Title: Sudden expansion and domain wall melting in clean and disordered optical lattices

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Abstract: <p>We numerically investigate the expansion of clouds of hard-core bosons in a 2D square lattice using a matrix-product state based method. This non-equilibrium setup is induced by quenching a trapping potential to zero and is specifically motivated by an experiment with ultracold atoms [1]. As the anisotropy for hopping amplitudes in different spatial directions is varied from 1D to 2D, we observe a crossover from a fast ballistic expansion in the 1D limit to much slower dynamics in the isotropic 2D lattice [2].</p>

<p> </p>

<p>Introducing a site-dependent disorder potential allows to study many body localization (MBL). In a very recent experiment, the melting of a domain wall gave evidence for an MBL transition in 2D [3]. We study 1D and quasi-1D models, for which the phase diagram in the presence of disorder is known, such as the Anderson insulator, Aubry-Andre model and interacting fermions in 1D and on a two-leg ladder [4]. By considering several observables, we demonstrate that the domain wall melting can indeed yield quantitative information on the transition from an ergodic to the MBL phase as a function of disorder.</p>

<p> </p>

<p>[1] J. P. Ronzheimer et al., PRL 110, 205301 (2013) [2] J. Hauschild et al., PRA 92, 053629 (2015) [3] J. Choi et al., arXiv:1604.04178 (2016) [4] J. Hauschild et al., in preparation</p>

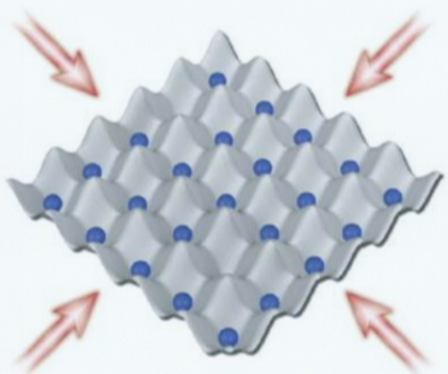
Sudden expansion and domain-wall melting in clean and disordered optical lattices

Johannes Hauschild[1], Fabian Heidrich-Meisner[2] and Frank Pollmann[1]

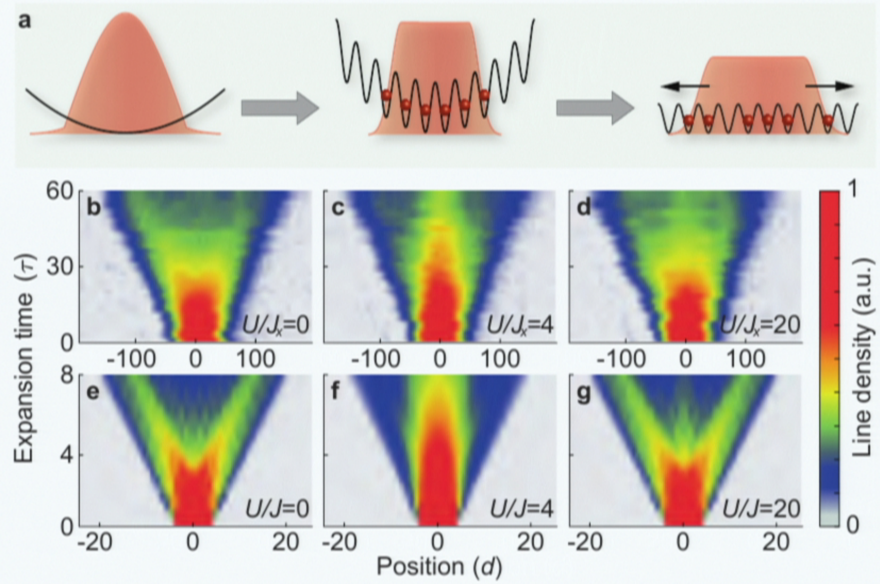
[1]Max Planck Institut für Physik komplexer Systeme Dresden
[2]Ludwig Maximilians Universtität München

Perimeter Institute 6.5.2016

Ultracold Atoms



Bloch (2008)



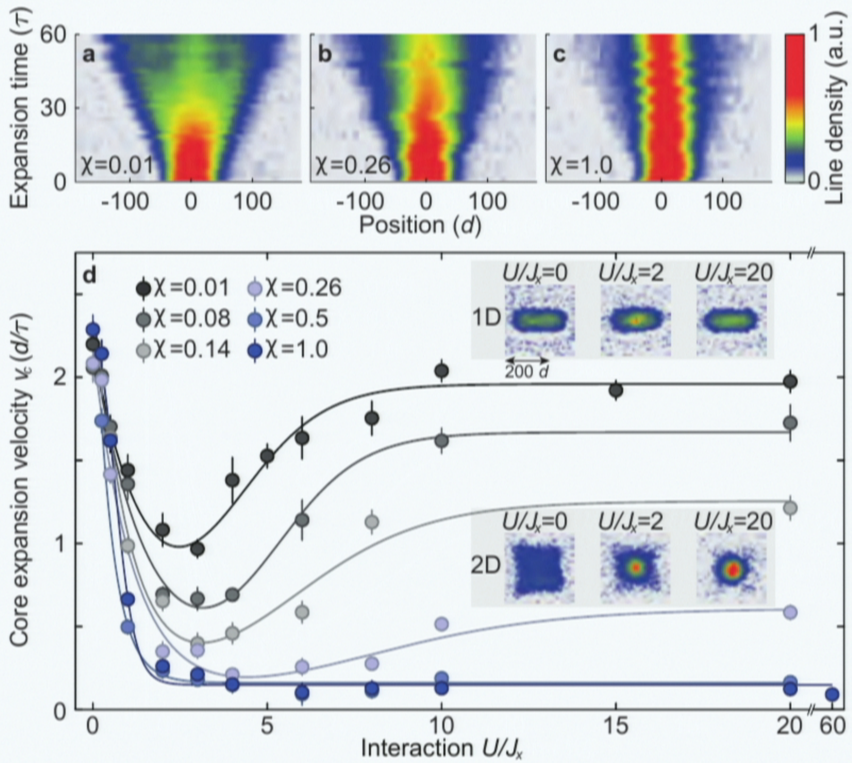
Ronzheimer *et al.* (2013)

- Hubbard model (bosons or fermions)
- highly tunable parameters
- quenches for non-equilibrium physics
- comparison of theory and expermient

$$H = -J_x \sum_{\langle i,j \rangle_x} (\hat{a}_i^\dagger \hat{a}_j + h.c.) - J_y \sum_{\langle i,j \rangle_y} (\hat{a}_i^\dagger \hat{a}_j + h.c.) + \frac{U}{2} \sum_j \hat{n}_j(\hat{n}_j - 1)$$

