Title: Network Gravity: A New Framework for Emergent Geometry

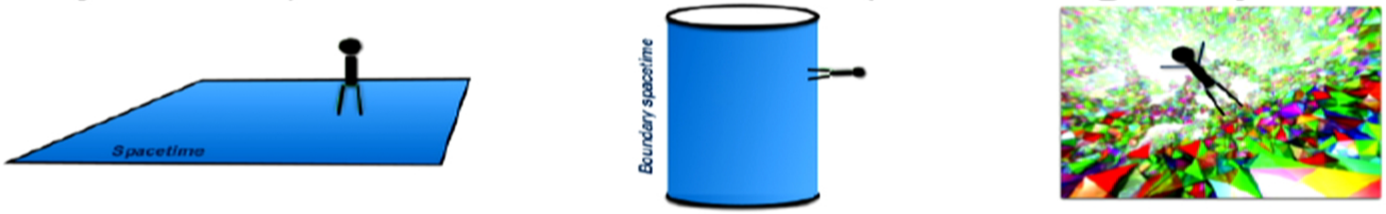
Date: May 19, 2016 02:30 PM

URL: http://pirsa.org/16050046

Abstract: <p>We introduce the construction of a new framework for probing discrete emergent geometry and boundary-boundary observables based on a fundamentally a-dimensional underlying network structure. Using a gravitationally motivated action with Forman weighted combinatorial curvatures and simplicial volumes relying on a decomposition of an abstract simplicial complex into realized embeddings of proper skeletons, we demonstrate model phenomenology such as a minimal volume-scale cutoff, a positive-definite cosmological constant as a regulator for non-degenerate geometries, and naturally emergent simplicial structures from Metropolis network evolution simulations with no restrictions on attachment rules or regular building blocks. Properties which echo results from both the spinfoam formalism and causal dynamical triangulations in quantum gravity will also be illustrated. We conclude with prospects for future investigations into the model, discussing ongoing simulations, modifications to the structure of the theory, and broader applications of the framework.</p>

Quantum Geometry

- May not expect manifolds at the quantum-gravity scale



Conventional quantum field theorist Post-Maldacena string theorist Genuine quantum-gravity physicist

- Strings and things perturb around a manifold topology
- We'd like a theory that builds foundationally without assuming such a structure
- Experimental consequences with geometric defects

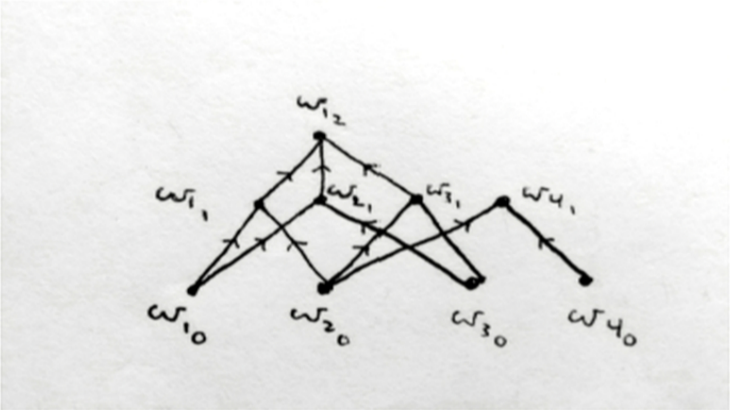
Rovelli and Vidotto, 2015

State Spaces

Combinatorial Space

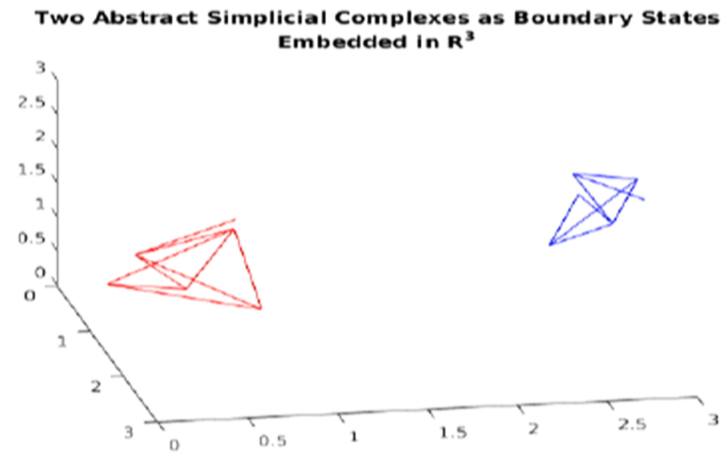
Given a state $\psi_m = \psi_m(\cup_{d=0}^{m}(K_d^*, \omega_{\alpha_d}))$:

$$Z_m = \int \mathscr{D}\psi \exp \sum_{d=0}^{m} -S(K_d^*, \omega_{\alpha_d})$$



Embedding

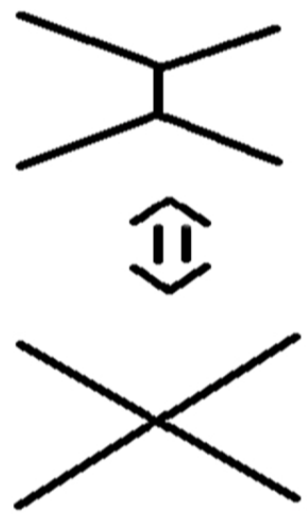
$$
\begin{aligned}
g &= g(V, E) \\
\chi_m(v \in V) &\mapsto p \in E^m \\
\chi_m((v_i, v_j) \in E) &\mapsto l_{[p_i, p_j]}
\end{aligned}
$$



- E^m embedding space, with investigations as $m \to \infty$
- Boundary states: embedded convex-hull disjoint graphs which are non-dynamical

