Title: Translating quantum gravity for the massless

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Abstract: <p>Recent developments in our understanding of black hole evaporation and the information paradox suggest that effects from quantum gravity are not necessarily hidden at the Planck scale. They might even one day be testable by gravitational wave measurements. To prepare ourselves, we must first understand what quantum gravity really means. Thankfully, we are pre-armed with a deep principle about gravityâ€”that spacetime is really a hologramâ€”and a powerful model for making this idea precise: gauge/gravity duality. The present challenge is to translate our questions about gravity into the natural language of the dual conformal field theory (CFT). I will describe the foundation for such a program that links the integral geometry of a gravitational spacetime to a CFT operator product expansion.</p>

What is the problem with quantum gravity?

Why is gravity a *hard* problem?

Gravity is **non-renormalizable** as a quantum field theory.

$$S_{EH} = \frac{1}{16\pi G}\int R dV \qquad [G] \sim L^2$$

To specify the theory in the **UV**, seem to require an **infinite number of counter-terms**: **the theory is not predictive.**

What is not the problem?

As an **effective theory** at **low energies**, perturbatively quantized gravity is a **fine framework** to answer many questions.

- At **low energies** we need only specify **a finite number of counter-terms** to achieve a desired accuracy.
  - **Higher-order terms** are **suppressed by powers of the cutoff**.

What is the problem with quantum gravity?

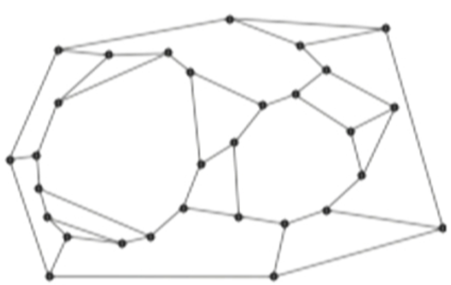
So the problem of quantum gravity has **historically** been thought of as the problem of finding the correct UV completion of the perturbative quantum theory.

There is an enormous body of work attempting to solve this problem.

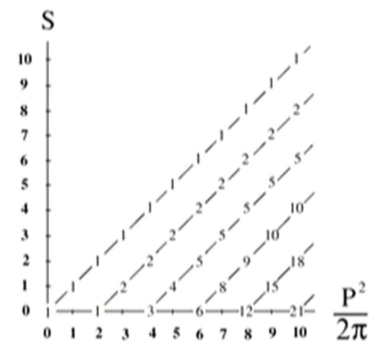
- Much of this work has been rooted in our understanding of **classical gravity** at long-wavelength and **perturbation theory**
- **UV complete** quantum theory of **metrics/geometry.**

One can go about this in many ways:

Modify geometry in the UV



Modify spectrum in the UV



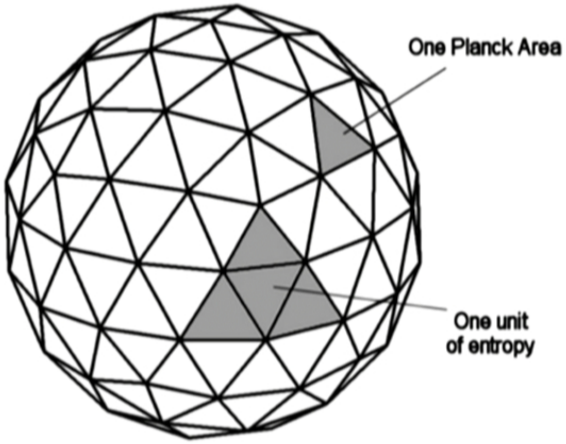
Microscopic quantum gravity

[Bekenstein; Hawking; 't Hooft; Susskind]

Yet in most of these approaches, the single most important fact about any microscopic theory of gravity is hidden in the choice of fundamental variables:

Gravity is a hologram.

Although our universe **appears to be 3+1 dimensional**, the **microscopic physics** behaves more like an **ordinary quantum system that lives in 2+1 dimensions.**



From this **microscopic perspective**, an **extra dimension emerges from the dynamics** of the theory.

Black Holes and Holography

[Bardeen, Carter, Hawking; Bekenstein; Hawking;]

It's worthwhile to recall why we believe this should be the case.

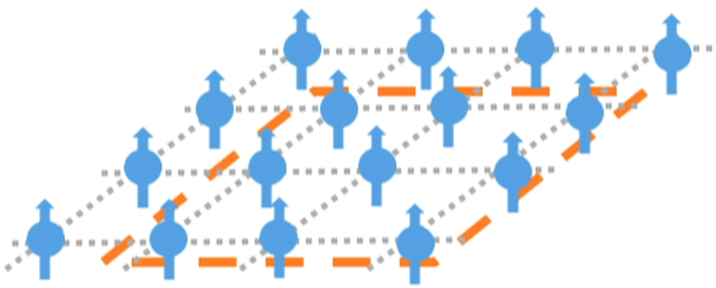
- In the 1970s, it was shown that black holes behave thermodynamically: they **Hawking radiate** at a temperature $T_H \sim 1/M$ and obey a **first Law**

$$\boxed{dM \sim T_H dA} \qquad dU = TdS$$

- The First Law of black hole thermodynamics implies a **microscopic entropy for black holes**

$$\boxed{S_{BH} = \frac{A}{4G}}$$

- This entropy is unlike most familiar QFTs



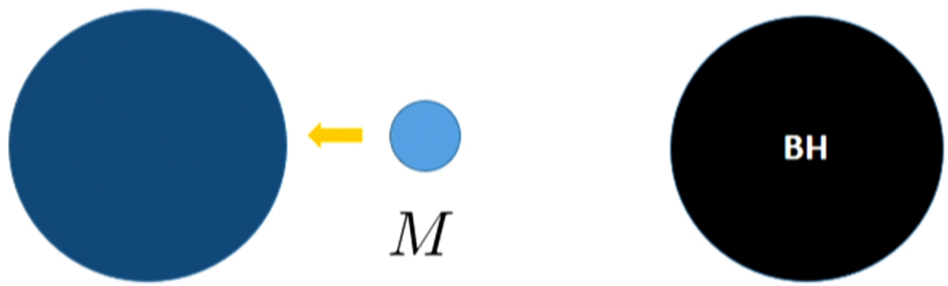
Holographic Principle

[Bekenstein; Hawking; 't Hooft; Susskind]

Entropy being given by area isn't *just* true for black holes.

- Suppose we had a region of spacetime with boundary area A and it had more entropy than a black hole of the same area

$S_A >> A/4G$



$S_{BH} = A/4G$

  - When we drop it into a black hole we would **violate the 2nd law of thermodynamics**!
- The **black hole entropy puts an upper bound on the entropy of any region of space.**

Thermodynamics isn't just a feature of gravity: equivalent! [Jacobson]

Holographic Principle

This holographic principle was **not derived from microscopic physics**.

It resulted from **infrared/thermodynamic behavior of black holes in any consistent theory of gravity**.

Holography seems like the most profound feature of gravity.

It then seems a much better idea to work with a theory of gravity where the **most important characteristic feature of gravity is built-in** to the microscopic description: we want an **explicitly holographic theory.**

1. **Old goal of QG: What is the UV completion of gravity?**

2. **New goal of QG? How does gravity emerge from a more ordinary quantum system in a lower dimension?**

Gauge/Gravity Duality

[Maldacena; Gubser, Klebanov, Polyakov; Witten]

For such an approach to be sensible, we're **required to understand** a **consistent UV complete theory of gravity**.

It seems beyond reach right now to construct such a model that describes our universe (**positive cosmological constant)**

But it is possible to construct UV complete theories of gravity that are slightly different from our universe (**negative cosmological constant)**

Gauge/Gravity Duality:

Certain quantum field theories are precisely equivalent to a theory of gravity in one higher dimension.

- You might call these **toy models for quantum gravity**, but they are rather sophisticated: they contain a **complete theory of quantum gravity.**

AdS/CFT Duality

Our best understood examples are certain conformal field theories (CFTs):

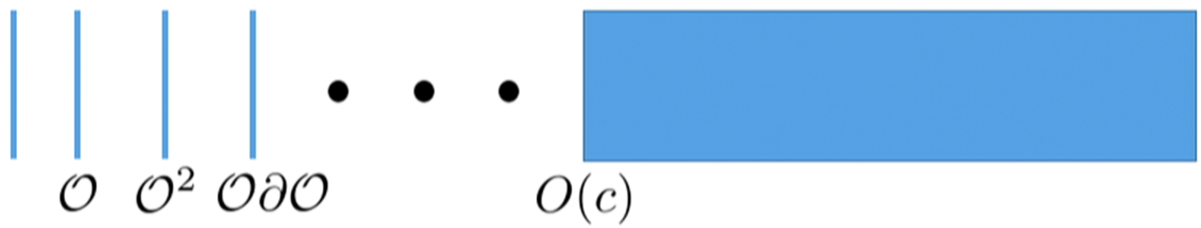
What are these CFTs?

