

Title: Disorder in spin liquids

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Abstract: Beyond their deceptively featureless ground states, spin liquids are particularly remarkable in the exotic nature of their (fractionalised and gauge charged) excitations. Quenched disorder can be instrumental in nucleating or localising defects with unusual properties, revealing otherwise hidden features of these topological many-body states. This talk discusses how to turn the nuisance of disorder into a powerful probe and origin of new collective behaviour.

# Disorder in frustrated magnets

Roderich Moessner

MPI-PKS

John Chalker and Adam Willans

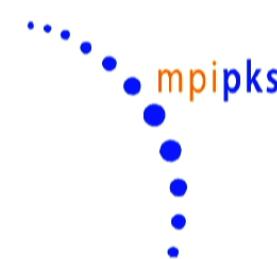
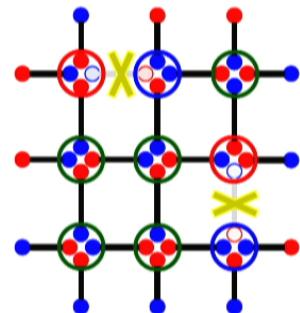
Oxford

Kedar Damle and Arnab Sen

TIFR/IACS Kolkata

Jorge Rehn

Dresden



# Outline

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## Disorder in magnetism

- ▶ distraction, probe, source of new physics

## Disorder and frustration

- ▶ history, disorder-free glassiness?

## Diluted spin ice

- ▶ phenomenology
- ▶ effective impurity Hamiltonian
  - ▶ topological spin glass

## Disorder and topology

- ▶ fractionalisation made visible
- ▶ example: Kitaev spin liquids

# Disorder

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Disorder is inevitably present in the solid state

- ▶ may be absent in cold atom systems

exists in different forms:

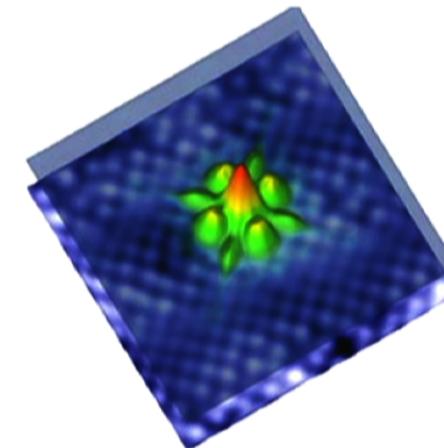
- ▶ substitution; random strain; vacancies;
- ...

can be useful as a probe

- ▶ can be introduced (increased) in some cases
- ▶ nature of the superconducting state
- ▶ nature of putative spin liquid states?

can yield new phenomena

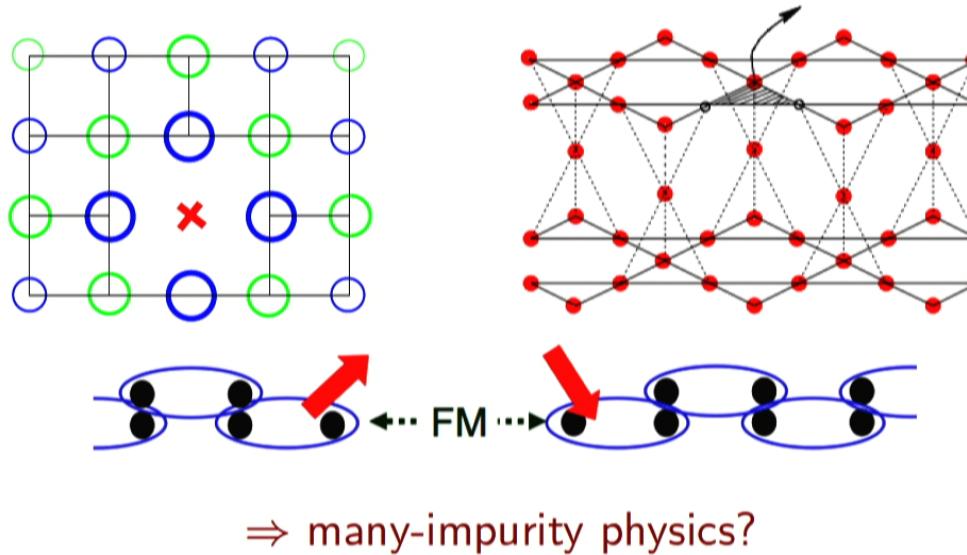
- ▶ glass transition; random singlets



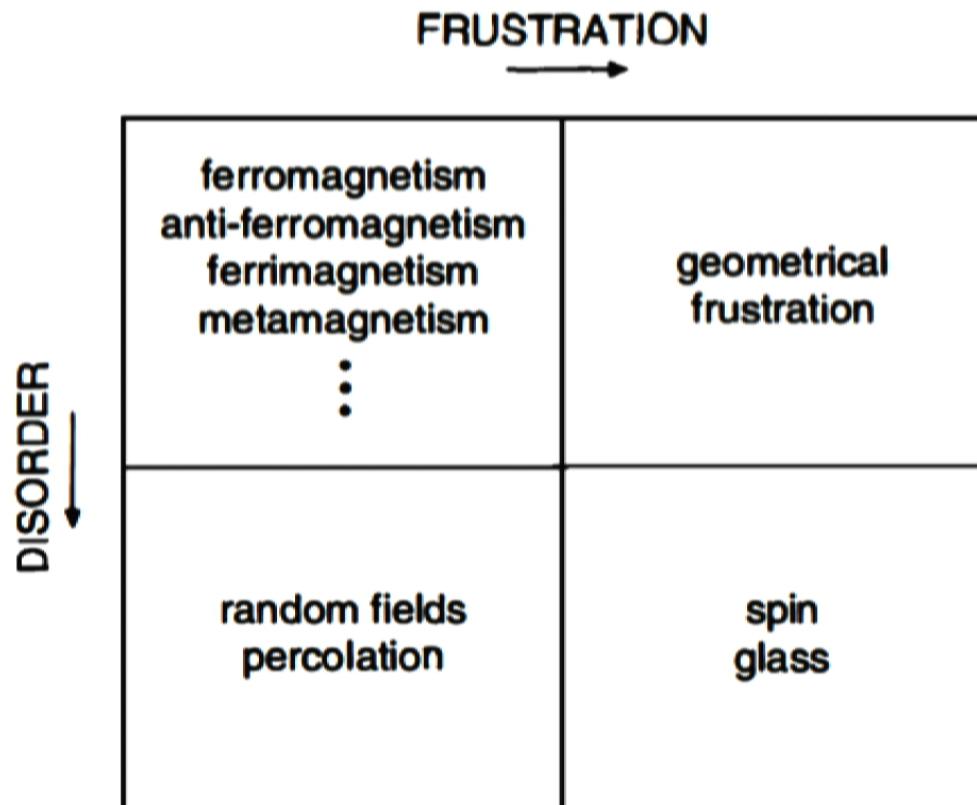
Davis et al.

