Title: Gluing in factorization homology via quantum Hamiltonian reduction

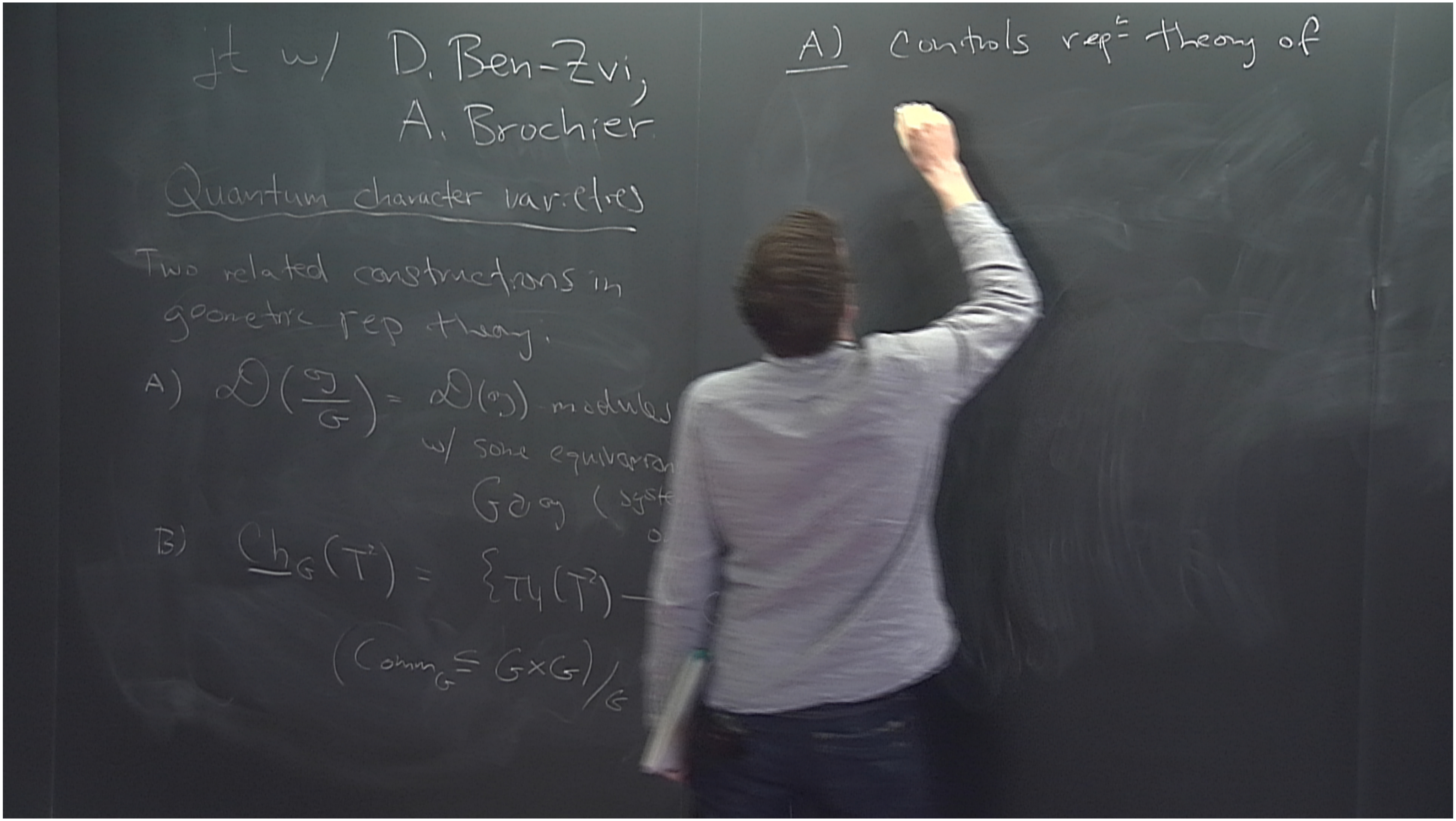
Date: May 26, 2016 03:00 PM

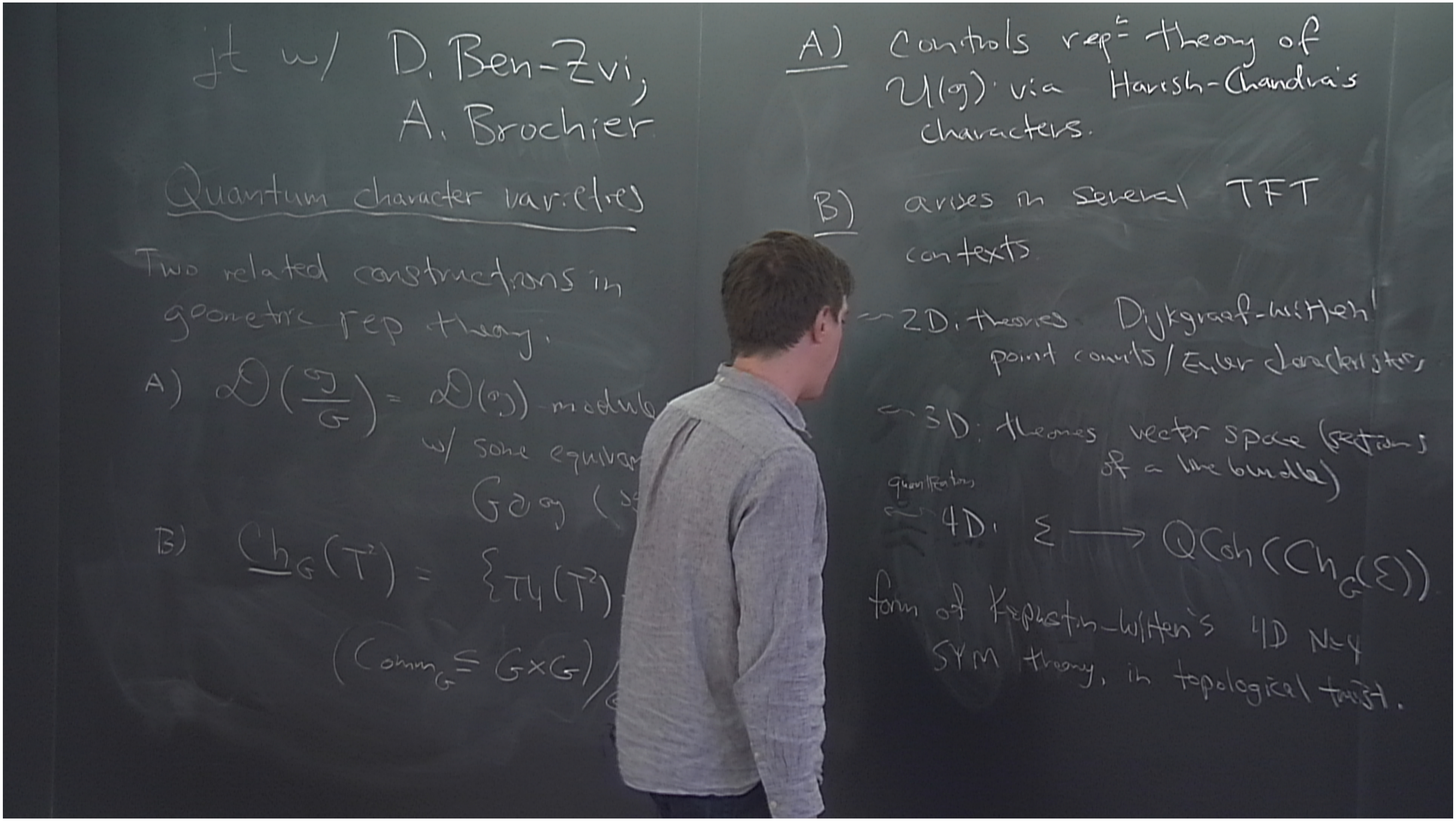
URL: http://pirsa.org/16050052

Abstract: <p>Topological factorization homology is an invariant of manifolds which enjoys a hybrid of the structures in topological field theory, and in singular homology. These invariants are especially interesting when we restrict attention to the factorization homology of surfaces, with coefficients in braided tensor categories. In this talk, I would like to explain a technique, related to Beck monadicity, which allows us to compute these abstractly defined categories, as modules for explicitly computable, and in many cases well-known, algebras.</p>

