

Title: Spinon Walk in Quantum Spin Ice

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Abstract: Quantum spin ice is a frustrated magnet that displays rich emergent phenomena. For example, the magnetic moments carried by the spins may separate into mobile magnetic charges, producing quantum fractional excitations known as spinons. The spinon moves in a background of disordered spins, and its motion is strongly coupled to the spin background. In this talk, I will demonstrate that the spinon dynamics can be described as a quantum walk with entropy-induced memory. Our numerical simulation shows that the dynamics of spinon exhibits a remarkable renormalization phenomenon: the spinon behaves as a massive free particle at low energy despite its strong coupling at the lattice scale.

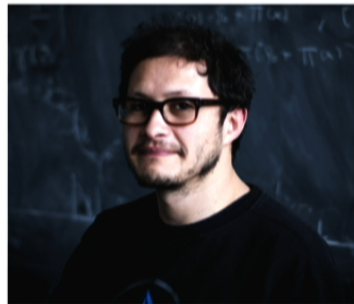
Four Corners, Waterloo, 2016



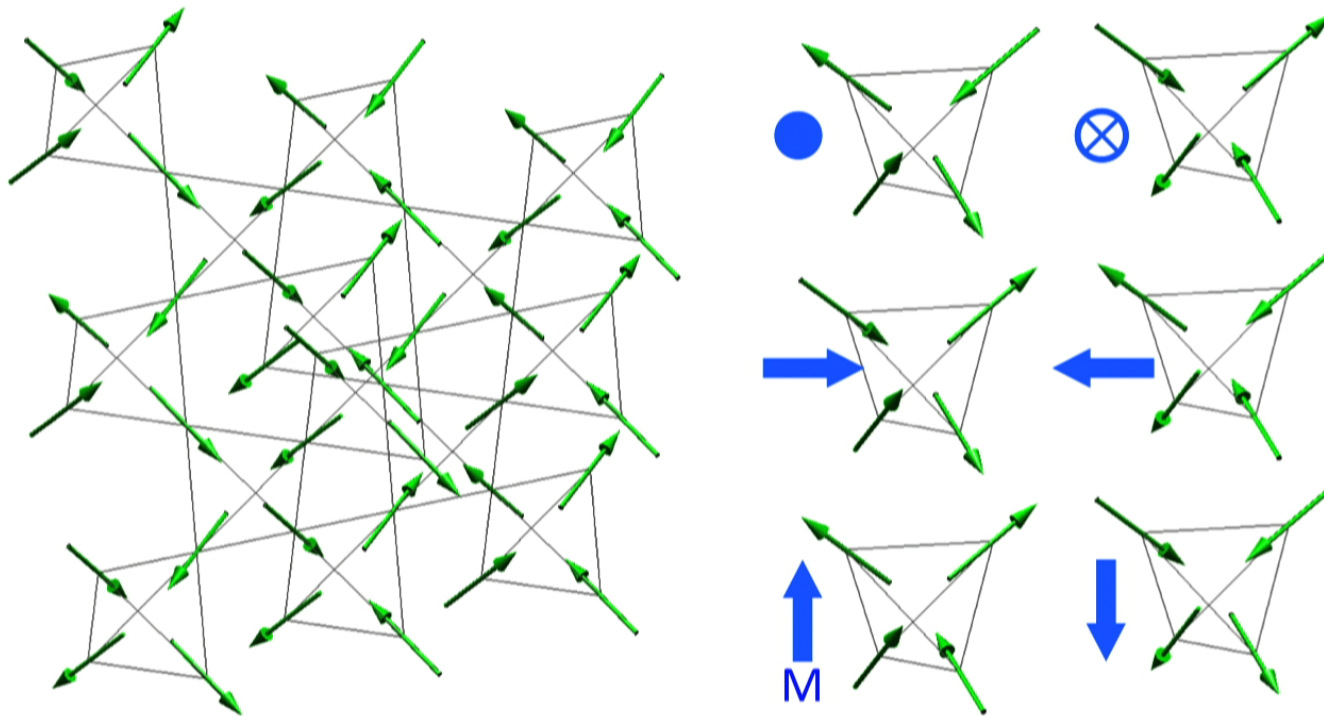
Spinon walk in quantum spin ice

Yuan Wan, Juan Carrasquilla, Roger Melko

[Phys. Rev. Lett. 116, 167202 \(2016\)](#)

