Title: Phenomenology of many-body localization

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Abstract: <p>I will review recent progress on theory of many-body localization, mostly focusing on properties of the many-body localized phase itself.</p>

<p>I will discuss explicit construction of effective Hamiltonians governing the dynamics of conserved quantities. The analysis reveals several inequivalent length scales in the system, some of which do not appear to diverge on the approach to the thermalized phase.</p>

<p>Experimental protocols to measure these length scales will also be discussed.</p>

- Classical vs. quantum thermalization
- Sharp alternatives to thermalization? Why do we need them?

- Many-body localization:
  - basic notions and tools
  - Phenomenology
  - spontaneous symmetry breaking

- beyond MBL?

Origins of classical statistical mechanics

- Boltzmann/Gibbs/Maxwell – dynamics generates entropy

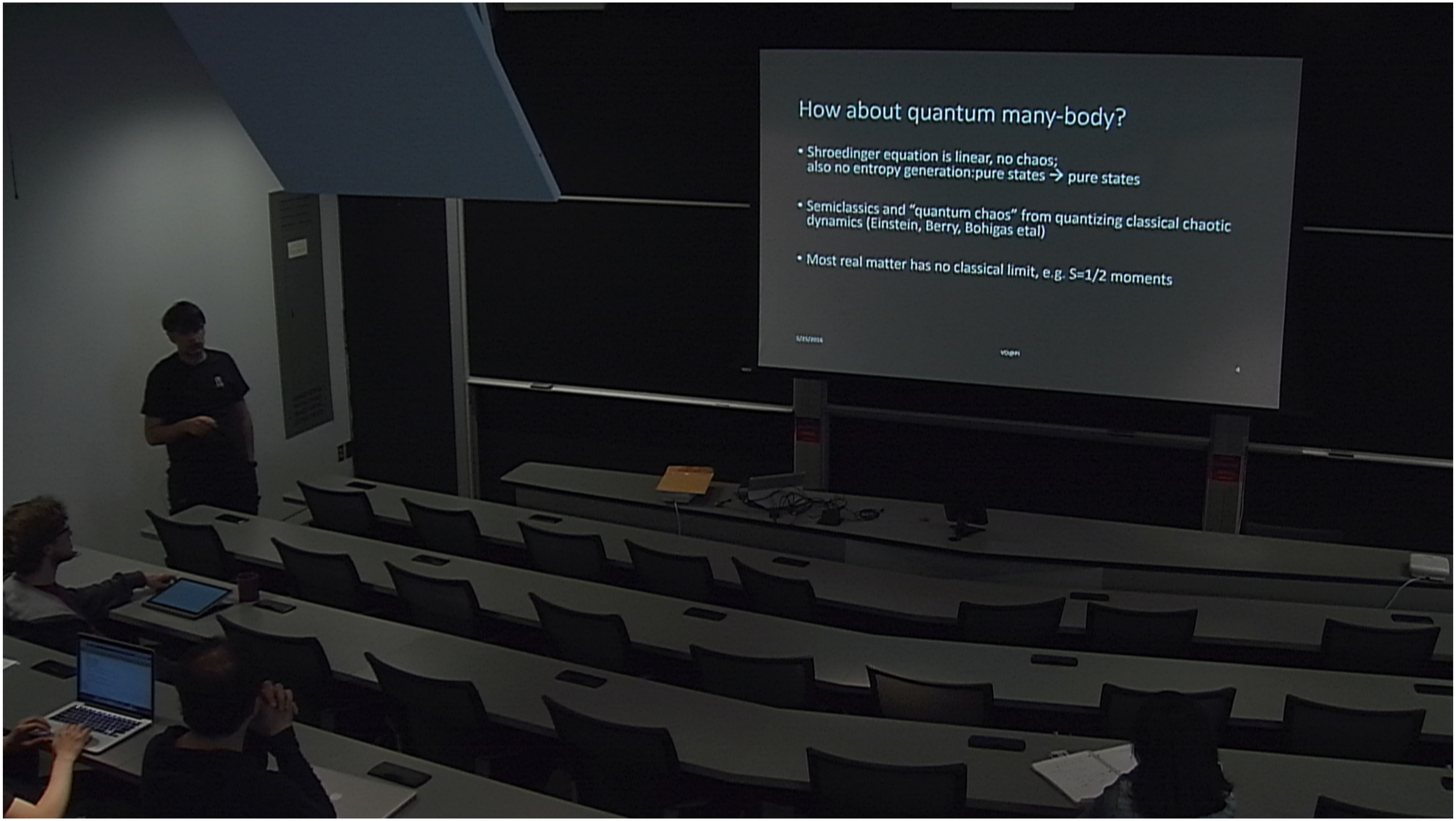
$$\ddot{x} = -V'(\{x\}) \Leftrightarrow P[\{x,p\}] = \exp\left[-\beta\left(\frac{p^2}{2} + V(x)\right)\right]$$

- Microscopic nature of temperature, chemical potential etc – fluctuations of energy and particles between subsystems

How about quantum many-body?

- Shroedinger equation is linear, no chaos;
  also no entropy generation:pure states → pure states

- Semiclassics and "quantum chaos" from quantizing classical chaotic dynamics (Einstein, Berry, Bohigas etal)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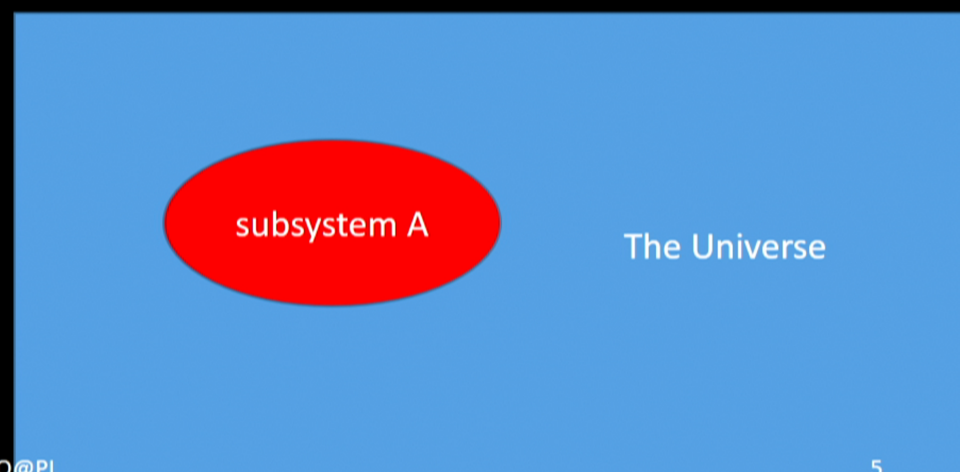

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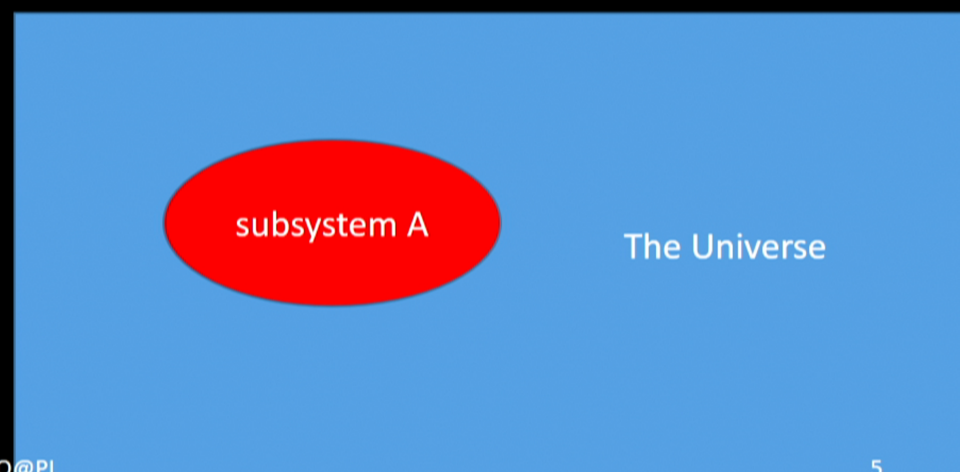

