Title: Symplectic covariant quantum Brownian motion: the harmonic oscillator of open systems

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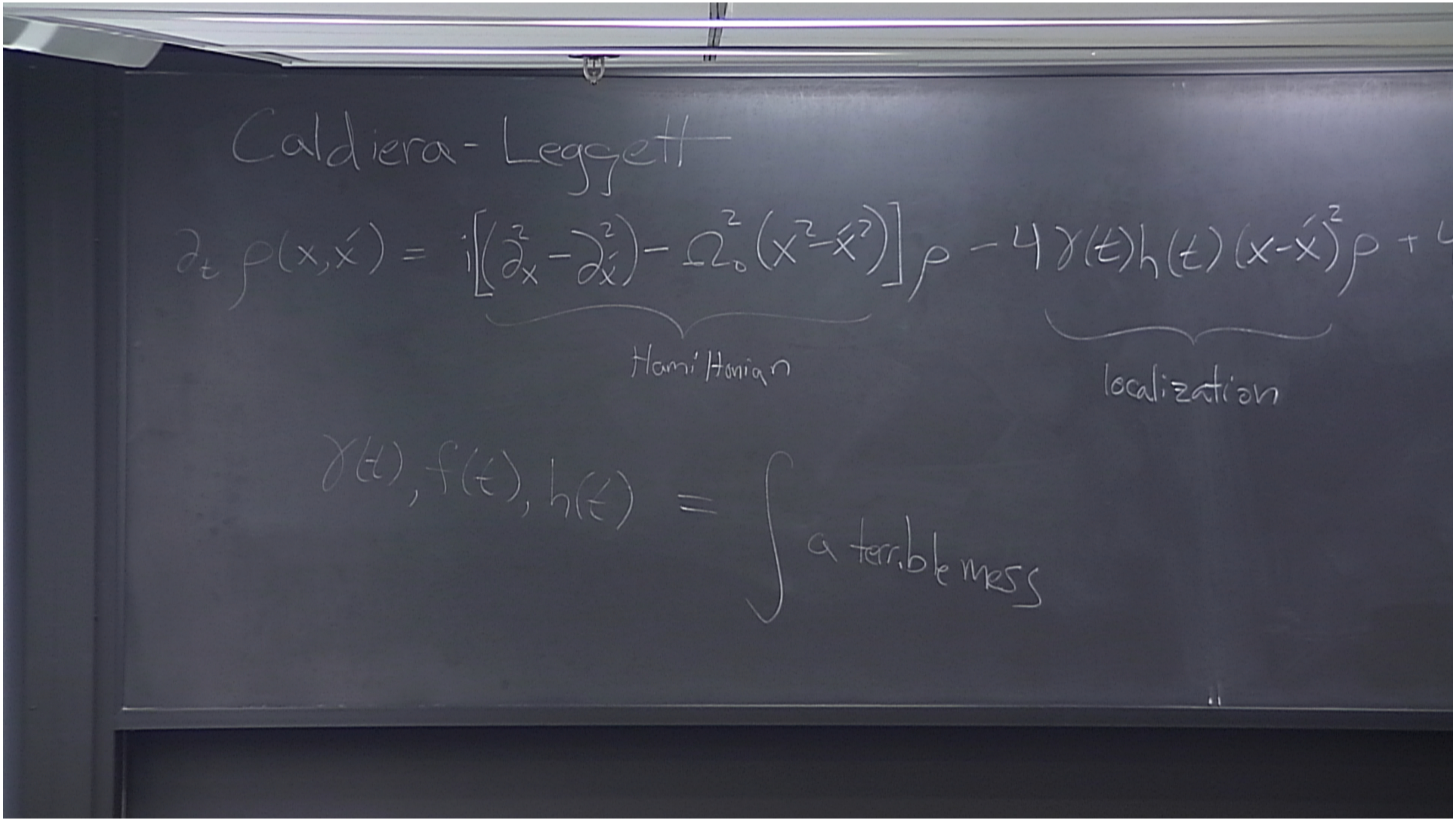
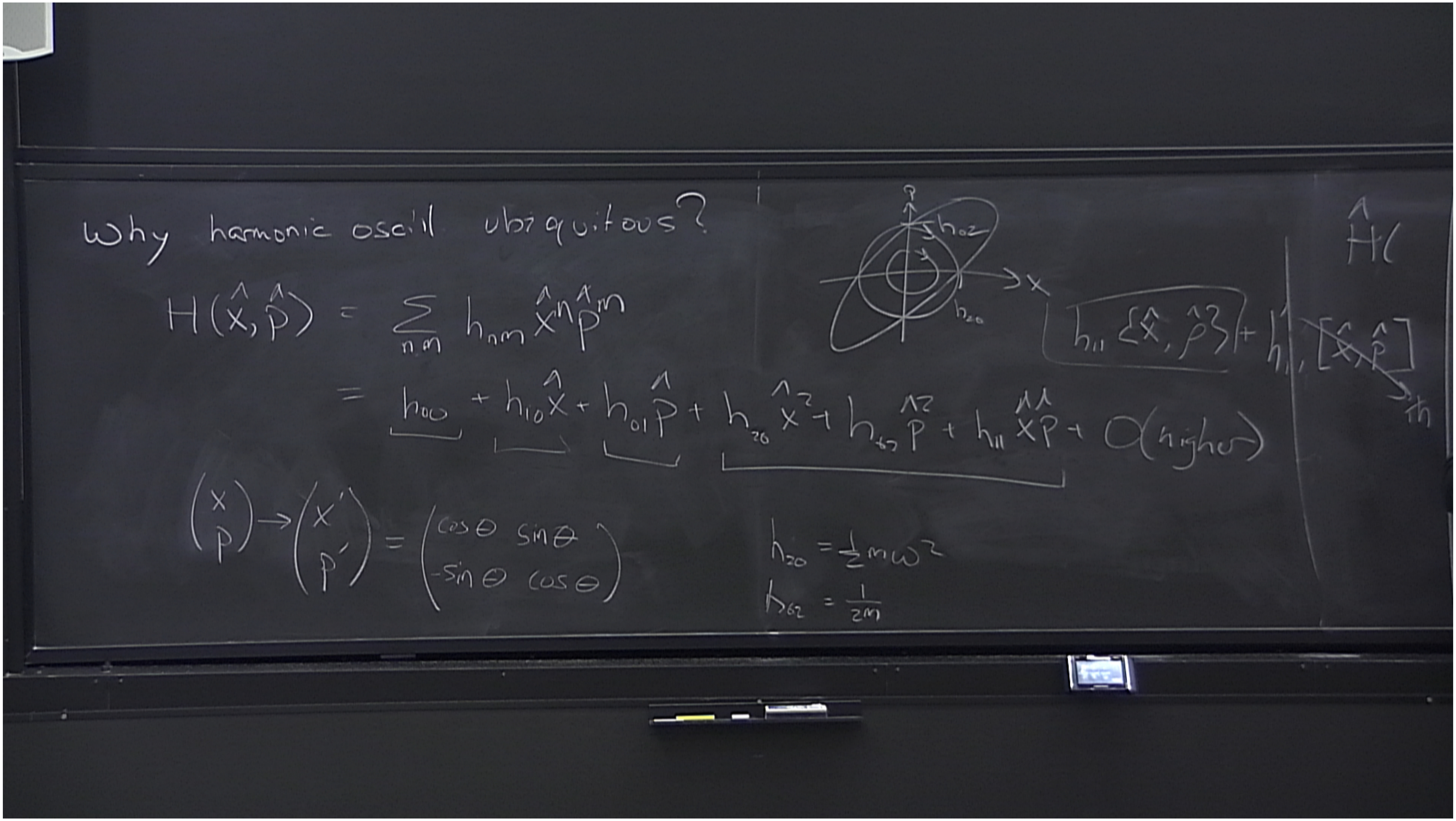
Abstract: <p>In the study of closed quantum system, the simple harmonic oscillator is ubiquitous because all smooth potentials look quadratic locally, and exhaustively understanding it is very valuable because it is exactly solvable. Although not widely appreciated, Markovian quantum Brownian motion (QBM) plays almost exactly the same role in the study of open quantum systems. QBM is ubiquitous because it arises from only the Markov assumption and linear Lindblad operators, and it likewise has an elegant and transparent exact solution. QBM is often introduced with specific non-Markovian models like Caldeira-Leggett, but this makes it very difficult to see which phenomena are universal and which are idiosyncratic to the model. Like frictionless classical mechanics or nonrenormalizable field theories, the exact Markov property is aphysical, but handling this subtlety is a small price to pay for the extreme generality. The widest class of QBM dynamics is symplectic invariant and includes Einstein-Smoluchowski diffusion, damped harmonic oscillations, and pure spatial decoherence as special cases, whose close relationship is often obscured. </p>