## Disorder in magnets: vacancies

- sublattice magnetisation near vacancy in Neel state Scalapino et al.
- Curie contribution to susceptibility (e.g. “orphan spins”, Schiffer)
- fractionalised excitations (e.g.  $S = 1/2$  d.o.f. at endpoints of Haldane chain Broholm et al.)
- effective moment formation at quantum criticality Sachdev et al.



## Disorder and frustration



Ramirez, 1994

# Frustration and glassiness

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Combination is very common

overview: Cepas, Canals PRB **86**, 024434 (2012)

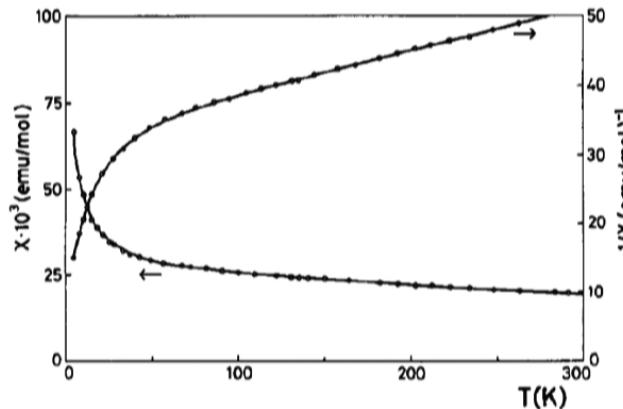
- ▶ Ising/Heisenberg
- ▶  $d = 2, d = 3$
- ▶ distortion, dilution

Origin?

- ▶ intrinsic: disorder-free glassiness
- ▶ extrinsic: a little disorder → a little glassiness
  - ▶ freezing often incomplete
- ▶ many possible mechanisms for slow dynamics

# SCGO, the archetypal frustrated magnet

Obradors 88<sup>+</sup>



started 'modern' frustrated magnetism

- ▶ frustration ratio  $f > 117$

order by disorder?

- ▶ kagome: yes

Chalker et al., Chandra et al., Berlinsky et al.,  
Huse et al. (early 90s)

- ▶ pyrochlore+SCGO: no

freezes at low temperature?

e.g. Lee et al. (1996), Limot et al. (2000)

temperature range. The high temperature susceptibility follow a Curie-Weiss law

C

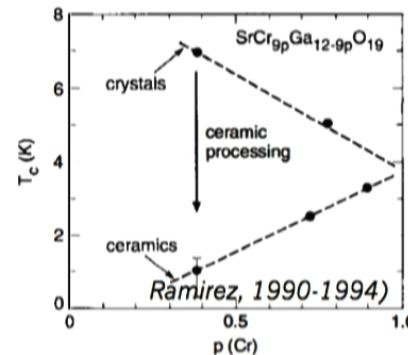
$\chi = \frac{C}{T - \theta}$  with an effective magnetic moment

corresponding to spin-only ( $S=3/2$ ) Cr<sup>3+</sup> magnetic moment and a Curie temperature  $\theta=-492\text{K}$  which shows that strong antiferromagnetic interactions do exist among the Cr ions. It is noteworthy that an antiferromagnetic or spin glass transition below 4.2K would give a very high  $|\theta|/T_g$  ratio ( $|\theta|/T_g > 117$ ). For the

# SCGO: frustration and freezing

Freezing without disorder?

$$\blacktriangleright \lim_{x \rightarrow 0} T_F(x) > 0$$



SCGO  $\Rightarrow$  kagome: dilution  $x$

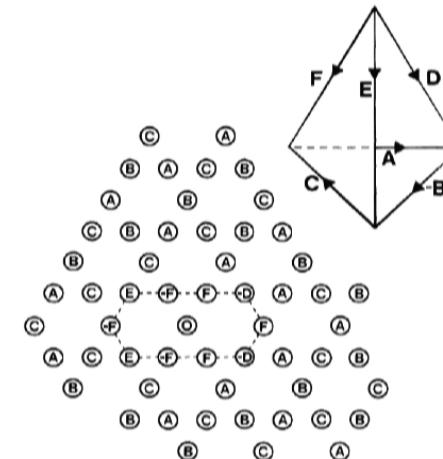
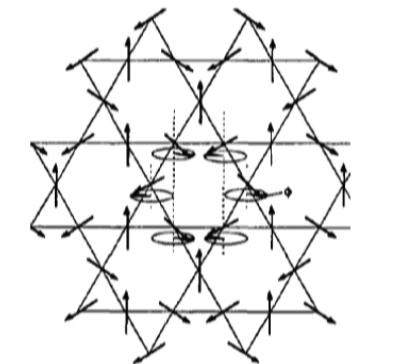
$$\blacktriangleright \text{weathervane modes / spin origami}$$