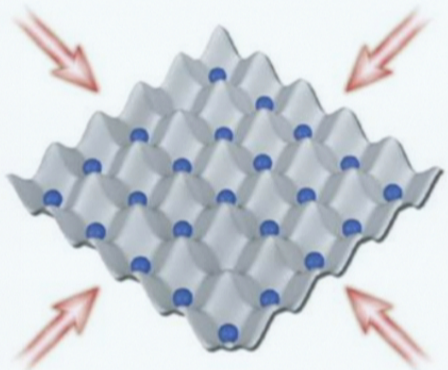
$\chi = J_y / J_x$

core expansion velocity $v_c = \frac{\partial}{\partial t} HWHM$

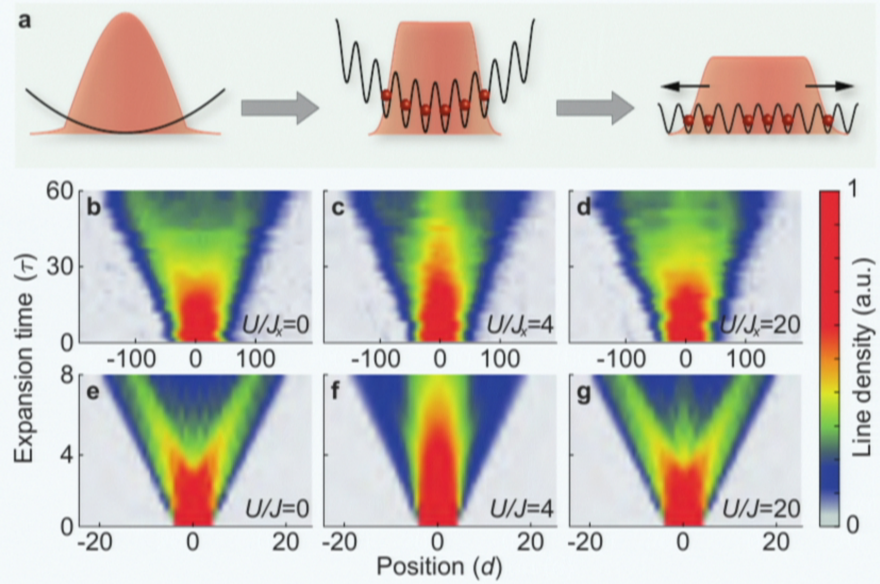


Ronzheimer *et al.* (2013)

Ultracold Atoms



Bloch (2008)



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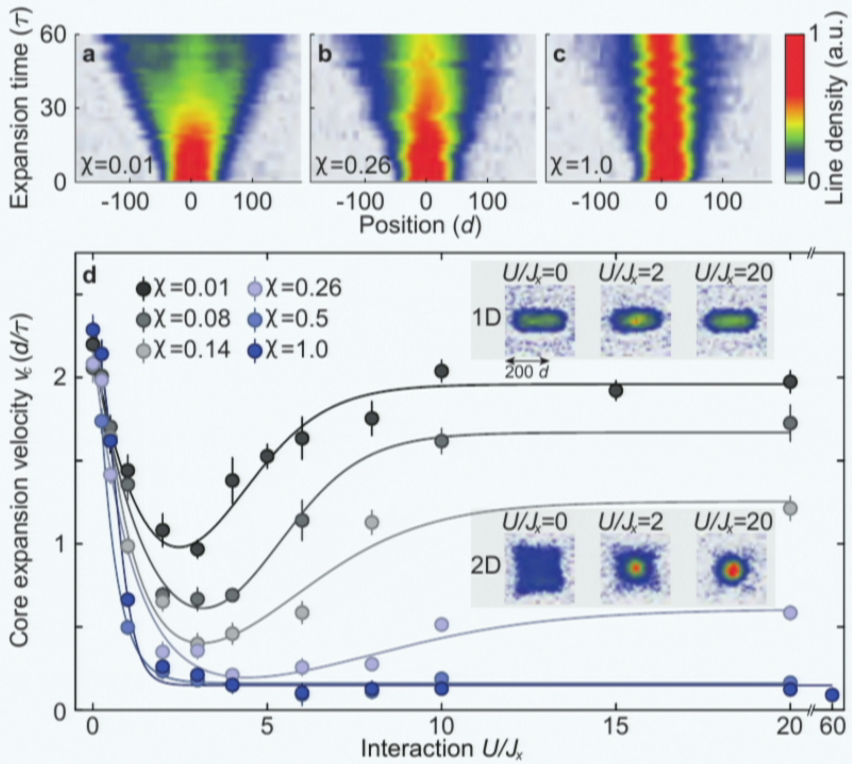
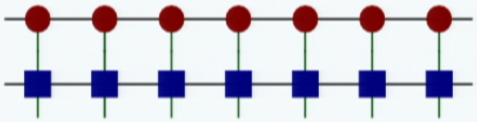
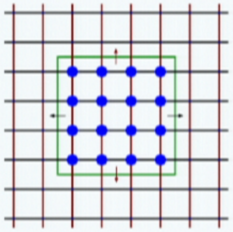


Ronzheimer *et al.* (2013)

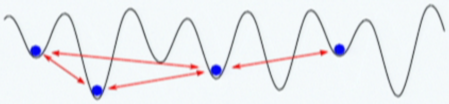
1. Method: time evolution with MPO



2. Sudden Expansion in clean lattices
   - Hauschild, Pollmann, Heidrich-Meisner, PRB (2015)



3. Disordered Systems
   - Hauschild, Pollmann, Heidrich-Meisner, in preparation

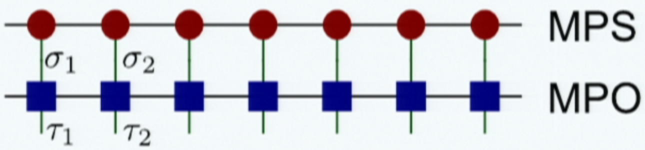


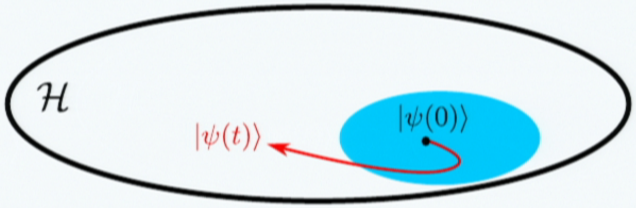
Method

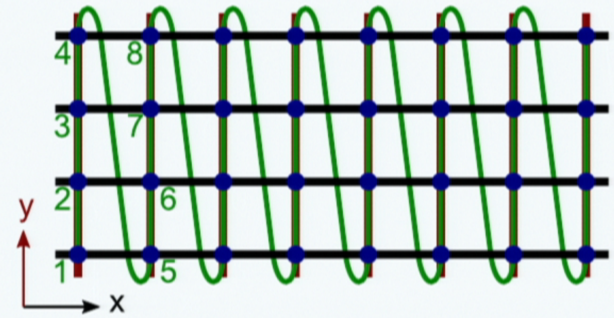
- goal: $|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle$
- efficient representation: matrix-product state (MPS)

$$|\psi\rangle = \sum_{\sigma_1\cdots\sigma_L} B^{\sigma_1}\cdots B^{\sigma_L}|\sigma_1\cdots\sigma_L\rangle$$