Consider a CFT that has:

1. **Large central charge**: $c >> 1$
2. Whose **correlators factorize**:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_1(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle = \langle \mathcal{O}_1(x_1)\mathcal{O}_1(x_2)\rangle \langle \mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle + O(1/c)$$

3. Whose **spectrum of conformal dimensions** is **sparse**:

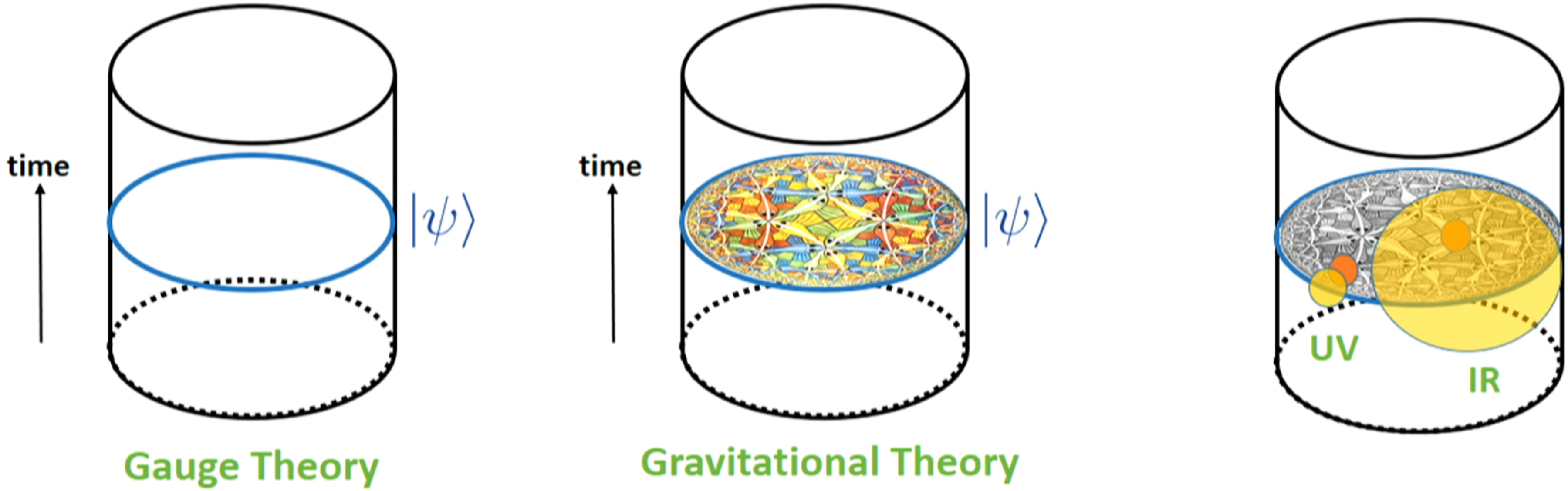


Then this CFT is dual to a theory of gravity whose low-energy energy description is gravity plus effective field theory of field ϕ dual to $\mathcal{O}$.

[Heemskerk, Penedones, Polchinski, JS]

AdS/CFT Duality

If we consider some **state, $|\psi\rangle$, in the CFT**



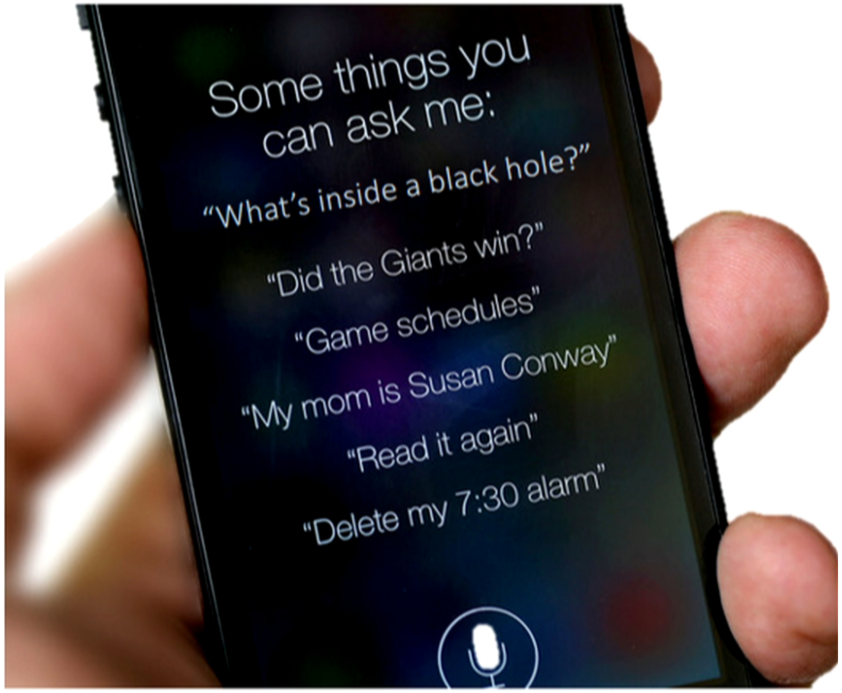
it also describes **some geometry in the gravitational theory**.

- Gauge theory lives on the **boundary** of the gravitational theory (**bulk**).
- In the case where $|\psi\rangle$ is the **vacuum**, the dual spacetime is maximally symmetric, negative curvature **anti-de Sitter Space (AdS)**

Siri for Quantum Gravity

The gauge/theory is like Siri, but for quantum gravity:

- The gauge theory has the **answer to any physical question about quantum gravity**
- But it can only give us answers to the **questions we know how to ask** in the right way.
- Even if we **completely solved** the gauge theory, we **wouldn't know the answers** to our questions about gravity.



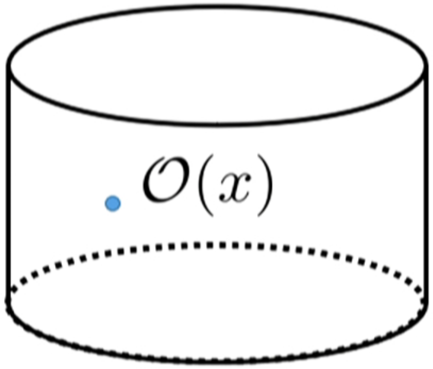
Siri and the gauge theory are both pretty rudimentary in speaking our language!

The real problem is that we need to know how to translate questions from the language of gravity to the language of gauge theory!

Different Languages

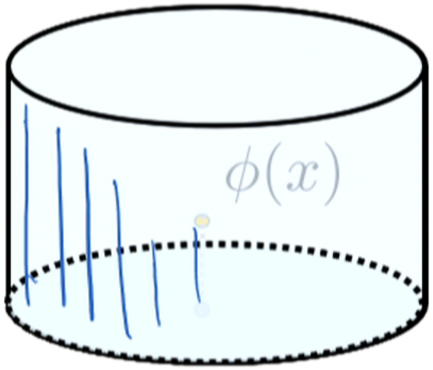
The **natural object** to work with in the **gauge theory** are **local operators**

- These are **UV operators** that **don't naturally see far into the bulk.**



The **natural objects** to work with **in gravity** might seem to be **local operators** in the bulk.

- Gravity has **no local operators** (diff-invariant)
- **Quasi-local operator** is **complicated** to specify in terms of boundary conditions on the cylinder.



Requirements for a new language

If we want to teach the gauge theory to speak gravity, we should start by **finding the simplest 'words' and 'grammar' to organize our translation** that look **natural on both sides of the duality.**

In the Gravitational Theory:

1) Should be **diffeomorphism-invariant**.
2) Necessarily will be **non-local**.

In the Gauge Theory:

1) Want a **non-local** variable that can see into the spacetime geometry.
2) Would like to be a **simple/natural object** (transform nicely under symmetries of the theory).

In this talk, I will describe exactly such a foundation for a better dictionary to translate between the CFT and gravity.

- *(It is a language very much still under construction.)*

We can already see that it is a **powerful framework**. It brings together many familiar ideas in holography, including:

1. The **Entanglement First Law** and **Einstein's Equations** [Faulkner, Guica, Hartman, Lashkari, McDermont, Myers, Swingle, Van Raamsdonk]
2. **Geodesic Witten diagrams** and **conformal blocks** [Hijano, Kraus, Perlmutter, Snively]
3. The **HKLL** construction of interacting **'local' bulk fields** [Hamilton, Kabat, Lifschytz, Lowe]+...
4. **de Sitter dynamics** for the variations of **EE** [de Boer, Heller, Myers, Neiman] [Nozaki, Numasawa, Prudenziati, Takayanagi], [Bhattacharya, Takayanagi]

Outline

1. **Why bother with quantum gravity?**

2. **A parable from forgotten physics**

3. **Teaching gauge theories to speak gravity**

When does QG matter?

