Title: Invariant Set Theory - A Realistic Causal Approach for Synthesising Quantum and Gravitational Physics?

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Abstract: <p>As discussed in last weekâ€™s colloquium, the use of the p-adic metric in state space provides a route to resolving the Bell Theorem in favour of realism and local causality, without fine tuning. Here the p-adic integers provide a natural way to describe the fractal geometry of Invariant Set Theoryâ€™s state space. In this talk I first explore the role of complex numbers in Invariant Set Theory (arXiv:1605.01051), and describe a novel realistic perspective on quantum interferometry. Then I will describe a programme of work to synthesise quantum and gravitational physics realistically and causally within the framework of Invariant Set Theory. I will describe a p-adic generalisation of the field equations of General Relativity, and discuss the consequent novel perspectives for understanding the dark (energy and matter) universe. </p>

Invariant Set Theory:

A realistic causal approach for synthesising quantum and gravitational physics?

http://arxiv.org/abs/1605.01051

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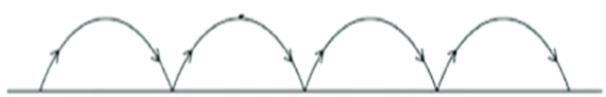
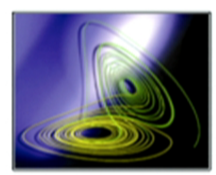
In a Nutshell

- Use Euclidean distance in space-time. 
- Use p-adic distance in state space. 
- This strategy is consistent with treating the universe as a nonlinear dynamical system evolving, in its state space, on a fractal invariant set homeomorphic to the *p*-adic integers, for large but finite *p*.



“cyclic” universe



In previous talk we used the p-adic metric + the number theoretic result:

$$\cos\theta,\ \cos\phi \text{ describable by finite \# bits} \Rightarrow \text{(almost certainly) } \cos(\theta-\phi) \text{ not describable by finite \# bits}$$

to provide a realistic locally causal account of the Bell Inequalities without fine tuning.

Theorem. Let $\frac{\theta}{2\pi} \in \mathbb{Q}$. Then $\cos\theta \notin \mathbb{Q}$ except when $\cos\theta = 0, \pm\frac{1}{2}, \pm 1$

Proof. (J. Jahnel). Assume $2\cos\theta = \frac{a}{b}$, where $a,b \in \mathbb{Z}$, $b \neq 0$ have no common factors

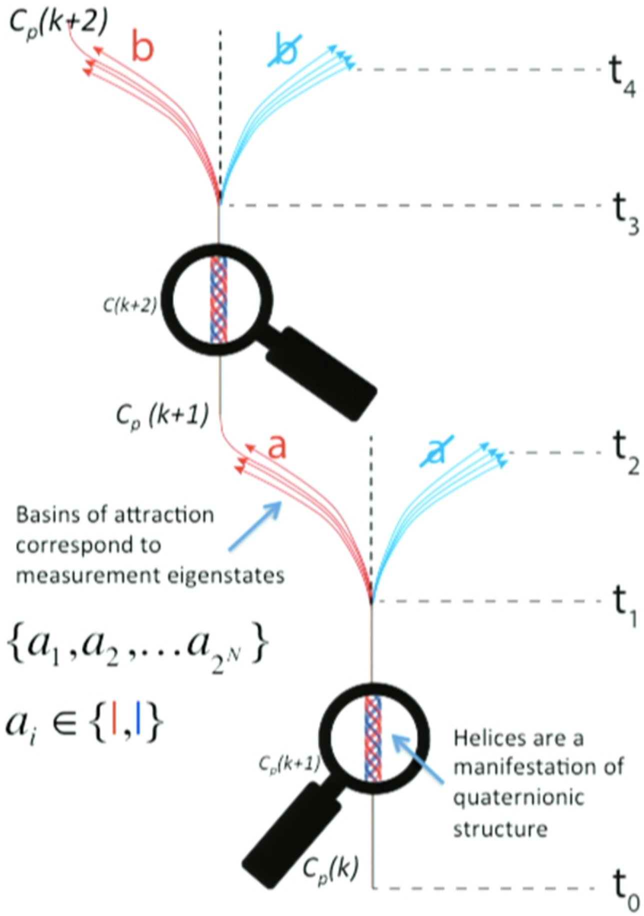
Since $2\cos 2\theta = (2\cos\theta)^2 - 2$ then

$$2\cos 2\theta = \frac{a^2 - 2b^2}{b^2}$$

Now $(a^2 - 2b^2)$ and b^2 can have no common factors. Hence, if $b \neq \pm 1$ then the denominators $b, b^2, b^4, b^8, \ldots$ in the sequence

