Title: From Wires to Cosmology

Date: May 12, 2016 01:00 PM

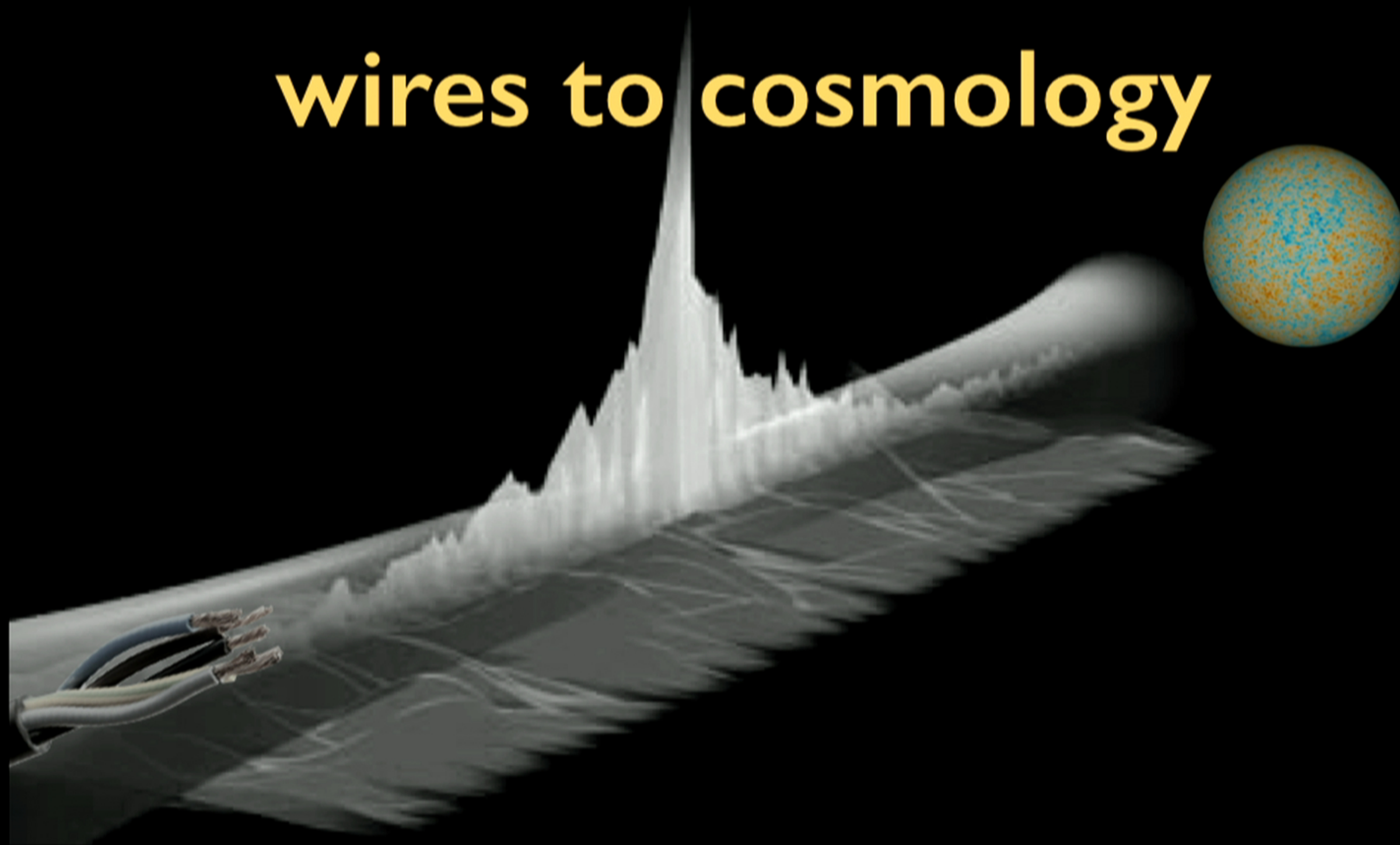
URL: http://pirsa.org/16050025

Abstract: <p>Current observations provide precise but limited information about inflation and reheating. Theoretical considerations, however, suggest that the early universe might be filled with a large number of interacting fields with unknown interactions. How can we quantitatively understand the dynamics of perturbations during inflation and reheating in such scenarios and when only limited</p>

<p>constraints are available from observations?   Based on a precise</p>

<p>mapping between particle production in cosmology to resistivity in disordered, quasi one-dimensional wires, I will provide a statistical framework to resolve such seemingly intractable calculations. A number of phenomenon in disordered wires find an analogue in particle production. For example, Anderson localization in quasi one-dimensional wires can be directly mapped to exponential particle production in the early universe. The talk will be focussed on the general framework and some toy examples, though in the end I will discuss possible (future) applications to calculating observables.</p>