<p> </p>

<p>The algebras produced in this way for the annulus and punctured torus are the so-called ``reflection equation algebra" and "quantum differential equation algebra", respectively. When we close up punctures, a variation on the our formalism naturally reproduces the framework of quantum Hamiltonian reduction, and leads to simultaneous deformations of categories D(g/G) of character sheaves on g, on the one hand, and categories QC(Ch_G(T^2)), of quasi-coherent sheaves on the character variety of T^2, on the other. We call these deformations "quantum character varieties", and they form the two-dimensional part of a four dimensional TFT related to Kapustin-Witten's geometric Langlands TFT, or "topologically twisted N=4 SYM".</p>


jt w/ D. Ben-Zvi,
A. Brochier
Quantum character varieties
Two related constructions in
geometric rep theory:
A) $\mathcal{D}\left(\frac{\mathfrak{g}}{G}\right) = \mathcal{D}(\mathfrak{g})$
w/ some
$G \circlearrowright \mathfrak{g}$
B) $\underline{Ch}_G(T^2) =$
$(\mathrm{Comm}_G \subseteq G \times G)/G$
A) Controls rep theory of


jt w/ D. Ben-Zvi,
A. Brochier
Quantum character varieties
Two related constructions in
geometric rep theory.
A) D(g/G) = D(g)-modules
w/ some equivar
G ⊙ g
B) Ch_G(T²) = {Tr(T²)
(Comm_G ⊆ G×G)
A) Controls rep theory of
U(g) via Harish-Chandra's
characters.
B) arises in several TFT
contexts.
2D theories Dijkgraaf-Witten
point counts / Euler characteristics
3D theories vector space (sections
of a line bundle)
quantization
4D Σ ⟶ QCoh(Ch_G(Σ))
form of Kapustin-Witten's 4D N=4
SYM theory, in topological twist.

In fact, the two questions have a common answer!

a's

T

itten!

aracteristics,

e (sections

dle)

$Ch_G(\mathcal{E}))$

N=4

al twist.

theory of
Harish-Chandra's

general TFT

Dijkgraaf-Witten
...nts / Euler characteristics

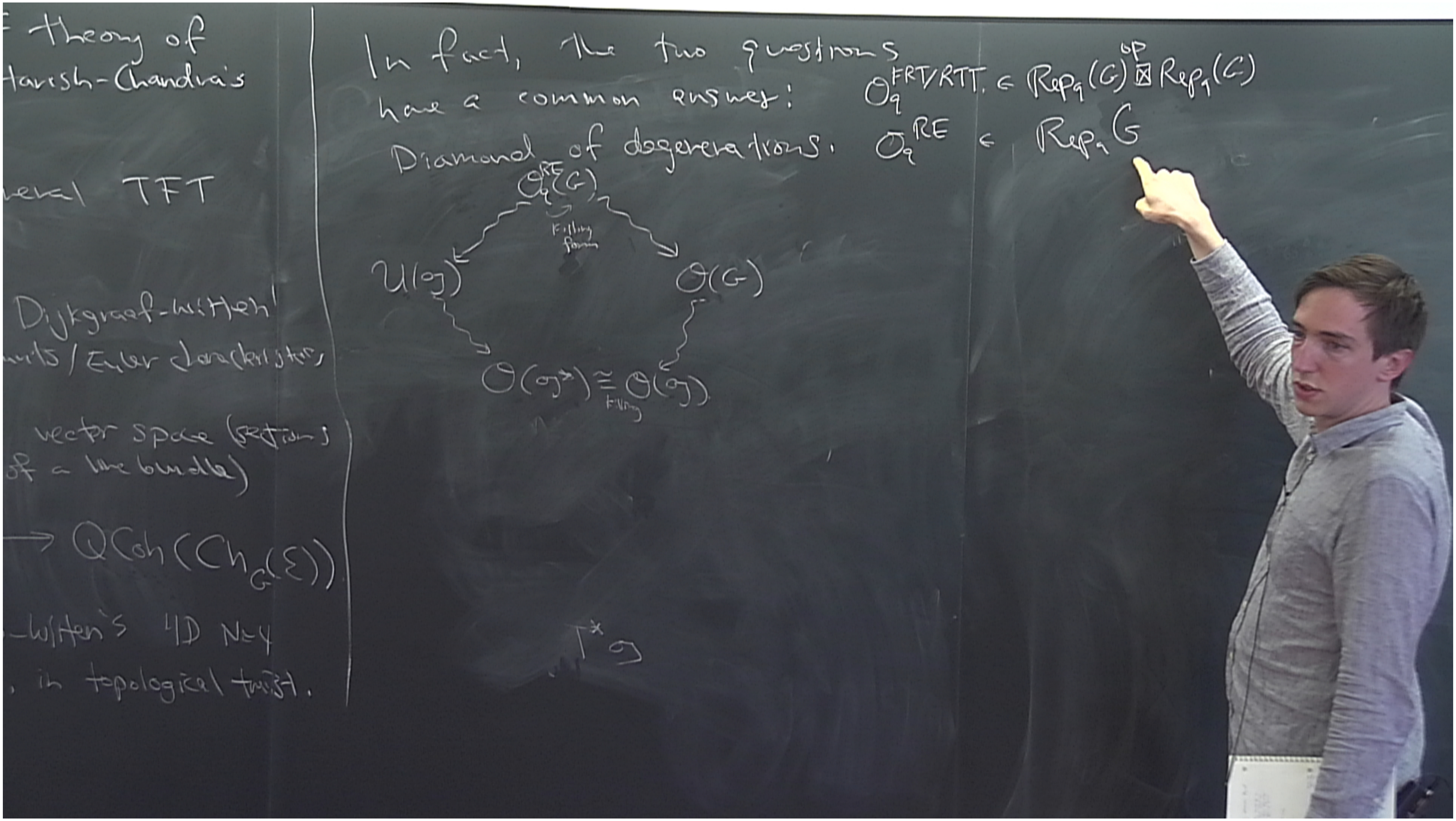
vector space (sections
of a line bundle)

