Title: Lattice quantum gravity and asymptotic safety

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Abstract: <p>We present results from a study of Euclidean dynamical triangulations in an attempt to make contact with Weinberg's asymptotic safety scenario. We find that a fine-tuning is necessary in order to recover semiclassical behavior, and that once this tuning is performed, our simulations provide evidence in support of the asymptotic safety scenario for gravity. We discuss our motivation for the tuning and present our numerical results. Finally, we discuss what our simulations imply for the dimension of the ultraviolet critical surface, which sets the number of free parameters in the theory.</p>

Quantum Gravity

Understanding quantum gravity is one of the outstanding problems in theoretical physics.

- Straightforward implementation as a perturbative quantum field theory is not renormalizable.
- Explicitly confirmed that a counter-term is necessary at 2-loop order for pure gravity [Goroff + Sagnotti, NPB266, 709, 1986] and 1-loop for gravity+matter [t'Hooft+Veltman].
- Can be formulated as an effective field theory at low energies, but new couplings at each order in perturbation theory lead to a loss of predictive power.
- Effective field theory arguments suggest cosmological constant should be 120 orders of magnitude larger than observed.

Asymptotic Safety

Weinberg proposed idea that gravity might be Asymptotically Safe in 1976 [Erice Subnucl. Phys. 1976:1]. This scenario would entail:

- Gravity is effectively renormalizable when formulated non-perturbatively. Problem lies with perturbation theory, not general relativity.
- Renormalization group flows of couplings have a non-trivial fixed point, with a finite dimensional ultraviolet critical surface of trajectories attracted to the fixed point at short distances.
- In a Euclidean lattice formulation the fixed point would show up as a second order critical point, the approach to which would define a continuum limit.

Lattice gravity

- Euclidean dynamical triangulations (EDT) is a lattice formulation that was introduced in the '90's. [Ambjorn, Carfora, and Marzuoli, The geometry of dynamical triangulations, Springer, Berlin, 1997] Lattice geometries are approximated by triangles with fixed edge lengths. The dynamics is contained in the connectivity of the triangles, which can be added or deleted.
- In lattice gravity, the lattice itself is a dynamical entity, which evolves in Monte Carlo time. The dimension of the building blocks can be fixed, but the effective fractal dimension must be calculated from simulations.
- EDT works perfectly in 2d, where it reproduces the results of non-critical bosonic string theory.
- The EDT formulation in 4d was shown to have two phases, a "crumpled" phase with infinite Hausdorff dimension and a branched polymer phase, with Hausdorff dimension 2. The critical point separating them was shown to be first order, so that new continuum physics is not expected. [Bialas et al, Nucl. Phys. B472, 293 (1996), hep-lat/9601024; de Bakker, Phys. Lett. B389, 238 (1996), hep-lat/9603024]

Einstein Hilbert Action

Continuum Euclidean path-integral:

$$Z = \int \mathscr{D}g \; e^{-S[g]}, \qquad (1)$$

$$S[g_{\mu\nu}] = -\frac{k}{2} \int d^d x \sqrt{\det g}(R - 2\Lambda), \qquad (2)$$

where $k = 1/(8\pi G_N)$.

Discrete action

Discrete Euclidean (Regge) action is

$$S_E = k\sum 2V_2\delta - \lambda \sum V_4, \tag{3}$$

where $\delta = 2\pi - \sum \theta$ is the deficit angle around a triangular face, V_i is the volume of an i-simplex, and $\lambda = k\Lambda$. Can show that

$$S_E = -\frac{\sqrt{3}}{2}\pi k N_2 + N_4\left(\frac{5\sqrt{3}}{2}k \arccos\frac{1}{4} + \frac{\sqrt{5}}{96}\lambda\right) \tag{4}$$

where N_i is the total number of i-simplices in the lattice. Conveniently written as

$$S_E = -\kappa_2 N_2 + \kappa_4 N_4. \tag{5}$$

Measure term

Continuum calculations suggest a form for the measure

$$Z = \int \mathscr{D}g \prod_x \sqrt{\det g}^{\beta} \; e^{-S[g]}, \tag{6}$$

Going to the discretized theory, we have

$$\prod_x \sqrt{\det g}^{\beta} \rightarrow \prod_{j=1}^{N_2} \mathscr{O}(t_j)^{\beta}, \tag{7}$$

where $\mathscr{O}(t_j)$ is the order of triangle t_j, i.e. the number of 4-simplices to which a triangle belongs. Can incorporate this term in the action by taking exponential of the log. β is a free parameter in simulations. Not fixed by diffeomorphism invariance, but is fixed, in principle, in canonical formulation. In our simulations it must be fine-tuned.