wires to cosmology

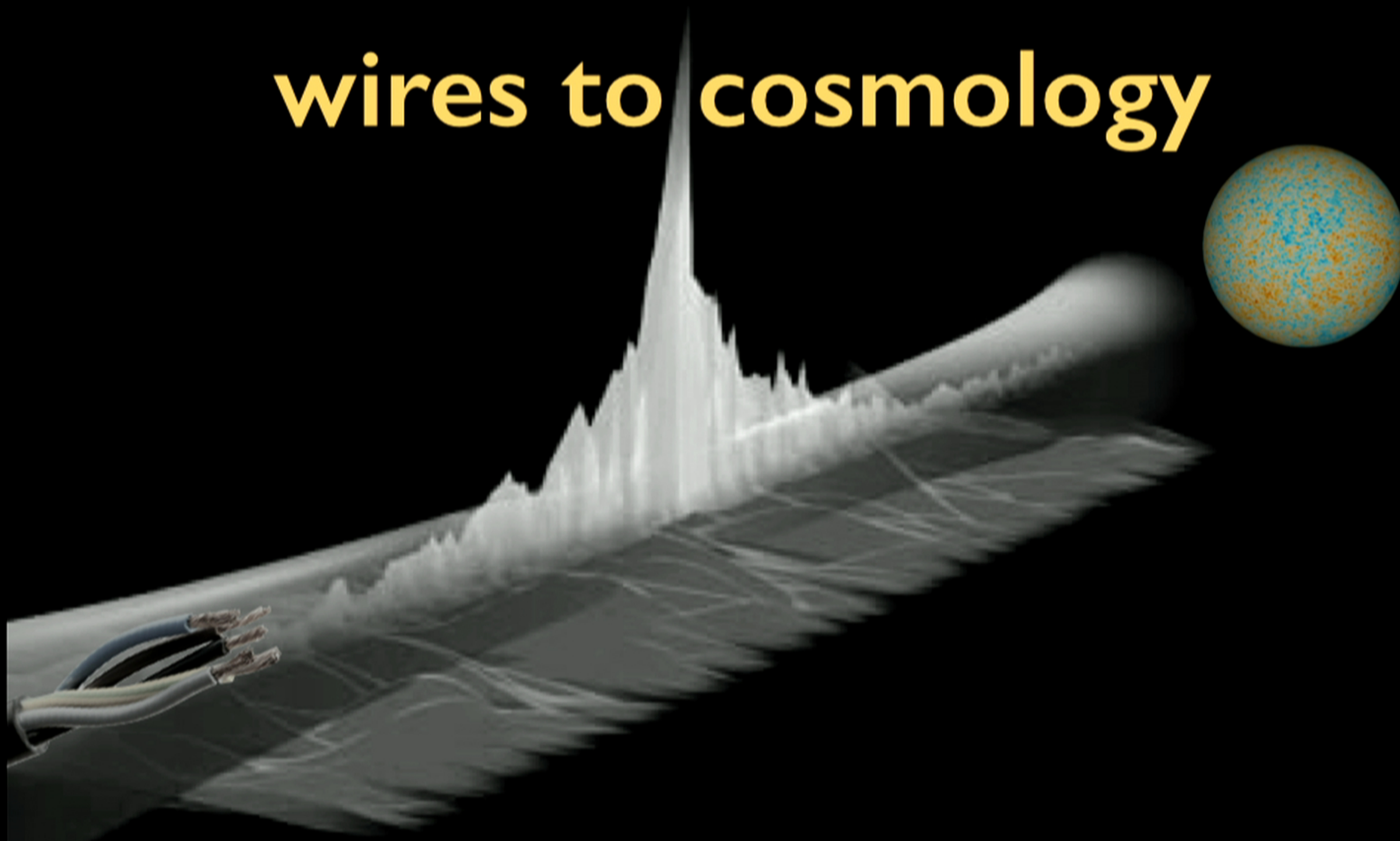


RICE

Mustafa A. Amin
[arXiv: 1512.02637] with D. Baumann

+ ongoing work with D. Green, H. Xie

wires to cosmology

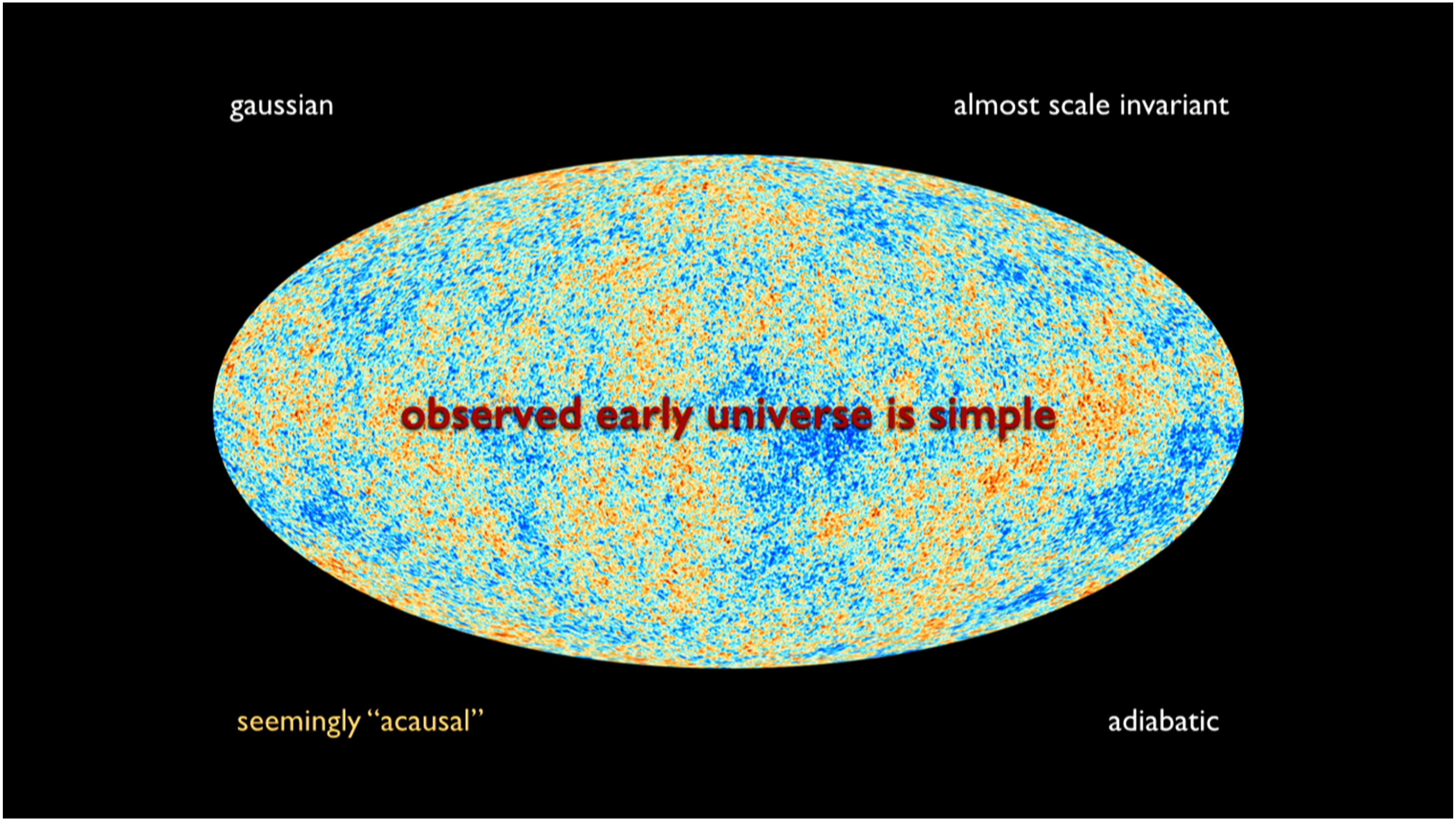


RICE

Mustafa A. Amin

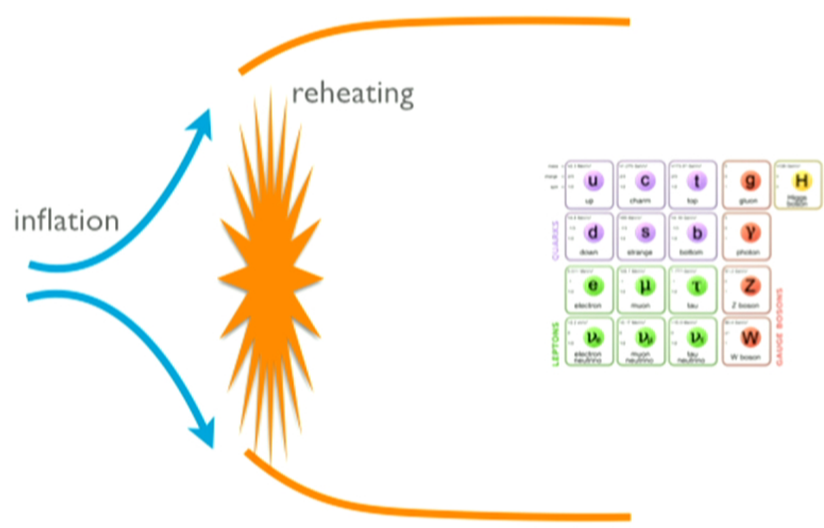
[arXiv: 1512.02637] with D. Baumann

+ ongoing work with D. Green, H. Xie


gaussian
almost scale invariant
observed early universe is simple
seemingly "acausal"
adiabatic

physics of inflation and reheating?

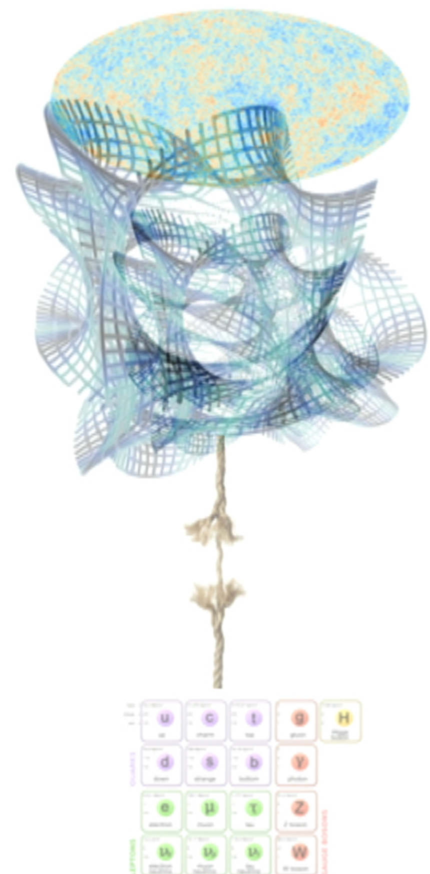
- what is the physics of inflation ?
- how did the universe get populated with particles after inflation ? (reheating)



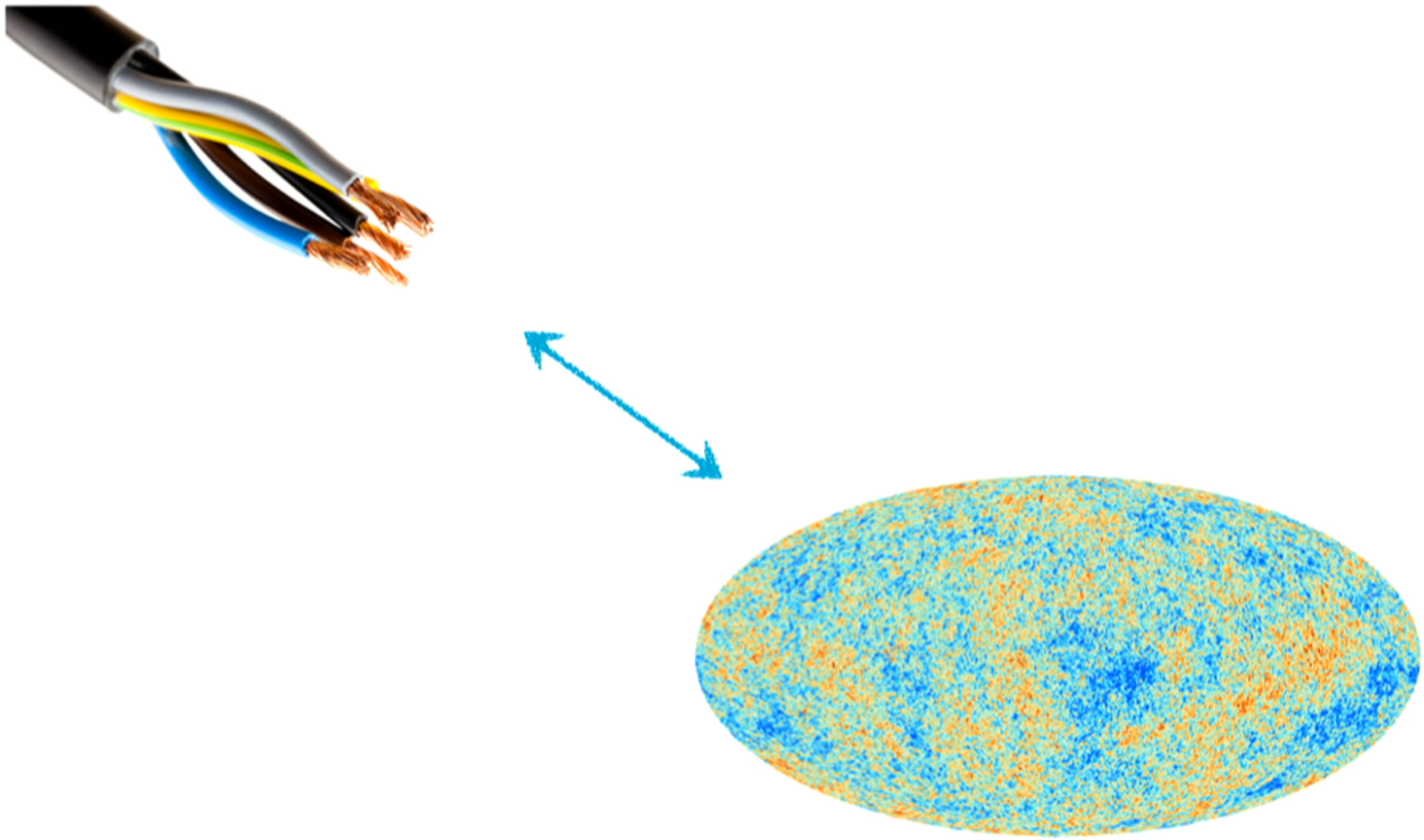
theory: it's complicated? (probably)

a statistical approach

- **observations**: early universe is simple
- **theory**: not so much …
  - inflation
  - reheating after inflation
- **coarse grained view ?**
- **calculational tools ?**



inspiration from wires



related work: condensed matter + cosmology

Anderson
Absence of diffusion in certain random matrices
(1957)

Mello, Pereyra Kumar
Macroscopic approach to multichannel disordered wires
(1987)

C. Beenakker,
Random matrix theory of quantum transport
(1997)

C. Muller and D. Delande,
Disorder and interference: localization phenomena
(2010)

Kofman, Linde & Starobinsky
(1997)

Traschen and Brandenberger
(1997)

Zanchin, Maia, Craig & Brandenberger
(1998)

Bassett
(1998)

Nacir, Porto, Senatore and Zaldarriaga
(2012)

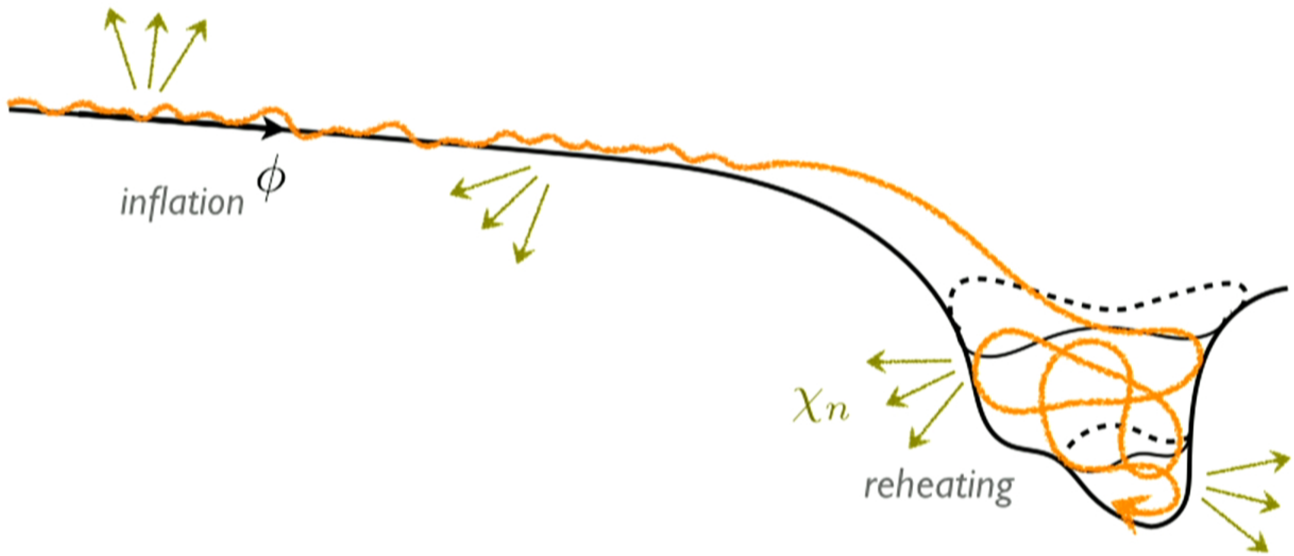
Marsh, McAllister, Pajer, Wrase
(2013)

Green
(2015)

+ many works on particle production during and after inflation.