- limitation: finite entanglement
- map 2D lattice to 1D MPS
  ⇒ long-range interaction/hopping
- generalization of MPS to operators: matrix-product operator (MPO)







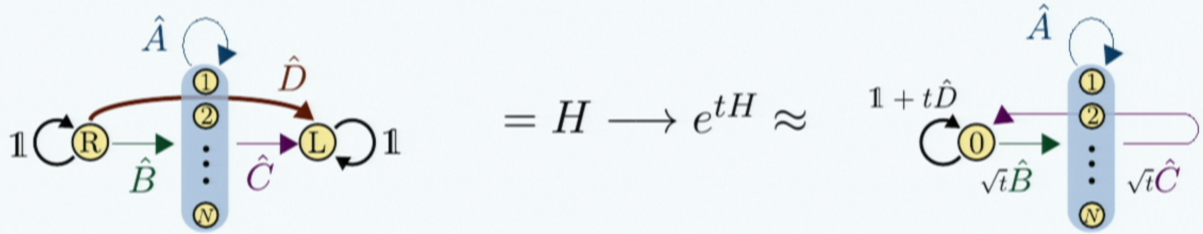
Method

- basic idea: improve

$$e^{-itH} \approx \underbrace{1 - it\sum_x H_x}_{\epsilon \propto L^2 t^2} \quad \longrightarrow \quad \underbrace{\prod_x (1 - itH_x)}_{\epsilon \propto L t^2}$$

- neglect overlapping terms

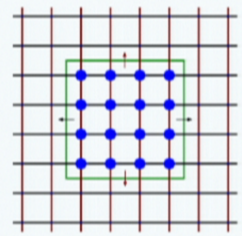
$$e^{-itH} \approx 1 - it\sum_x H_x - t^2 \sum_{x<y} H_x H_y + it^3 \sum_{x<y<z} H_x H_y H_z + \dots$$



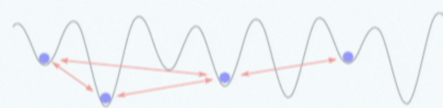
Zaletel *et al.* (2015)

1. Method: time evolution with MPO

2. **Sudden Expansion in clean lattices**
   - Hauschild, Pollmann, Heidrich-Meisner, PRB (2015)



3. Disordered Systems
   - Hauschild, Pollmann, Heidrich-Meisner, in preparation

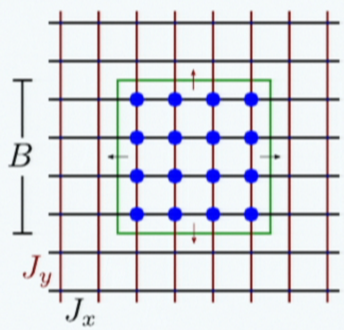


Model

$$\hat{H} = -J_x \sum_{\langle i,j \rangle_x} (\hat{a}_i^\dagger \hat{a}_j + h.c.) - J_y \sum_{\langle i,j \rangle_y} (\hat{a}_i^\dagger \hat{a}_j + h.c.)$$

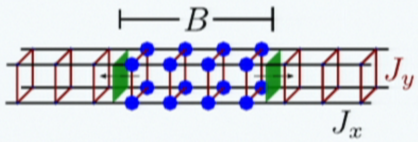
- strongly interacting: hard-core bosons