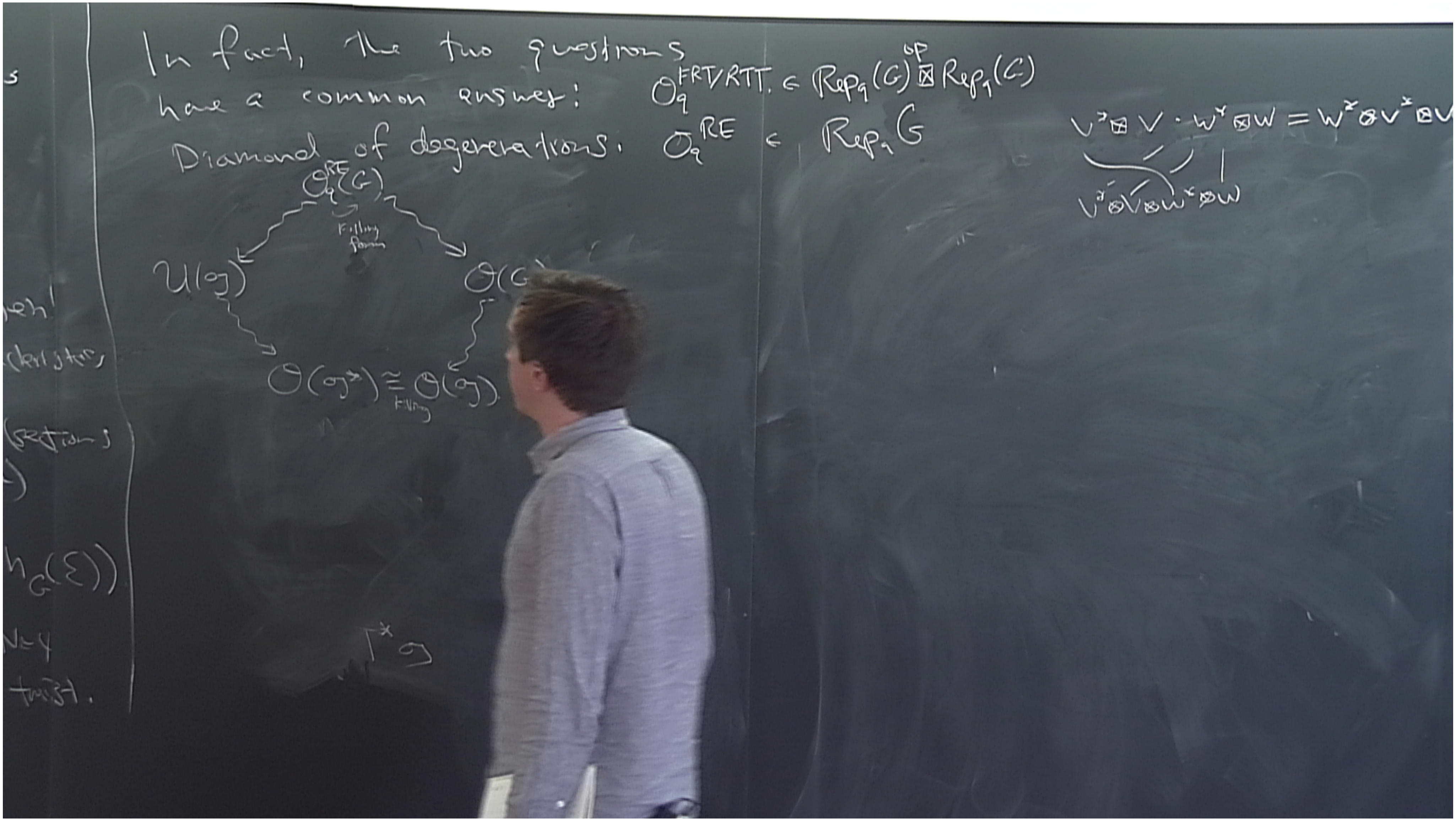
$\Longrightarrow QCoh(Ch_G(\Sigma))$

-Witten's 4D N=4
, in topological twist.

In fact, the two questions
have a common answer:
Diamond of degenerations.

$$\begin{tikzcd}
 & \mathcal{O}_q^{RE}(G) \arrow[dl, rightsquigarrow] \arrow[dr, rightsquigarrow] & \\
\mathcal{U}(\mathfrak{g}) \arrow[dr, rightsquigarrow] & & \mathcal{O}(G) \arrow[dl, rightsquigarrow] \\
 & \mathcal{O}(\mathfrak{g}^*) \underset{\text{Killing}}{\cong} \mathcal{O}(\mathfrak{g}). &
\end{tikzcd}$$


theory of
Harish-Chandra's
eral TFT
Dijkgraaf-Witten
its / Euler characteristics
vector space (sections
of a line bundle)
$\to QCoh(Ch_G(\mathcal{E}))$
-Witten's 4D N=4
, in topological twist.
In fact, the two questions
have a common answer!
$\mathcal{O}_q^{FRT/RTT} \in Rep_q(G)^{op} \boxtimes Rep_q(G)$
Diamond of degenerations.
$\mathcal{O}_q^{RE} \in Rep_q G$
$\mathcal{O}_q^{RE}(G)$
Filling form
$\mathcal{U}(\mathfrak{g})$
$\mathcal{O}(G)$
$\mathcal{O}(\mathfrak{g}^*) \cong \mathcal{O}(\mathfrak{g})$
Filling
$T^*\mathfrak{g}$


In fact, the two questions
have a common answer!
$\mathcal{O}_q^{FRT/RTT} \in \mathrm{Rep}_q(G)^{op} \boxtimes \mathrm{Rep}_q(G)$
Diamond of degenerations:
$\mathcal{O}_q^{RE} \in \mathrm{Rep}_q G$
$\mathcal{O}_q^{RE}(G)$
$U(\mathfrak{g})$
$\mathcal{O}(G)$
$\mathcal{O}(\mathfrak{g}^*) \cong \mathcal{O}(\mathfrak{g})$
$V^* \boxtimes V \cdot W^* \boxtimes W = W^* \otimes V^* \otimes V$
$V^* \otimes V \otimes W^* \otimes W$
$T^*\mathfrak{g}$

In fact, the two questions have a common answer!

$\mathcal{O}_q^{FRT/RTT} \in \mathrm{Rep}_q(G)^{op} \boxtimes \mathrm{Rep}_q(G)$

Diamond of degenerations: $\mathcal{O}_q^{RE} \in \mathrm{Rep}_q G$

$$\begin{tikzcd}
 & \mathcal{O}_q^{RE}(G) \arrow[dl, squiggly] \arrow[dr, squiggly] \arrow[d, phantom, "\text{Killing form}"] & \\
U(\mathfrak{g}) \arrow[dr, squiggly] & & \mathcal{O}(G) \arrow[dl, squiggly] \\
 & \mathcal{O}(\mathfrak{g}^*) \underset{\text{Killing}}{\cong} \mathcal{O}(\mathfrak{g}) &
\end{tikzcd}$$

$V^* \boxtimes V \cdot W^* \boxtimes W = W^* \otimes V^* \boxtimes V \otimes W$

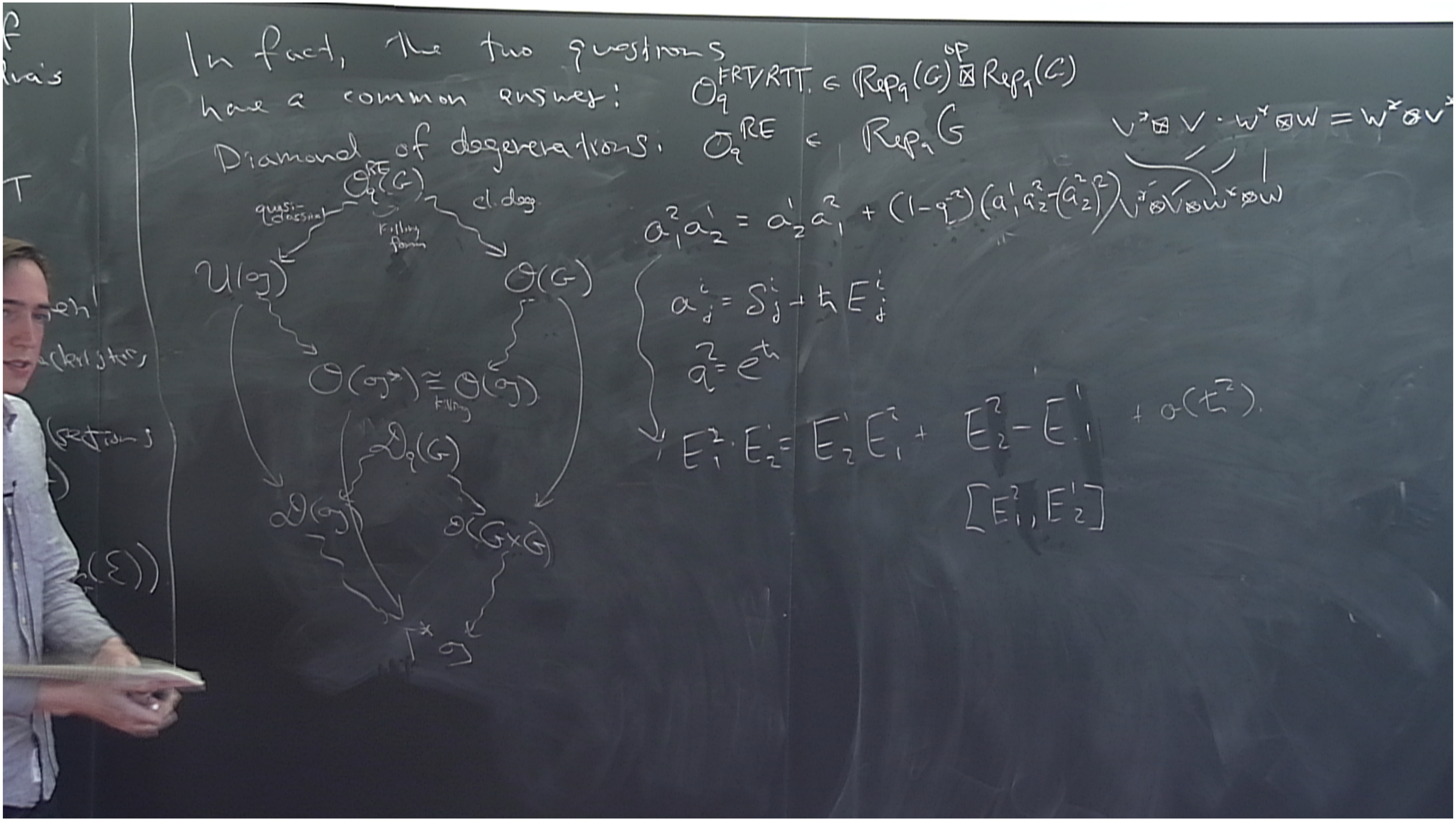
$$a^2_1 a^1_2 = a^1_2 a^2_1 + (1-q^{-2})(a^1_1 a^2_2 - (a^2_2)^2) \quad V^* \otimes V \otimes W^* \otimes W$$