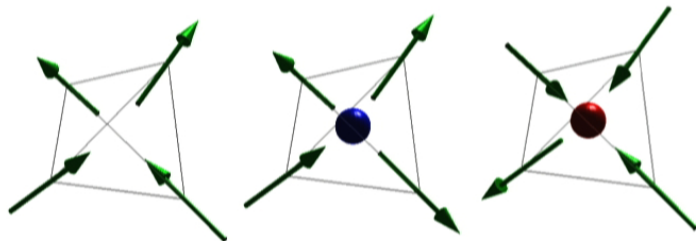


Spin ice: a frustrated ferromagnet



Freedom of poles

$$Q_m = - \int \mathbf{M} \cdot d\mathbf{S} = \int \mathbf{H} \cdot d\mathbf{S}.$$

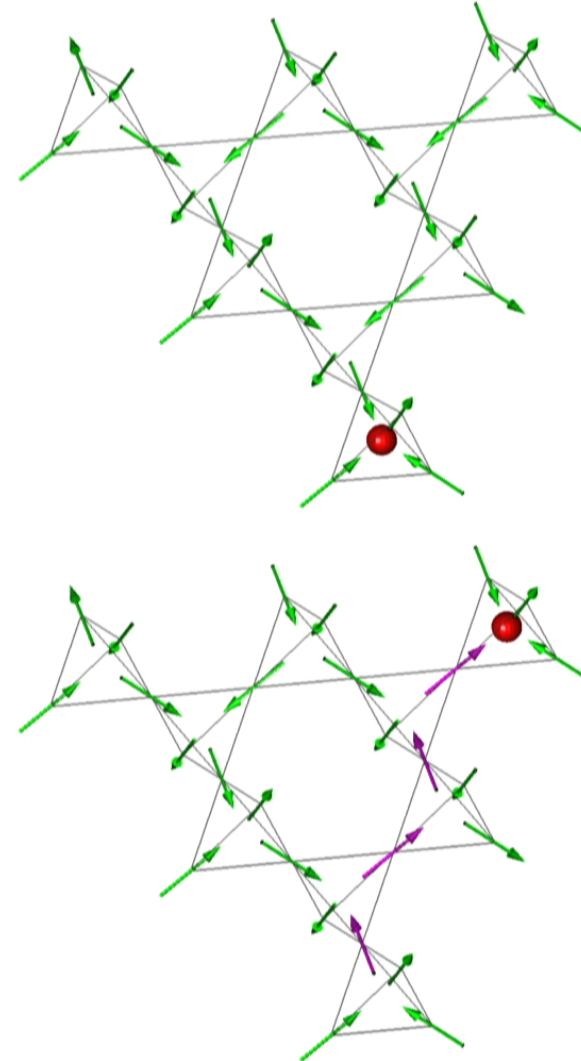


$$Q_m = 0.$$

$$Q_m = -1; \\ \Delta E = J/2.$$

$$Q_m = 1; \\ \Delta E = J/2.$$

Ivan Ryzhkin, JETP 2005.
C. Castelnovo, R. Moessner, & S.
Sondhi, Nature 2008.



Quantum spin ice

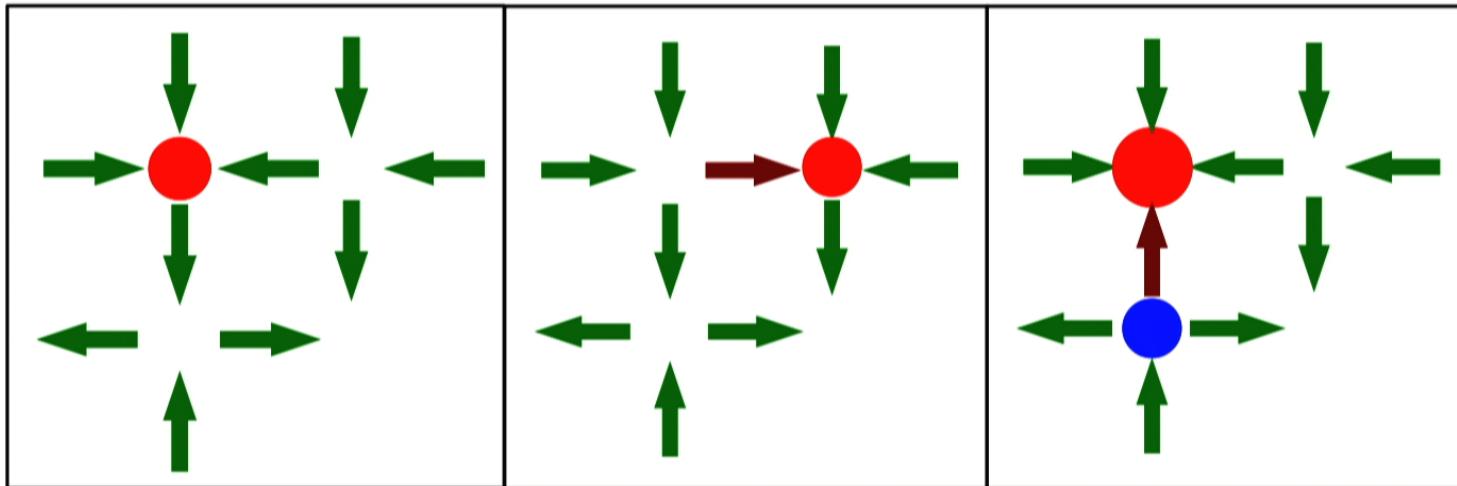
- Substantial quantum fluctuations.
- **Quantum tunneling** of magnetic charges.
- Classical magnetic charges → **Spinons**
- Spinons are responsible for many physical properties of QSI. **But how does a spinon move?**
- Candidates: $\text{Tb}_2\text{Ti}_2\text{O}_7$, $\text{Yb}_2\text{Ti}_2\text{O}_7$, $\text{Pr}_2\text{Zr}_2\text{O}_7$, $\text{Yb}_2\text{Sn}_2\text{O}_7$...