Chandra et al., Shender et al., 1993

$$\blacktriangleright \text{classical kagome orders Chern+RM}$$

Disorder level in SCGO underestimated

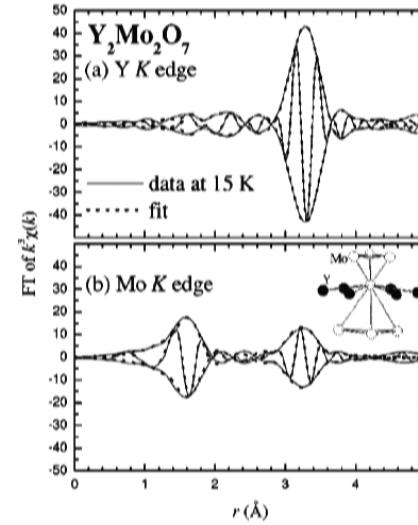
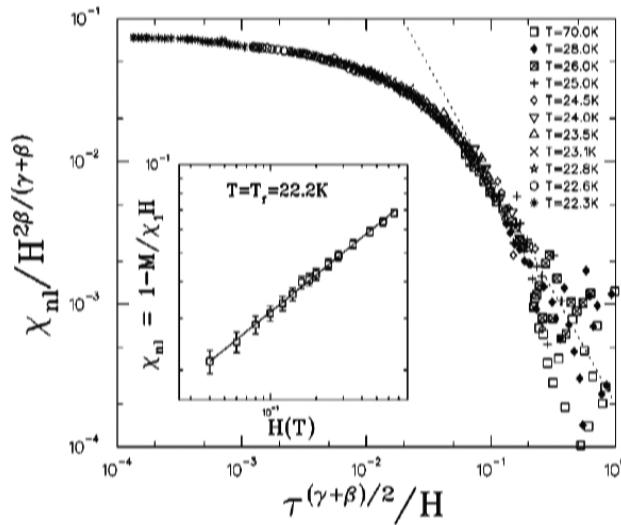
$$\blacktriangleright \text{diluted SCGO does not freeze Rehn et al.}$$



# $\text{Y}_2\text{Mo}_2\text{O}_7$ pyrochlore: bona fide spin glass

Conventional freezing for immeasurably small disorder Gingras et al., 1997

- disorder level in  $\text{Y}_2\text{Mo}_2\text{O}_7$  underestimated? Booth et al., 2000
- bond disorder leads to freezing Bellier-Castella, 2001; ..., Andreanov et al., 2010



# Open questions

How well do we estimate disorder?

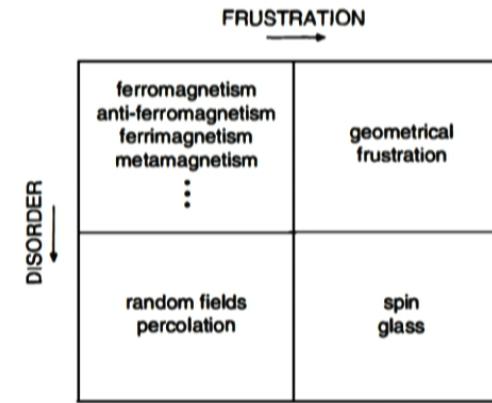
- ▶ important input parameter!

Coexistence of liquidity and glassiness

- ▶ “who” freezes?

Features of freezing specific to spin liquids?

- ▶ topology + disorder



# Spin freezing in spin ice

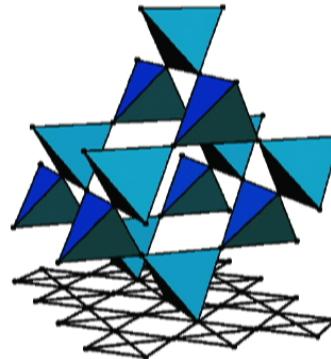
Many experimental studies

- ▶ “how spin ice freezes” Matsuhiro, Schiffer, Bramwell, ..., Petrenko, Gaulin, Pomaranski
- ▶ slow dynamics not properly understood

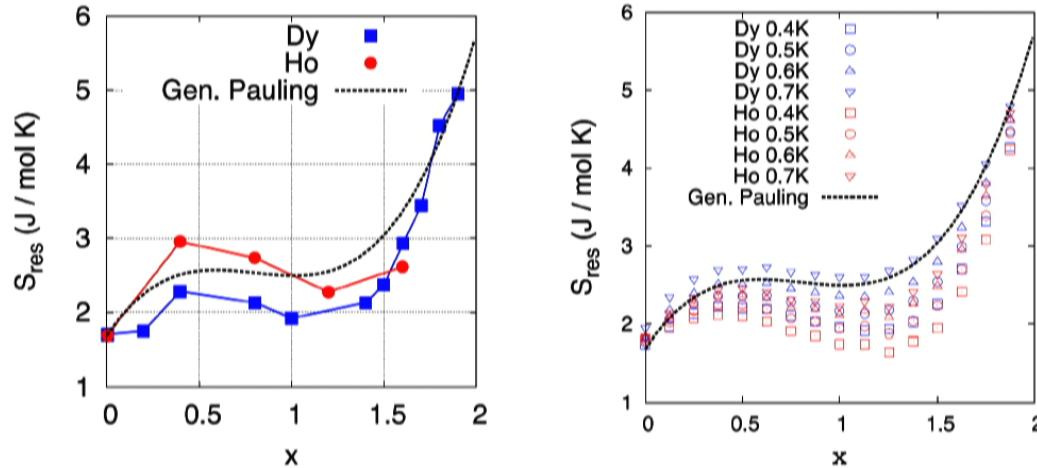
Pristine dipolar spin ice

- ▶ (elusive) thermodynamic ordering transition Melko et al., 2004
- ▶ Coulomb phase can persist to  $T = 0$  upon fine-tuning

Isakov et al., 2005



## Diluted spin ice



Non-monotonic “zero-point entropy” with dilution  $\times$  Ke et al., 2007

- ▶ need to consider interactions between defects Lin et al., 2013
  - ▶ effective impurity Hamiltonian?