$$\begin{cases} a^i_j = \delta^i_j + \hbar E^i_j \\ q^2 = e^{\hbar} \end{cases}$$

$$E^2_1 \cdot E^1_2 = E^1_2 E^2_1 + E^2_2 - E^1_1 + o(\hbar^2).$$

$[E^2_1, E^1_2]$

$T^*\mathfrak{g}$


In fact, the two questions
have a common answer!
$\mathcal{O}_q^{FRT/RTT} \in \mathrm{Rep}_q(G)^{op} \boxtimes \mathrm{Rep}_q(G)$
Diamond of degenerations:
$\mathcal{O}_q^{RE} \in \mathrm{Rep}_q G$
$V^* \boxtimes V \cdot W^* \boxtimes W = W^* \otimes V^*$
$\mathcal{O}_q^{RE}(G)$
quasi-classical
cl. deg
Filling form
$U(\mathfrak{g})$
$\mathcal{O}(G)$
$\mathcal{O}(\mathfrak{g}^*) \cong \mathcal{O}(\mathfrak{g})$
Filling
$\mathcal{D}_q(G)$
$\mathcal{D}(\mathfrak{g})$
$\mathcal{O}(G \times G)$
$T^*\mathfrak{g}$
$a_1^2 a_2^1 = a_2^1 a_1^2 + (1-q^{-2})(a_1^1 a_2^2 - (a_2^2)^2)$
$V^* \otimes V \otimes W^* \otimes W$
$a_j^i = \delta_j^i + \hbar E_j^i$
$q^2 = e^{\hbar}$
$E_1^2 \cdot E_2^1 = E_2^1 E_1^2 + E_2^2 - E_1^1 + o(\hbar^2)$
$[E_1^2, E_2^1]$

...ry of
Chandra's

TFT

...eaf-Witten!

...ler characters,

... space (sections
...e bundle)

$\mathrm{Coh}(\mathrm{Ch}_G(\mathcal{E}))$

...s 4D N=4

...pological twist.

In fact, the two questions have a common answer!

$\mathcal{O}_q^{FRT/RTT} \in \mathrm{Rep}_q(G)^{op} \boxtimes \mathrm{Rep}_q(G)$

Diamond of degenerations: $\mathcal{O}_q^{RE} \in \mathrm{Rep}_q G$

$$\begin{tikzcd}
 & \mathcal{O}_q^{RE}(G) \arrow[dl, "\text{quasi-classical}"'] \arrow[dr, "\text{cl. deg}"] \arrow[d, "\text{Filtering}"] & \\
U(\mathfrak{g}) \arrow[d] \arrow[dd, bend right] & \mathcal{O}(\mathfrak{g}^*) \cong \mathcal{O}(\mathfrak{g}) & \mathcal{O}(G) \arrow[l] \arrow[dd, bend left] \\
 & \mathcal{D}_q(G) \arrow[dl] \arrow[dr] & \\
\mathcal{D}(\mathfrak{g}) \arrow[dr] & & \mathcal{O}(G\times G) \arrow[dl] \\
 & \mathfrak{g} &
\end{tikzcd}$$

Filtering

$G\times G = \{\pi_1(T^2) \to G\}$

$G\times G \to G$

$(A,B) \mapsto (ABA^{-1}B^{-1})$.

$V^* \boxtimes V \cdot W^* \boxtimes W = V$

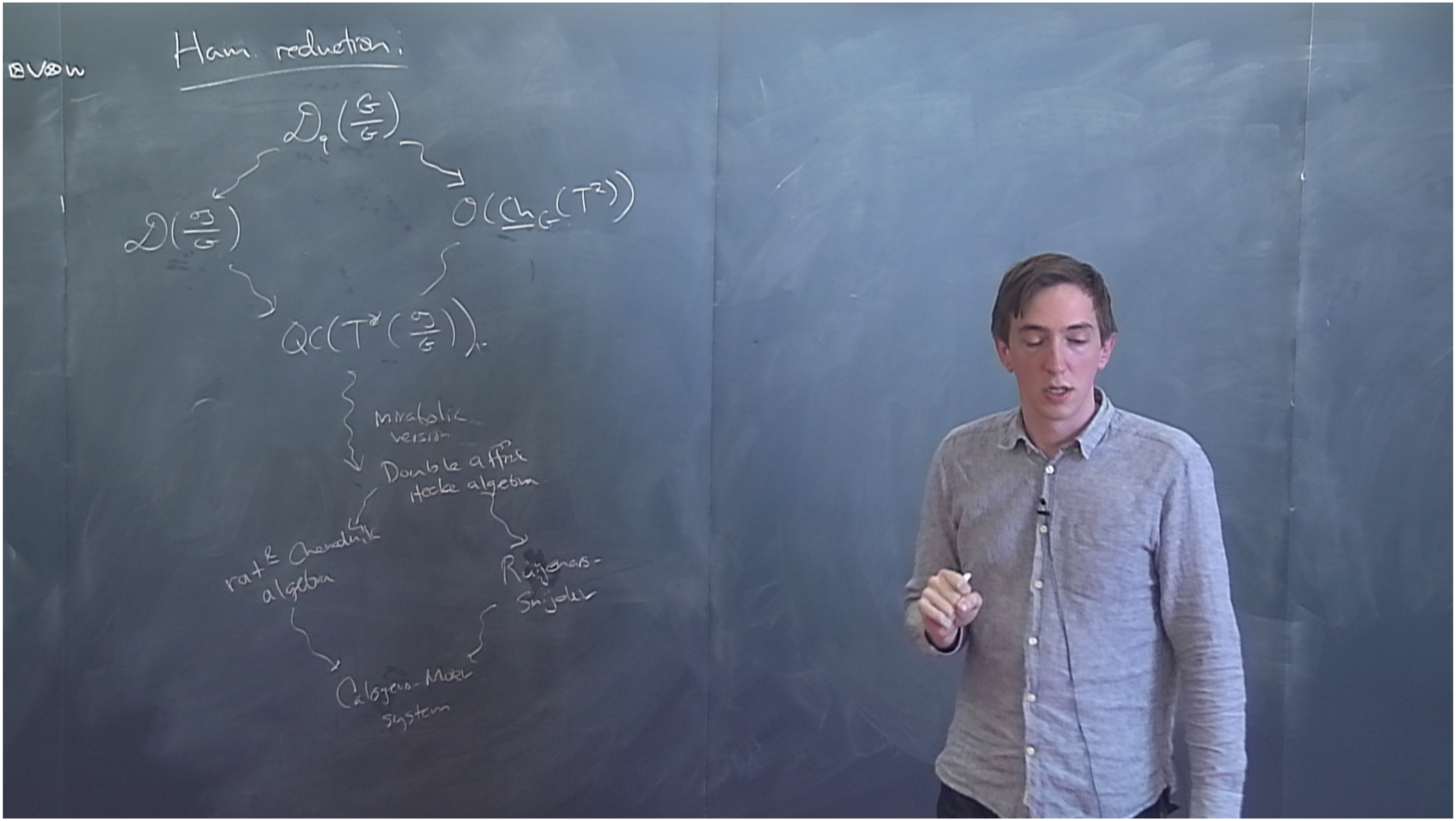
$a^2_1 a^1_2 = a^1_2 a^2_1 + (1-q^{-2})(a^1_1 a^2_2 - (a^2_2)^2)$ $V^* \otimes V \otimes W^* \otimes W$