QSI Survey: M. Gingras and P. McClarty, Rep. Prog. Phys. 2014.

Single-spinon dynamics

$$H = - \sum_{\langle ij \rangle} (|\oplus_i \leftarrow \cdot_j\rangle \langle \cdot_i \rightarrow \oplus_j| + |\ominus_i \rightarrow \cdot_j\rangle \langle \cdot_i \leftarrow \ominus_j|)$$

Hardcore
Bosons

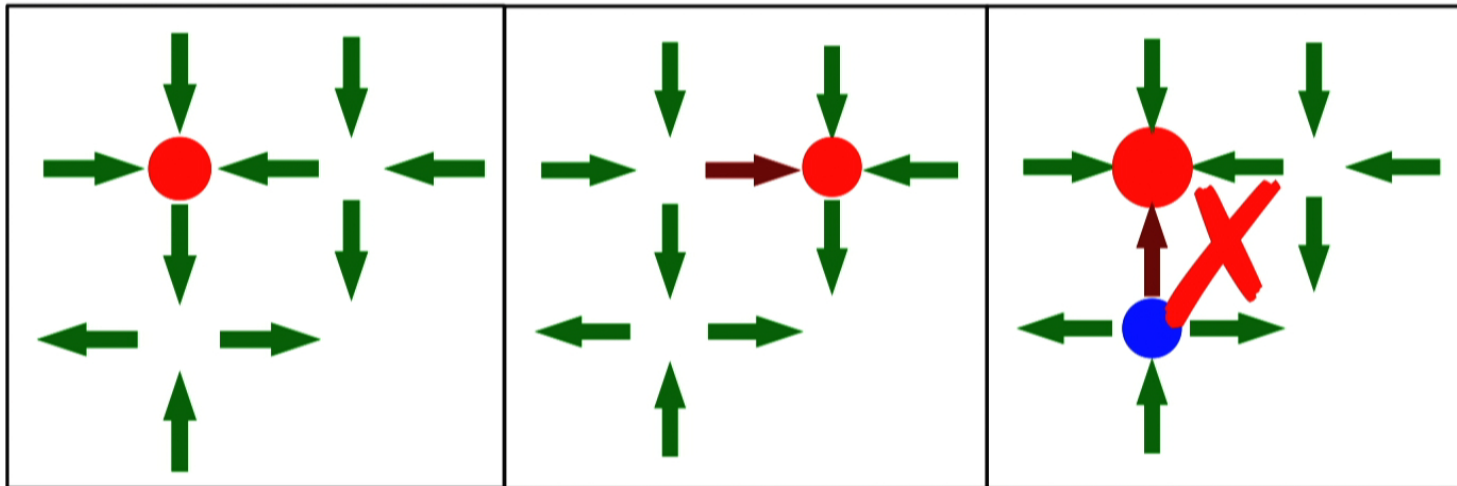


M. Chen, L. Onsager, J. Bonner, and J. Nagle, JCP, 1974.
O. Petrova, R. Moessner, S. Sondhi, PRB, 2015.

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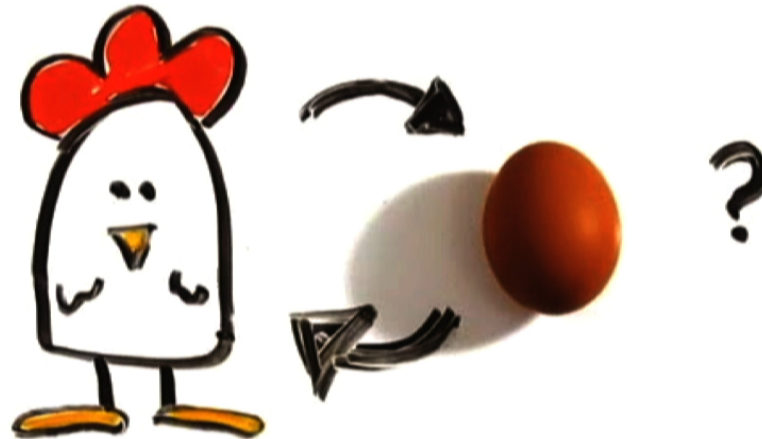
Hardcore
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M. Chen, L. Onsager, J. Bonner, and J. Nagle, JCP, 1974.
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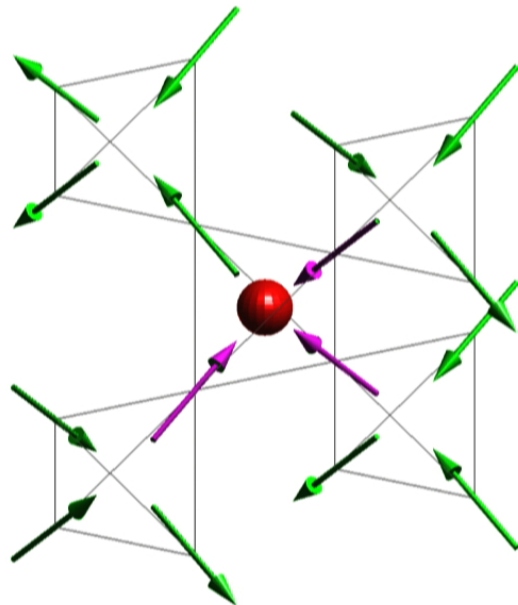
A chicken and egg problem

- Background spins guide spinon motion.
- Lattice contains **loops**.
- **New** spin background each time spinon **revisits** a site.



Ground State

$$\sum_{\alpha} \langle \alpha | H | \beta \rangle = -3. \quad \Rightarrow \quad |G.S.\rangle = \sum_{\alpha} |\alpha\rangle$$



$$\begin{aligned} H|G.S.\rangle &= \sum_{\alpha} H|\alpha\rangle \\ &= \sum_{\alpha, \beta} |\beta\rangle \langle \beta | H | \alpha \rangle \\ &= -3 \sum_{\beta} |\beta\rangle \\ &= -3|G.S.\rangle. \end{aligned}$$

Characterizing single-spinon dynamics

$$C_{ij}(t) = \frac{\langle \text{G.S.} | e^{i\hat{H}t} \hat{n}_j e^{-i\hat{H}t} \hat{n}_i | \text{G.S.} \rangle}{\langle \text{G.S.} | \hat{n}_i | \text{G.S.} \rangle} \xrightarrow{it \rightarrow \tau} C_{ij}(\tau) = \frac{\langle \text{G.S.} | e^{\hat{H}\tau} \hat{n}_j e^{-\hat{H}\tau} \hat{n}_i | \text{G.S.} \rangle}{\langle \text{G.S.} | \hat{n}_i | \text{G.S.} \rangle}$$

$$\boxed{C_{ij}(t)}$$

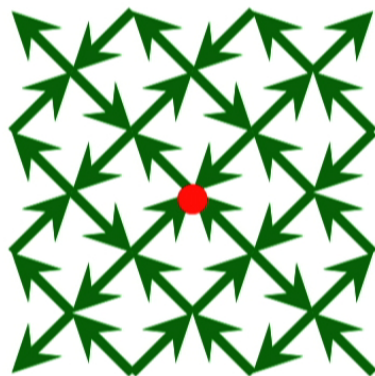
Probability of observing the spinon on site **j** at time **t** provided it was observed on **i** at time **0**.