When do quantum gravity effects become relevant?

Since it’s hard calculate directly, we must use dimensional analysis to estimate.

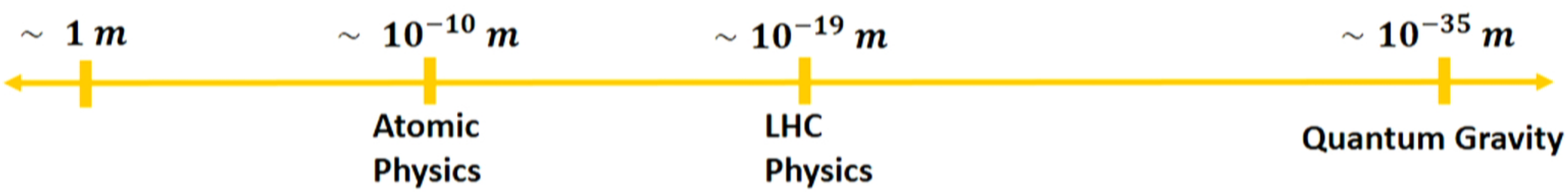
- There is **only one scale** (that we are certain of) in the theory: **G**

$$G = 6.67 \times 10^{11} Nm^2/kg^2$$

Planck Length:

$$l_p = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35} m$$

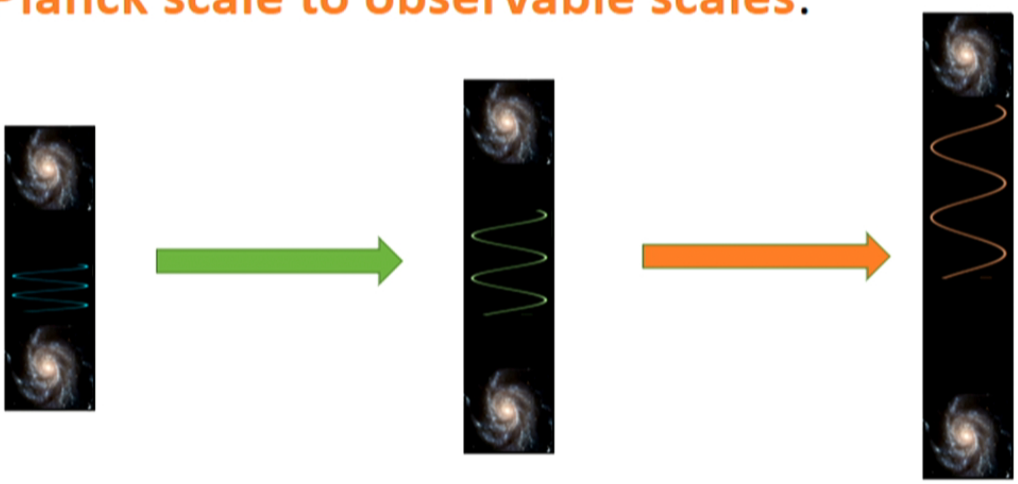
Hopelessly small!!!



Why bother?

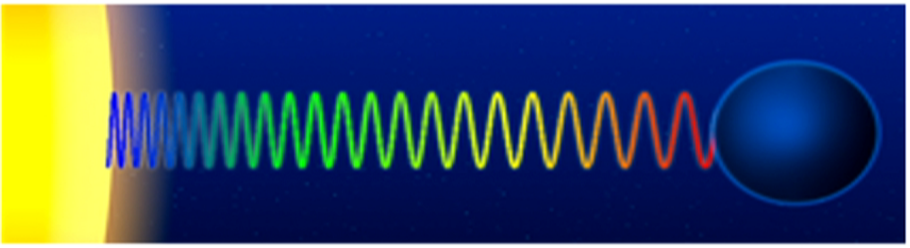
If we don't expect to see effects of quantum gravity until unobservably small scales, what's the point in studying quantum gravity?

We have **neglected one important fact about gravity**: the **warping of spacetime** can **shift the frequencies** of excitations all the way **down from the Planck scale to observable scales**:



Quantum Gravity and Black Holes

In particular radiation from near a massive object is redshifted as it escapes the gravitational potential.



- The **strongest such redshift is produced by black holes.**
  - Radiation produced near the horizon of a black hole can be **redshifted from Planckian energies** before it escapes away.

Can we make use of this fact to study quantum gravity?

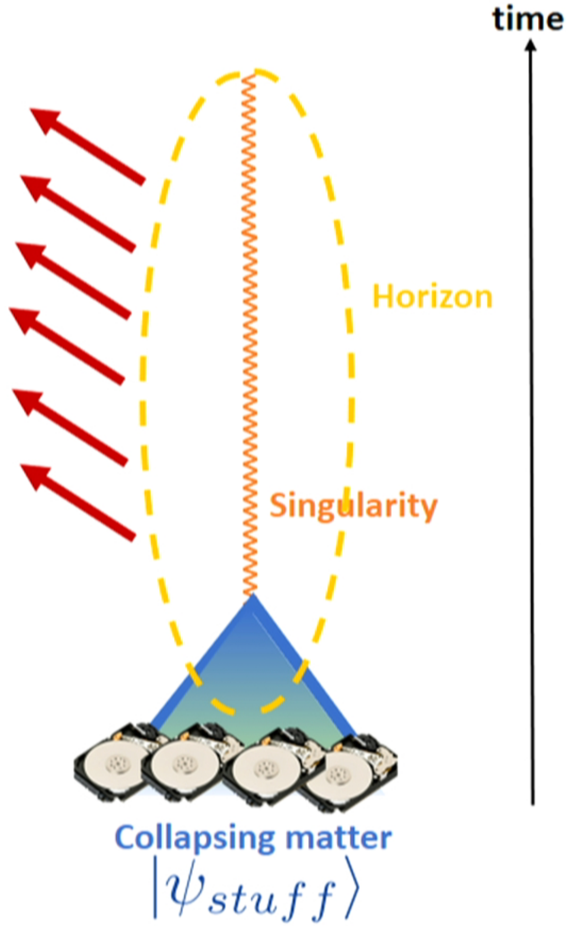
Not so black holes

- In the 1970s, Hawking showed that **black holes evaporate** away quantum mechanically by emitting radiation.
- One feature of QFT in curved space: different observers don't agree on whether a quantum state is the vacuum.
  - **Horizon appears empty to infalling observer**. Vacuum decomposed into '**Rindler**' basis:

$$|0\rangle = \frac{1}{N}\sum_E e^{-\beta E/2}|E\rangle_{out}|E\rangle_{in}$$

  - This same state that looks empty as an observer falls across the horizon, **appears hot to distant observers**, (like accelerated Rindler observers).
  - They see the black hole radiate with temperature

$$T_H \sim 1/M$$



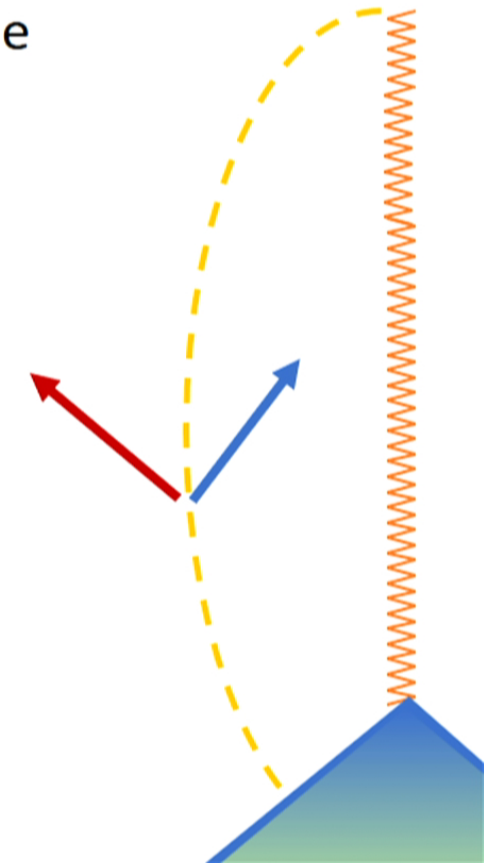
Not so black holes

- For purposes of expository **simplicity**, we can think of the state of the radiation as a **pair of qubits** in the state

$$|\psi_H\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_{out}|0\rangle_{in} + |1\rangle_{out}|1\rangle_{in}\right)$$

- The **outgoing Hawking radiation is entangled** with its partner inside the black hole.