Two-dimensional expansion



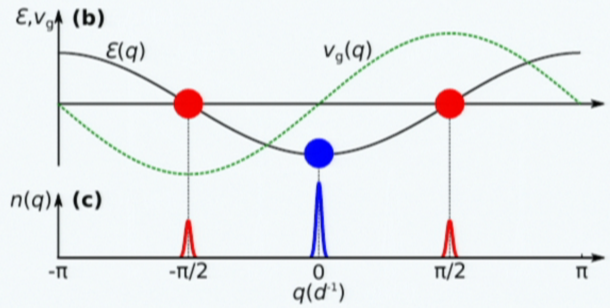
Ladders and cylinders

central block

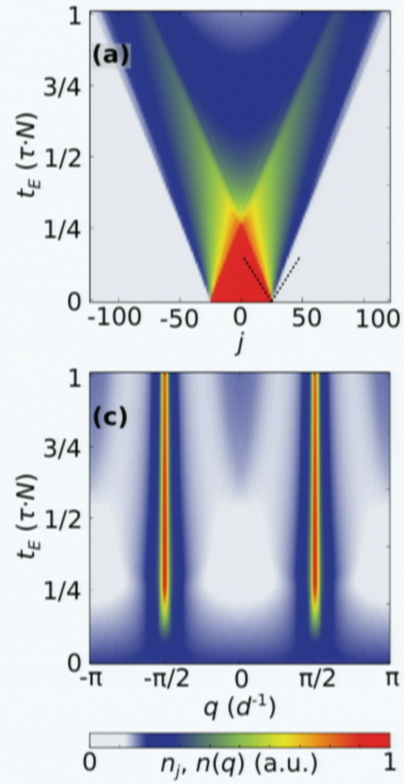


domain wall

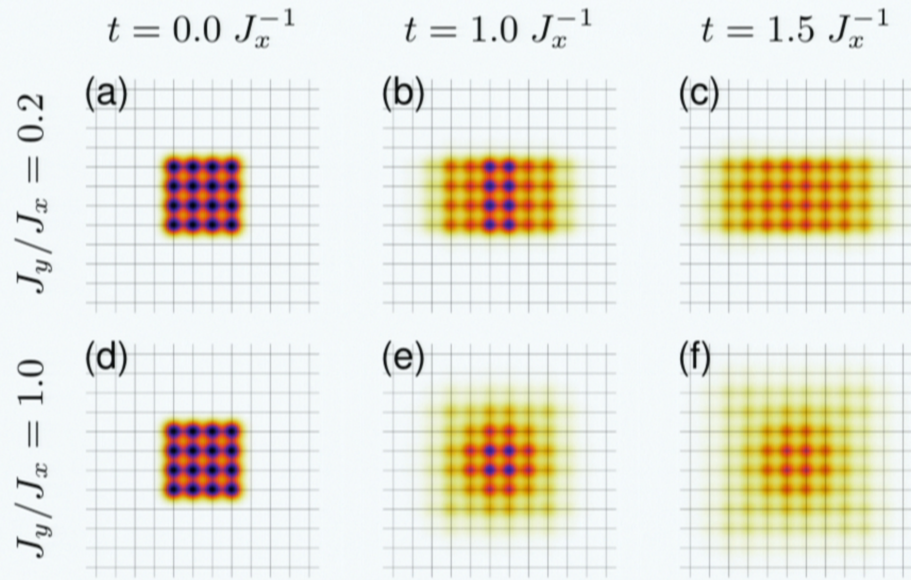
1D Limit $J_y = 0$



- maps to free fermions
  - $H = \sum_k -2J_x \cos(k)\hat{c}_k^\dagger \hat{c}_k$
  - ⇒ momentum occupation fixed by initial state
  - ⇒ ballistic expansion
  - Antal *et al.* (1999)
- hard-core boson picture:
  - quasicondensation at finite momenta $k_x = \pm\frac{\pi}{2}$
  - Rigol *et al.* (2004), Jreissaty *et al.* (2011), Vidmar *et al.* (2015)



2D Expansion: Density Profile $n_j(t)$

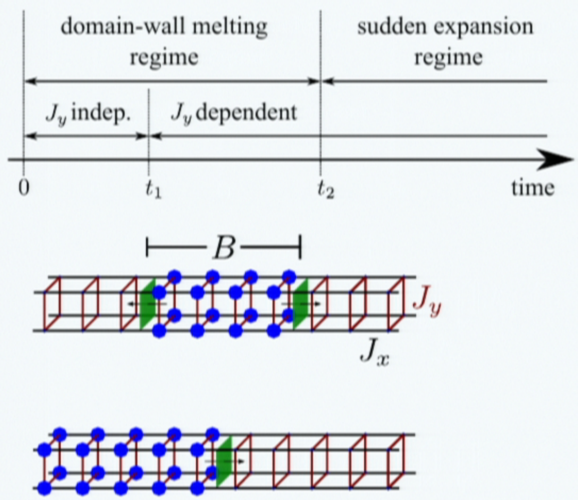


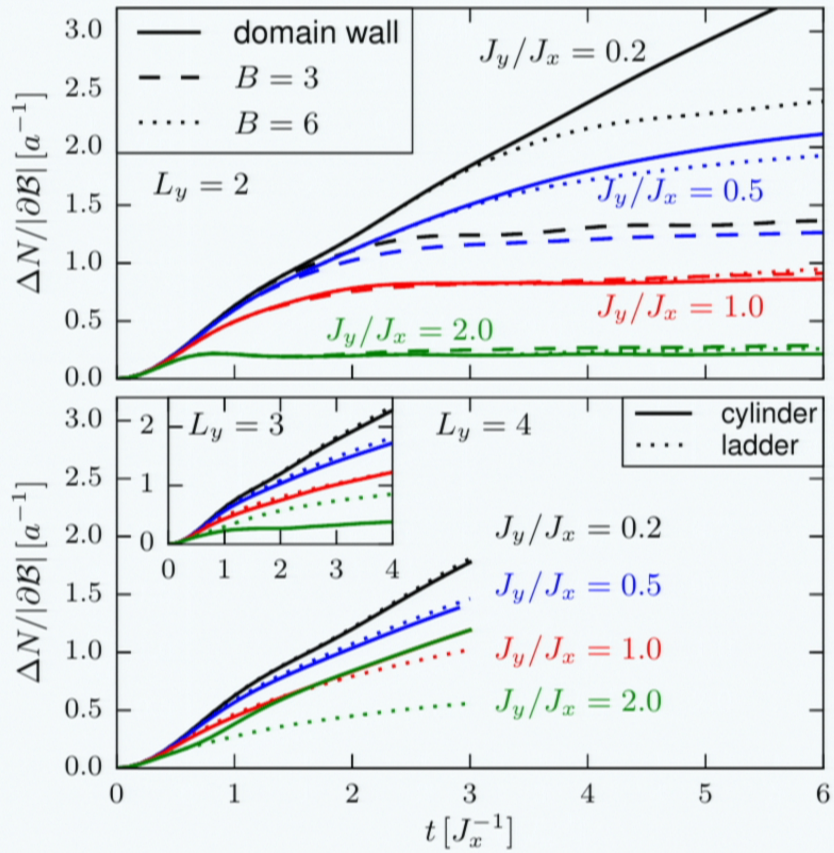
⇒ qualitative agreement of shapes with experiment

- expansion in x-direction suppressed by hopping in y-direction
- dominated by surface effects due to small boson number

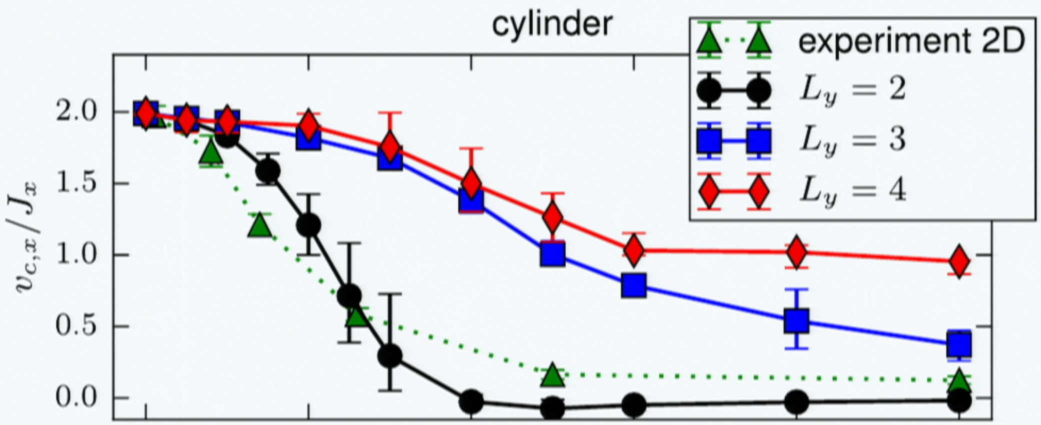
Cylinders and Ladders

$$\Delta N = \sum_{i \notin \mathcal{B}} n_i = \int_0^t \mathrm{d}\tilde{t}\,(j_r^x(\tilde{t}) - j_l^x(\tilde{t}))$$





Cylinders and Ladders



- enhanced expansion on wide cylinders
- lack of propagating modes for $L_y = 2$
- momentum distribution function: no condensation