## Analytical approach: dumbbell model

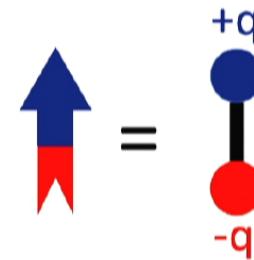
Spin = dipole = pair of charges

- ▶  $|\mu| = |q|a$  ( $a \dots$  premedical lattice constant)

Transparently accounts for Coulomb phase

Castelnovo et al., 2008

- ▶ easily visualised
- ▶ stable to  $T = 0$
- ▶ monopole excitations



## Genesis of ghost spins

Visualize in  $d = 2$

- ▶ tetrahedron = vertex

Satisfied tetrahedron

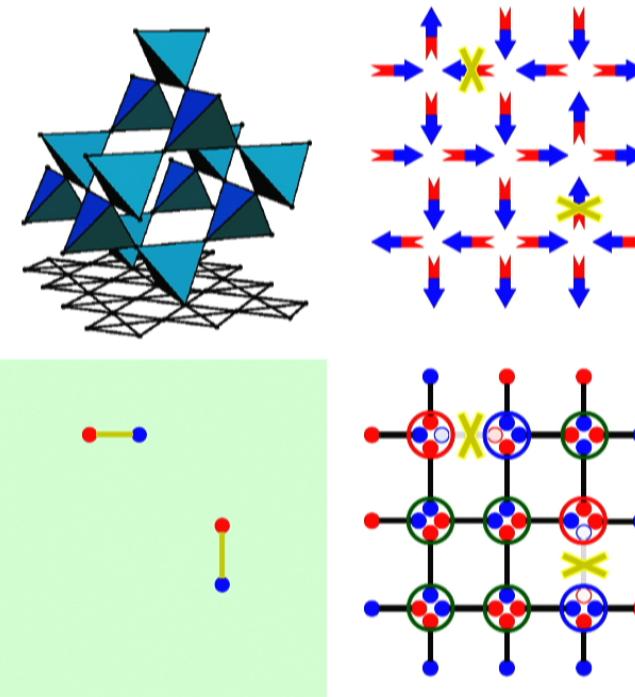
- ▶ charge neutral

Missing spin

- ▶ pair of charged tetrahedra
- ▶ with dipole moment

Delete neutral sites

- ▶ ghost spins left over
- ▶ moment opposite to missing spin
- ▶ embedded in medium



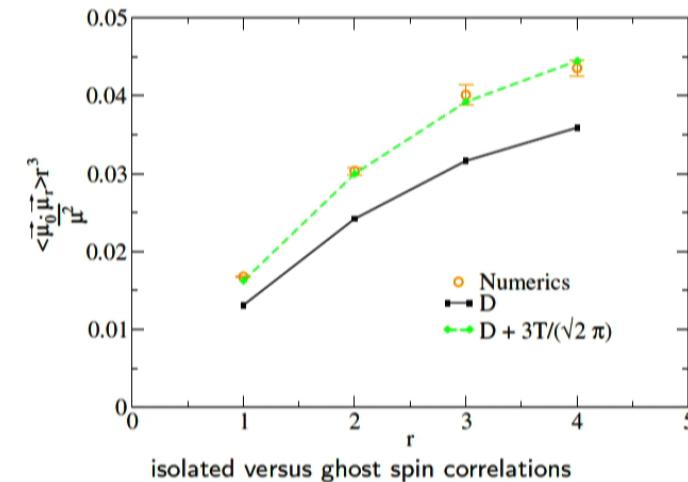
# Interactions between ghost spins

Compare free and ghost spin pair

- enhanced (entropic) dipolar interactions

Entropic interaction

- as in pristine n.n. spin ice
- missing dipoles interact as if
  - not missing
  - not dipoles



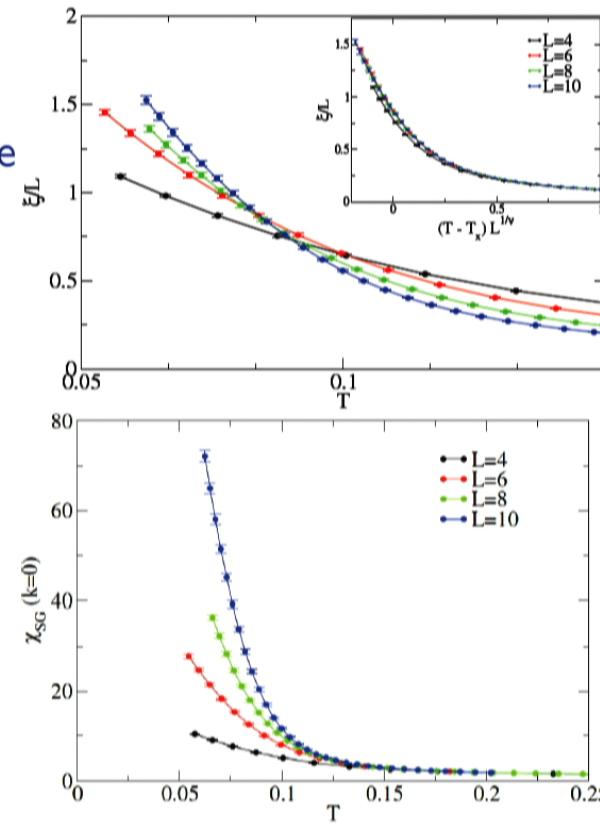
# Freezing of ghost spins

Low dilution = sparse ghost spins

- ▶ simulation with effective  $\mathcal{H}$  possible

Spin freezing of ghost spins

- ▶ dipolar spin glass on pyrochlore lattice
- ▶ scaling collapse of  $\xi$
- ▶ non-linear susceptibility
  - ▶  $T_x = 0.95Dx$
  - ▶  $T_c(x) = T_x/(1 - 3T_x/\sqrt{2}\pi D)$
  - ▶  $D \approx 1.41K$  for Dy/Ho spin ice



