Title: Towards postquantum information relativity: a status report

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Abstract: <p>In this talk I will: 1) review the results of my work on a geometric approach to foundations for a postquantum information theory; 2) discuss how it is related to other foundational approaches, including some resource theories of knowledge and quantum histories; 3) present some of my research on a category theoretic framework for a multi-agent information relativity. More details on part 1: this approach does not rely on probability theory, spectral theory, or Hilbert spaces. Normalisation of states, convexity, and tensor products are allowed but not assumed foundationally. Nonlinear generalisation of quantum kinematics and dynamics is constructed using geometric structures (quantum relative entropies and Banach Lie--Poisson structure) over the sets of quantum states on W*-algebras. In particular, unitary evolution is generalised to nonlinear hamiltonian flows, while Lueders' rules are generalised to constrained relative entropy maximisations. Combined together, they provide a framework for causal inference that is a generalisation and replacement for completely positive maps, with information dynamics determined directly by epistemic constraints, and no requirement for lack of correlation. Orthodox probability theory and quantum mechanics are special cases of this framework. I will also give the progress report on the reconstruction conjecture: given the category of sets of abstract "states" equipped with the suitably defined entropic distances and BLP structure, how one reconstructs the W*-algebraic case? The discussion of the consistent operational semantics for this approach will lead us to the parts 2 and 3.</p>

Plan

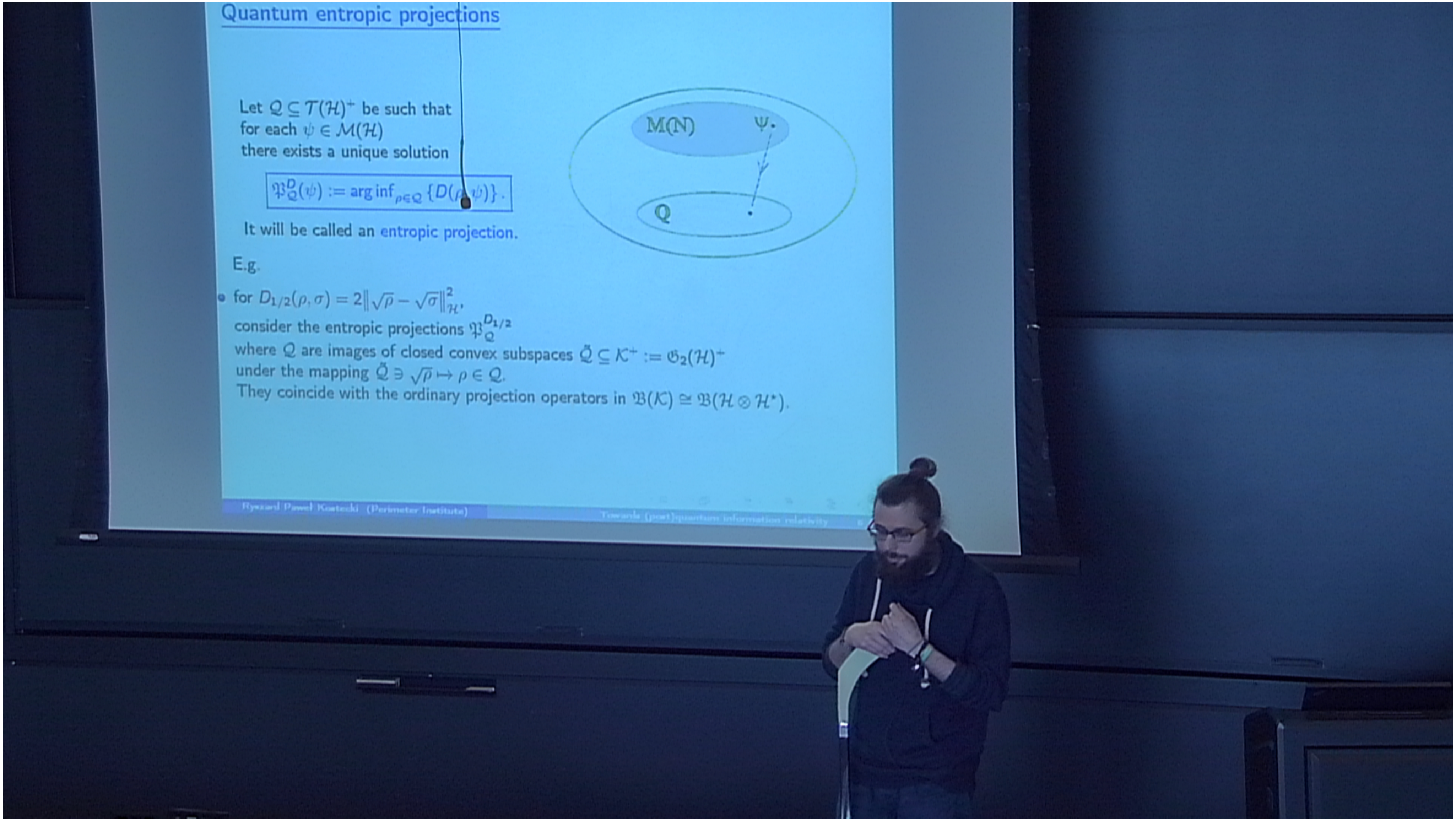
1. **Nonlinear generalisations of quantum dynamics:**
   - Geometric structures on quantum state spaces → relative entropies & Poisson brackets
   - Lüders' rules → constrained relative entropy maximisations
   - Unitary evolution → nonlinear hamiltonian flows
2. **Geometric (post)quantum information foundations:**
   - Mathematical and physical principles
   - Global and local dynamics
   - Global and local reconstruction of QM
3. **Category-theoretic operational semantics:**
   - Adjointness in foundations, functorial localisation
   - Resource theories a la LdR–LK–RR
   - Beyond adjointness: local monad–comonad systems
4. **Towards [(post)quantum] local information relativity:**
   - From equlibrium to nonequilibrium space-time thermodynamics
   - Two-dimensional surfaces (θ, σ) and geometry in Klauder–Daubechies quantisation
   - Quantum dynamics of (θ, σ)-spaces

trace class operators: $\mathcal{T}(\mathcal{H}) := \{\rho \in \mathfrak{B}(\mathcal{H}) \mid \rho \geq 0,\ \mathrm{tr}_{\mathcal{H}}|\rho| < \infty\}$
we will consider arbitrary sets of denormalised quantum states: $\mathcal{M}(\mathcal{H}) \subseteq \mathcal{T}(\mathcal{H})^+$

Quantum information distances $D : \mathcal{M}(\mathcal{H}) \times \mathcal{M}(\mathcal{H}) \to [0,\infty]$s.t. $D(\rho,\sigma) = 0 \iff \rho = \sigma$.