Caldiera-Leggett
$\partial_t \rho(x,x') = i\underbrace{\left[(\partial_x^2 - \partial_{x'}^2) - \Omega_0^2 (x^2 - x'^2)\right]}_{\text{Hamiltonian}}\rho - 4\underbrace{\gamma(t) h(t)(x-x')^2}_{\text{localization}}\rho + 4$
$\gamma(t), f(t), h(t) = \int$ a terrible mess

$$\underbrace{(t)(x-x')^2}_{\text{ization}}\rho + \underbrace{4i\gamma(t)f(t)(x-x')}_{\text{friction}}(\partial_x+\partial_{x'})\rho \underbrace{-2\gamma(t)(x-x')}_{\text{"anti-friction"}}(\partial_x-\partial_{x'})\rho$$


Why harmonic oscil ubiquitous?
$H(\hat{x},\hat{p}) = \sum_{n,m} h_{nm}\hat{x}^n\hat{p}^m$
$= h_{00} + h_{10}\hat{x} + h_{01}\hat{p} + h_{20}\hat{x}^2 + h_{02}\hat{p}^2 + h_{11}\hat{x}\hat{p} + O(\text{higher})$
$\begin{pmatrix} x \\ p \end{pmatrix} \to \begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$
p
x
h_{02}
h_{20}
$h_{11}\{\hat{x},\hat{p}\} + h_{11}'[\hat{x},\hat{p}]$
th
$h_{20} = \frac{1}{2}m\omega^2$
$h_{02} = \frac{1}{2m}$
$\hat{H}($

$\hat{H}($

$\{\hat{x}, \hat{p}\} + h_1 \cancel{[\hat{x}, \hat{p}]}$ th

$O(\text{higher})$

$$\partial_t \rho = -i[\hat{H}, \rho] = -\frac{i}{2} H_{ab}[\hat{\alpha}^a \hat{\alpha}^b, \rho]$$