Markov Process

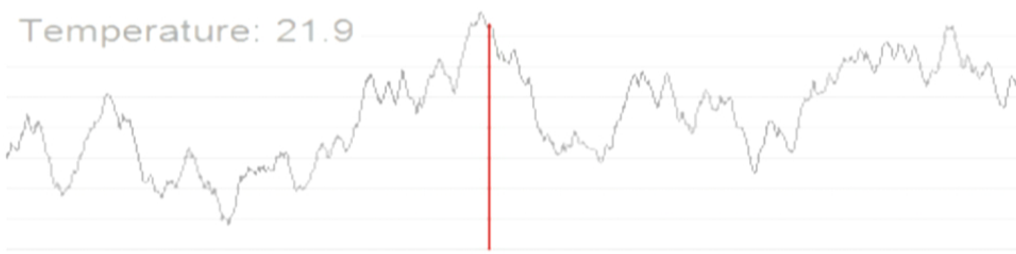
- Finite horizon Markov chain
- All discrete moves equally likely:
  - Nodal Displacement
  - Nodal/Edge splitting/recombining
  - Nodal/Edge addition/subtraction



Metropolis Annealing

Simulation temperature controls growth fluctuations

- Large initial fluctuations allowing for perturbations into a boundary-biased ‘random’ configuration
- The simulation will begin to settle on a local branch minimum as the temperate is lowered
- ‘Freeze-out sampling’ of minima of the state space



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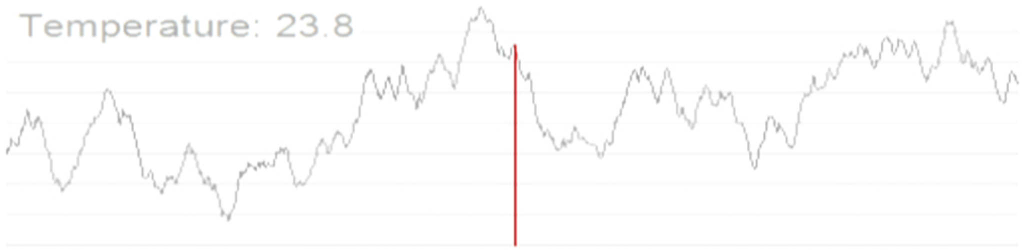


Discretizing Gravity

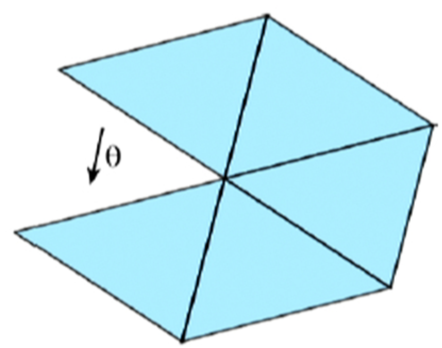
Einstein-Hilbert: Applicable to smooth manifolds

$$S_{EH} = \int_{M^n} \sqrt{-g}(R_n - \Lambda)$$

Regge: Applicable for a triangulated d-dimensional manifold

$$S_R = \sum_h V_h \theta_h + \Lambda \sum_\sigma V_\sigma$$

- d-simplexes indexed by σ
- $(d-2)$-hinges indexed by h
- simplicial volumes V
- deficit angles θ



Discretizing Gravity

Question

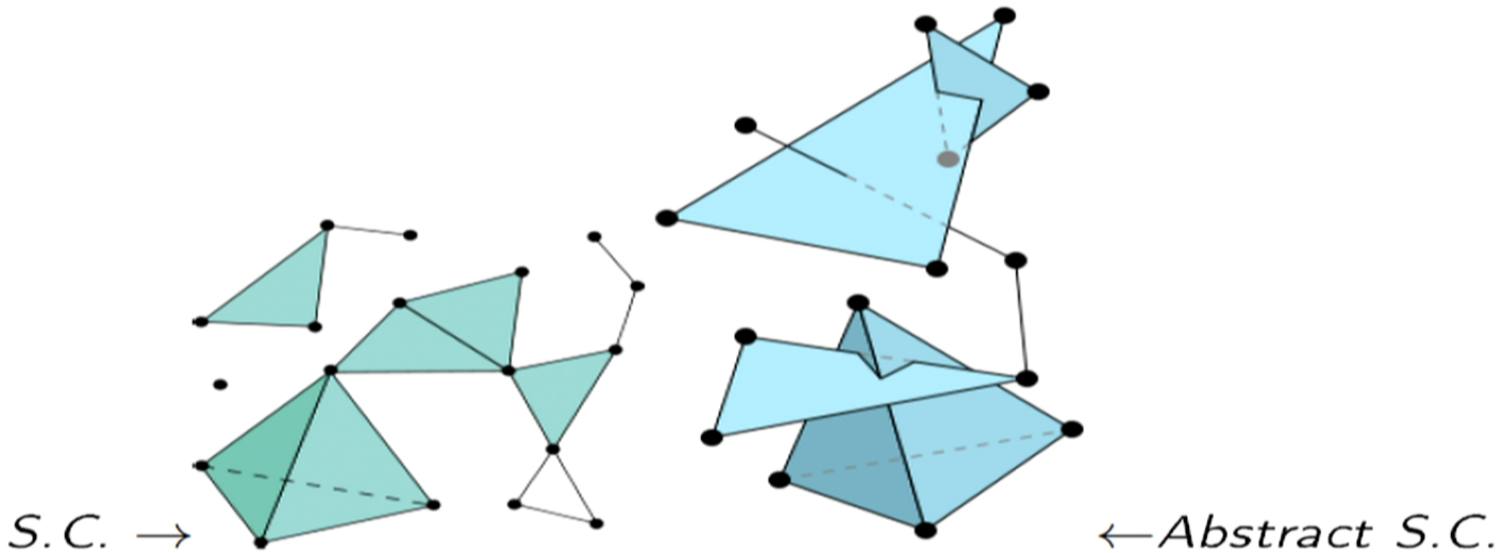
When we don't even have a triangulated manifold, how do we start to build geometry?