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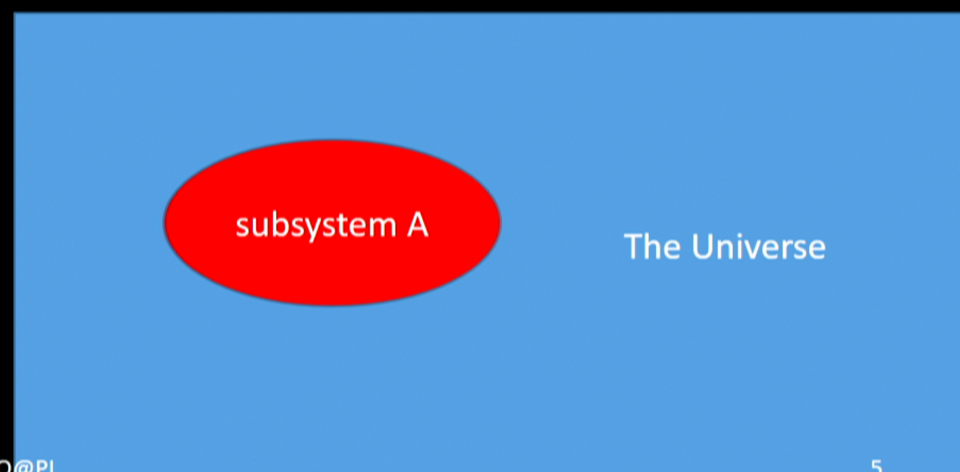

- Compute eigenstates?!
  Rigol etal 2008

Thermalization in quantum quenches

- Prepare a simple initial state, e.g. a product state, and apply $e^{-i\,H\,t/\hbar}$
- Athermal (dynamical) equilibrium at late times from the "diagonal ensemble":

$$\psi(t) = \sum_n a_n \mathrm{e}^{-iE_n t/\hbar} |n\rangle$$

$$\rho_A(t \to \infty) \approx \sum_n |a_n|^2 \rho_{nA}$$

- In thermalizing systems $\rho_A(t \to \infty) \propto \exp[-\beta_{\psi(0)} H_A]$ generically
- The process of thermalization?
  Quantum time evolution builds non-local patterns of entanglement in initially simple states; entanglement is the mechanism of thermalization

Why all the recent activity and progress?

- Cold atoms
  - availability of well controlled exp. setups, simple model systems
  - real time control and monitoring (intrinsic dynamics is nice and slow)
  - Eventually, experiments started pushing to strong correlations beyond BEC
  - ETH is most dramatic away from GS – cold atoms don't cool well anyway
- Gestation of ideas and training – accumulation of insights from statmech, critical phenomena, renormalization group, q.information
- Smarter use of (slightly) better computers

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- Theoretical examples of athermal states in integrable models, Floquet thermalization, <u>many-body localization</u>
- Experimental examples?

(absence of) spin diffusion in Si:P?

- Charge delocalizes around concentration 10^{18}
- Non-interacting spin regime (aka spintronics) below $10^{16}/\mathrm{cm}^3$?

- Localized electron moments (P) coupled to <u>quasistatic nuclei</u>:

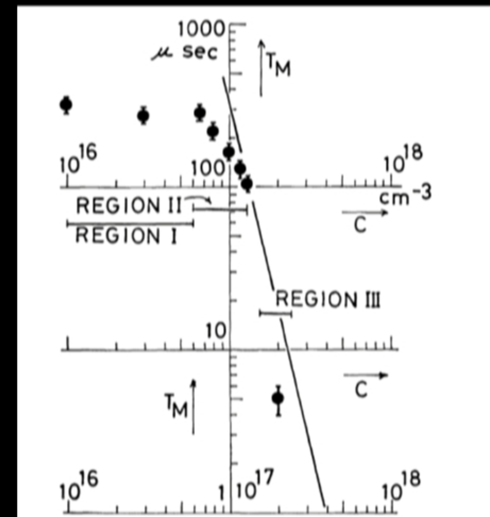
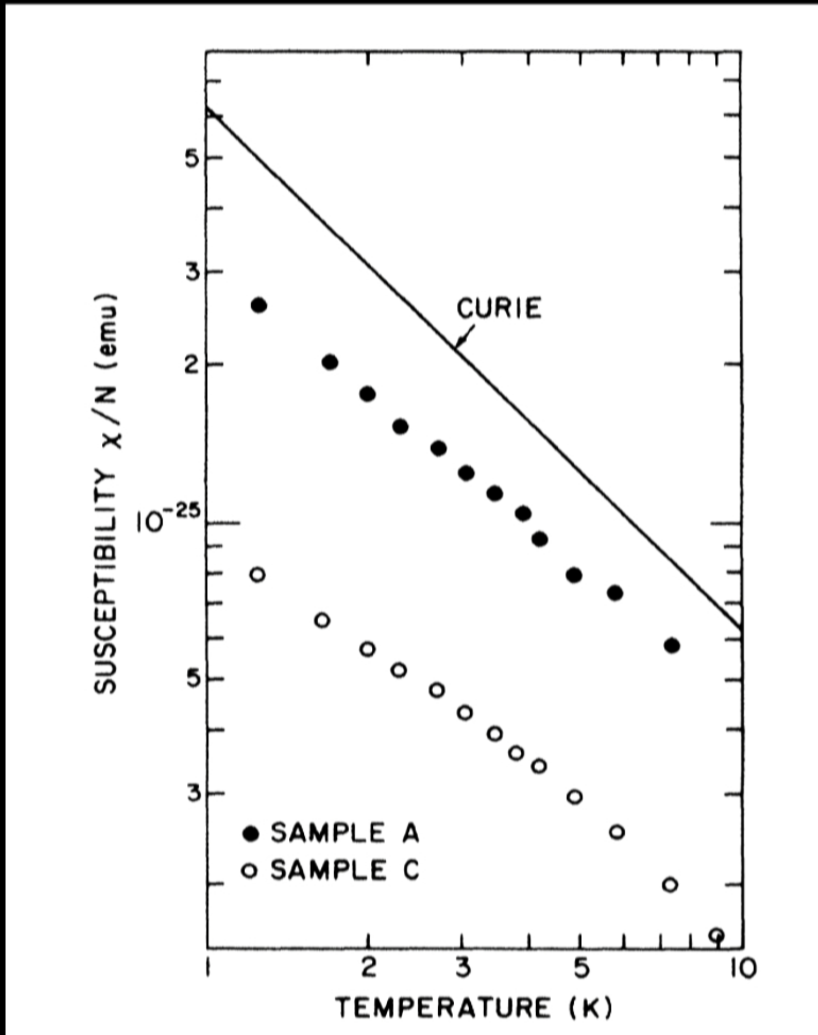


Fig. 2. The concentration dependence of the spin echo decay time at 1.6 K. The solid line is the curve obtained by using the experimental data of T_1 (see § 4 (3) in the text).

$$H = A \sum_j S_j \cdot \sum_{eta} I_{j+\eta} + phonons + \sum_{ij} J_{ij} S_i \cdot S_j$$

- What is the spin-diffusion constant?
  No direct experiments as of yet! But soon, hopefully.
- Lore in spintronics – exchange is weak and negligible, nuclear dynamics is more important



Here is the magnetic susceptibility of strongly insulating P doped Si

A) $n_P = 6.7 \times 10^{17}$ cm^{-3}

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M.P. Sarachik *et al*
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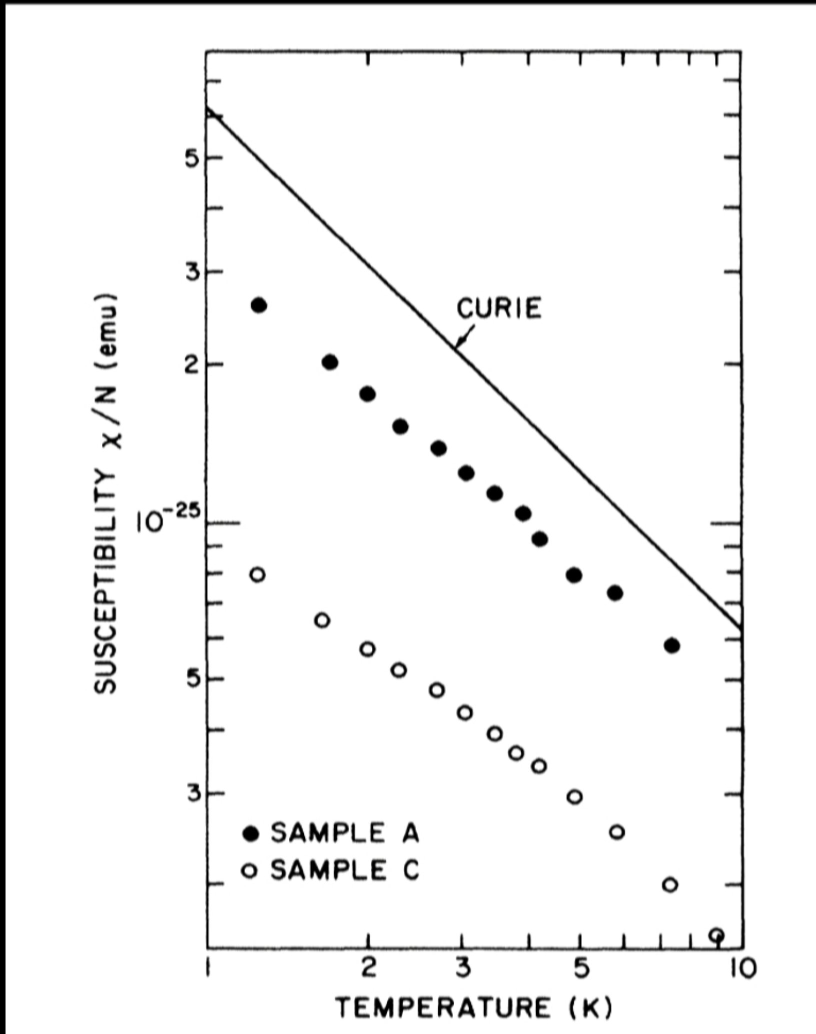