- E.g.
  - $D_1(\rho,\sigma) := \mathrm{tr}_{\mathcal{H}}(\rho \log \rho - \rho \log \sigma)$ [Umegaki'62]
  - $D_{1/2}(\rho,\sigma) := 2\|\sqrt{\rho} - \sqrt{\sigma}\|^2_{\mathfrak{S}_2(\mathcal{H})} = 4\mathrm{tr}_{\mathcal{H}}(\frac{1}{2}\rho + \frac{1}{2}\sigma - \sqrt{\rho}\sqrt{\sigma})$ (Hilbert–Schmidt norm^2)
  - $D_{L_1(\mathcal{N})}(\rho,\sigma) := \frac{1}{2}\|\rho - \sigma\|_{\mathcal{T}(\mathcal{H})} = \frac{1}{2}\mathrm{tr}_{\mathcal{H}}|\rho - \sigma|$ (L_1/predual norm)
  - $D_\gamma(\rho,\sigma) := \frac{1}{\gamma(1-\gamma)}\mathrm{tr}_{\mathcal{H}}(\gamma\rho + (1-\gamma)\sigma - \rho^\gamma\sigma^{1-\gamma})$; $\gamma \in \mathbb{R} \setminus \{0,1\}$ [Hasegawa'93]
  - $D_{\alpha,z}(\rho,\sigma) := \frac{1}{1-\alpha}\log \mathrm{tr}_{\mathcal{H}}(\rho^{\alpha/z}\sigma^{(1-\alpha)/z})^z$; $\alpha, z \in \mathbb{R}$ [Audenauert–Datta'14]
  - $D_{\mathfrak{f}}(\rho,\sigma) := \mathrm{tr}_{\mathcal{H}}(\sqrt{\rho}\,\mathfrak{f}(\mathfrak{L}_\rho\mathfrak{R}_\sigma^{-1})\sqrt{\rho})$; $\mathfrak{f}$ operator convex, $\mathfrak{f}(1) = 0$ [Kosaki'82,Petz'85]

  for $\mathrm{ran}(\rho) \subseteq \mathrm{ran}(\sigma)$, and with all $D(\rho,\sigma) := +\infty$ otherwise.


Quantum entropic projections
Let $\mathcal{Q} \subseteq \mathcal{T}(\mathcal{H})^+$ be such that
for each $\psi \in \mathcal{M}(\mathcal{H})$
there exists a unique solution
$\mathfrak{P}^D_{\mathcal{Q}}(\psi) := \arg\inf_{\rho\in\mathcal{Q}} \{D(\rho,\psi)\}$.
It will be called an entropic projection.
M(N)
Ψ
Q
E.g.
for $D_{1/2}(\rho,\sigma) = 2\|\sqrt{\rho}-\sqrt{\sigma}\|^2_{\mathcal{H}}$,
consider the entropic projections $\mathfrak{P}^{D_{1/2}}_{\mathcal{Q}}$
where $\mathcal{Q}$ are images of closed convex subspaces $\breve{\mathcal{Q}} \subseteq \mathcal{K}^+ := \mathfrak{G}_2(\mathcal{H})^+$
under the mapping $\breve{\mathcal{Q}} \ni \sqrt{\rho} \mapsto \rho \in \mathcal{Q}$.
They coincide with the ordinary projection operators in $\mathfrak{B}(\mathcal{K}) \cong \mathfrak{B}(\mathcal{H}\otimes\mathcal{H}^*)$.
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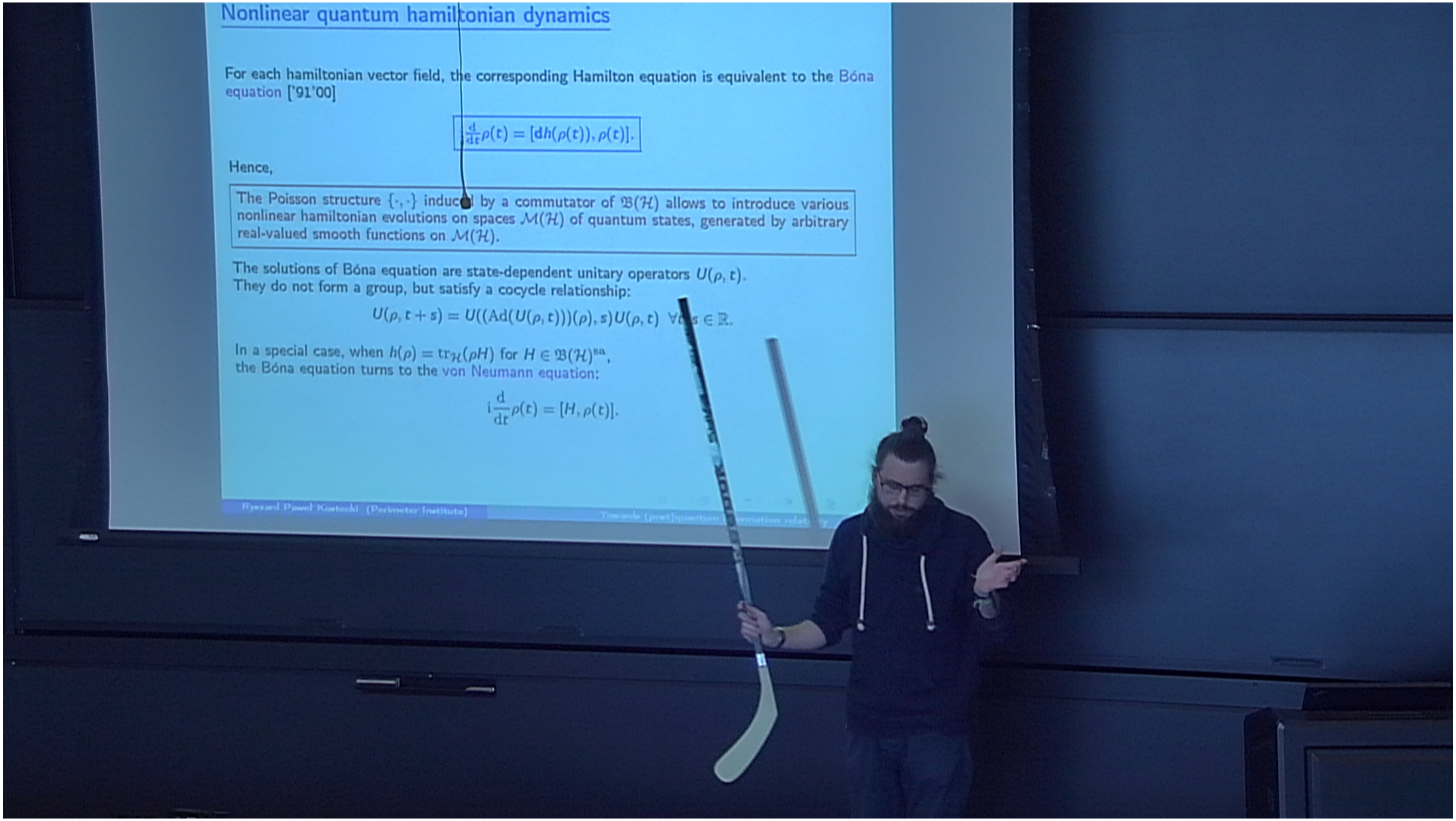
Towards (post)quantum information relativity