multifield inflation/reheating

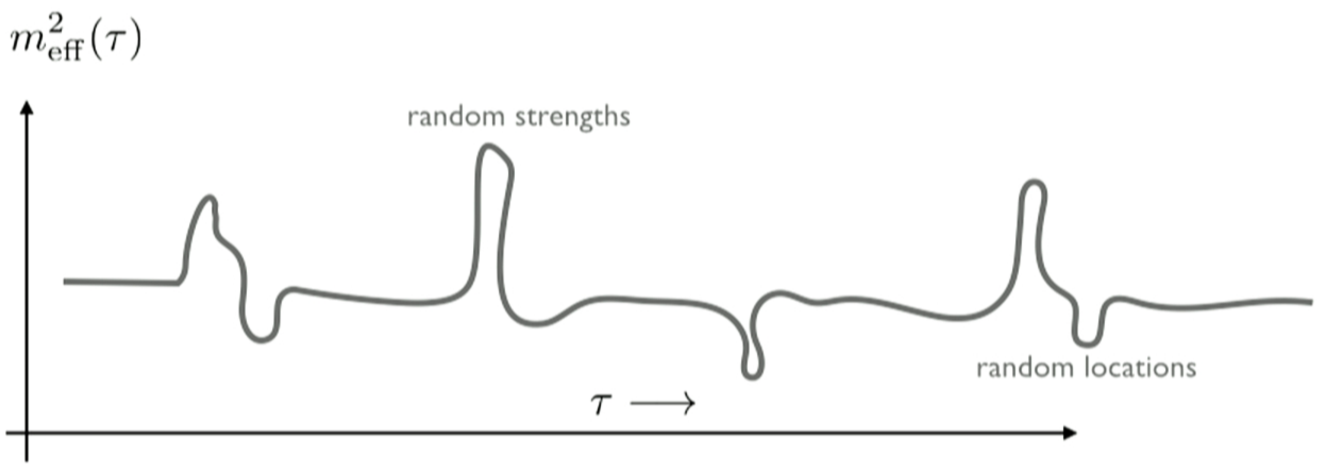
- inflation/reheating: many interacting fields
- fluctuations: coupled, non-perturbative



complexity in time: cosmology

$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\text{eff}}^2(\tau)\right]\chi_k(\tau) = 0$$

$$m_{\text{eff}}^2(\tau) = -\frac{\ddot{a}(\tau)}{a(\tau)} + a^2(\tau)m_\varphi^2 + a^2(\tau)g^2(\phi(\tau) - \phi_*)^2 + \ldots$$



simplified version!

complexity in time ⟷ complexity in space

cosmology ⟷ wires

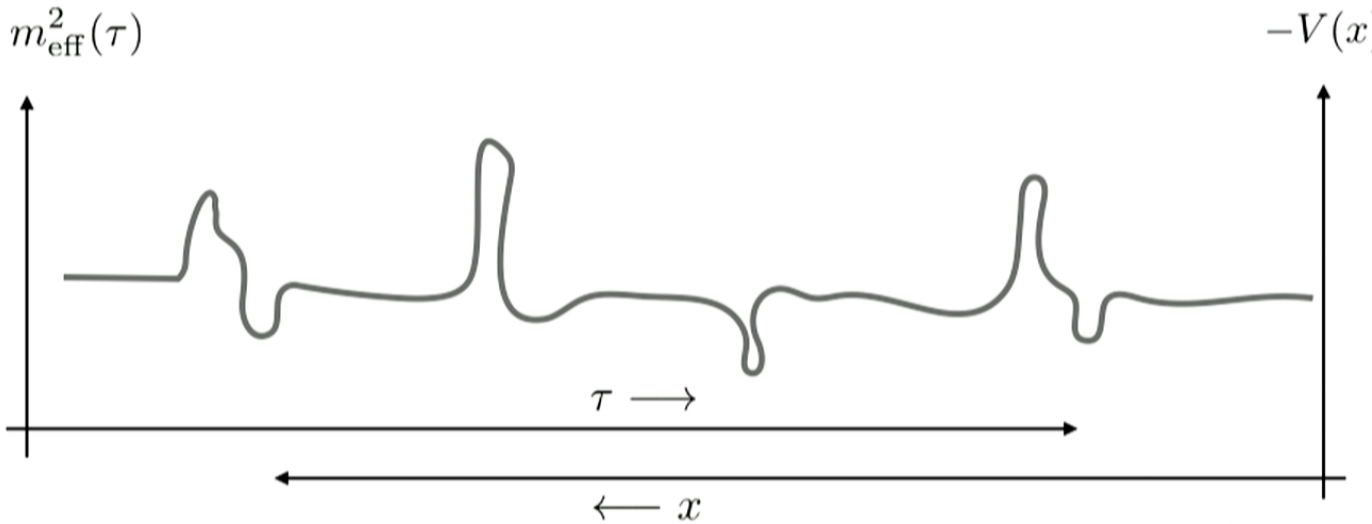
particle production

$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$$

$\longleftrightarrow$

Schrodinger

$$\psi''(x) + \left[k^2 - V(x)\right]\psi(x) = 0$$

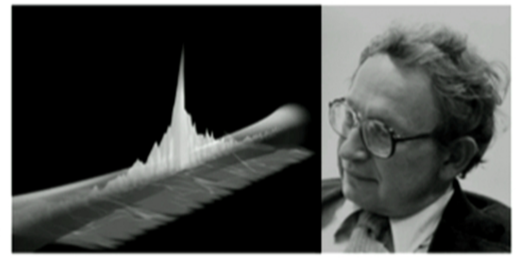


simplified version!

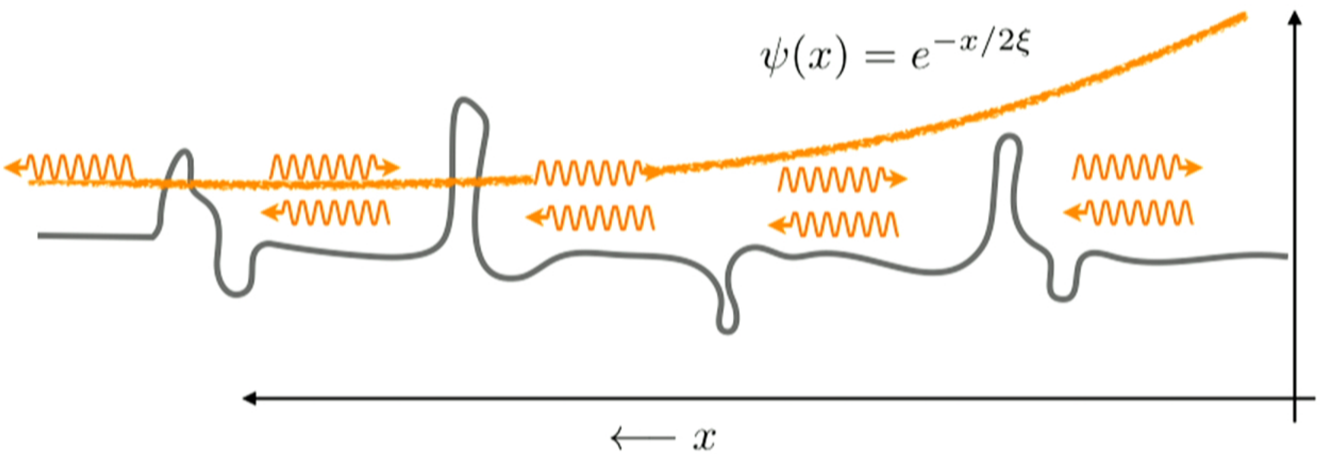
Anderson localization!
complexity in space — emergent simplicity

$$\psi''(x) + \left[k^2 - V(x)\right]\psi(x) = 0$$

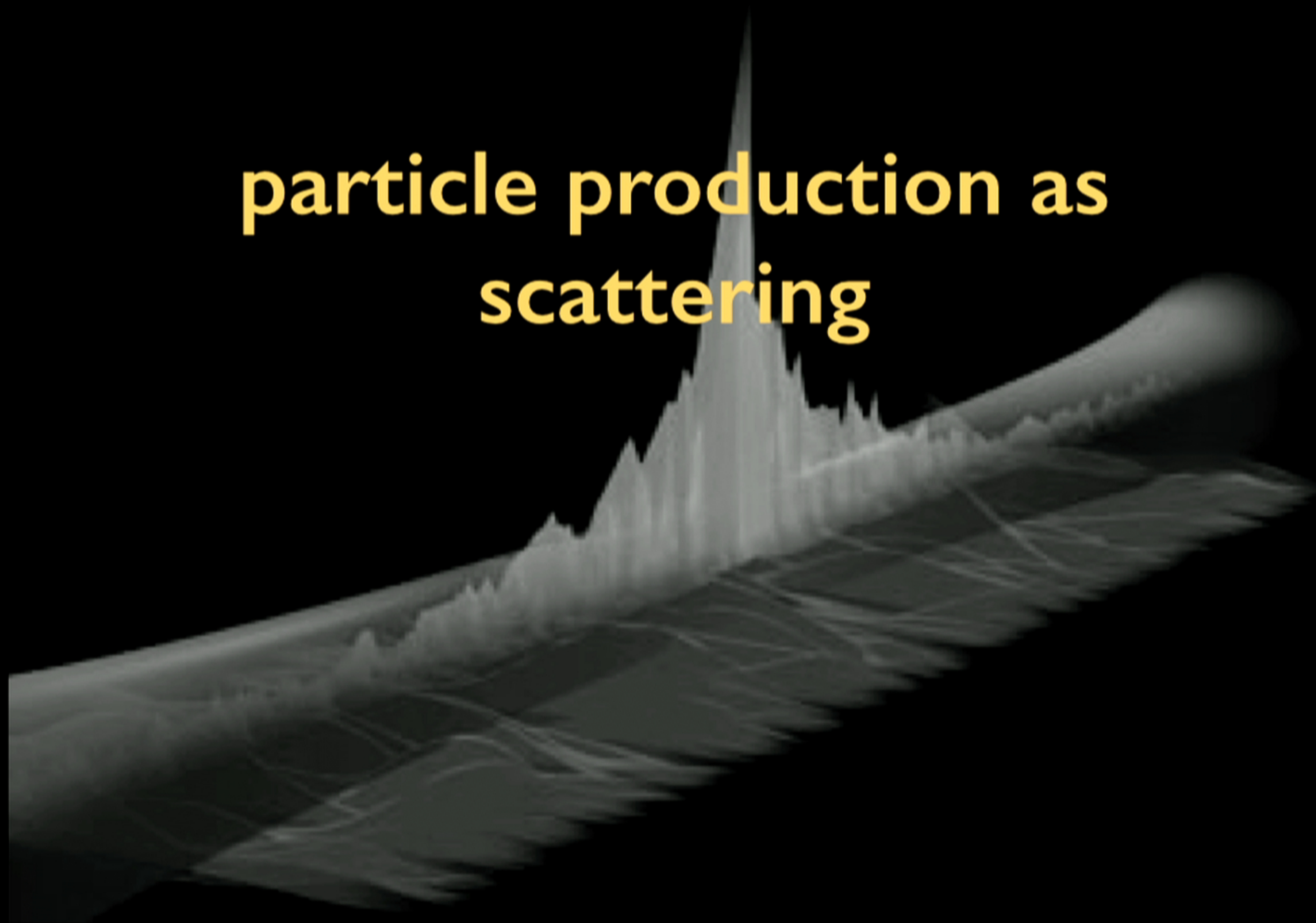
at low temperatures, one dimensional wires are insulators