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(positive sign corresponds to paramagentism)

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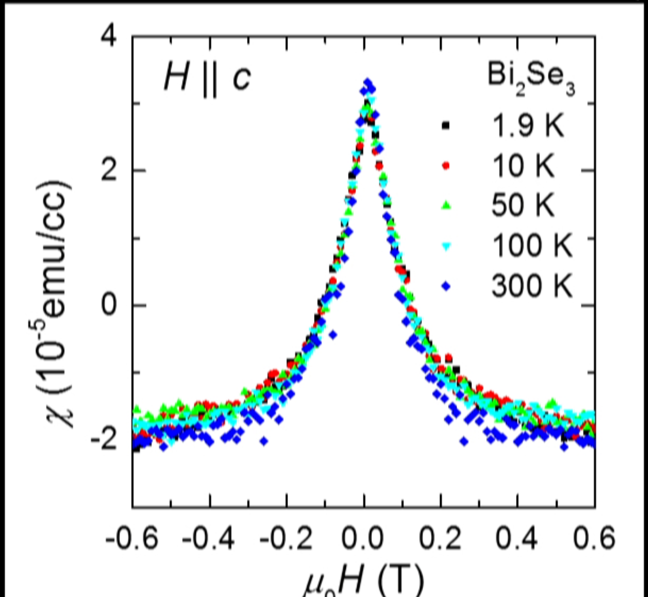

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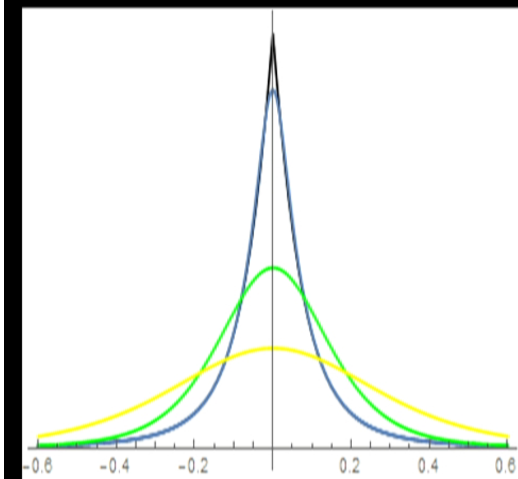
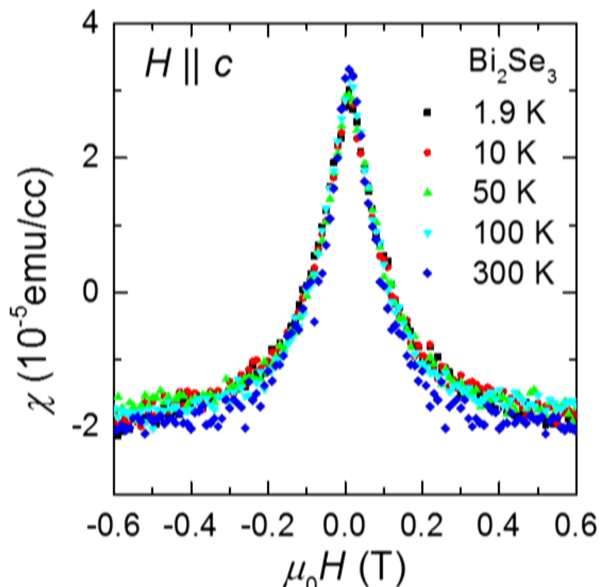


Equilibrium theory at T=0,1,5,10 K



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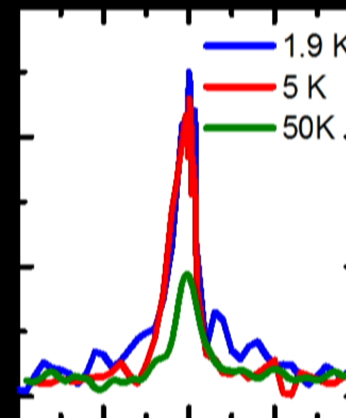

DC magnetometry
SQUID

- Paramagnetic anomaly from surfaces of 3D topological insulator crystals of $Bi_2Te_3, Bi_2Se_3, Sb_2Te_3$, new disorder physics (see Zhao etal, NatMat 2014)
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MBL outline

- What is MBL? A flavor of perturbation theory and beyond
- Phenomenology of the localized phase
- MBL and spontaneous symmetry breaking
- MBL futures

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Why there is (probably) no such thing as an weakly interacting/correlated Anderson insulator (**low T**)

Effect of weak repulsive interactions:

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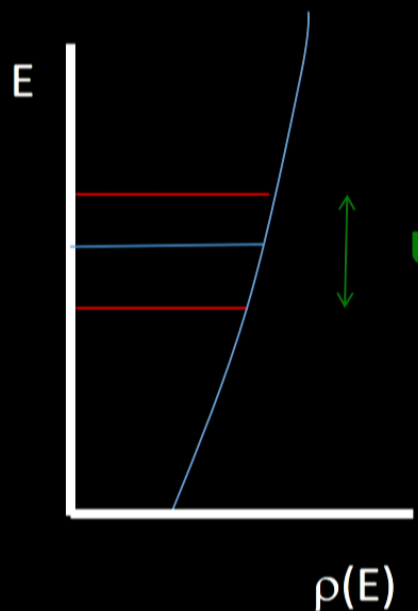

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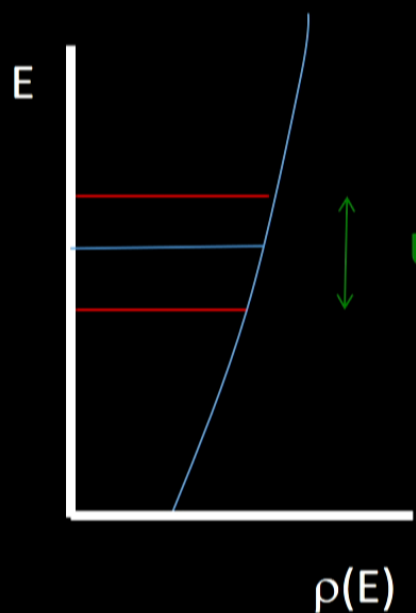

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Probably leads to spin-glass or random singlet phase.