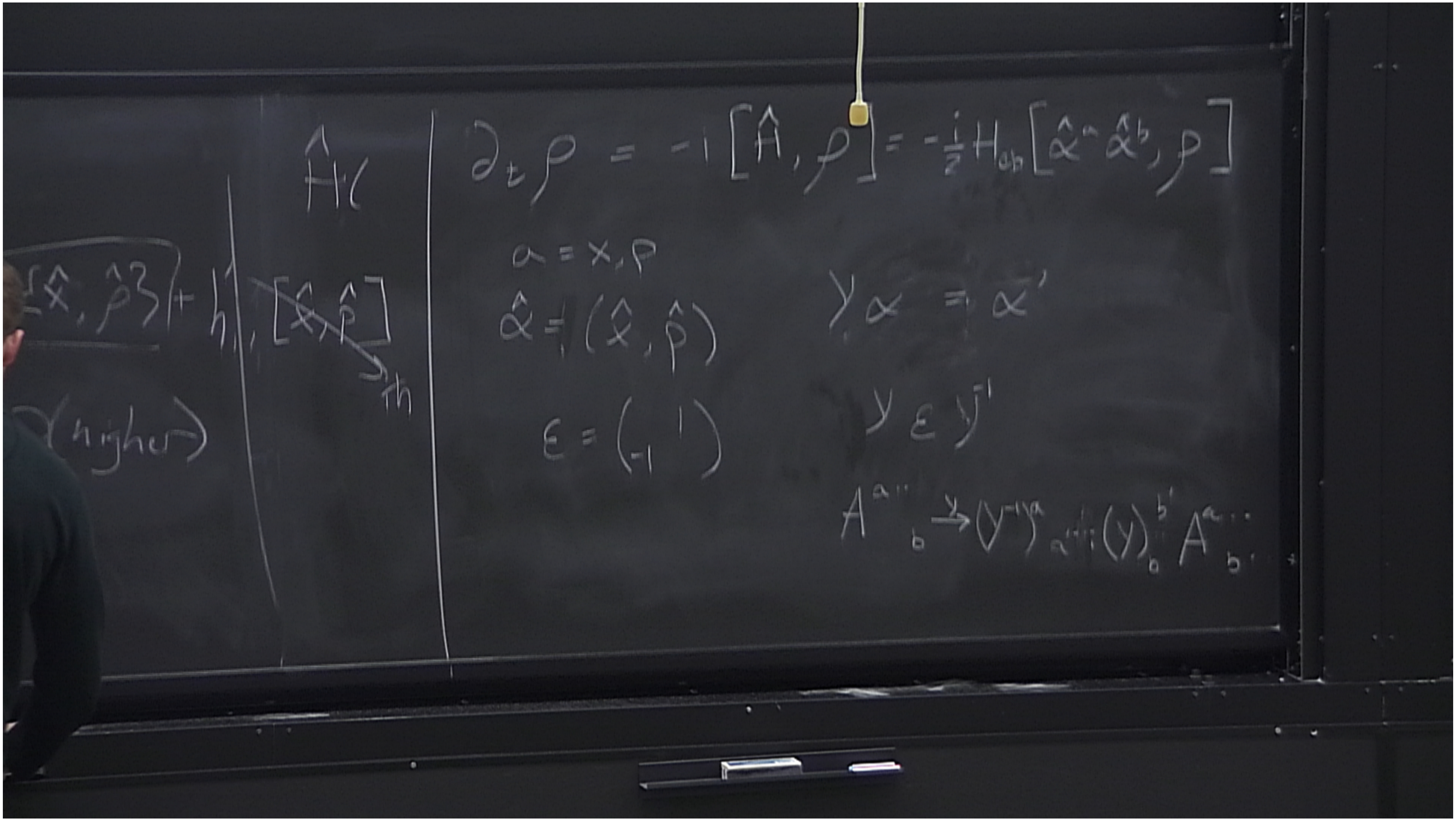
$\alpha = x, p$

$\hat{\alpha} = (\hat{x}, \hat{p})$

$\varepsilon = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}$

$y \alpha = \alpha'$

$y \varepsilon y^{-1}$



$\hat{A}:($
$\{\hat{x},\hat{p}\} + \hbar\,[\hat{x},\hat{p}]$
$O(\text{higher})$
$\partial_t \rho = -i[\hat{A}, \rho] = -\frac{i}{2} H_{ab}[\hat{\alpha}^a \hat{\alpha}^b, \rho]$
$\alpha = x, p$
$\hat{\alpha} = (\hat{x}, \hat{p})$
$\epsilon = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}$
$y\,\alpha = \alpha'$
$y\,\epsilon\,y^{-1}$
$A^{a\cdots}{}_{b} \xrightarrow{y} (y^{-1})^{a}{}_{a'} \cdots (y)_{b}{}^{b'} A^{a'\cdots}{}_{b'\cdots}$

$$\begin{pmatrix} x \\ p \end{pmatrix} \to \begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$h_{20} = \frac{1}{2} m \omega^2$$

$$h_{02} = \frac{1}{2m}$$

$$W_O(x,p) = \int d\Delta x \, e^{-i\Delta x p} \langle x + \Delta x/2 | O | x - \Delta x/2 \rangle$$

$$\hat{W}_O(\xi) = \mathrm{Tr}\left[T_\alpha^\dagger O \right] \quad \text{w/} \quad T_\alpha = e^{i\alpha^a \hat{\alpha}_a} = e^{i(x\hat{p} - p\hat{x})}$$

$$W_O(\alpha) = \int d\xi \, e^{-i\alpha\xi} \hat{W}_O(\xi)$$

Classical EOM: $\vec{\alpha}(t) = e^{tH}\vec{\alpha}(0)$

$$H = H^a{}_b = \varepsilon^{ac} H_{cb}$$

$$-p\hat{x})$$

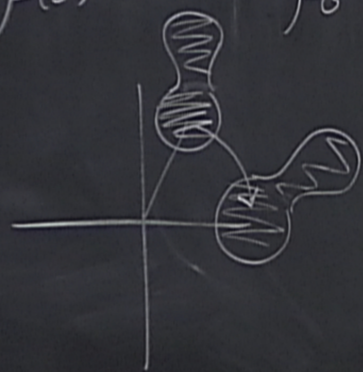
$h_{11}=0$

$$H_{ab} = \begin{pmatrix} m\omega^2 & 0 \\ 0 & \frac{1}{m} \end{pmatrix} \qquad H^a{}_b = \begin{pmatrix} 0 & 1/m \\ -m\omega^2 & 0 \end{pmatrix}$$