Proposition

Combinatorial analogues

1. State-space $\longrightarrow$ abstract simplicial complexes
2. Triangulations $\longrightarrow$ disjoint union of 'proper-pruned' skeletons
3. Volumes $\longrightarrow$ combinatorial weights from geometric embeddings
4. Curvatures $\longrightarrow$ Forman weighted combinatorial curvature

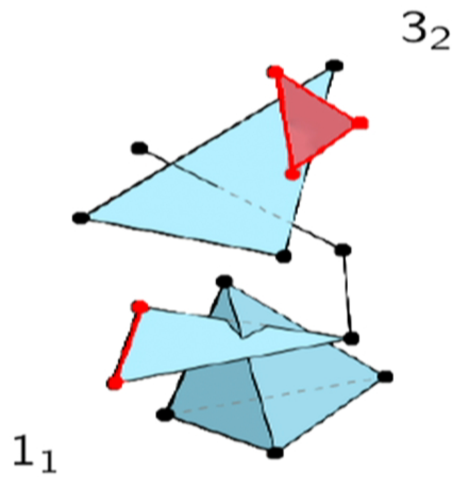
Abstract Simplicial Complexes



Substructures

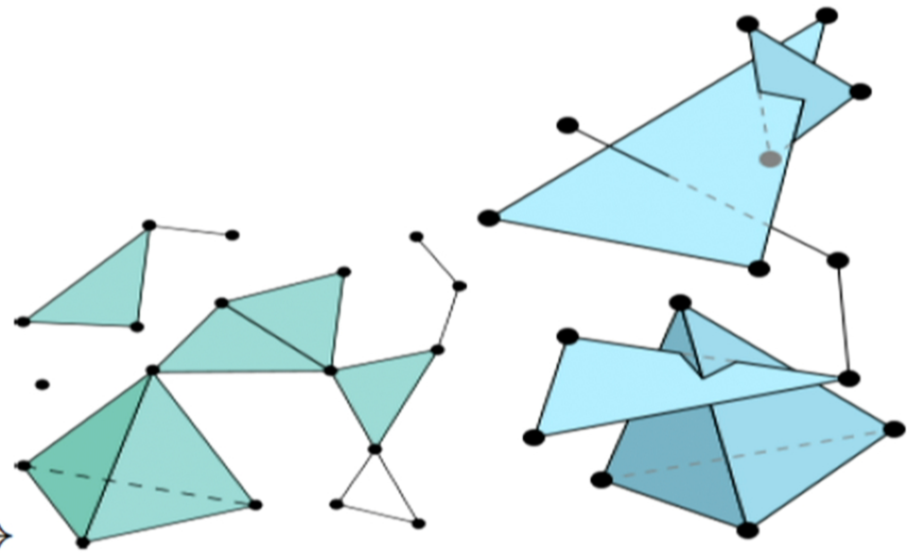
Definition

Complete subgraphs of order $d+1$ are denoted α_d, where α runs over the indexing set of all $(d+1)$-dimensional complete subgraphs in the state.



"Clique Complex"

Abstract Simplicial Complexes

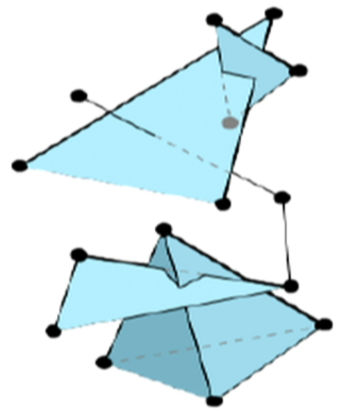


Substructures

Definition

We define K_d, the 'proper d-skeleton', to be the disjoint union of all the complete subgraphs at order d.

$$K_d = \sqcup_\alpha \alpha_d$$

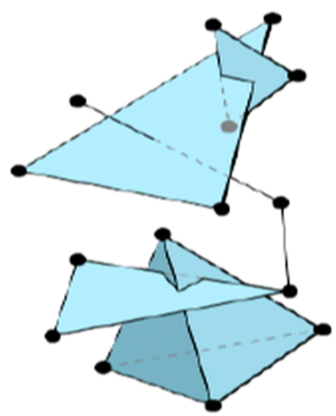


Realized Proper-Pruned Skeletons

It is not true that all α_d have geometric realizations in a given embedding.

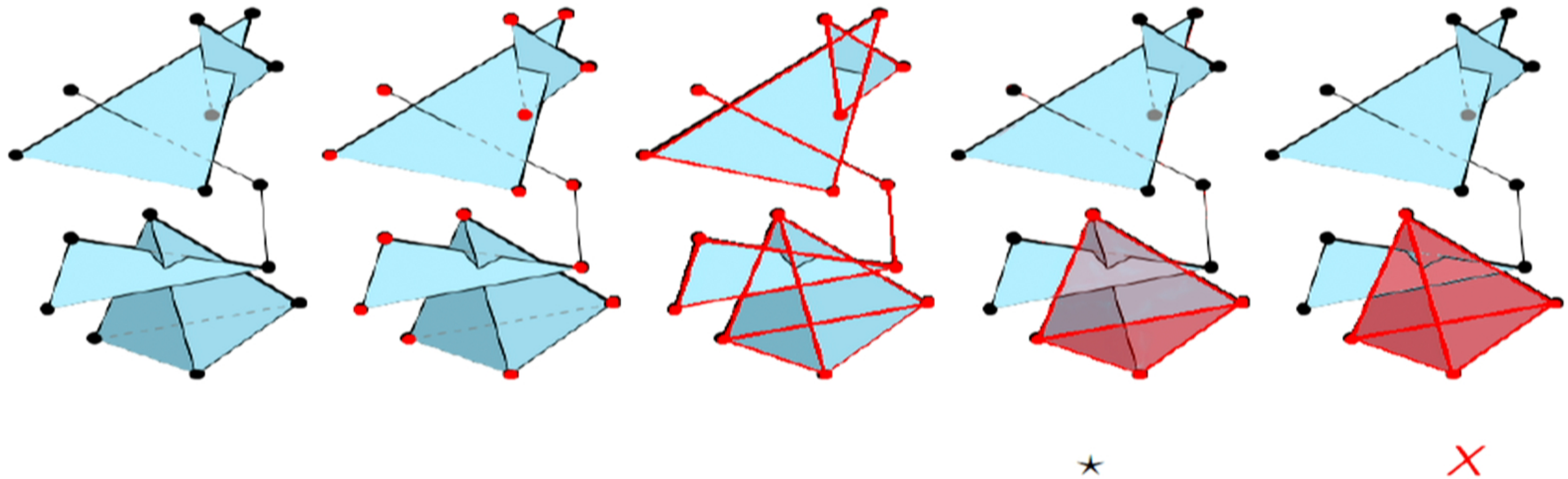
Strictly,

Not every intersection of two simplexes occurs in a subset of the union of their boundaries which is also a lower dimensional simplex.