Anderson 1957

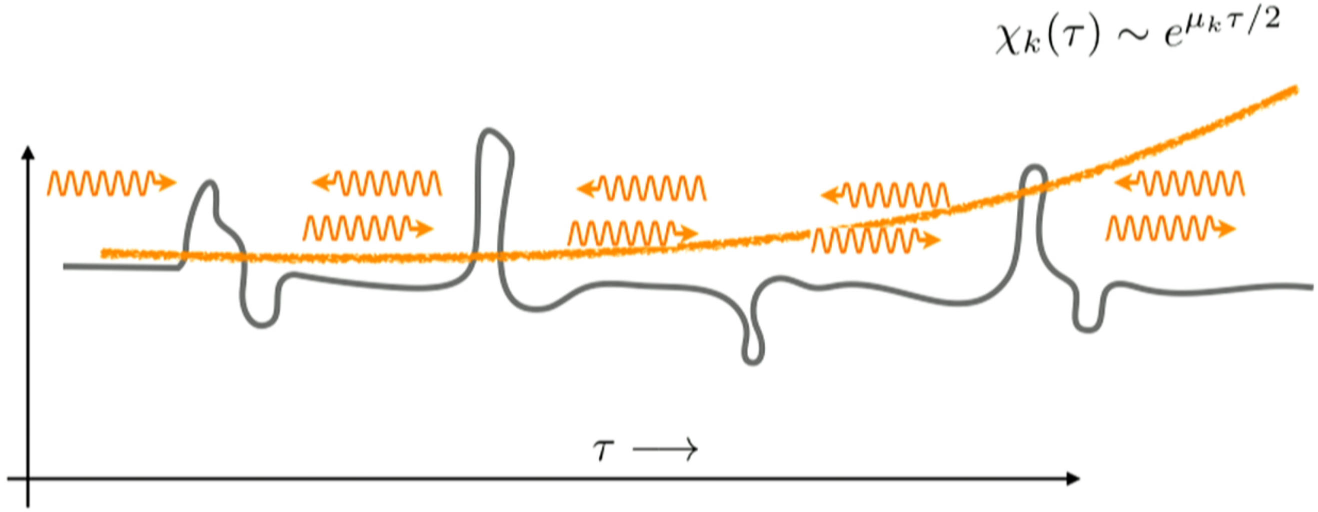


particle production as scattering

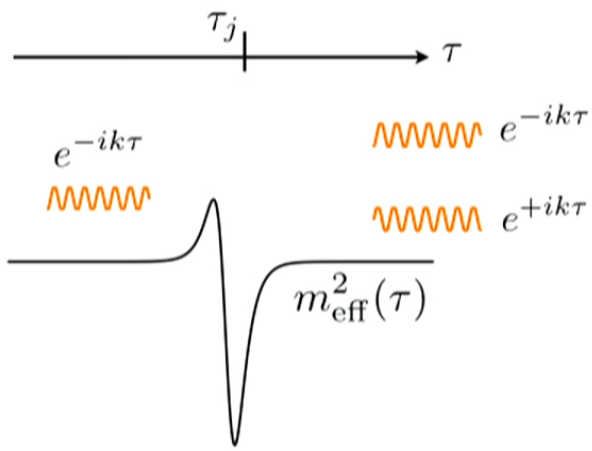


complexity in time —
exponential particle production

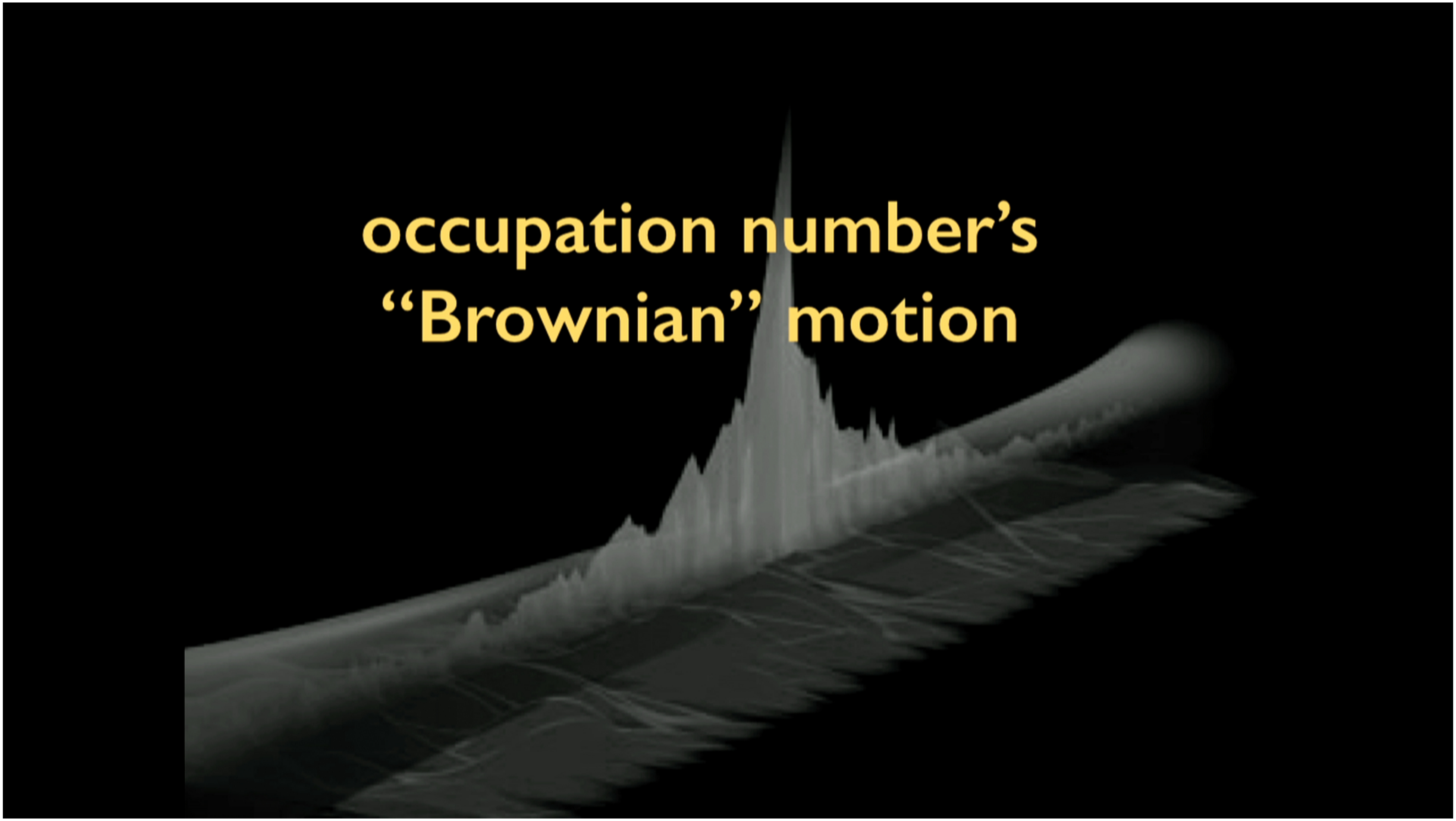
$$\ddot{\chi}_k(\tau) + \left[k^2 + m^2_{\text{eff}}(\tau)\right] \chi_k(\tau) = 0$$



particle production as "scattering"

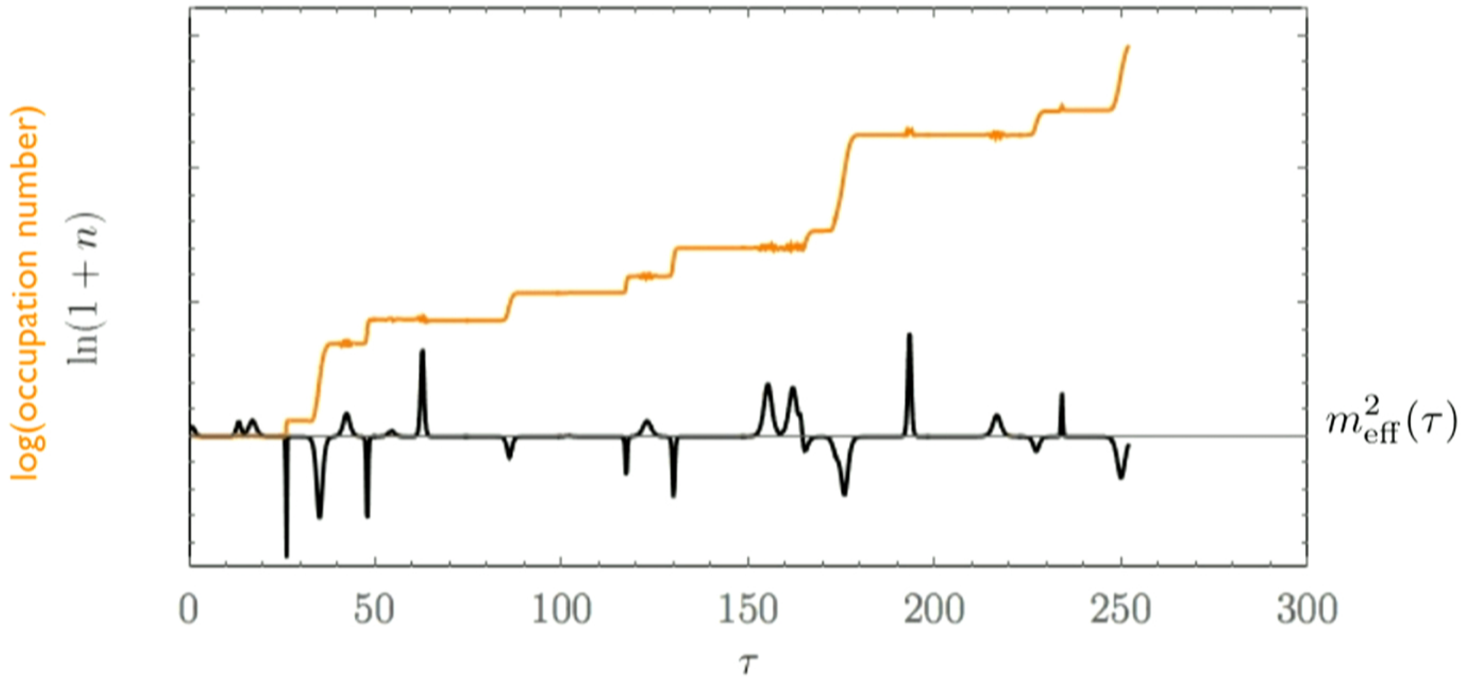


Kofman, Linde & Starobinsky 1997


occupation number's
"Brownian" motion

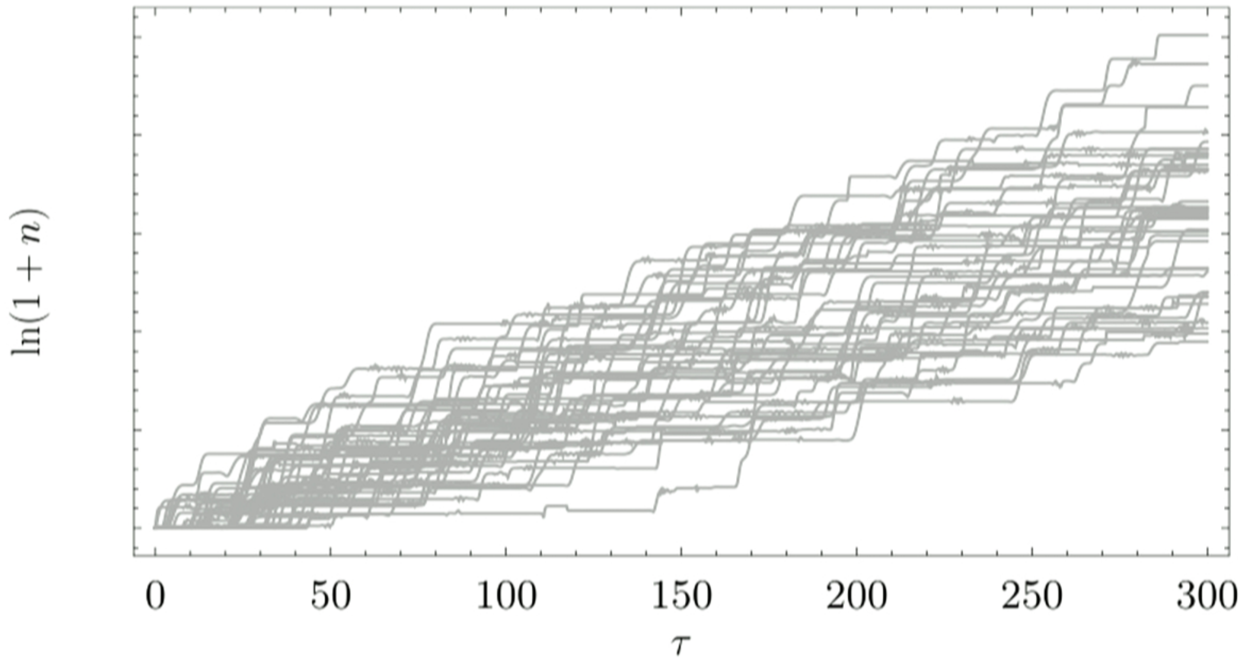
occupation number performs a drifted random walk

$$n(k,\tau) = \frac{1}{2\omega_k}\left(|\dot{\chi}_k|^2 + \omega_k^2|\chi_k|^2\right)$$