(1) $2\cos\theta,\ 2\cos 2\theta,\ 2\cos 4\theta,\ 2\cos 8\theta \ldots$

get bigger without limit. However, if $\frac{\theta}{2\pi} = \frac{m}{n}$, $m,n \in \mathbb{Z}$ then (1) can admit at most n values. Contradiction. Hence $b = \pm 1$.



Decoherence, Measurement and Fractal Structure

$$I_U \approx \mathbb{R} \times C_p$$

$$C_p = \bigcap_k C_p(k)$$

$$p = 2^N + 1$$

A bit reminiscent of Many Worlds, but no branching!

Complex numbers as bit-string permutation operators

$S = \{a_1 a_2 a_3 a_4\}$

$a_i \in \{|,|\} \qquad \neg | = | \qquad \neg | = |$

Define $e^{i\pi/4}(S) \equiv \{\neg a_4 a_3 a_1 a_2\}$

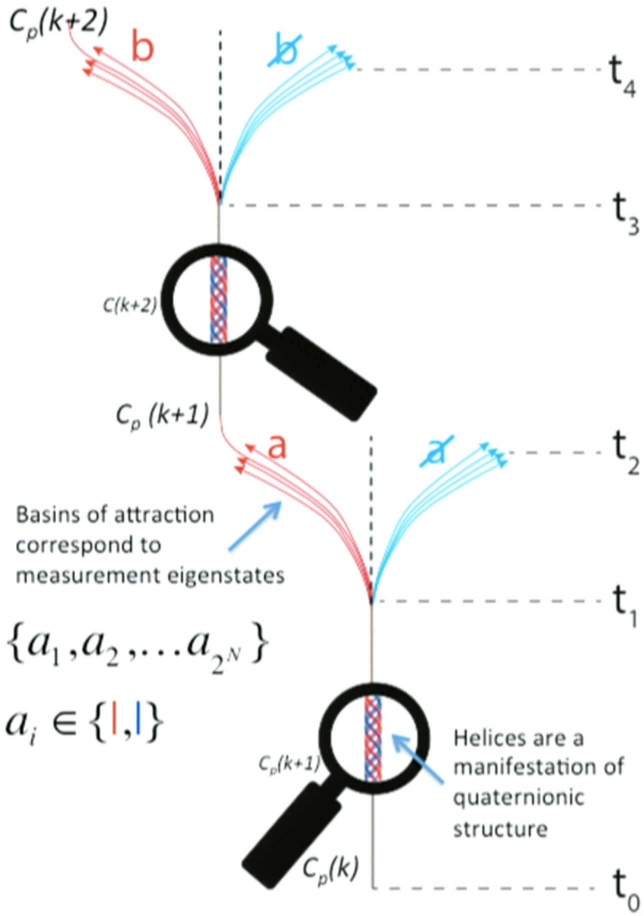
$\Rightarrow \; \mathrm{e}^{i\pi/4} \circ e^{i\pi/4}(S) \equiv e^{i\pi/2}(S) = \{\neg a_2 a_1 \neg a_4 a_3\}$

$\Rightarrow e^{i\pi/2} \circ e^{i\pi/2}(S) \equiv e^{i\pi}(S) = \{\neg a_1 \neg a_2 \neg a_3 \neg a_4\} \equiv -S$

Generalises to $e^{i\phi}(S)$ with $S = \left\{a_1 a_2 \ldots a_{2^N}\right\}$

for all $\phi / 2\pi$ describable by finite N bits.