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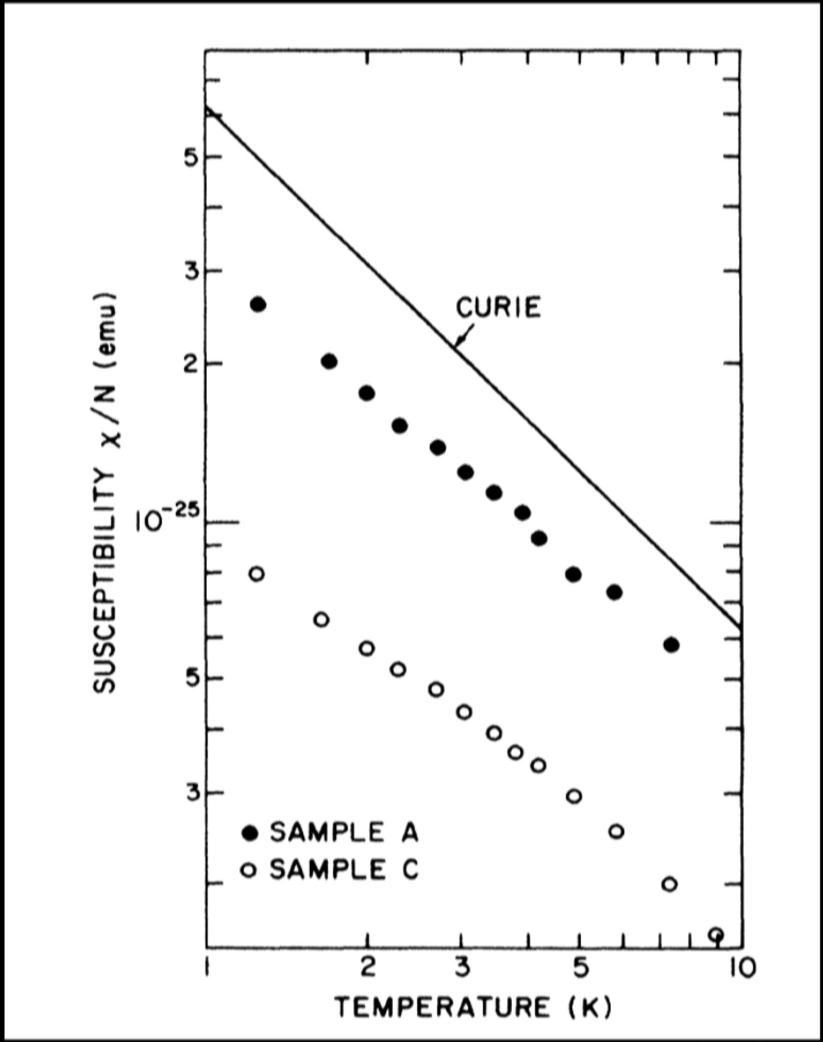

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- Common (?) belief until mid2000's:
  σ is finite but is (probably) highly non-perturbative in interaction and temperature (leading term in p.t. Fleischman/Anderson, 1980)

Basko/Aleiner/Altshuler 2006

- The stability of the weakly interacting insulator is controlled by excitation density (quasiparticle density), pert. theory converges for low but finite T
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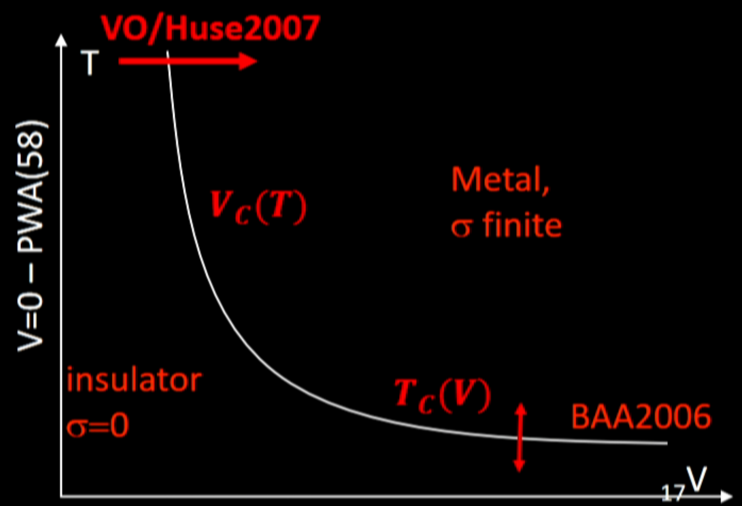
$$H = V\Sigma_j c_j^+ c_j c_{j+1}^+ c_{j+1} + U_j c_j^+ c_j + \sum_\eta t_\eta c_j^+ c_{j+\eta}$$

- (at least) two phases:ergodic metal and localized interacting phase "connected" to Anderson insulator, $|U_j| \gg t, V$
- A purely dynamical transition at nominally INFINITE temperature!



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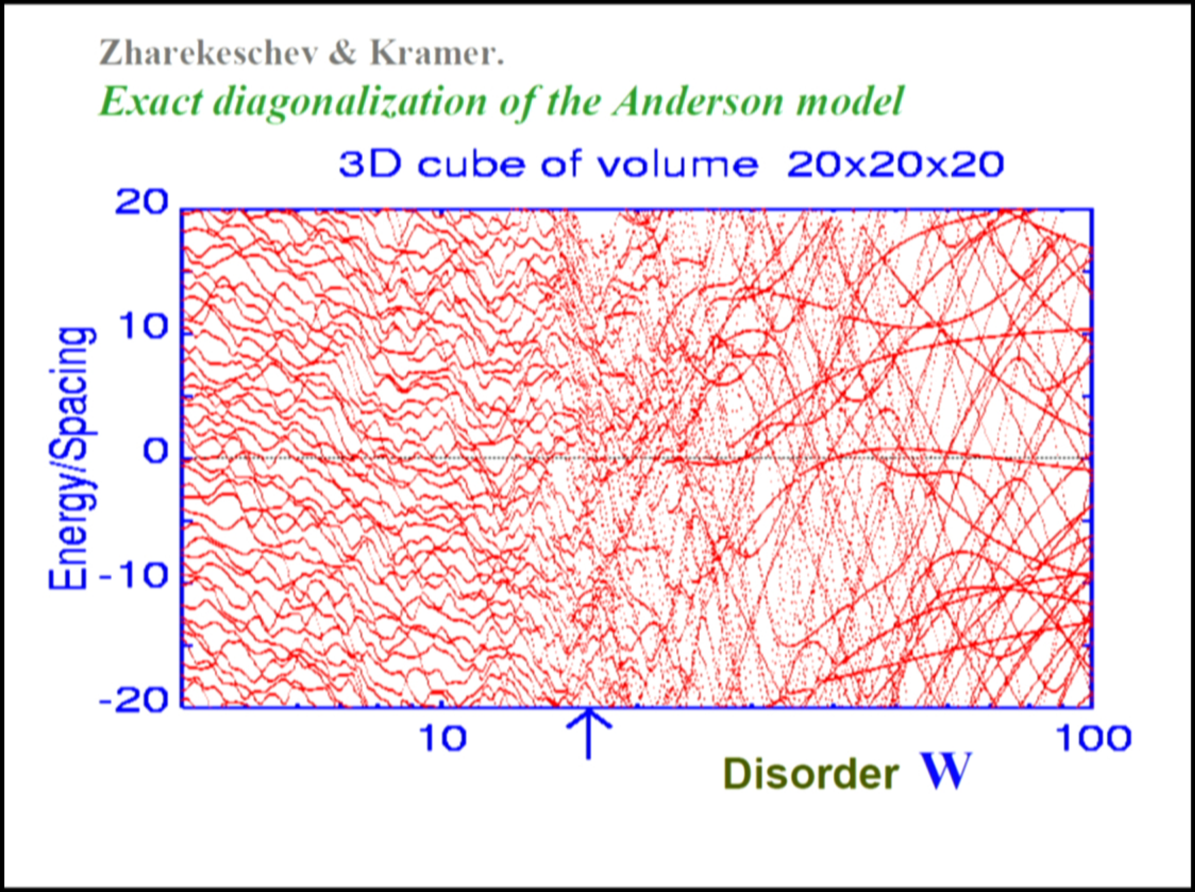
- Strongly disordered states should have weaker finite size effects
- Is there enough range to do scaling and identify the transition between L=4~6 to L=16?

What to compute? Insight from 3D Anderson

universality in (mini-) gap statistics – essentially DOS fluctuations



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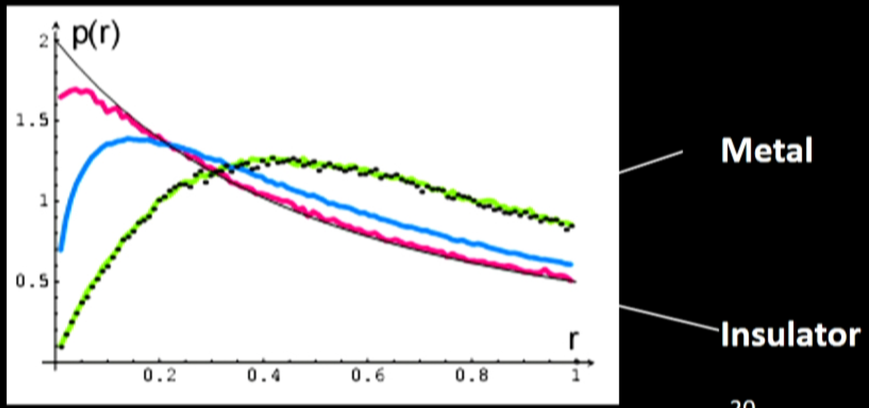
Level statistics as “order parameter” for many-body systems?