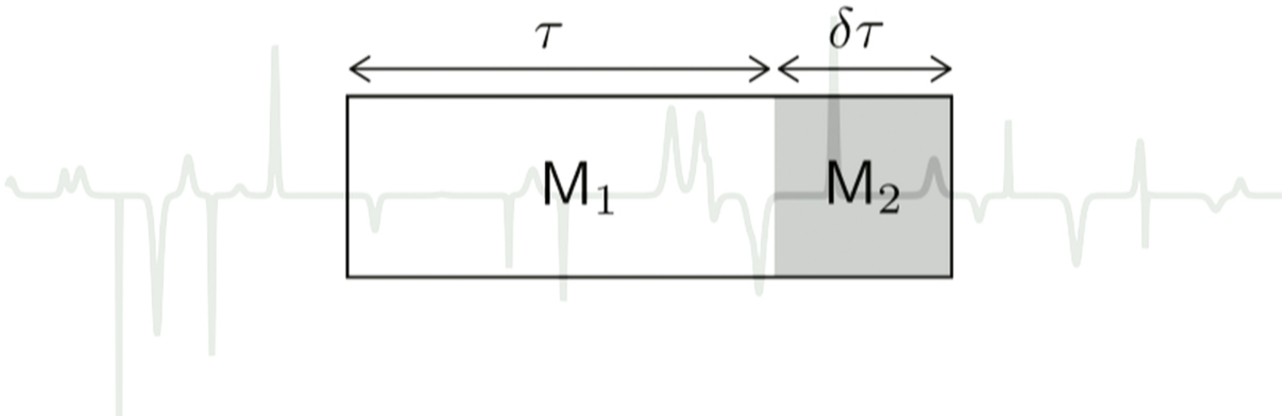


realizations: ensemble of trajectories

$$n(k,\tau) = \frac{1}{2\omega_k}\left(|\dot{\chi}_k|^2 + \omega_k^2|\chi_k|^2\right)$$



Fokker Planck eq: heuristic derivation



$$P(\mathsf{M};\tau+\delta\tau) = \int P(\mathsf{M}_1;\tau)\,P(\mathsf{M}_2;\delta\tau)\,\mathrm{d}\mathsf{M}_2 \;\equiv\; \langle P(\mathsf{M}_1;\tau)\rangle_{\mathsf{M}_2}$$

Smoluchowski eq.

$$\partial_\tau P(\mathsf{M};\tau) = \frac{\langle \delta\mathsf{M}\rangle_{\mathsf{M}_2}}{\delta\tau}\,\partial_{\mathsf{M}} P(\mathsf{M};\tau) + \frac{\langle \delta\mathsf{M}\delta\mathsf{M}\rangle_{\mathsf{M}_2}}{\delta\tau}\,\partial_{\mathsf{M}}\partial_{\mathsf{M}} P(\mathsf{M};\tau) + \cdots$$

"formal" Fokker Planck eq.

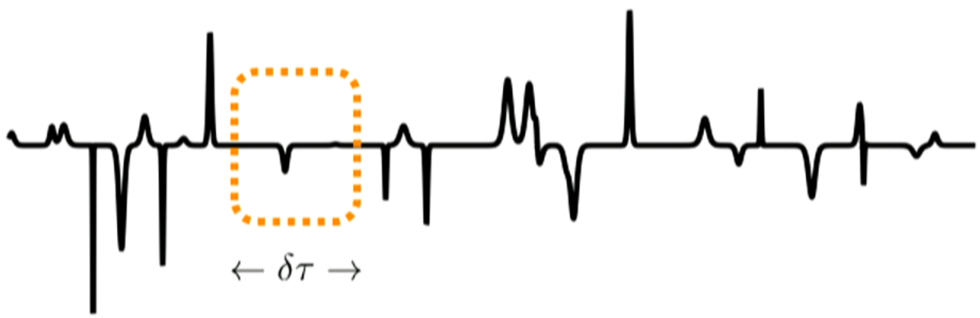
"local" mean particle production rate

$$\boxed{\frac{1}{\mu_k}}\frac{\partial}{\partial \tau}P(n,\tau) = \frac{\partial}{\partial n}\left[n(1+n)\frac{\partial}{\partial n}P(n,\tau)\right]$$

μ_k local mean particle production rate

$$\mu_k = \frac{\langle n \rangle_{\delta\tau}}{\delta\tau}$$

analogue: mean free path
calculate from 'local'' microphysics



moments: Fokker Planck equation

$$\langle n \rangle = \frac{1}{2}\left(e^{2\mu_k \tau} - 1\right)$$

$$\frac{\mathrm{Var}(n)}{\langle n \rangle^2} \xrightarrow{\mu_k \tau \gg 1} e^{2\mu_k \tau}$$

moments: Fokker Planck equation

$$\langle n \rangle = \frac{1}{2}\left(e^{2\mu_k \tau} - 1\right)$$

$$\langle \ln(1+n) \rangle = \mu_k \tau$$

$$\frac{\mathrm{Var}(n)}{\langle n \rangle^2} \xrightarrow{\mu_k \tau \gg 1} e^{2\mu_k \tau}$$

$$\frac{\mathrm{Var}[\ln(1+n)]}{\langle \ln(1+n) \rangle^2} \longrightarrow \frac{2}{\mu_k \tau}$$

The most probable value of the occupation number

$$n_{\mathrm{typ}} \equiv \exp\langle \ln(1+n) \rangle = e^{\mu_k \tau}$$

moments: Fokker Planck equation

$$\langle n \rangle = \frac{1}{2}\left(e^{2\mu_k \tau} - 1\right)$$

$$\frac{\mathrm{Var}(n)}{\langle n \rangle^2} \xrightarrow{\mu_k \tau \gg 1} e^{2\mu_k \tau}$$

$$\langle \ln(1+n) \rangle = \mu_k \tau$$

$$\frac{\mathrm{Var}[\ln(1+n)]}{\langle \ln(1+n) \rangle^2} \longrightarrow \frac{2}{\mu_k \tau}$$

The most probable value of the occupation number

$$n_{\mathrm{typ}} \equiv \exp\langle \ln(1+n) \rangle = e^{\mu_k \tau}$$

moments: Fokker Planck equation

$$\langle n \rangle = \frac{1}{2}\left(e^{2\mu_k \tau} - 1\right)$$

$$\langle \ln(1+n) \rangle = \mu_k \tau$$

$$\frac{\mathrm{Var}(n)}{\langle n \rangle^2} \xrightarrow{\mu_k \tau \gg 1} e^{2\mu_k \tau}$$

$$\frac{\mathrm{Var}[\ln(1+n)]}{\langle \ln(1+n) \rangle^2} \longrightarrow \frac{2}{\mu_k \tau}$$

The most probable value of the occupation number

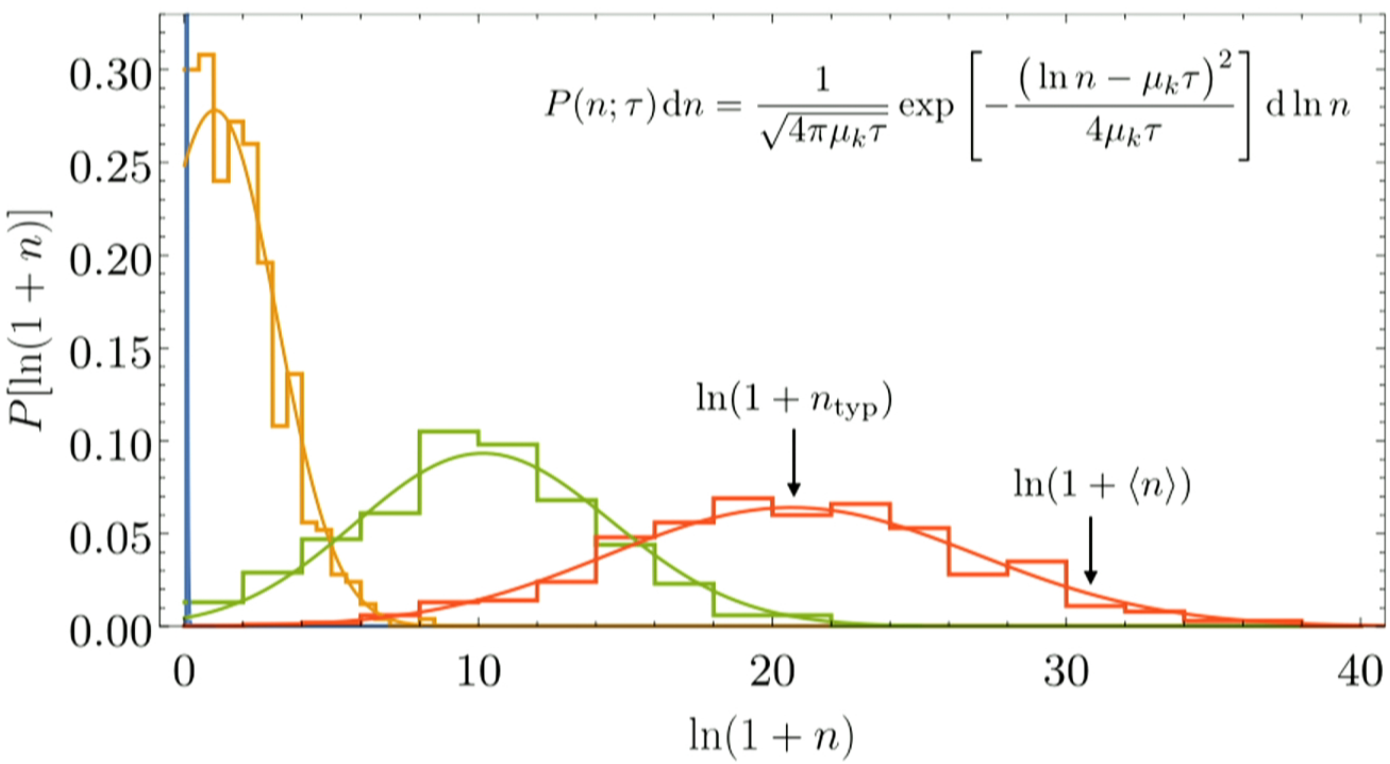
$$n_{\rm typ} \equiv \exp\langle \ln(1+n) \rangle = e^{\mu_k \tau}$$