Realized Proper-Pruned Skeletons

Decompose a state into a superposition of skeletons $K_d^* \subseteq K_d$ from volumetric elements α_d which have empty convex intersection.



Weightings

Volumes become combinatorial weights derived from the embedded simplicial volumes of the subgraphs

Definition

Each complete subgraph $\alpha_d \in K_d^*$ is equipped with a finite weighting ω_{α_d}, provided by a map

$$\Omega : \alpha_d \to \mathbb{R}^+$$

Forman Curvature

- Weighted combinatorial curvature on any d-cell of a quasi-convex cellular complex
- Strong homological analogue to Riemannian Ricci curvature
- Depends on the near-nonlocal data of the $\alpha_{d\pm1}$ neighbors

$$\alpha_d \subset \alpha_{d+1} \text{ or } \alpha_{d-1} \subset \alpha_d$$

- Naturally defined inner product from chain complex gives rise to positive combinatorial weights

Forman Curvature

$$F(\alpha_d) = \omega_{\alpha_d}\left\{\sum_{\alpha_{d+1}\supset\alpha_d}\frac{\omega_{\alpha_d}}{\omega_{\alpha_{d+1}}} + \sum_{\alpha_{d-1}\subset\alpha_d}\frac{\omega_{\alpha_{d-1}}}{\omega_{\alpha_d}} - \sum_{\tilde{\alpha}_d\neq\alpha_d}\left|\sum_{\substack{\alpha_{d+1}\supset\alpha_d\\ \alpha_{d+1}\supset\tilde{\alpha}_d}}\frac{\sqrt{\omega_{\alpha_d}\omega_{\tilde{\alpha}_d}}}{\omega_{\alpha_{d+1}}} - \sum_{\substack{\alpha_{d-1}\subset\alpha_d\\ \alpha_{d-1}\subset\tilde{\alpha}_d}}\frac{\omega_{\alpha_{d-1}}}{\sqrt{\omega_{\alpha_d}\omega_{\tilde{\alpha}_d}}}\right|\right\}$$

Action

Definition

$$S_N = \sum_{d=0} \xi_d \sum_{\alpha_d \subset K_d^*} \omega_{\alpha_d} (F(\alpha_d) + \Lambda)$$

- Sum over simplicial scales
- ξ_d are coupling constants which differentially weight the slices of the network
- Λ is a scalar offset to the Forman curvature

0-Skeleton Weights

- Another free-parameter
- All points are geometrically equivalent for our model
- Assign a single uniform weighting ω_{α_0} to each node

Point Volume

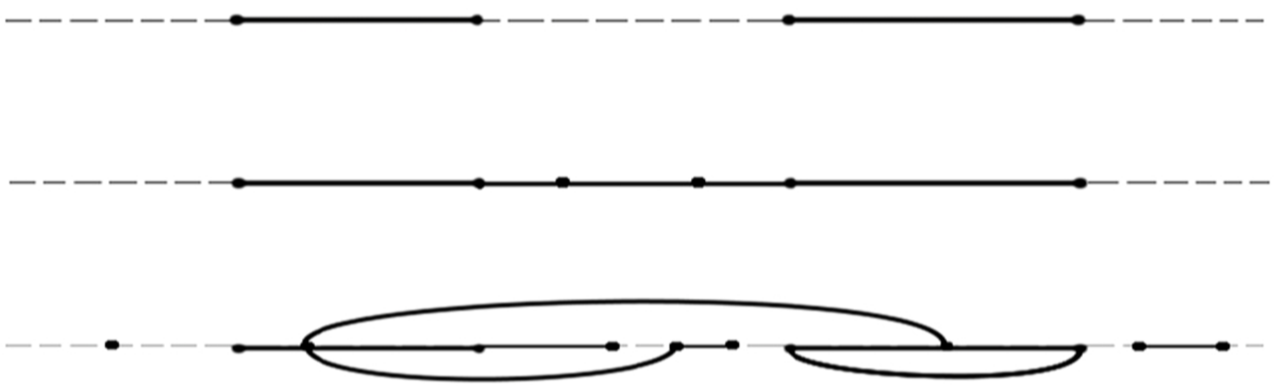
- Naturally 0 from a volume perspective
- Equivalence class under disconnected point addition
- Nontrivial graphs under an infinite sea of points

Cosmological Constant

Once we establish a strictly positive point-volume, we can use a similar argument on Λ.