1. Method: time evolution with MPO

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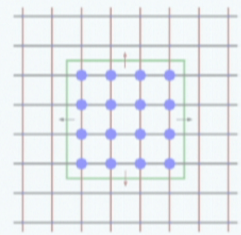
   Hauschild, Pollmann, Heidrich-Meisner, PRB (2015)



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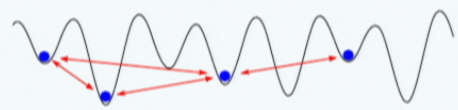
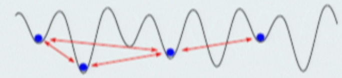
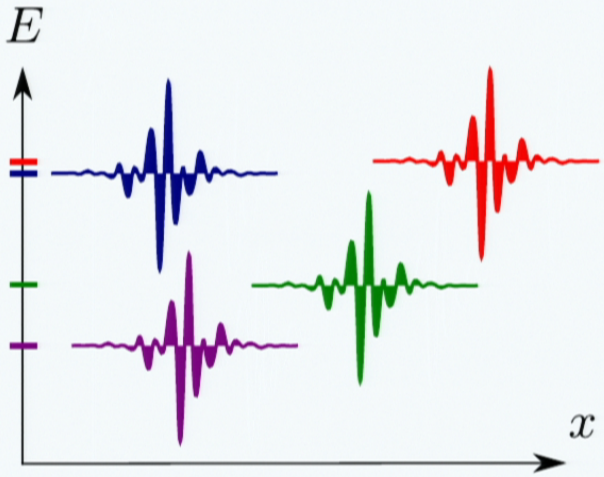
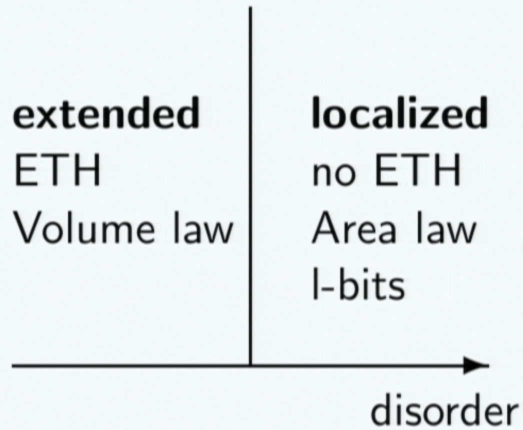
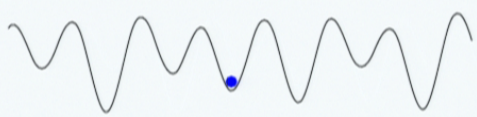


Disordered Systems



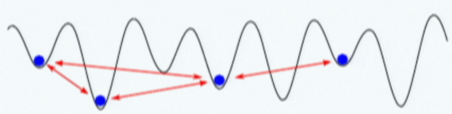






Anderson (1958)
Kramer, MacKinnon (1993)

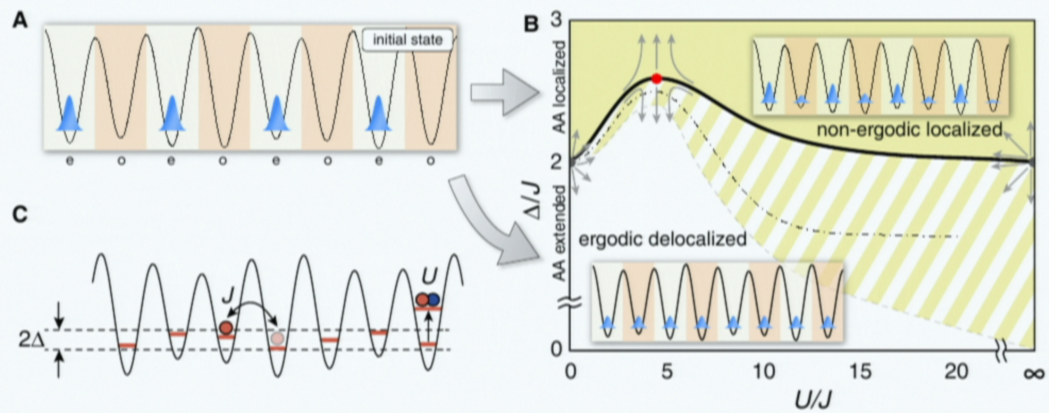
...



Basko *et al.* (2006), Gornyi *et al.* (2005)
Nandkishore, Huse (2015)

...

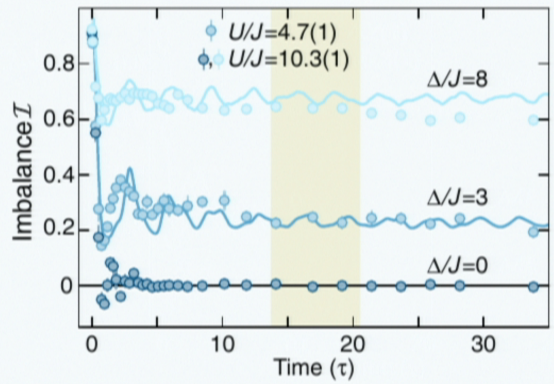
Experiment: Correlated Disorder (Aubry-André)



$$H = -J \sum_{\langle i,j\rangle,\sigma} (\hat{c}^\dagger_{i,\sigma}\hat{c}_{j,\sigma} + h.c.) + U \sum_j \hat{n}_{j,\uparrow}\hat{n}_{j,\downarrow}$$