Decoherence, Measurement and Fractal Structure



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Correspondence between trajectories on I_U and complex Hilbert Space vectors

$$\{a_1, a_2 \ldots a_{2^N}\} \mapsto \cos\frac{\theta}{2}|\ |\ \rangle + \sin\frac{\theta}{2} e^{i\phi} |\ |\ \rangle$$

$$a_i \in \{|,|\}$$

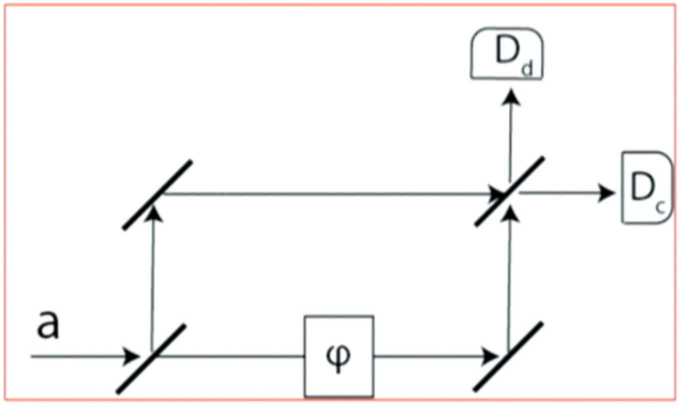
but only when

$$\cos\theta \in \mathbb{Q} \qquad \frac{\phi}{2\pi} \in \mathbb{Q}$$

N qubits $\Rightarrow$ N bit strings (see *arXiv*:1605.01051)

Quantum Interference

$U_1 =$



$$U = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad V = \begin{pmatrix} 1 & \\ & e^{i\phi} \end{pmatrix}$$

$$|a\rangle \xrightarrow{UVU} \cos\frac{\phi}{2}|c\rangle + \sin\frac{\phi}{2}|d\rangle$$

$$\Rightarrow \cos\phi \in \mathbb{Q}$$

Quantum Interference

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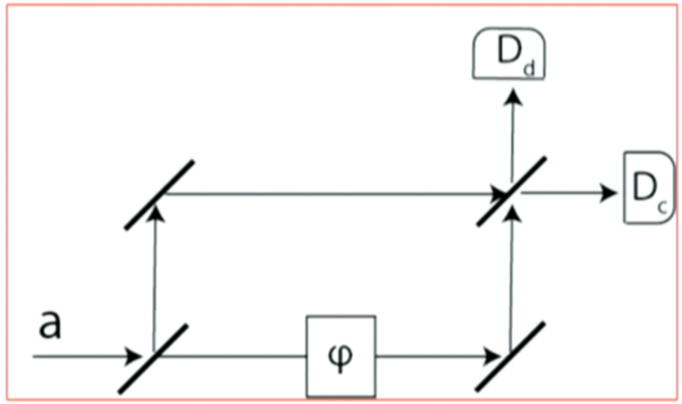


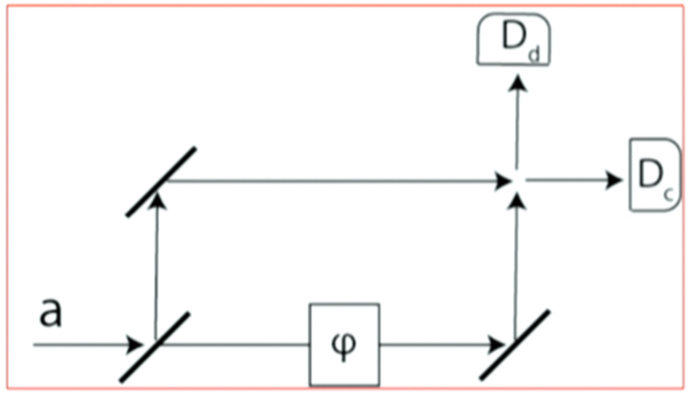
$\in I_U$

$$U = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad V = \begin{pmatrix} 1 & \\ & e^{i\phi} \end{pmatrix}$$

$$|a\rangle \xrightarrow{UVU} \cos\frac{\phi}{2}|c\rangle + \sin\frac{\phi}{2}|d\rangle$$ ✔

$$\Rightarrow \cos\phi \in \mathbb{Q}$$

Welcher Weg?

