Title: The Classicality Puzzle

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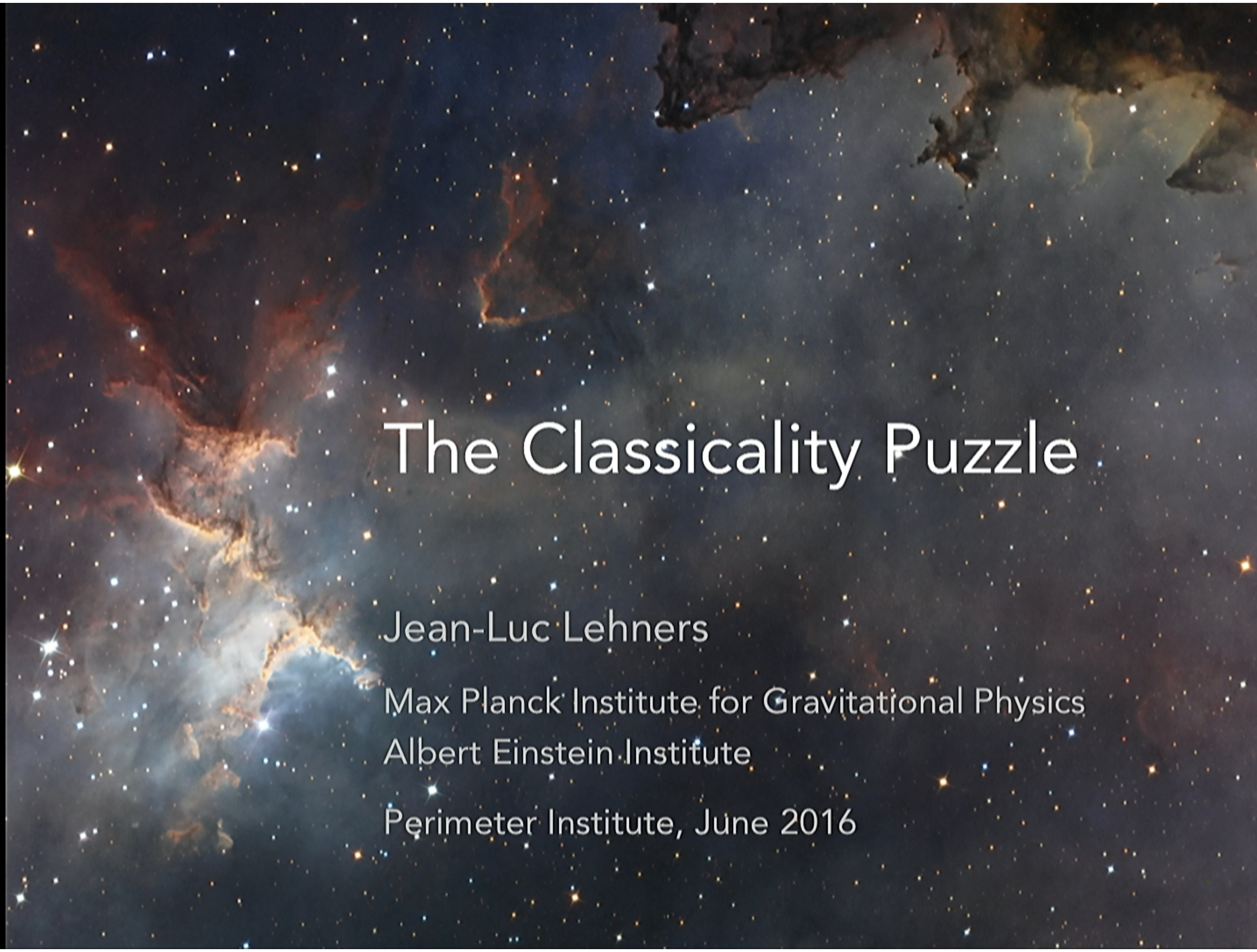
Abstract: Why was the early universe classical? Along with the big bang singularity problem and the flatness, horizon and inhomogeneity puzzles, this is one of the big unexplained features of the hot big bang scenario. In this talk I will discuss how inflation and ekpyrosis, which have mainly been considered as models that can address some of the other puzzles, can both drive the early universe towards classicality. The remarkable aspect is that classicality is achieved via the intrinsic dynamics of inflation and ekpyrosis, without invoking decoherence.


The Classicality Puzzle
Jean-Luc Lehners
Max Planck Institute for Gravitational Physics
Albert Einstein Institute
Perimeter Institute, June 2016

Why was the early universe classical?