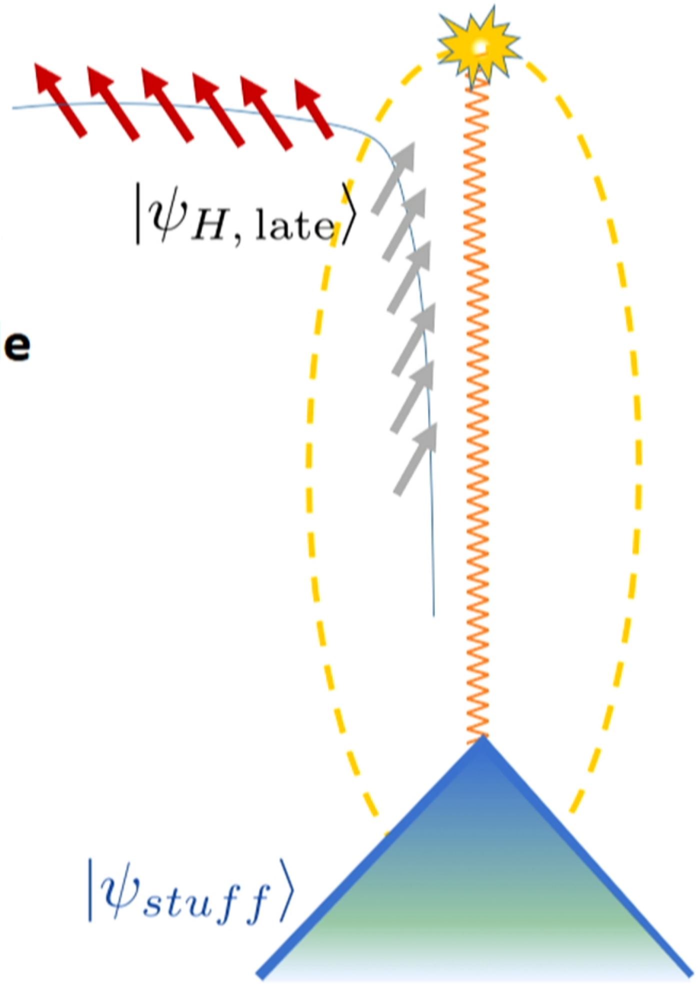
$$|\psi_H\rangle \neq |\psi_a\rangle_{out} \otimes |\psi_b\rangle_{in}$$



When the black hole has almost radiated away, there is a **large amount of outgoing Hawking radiation**, **entangled with its counterpart inside the black hole**

$$|\psi_{H,\,\mathrm{late}}\rangle = \frac{1}{N} \otimes_i (|00\rangle_i + |11\rangle_i)$$

And then the black hole disappears….



So we have an evolution from the **pure state of collapsed matter**, which can contain a lot of information

$$|\psi_{stuff}\rangle$$

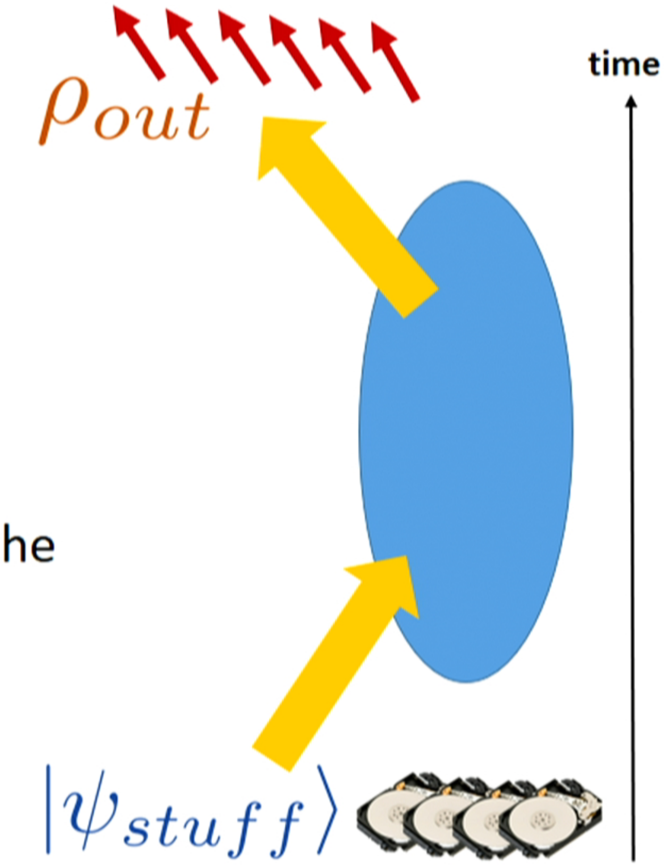
to just the **outgoing radiation, a mixed state**

$$\rho_{out}$$

- The **radiation** is, to good approximation **featureless** (it contains only a small amount of information like the mass and charges of the black hole).

- The rest of the state has disappeared---the outgoing radiation is not pure:

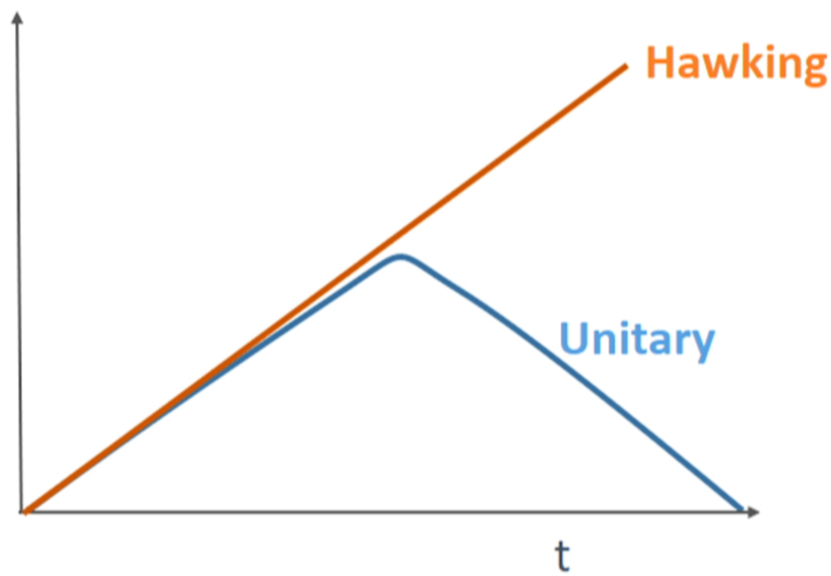
INFORMATION LOSS and VIOLATION OF UNITARITY



[Page; Mathur]

The basic issue:

- **EFT/Hawking**: emit thermal, entangled radiation for the lifetime of the black hole
- **Unitary process:** entropy turns over when the Hilbert space of the black hole is half its initial size, then decreases maximally.
    - Every late emitted bit is maximally entangled with earlier emitted outside bits



These two curves are radically different.

- You won't be saved by small corrections to thermal radiation.

Firewalls

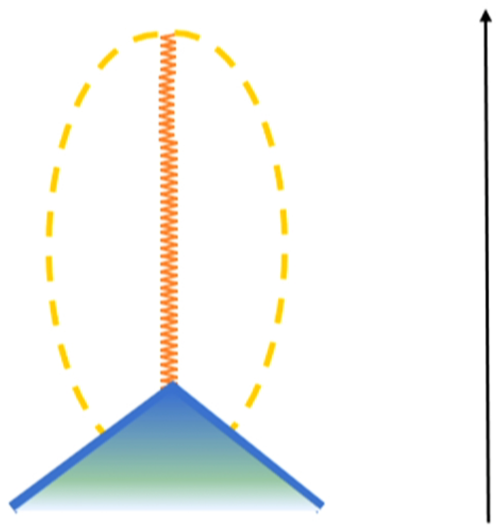
[Almheiri, Marolf, Polchinski, JS; AMPS+Stanford; Mathur; Braunstein]

- Recall, it was the **thermally entangled state** that looked **smooth** to an observer passing through the horizon.
- In the frame of an infalling observer, the unitarily-evolved state is **very different than the empty vacuum**.

Because, we don't understand quantum gravity well enough, we can't really say what they would **oberserve**, but unitarity **requires** a **failure of 'physics as usual' (ie. the use of low-energy EFT).**

We can call this a **firewall**.

- *One possible result:*
  Spacetime is torn to pieces near the horizon of a black hole. It has no interior and **spacetime ends**.