$U_2 =$  $\notin I_U$

$$|a\rangle \xrightarrow{VU} \frac{1}{\sqrt{2}}\{|c\rangle + e^{i\phi}|d\rangle\}$$

$$\cos\phi \in \mathbb{Q} \Rightarrow \text{a.c. } \phi/2\pi \notin \mathbb{Q}$$

$$\Rightarrow \text{no } \{a'_1, a'_2 \ldots a'_{2^N}\} \text{ corresponding to } |c\rangle + e^{i\phi}|d\rangle$$ ✗

Uncertainty Principle from Number Theory!!

Is this fine tuning gone mad? No!

- $U_1 \in I_U$ denotes the universe where an interferometric experiment is performed in MZ(ϕ) on some particle p.
- $U_2 \notin I_U$ denotes the counterfactual universe where a Which Way experiment is performed in MZ(ϕ) on p.
- Let $U_{2'} \in I_U$ denote the universe where a Which Way experiment is performed in MZ(ϕ') on p and $\frac{\phi'}{2\pi}$ finitely describable.
- $U_{2'}$ is not p-adically close to the counterfactual U_2 even if $|\phi' - \phi| \ll 1$.

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Mysteries of Quantum Physics

- Bell
- Quantum interferometry
- Sequential Stern-Gerlach
- Pusey et al

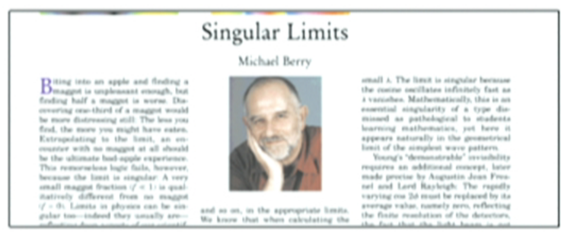
All resolved from number-theoretic properties of the cosine function + use of p-adic distance in state space.

Where Does Quantum Theory Fit ?

- Fractal dimension of C_p is log p/ log (2p-1) ➔1 as p ➔∞. But [0,1] is the **singular limit** of C_p at p=∞ because the measure of C_p is strictly zero for all finite p.

- Similarly, the complex Hilbert Space arises as the **singular limit** of IS state space at p=∞.

- Old theories are generically the **singular limit** of new theories (Michael Berry).



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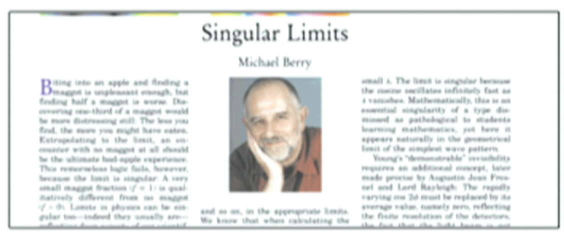


Singular Limits

Michael Berry

Biting into an apple and finding a maggot is unpleasant enough, but finding half a maggot is worse. Discovering one-third of a maggot would be more distressing still: The less you find, the more you might have eaten. Extrapolating to the limit, an encounter with no maggot at all should be the ultimate bad-apple experience. This remorseless logic fails, however, because the limit is singular: A very small maggot fraction ($f \ll 1$) is qualitatively different from no maggot ($f = 0$). Limits in physics can be singular too—indeed they usually are—

and so on, in the appropriate limits. We know that when calculating the

small λ. The limit is singular because the cosine oscillates infinitely fast as λ vanishes. Mathematically, this is an essential singularity of a type dismissed as pathological to students learning mathematics, yet here it appears naturally in the geometrical limit of the simplest wave pattern.