$$+ \Delta \sum_{j,\sigma} \cos(2\pi r\, j + \phi)\hat{n}_{j,\sigma}$$

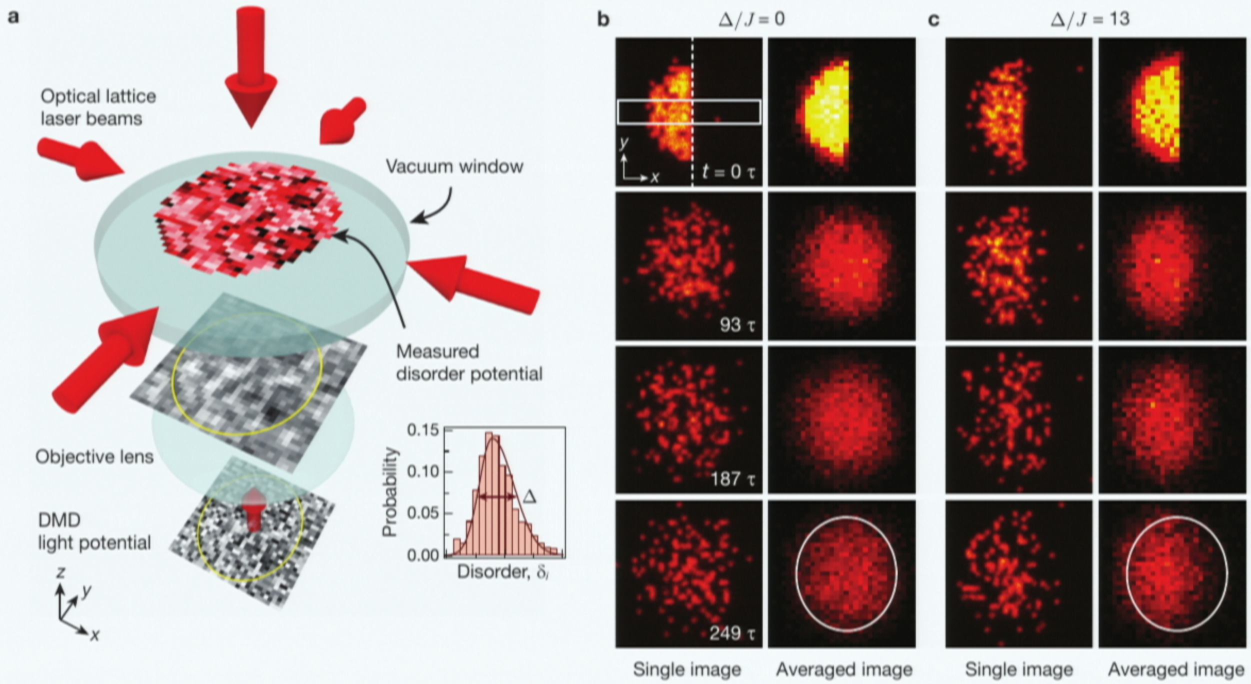
$$\text{Imbalance } \mathcal{I} = \frac{N_e - N_o}{N_e + N_o}$$



Schreiber *et al.* (2015)

Experiment: Uncorrelated Disorder

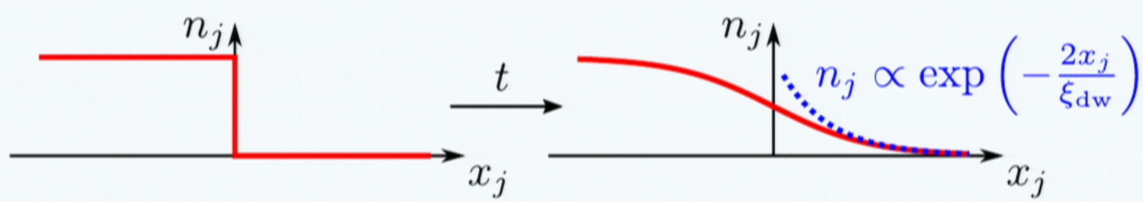
$$H = -J \sum_{\langle i,j \rangle} (\hat{a}_i^\dagger \hat{a}_j + h.c.) + \frac{U}{2} \sum_j \hat{n}_j(\hat{n}_j - 1) + \sum_j (\delta_j + V_j)\hat{n}_j$$



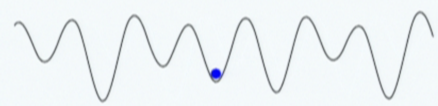
Choi *et al.* arXiv:1604.04178

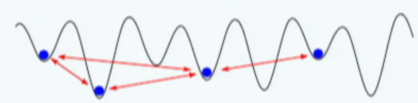
Model

$$H = -\frac{J}{2}\sum_{\langle i,j\rangle}(\hat{c}_i^\dagger \hat{c}_j + h.c.) + U\sum_{\langle i,j\rangle}\hat{n}_i\hat{n}_j + \sum_j \epsilon_j \hat{n}_j$$

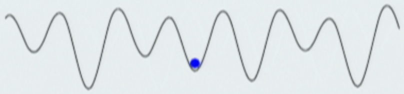


- first check: $U = 0$
  - exact diagonalization
  - uncorrelated $\epsilon_j \in [-W, W]$
  - correlated $\epsilon_j = W\cos(2\pi r\, j)$
- later: $U = J$
  - time evolution with MPS
  - chain: $W_c/J = 3.5 \pm 0.5$
  - two-leg ladder