Classical EOM: $\vec{\alpha}(t) = e^{tH}\vec{\alpha}(0)$

$\hat{p} - p\hat{x})$

$$\boxed{W_{\rho(t)}(\alpha) = W_{\rho_0}(e^{-tH}\alpha)}$$



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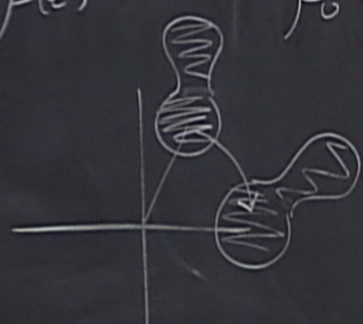


$$W_{\rho(t)} = (W_{\rho_0})\circ(e^{-tH})$$

$H = H^a{}_b = \varepsilon^{ac}H_{cb}$

$h_{11}=0$

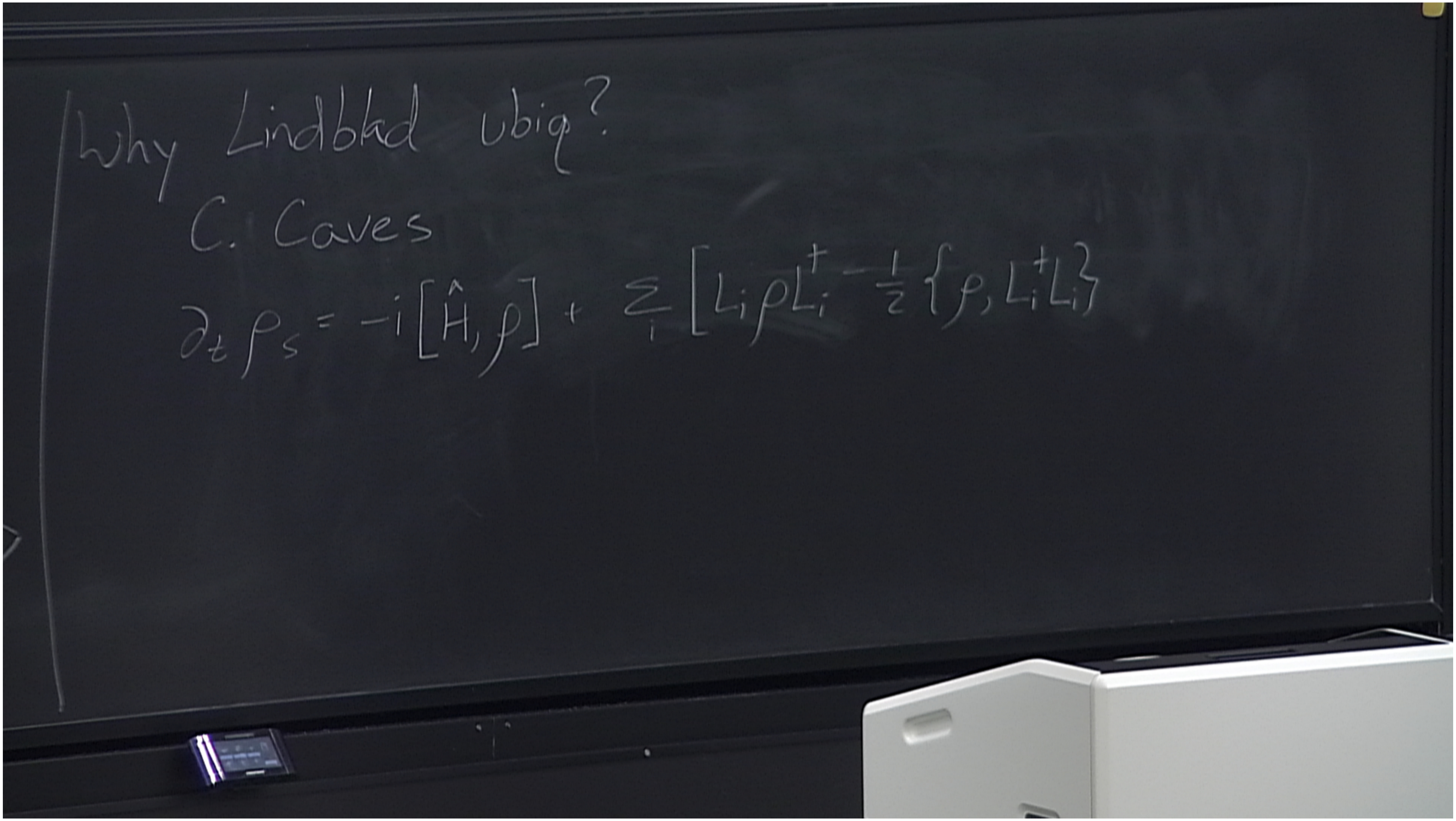
$H_{ab} = \begin{pmatrix} m\omega^2 & 0 \\ 0 & \frac{1}{m} \end{pmatrix}$ $\quad H^a{}_b = \begin{pmatrix} 0 & 1/m \\ -m\omega^2 & 0 \end{pmatrix}$