Young's "demonstrable" invisibility requires an additional concept, later made precise by Augustin Jean Fresnel and Lord Rayleigh: The rapidly varying cos 2δ must be replaced by its average value, namely zero, reflecting the finite resolution of the detectors,

$$\psi(0) = \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{pmatrix} \text{ where } S_i = (a_1, a_2, a_3 \ldots a_{2^N}) \text{ etc.}$$

Dirac Equation

$$\psi(\Delta t) = \begin{pmatrix} e^{i\omega\Delta t} & & & \\ & e^{i\omega\Delta t} & & \\ & & e^{-i\omega\Delta t} & \\ & & & e^{-i\omega\Delta t} \end{pmatrix} \psi(0)$$

In the singular limit $N = \infty$ and with $\hbar\omega = mc^2 \Rightarrow$

$$i\hbar\gamma_0\, \partial_t \psi + mc\psi = 0$$

Spin Structure

$S = \{a_1, a_2, a_3, a_4\}$

$a_i \in \{|,|\}$

$\mathbf{E_1}(S) = \{\neg a_2 a_1 a_4 \neg a_3\}$

$\mathbf{E_2}(S) = \{\neg a_4 a_3 \neg a_2 a_1\}$

$\mathbf{E_3}(S) = \{\neg a_3 \neg a_4 a_1 a_2\}$

$\Rightarrow$

$\mathbf{E}^2{}_j(S) = -S$

$\mathbf{E}_1 \circ \mathbf{E}_2(S) = \mathbf{E}_3(S)$

i.e. $\mathbf{E}_j$ are quaternions

Easily generalised when S has 2^N elements.

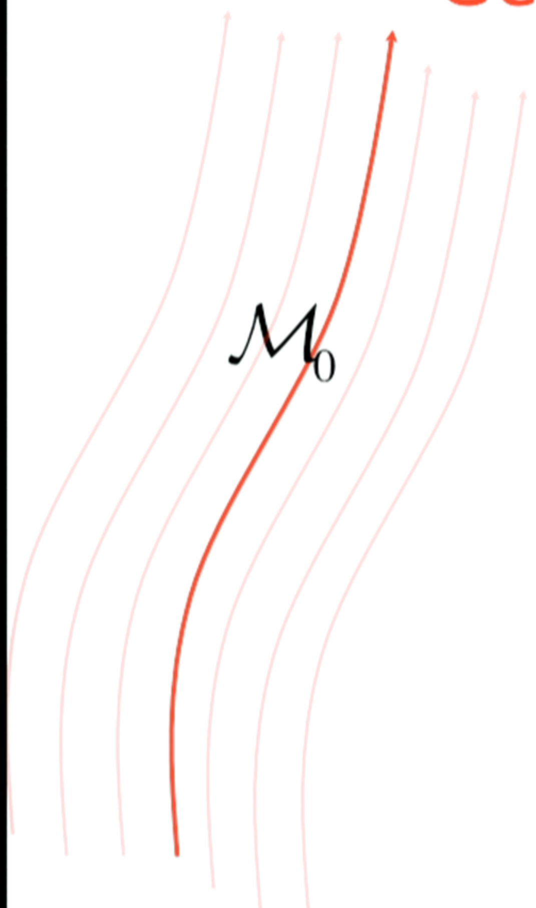
Easily related to Pauli Spin matrices and Dirac Gamma matrices

De Broglie Relationships

$$E = \hbar\omega \quad p = \hbar k$$

- In GR, a test particle's inertial properties are inherited from the neighbouring geometry of space-time.
- In Invariant Set Theory, a particle's energy-momentum in space-time is inherited from the neighbouring periodic geometry of I_U.
- Is the quantum potential of Bohmian theory a coarse-grain (L^2) representation of $d_p(x,y)$ on I_U ?