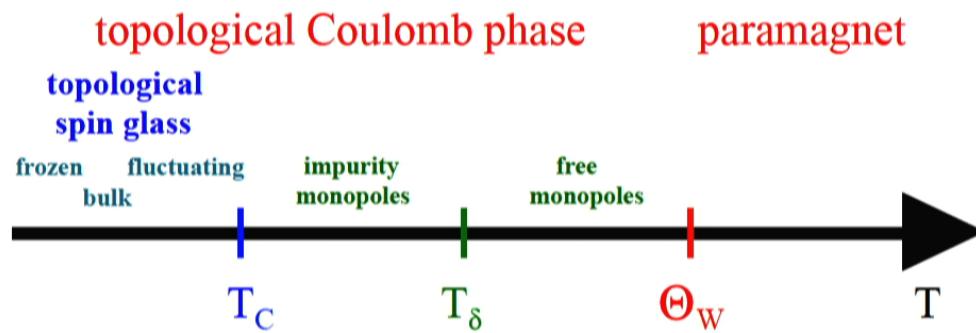
# Phase diagram

Different regimes and crossovers

- ▶ paramagnet → regular spin ice
- ▶ bulk → impurity monopoles

Glass phase

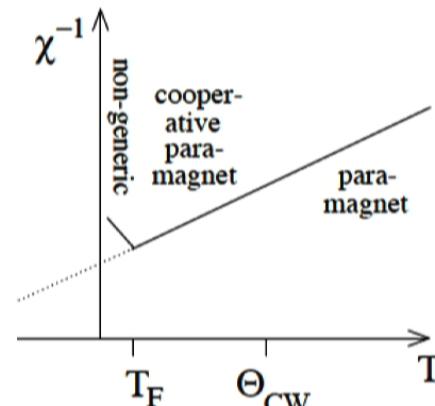
- ▶ continuous onset of frozen moment
- ▶ competition: entropy vs. random frozen fields



## Relation to experiment

Competing instabilities to freezing

- ▶ may always be present
  - ▶ low-T instabilities rarely 'generic'
- ▶ no high-T instability
- ▶ but  $\exists$  microscopic  $\mathcal{H}$  exhibiting
  - ▶ partial freezing
  - ▶ coexistence topological phase + glass



Topological phase produces d.o.f. + mediates their interaction

- ▶ 'generic'

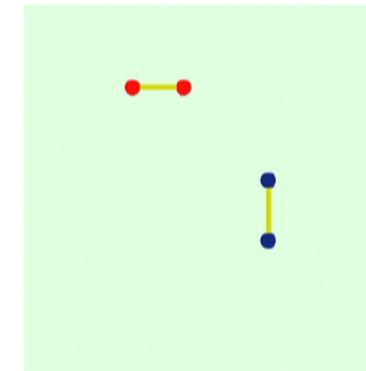
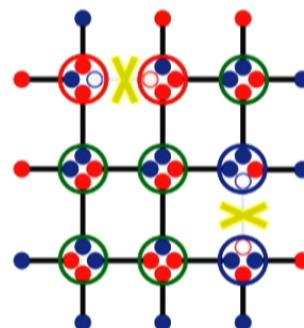
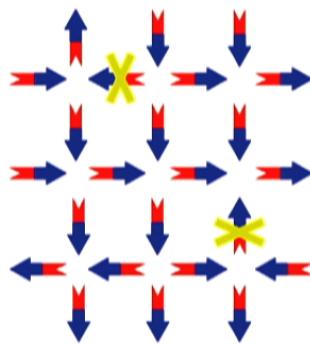
Slow dynamics can have many origins

- ▶ oxygen disorder Sala et al., 2014
- ▶ kinematic constraints Ryzhkin 2005, Jaubert + Holdsworth 2009, ...
- ▶ cluster formation Biltmo + Henelius, 2012

## Impurity monopoles

Flip “Dirac string” between ghost spins

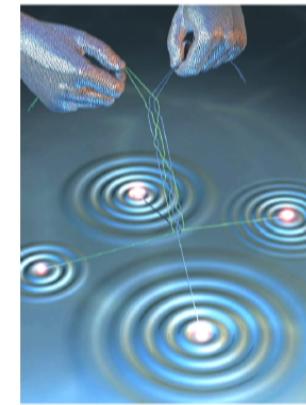
- ▶ monopoles with usual magnetic charge
- ▶ cheaper excitation energy  $\delta$  than bulk  $\Delta$ 
  - ▶ dominate for  $x e^{-\delta/T} > e^{-\Delta/T}$



# Disorder in topological physics

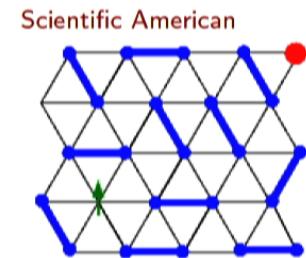
Impurities/defects can be (multiply) gauge charged

- ▶ interactions (poss. quantum statistical) via bulk
  - ▶ underpins topological quantum computation



Spin liquids are such topological phases

- ▶ impurity physics generally interesting
  - ▶ diagnosing spin liquids
  - ▶ access fractional particles



