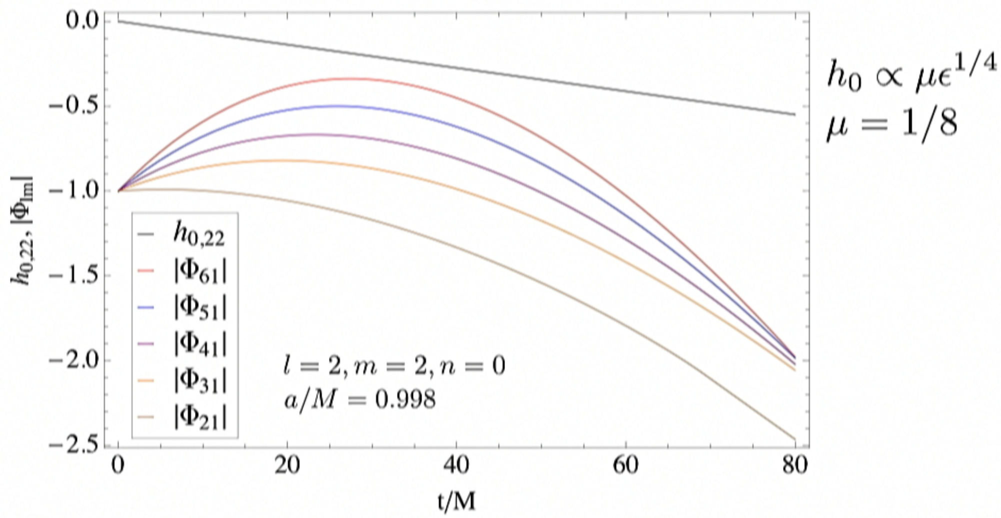
- Evolve test scalar field on dynamic background

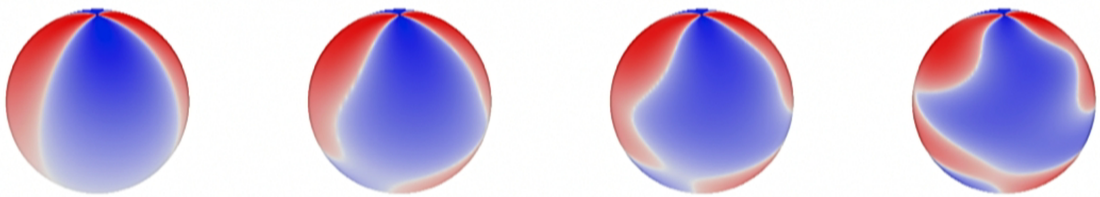
$$\Box_{g+h}\Phi = \Box_g\Phi + \mathcal{D}_h\Phi$$

- Instability can occur
- Oscillating background enforces freq and spatial harmonic matching

Yang, AZ, Lehner (2015) 31

Parametric instability of BHs

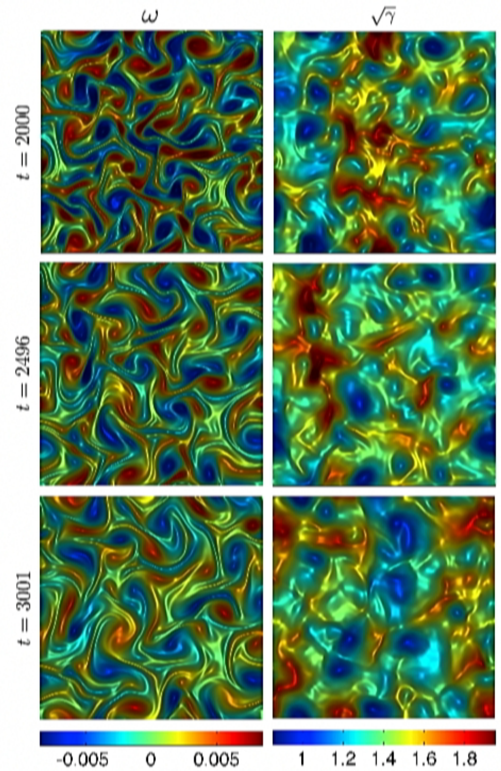




Yang, AZ, Lehner (2015)

Fluid-gravity duality: gravitational turbulence

- Turbulence prev seen in AdS black holes
- CFT dual: 2+1 d, long wavelength → fluid
- 2+1 d fluid has turbulence, inverse cascade
- Kerr case: gravity, no AdS boundary, mode cascade