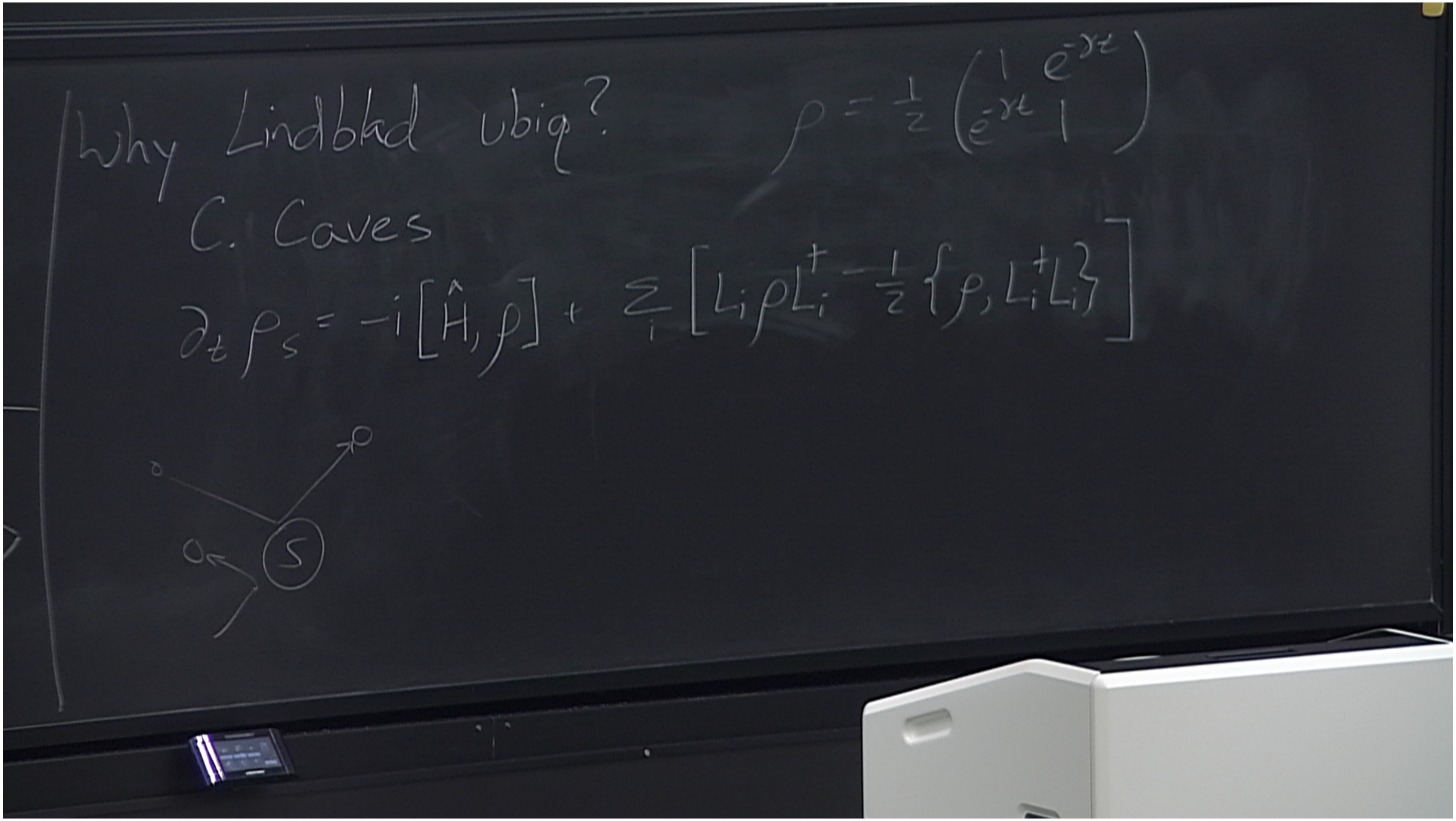
Why CP ubiq.?

$$\rho_s \otimes |E_0\rangle\langle E_0| \longrightarrow U(\rho_s \otimes |E_0\rangle\langle E_0| U^\dagger$$

$$\rho_s \to \mathrm{Tr}_E\left[U(\rho_s \otimes |E_0\rangle\langle E_0|)U^\dagger\right]$$

$$= \sum_i L_i \rho_s L_i^\dagger \quad \text{w/ } L_i = \langle i|U|E_0\rangle$$


Why Lindblad ubiq?
C. Caves
$\partial_t \rho_s = -i[\hat{H}, \rho] + \sum_i [L_i \rho L_i^\dagger - \frac{1}{2}\{\rho, L_i^\dagger L_i\}$



Why Lindblad ubiq?
C. Caves
$\rho = \frac{1}{2}\begin{pmatrix} 1 & e^{-\gamma t} \\ e^{-\gamma t} & 1 \end{pmatrix}$
$\partial_t \rho_s = -i[\hat{H}, \rho] + \sum_i \left[L_i \rho L_i^\dagger - \frac{1}{2}\{\rho, L_i^\dagger L_i\} \right]$
S