# Topological excitations in Kitaev spin liquid

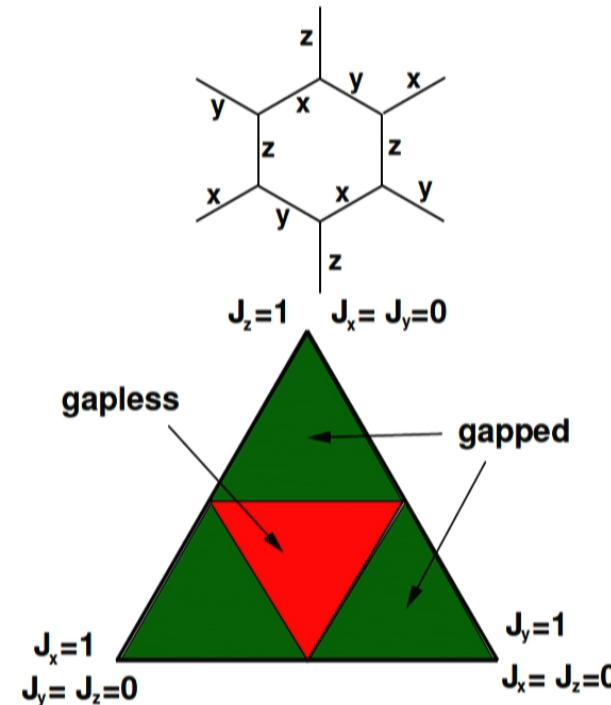
$$H = -J_x \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-links}} \sigma_j^z \sigma_k^z$$

Spins-1/2 on honeycomb lattice

- ▶ includes strong “spin-orbit coupling”

Two spin liquid phases

- ▶ one gapped, one gapless
- ▶ short-range spin correlations throughout Baskaran et al.
- ▶ model is exactly soluble

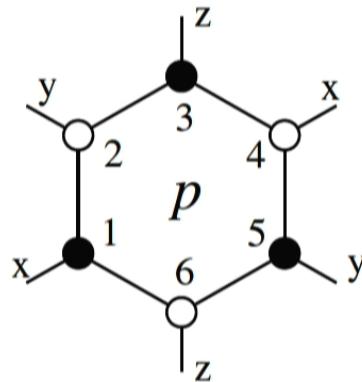


## Emergent degrees of freedom: fluxes and fermions

Local conserved quantities:

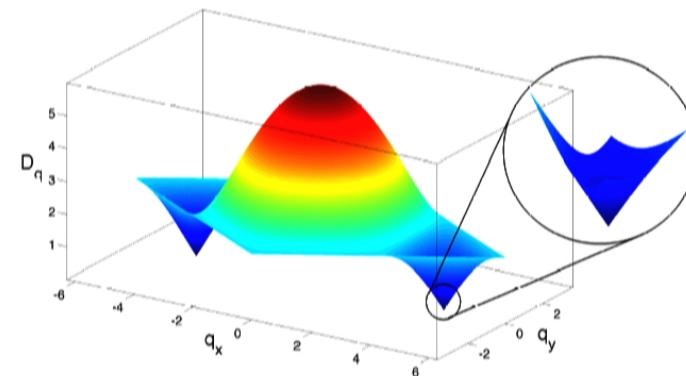
- ▶ plaquette fluxes  $[W_b, H] = 0$
- ▶ emergent  $Z_2$  gauge field

$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$



In each flux sector

- ▶ quadratic Majorana Fermion Hamiltonian
- ⇒ soluble honeycomb tight-binding model



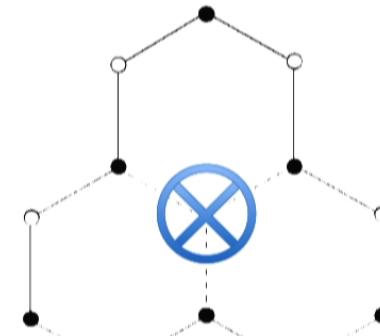
Signatures???

T/S matrices — dynamics — disorder

# Spontaneous flux attachment

Ground state flux sector

- ▶ No vacancies: flux free Lieb
- ▶ Each vacancy binds one  $Z_2$  flux
- ⇒ elegant way to see emergent fluxes in ground state!



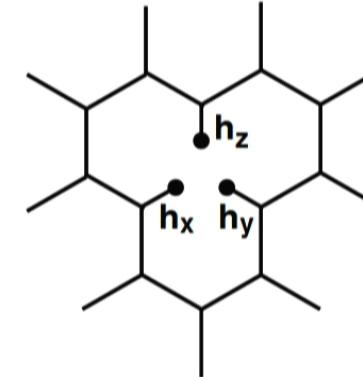
## Local moments and their interactions: gapless phase

Vacancy has 'field-dependent local moment'

- ▶ divergence of  $\chi \sim |\log h|$
- ▶ components of magnetisations sit on different sites
- ▶ qualitatively enhanced from the bulk

Interactions between vacancy moments

- ▶ depends on vacancy sublattices
- ▶ signature of fluxes!