moments: Fokker Planck equation

$$\langle n \rangle = \frac{1}{2}\left(e^{2\mu_k \tau} - 1\right)$$

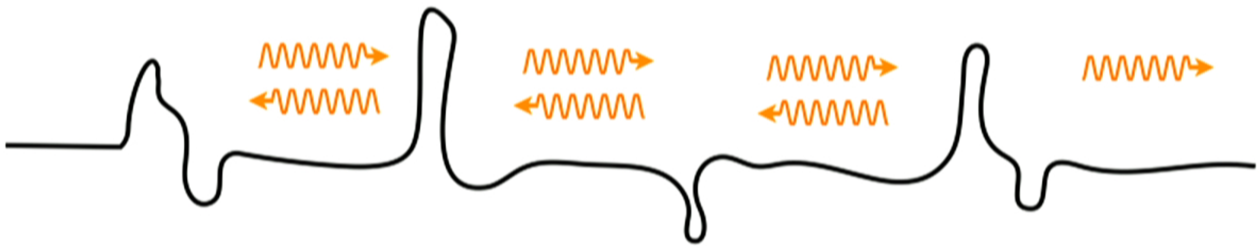
$$\frac{\mathrm{Var}(n)}{\langle n \rangle^2} \xrightarrow{\mu_k \tau \gg 1} e^{2\mu_k \tau}$$

approaching "universal" distributions



dictionary

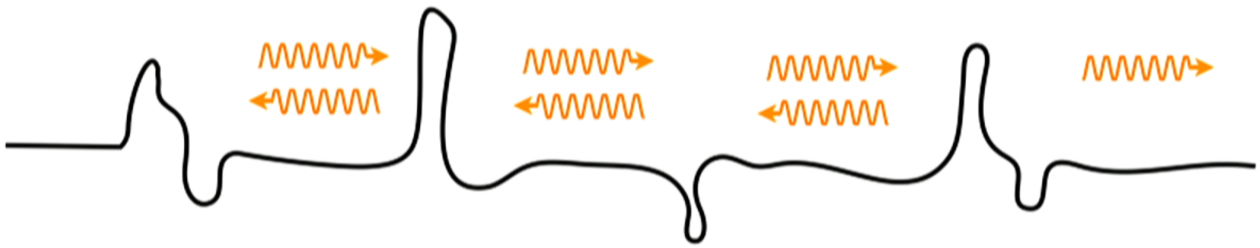
Time-dependent "Klein-Gordon"	Time-independent Schrödinger
$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$	$\dfrac{d^2\psi}{dx^2} + \big(E - V(x)\big)\psi = 0$



occupation number	specific resistance
(local) particle production rate	mean free path
multiple fields	multiple channels

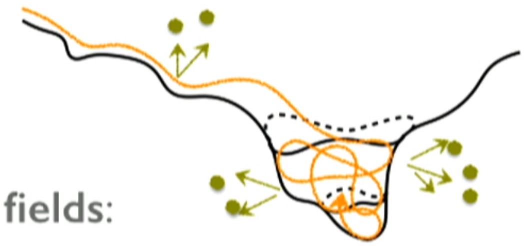
dictionary

Time-dependent “Klein-Gordon”	Time-independent Schrödinger
$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\text{eff}}^2(\tau)\right]\chi_k(\tau) = 0$	$\frac{d^2\psi}{dx^2} + \left(E - V(x)\right)\psi = 0$



occupation number	specific resistance
(local) particle production rate	mean free path
multiple fields	multiple channels

many interacting fields (thick wires)



early universe: **multiple interacting fields**:

$$\ddot{\chi}_a + \left[k^2 \delta^b_a + \mathcal{M}^b{}_a(\tau)\right] \chi_b = 0$$

$$a, b = 1, \ldots, N_{\rm f}$$

↕

real wires are not one-dimensional.

current conduction: **multiple channels**.

multifield Fokker Planck equation

joint probability for occupation numbers satisfies the DMPK equation:

$$\frac{1}{\mu_k}\frac{\partial}{\partial \tau}P(n_a;\tau) = \sum_{a=1}^{N_\mathrm{f}}\left[(1+2n_a) + \frac{1}{N_\mathrm{f}+1}\sum_{b\neq a}\frac{n_a+n_b+2n_a n_b}{n_a-n_b}\right]\frac{\partial P}{\partial n_a} + \frac{2}{N_\mathrm{f}+1}\sum_{a=1}^{N_\mathrm{f}} n_a(1+n_a)\frac{\partial^2 P}{\partial n_a^2}$$

Dokhorov, Mello, Pereyra & Kumar

multifield Fokker Planck equation

joint probability for occupation numbers satisfies the DMPK equation:

$$\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n_a;\tau) = \sum_{a=1}^{N_\mathrm{f}}\left[(1+2n_a)+\frac{1}{N_\mathrm{f}+1}\sum_{b\neq a}\frac{n_a+n_b+2n_an_b}{n_a-n_b}\right]\frac{\partial P}{\partial n_a}$$
$$+\frac{2}{N_\mathrm{f}+1}\sum_{a=1}^{N_\mathrm{f}}n_a(1+n_a)\frac{\partial^2 P}{\partial n_a^2}$$

Dokhorov, Mello, Pereyra & Kumar

multifield Fokker Planck equation

joint probability for occupation numbers satisfies the DMPK equation:

$$\frac{1}{\mu_k}\frac{\partial}{\partial \tau}P(n_a;\tau) = \sum_{a=1}^{N_\mathrm{f}}\left[(1+2n_a)+\frac{1}{N_\mathrm{f}+1}\sum_{b\neq a}\frac{n_a+n_b+2n_a n_b}{n_a-n_b}\right]\frac{\partial P}{\partial n_a} + \frac{2}{N_\mathrm{f}+1}\sum_{a=1}^{N_\mathrm{f}} n_a(1+n_a)\frac{\partial^2 P}{\partial n_a^2}$$

Dokhorov, Mello, Pereyra & Kumar

multifield Fokker Planck equation

joint probability for occupation numbers satisfies the DMPK equation:

$$\frac{1}{\mu_k}\frac{\partial}{\partial \tau}P(n_a;\tau) = \sum_{a=1}^{N_\mathrm{f}}\left[(1+2n_a)+\frac{1}{N_\mathrm{f}+1}\sum_{b\neq a}\frac{n_a+n_b+2n_a n_b}{n_a-n_b}\right]\frac{\partial P}{\partial n_a} + \frac{2}{N_\mathrm{f}+1}\sum_{a=1}^{N_\mathrm{f}} n_a(1+n_a)\frac{\partial^2 P}{\partial n_a^2}$$

Dokhorov, Mello, Pereyra & Kumar

local mean particle production rate

$$\mu_k \equiv \frac{1}{N_\mathrm{f}}\lim_{\delta\tau\to 0}\frac{\langle n\rangle}{\delta\tau} \quad \text{where} \quad n = \sum_{a=1}^{N_\mathrm{f}} n_a$$

moments: Fokker Planck equation

$$\langle n \rangle = \frac{N_\mathrm{f}}{2}\left(e^{2\mu_k \tau} - 1\right)$$

$$\langle \ln(1+n) \rangle = \mu_k \tau$$

$$\frac{\mathrm{Var}[n]}{\langle n \rangle^2} \xrightarrow{\mu_k \tau \gg 1} \left(\frac{1+N_\mathrm{f}}{3N_\mathrm{f}}\right) e^{\frac{4}{1+N_\mathrm{f}}\mu_k \tau}$$

$$\frac{\mathrm{Var}[\ln(1+n)]}{\langle \ln(1+n) \rangle^2} \xrightarrow{\mu_k \tau \gg 1} \frac{N_\mathrm{f}+1}{N_\mathrm{f}^2} \frac{1}{\mu_k \tau}$$

most probable total occupation number

$$n_\mathrm{typ} \equiv e^{\langle \ln(1+n) \rangle} \longrightarrow e^{\frac{2N_\mathrm{f}}{1+N_\mathrm{f}}\mu_k \tau}$$

moments: Fokker Planck equation

$$\langle n \rangle = \frac{N_\mathrm{f}}{2}\left(e^{2\mu_k \tau} - 1\right)$$

$$\langle \ln(1+n) \rangle = \mu_k \tau$$

$$\frac{\mathrm{Var}[n]}{\langle n \rangle^2} \xrightarrow{\mu_k\tau \gg 1} \left(\frac{1+N_\mathrm{f}}{3N_\mathrm{f}}\right) e^{\frac{4}{1+N_\mathrm{f}}\mu_k \tau}$$

$$\frac{\mathrm{Var}[\ln(1+n)]}{\langle \ln(1+n) \rangle^2} \xrightarrow{\mu_k\tau \gg 1} \frac{N_\mathrm{f}+1}{N_\mathrm{f}^2}\frac{1}{\mu_k \tau}$$

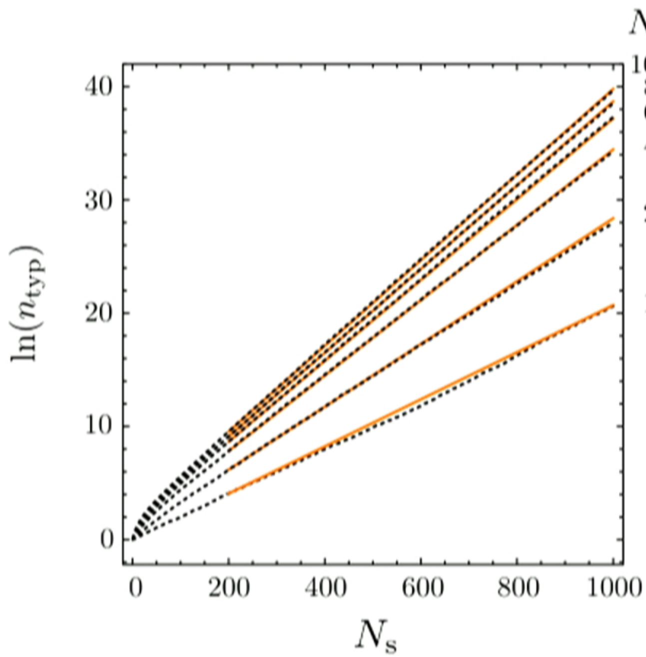
most probable total occupation number

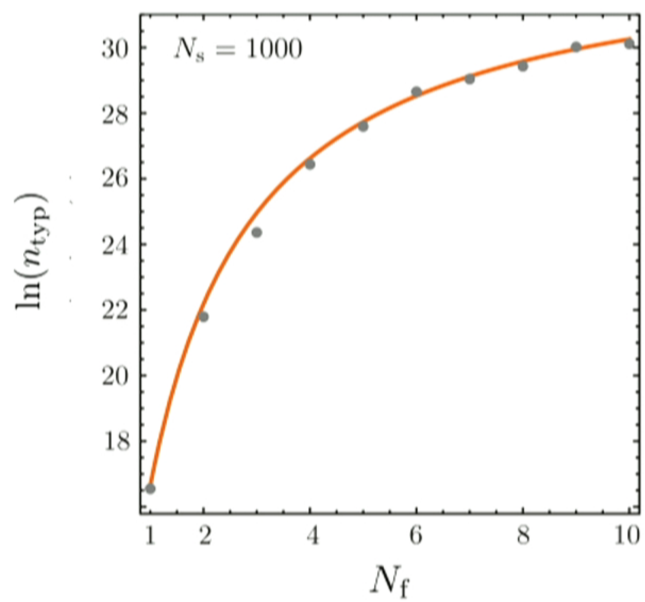
$$n_\mathrm{typ} \equiv e^{\langle \ln(1+n) \rangle} \longrightarrow e^{\frac{2N_\mathrm{f}}{1+N_\mathrm{f}}\mu_k \tau}$$

numerical tests
[typical occupation numbers]

$$\ln(n_{\mathrm{typ}}) \propto \frac{2N_{\mathrm{f}}}{1+N_{\mathrm{f}}} N_{\mathrm{s}}$$

where N_{f} = number of fields
N_{s} = number of scatterings





universality?