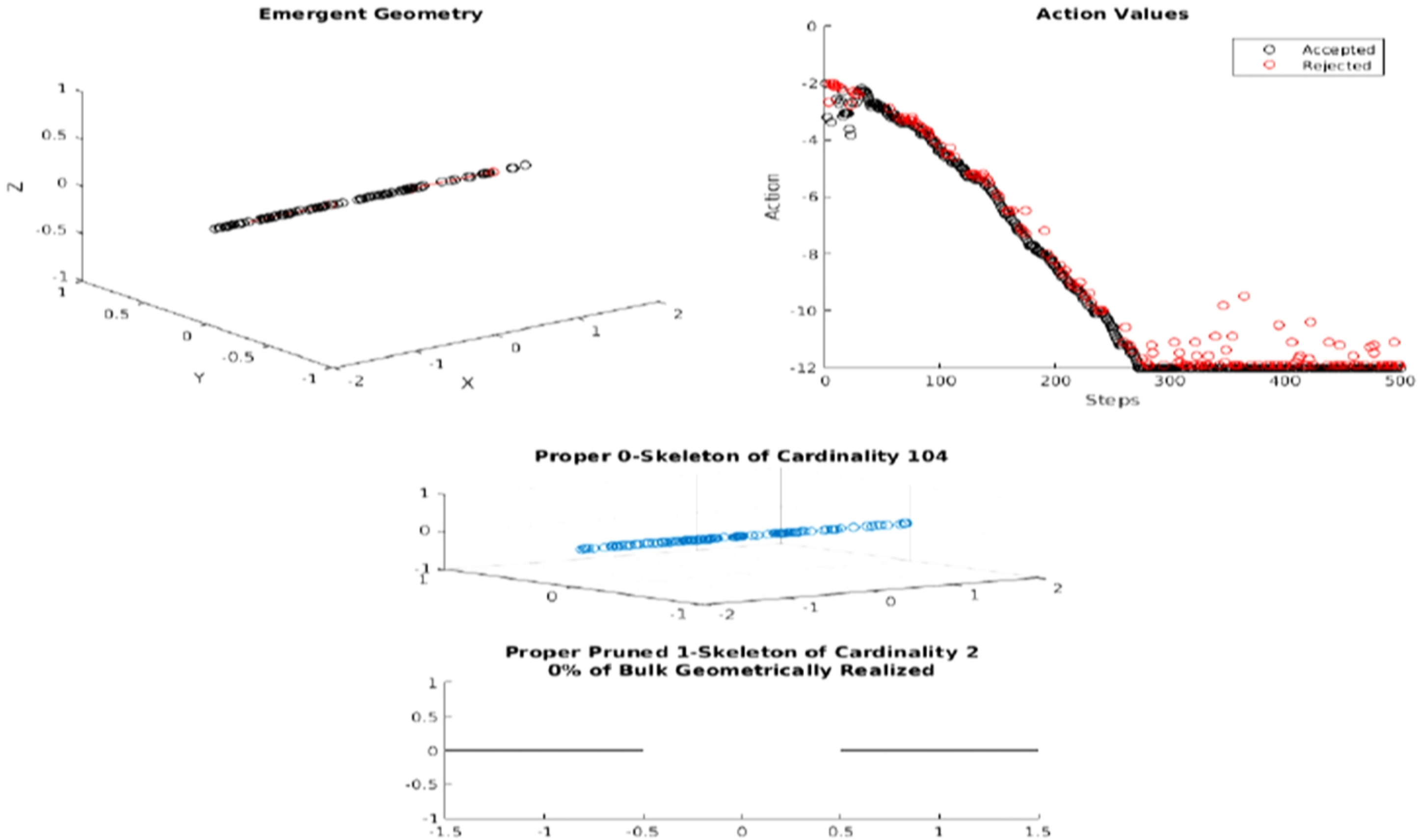
If we desire suppression against the emergence of infinite disconnected components, we are forced into a regime of $\Lambda > 0$.

1-D Simulations

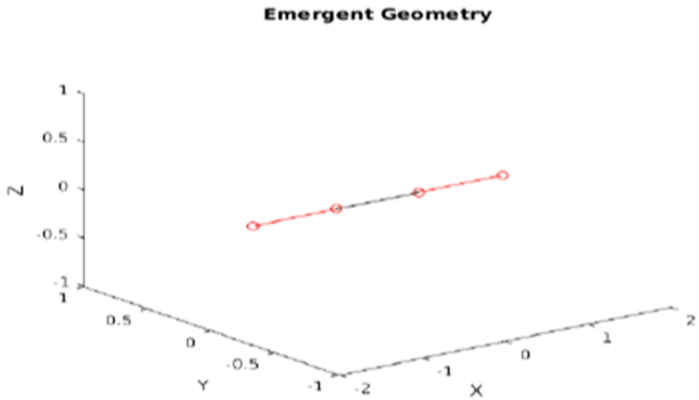


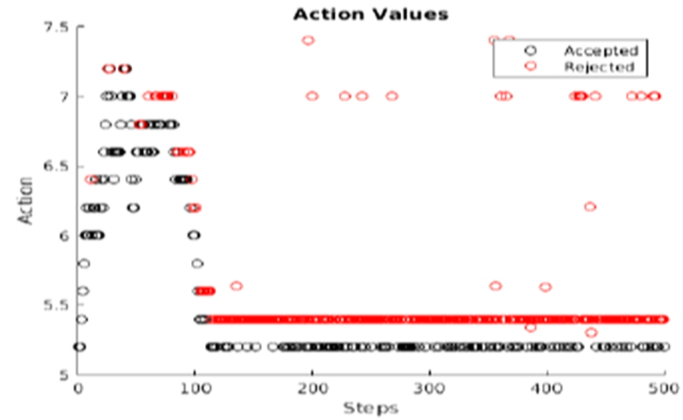
1. 1-D boundary setup
2. Example geometric solution with simplicial attachments
3. Example abstract solution with non-simplicial attachments (shown with curved lines for illustrating overlap)

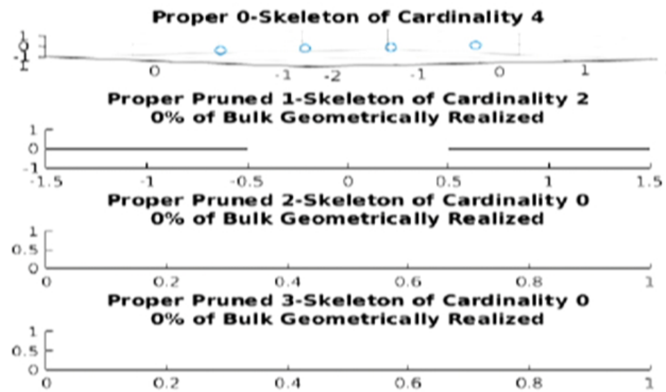
1-D Simulation Results: $\Lambda < 0$



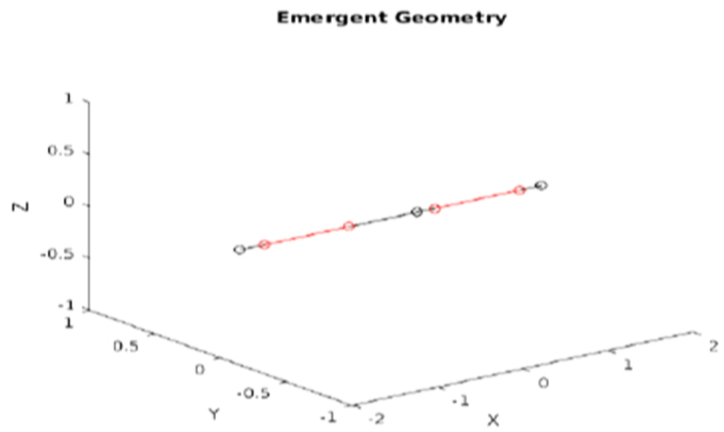
1-D Simulation Results: $\Lambda > \Lambda_c$