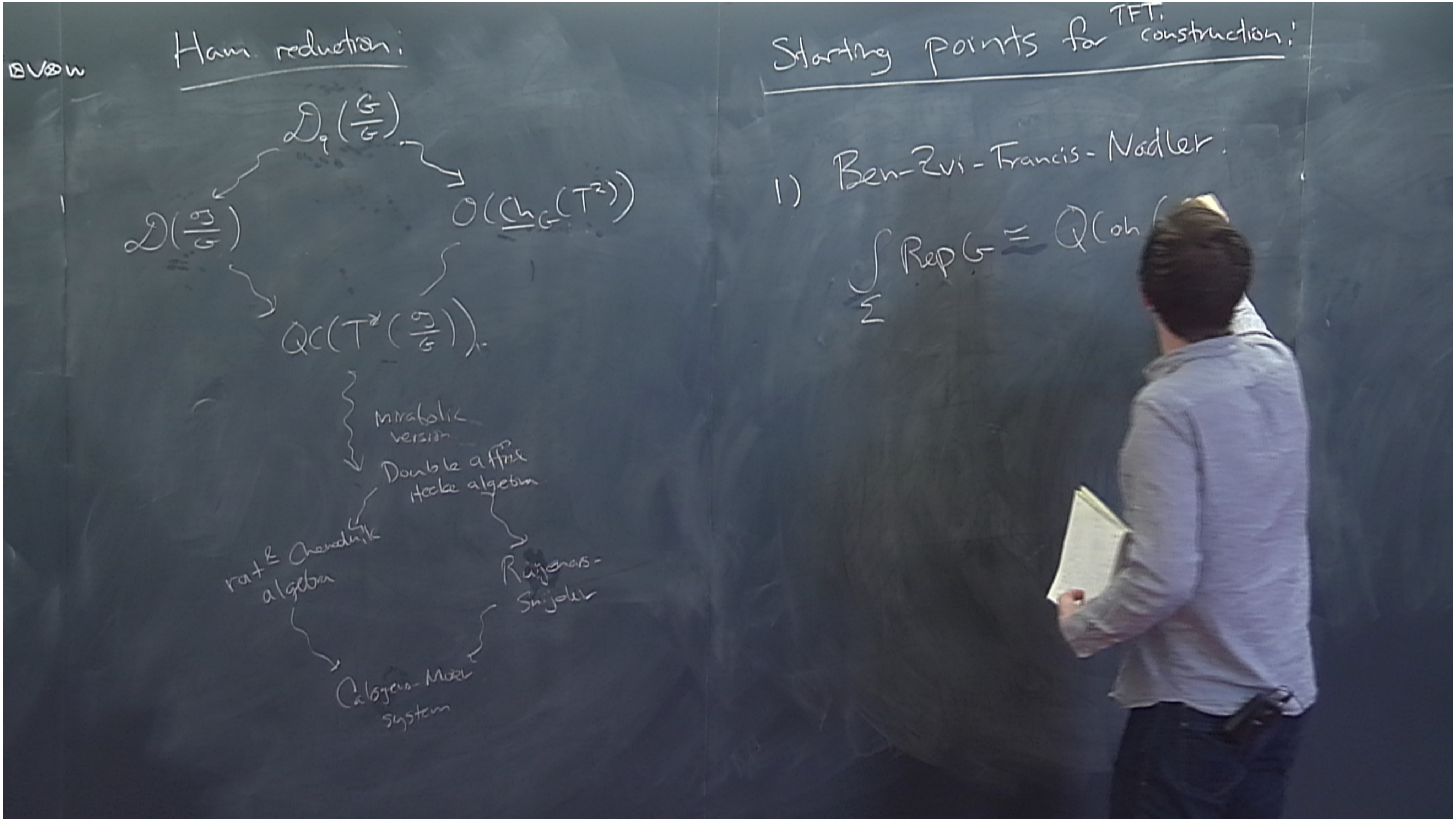
$a^i_j = \delta^i_j + \hbar E^i_j$

$q = e^{\hbar}$

$E^2_1 \cdot E^1_2 = E^1_2 E^2_1 + E^2_2 - E^1_1 + o(\hbar^2)$.

$[E^2_1, E^1_2]$


Ham. reduction:
$\mathcal{D}_q(\frac{G}{G})$
$\mathcal{D}(\frac{\mathfrak{g}}{G})$
$\mathcal{O}(\underline{Ch}_G(T^2))$
$QC(T^*(\frac{\mathfrak{g}}{G}))$
Mirabolic version
Double affine Hecke algebra
rat^{l} Cherednik algebra
Ruijsenaars-Schneider
Calogero-Moser system


Ham reduction:
$\mathcal{D}_q\left(\frac{G}{G}\right)$
$\mathcal{D}\left(\frac{\mathfrak{g}}{G}\right)$
$\mathcal{O}(\underline{Ch}_G(T^2))$
$QC\left(T^*\left(\frac{\mathfrak{g}}{G}\right)\right)$
Mirabolic version
Double affine Hecke algebra
rat'l Cherednik algebra
Ruijsenaars-Schneider
Calogero-Moser system
Starting points for TFT construction:
1) Ben-Zvi - Francis - Nadler:
$\int_\Sigma \mathrm{Rep}\, G \cong QCoh($

Starting points for TFT construction:

$T^2))$

1) Ben-Zvi - Francis - Nadler.

$$\int_\Sigma \operatorname{Rep} G \simeq QCoh(Ch_G(\Sigma))$$

$$B_n(\Sigma) = \pi_1(Conf_n(\Sigma)).$$

2) w/ A. Brochier:

$\mathcal{D}_q(G)$ is a universal source of rep^{ns}'s of the braidgroup of .

eduction:

$\mathcal{D}_q\left(\frac{G}{G}\right)$

$\rightsquigarrow \mathcal{O}(\underline{Ch}_G(T^2))$

$QC\left(T^*\left(\frac{\mathfrak{g}}{G}\right)\right)$.

mirabolic version

Double affine Hecke algebra

Cherednik algebra

Ruijsenaars-Snijder

Calogero-Moser system

Starting points for TFT construction:

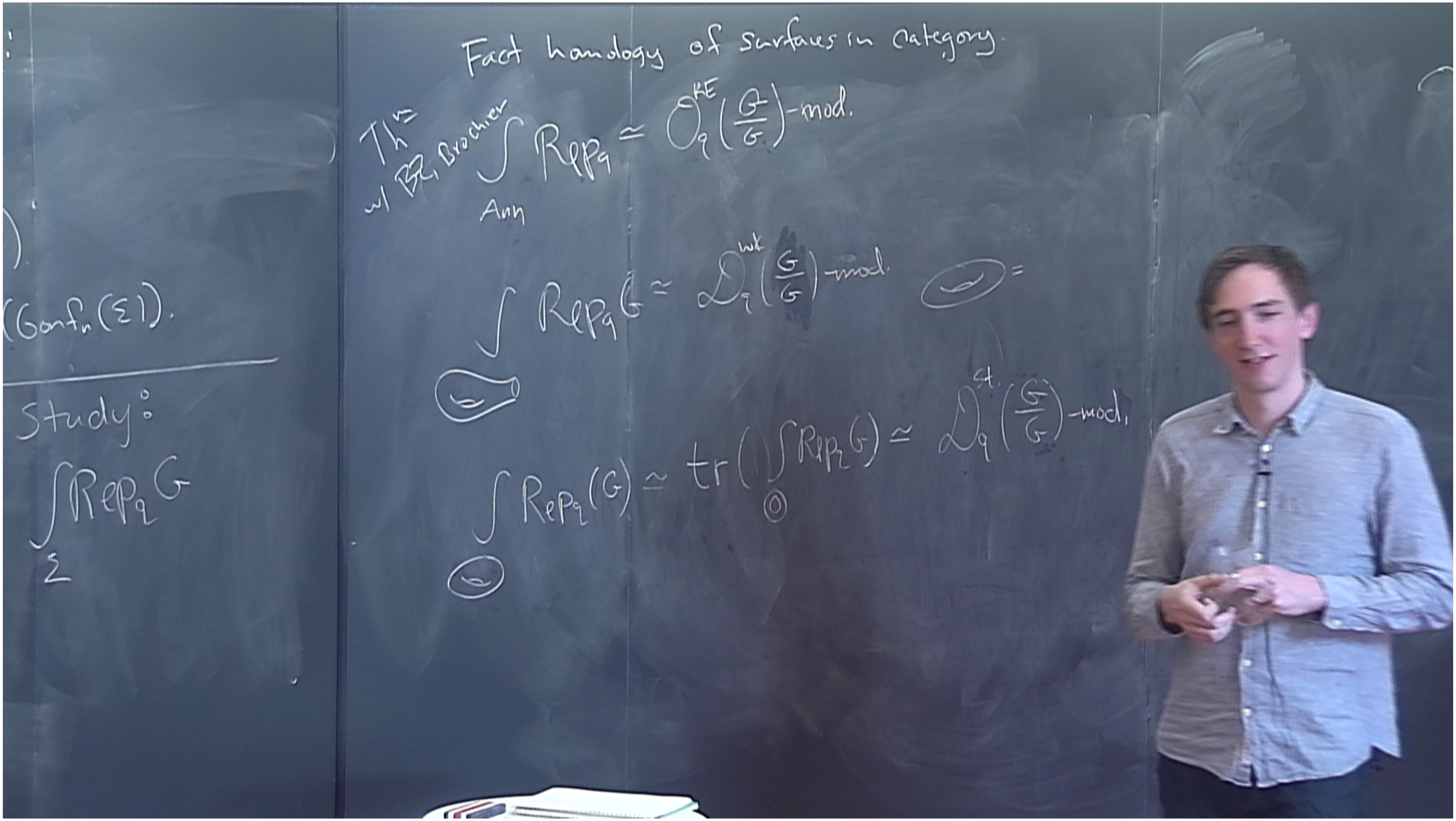
1) Ben-Zvi - Francis - Nadler.