Spinon path integral

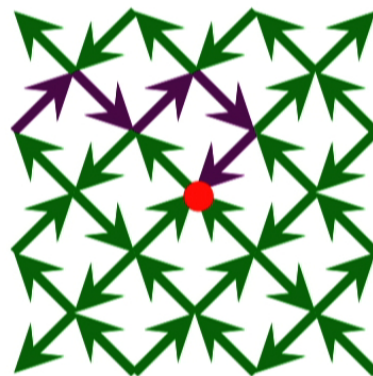
$$C_{ij}(\tau) \propto \frac{1}{\sum_{\alpha} 1} \sum_{\alpha} \sum'_{\gamma: (i,0) \rightarrow (j,\tau)} (\delta\tau)^{L_{\gamma}}$$

γ : All paths that are allowed by initial spin config. α .

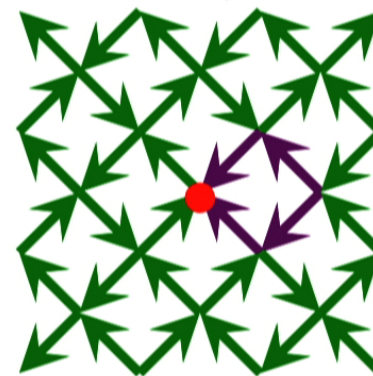
α .