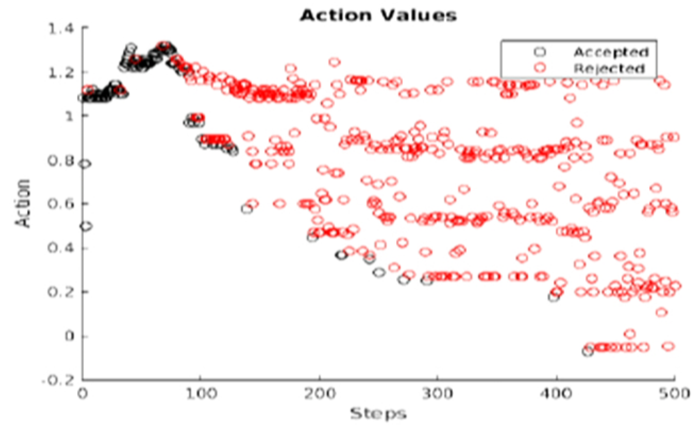


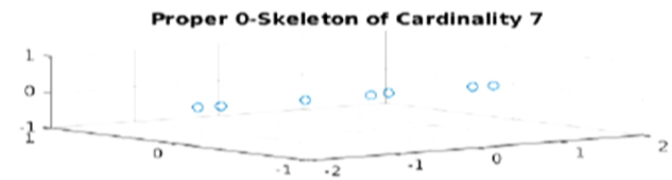


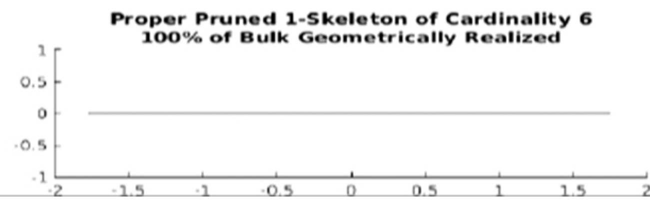


1-D Simulation Results: $0 < \Lambda < \Lambda_c$

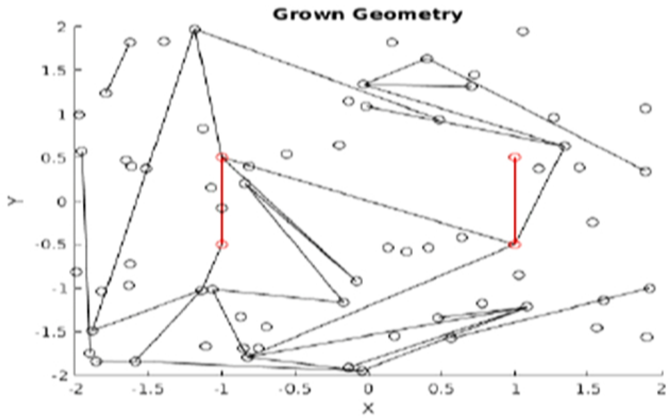


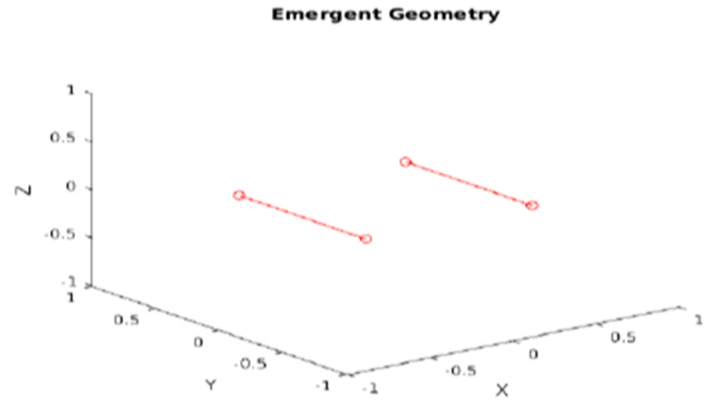


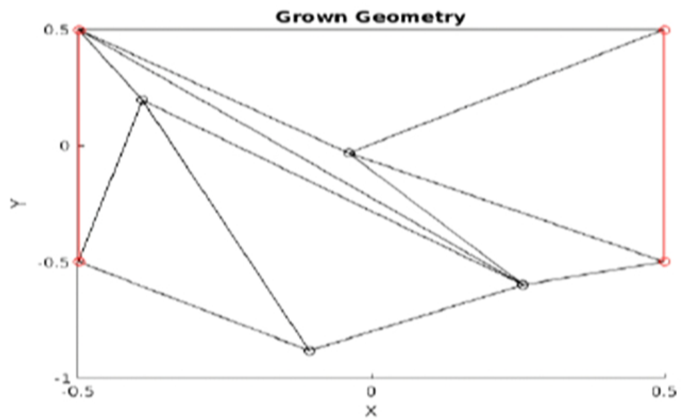




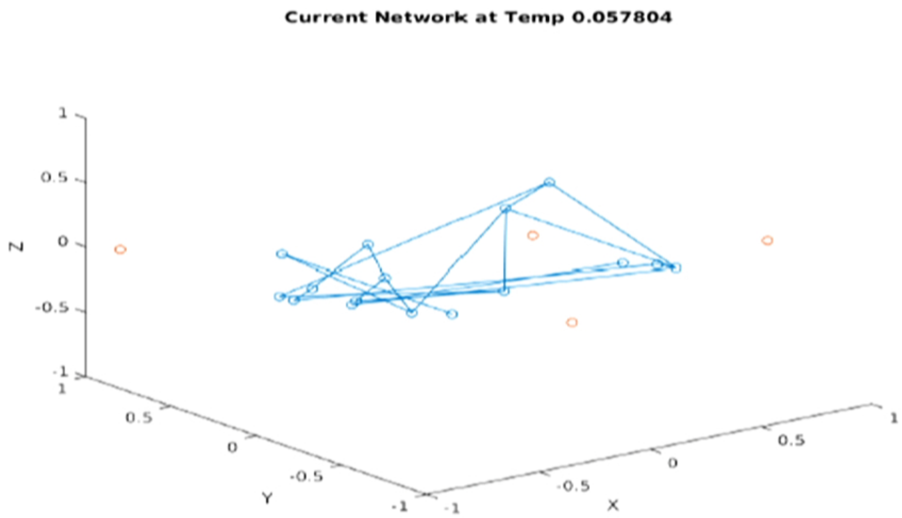
2D Embedding Example

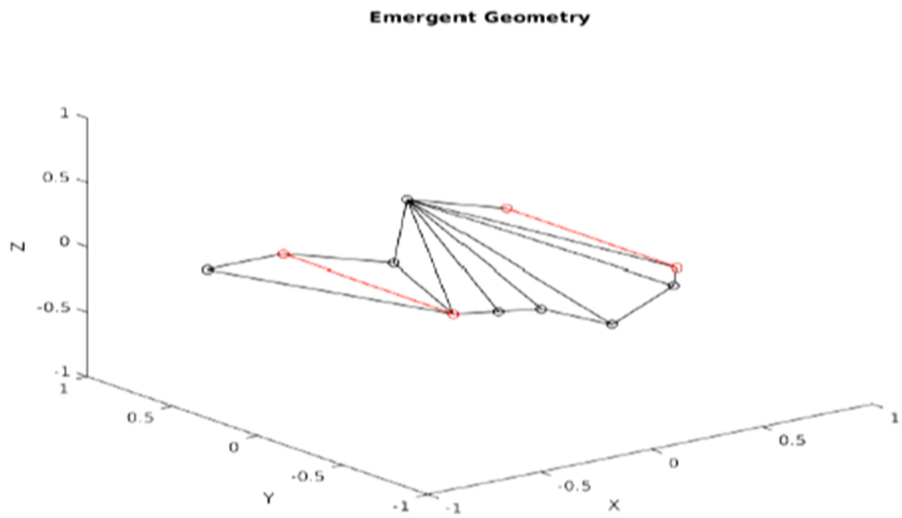






2D Embedding Movie