Ideal QBM is time-continuous, Markovian, open-sys evolution

$$\hat{H} = h_{00}\hat{I} + h_{10}\hat{x} + h_{01}\hat{p} + h_{20}\hat{x}^2 + \dots = \frac{1}{2} H_{ab}\hat{\alpha}^a\hat{\alpha}^b$$

$$\hat{L}^{(i)} = \underset{\sim}{L^{(i)}_0} \hat{I} + L^{(i)}_x \hat{X} + L^{(i)}_p \hat{P} + \cancel{\text{higher order}} = L^{(i)}_a \hat{\alpha}^a$$

$$\partial_t \rho = -\frac{i}{2} H_{ab}[\hat{\alpha}^a \hat{\alpha}^b, \rho] + \sum_i L^{(i)*}_a L^{(i)}_b \left(\hat{\alpha}_b \rho \hat{\alpha}_a - \frac{1}{2}\{\hat{\alpha}_a \hat{\alpha}_b, \rho\}\right)$$

$$D_{ab} = \mathrm{Re} \sum_i L$$

…arkovian, open-sys evolution

$$= \frac{1}{2} H_{ab} \hat{\alpha}^a \hat{\alpha}^b$$

$$\ldots = L_a^{(i)} \hat{\alpha}^a$$

$$\ldots \hat{\alpha}_a - \frac{1}{2}\{\hat{\alpha}_a \hat{\alpha}_b, \rho\}\Big)$$

$$D_{ab} = \mathrm{Re} \sum_i L_a^{(i)*} L_b^{(i)} \qquad \gamma = \frac{1}{2i} \varepsilon^{ab} \sum_i L_a^{(i)*} L_b^{(i)}$$

$$K_{ab} = H_{ab} + \varepsilon_{ab} \gamma \qquad \text{w/ } \det(D) \geq \gamma^2\Big)$$

$$\partial_t \rho = \frac{-i}{2} K_{ab} \left[\hat{\alpha}^a, \{\hat{\alpha}^b, \rho\}\right] - \frac{1}{2} D_{ab} \left[\hat{\alpha}^a, [\hat{\alpha}^b, \rho]\right]$$

$$\partial_t W(\alpha) = \left[-K^a{}_b \partial_a \alpha^b + \frac{1}{2} D^{ab} \partial_a \partial_b\right] W_\rho(\alpha) \qquad \partial_a = \frac{\partial}{\partial \alpha^a}$$

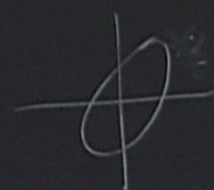
$$(-K^a{}_a - K^a_b \alpha^b \partial_a)$$

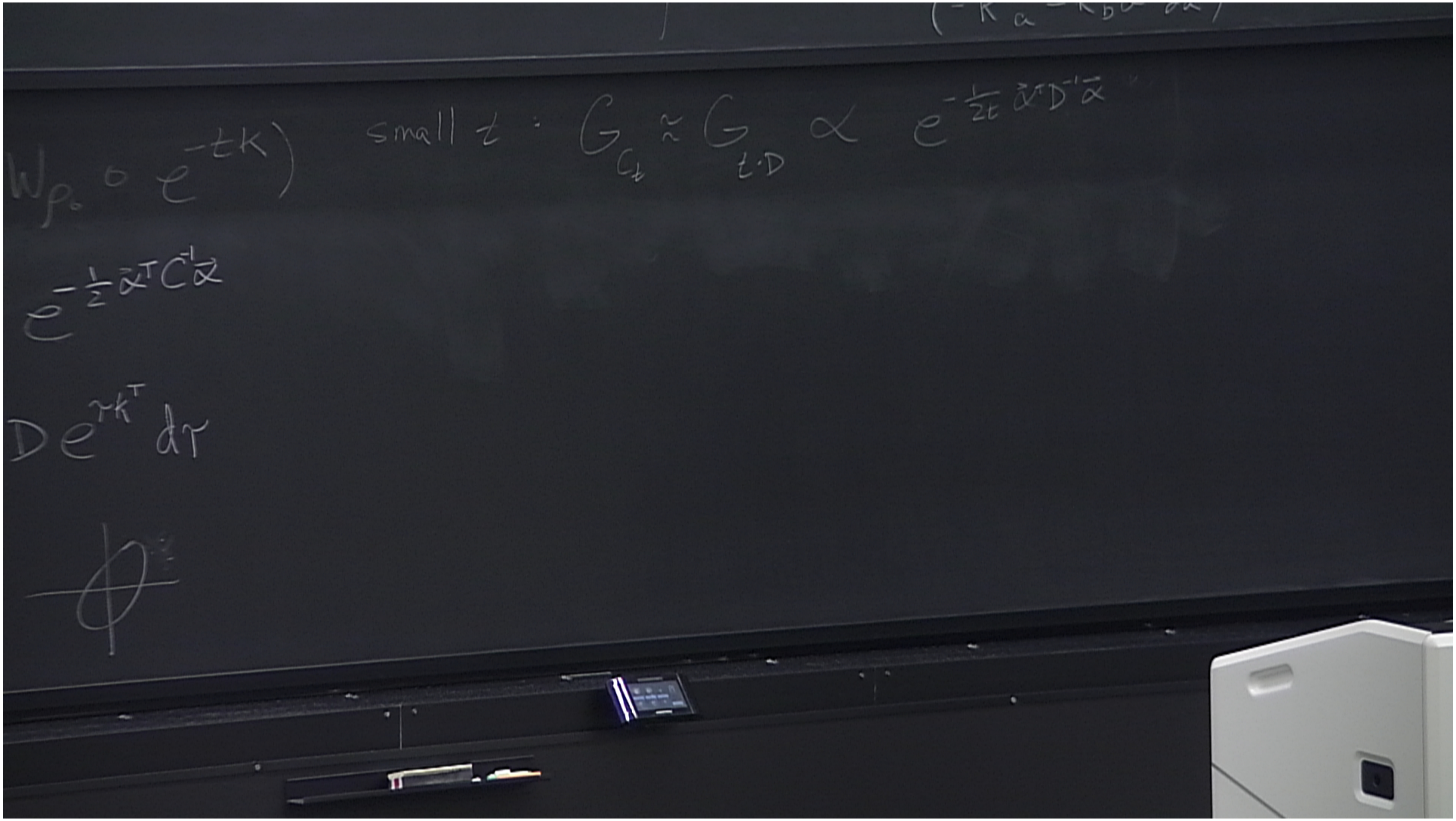
$$W_{\rho(t)} = G_{C_t} * \left(e^{2\gamma t} W_{\rho_0} \circ e^{-tK}\right)$$

where $G_C = \frac{1}{2\pi\sqrt{\det(C)}} e^{-\frac{1}{2}\vec{\alpha}^T C \vec{\alpha}}$