New Idea

Revisiting the EDT approach because other formulations (renormalization group and other lattice approaches) suggest that gravity is asymptotically safe.

New work done in collaboration with students (past and present) and postdoc: JL, S. Bassler, D. Coumbe, Daping Du, J. Neelakanta, (arXiv:1604.02745).

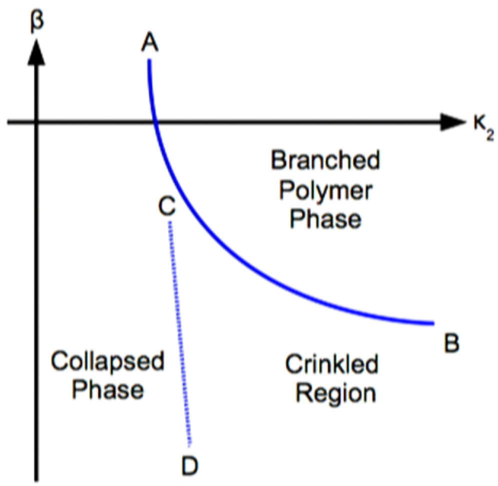
- Key new idea is that a fine-tuning of bare parameters in EDT is necessary to recover the correct continuum limit.
- Previous work did not implement this fine-tuning, leading to negative results.

Simulations

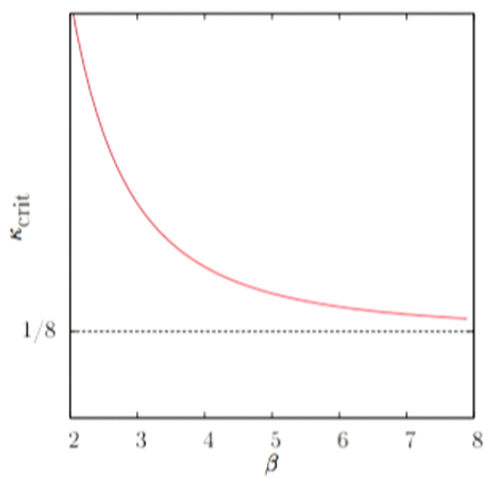
Methods for doing these simulations were introduced in the 90's. We wrote new code from scratch.

- The Metropolis Algorithm is implemented using a set of local update moves.
- We introduce a new algorithm for parallelizing the code, which we call parallel rejection. Exploits the low acceptance of the model, and partially compensates for it. Checked that it reproduces the scalar code configuration-by-configuration. Buys us a factor of ~ 10.

Phase diagram EDT vs. QCD with Wilson fermions



EDT



QCD

Main problems to overcome

- Must show recovery of semiclassical physics in 4 dimensions.
- Must show existence of continuum limit at 2nd order critical point.
- Argument against renormalizability of gravity due to Banks. Tension between renormalizability and holography.

Argument against asymptotic safety

Holographic argument against asymptotic safety due to Banks and Shomer (arXiv:0709.3555):
For a renormalizable theory with an ultraviolet fixed point the theory is a CFT at very high energies.
One finds an entropy equation of state