- Along with singularity, flatness, horizon, inhomogeneity puzzles, one of the big unexplained features of hot big bang cosmology

- Two aspects:
  - Quantum-to-classical transition of perturbations *(fairly well understood)*
  - Quantum-to-classical transition of background *(main interest in this talk)*

Background classicality

- Usual explanation:
  Interactions between perturbations and background decohere the background [Zeh '86, Kiefer '87,...]

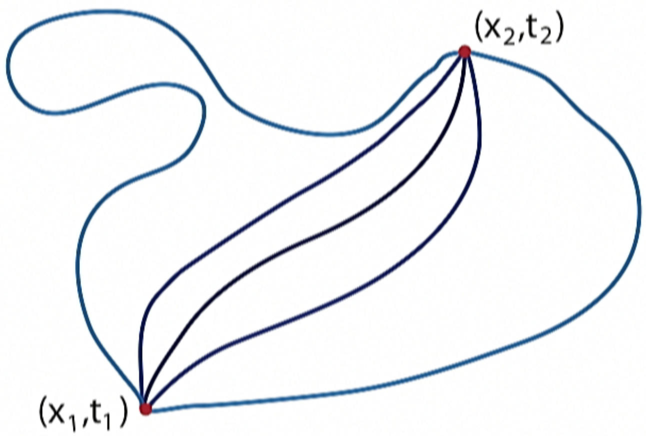
- Reasons not to be fully satisfied with this:
  - Decoherence happens in classical time, while we want to understand why time is classical in the first place [Butterfield & Isham '98]
  - Imagine a beginning: there might not be a background/perturbations split, i.e. no perturbations might have been produced yet

- Here discuss how classicality can be achieved via the "*intrinsic dynamics*"
  [Hartle, Hertog & Hawking '08, Battarra & JLL '15, Bramberger, Farnsworth & JLL, *in preparation*]

Quantum Mechanics

- Path Integral formulation

In quantum mechanics, we are used to calculating transition amplitudes between two events – according to Feynman's prescription we must *sum over all interpolating paths:*

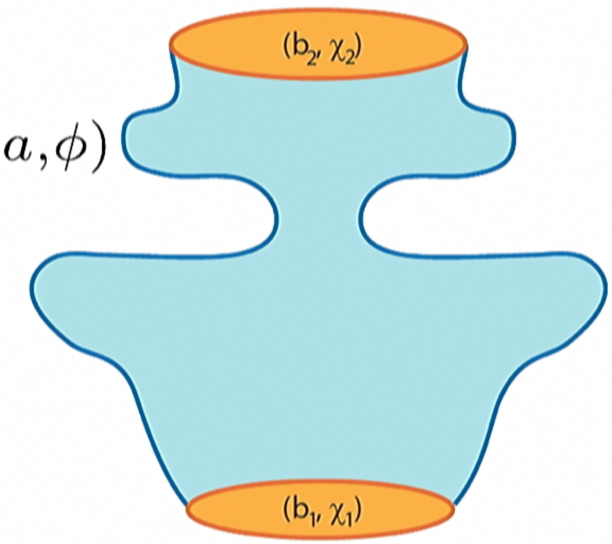


Quantum cosmology

- Semi-classically we can generalize the path integral:

$$\Psi(b,\chi) = \int_{\mathcal{C}} \mathcal{D}a\mathcal{D}\phi e^{-S_E(a,\phi)}$$

$$\approx e^{-S_{E,ext}(b,\chi)}$$



Here the action describes gravity plus a scalar field:

$$S_E = 2\pi^2 \int d\tau \left(-3aa'^2 - 3a + a^3\left(\frac{1}{2}\phi'^2 + V\right)\right)$$