Quantum bayesian inference from quantum entropic projections

- RPK'13'14, F.Hellmann–W.Kamiński–RPK'14:
  1. weak Lüders' rule is a special case of
  $$\rho \mapsto \arg\inf_{\sigma\in\mathcal{Q}} \{D_1(\rho,\sigma)\}$$
  with
  $$\mathcal{Q} = \{\sigma \in \mathcal{T}(\mathcal{H})^+ \mid [P_i,\sigma] = 0\ \forall i\}$$
  2. strong Lüders' rule derived from
  $$\rho \mapsto \arg\inf_{\sigma\in\mathcal{Q}} \{D_1(\rho,\sigma)\}$$
  with
  $$\mathcal{Q} = \{\sigma \in \mathcal{T}(\mathcal{H})^+ \mid [P_i,\sigma] = 0,\ \mathrm{tr}_{\mathcal{H}}(\sigma P_i) = p_i\ \forall i\}$$
  under the limit $p_2,\ldots,p_n \to 0$.
  3. hence, weak and strong Lüders' rules are special cases of quantum entropic projection $\mathfrak{P}^{D_0}_{\mathcal{Q}}$ based on relative entropy $D_0(\sigma,\rho) = D_1(\rho,\sigma)$.

Bayes–Laplace and Lüders' conditionings are special cases of entropic projections
$\Rightarrow$ "quantum bayesianism $\subseteq$ quantum relative entropism".

Meaning: the rule of maximisation of relative entropy (entropic projection on the subspace of constraints) can be considered as a nonlinear generalisation of the dynamics describing "quantum measurement". [RPK'10'11]


Nonlinear quantum hamiltonian dynamics
For each hamiltonian vector field, the corresponding Hamilton equation is equivalent to the Bóna equation ['91'00]
$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = [\mathbf{d}h(\rho(t)), \rho(t)].$
Hence,
The Poisson structure $\{\cdot,\cdot\}$ induced by a commutator of $\mathfrak{B}(\mathcal{H})$ allows to introduce various nonlinear hamiltonian evolutions on spaces $\mathcal{M}(\mathcal{H})$ of quantum states, generated by arbitrary real-valued smooth functions on $\mathcal{M}(\mathcal{H})$.
The solutions of Bóna equation are state-dependent unitary operators $U(\rho, t)$.
They do not form a group, but satisfy a cocycle relationship:
$U(\rho, t+s) = U((\mathrm{Ad}(U(\rho,t)))(\rho), s)U(\rho, t)$
In a special case, when $h(\rho) = \mathrm{tr}_{\mathcal{H}}(\rho H)$ for $H \in \mathfrak{B}(\mathcal{H})^{\mathrm{sa}}$,
the Bóna equation turns to the von Neumann equation:
$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = [H, \rho(t)].$
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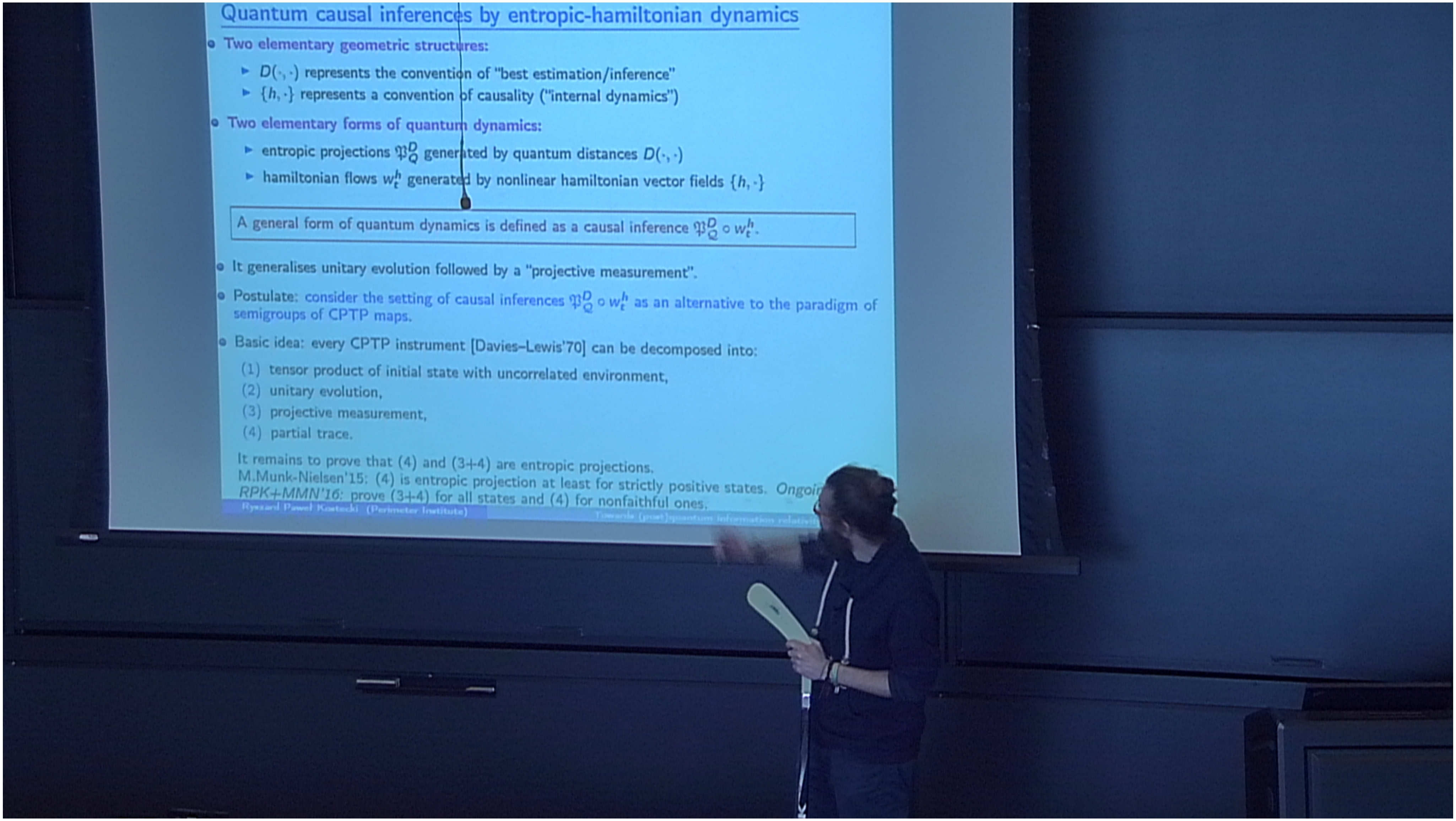