- Construct a “good” dimensionless observable from $\{\delta_n = E_{n+1} - E_n\}$

$$r_n = \min(\delta_n, \delta_{n+1}) / \max(\delta_n, \delta_{n+1})$$

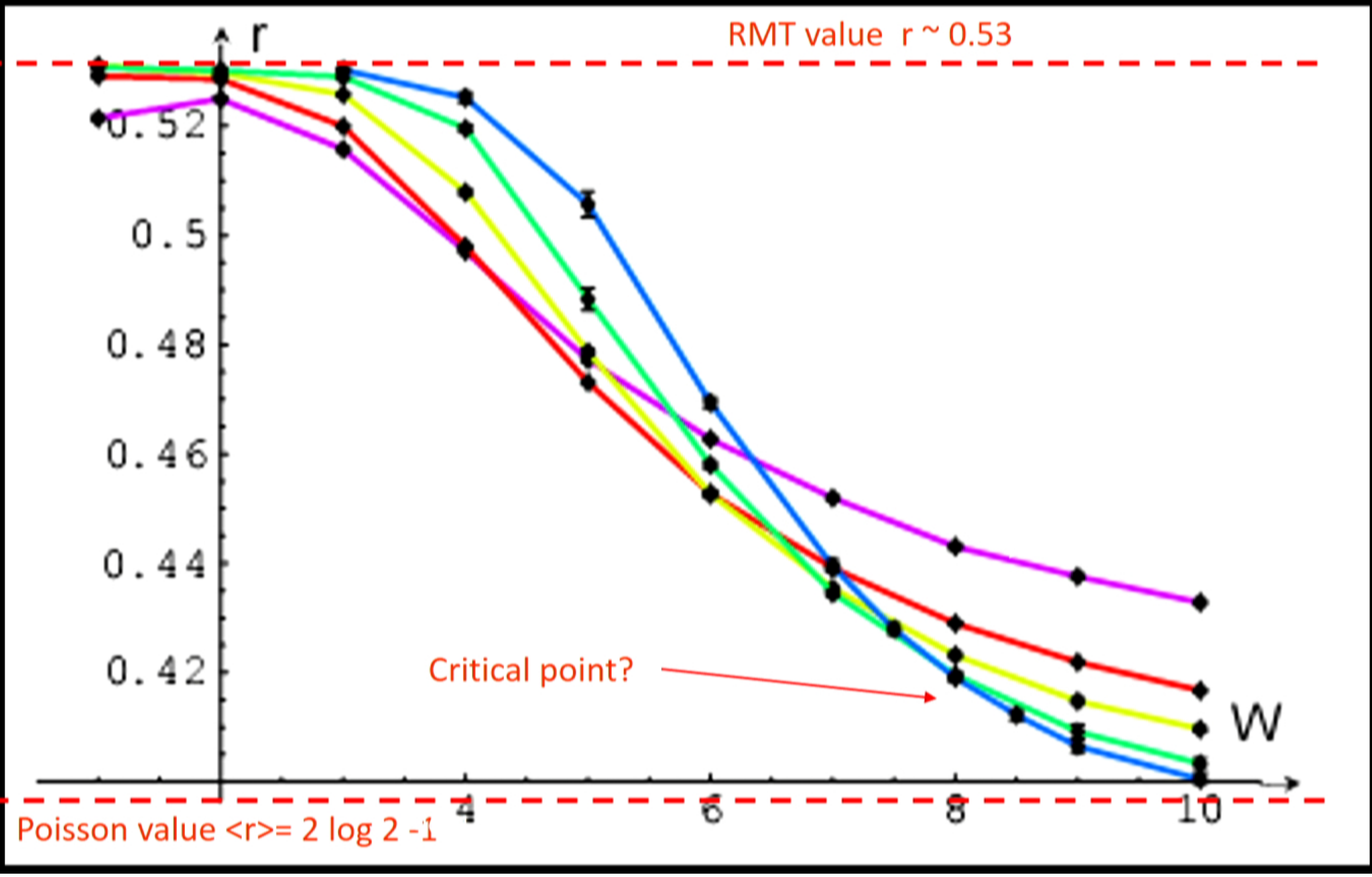
- Two universal distributions can be identified with the metal, Wigner-Dyson, $\langle r \rangle \approx 0.53$, and the insulator, Poisson, $\langle r \rangle = 2\log 2 - 1 \approx 0.39$



Numerical evidence for criticality

Sharpening of the crossover points to a phase transition at finite disorder

The critical value of the "order parameter" is surprisingly close to the Poison value – is the critical point also the endpoint of the MBL phase?



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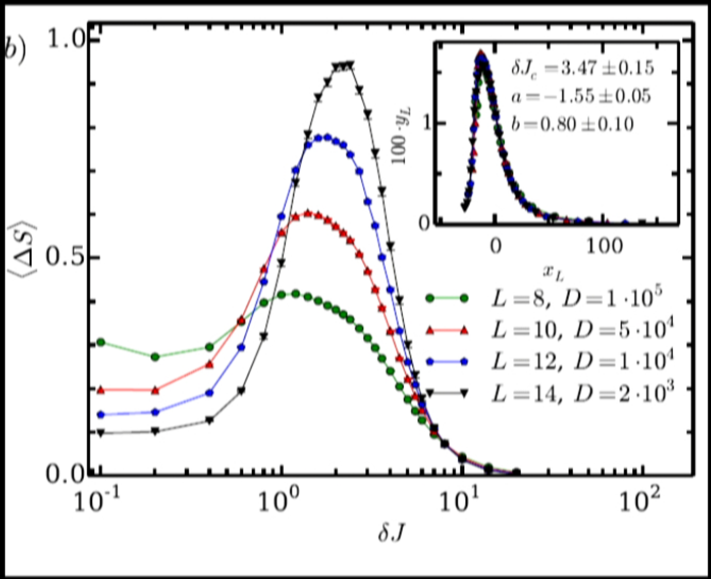
Future of ED+scaling approach to MBL?

- An extremely powerful approach for exploration of ideas: easy to setup, easy to avoid technical mistakes; anybody with ideas for better observables can test them quickly! Also, ideally suited for $T = \infty$, good averaging, least noise
- E.g. testing ETH by monitoring fluctuations in entanglement (Pal/Huse 2010, Kjall etal, 2014)
- Does it produce reliable information? Bounds on exponents (Chandran etal 2015)
- Future probably belongs to more efficient methods –MPS, cluster expansions,???



Phenomenology deep inside MBL

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- Qualitative and non-microscopic lines of attack
- Often helps guess new physics

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  Are MBL phases adiabatic continuations of Anderson localization or are there qualitatively new phenomena?

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- Analogy:
  Fermi liquid vs. Fermi gas → good phenomenology can give quick access to essential physics, e.g. collective modes

Phenomenology of Anderson insulators

$$H = \sum_{j=1..L-1} t\,(C_j^+ C_{j+1} + h.c.)\; + V_j C_j^+ C_j$$

- Single particle propertiles: $\langle C_i^+ C_j \rangle_n \sim \exp[-\alpha|i-j|]\,;\ \sigma(T) = 0$
- Occupations (0 or 1) of localized states are local quantum numbers
- Many-body spectrum is equivalent to L non-interacting S=1/2

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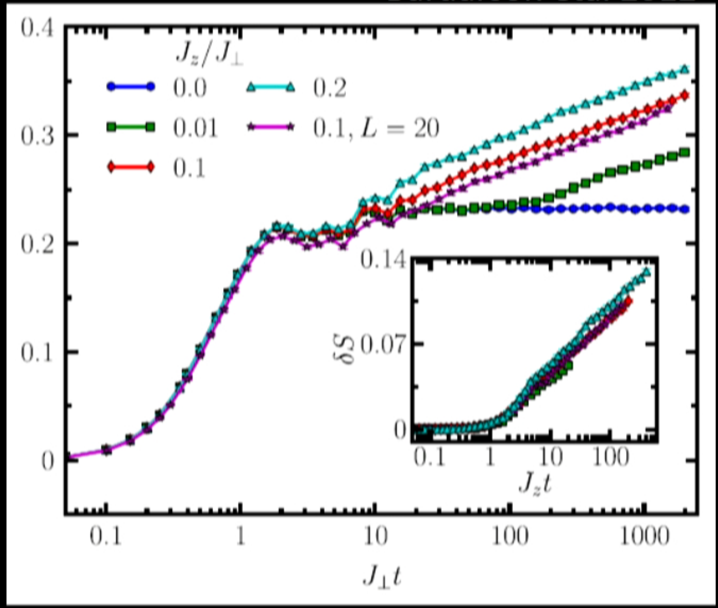
- MBL phase – couplings decay rapidly $\mathrm{J_{eff}} \propto \exp[-cluster\ size/\xi]$
- Local relationship to physical degrees of freedom, P-bits, e.g.