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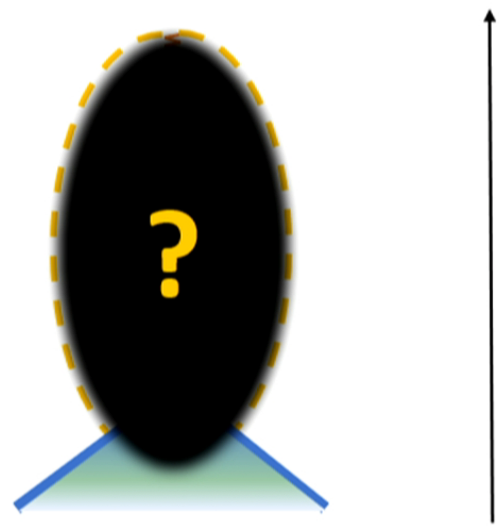


Firewalls

For a large black hole, the **curvature of spacetime is small at the horizon.**

- **Equivalence principle**: It is largely indistinguishable (locally) from any other normal point in spacetime

- If a firewall had appeared **near the singularity**, where curvature is Planck scale, this would be **nothing unexpected**.
- **A firewall at the horizon of a large black hole is a quantum gravity effect now visible at low-energy—this is a crisis!**

- **One has to give up something precious:** locality, effective field theory, measurement theory in quantum mechanics.
- **And/or invoke new principles:** error correction, entanglement and wormholes, …

Firewalls

One can interpret the crisis two ways:

1. **We're doomed.**
   We don't know what we're doing. We've lost control of our calculations and we don't know why.

2. **We could never have hoped to be so lucky.**
   Quantum gravity matters and it matters at low energy.

If we discover the right answer, with powerful enough tools to study black holes, we might even be able to measure these effects without having to fall into a black hole ourselves.

- For example, the precise **gravitational wave spectrum** of a black hole merger *might* **depend on the geometry near the horizon**. *Maybe* this will be measurable in future detectors.

While the information paradox is our justification, we don't know yet how to choose the right solution. **We need to think hard about emergent geometry in simpler situations where the stakes are a little lower…**

A parable from forgotten physics

Lessons from gauge theories

We typically formulate a gauge theory in terms of a **gauge potential** $A^a_\mu(X)$ and the corresponding **field strength** $F^a_{\mu\nu}$, where the equation of motion is

$$(\nabla^\mu F_{\mu\nu})^a = 0$$

But the inclusion of the gauge field is just a **redundancy of the description**. It's unfortunate to have to use such an **inefficient representation** of the physics.

However, we do have a **complete set of gauge-invariant observables**

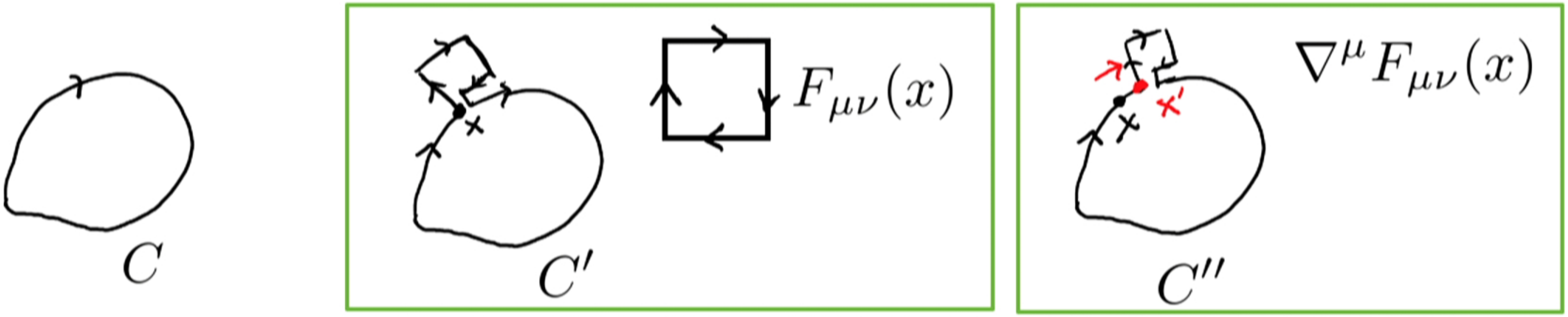
$$\Psi(C) = \mathrm{Tr}\left[\mathrm{P}\exp\left(\oint_c A_\mu dx^\mu\right)\right] \qquad W(C) = \langle \Psi(C) \rangle$$

Can we express the dynamics of the theory in terms of these variables alone? (That is, can we write an equivalent EOM in the **space of loops?)**

Loops

[Polyakov; Migdal, Makeenko]

How do we insert the equation of motion into a Wilson loop?



$$\frac{\delta W(C)}{\delta x_\mu(s)} = \langle \mathrm{Tr}\ \mathrm{P} F_{\mu\nu}(x(s))\dot{x}(s)^\nu \exp\left(\oint A_\rho dx^\rho\right)\rangle$$

$$\frac{\delta^2 W(C)}{\delta x_\mu(s))\delta x^\mu(s')} = \langle \mathrm{Tr}\ \mathrm{P} F_{\mu\nu}(x(s))\dot{x}(s)^\nu F_{\mu\sigma}(x(s'))\dot{x}(s')^\sigma \exp\left(\oint A_\rho dx^\rho\right) + \delta(s-s')\dot{x}_\nu(s)\nabla^\mu F_{\mu\nu} \exp\left(\oint A_\rho dx^\rho\right)\rangle$$

$$\frac{\partial^2 W(C)}{\partial x(s)^2} = \lim_{\epsilon\to 0}\int_{-\epsilon}^{\epsilon} dt\, \frac{\delta^2 W(C)}{\delta x_\mu(s+t/2))\delta x^\mu(s-t/2)} = \langle \mathrm{Tr}\ \mathrm{P}\ \nabla^\mu F_{\mu\nu}\dot{x}_\nu(s) \exp\left(\oint A_\rho dx^\rho\right)\rangle$$

Loop Equations

[Polyakov; Migdal, Makeenko]

We now have the classical equations of motion in written purely in terms of Wilson loops:

$$\frac{\partial^2 W(C)}{\partial x(s)^2} = \left\langle \mathrm{Tr}\ \mathrm{P}\ \nabla^\mu F_{\mu\nu} \dot{x}_\nu(s) \exp\left(\oint A_\rho dx^\rho \right) \right\rangle = 0$$

But we can do better and find the **correct quantum equation**:

- Consider the **Schwinger-Dyson equation for $W(C)$** under the variation $\mathrm{A} \rightarrow A + \delta A$ *(for the case G = SU(N))*:

$$\left\langle -\frac{1}{e_0^2} \mathrm{Tr} \nabla^\mu F_{\mu\nu}(z) \Psi(C_{xx}) \right\rangle = \oint_C dy_\nu \delta(z-y) \left(\langle \mathrm{Tr}\ \Psi(C_{xy}) \mathrm{Tr}\ \Psi(C_{yx}) \rangle + \frac{1}{N} \langle \mathrm{Tr}\ \Psi(C_{xx}) \rangle \right)$$

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[Polyakov; Migdal, Makeenko]

We can again rewrite the left hand side using the loop derivative to find the complete **equation of motion for $W(C)$**:

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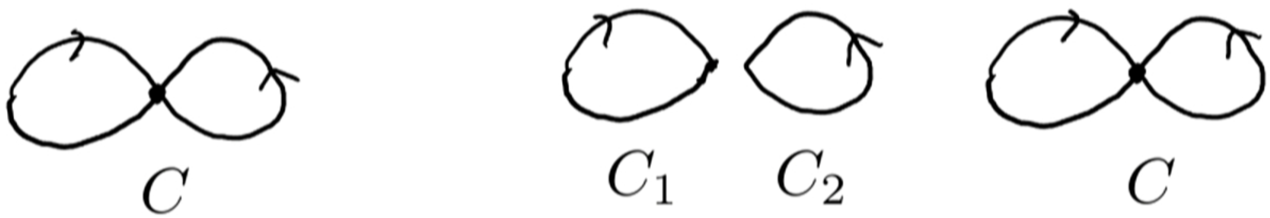
Loop Equations

[Polyakov; Migdal, Makeenko]

We can again rewrite the left hand side using the loop derivative to find the complete **equation of motion for $W(C)$**:

$$\frac{\partial^2 W(C)}{\partial x(s)^2} = -e_0^2 \oint_C dy_\nu \delta(z-y) \left(W_2(C_1, C_2) - \frac{1}{N} W(C) \right)$$



This equation **simplifies at large N**:

$$\frac{\partial^2 W(C)}{\partial x(s)^2} = -\lambda \oint_C dy_\nu \delta(z-y) W_2(C_1) W(C_2)$$

$$\lambda = e_0^2 N$$

Intertwinement

[Halpern, Makeenko]

One can also define a **Laplacian on field space** and a **Laplacian on loop space**

$$\Box_F = \int dx dy R^2 (D^2)^{ab}_{xy} \frac{\delta}{\delta A^a_\mu(x)} \frac{\delta}{\delta A^{b\mu}(x)} \qquad \Box_L = \oint ds \frac{\partial^2}{\partial x(s)^2}$$

Then the loop equation is just

$$(\Box_L + \Box_F)\, W(C) = 0$$

If we view the **Wilson loop as a** **loop transform W** of the gauge field , then this is a statement

$$\Box_L W = -W \Box_F$$

Intertwining Operators

The loop equations are **conceptually beautiful** (although *rather singular* and *hard to actually calculate* with).