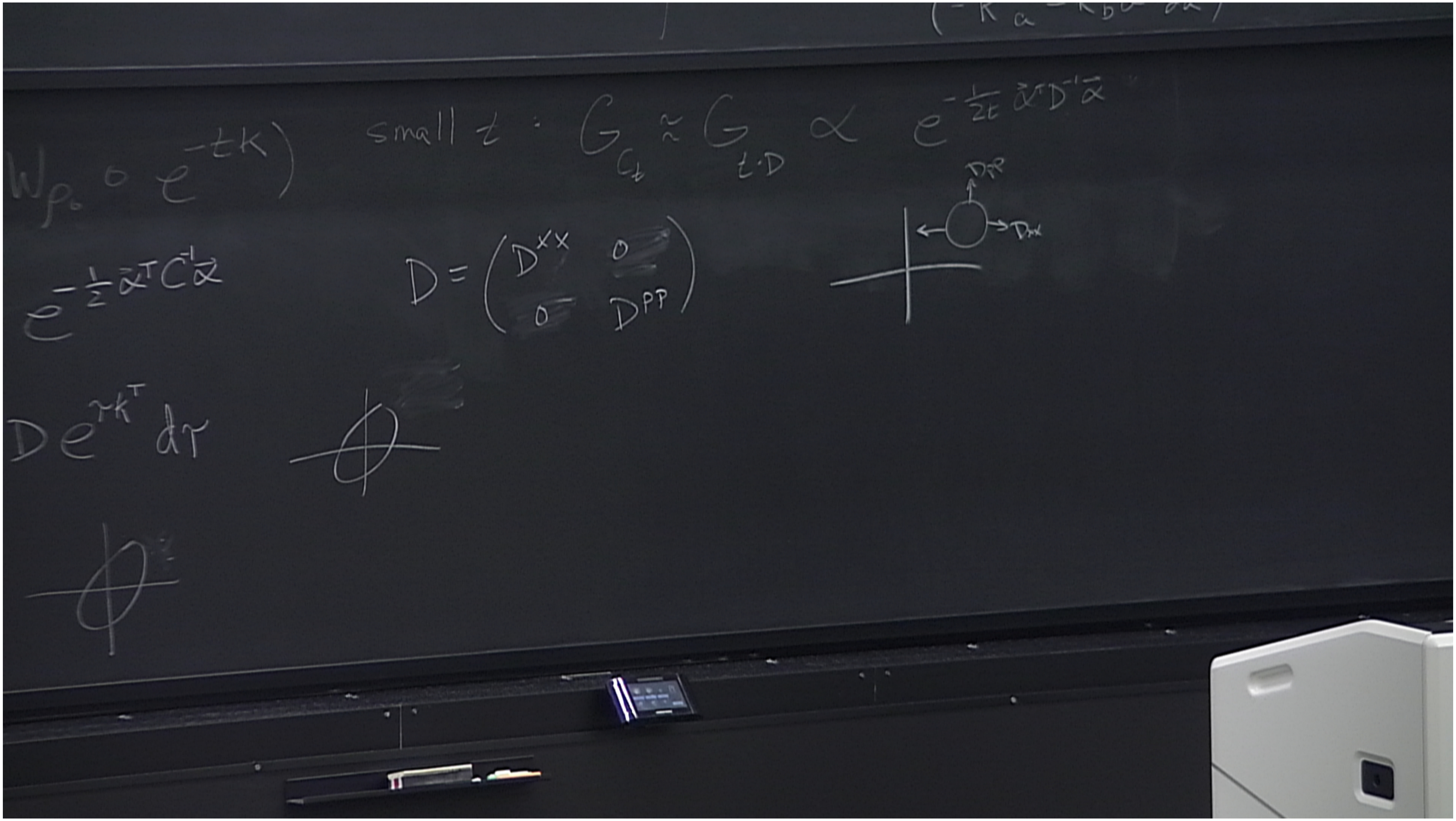
$$C_t = \int_0^t e^{\tau K} D e^{\tau K^T} d\tau$$