$$\tau_j^z = (1 - \cdots)\sigma_j^z + .0017\sigma_{j-1}^+ \sigma_{j+1}^- + \cdots$$

Applications of MBL phenomenology

- Consider making 2 excitations: $|\uparrow\uparrow\downarrow\searrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\nearrow\downarrow\uparrow\uparrow\rangle$
- Time – length relationship $J_{eff} \cdot t = \pi \rightarrow r(t) = \xi \log t$

Znidaric etal 2008
Bardarson etal 2012



- Explains log[t] growth of entanglement in DMRG
- Powerlaw decay temporal decays (Serbyn etal) e.g. $\langle \sigma_j^+(t)\sigma_j^-(0)\rangle \sim \cos(\ldots)/t\hat{}\gamma$
- Non-Mott AC conductivity $\propto \omega^\eta, 1 < \eta < 2$ (Gopalakrishnan etal)

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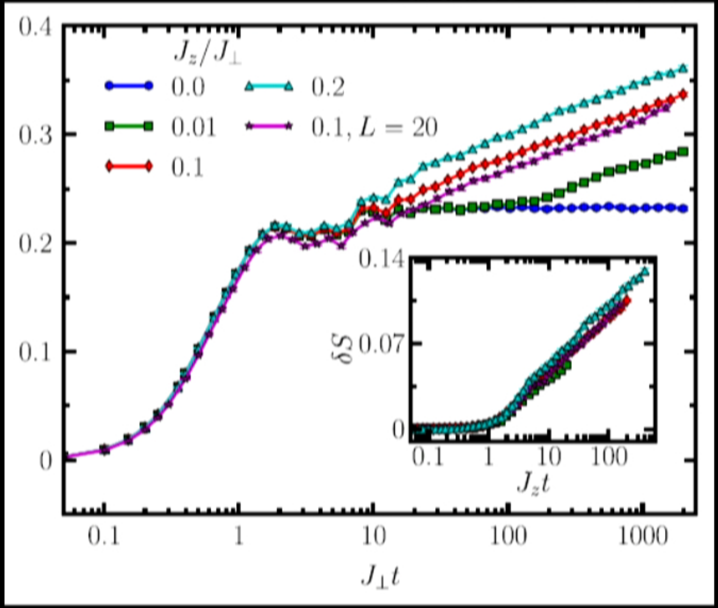
Bardarson etal 2012

- Explains log[t] growth of entanglement in DMRG
- Powerlaw decay temporal decays (Serbyn etal) e.g. $\langle \sigma_j^+(t)\sigma_j^-(0)\rangle \sim \cos(\ldots)/t\hat{}\gamma$
- Non-Mott AC conductivity $\propto \omega^\eta, 1 < \eta < 2$ (Gopalakrishnan etal)



Engineering L-bits: why and how

- Need to confirm that this phenomenology is basically correct
- Exponential ansatz for couplings $J_{eff} \propto \exp[-r/\xi]$ and log[t] "lightcone" are of mean-field nature. Are there "fluctuations"?

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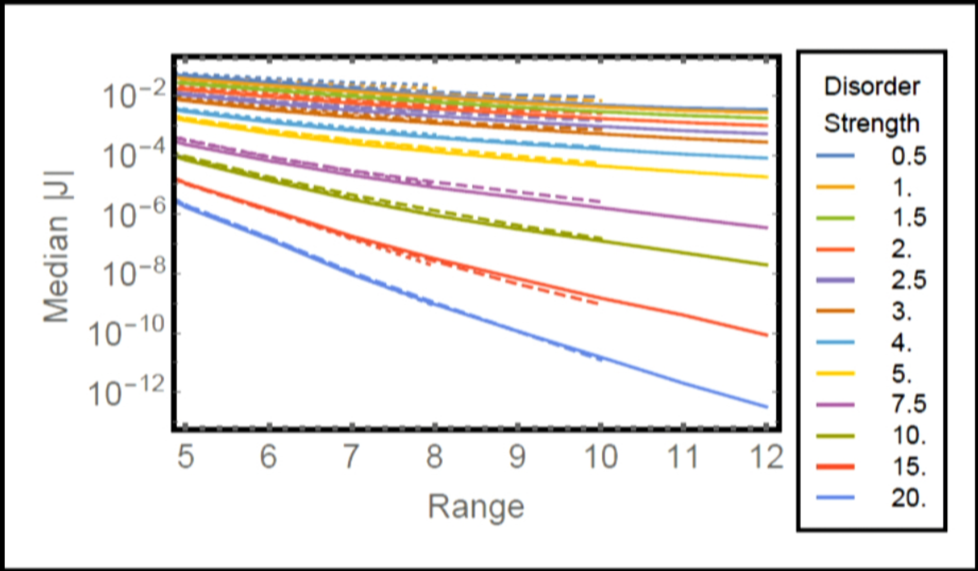
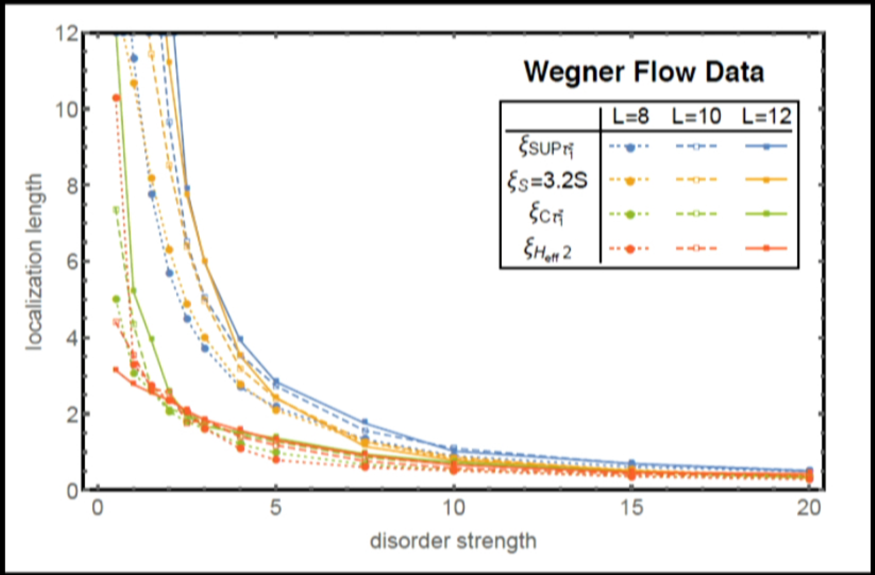
- While very similar for weak interactions these lengths differ significantly away from perturbative limit inside MBL, some remain clearly finite at the transition, others less clearly…
- In fact, “nobody has seen any clean evidence yet of a diverging length scale as the MBL transition is approached from within the MBL phase”

Preliminary results – critical divergence(s)? (Pekker etal)



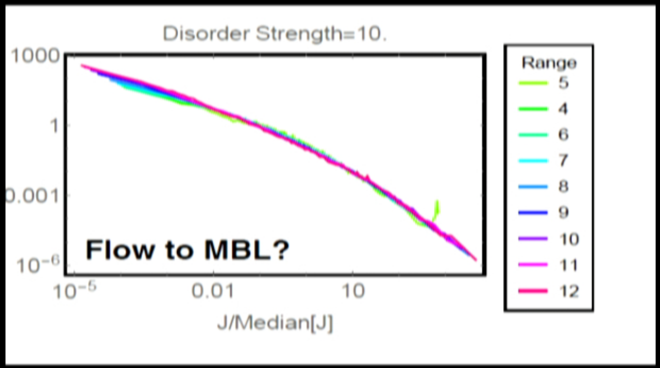


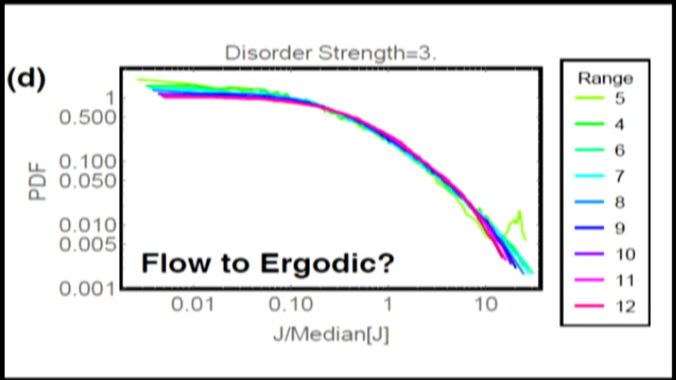
Fluctuation effects inside the MBL?