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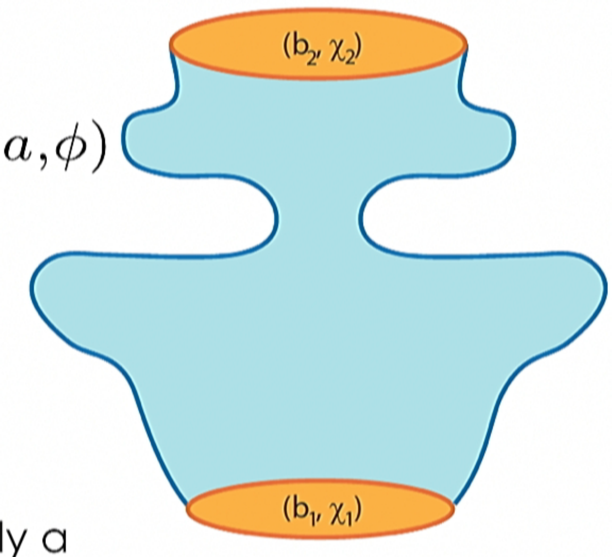
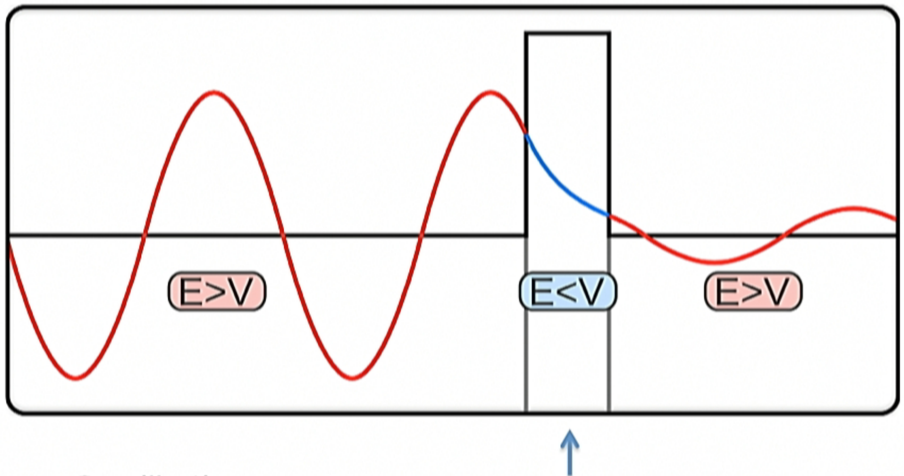
Saddle point of the action, generally a complex-valued solution of the classical equations of motion



WKB

- How do we tell whether the wavefunction describes a classical universe?



Source:
wikimedia commons

Oscillating:
phase changes fast

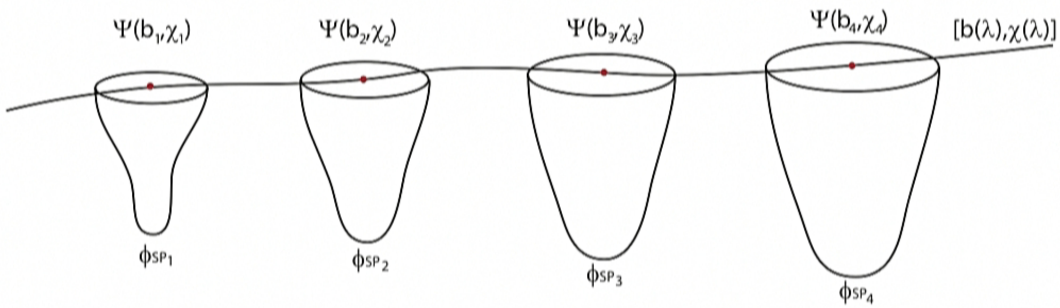
Decaying:
amplitude changes fast

- WKB criterion: $$\frac{\partial(\text{amplitude})}{\partial(\text{phase})} = \frac{\partial S_E^R}{\partial S_E^I} \ll 1$$

$\Psi = e^{-S_E}$

WKB

- Note that the WKB conditions concern the evolution of the transition amplitude as the final hypersurface is varied



- Denote final value of scale factor a by b
- Denote final value of scalar ϕ by χ

Models

- Consider gravity plus a minimally coupled scalar field with potential

$$V = \pm V_0 e^{-\sqrt{2\epsilon}\phi} \qquad \epsilon = constant$$

- Then, for a positive potential and $\epsilon < 1$ we obtain *inflation*
- For a negative potential and $\epsilon > 3$ we obtain *ekpyrosis*

Asymptotic solutions

- A constant equation of state is obtained with the potentials $V = \pm V_0 e^{-\sqrt{2\epsilon}\phi}$

- Then there exist the asymptotic scaling solutions (which are attractors) for both inflation and ekpyrosis:

$$a = a_0|\lambda|^{1/\epsilon}, \quad \phi = -\sqrt{\frac{2}{\epsilon}} \ln\left(\sqrt{\frac{\epsilon^2 V_0}{3-\epsilon}}\,|\lambda|\right), \quad V = \frac{3-\epsilon}{\epsilon^2\lambda^2}$$

- Time ranges: inflation $0 < \lambda < \infty$; ekpyrosis $-\infty < \lambda < 0$

- The asymptotic solution immediately allows us to determine the asymptotic behaviour of the imaginary part of the Euclidean action (i.e. the real part of the ordinary action)

$$S_E^I \sim i \int d\lambda\, a^3\, V$$

$$\sim i\, a_0^3\, (\lambda)^{-1+3/\epsilon}$$

$$\sim i\, a_0^3\, V^{\frac{1}{2}-\frac{3}{2\epsilon}}$$

$$\sim i\, b^3\, V(\chi)^{1/2}$$

- But how do we find S_E^R ?