Adams et al (2013)

Turbulent instability

- Growth for critical "grav Re"

$$\frac{h_0(t)}{m'\sqrt{\epsilon}} > \left|\frac{Q}{H}\right| \sqrt{\frac{(\omega_R' - \omega_R/2)^2 + {\omega_I'}^2}{\epsilon}}$$

- Compare to condition in fluids:

$$\frac{u}{L\nu} = \mathrm{Re} > \mathrm{Re}_c$$

- Inverse cascade (in ω and m)

$$\omega_R' = \omega_R/2 \qquad\qquad m' = m/2$$

- Hallmarks of transient turbulence

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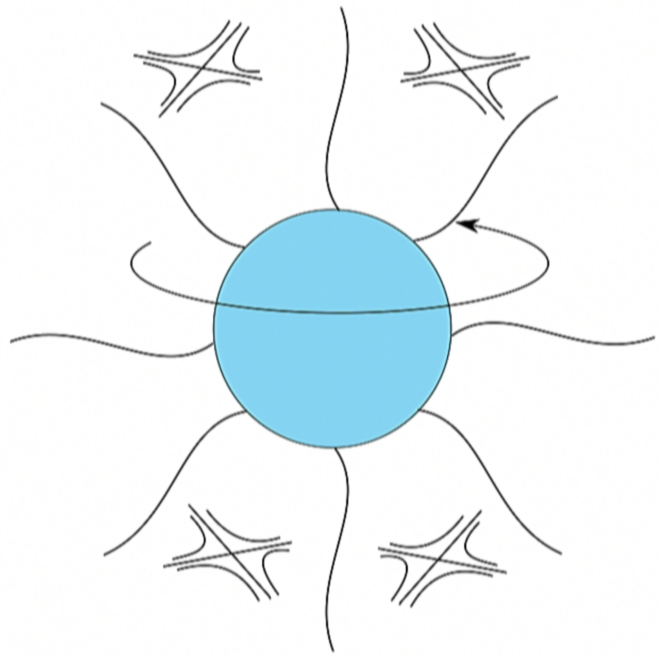
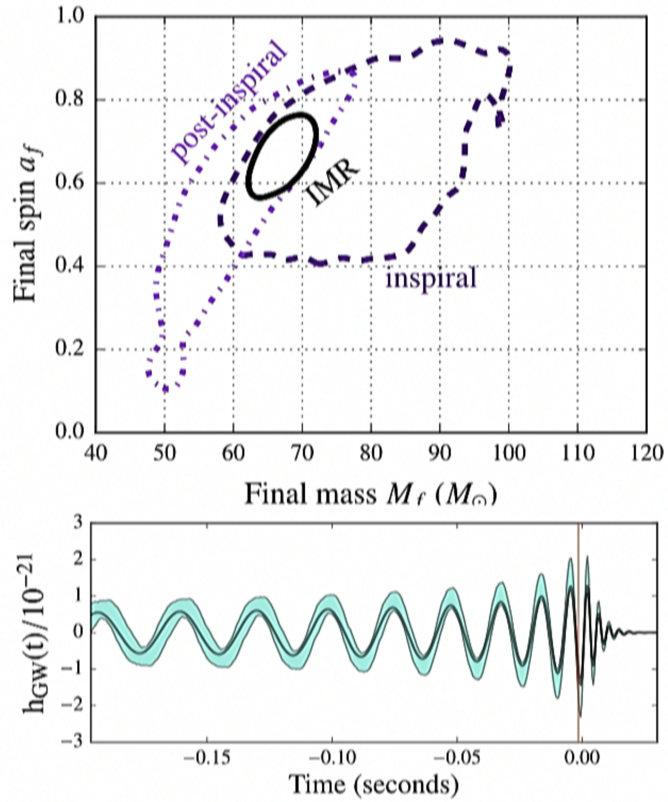
QNMs of Kerr-Newman

- Grav and EM perts coupled
- No separation
- Small spin solved (Pani, Berti, Gualtieri 2013)
- Small charge $q = Q^2$
- EV perts, allows for separation
- Can compute shifts to freqs (Mark, Yang, AZ, Chen 2015)



Outlook: Ringdowns and GW astronomy

- Ringdown observed!
- Much more to come: $R \sim 2 - 400\,\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$
- Statistical power: tests of GR
- EV pert: tailored ringdown tests of GR
- Application to other systems (e.g. Yang and Zhang arXiv:1406.4602)



LSC, arXiv:1602.03841

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