$$\int_\Sigma \mathrm{Rep}\, G \simeq QCoh(Ch_G(\Sigma))$$

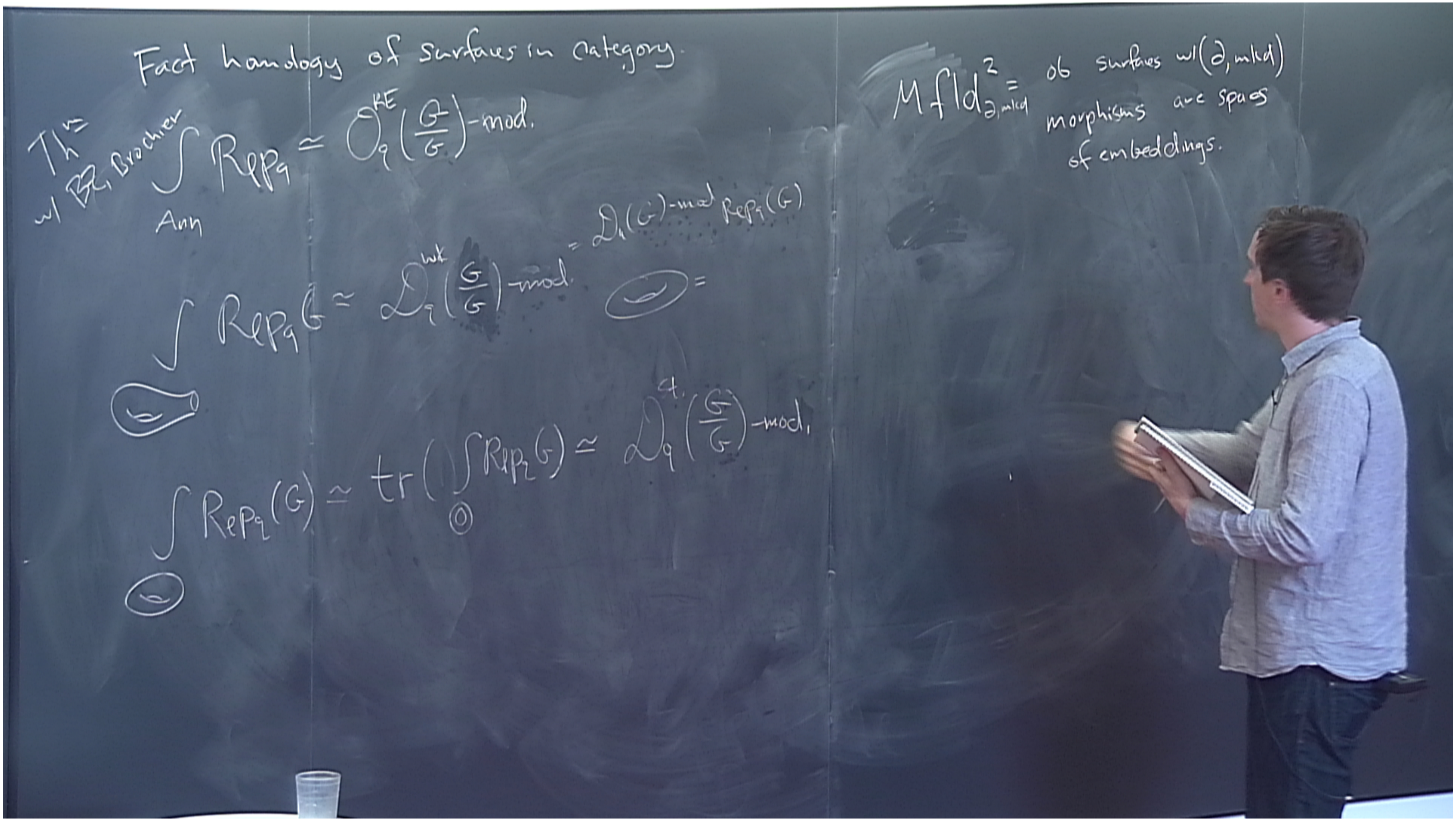
$$B_n(\Sigma) = \pi_1(Conf_n(\Sigma)).$$

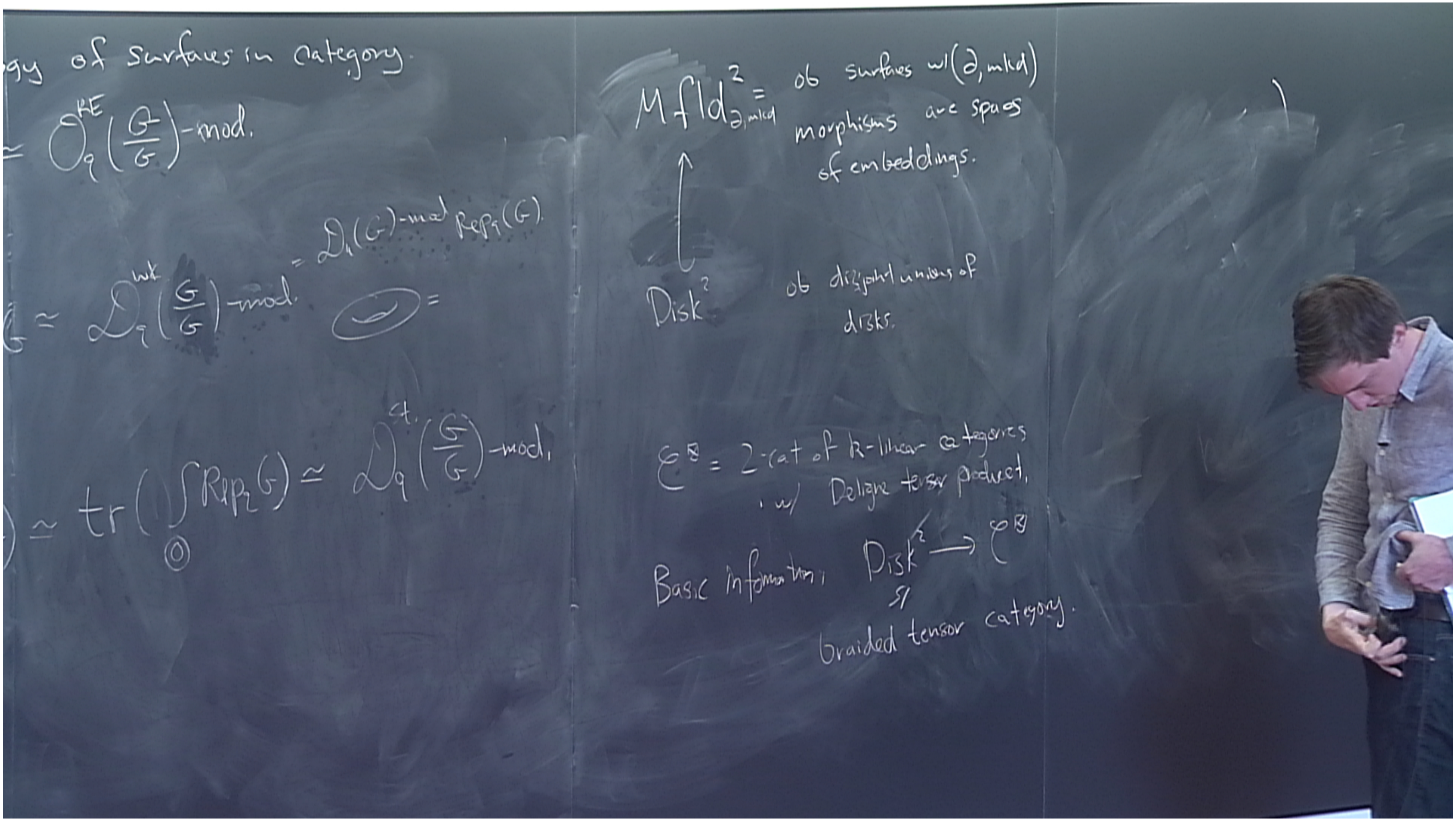
2) w/ A. Brochier:

$\mathcal{D}_q(G)$ is a universal source of rep^{ns} of the braid group of  .