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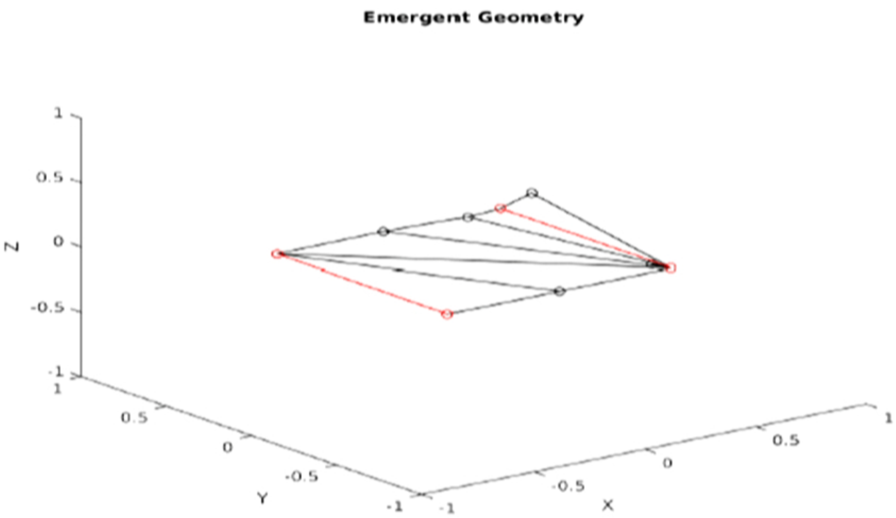
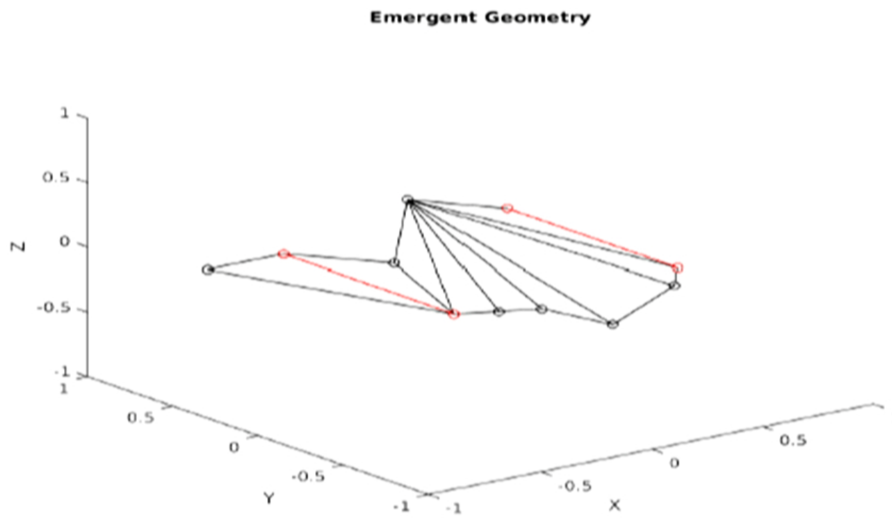




Regulation

For a finite network, the action necessarily contains a finite number of finite terms by construction.

However, the action is intrinsically not positive-definite.

Global minimum may be unbounded with network growth...

UV

Without the point volume regulator in the volumetric cutoff, get divergence to $-\infty$.