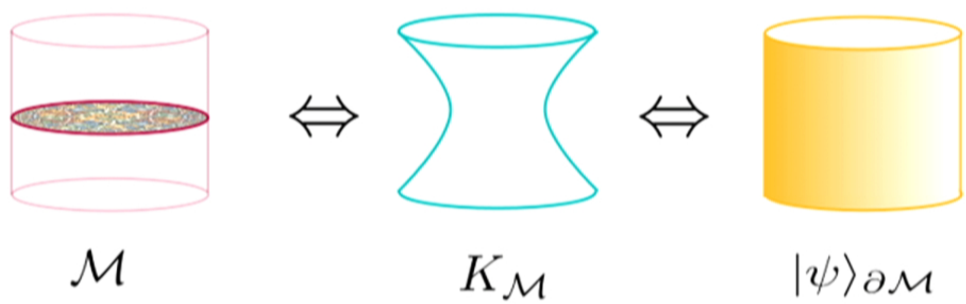
- *None of us learn gauge theory this way.*
- **Perhaps this idea's time just hadn't come yet… and perhaps this was the *wrong theory*.**

Requirements for a new language

The loop equations seem just like the way of re-expressing gravitational physics we have been looking for.

We might re-express our search for the correct language:

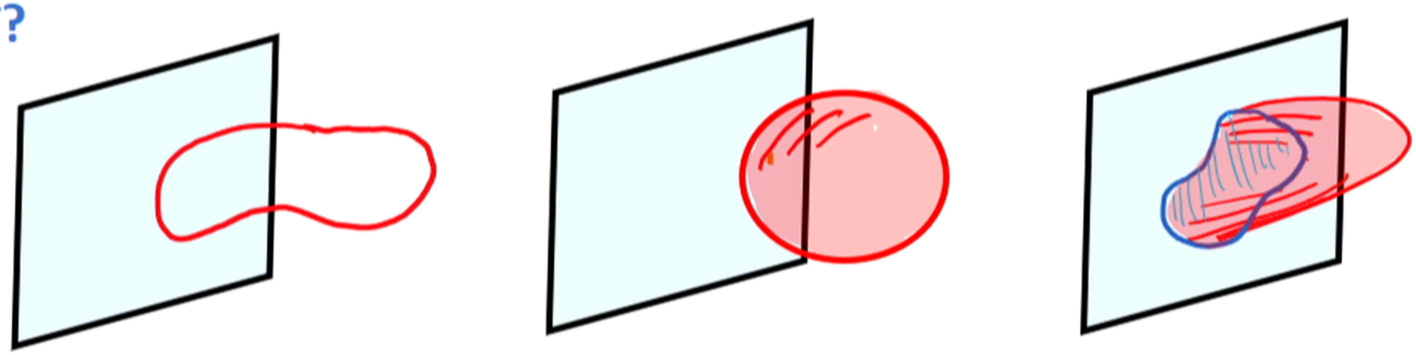
1. What is the right **loop space for gravity**?
2. What are the **loop operators** and their **loop equations for bulk physics**?
3. What do the loop equations and loop operators **look like in the gauge theory**?



Loop Space

Loop Space = ?

We need to understand: **what is the right loop space for gravitational physics in AdS/CFT?**



Those that **aren't anchored on the boundary** are **difficult to specify in a diff-invariant way** (they must be specified by boundary data)

- **Unlikely** to have a **simple representation** in the gauge theory

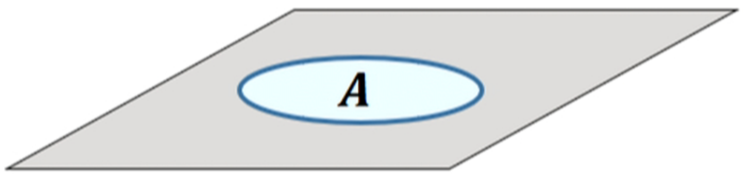
If they are **boundary-anchored**, would like to be of a type simply specified in terms their **geometric boundary data**.

What is a natural choice of boundary-anchored objects?

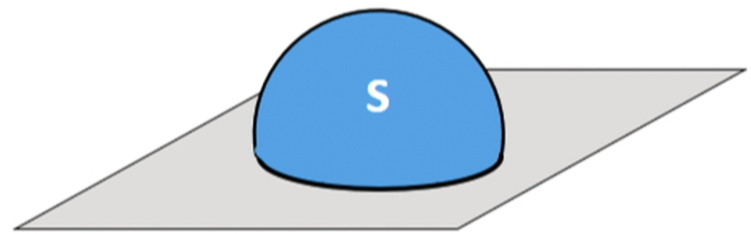
Geometry and Entropy

Thankfully an example of the type of construction we're searching for has already been found: Ryu-Takayanagi Proposal.

The **Ryu-Takayanagi (RT/HRT) proposal** connects:



The **entanglement entropy** of a region A on the boundary.



The **area of a minimal surface** S in the gravitational dual

Integral Geometry

What is the lesson from Ryu-Takayanagi?

The CFT sees the bulk geometry naturally through codimension-2 minimal surfaces.

- Boundary conditions are codimension-2 surfaces on the CFT cylinder

One could define the relevant **loop space** to be **all possible spacelike surfaces.**

- Probably **enormously redundant:** **infinite-dimensional space**

  Bulk: d+1 dimensional

  Space of spherical regions: d+1 dimensional (d: center 1: radius)

Let's define our loop space to be the space of minimal surfaces with spherical boundary conditions.

We will give it a new name: Kinematic Space

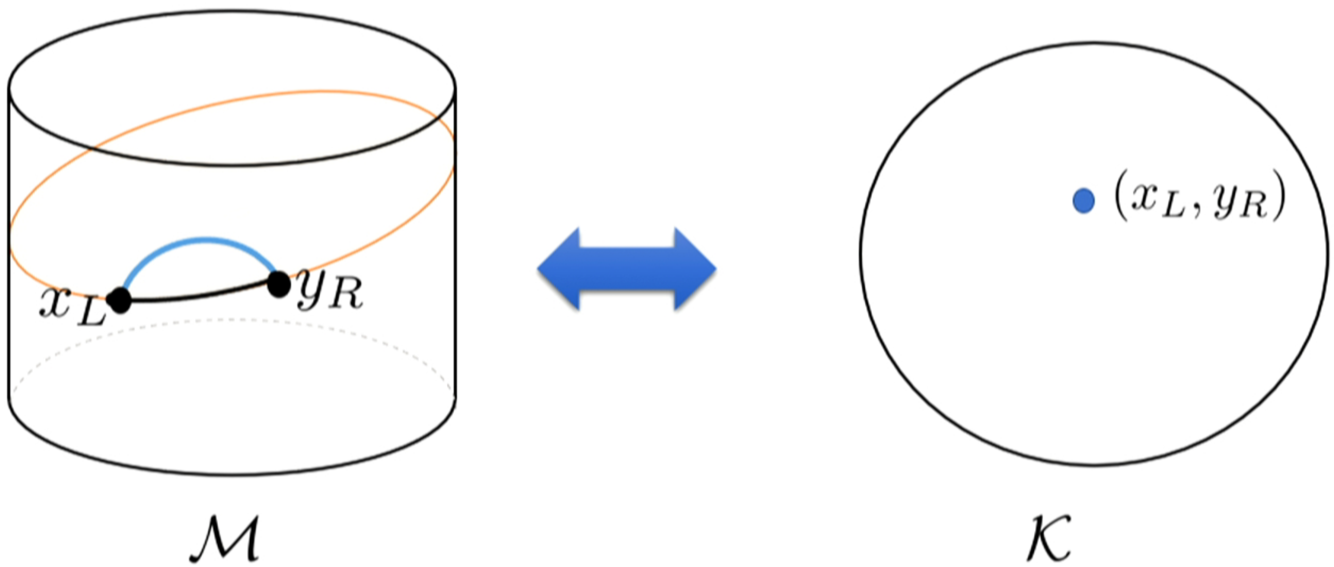
- This is a familiar object of study for some mathematicians: it is a primary object in the field of **integral geometry.**

Kinematic Space

What does kinematic space look like?