Quantum causal inferences by entropic-hamiltonian dynamics
Two elementary geometric structures:
$D(\cdot,\cdot)$ represents the convention of "best estimation/inference"
$\{h,\cdot\}$ represents a convention of causality ("internal dynamics")
Two elementary forms of quantum dynamics:
entropic projections $\mathfrak{P}^D_Q$ generated by quantum distances $D(\cdot,\cdot)$
hamiltonian flows w^h_t generated by nonlinear hamiltonian vector fields $\{h,\cdot\}$
A general form of quantum dynamics is defined as a causal inference $\mathfrak{P}^D_Q \circ w^h_t$.
It generalises unitary evolution followed by a "projective measurement".
Postulate: consider the setting of causal inferences $\mathfrak{P}^D_Q \circ w^h_t$ as an alternative to the paradigm of semigroups of CPTP maps.
Basic idea: every CPTP instrument [Davies–Lewis'70] can be decomposed into:
(1) tensor product of initial state with uncorrelated environment,
(2) unitary evolution,
(3) projective measurement,
(4) partial trace.
It remains to prove that (4) and (3+4) are entropic projections.
M.Munk-Nielsen'15: (4) is entropic projection at least for strictly positive states. *Ongoi*
RPK+MMN'16: prove (3+4) for all states and (4) for nonfaithful ones.