*bulk*

*single vacancy*

*AB pair*

*AA pair*

$\chi \sim \text{const}$

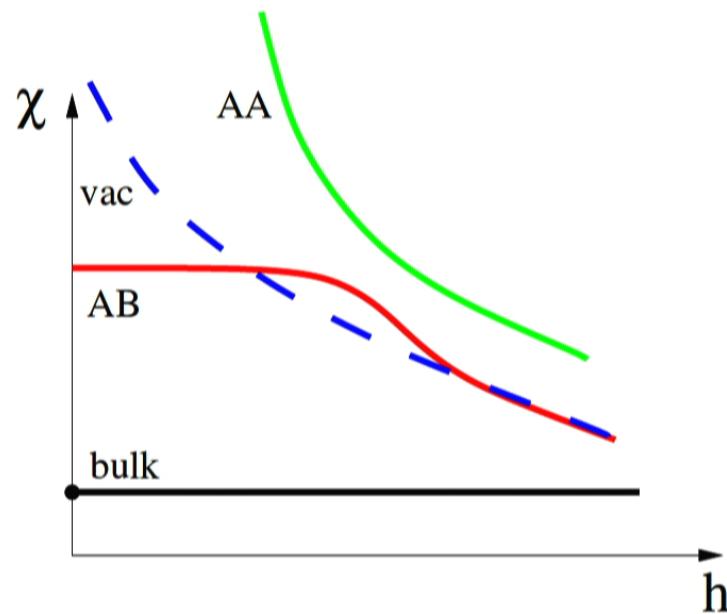
$\chi \sim |\log h|$

$\chi \sim (\text{large}) \text{ constant}$

$\chi \sim 1/[h|\log h|^{3/2}]$

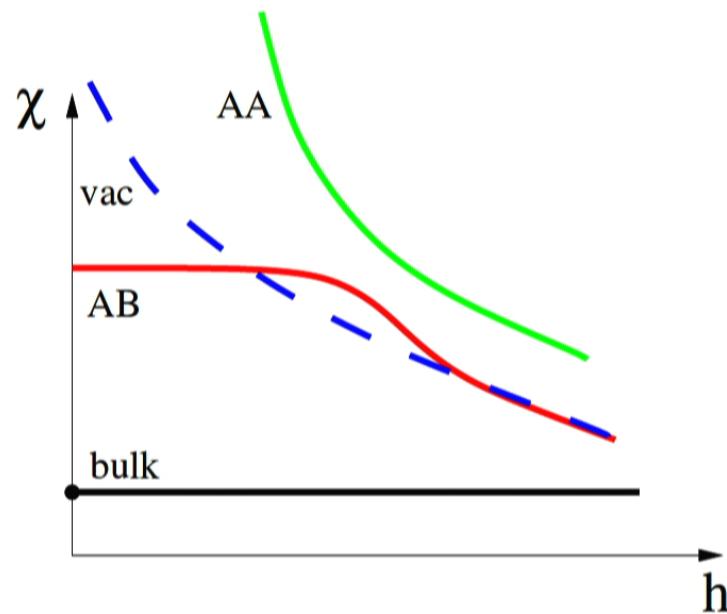
response for AA pair parametrically enhanced

## Susceptibility of vacancy configurations



<i>bulk</i>	<i>single vacancy</i>	<i>AB pair</i>	<i>AA pair</i>
$\chi \sim const$	$\chi \sim  \log h $	$\chi \sim (large) \; constant$	$\chi \sim 1/[h \log h ^{3/2}]$

## Susceptibility of vacancy configurations



<i>bulk</i>	<i>single vacancy</i>	<i>AB pair</i>	<i>AA pair</i>
$\chi \sim const$	$\chi \sim  \log h $	$\chi \sim (large) \; constant$	$\chi \sim 1/[h \log h ^{3/2}]$

## Local moments and their interactions: gapped phase

Vacancy zero mode: uncompensated bipartite lattice

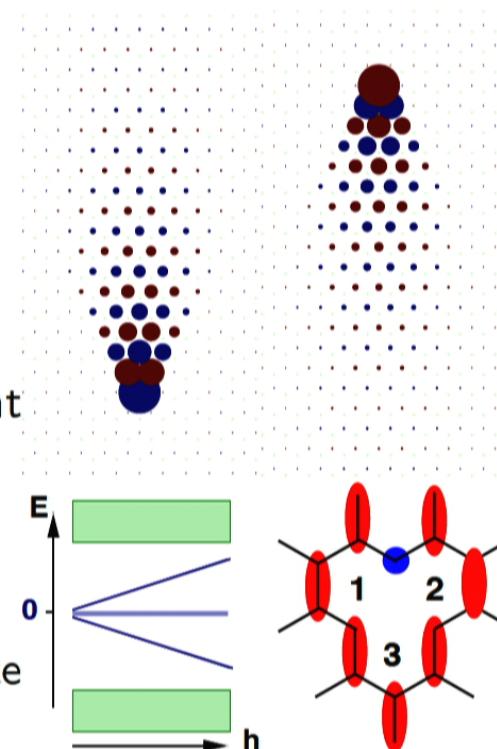
- ▶ confined to a wedge
- ▶ amplitude exponentially decaying along wedge
- ▶ wavefunction given by Pascal's triangle

Vacancy moment given by adjacent zero-mode amplitude

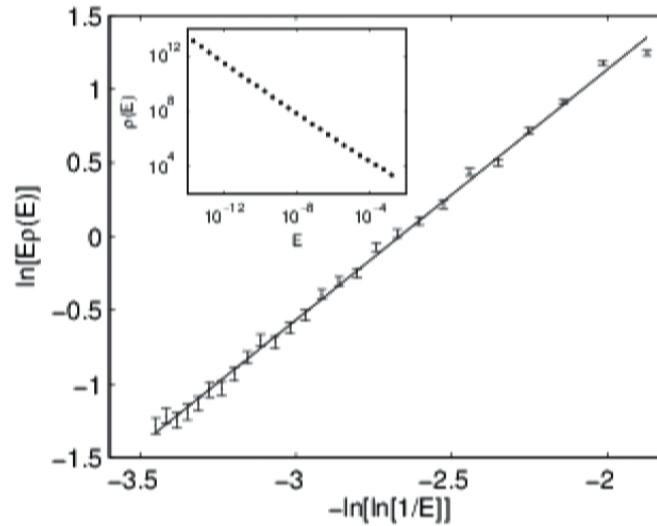
- ▶  $g_{\text{eff}} \rightarrow 1$  for  $J_{x,y}/J_z \rightarrow 0$
- ▶  $g_{\text{eff}} \rightarrow 0$  for  $J_x + J_y \rightarrow J_z$

Many vacancies: random bipartite hopping problem

- ▶  $\rho(E) \sim 1/[E(\log E)^{1.7}]$



## Numerical results for d.o.s. of the Kitaev model



- ▶ divergent density of states near zero energy:  
 $\rho(E) \approx 1/[E(\log E)^{1.7}]$
- ▶ divergent correlation length near zero energy