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- But how do we find S_E^R ?

Symmetry of the action

- The action

$$S_E = -\int d^4x\sqrt{g}\left(\frac{R}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - e^{-c\phi}\right)$$

has a classical shift/scaling symmetry

$$\phi \equiv \bar{\phi} + \Delta\phi\,, \quad g_{\mu\nu} \equiv e^{c\Delta\phi}\bar{g}_{\mu\nu}$$

$$S_E = -\,e^{c\Delta\phi}\int d^4x\sqrt{\bar{g}}\left(\frac{\bar{R}}{2} - \frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\phi} - e^{-c\bar{\phi}}\right)$$

Under the symmetry, the scaling solutions

$$a = a_0|\lambda|^{1/\epsilon}, \quad \phi = -\sqrt{\frac{2}{\epsilon}} \ln\left(\sqrt{\frac{\epsilon^2 V_0}{3-\epsilon}}\,|\lambda|\right), \quad V = \frac{3-\epsilon}{\epsilon^2\lambda^2}$$

are transformed such that

$$\bar{a} = \bar{a}_0\,(\bar{\lambda})^{1/\epsilon}\,, \qquad \bar{a}_0 = \exp\left(\frac{\epsilon-1}{\epsilon}\frac{c\,\Delta\phi}{2}\right) a_0\,, \qquad V(\bar{\phi}) = \frac{3-\epsilon}{\epsilon^2}\frac{1}{\bar{\lambda}^2}$$

It is clear that $a_0 = a\left(\frac{\epsilon^2}{3-\epsilon}V\right)^{1/2\epsilon}$ serves as a label to distinguish solutions

The scaling of the action then implies

$$\bar{S}_E = e^{c\Delta\phi} S_E^R = \left(\frac{\bar{a}_0}{a_0}\right)^{\frac{2\epsilon}{\epsilon-1}} S_E^R \qquad \to S_E^R \propto a_0^{\frac{2\epsilon}{\epsilon-1}}$$

Asymptotic solutions

- Using these we can find the asymptotic dependence of the action upon varying the end points along a classical history:

$$S_E^R \sim a_0^{\frac{2\epsilon}{\epsilon-1}} \sim b^{\frac{2\epsilon}{\epsilon-1}} V(\chi)^{1/(\epsilon-1)}$$

$$S_E^I \sim i\, b^3\, V(\chi)^{1/2}$$

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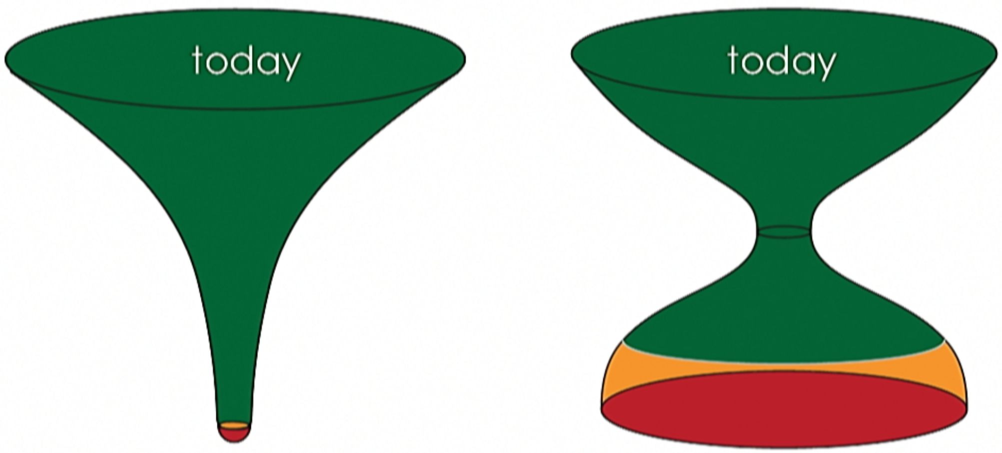
- This leads to the WKB expressions (with $\mathcal{N} = \ln|aH|$)

$$\frac{\partial_\chi S_E^R}{\partial_\chi S_E^I} \sim \frac{b^{\frac{2\epsilon}{\epsilon-1}} V(\chi)^{1/(\epsilon-1)}}{b^3\, V(\chi)^{1/2}} \sim b^{\epsilon-3} \sim e^{-(\epsilon-3)\mathcal{N}/(\epsilon-1)}$$

$$\left|\frac{\partial_b S_E^R}{\partial_b S_E^I}\right| \sim \lambda^{\frac{\epsilon-3}{\epsilon}} \sim b^{\epsilon-3} \sim e^{-(\epsilon-3)\mathcal{N}/(\epsilon-1)}$$