Let's consider AdS3 for simplicity (***minimal surface = geodesic***):

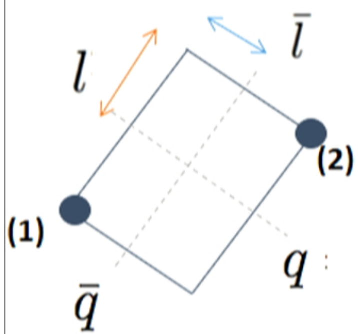
- Consider a geodesic. This is specified by an **ordered pairs of spacelike separated points** on the boundary of a 3D spacetime:



Kinematic Space

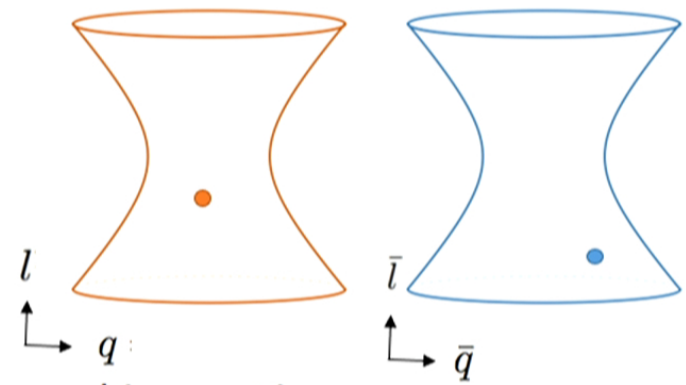
Can we assign a metric to kinematic space?

- The metric on $\mathcal{K}$ should be **invariant under isometries of the geometry.**
- This **uniquely fixes the metric** when we consider the **ground state** AdS_3 :



$$ds^2 = \frac{1}{2}\left[\frac{dq^2 - dl^2}{l^2} + \frac{d\bar{q}^2 - d\bar{l}^2}{\bar{l}^2}\right] = \frac{1}{2}\left[ds_z^2 + ds_{\bar{z}}^2\right]$$

dS_z $\quad$ $\mathrm{dS}_{\bar{z}}$



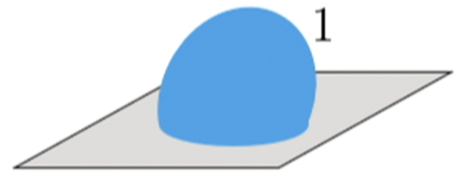
- In some cases, we understand how to specify a metric on kinematic space even when it's not fixed by symmetry
- Geometries are equivalent ⇔ **map from real space to kinematic space** is **invertible**.
  - This is an active area of mathematical research! (*Boundary Rigidity / Lens Rigidity*)

Kinematic Fields

We want to generalize the Ryu-Takayanagi example to describe all of the bulk physics. We also need the right 'Wilson Loops' in addition to the right space.

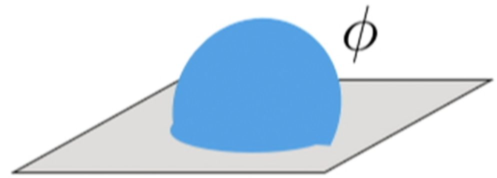
- We can think of their areas as integrating the unit operator over the minimal surface:

$$A = \int d^n x \sqrt{h}(1)$$



- A natural generalization then is:

$$\tilde{\phi} = \int d^n x \sqrt{h}(\phi)$$

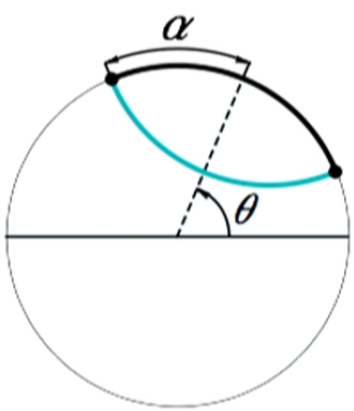


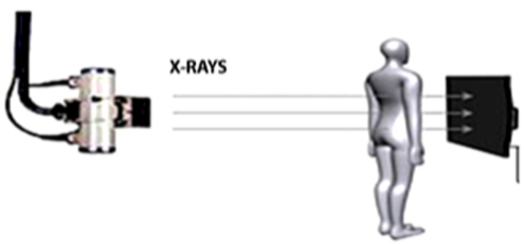
- This bulk surface/geodesic operator is a **non-local** and **diff-invariant** bulk probe.

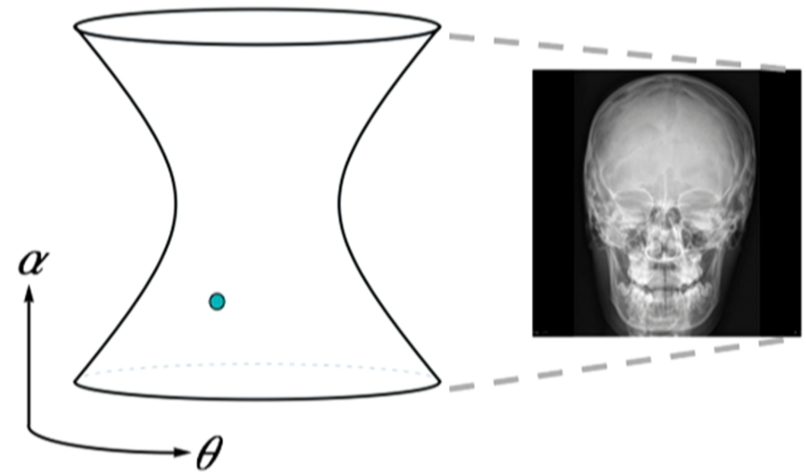
X-Ray Transform

In the case of a 2D boundary, this a well-known operation, the **X-ray transform:**

$$f(x) \xrightarrow{Rf} \tilde{f}(\alpha, \theta_c) = Rf = \int_{\gamma(\alpha,\theta_c)} ds f(x)$$

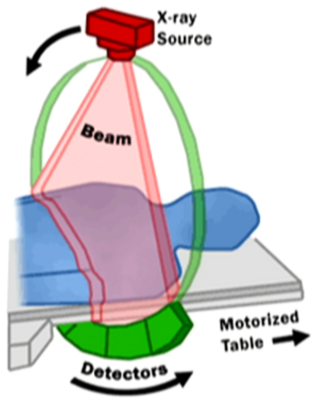


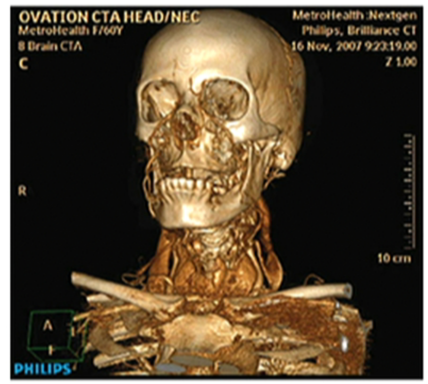




You might worry our kinematic space only sees a projection of the gravitational theory, like the X-ray image.

- *But*, we don't have just one projection of the bulk data—**we have the projections in all 'angles'**
- Reconstructing a complete 3D image from all of these projections (geodesic integrals) is a well-understood problem. It's what allows this:

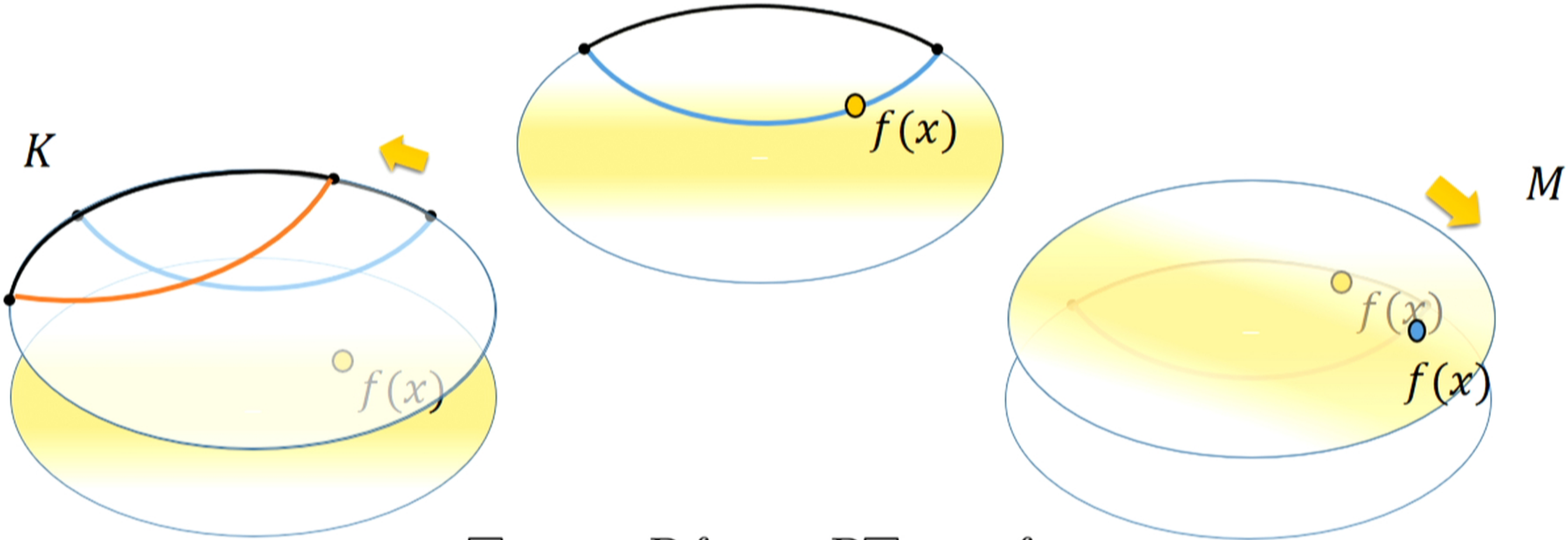




We can do '**computed tomography' (CT)** in the gravitational theory and reconstruct the complete physics in the extra dimension.