γ . ✓



γ . ✗

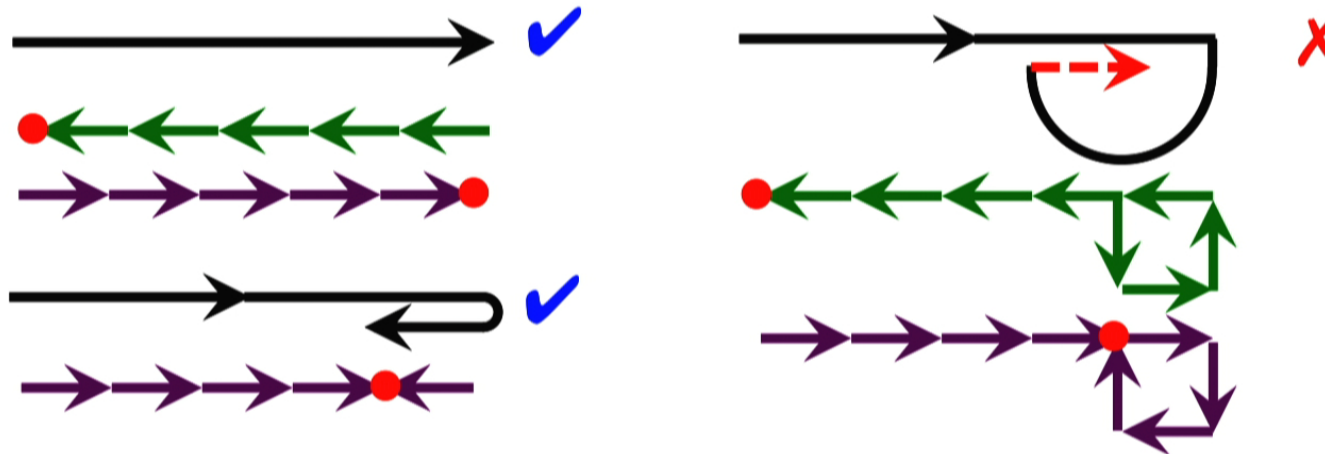


Trace out spins, acquire memory

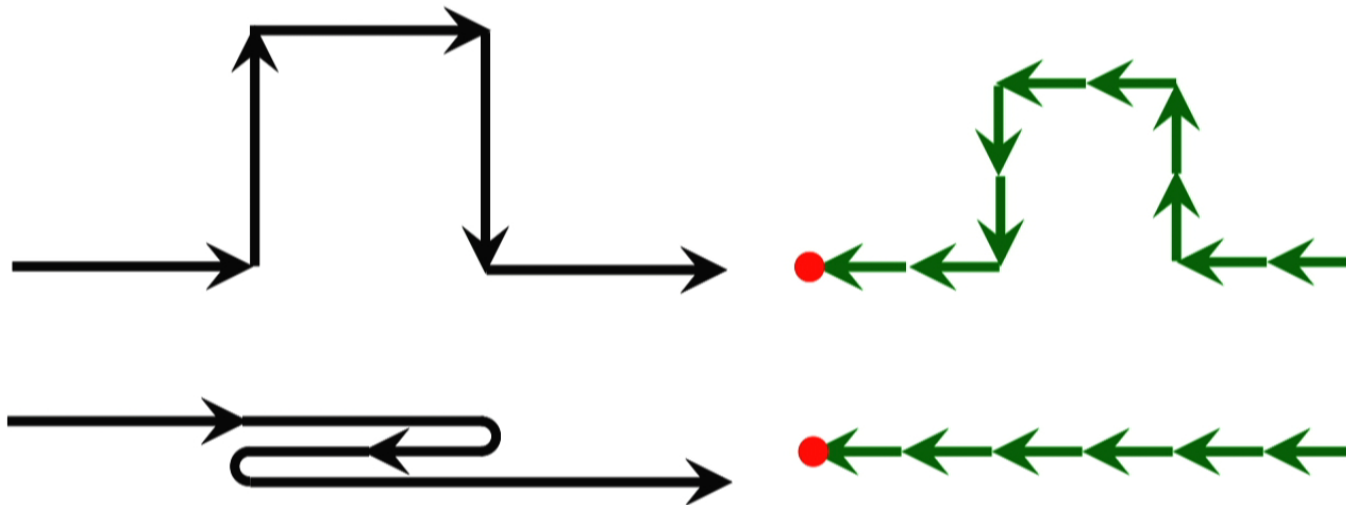
$$C_{ij}(\tau) \propto \frac{1}{\sum_{\alpha} 1} \sum_{\alpha} \sum'_{\gamma: (i,0) \rightarrow (j,\tau)} (\delta\tau)^{L_{\gamma}} = \sum_{\gamma: (i,0) \rightarrow (j,\tau)} W_{\gamma} (\delta\tau)^{L_{\gamma}}$$

$$W(\gamma) = \frac{\text{Number of initial spin states for which } \gamma \text{ is feasible}}{\text{Number of all initial spin states}}$$

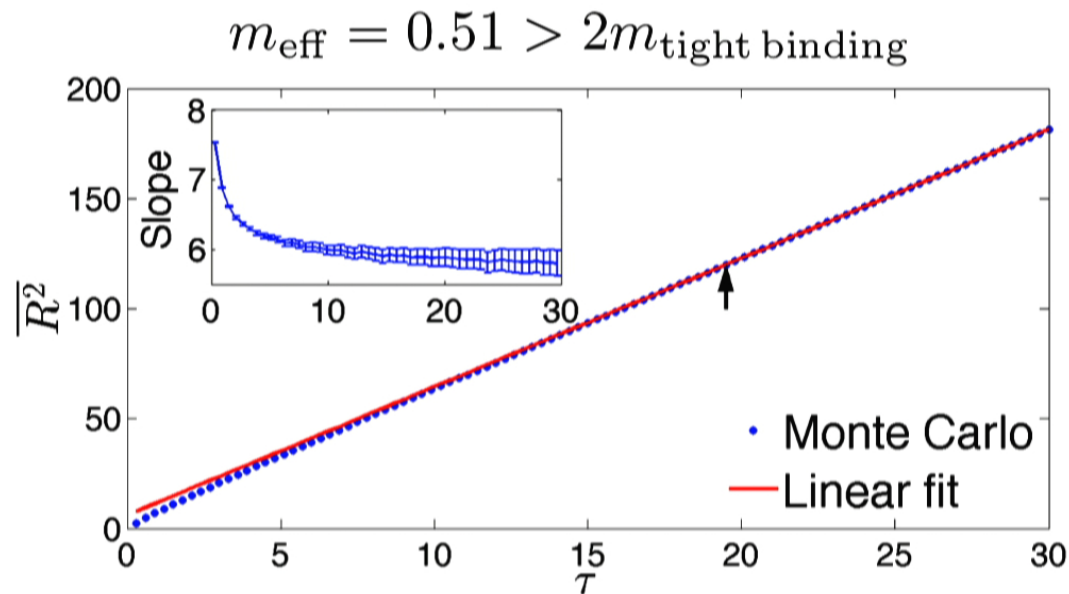
$= e^{\text{Entropy cost of } \gamma}.$



Spinon likes retracing its steps.



Mean Displacement Squared



$$i \frac{\partial \psi}{\partial t} = -\frac{1}{2m} \nabla^2 \psi \Rightarrow \frac{\partial \psi}{\partial \tau} = \frac{1}{2m} \nabla^2 \psi$$

“Diffusion Constant”: $D = \frac{1}{2m}.$

Conclusion

- Spinon dynamics is (infinitely) strong-coupled at lattice scale.
- Spinon behaves as a nearly-free, massive particle at low energy.
- Spinon walk is self-attractive; mass heavily renormalized.