Numerical verification

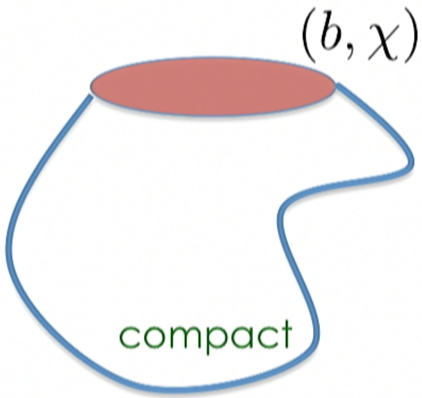
- We can verify these simple scaling laws numerically, for instance by starting the universe in the no-boundary quantum state



The No-Boundary Proposal

$$\Psi(b,\chi) = \int_{\mathcal{C}} \mathcal{D}a\mathcal{D}\phi e^{-S_E(a,\phi)}$$

$$\approx e^{-S_{E,ext}(b,\chi)}$$



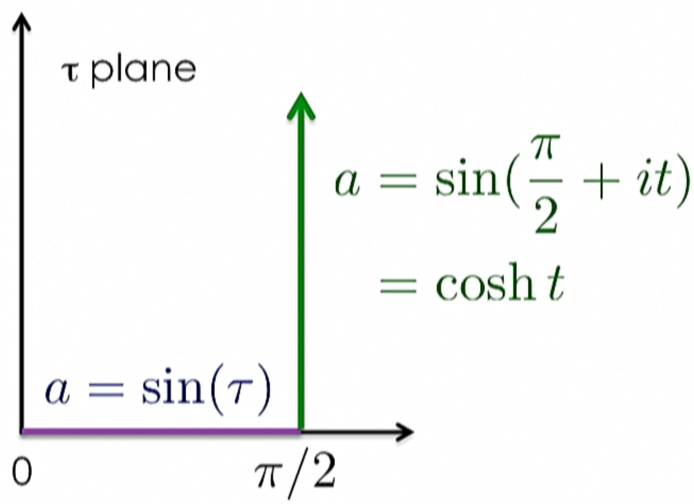
- The wavefunction is given by a path integral over all possible four-geometries that are compact in the past (i.e. the possible paths are *restricted*)
- Hartle-Hawking b.c.: the universe is *finite and self-contained*
- Saddle point approximation: the geometries that are an extremum of the action with the required boundary conditions are typically *complex*

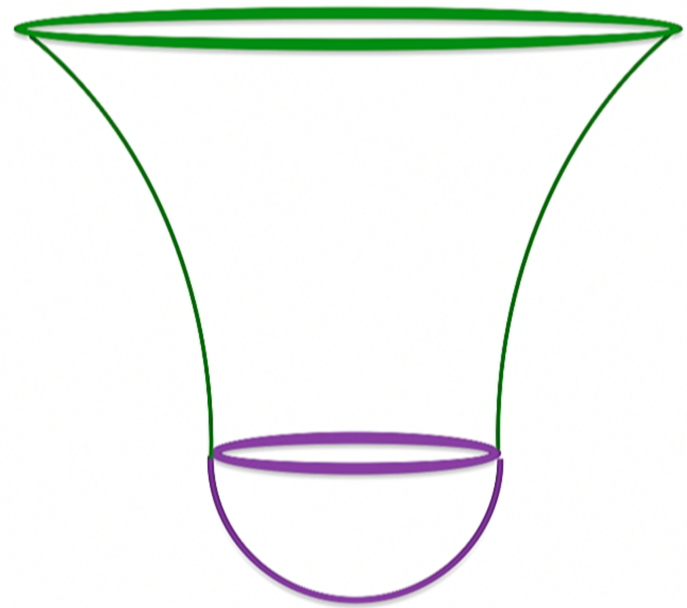
[Hartle & Hawking]

Hawking's Prototype Instanton: Pure dS

- Here there is no scalar field, only a pure de Sitter space with cosmological constant $\Lambda = 3\,H^2$
- Probability