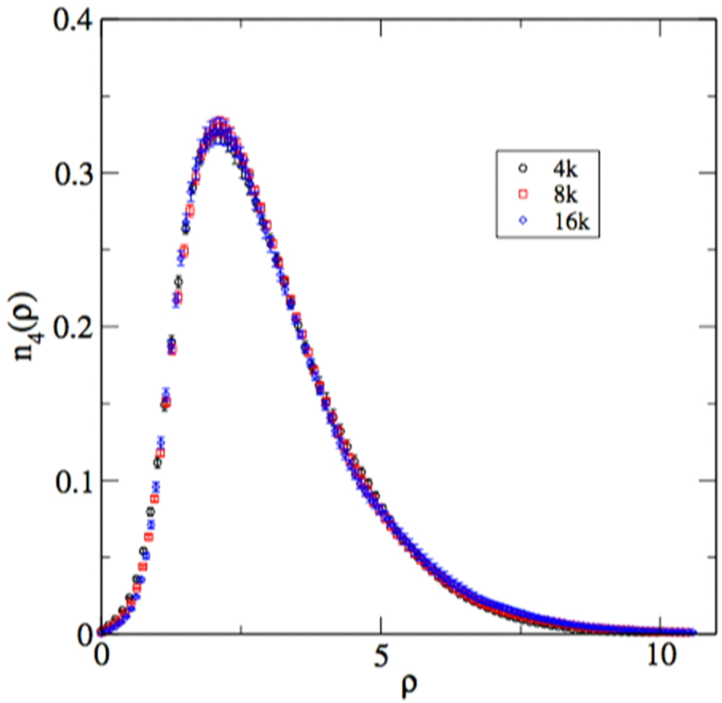
$$S \sim E^{\frac{d-1}{d}}, \tag{8}$$

where S is entropy and E is energy.
For gravity one expects that the high energy spectrum will be dominated by black holes. The Beckenstein-Hawking entropy formula leads to

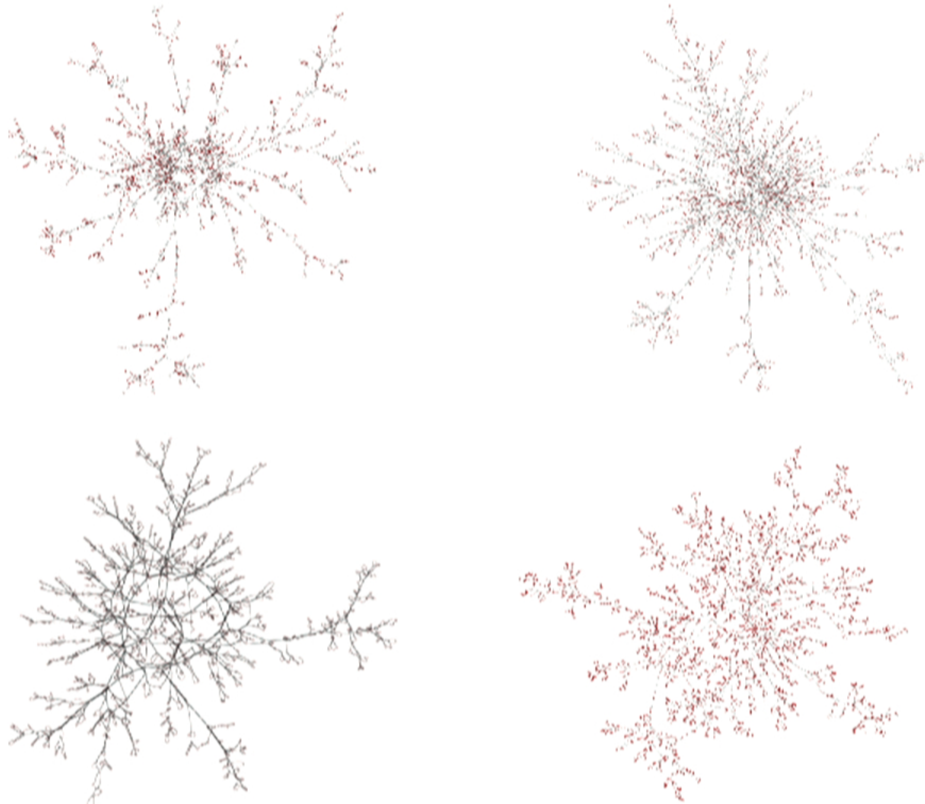
$$S \sim E^{\frac{d-2}{d-3}}. \tag{9}$$

These disagree for $d = 4$.