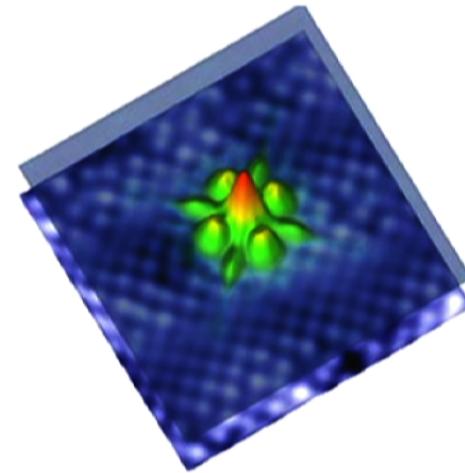
## Disorder as a probe: diagnosing spin liquids

What kind of spin liquid is gapless phase?

- ▶ We lack systematic understanding of gapless spin liquids
- ⇒ We do not really know

Response to disorder as diagnostic?

- ▶ flux attachment
- ▶ local moment formation



Davis et al.

## Bond randomness in gapless phase, $\sigma = \langle (\delta J)^2 \rangle$

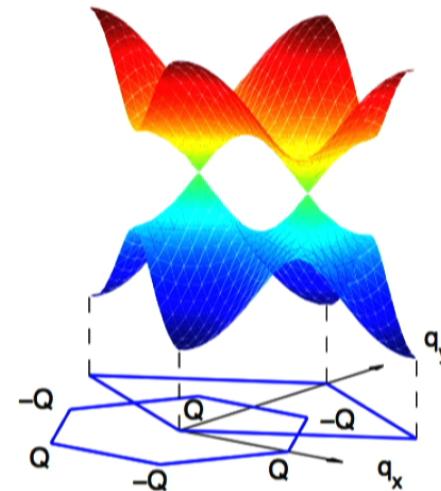
Fermionic excitations: massless Dirac particles

- exchange disorder translates into random vector potential

Power law exponents vary with disorder strength  $\sigma$  Ludwig et al, 1994

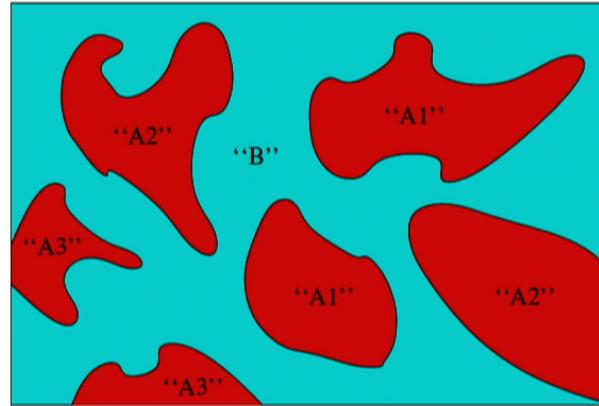
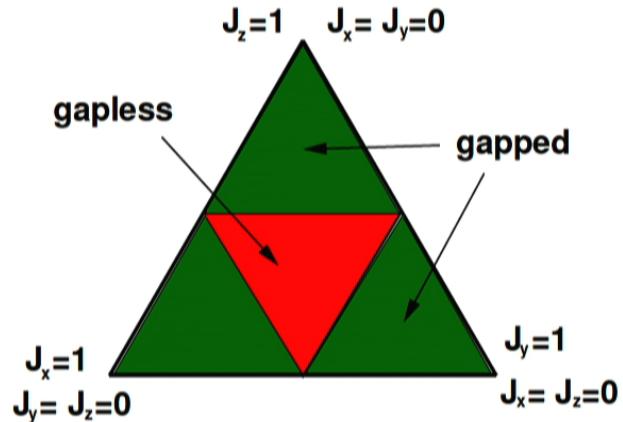
- density of states:  $\rho(E) \sim E^{\frac{1-\sigma}{1+\sigma}}$
- specific heat:  $C \sim T^{\frac{2}{1+\sigma}}$

No qualitative change to susceptibility  
⇒ implications for other spin liquids?



## More disorder

- ▶  $\pm J$  disorder  $\Rightarrow$  can be 'gauged away'
- ▶ Slowly varying, strong disorder - percolating gapless background



$\implies$  implications for transport?

# Topological spin glass in diluted spin ice

Dilution leaves Coulomb phase intact

- ▶ but generates ghost spins

Ghost spins interact like normal spins

- ▶ dipolar interactions
- ▶ entropic interactions

Ghost spins freeze

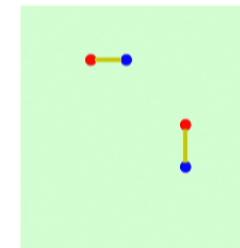
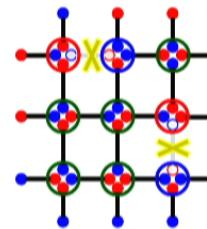
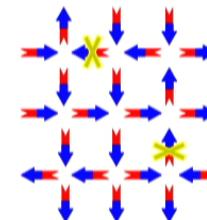
- ▶ glass + topological phase

Coulomb phase acts as substrate for

- ▶ emergent ghosts
- ▶ entropic interactions

General feature of topological phases

- ▶ cf. topological quantum computing



# Disorder in frustrated magnets

Many roles and types

- ▶ useful, interesting, nuisance

Topological glass in spin ice

- ▶ dipolar glass of ghost spins

Vacancies in Kitaev QSL

- ▶ ground state: flux nucleation
- ▶ excitations: local moment formation
- ▶ new collective physics

Many other instances

- ▶ orphan physics in classical SL
  - ▶ fractionalised 1/2 or 1/3 moment
  - ▶ random Coulomb antiferromagnet