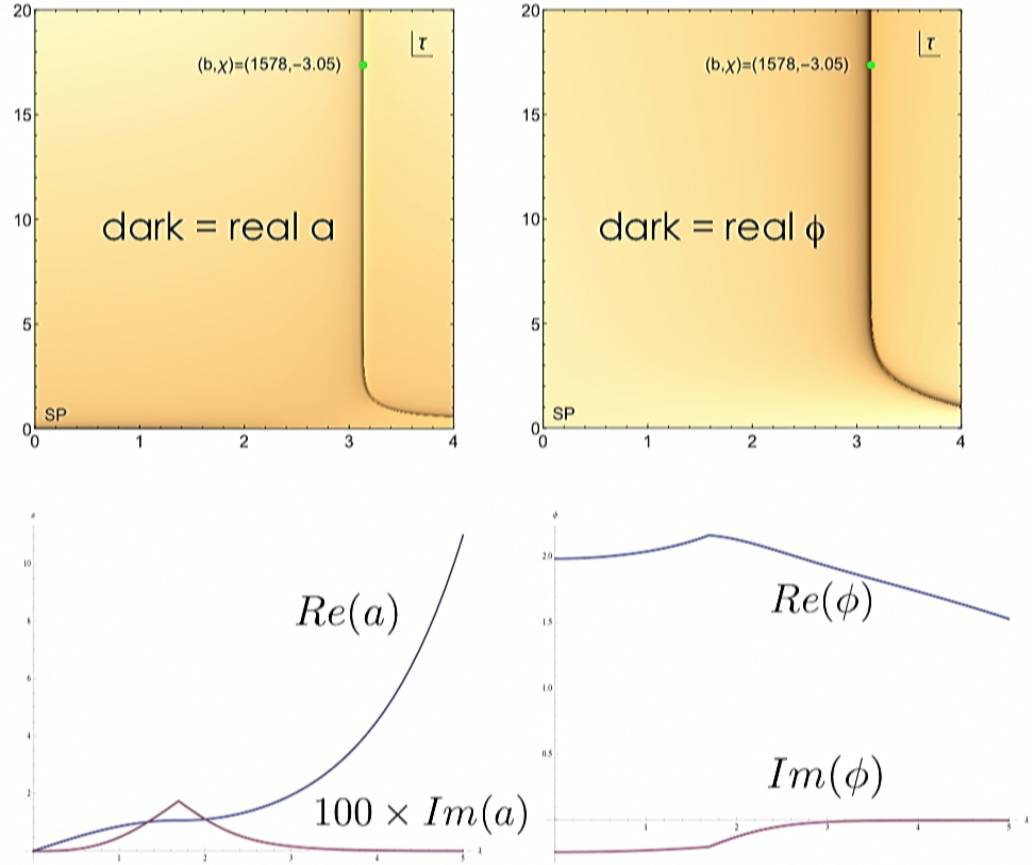
$$e^{-2Re(S)} = e^{\frac{24\pi^2}{H^2}}$$





Example of Inflationary Instanton

- This is the complex generalisation of Hawking's pure de Sitter instanton
- *Classicality* is reached along a coincident vertical line of real field values

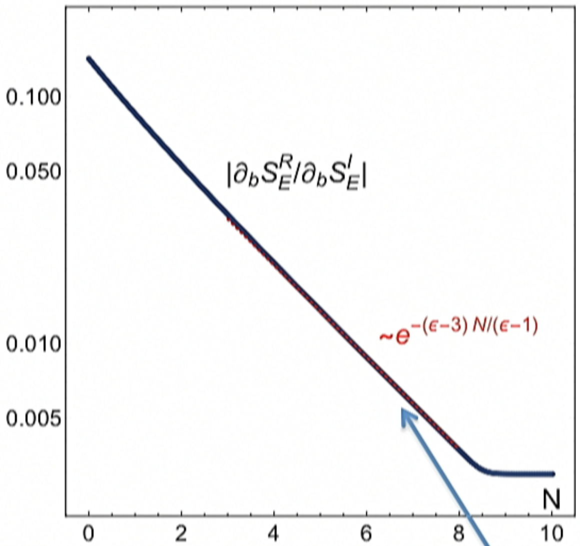


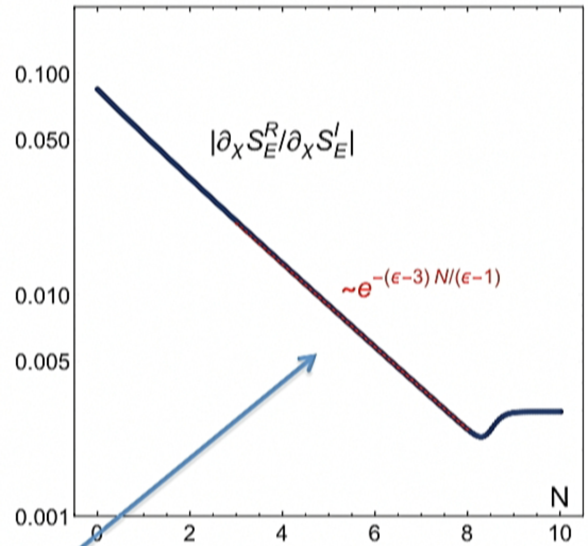
[Hartle, Hertog & Hawking; JLL]