With the cutoff,

$$\lim_{\omega_B \to \omega_0} \Delta S = 2\omega_0(\Lambda + \omega_0 - \frac{(\omega_A^2 + \omega_0^2)}{2\sqrt{\omega_A \omega_0}})$$

In the limit $\omega_{A,B} \to \omega_0$, recover the action equivalent of the addition of two isolated points

IR

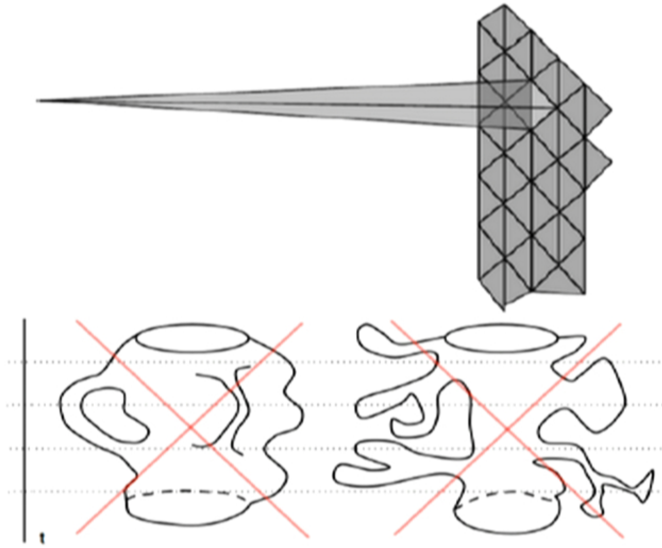
The limit

$$\lim_{\omega_B \to \infty} \Delta S \to -\infty$$

signals a large structure divergence in the ‘infrared’ regime of the theory.

Bubbles and Branches

- Similar to bubble/spike divergence in some spinfoam models
- Polymer behavior qualitatively similar to branching universes in CDTs



Rovelli and Vidotto, 2015; Loll, 2008

Flat Action Direction, $(\omega_{\alpha_0}, \Lambda) > 0$

We ask whether there are equivalence classes of states under the action.

Probe independent regular simplicial building blocks to provide a 3-dimensional configuration space: $\omega_{\alpha_0}, \Lambda$ and ω denoting the uniform edge length.

Flat Action Direction, $(\omega_{\alpha_0}, \Lambda) > 0$

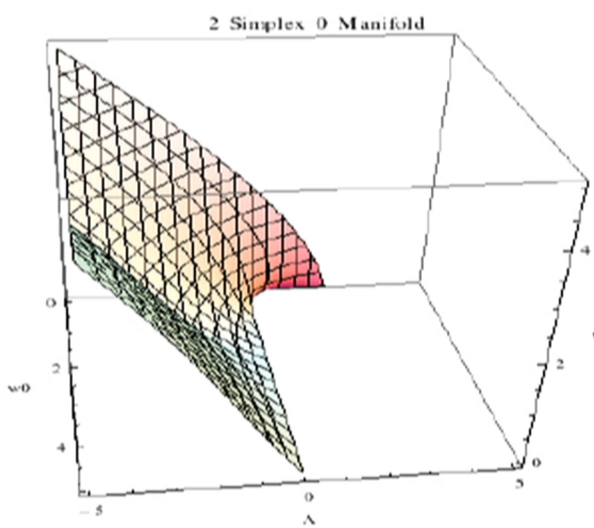
We can easily generalize to the case of an arbitrary regular n-simplex of side-length ω, and find the action in closed form:

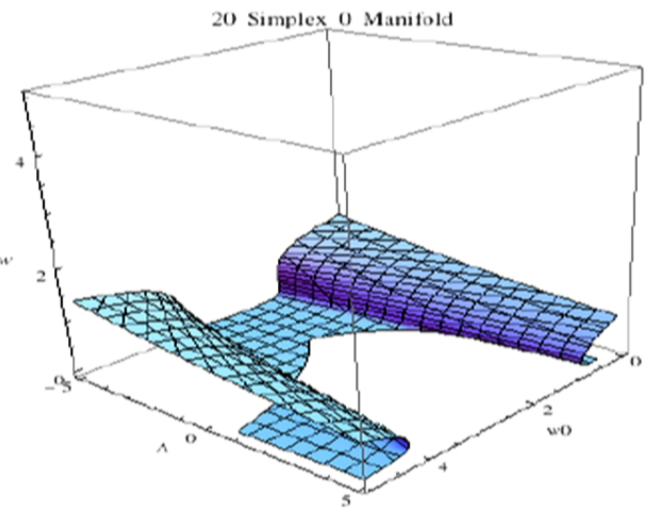
$$
\begin{aligned}
\omega_d &= \frac{\omega^d}{d!}\sqrt{\frac{d+1}{2^d}}\left(1+\delta_0^d(\omega_0-1)\right) \\
S_n &= \sum_{d=0}^{n}\binom{n+1}{d+1}\omega_d\left\{\Lambda+(1-\delta_0^d)\right. \\
&\times\quad \omega_d\left((n-d)\frac{\omega_d}{\omega_{d+1}}+(d+1)\frac{\omega_{d-1}}{\omega_d}\right. \\
&-\quad \left.\left.(2n-1-d)\Theta(2n-1-d-n)\left|\frac{\omega_d}{\omega_{d+1}}-\frac{\omega_{d-1}}{\omega_d}\right|\right)\right\}
\end{aligned}
$$

We search for degenerate solutions, in this case to $S=0$.

Regular Simplex S=0 Manifolds

Multivalued behavior in certain parameter ranges...





Implications for spontaneous growth, as well as simulation ergodicity

Exceptional zeros of 3nj symbols

Summary

- We have presented a formalism with strong analytic handles which can be used to investigate the nature of network interactions and probe emergent geometry considerations

Foundational Questions

1. Regge Reduction?
2. Density of space of action-equivalent configurations under fixed global parameters?
3. IR regulation?
4. Constraints on free parameters?

Non Geometric Considerations

Can we use the framework to model other types of network growth and dynamics?

- Social networks
- Neural networks
- Distributed computing
- Shipping and commerce

The study of emergent complex networks is an active field with wide applicability