$$\frac{\mathrm{Var}\left[\ln(n_{\mathrm{typ}})\right]}{\ln(n_{\mathrm{typ}})} \longrightarrow \frac{1}{2N_{\mathrm{f}}}$$

analog of universal conductance fluctuations?

observables where dependence on number of fields and/or time vanishes?

universality?

$$\frac{\mathrm{Var}\left[\ln(n_{\mathrm{typ}})\right]}{\ln(n_{\mathrm{typ}})} \longrightarrow \frac{1}{2N_{\mathrm{f}}}$$

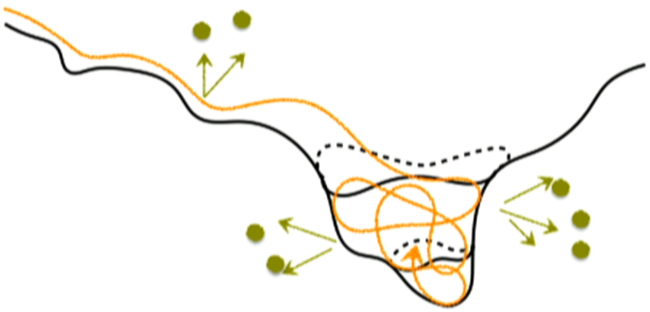
analog of universal conductance fluctuations?

observables where dependence on number of fields and/or time vanishes?

simplicity/universality

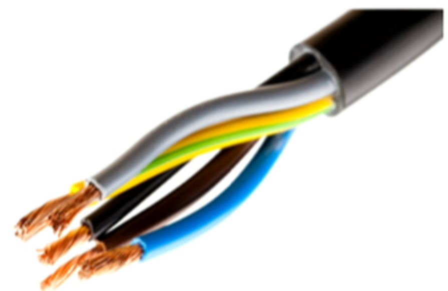
μ_k local mean particle production rate

N_f number of fields



l_mf mean ballistic mean free path

N_c number of channels



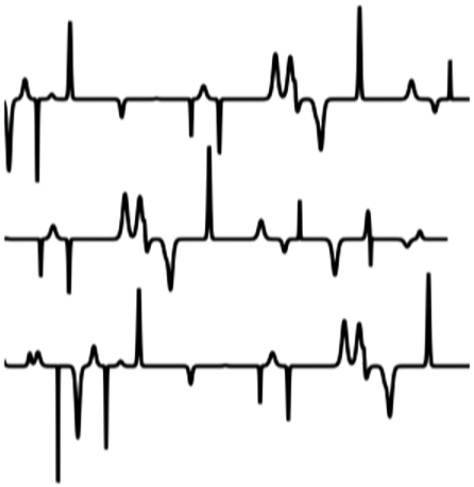
μ_k - calculate from 'local'' microphysics or parametrize

N_f - regimes exist where dependence vanishes

caveats

- assumption of *maximum entropy*
    - all fields statistically equivalent

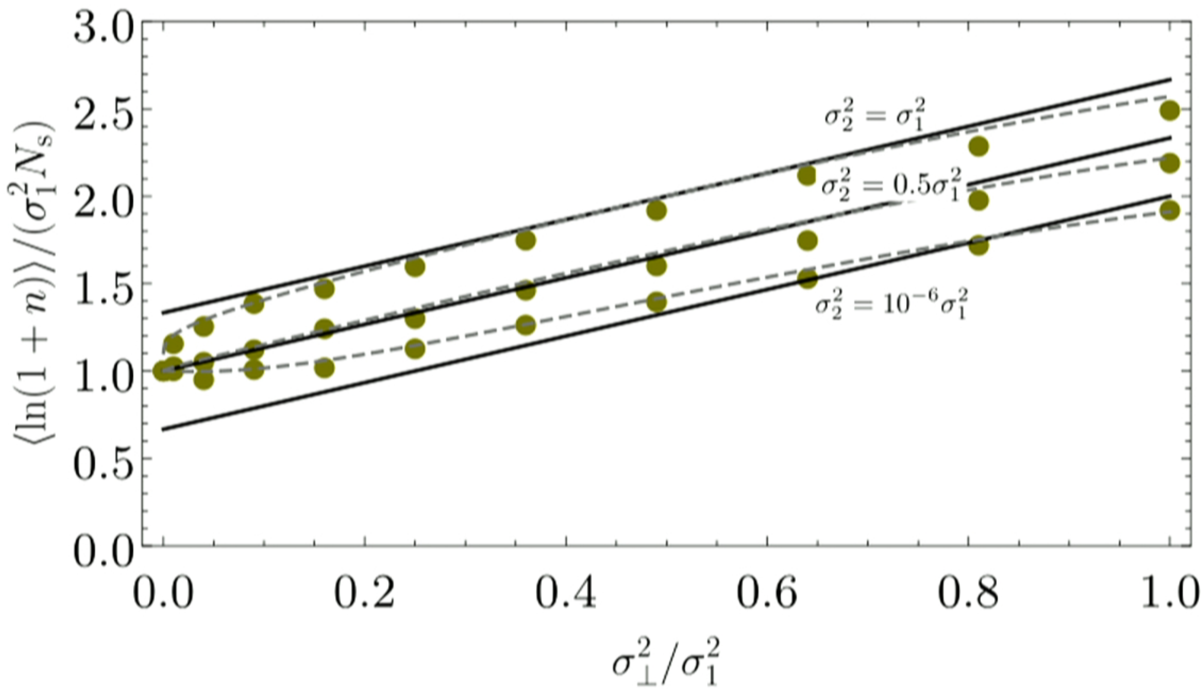
$$\ddot{\chi}_a + \left[k^2\delta^b_a + \mathcal{M}^b{}_a(\tau)\right]\chi_b = 0$$



- adiabatic evolution of variables

WORK IN PROGRESS

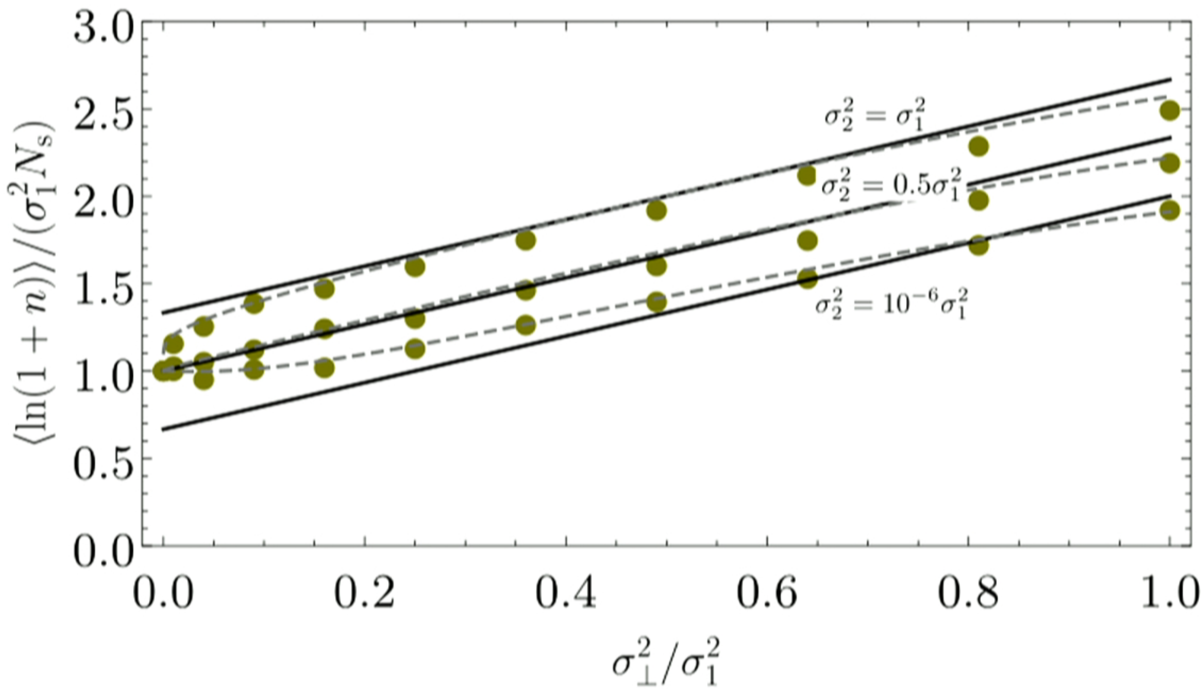
beyond "maximum entropy" ansatz



MA, H . Xie & O.Wen

beyond "maximum entropy" ansatz

WORK IN PROGRESS



MA, H . Xie & O. Wen

Universality from Random Matrix Theory

two large N's to make life easier:

	• large number of fields: N_f	• large number of scatterings: N_s
	↕	↕
from RMT:	• eigenvalue spectrum of M_j	• non-random limit of $\mathsf{M} = \prod_{j=1}^{N_\mathrm{s}} \mathsf{M}_j$

Pichard and Sarma

prediction for exponential behavior in time

non-random behavior of the exponent

Universality from Random Matrix Theory

two large N's to make life easier:

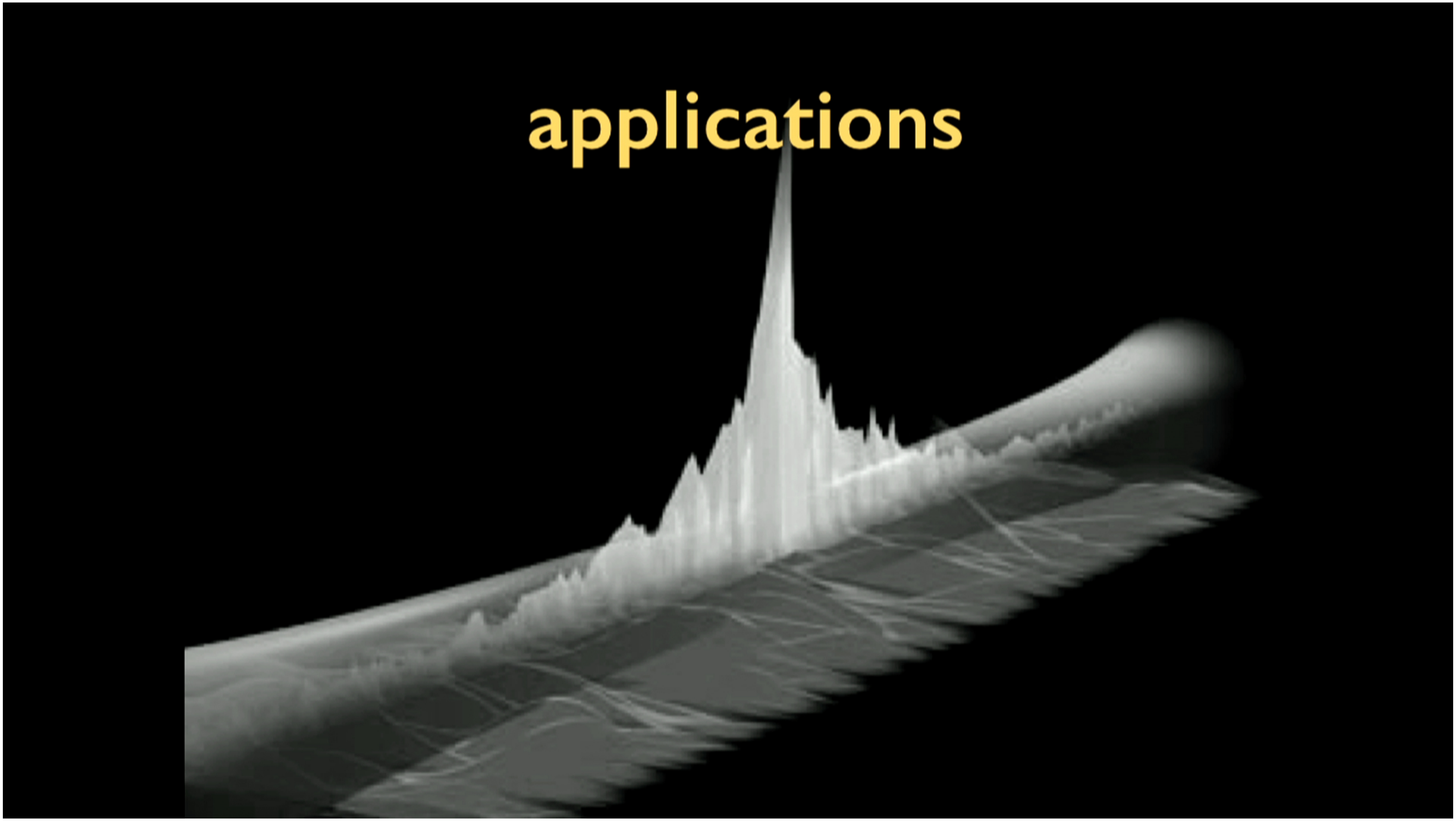
from RMT:

• large number of fields: N_f ↕ • eigenvalue spectrum of M_j	• large number of scatterings: N_s ↕ • non-random limit of $\mathsf{M} = \prod_{j=1}^{N_\mathrm{s}} \mathsf{M}_j$

Pichard and Sarma

prediction for exponential behavior in time

non-random behavior of the exponent


applications

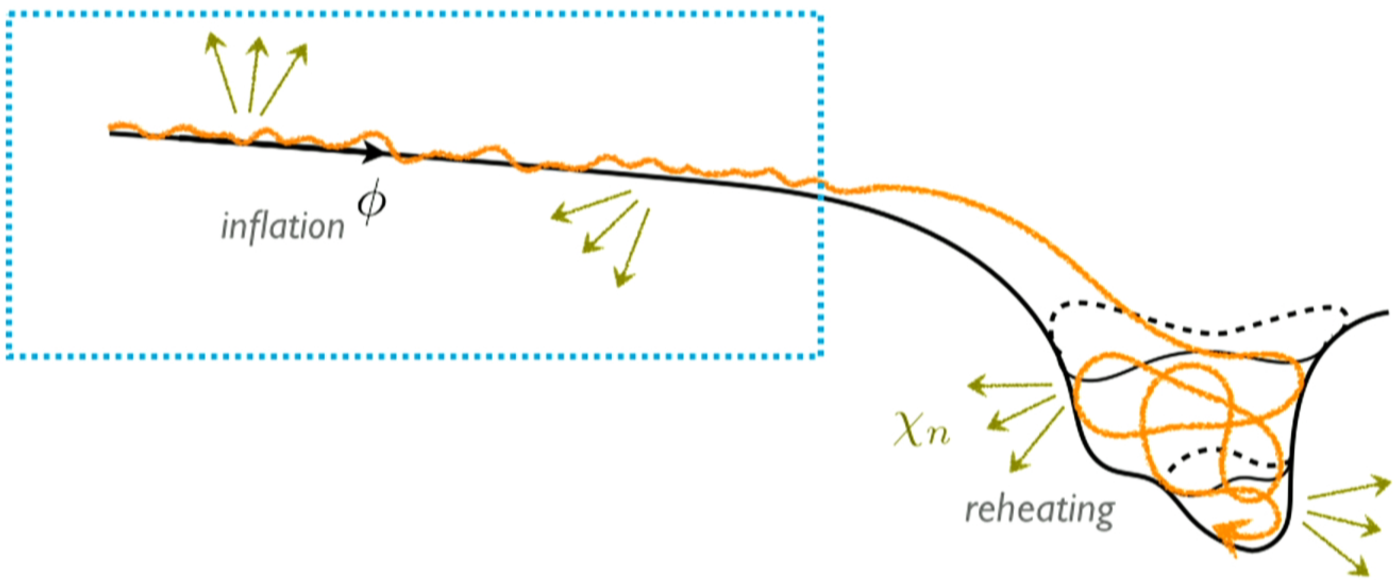
WORK IN PROGRESS

applications: inflation

MA, Baumann & Green

background dynamics → particle production ↔ curvature fluctuations

$\langle n_{k_1} n_{k_2} \ldots \rangle$ $\langle \zeta_{k_1} \zeta_{k_2} \ldots \rangle$



WORK IN PROGRESS

combine particle production & EFT with driving and dissipation

Nacir, Porto, Senatore, and Zaldarriaga
Green, Horn, Senatore, and Silverstein

$$\mathcal{L} = \mathcal{L}_{\rm sr} - m^2(t+\pi)\,\chi^2$$

Goldstone boson $\zeta = -H\pi$

$$\left(\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right)\pi = \frac{dm^2}{dt}\,\chi^2$$

source

stochastic noise

$$(\chi^2)_S \equiv \langle \chi^2 \rangle_{\pi=0}$$

linear response

$$(\chi^2)_R \equiv \int^t {\rm d}t'\, G_{\rm ret}^{\langle \chi^2 \rangle}(t,t')\,\pi(t')$$

background dynamics

↓

particle production

↕ $\langle n_{k_1} n_{k_2} \ldots \rangle$

curvature fluctuations

$\langle \zeta_{k_1} \zeta_{k_2} \ldots \rangle$

MA, Baumann & Green

combine particle production & EFT with driving and dissipation

WORK IN PROGRESS

Nacir, Porto, Senatore, and Zaldarriaga
Green, Horn, Senatore, and Silverstein

$$\mathcal{L} = \mathcal{L}_{\rm sr} - m^2(t+\pi)\,\chi^2$$

Goldstone boson $\zeta = -H\pi$

$$\left(\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right)\pi = \frac{dm^2}{dt}\,\chi^2$$

source

stochastic noise

$$(\chi^2)_S \equiv \langle \chi^2 \rangle_{\pi=0}$$

linear response

$$(\chi^2)_R \equiv \int^t \mathrm{d}t'\, G_{\rm ret}^{\langle\chi^2\rangle}(t,t')\,\pi(t')$$

background dynamics

particle production

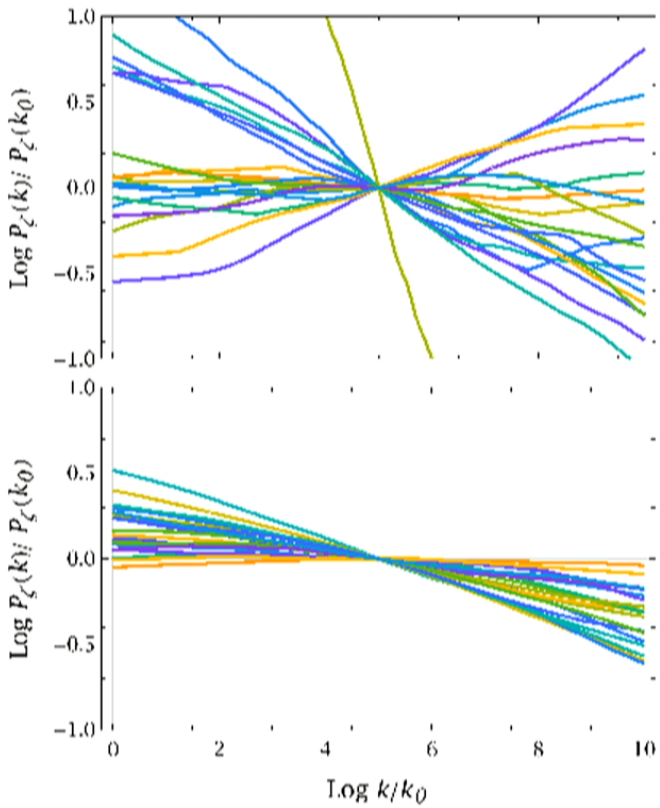
$\langle n_{k_1} n_{k_2} \dots \rangle$

curvature fluctuations

$\langle \zeta_{k_1} \zeta_{k_2} \dots \rangle$

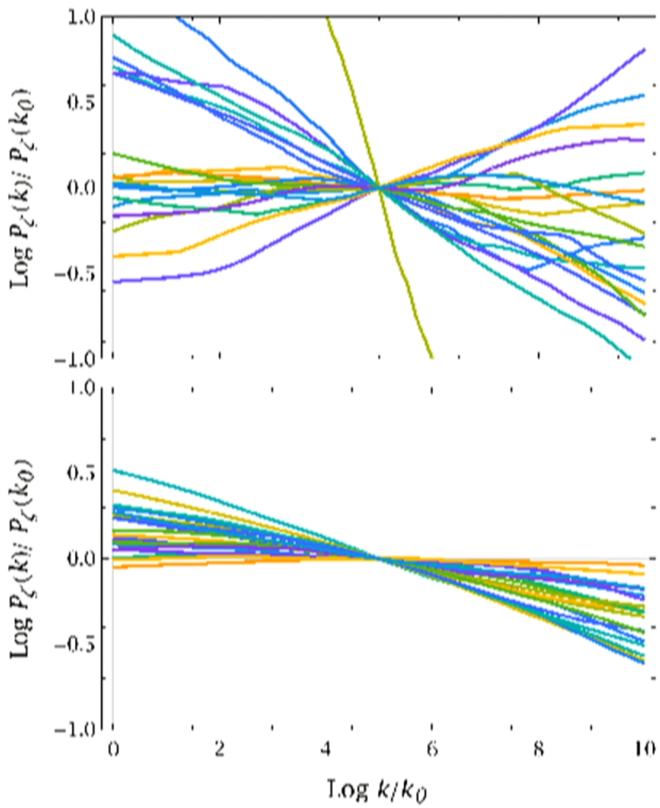
MA, Baumann & Green

a related example



Diaz, Frasier & Marsh 2016

a related example



Diaz, Frasier & Marsh 2016

potential "future" outputs

- characterizable "variance" in the power spectrum
- multifield — outside horizon evolution ?
- efficiency of reheating, scale dependence (but not "nonlinear" physics/thermalization)
- applications beyond inflation and reheating

summary

- statistical tool for theoretical complexity
- simplicity & hints of universality
- *observed simplicity* in spite of underlying complexity ?

collaborators

H. Xie:
postdoc in condensed
matter physics

J. Shen + O. Wen
+ R. Fang + S. Carleston …
undergrads and
graduate students

D. Baumann

D. Green