WKB Classicality - Ekpyrosis

- In this case also, the wavefunction becomes increasingly classical in a WKB sense

$$|\partial_b S_E^R \,/\, \partial_b S_E^I| \ll 1, \qquad |\partial_\chi S_E^R \,/\, \partial_\chi S_E^I| \ll 1$$





$e^{-(\varepsilon-3)N/(\varepsilon-1)}$

$\sim e^{-N}$

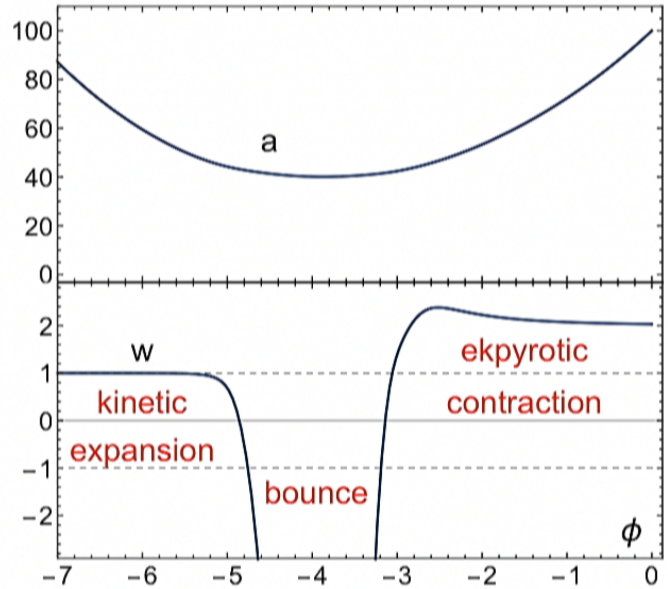
[Battarra & JLL]

Ekpyrotic instantons – with a bounce

- Extend the theory to include a ghost condensate after the ekpyrotic phase

$$S = \int \sqrt{-g} \left[\frac{R}{2} + P(X, \phi) \right]$$

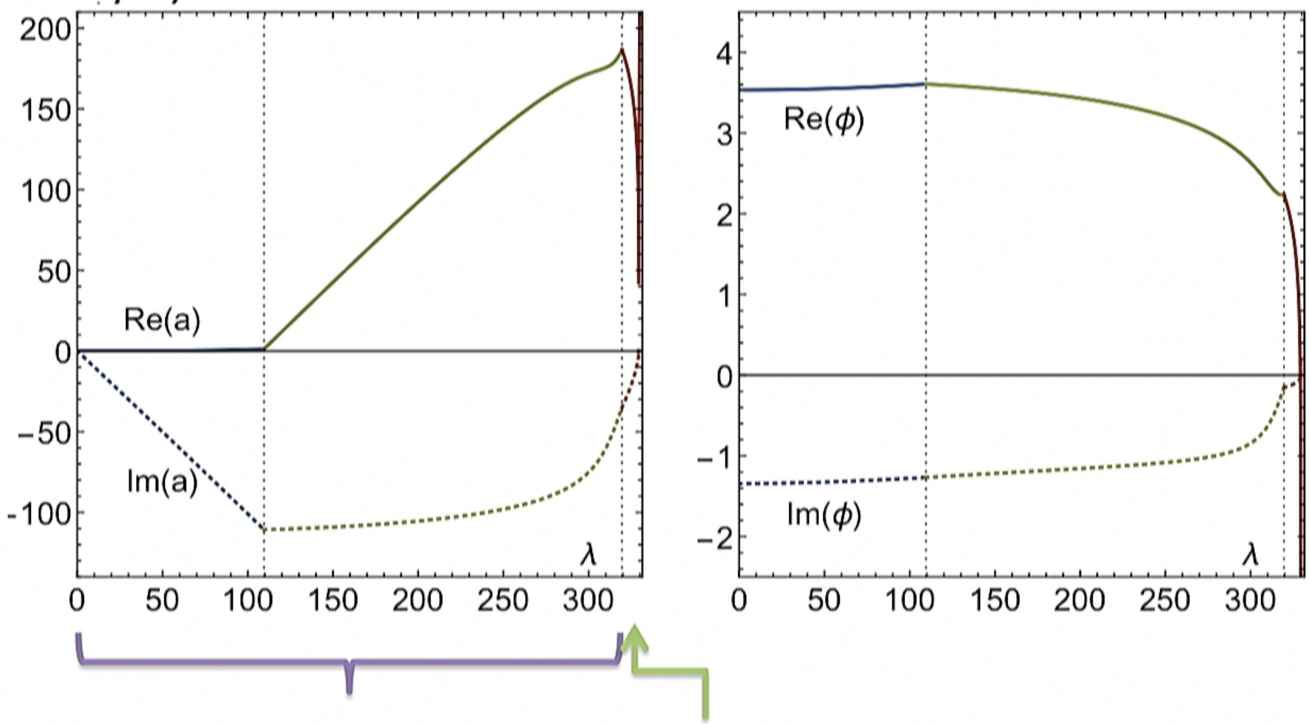
$$P(X, \phi) = k(\phi) X + q(\phi) X^2 - V(\phi).$$