Fact homology of surfaces in category.
Thm w/ BZ, Brochier
$\int_{Ann} \mathrm{Rep}_q \simeq \mathcal{O}_q^{LE}\left(\frac{G}{G}\right)$-mod.
$\int \mathrm{Rep}_q G \simeq \mathcal{D}_q^{wk}\left(\frac{G}{G}\right)$-mod.
$\int \mathrm{Rep}_q(G) \simeq \mathrm{tr}\left(\int \mathrm{Rep}_q G\right) \simeq \mathcal{D}_q^{st.}\left(\frac{G}{G}\right)$-mod.
$(\mathrm{Conf}_n(\Sigma))$.
Study:
$\int_\Sigma \mathrm{Rep}_q G$


Fact homology of surfaces in category.
Thm w/ BZ, Brochier
∫ Rep_q ≃ O_q^KE (G/G)-mod.
Ann
∫ Rep_q G ≃ D_q^wk (G/G)-mod. = D_q(G)-mod Rep_q(G)
∫ Rep_q(G) ≃ tr(∫ Rep_q G) ≃ D_q^st (G/G)-mod.
Mfld^2_∂,mkd = ob surfaces w/(∂, mkd)
morphisms are spaces
of embeddings.


gy of surfaces in category.
$\simeq \mathcal{O}_q^{KE}\left(\frac{G}{G}\right)$-mod.
$\mathcal{D}_q(G)$-mod $Rep_q(G)$
$G \simeq \mathcal{D}_q^{wk}\left(\frac{G}{G}\right)$-mod.
$\simeq tr\left(\int_{\circledcirc} Rep_q G\right) \simeq \mathcal{D}_q^{st}\left(\frac{G}{G}\right)$-mod.
$Mfld^2_{\partial, mkd}$ = ob surfaces w/(∂, mkd)
morphisms are spaces of embeddings.
$Disk^2$ ob disjoint unions of disks.
$\mathcal{C}^{\otimes}$ = 2-cat of k-linear categories w/ Deligne tensor product.
Basic information: $Disk^2 \longrightarrow \mathcal{C}^{\otimes}$
braided tensor category.

G).

nod,

$\mathrm{Mfld}^2_{\partial,\mathrm{mkd}}$ = ob surfaces w/ (∂, mkd)

morphisms are spaces of embeddings.

$\uparrow$

Disk^2 ob disjoint unions of disks.

$\mathcal{C}^{\otimes}$ = 2-cat of k-linear categories w/ Deligne tensor product.

Basic information: $\mathrm{Disk}^2 \longrightarrow \mathcal{C}^{\otimes}$

$\cong$

braided tensor category.

$\bigcirc \longmapsto A$

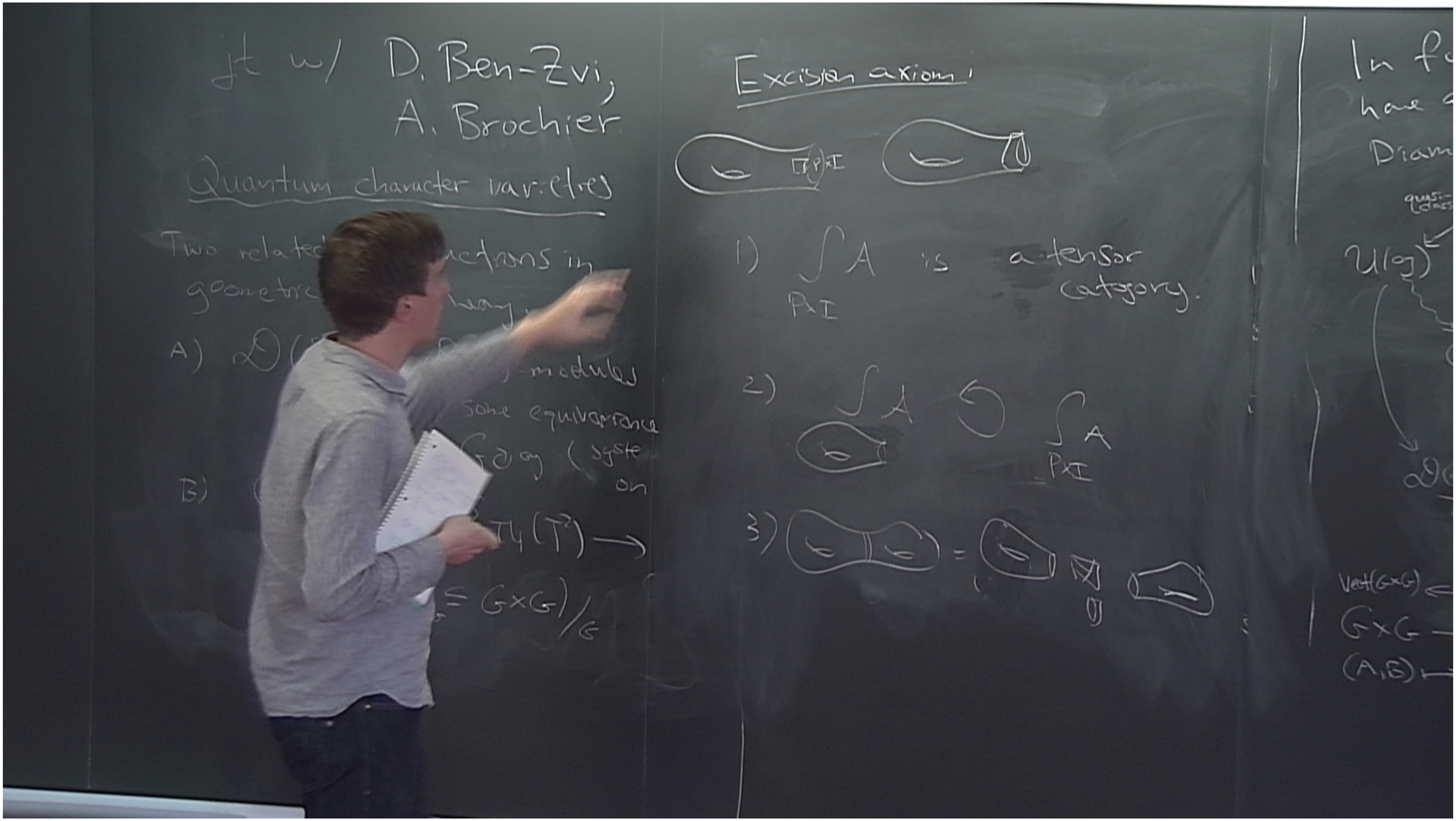
$$\begin{array}{ccc} \circ\ \circ & & A \boxtimes A \\ \downarrow & \longmapsto & \downarrow \\ \bigcirc & & A \end{array} \quad \text{associative}$$