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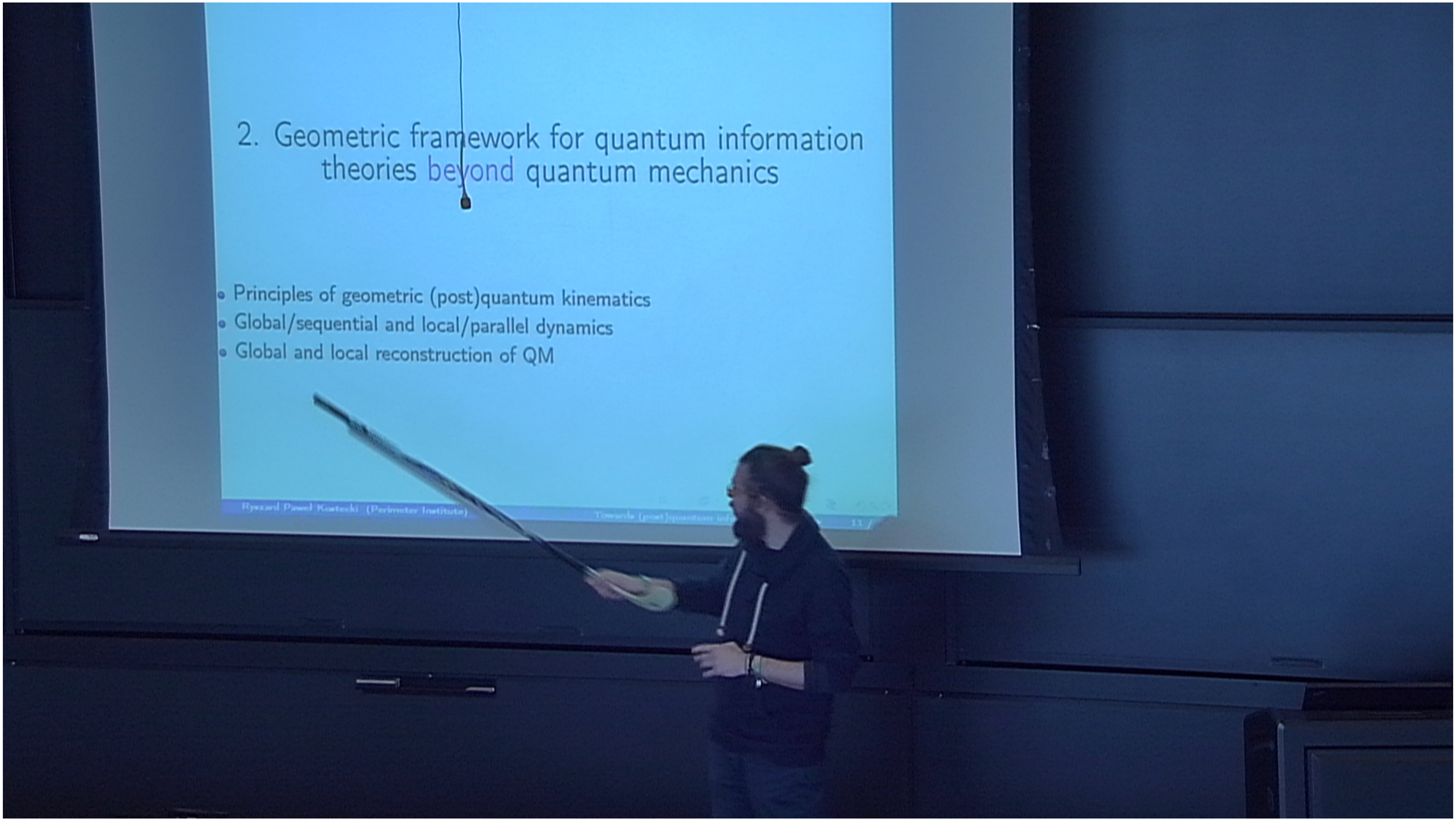
2. Geometric framework for quantum information theories beyond quantum mechanics
Principles of geometric (post)quantum kinematics
Global/sequential and local/parallel dynamics
Global and local reconstruction of QM
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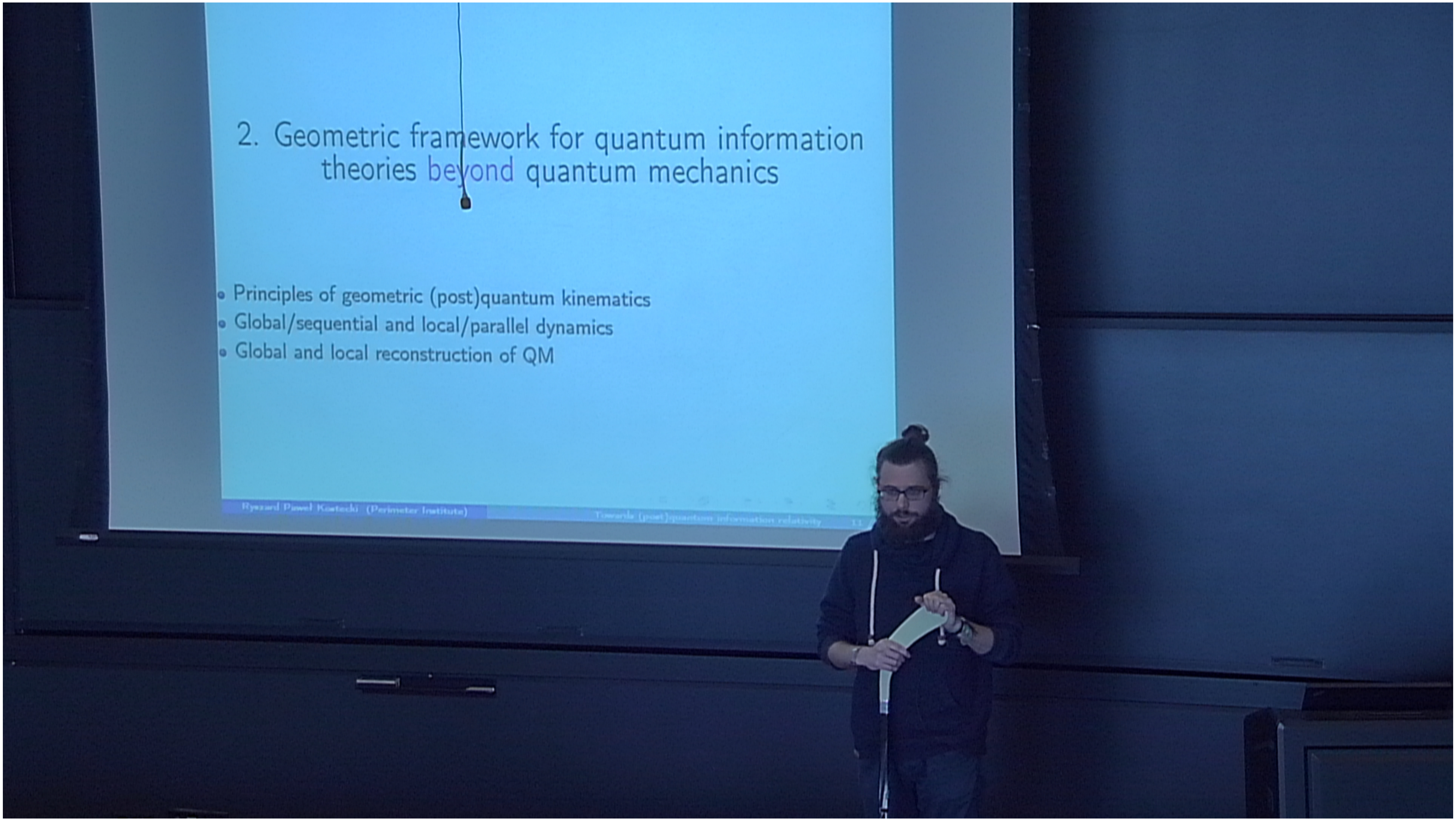
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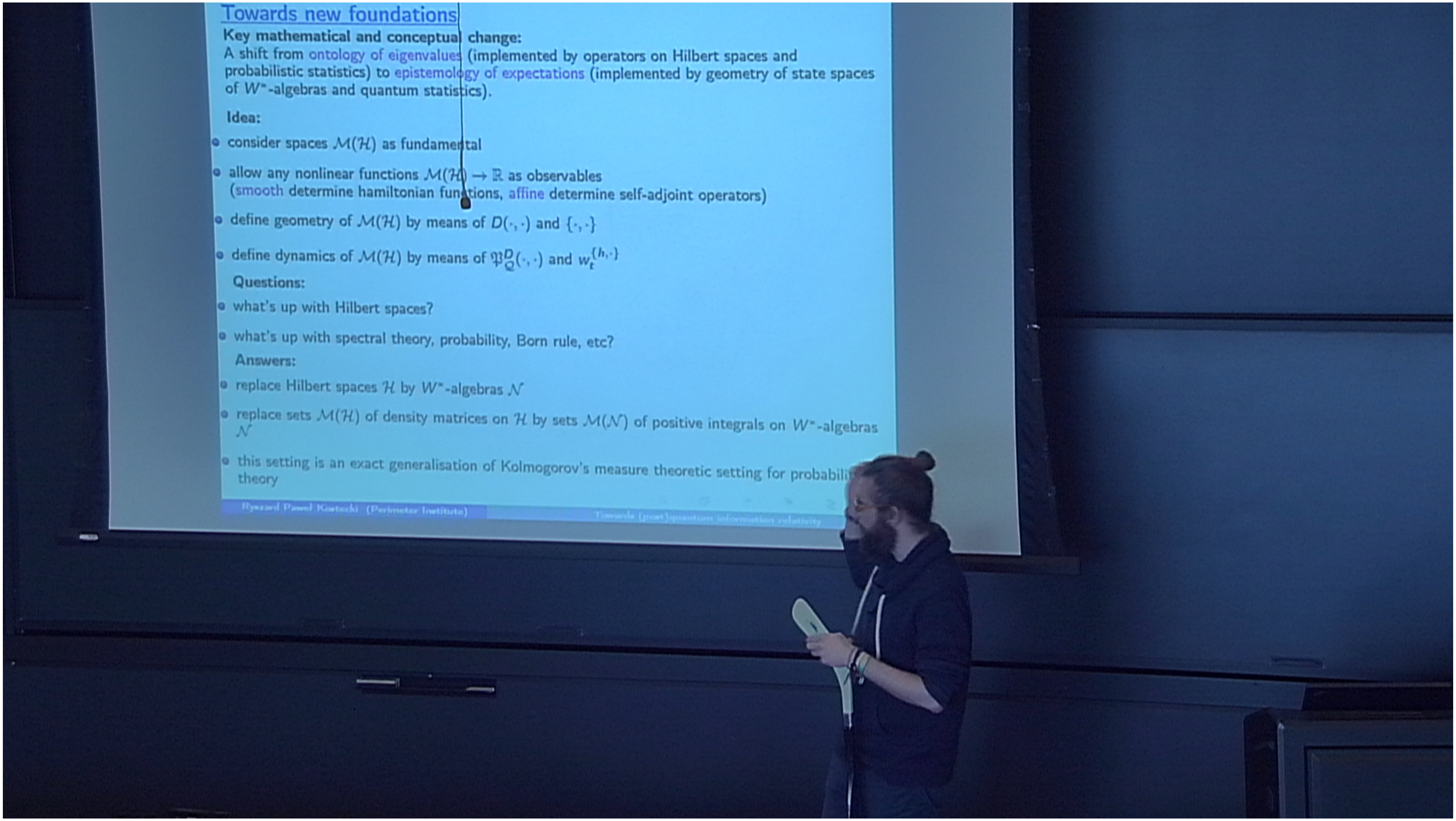
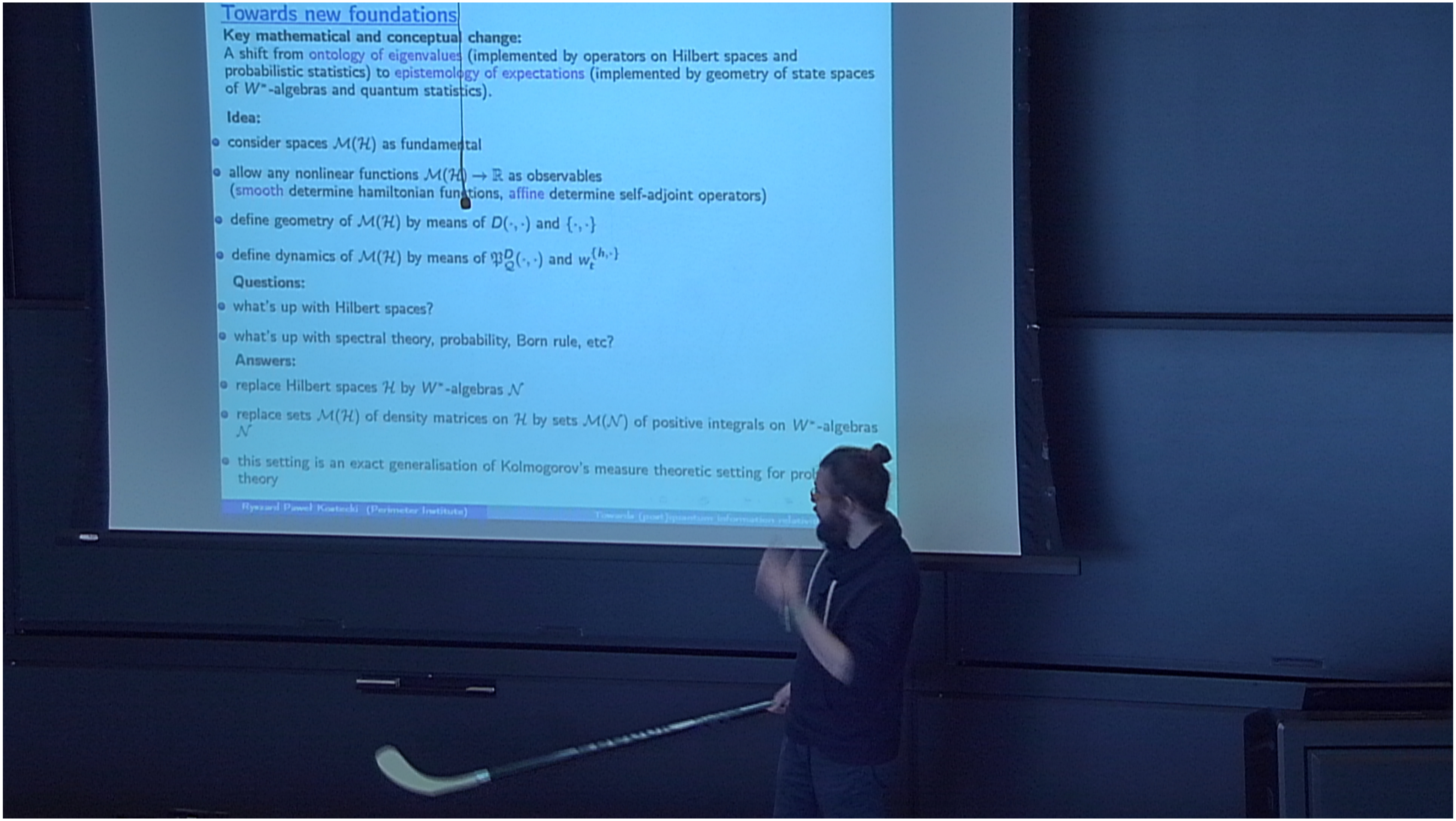
Towards (post)quantum information relativity