- Classical solution:
(time runs from right to left)
- Now try to evaluate the wavefunction $\Psi(b, \chi)$ with arguments evolving along this classical solution

Ekpyrotic instantons – with a bounce

- Here is a typical instanton shape: ("time" from left to right)



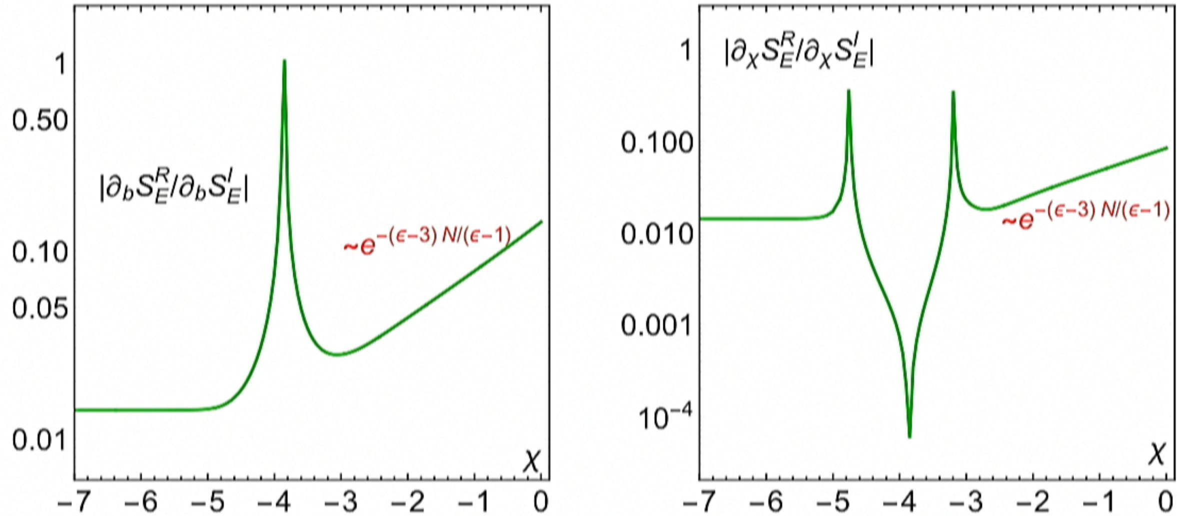
Long complex phase

Ekpyrotic phase & bounce

[JLL]

WKB Classicality of the Wavefunction

("time" from right to left)



- After the bounce the universe is even slightly more classical than before

[JLL]

WKB Scaling

$$WKB \propto e^{-\frac{\epsilon-3}{\epsilon-1}\mathcal{N}}$$
$$\propto a^{\epsilon-3}$$

- Implications depend on what is varied and what is held fixed:
  – For a fixed number of e-folds, prefers a strong ekpyrotic phase and a weak inflationary phase
  – For a fixed amount of expansion/contraction, prefers strong inflationary and strong ekpyrotic phases

Summary

- Smoothness is achieved in proportion to

$$\frac{1}{(aH)^2} = e^{-2\mathcal{N}}$$

as long as $\epsilon < 1$ or $\epsilon > 3$

- Classicality is achieved as the WKB factor becomes small

$$e^{-\frac{\epsilon-3}{\epsilon-1}\mathcal{N}}$$

which again requires $\epsilon < 1$ or $\epsilon > 3$

Open questions

- Are there any other ways of obtaining smoothness or classicality?
- Can there exist interferences between the different branches/universes before decoherence sets in?
- What happens when anisotropies are included?
- What was the initial quantum state? Was it the no-boundary state?