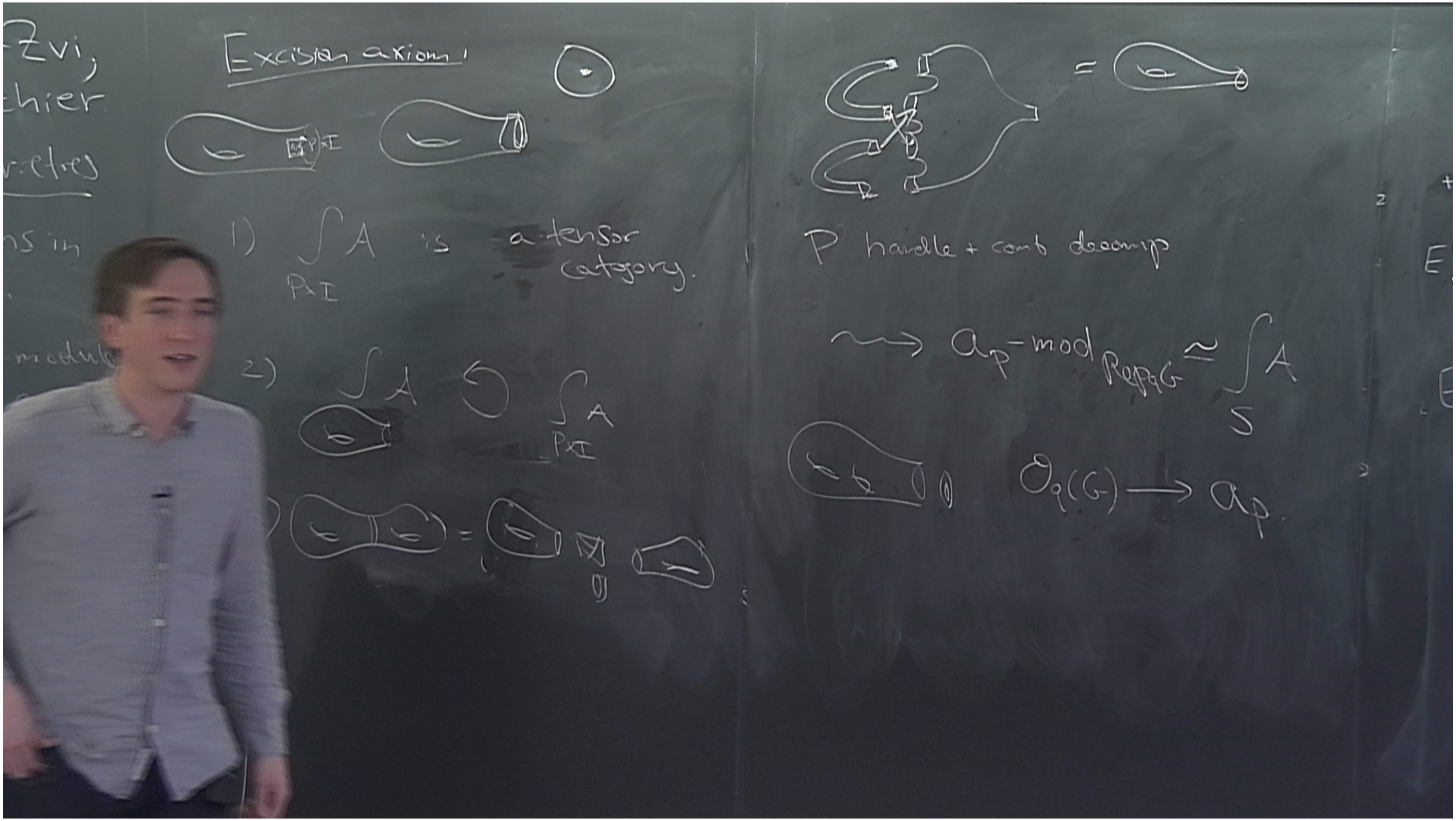
$\bigcirc \longmapsto$ br

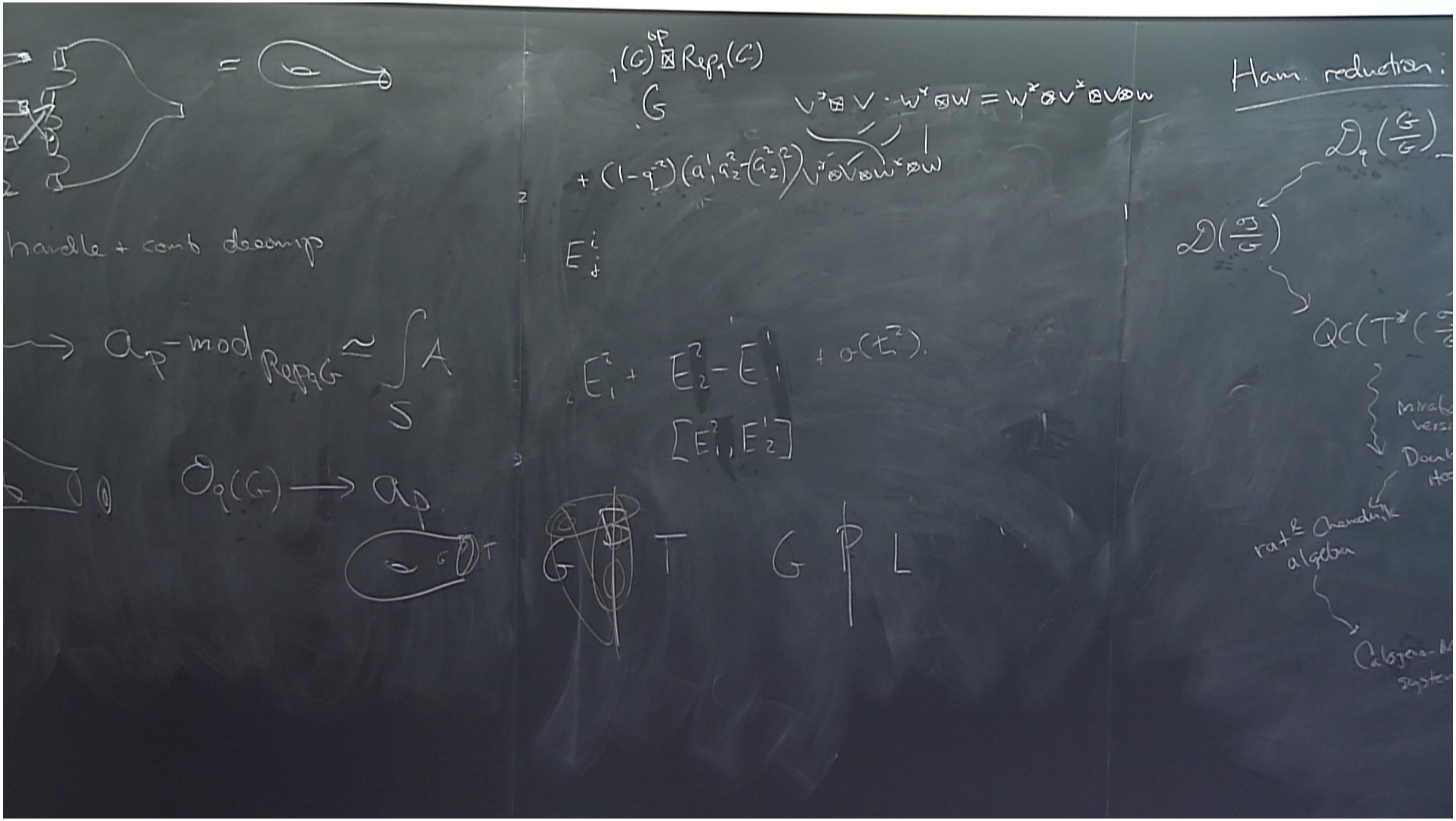
Def$^{\underline{n}}$ (Lurie, Ayala-Francis)

$$\begin{tikzcd} \mathrm{Disk}^2 \arrow[r, hook] \arrow[d, "A"'] & \mathrm{Mfld}^2 \arrow[dl, dashed] \\ \mathcal{C}^{\otimes} & \end{tikzcd}$$

$\int_{\Sigma} A$


jt w/ D. Ben-Zvi,
A. Brochier
Quantum character varieties
Excision axiom:
1) $\int_{P\times I} A$ is a tensor category.
2)
3)
In f
Diam
$\mathcal{U}(\mathfrak{g})$
Vect(G×G)
G×G
(A,B)


Excision axiom:
1) ∫ A is a tensor category.
P×I
2)
P×I
P handle + comb decomp
⇝ a_P-mod_{Rep_q G} ≃ ∫_S A
O_q(G) ⟶ a_P.


handle + comb decomp
$\mathcal{a}_p$-mod$_{\mathrm{Rep}_q G} \simeq \int_S A$
$\mathcal{O}_q(G) \longrightarrow \mathcal{a}_p$
$(G)^{op} \boxtimes \mathrm{Rep}_q(G)$
$V^* \boxtimes V \cdot W^* \boxtimes W = W^* \otimes V^* \boxtimes V \otimes W$
$+ (1-q^{-2})(a^1_1 a^2_2 - (a^2_2)^2) V^* \otimes V \otimes W^* \otimes W$
E^i_j
$E^2_1 + E^2_2 - E^1_1 + o(t^2)$
$[E^2_1, E^1_2]$
G T
G P L
Ham. reduction:
$\mathcal{D}_q(\frac{G}{G})$
$\mathcal{D}(\frac{\mathfrak{g}}{G})$
$QC(T^*(\frac{\mathfrak{g}}{G}$
rat^l Cherednik algebra
Calogero-M system