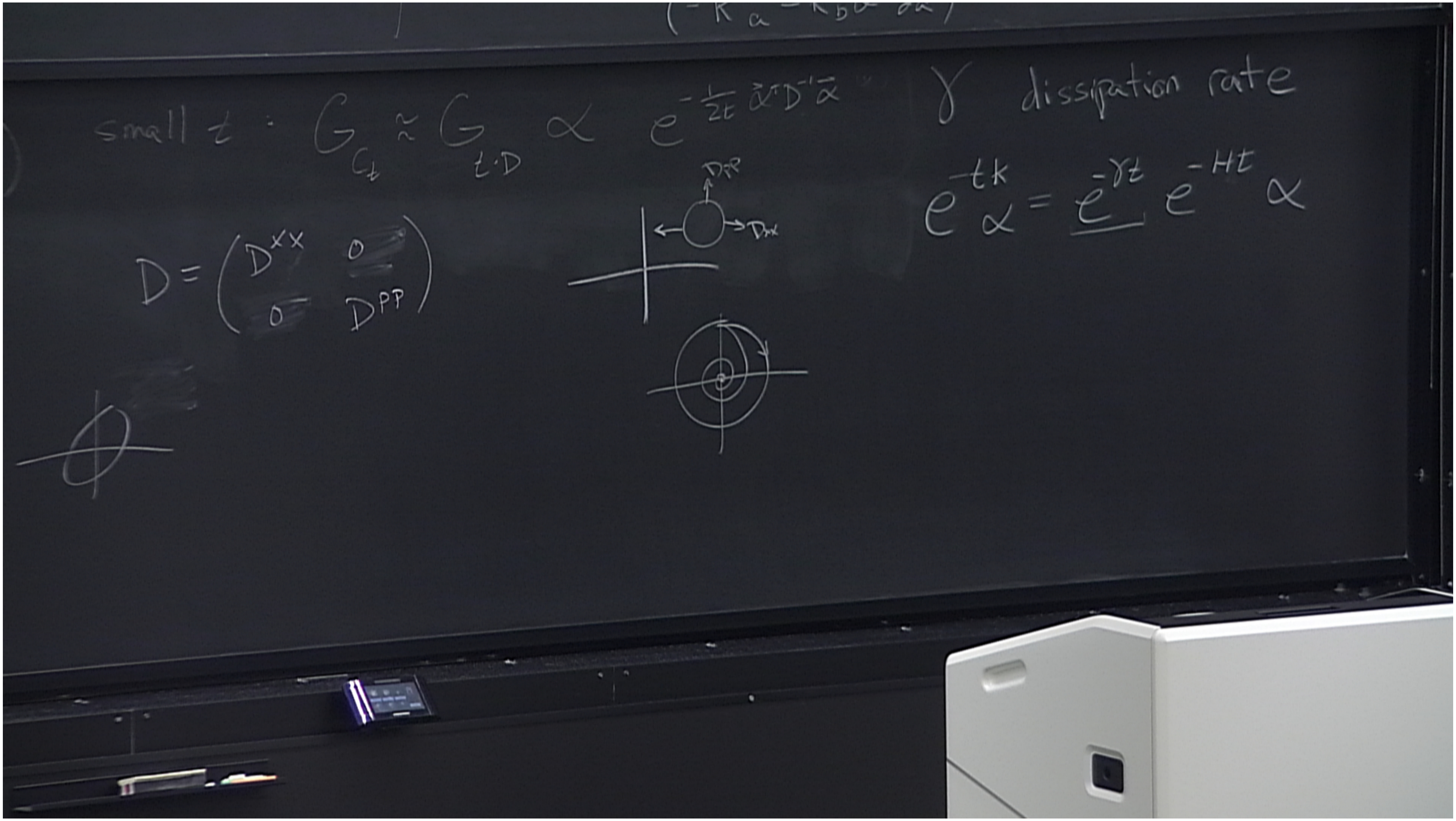
$$(f * g)(\alpha) = \int d\beta\, f(\beta)\, g(\alpha - \beta)$$





small t : $G_{C_t} \approx G_{t\cdot D} \propto e^{-\frac{1}{2t}\vec{\alpha}^T D^{-1}\vec{\alpha}}$
$W_{\rho_0} \circ e^{-tK})$
$e^{-\frac{1}{2}\vec{\alpha}^T C^{-1}\vec{\alpha}}$
$De^{\tau K^T} d\tau$


small t : G_{C_t} ≈ G_{t·D} ∝ e^{-1/(2t) α^T D^{-1} α}
W_{ρ_0} ∘ e^{-tK})
e^{-1/2 α^T C^{-1} α}
D = (D^{xx} 0 ; 0 D^{pp})
D^{pp}
D^{xx}
D e^{τK^T} dτ


small t : $G_{C_t} \approx G_{t \cdot D} \propto e^{-\frac{1}{2t}\vec{\alpha}^T D^{-1} \vec{\alpha}}$
γ dissipation rate
$e^{-tK}\alpha = e^{-\gamma t} e^{-Ht}\alpha$
$D = \begin{pmatrix} D^{xx} & 0 \\ 0 & D^{pp} \end{pmatrix}$
D^{pp}
D^{xx}

$(x-x')(\partial_x - \partial_{x'})\rho$

"friction"

Damped HO

$\gamma > 0, \quad D = \begin{pmatrix} D_0 & \\ & D_0 \end{pmatrix}$

pure spatial decoh.

$\Longleftrightarrow$ pure mom. diff

$H=0, \quad \gamma=0, \quad D = \begin{pmatrix} D_0 & 0 \\ 0 & 0 \end{pmatrix}$

$\partial_t \rho = -\frac{1}{2} D_0 [\hat{x}, [\hat{x}, \rho]]$

$\partial_t \rho(x,x') = \rho_0(x,x') e^{-D_0(x-x')t}$

$W_{\rho(t)} =$

where

$(f * g)(x) = \int d$

$$|\psi,\alpha\rangle = T_\alpha |\psi\rangle$$

$$Q^\psi_\rho(\alpha) = \langle\psi,\alpha|\,\rho\,|\psi,\alpha\rangle$$

$$\rho = \int d\alpha\; P^\psi_\rho(\alpha)\, |\psi,\alpha\rangle\langle\psi,\alpha|$$

$$Q^\psi_\rho = W_\rho \star f^\psi$$

$$W_\rho = P^\psi_\rho \star f^\psi$$