Rather than looking at mean/median of couplings in

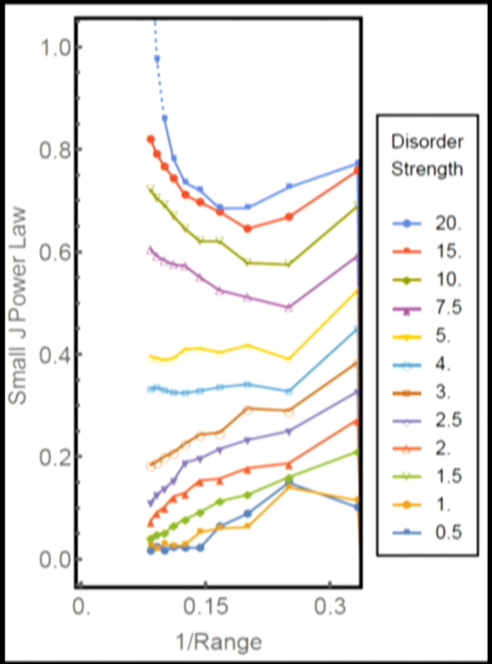
$$H = \sum_j B_j \tau_j^z + \sum_{jk} C_{jk} \tau_j^z \tau_k^z + \sum_{jkl} D_{jkl} \tau_j^z \tau_k^z \tau_l^z + \cdots \ldots$$

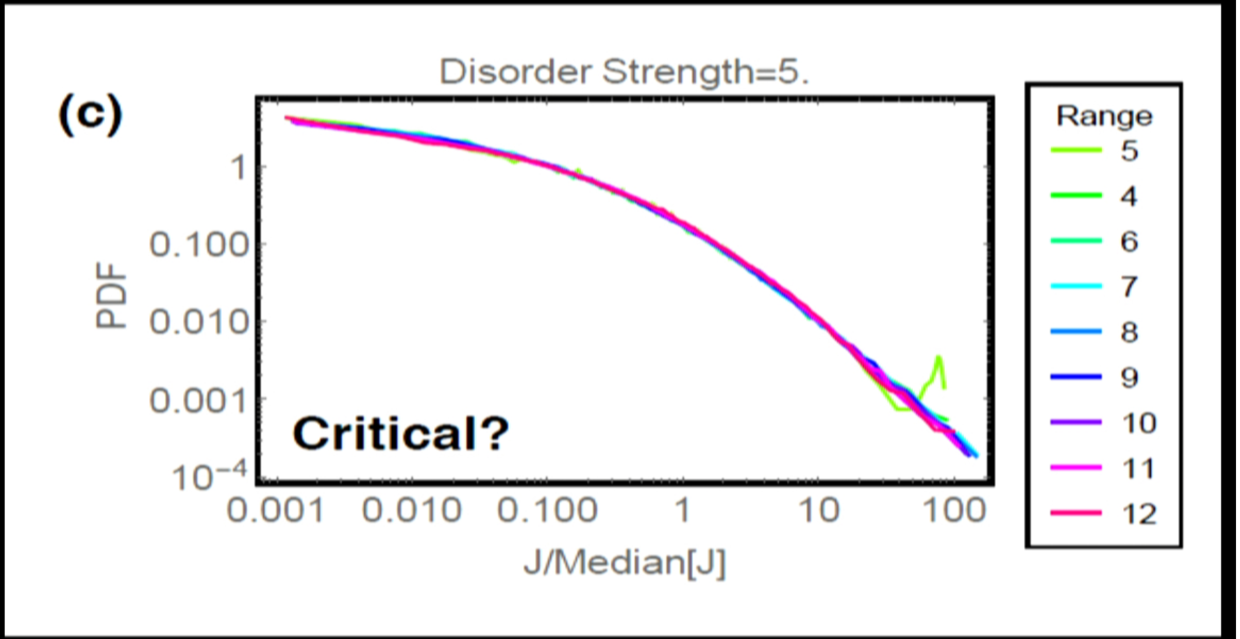
lets examine the distributions. They are broad and evolve differently in the two phases – towards 1/J (MBL) vs. uniform (ergodic)



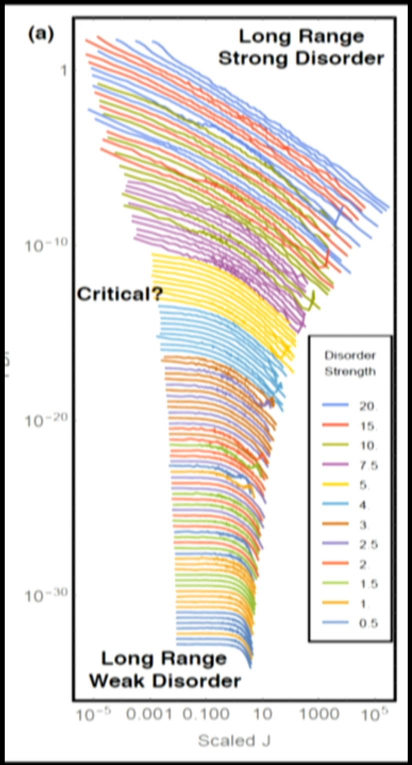


Critical distribution?





data dump



(a)
Long Range
Strong Disorder
1
10⁻¹⁰
Critical?
10⁻²⁰
10⁻³⁰
Disorder Strength
20.
15.
10.
7.5
5.
4.
3.
2.5
2.
1.5
1.
0.5
Long Range
Weak Disorder
10⁻⁵ 0.001 0.100 10 1000 10⁵
Scaled J

Future of L-bits?

- Measuring length scales (and fluctuation effects?) in quenches – Kapit/VO (in prep)
- L-bits in other models: quasiperiodic systems? With more conserved quantities?

- The mobility edge and "partial" L-bits
- Destruction of L-bits by bath and/or finite size effects (Chandran etal 2016)

- L-bits and spontaneous symmetry breaking?

Crashcourse on spontaneous symmetry breaking

- Quantum Ising chain

$$H = \sum_j \Delta_j \sigma_j^x + J \sigma_j^z \sigma_{j+1}^z$$

- Ground state for $\Delta_\mathrm{j} \gg J$: $|0\rangle = |{\rightarrow\rightarrow\rightarrow} \cdots \rightarrow\rangle$
- Ground states for $\Delta_\mathrm{j} \ll J$: $|\pm\rangle = |{\uparrow\uparrow\uparrow} \cdots \uparrow\rangle \pm |{\downarrow\downarrow\downarrow} \cdots \downarrow\rangle$
- Small field along z polarizes the system, susc. $\chi \propto \exp[L]$
- Finite <u>temperature</u> – paramagnet with finite density of domain walls (Landau/Peierls), finite χ
- Are the domain walls mobile? If not we should expect trouble!

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MBL spin-glass = FM+frozen defects

- Quantum Ising chain

$$H = \sum_j \Delta_j \sigma_j^x + J\sigma_j^z \sigma_{j+1}^z$$

- Domain walls in the FM background are localized
- Excited states for $\Delta_j \ll J$: $|\pm\rangle = |\uparrow\downarrow\uparrow \cdots \uparrow\rangle \pm |\downarrow\uparrow\downarrow \cdots \downarrow\rangle$
- Small field B along z, χ is still exp. large!

$$H_{eff} = \begin{pmatrix} E & B\sqrt{L} \\ B\sqrt{L} & E + \Delta \exp[-L/\xi] \end{pmatrix}$$

- Localization protected order – no Landau/Peierls

 (Huse etal 2013)

MBL spin-glass in the language of L-bits

$$H = \sum_j \Delta_j \sigma_j^x + J_j \sigma_j^z \sigma_{j+1}^z + \tilde{J} \sigma_j^x \sigma_{j+1}^x$$

$\Delta_j \gg J$:

$H = H_{PM} = \sum_j b_j^1 \tau_j^z + \sum_{jk} b_{jk}^2 \tau_j^z \tau_k^z + \sum_{jk} b_{jkl}^3 \tau_j^z \tau_k^z \tau_l^z + \ldots$

$\Delta_j \ll J$:

$H = H_{FM} = \sum_{jk} b_{jk}^2 \tau_j^z \tau_k^z + \sum_{jk} b_{jkl}^3 \tau_j^z \tau_k^z \tau_l^z + \ldots$

Spin-glass behavior arises from a phase transition in the effective L-bit Hamiltonian!