General Relativity



$$G_{\mu\nu}(\mathcal{M}_0) = T_{\mu\nu}(\mathcal{M}_0)$$

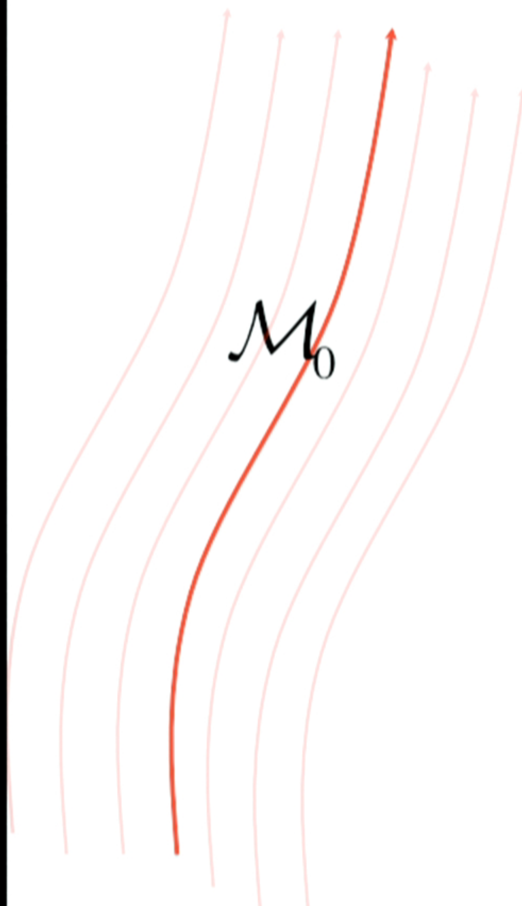
$$G_{\mu\nu}(\mathcal{M}_0) = \int T_{\mu\nu}(\mathcal{M})\delta(\mathcal{M},\mathcal{M}_0)d\mu$$

$$G_{\mu\nu}(\mathcal{M}_0) = \lim_{p\to\infty}\int T_{\mu\nu}(\mathcal{M})\Delta_p(\mathcal{M},\mathcal{M}_0)d\mu$$

where e.g.

$$\delta(x - x_0) = \lim_{p\to\infty}\Delta_p(x,x_0) = \lim_{p\to\infty}\frac{\sin px}{\pi x}$$

Generalised General Relativity



GR:

$$G_{\mu\nu}(\mathcal{M}_0) = \lim_{p\to\infty} \int_{I_U} T_{\mu\nu}(\mathcal{M})\Delta_p(\mathcal{M},\mathcal{M}_0)\,d\mu$$

where μ is a Haar measure. We now generalise the field eqns of GR by letting p be large but finite.

$$G_{\mu\nu}(\mathcal{M}_0) = \int T_{\mu\nu}(\mathcal{M})\Delta_p(\mathcal{M},\mathcal{M}_0)\,d\mu$$

Removing Space-Time Singularities?

$$G_{\mu\nu}(\mathcal{M}_0) = \int T_{\mu\nu}(\mathcal{M})\Delta_p(\mathcal{M},\mathcal{M}_0)d\mu$$

Smearing out over l_U

The Dark Universe:

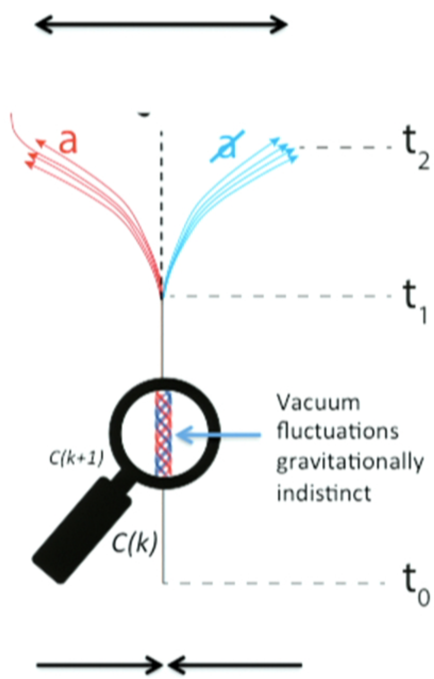
Where the Euclidean metric meets the p-adic metric?

$$G_{\mu\nu}(\mathcal{M}_0) = \int T_{\mu\nu}(\mathcal{M})\Delta_p(\mathcal{M},\mathcal{M}_0)d\mu$$

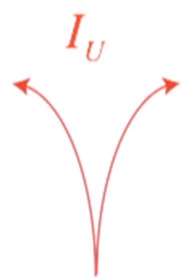
Curvature in our space-time influenced by energy-momentum in neighbouring space-times on the invariant set.