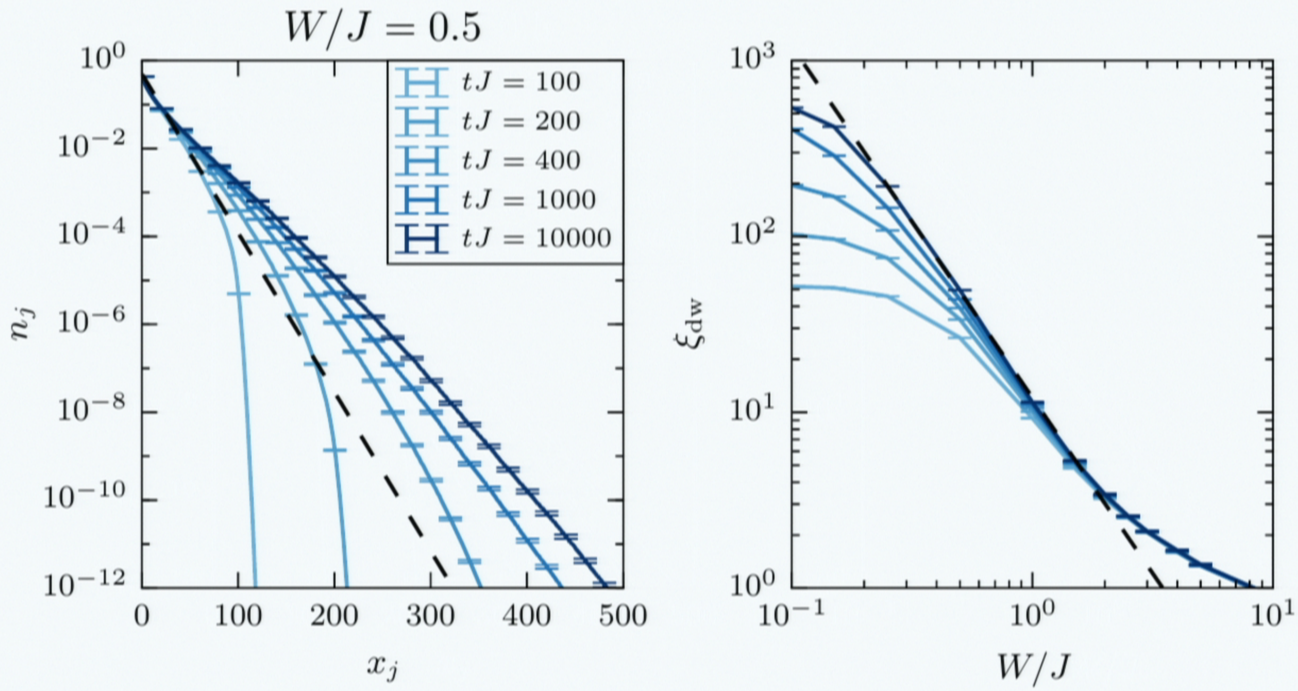




1D Anderson Insulator

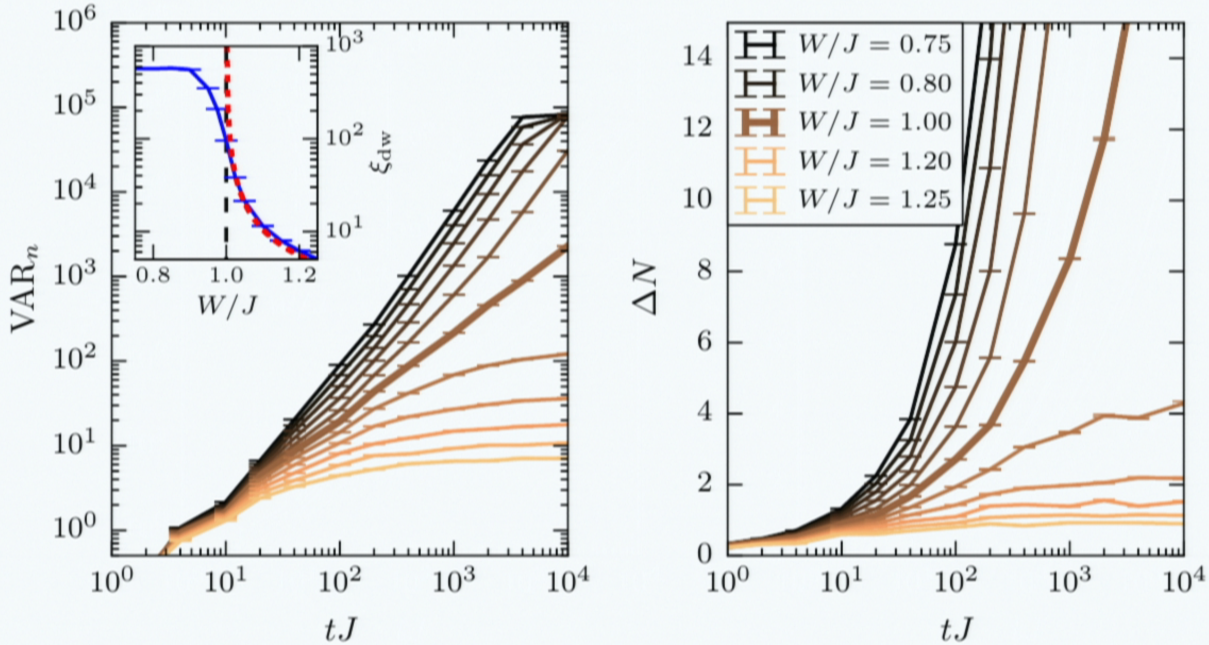


extract ξ_{dw} from $n_j \propto \exp\left(-\frac{2x_j}{\xi_{\mathrm{dw}}}\right)$



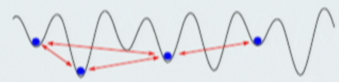
Aubry-André model

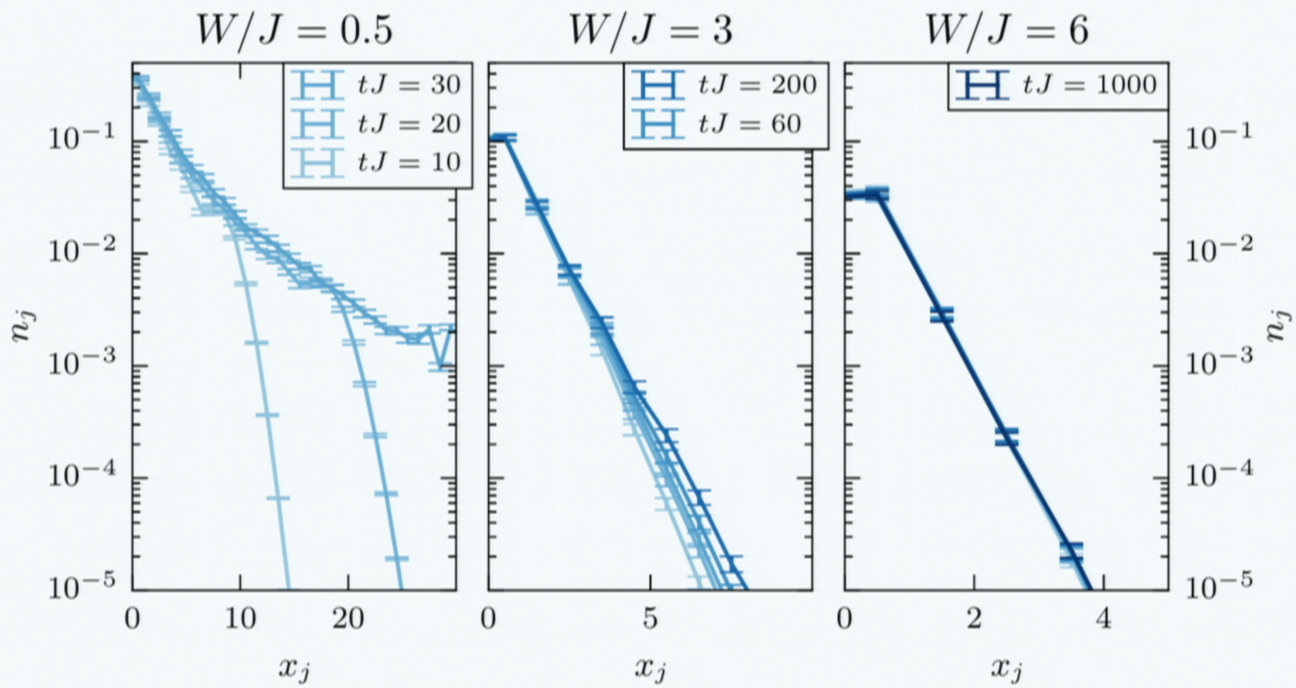
$$\mathrm{VAR}_n = \langle x_j^2 \rangle_n - \langle x_j \rangle_n^2 \qquad \langle \cdot \rangle_n = \frac{1}{\Delta N} \sum_{j:\, x_j > 0} n_j \cdot \qquad \Delta N = \sum_{j:\, x_j > 0} n_j$$