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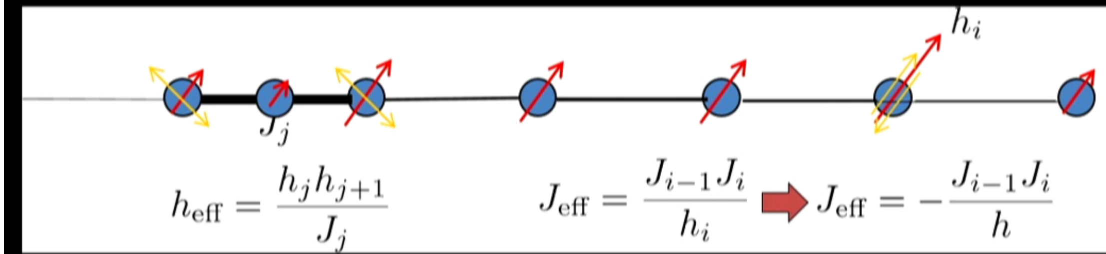
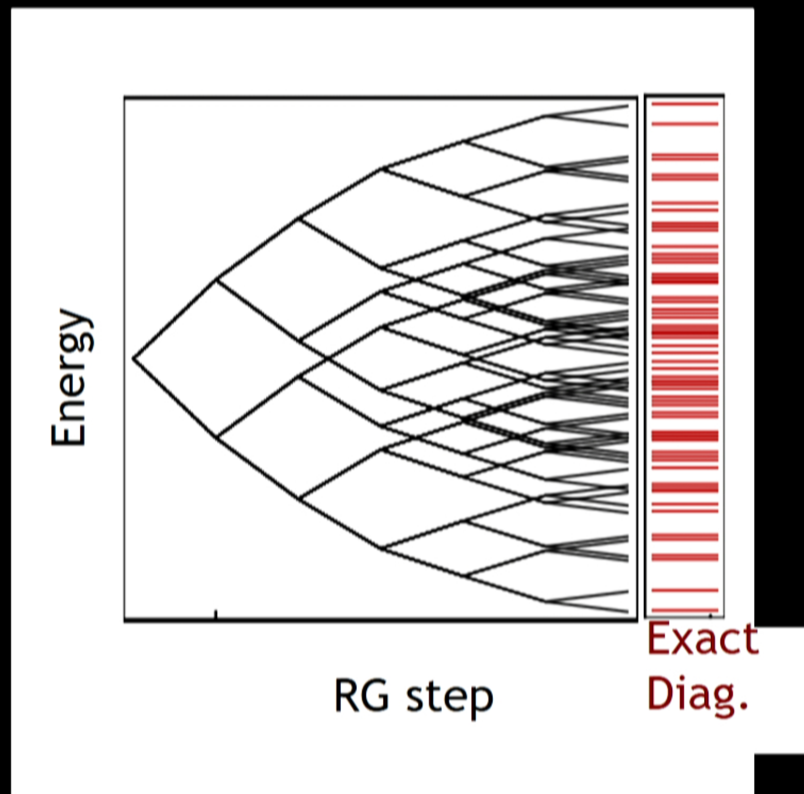
MBL spin-glass from real-space RG

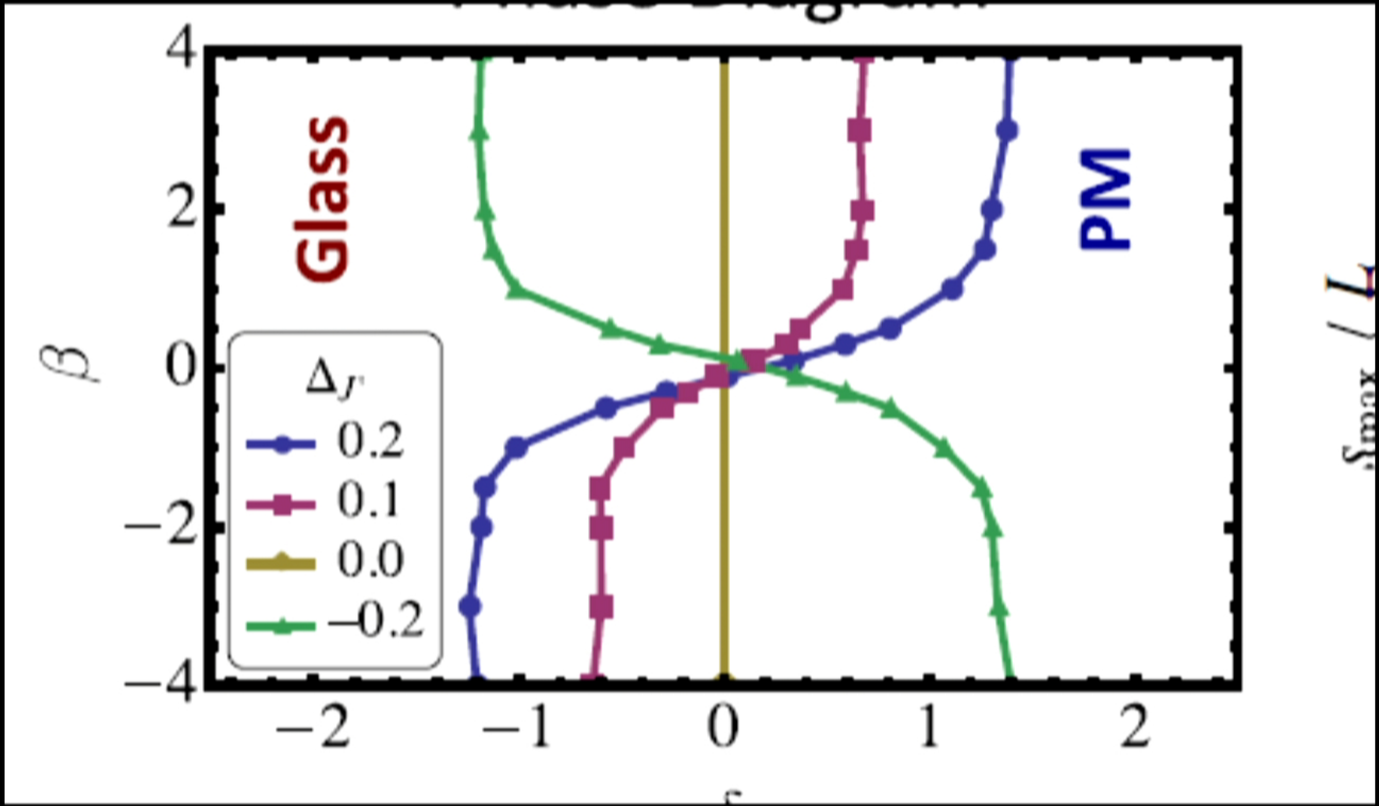


Dasgupta/Ma, Fisher,....



- Why does this work?
  RG is controlled by smallness of local susceptibility – locally GS and excited states have the same susceptibility!
- The method should be generalizeable for excited states;
- Localization of domain walls is exact for the nn Ising chain (free fermions)

“Thermally” driven spin-glass transition with n.n.n.



Pekker etal 2014

Open questions re: symmetry breaking

- Is the transition region itself stable against thermalization?
- Is this generalizeable to less disordered models, e.g. random bond Heisenberg? XXZ? Violations of mermin-wagner thm?
- Other glass states? Can we find analogs of classical spin-glasses, i.e. thermalized glassy states?

- Interplay between MBL and traditional low T correlation physics in disordered interacting systems?

MBL and beyond

- Experiments:

  systems: cold atoms, q-bits, charge conduction, defect states (Ho, Si:P);
  probes beyond DC transport: echoes/quenches; hole burning?

- Theory:
  practical challenges: need better tools
  results: more different models and observables
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- Beyond MBL:
  interplay with symmetry breaking, esp. in high dimensions?
  quantum computing – is localization good or bad for it?
  Self-localizing glassy behavior, e.g. Josephson junction arrays?