Three volume distribution

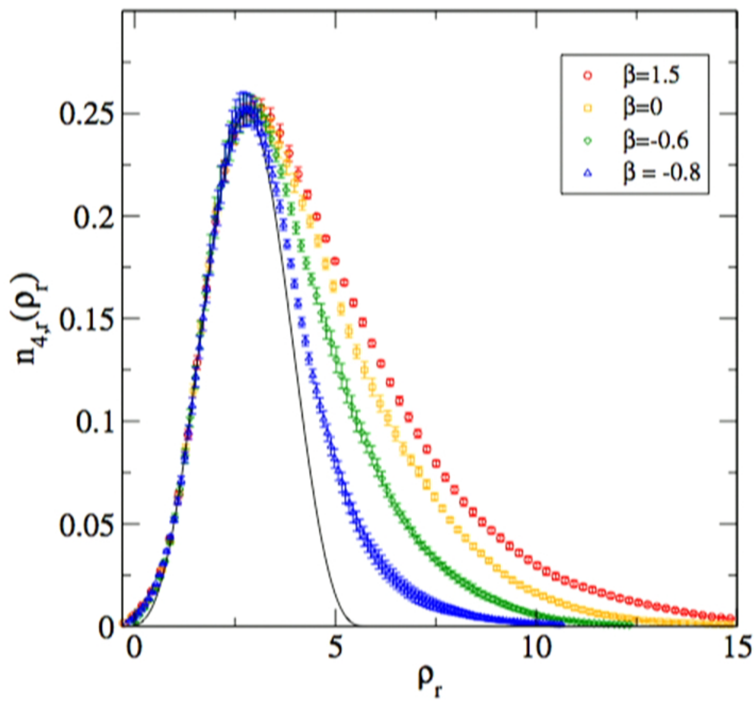


Visualization of geometries



Coarser to finer, left to right, top to bottom.

Three volume distribution



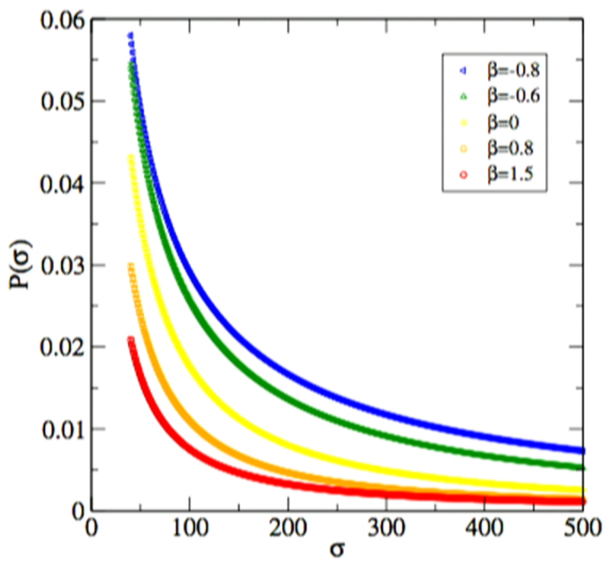
Diffusion process and the spectral dimension

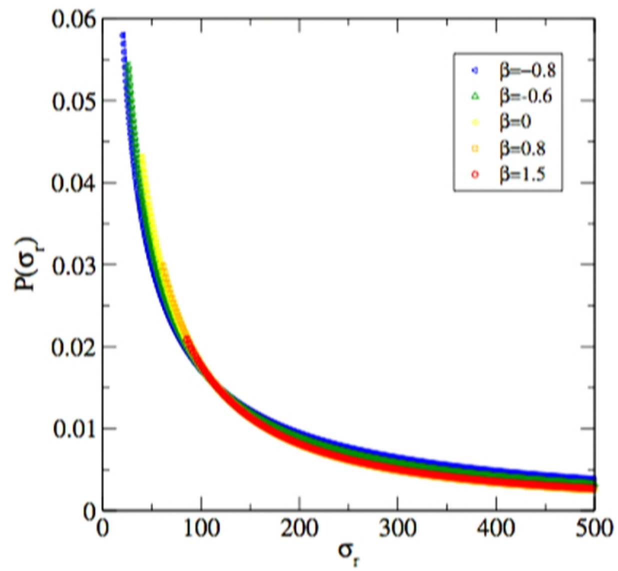
Spectral dimension is defined by a diffusion process

$$D_S(\sigma) = -2\frac{d\log P(\sigma)}{d\log\sigma}, \tag{10}$$

where σ is the diffusion time step on the lattice, and $P(\sigma)$ is the return probability, i.e. the probability of being back where you started in a random walk after σ steps.