⇒ Domain-wall melting/ξ_{dw} captures transition at $W_c/J = 1$

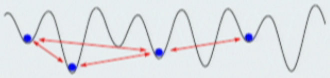
Many-Body Localization

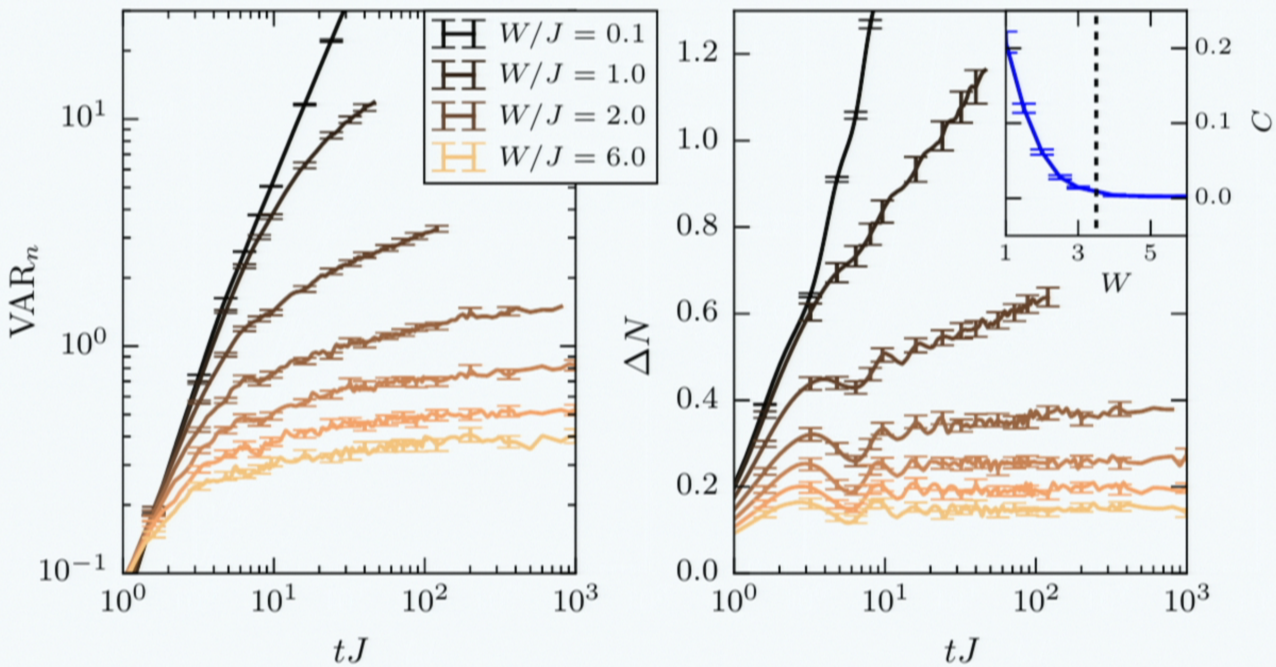




$\Rightarrow$ slow dynamics for $W \lesssim W_c \approx 3.5J$

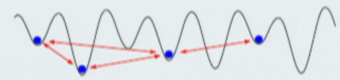
Many-Body Localization

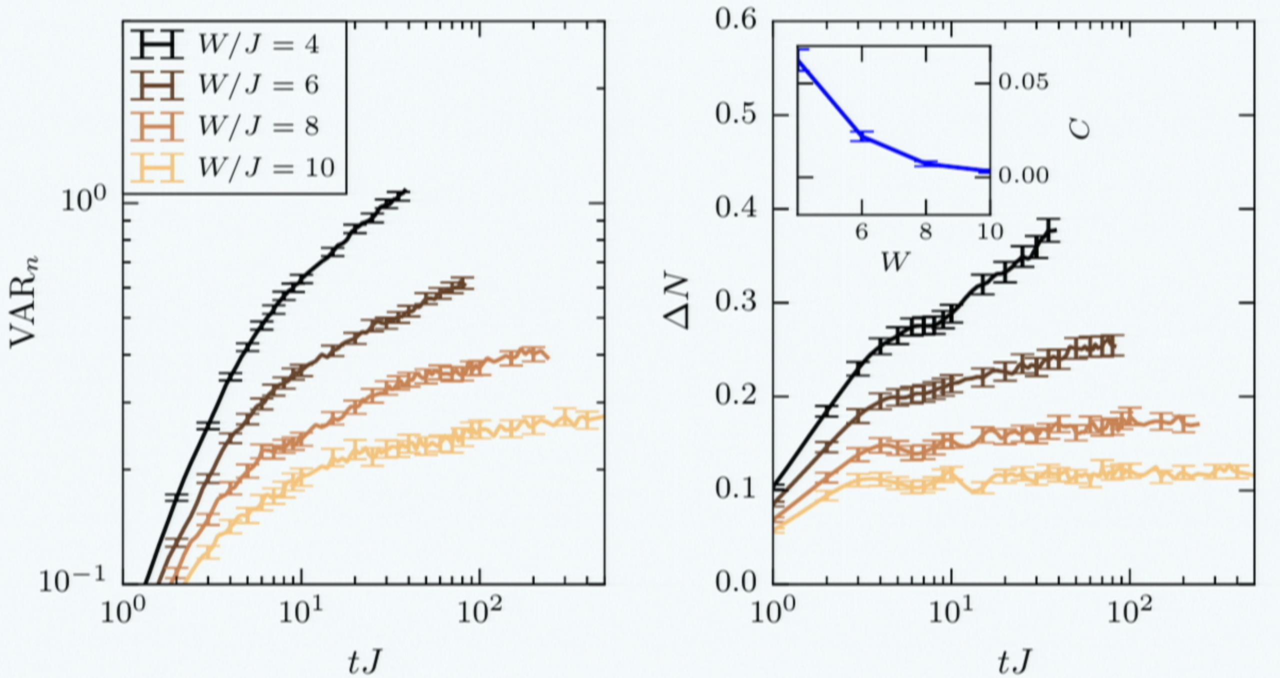




Fit $\Delta N = C(W) \log(t) + \mathrm{const}$ $\Rightarrow$ can extract W_c

Many-Body Localization: Ladder





combatible with $W_c \approx 8.5$ from ED study of Heisenberg model on ladder

Summary

Clean lattice: 1D-to-2D crossover

- fast expansion in 1D-limit, diffusive core in 2D
- larger times/boson numbers for ladders and cylinders
- expansion on $L_y = 2$ ladder suppressed: no 'propagating modes'
- no (quasi-)condensation on cylinders

Hauschild *et al.* PRA **92** 053629 (2015)

Disordered System: domain wall melting

- larger times reachable: slow entanglement growth
- exponential density profiles
- capture Anderson and MBL transition
- slow dynamics around transition

Hauschild, Heidrich-Meisner, Pollmann, in preparation