Dark Matter?

Gravitationally distinct
(Penrose, Diósi) d_p=1



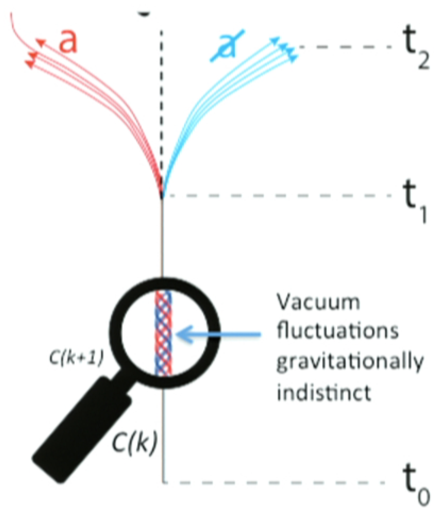
Gravitationally
indistinct d_p=1/p



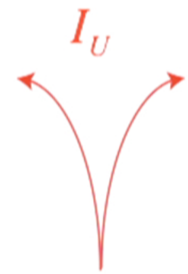
Dark Energy?

$$G_{\mu\nu}(\mathcal{M}_0) = \int T_{\mu\nu}(\mathcal{M})\Delta_p(\mathcal{M},\mathcal{M}_0)d\mu$$

Gravitationally distinct
(Penrose, Diósi) $d_p=1$

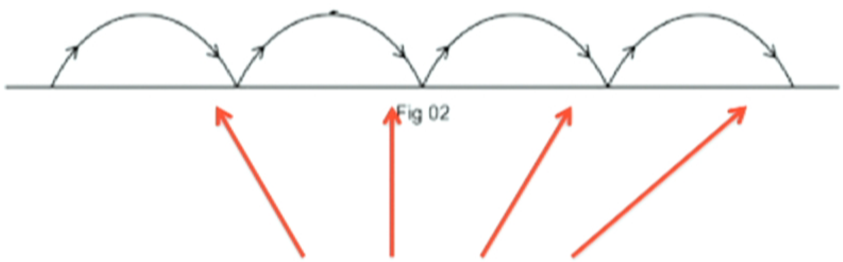


Gravitationally
indistinct $d_p=1/p$



$$G_{\mu\nu}(\mathcal{M}_0) = \int T_{\mu\nu}(\mathcal{M})\Delta_p(\mathcal{M},\mathcal{M}_0)d\mu$$

Quasi-Cyclic Universe



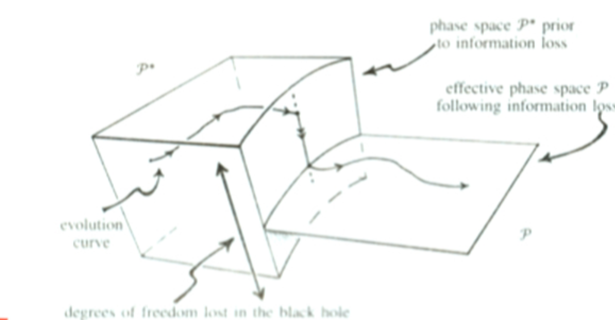


Fig. 3.14 Evolution in phase space following black-hole information loss.

Penrose, 2010

Information compression
but not loss

Conclusions

- Invariant Set Theory is a geometric/number theoretic way to interpret all the mysteries of quantum physics causally and realistically.
  - Nothing exotic like wormholes needed (e.g. ER=EPR explanation of entanglement).
  - Merely based on the state-space geometry associated with a generic class of nonlinear dynamical systems.
- IST is holistic but locally causal. Top-down constraint from cosmology to quantum physics implies we may need to abandon strictly reductionist approaches to fundamental physics.
- Easy to generalise GR to combine space-time pseudo-riemannian geometry with state-state p-adic geometry.
- Implies a new graviton-free approach to synthesise gravitational and quantum physics. Possible explanation of the dark universe, singularities, black-hole information paradox.