Relative lattice spacing



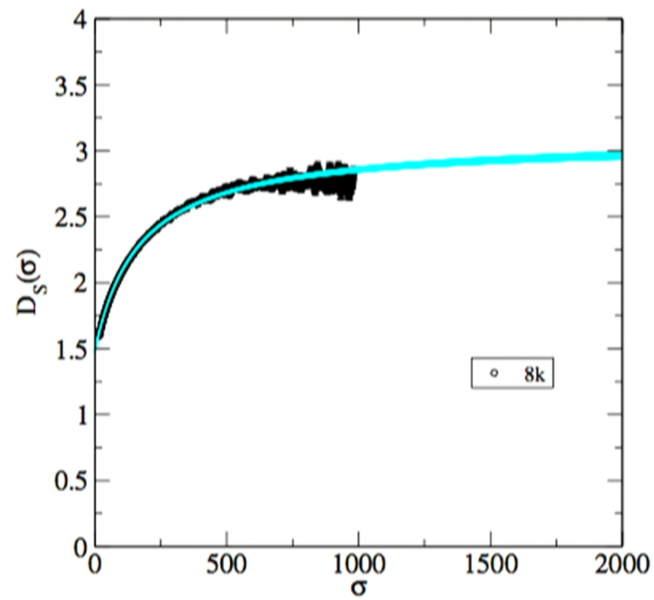


Return probability left and rescaled return probability right.

Spectral Dimension

χ^2/dof=1.25, p-value=17%

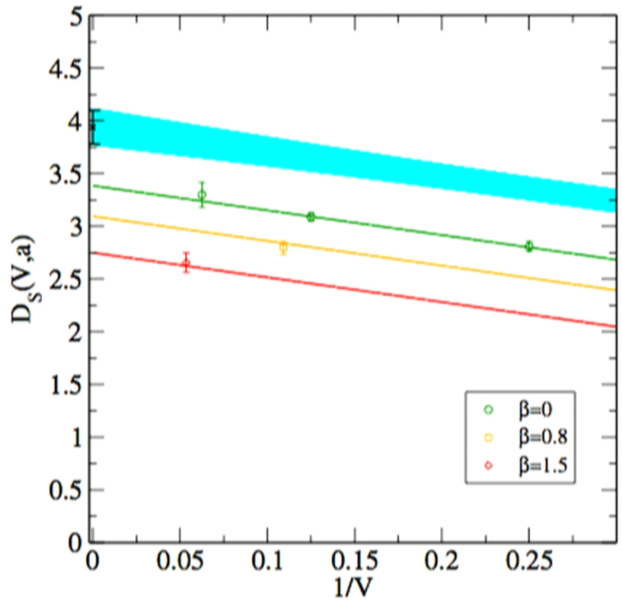
$D_S(\infty) = 3.090 \pm 0.041$, $D_S(0) = 1.484 \pm 0.021$



Infinite volume, continuum extrapolation

χ^2/dof=0.52, p-value=59%

$D_S(\infty) = 3.94 \pm 0.16$



Consistent?

$$S \sim E^{\frac{d-1}{d}}, \quad CFT \tag{11}$$

$$S \sim E^{\frac{d-2}{d-3}}, \quad GR \tag{12}$$

For these relations the relevant dimension is the spectral dimension if one lives on a fractal space.
The scaling agrees when $d = 3/2$. This is consistent with our result $D_S(0) = 1.44 \pm 0.19$.

The number of relevant parameters

Three adjustable parameters in the action: G, Λ, β.

Nontrivial evidence that G and Λ are not separately relevant couplings. One of these is redundant, with $G\Lambda$ a relevant coupling. Only $G\Lambda$ approaches a constant near the fixed-point.

Further evidence that β is only relevant because the lattice regulator breaks the canonical symmetry. This symmetry should be an exact symmetry of the quantum theory, so β should not be a relevant parameter in the target continuum theory. Makes sense, since the local measure should not run.

This means there is only one relevant coupling! Maximally predictive theory with no adjustable parameters once the scale is set.

Field strength renormalization

For convenience in the following argument, we can rewrite the Lagrangian corresponding to the Einstein-Hilbert action as

$$\mathscr{L} = \frac{\omega}{16\pi}\sqrt{g}(R - 2\omega\Lambda') \tag{13}$$

where ω and Λ' replace G and Λ. Then

$$\frac{\partial \mathscr{L}}{\partial \omega} = \frac{1}{16\pi}\sqrt{g}(R - 4\omega\Lambda'). \tag{14}$$

which vanishes by the equations of motion, suggesting that ω is a redundant parameter.

Conclusions

Important to test the picture presented here against other approaches, renormalization group and other lattice formulations.

If this holds, lattice provides a nonperturbative definition of a renormalizable quantum field theory of general relativity with no adjustable parameters and a cosmological constant that is small in the infrared.

Can we make contact with experiment? Must add matter! Predictions for $G\Lambda$ run to low energy to understand dark energy? Early universe?