X-Ray Transform

The X-ray transform has nice properties under isometries of the geometry:



$$\Box_{dS\times dS} Rf = -R\Box_{AdS_3} f$$

"Intertwining Operators"

Loop Equations in kinematic space

Intertwinement allows us to **rewrite the dynamics of the gravitational** theory in terms of **dynamics on kinematic space**:

Free Scalar Field:

$$\left(\Box_{AdS} - m^2\right)\phi(x) = 0 \quad \Leftrightarrow \quad \Box_{dS\times dS} Rf = -R\Box_{AdS_3} f$$

$$(\Box_{KS} + m^2)\tilde{\phi}(\gamma) = 0$$

Loop EOM for free scalar
= Kinematic Free scalar EOM

Linear Einstein Equations:

$$\left(\Box_{AdS} + 2\right)\delta g_{\mu\nu}(x) = 0$$

$$\Updownarrow$$

$$(\Box_{KS} + 2d)\,\tilde{\delta} g(x) = 0$$

[Faulkner, Guica, Hartman, Lashkari, McDermont, Myers, Swingle, Van Raamsdonk]

[de Boer, Heller, Myers, Neiman]
[Nozaki, Numasawa, Prudenziati, Takayanagi], [Bhattacharya, Takayanagi]

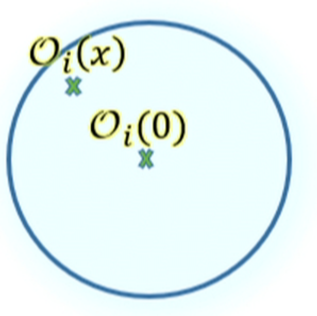
Loop/kinematic Einstein's Equations

The Operator Product Expansion

We've found a **natural set of gravitational variables** that are non-local, and that contain the information normally specified using local 'differential' description.

We **haven't succeeded** unless these appear **naturally in the dual gauge theory**:

- Consider a quasi-primary operator $\mathcal{O}_i(x)$ of dimensions Δ_i. We can expand the product of two such operators using a local basis of operators:



$$\mathcal{O}_i\,(x)\,\mathcal{O}_i\,(0) = \underbrace{\sum_k C_{iik}}_{\text{Dynamical Parameters}}\,\underbrace{|x|^{\Delta_k - 2\Delta_i}\,(1 + b_1\,x^\mu\partial_\mu + b_2\,x^\mu x^\nu\partial_\mu\partial_\nu + \ldots)\,\mathcal{O}_k\,(0)}_{\text{Conformal Kinematics}}$$

- Let us introduce a more compact notation for this expansion

$$\mathcal{O}_i\,(x)\,\mathcal{O}_i\,(y) = |x - y|^{-2\Delta_i}\sum_k C_{iik}\mathcal{B}_k\,(x, y)$$

- We will call $\mathcal{B}_k\,(x, y)$ the '**OPE Block**'

OPE Blocks as Kinematic Fields

- The OPE block carries coordinates of two points (x, y), so we might naturally identify it with a **'loop operator' living in our kinematic space**.

Consider a scalar block $(\Delta_k, l = 0)$. Let's characterize this field:

1) What type of field is an OPE block on KS?

- Consider a conformal transformation $x \to x'$. Then $\mathcal{B}_k(x, y) \to \mathcal{B}_k(x', y')$
  So the OPE block is a scalar operator.

2) What is its equation of motion?

- Eigenoperator of the conformal Casimir: $\left[L^2, \mathcal{B}_k(x, y)\right] = C_{\mathcal{O}_k} \mathcal{B}_k(x, y)$

$$C_{\mathcal{O}_k} = -\Delta(\Delta - d)$$

- We represent this as

$$\boxed{\mathcal{L}^2_{(B)} = 2\Box_{\mathrm{KS}}}$$

OPE Blocks as Kinematic Fields

We thus find an equation of motion for the OPE block

$$\left[\Box_{\mathrm{KS}} + m_{\Delta_k}^2\right] \mathcal{B}_k\,(x, y) = 0$$

$$m_{\Delta_k}^2 = -C_{\Delta_k}$$

- This is precisely the **wave equation for a scalar field** propagating in our kinematic space.

This is the same wave equation obeyed by a bulk geodesic operator dual to the operator $\mathcal{O}_k$:

$$(\Box_{KS} + m_{\Delta_k}^2)\tilde{\phi}_k(\gamma) = 0$$

- They also **obey** the **same boundary conditions** (and satisfy the **same constraints**).

The Kinematic Dictionary

We have now established a **new organization of the holographic dictionary**

$$\mathcal{B}_k\,(x,y) = \tilde{\phi}(\gamma) = \int\, ds\;\phi(x,z)\Big|_{\gamma}$$

between OPE Blocks and geodesic operators (and extensions to surface operators)

We have **equivalent,** suitably **invariant, non-local** building blocks on both sides.

How derailed was this talk by questions about firewalls?

Mentioning them was very ill-advised

Everyone behaved

Where next?

So far we have been primarily occupied with kinematics (choosing appropriate variables for the symmetries of our problem).

We also need to understand how to **incorporate dynamics and interactions** into this picture.

- This can be done quite easily for interacting fields.
- More **challenging to incorporate interactions with gravity**.
  - While the bulk picture is confusing, the gauge theory OPE should instruct us how to organize these corrections.
- We still need to understand the kinematic space equivalent of **the full non-linear Einstein Equations**, not just the linearized equations.

Once we have understood how to see gravitational physics emerge, we can turn our tools to black holes. Perhaps there is a signal waiting for us…

Summary

1. Understanding quantum gravity is a difficult problem, but **one that we should still care about**!
    - We showed that our naïve understanding of **quantum gravity breaks down near the horizons of black holes**—this might lead to effects that are **measurable** one day.

2. To make progress, we must make use of the deepest and most surprising fact about gravity**: it is holographic and emergent**!
    - We have **powerful models** of holographic gravity using **gauge/gravity duality.**
    - The problem is that we **don't know how to translate** many results from gauge theory to ask about the dual gravitational system.

3. We can build a **better gauge/gravity dictionary!**
    - Can use a basis of **non-local (surface) operators in the gravitational theory** and a corresponding basis of **OPE blocks in the dual gauge theory**.

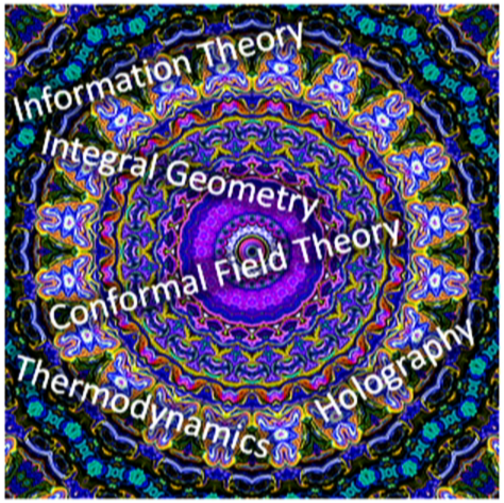
Outlook

Where are we now?

"In recent years I have worked, in part together with my friend Grossman, on a generalization of the theory of relativity. During these investigations, a kaleidoscopic mixture of postulates from physics and mathematics has been introduced and used as heuristical tools; as a consequence it is not easy to see through and characterize the theory from a formal mathematical point of view, that is, only based on these papers. The primary objective of the present paper is to close this gap."

-Einstein, 1914

- Armed with the holographic principle, **there are now too have a 'kaleidoscopic mixture' of techniques** used to understand emergent gravitational physics.



Perhaps the shift from **local differential geometry** to **non-local integral geometry** is the tool that will be needed to close this gap?