Towards new foundations
Key mathematical and conceptual change:
A shift from ontology of eigenvalues (implemented by operators on Hilbert spaces and probabilistic statistics) to epistemology of expectations (implemented by geometry of state spaces of W^*-algebras and quantum statistics).
Idea:
consider spaces $\mathcal{M}(\mathcal{H})$ as fundamental
allow any nonlinear functions $\mathcal{M}(\mathcal{H}) \to \mathbb{R}$ as observables (smooth determine hamiltonian functions, affine determine self-adjoint operators)
define geometry of $\mathcal{M}(\mathcal{H})$ by means of $D(\cdot,\cdot)$ and $\{\cdot,\cdot\}$
define dynamics of $\mathcal{M}(\mathcal{H})$ by means of $\mathfrak{P}^D_{\mathcal{Q}}(\cdot,\cdot)$ and $w_t^{\{h,\cdot\}}$
Questions:
what's up with Hilbert spaces?
what's up with spectral theory, probability, Born rule, etc?
Answers:
replace Hilbert spaces $\mathcal{H}$ by W^*-algebras $\mathcal{N}$
replace sets $\mathcal{M}(\mathcal{H})$ of density matrices on $\mathcal{H}$ by sets $\mathcal{M}(\mathcal{N})$ of positive integrals on W^*-algebras $\mathcal{N}$
this setting is an exact generalisation of Kolmogorov's measure theoretic setting for probabili theory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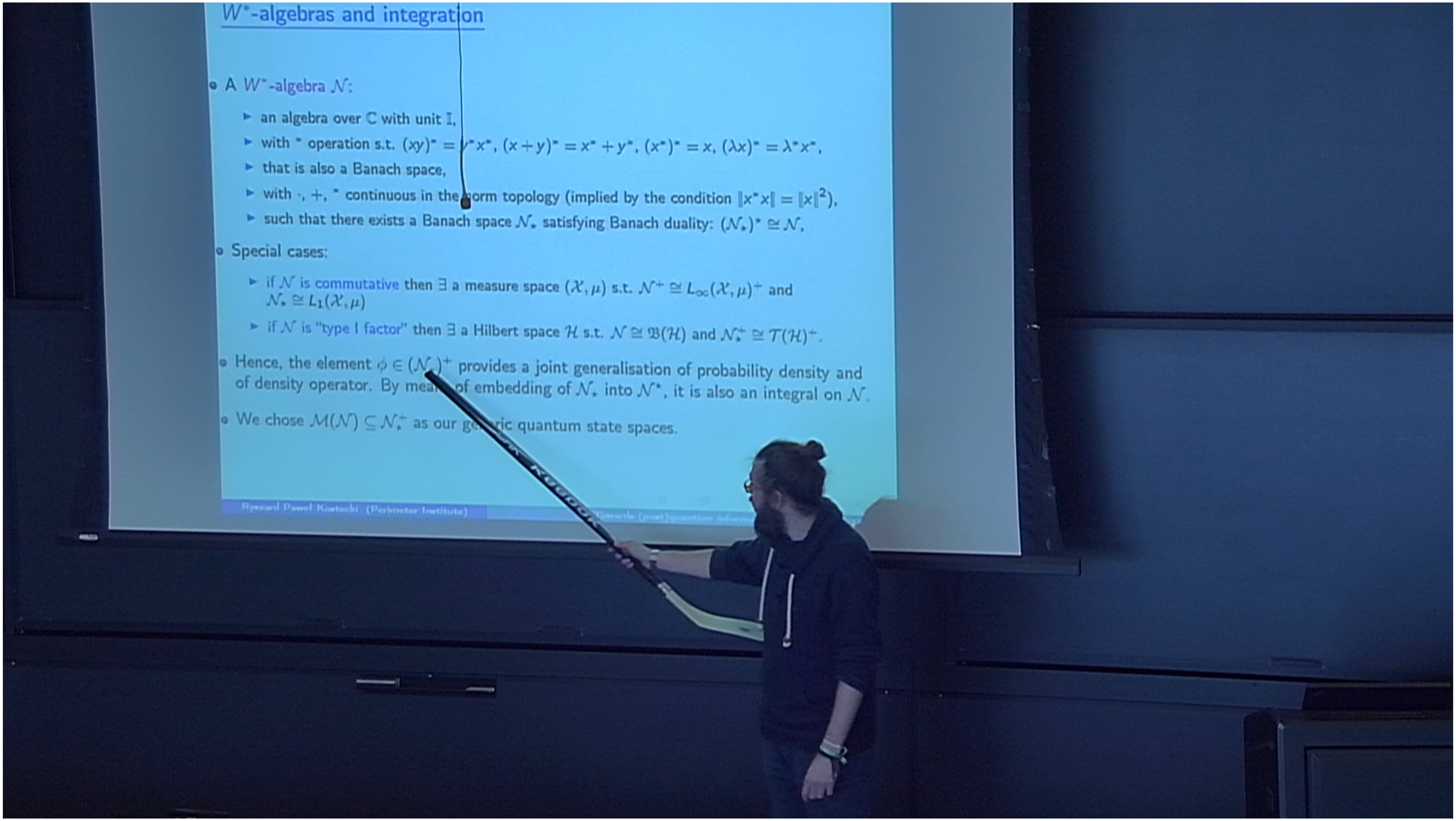
W^*-algebras and integration
A W^*-algebra $\mathcal{N}$:
an algebra over $\mathbb{C}$ with unit $\mathbb{I}$,
with * operation s.t. $(xy)^* = y^*x^*$, $(x+y)^* = x^* + y^*$, $(x^*)^* = x$, $(\lambda x)^* = \lambda^* x^*$,
that is also a Banach space,
with $\cdot$, $+$, * continuous in the norm topology (implied by the condition $\|x^*x\| = \|x\|^2$),
such that there exists a Banach space $\mathcal{N}_*$ satisfying Banach duality: $(\mathcal{N}_*)^* \cong \mathcal{N}$,
Special cases:
if $\mathcal{N}$ is commutative then $\exists$ a measure space $(\mathcal{X}, \mu)$ s.t. $\mathcal{N}^+ \cong L_\infty(\mathcal{X}, \mu)^+$ and $\mathcal{N}_* \cong L_1(\mathcal{X}, \mu)$
if $\mathcal{N}$ is "type I factor" then $\exists$ a Hilbert space $\mathcal{H}$ s.t. $\mathcal{N} \cong \mathfrak{B}(\mathcal{H})$ and $\mathcal{N}_*^+ \cong \mathcal{T}(\mathcal{H})^+$.
Hence, the element $\phi \in (\mathcal{N}_*)^+$ provides a joint generalisation of probability density and of density operator. By mea of embedding of $\mathcal{N}_*$ into $\mathcal{N}^*$, it is also an integral on $\mathcal{N}$.
We chose $\mathcal{M}(\mathcal{N}) \subseteq \mathcal{N}_*^+$ as our ge ric quantum state spaces.
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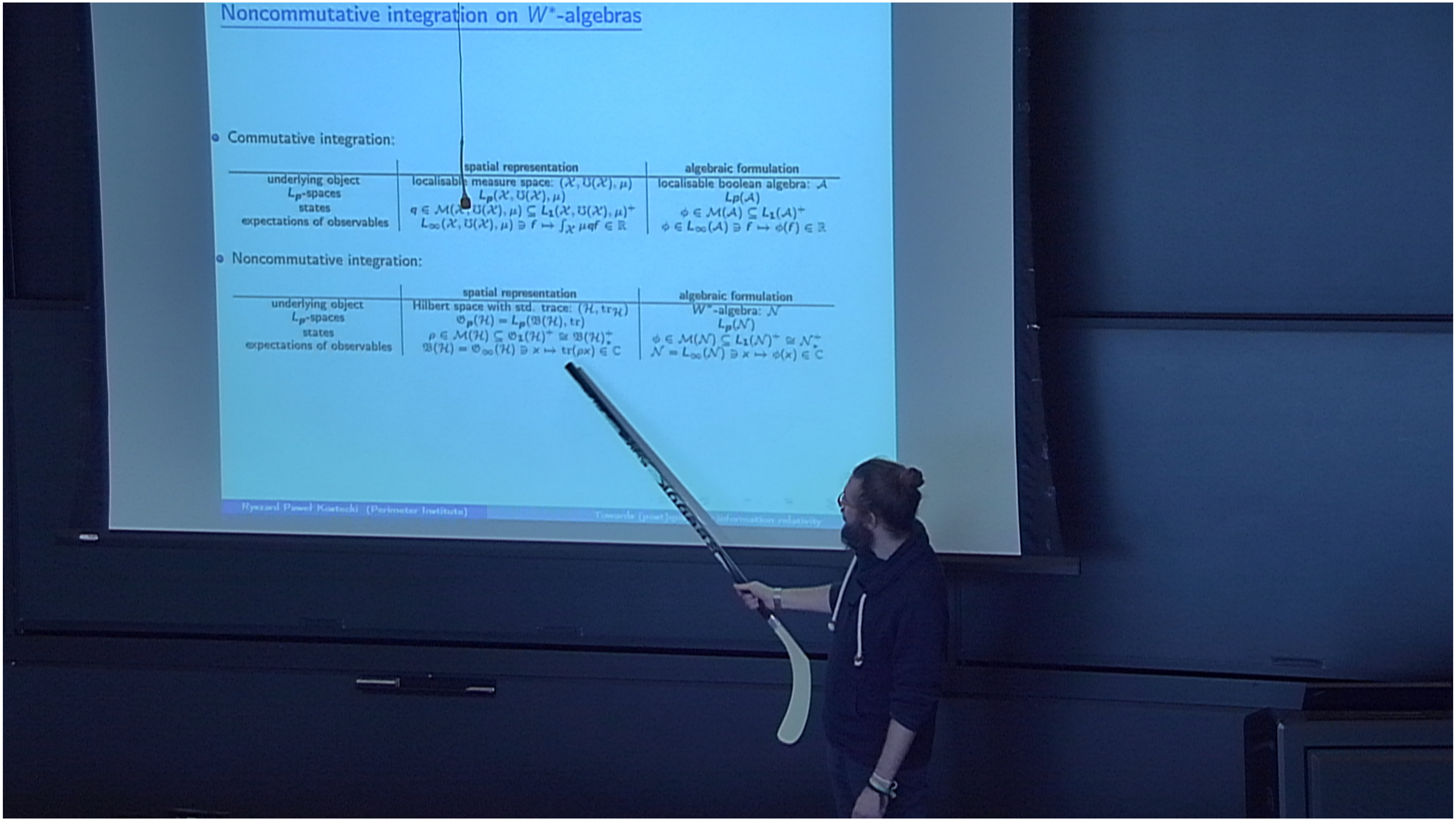
Noncommutative integration on W^*-algebras
Commutative integration:
	spatial representation	algebraic formulation
underlying object	localisable measure space: $(\mathcal{X}, \mho(\mathcal{X}), \mu)$	localisable boolean algebra: $\mathcal{A}$
L_p-spaces	$L_p(\mathcal{X}, \mho(\mathcal{X}), \mu)$	$L_p(\mathcal{A})$
states	$q \in \mathcal{M}(\mathcal{X}, \mho(\mathcal{X}), \mu) \subseteq L_1(\mathcal{X}, \mho(\mathcal{X}), \mu)^+$	$\phi \in \mathcal{M}(\mathcal{A}) \subseteq L_1(\mathcal{A})^+$
expectations of observables	$L_\infty(\mathcal{X}, \mho(\mathcal{X}), \mu) \ni f \mapsto \int_{\mathcal{X}} \mu q f \in \mathbb{R}$	$\phi \in L_\infty(\mathcal{A}) \ni f \mapsto \phi(f) \in \mathbb{R}$

Noncommutative integration:
	spatial representation	algebraic formulation
underlying object	Hilbert space with std. trace: $(\mathcal{H}, \mathrm{tr}_{\mathcal{H}})$	W^*-algebra: $\mathcal{N}$
L_p-spaces	$\mathfrak{G}_p(\mathcal{H}) = L_p(\mathfrak{B}(\mathcal{H}), \mathrm{tr})$	$L_p(\mathcal{N})$
states	$\rho \in \mathcal{M}(\mathcal{H}) \subseteq \mathfrak{G}_1(\mathcal{H})^+ \cong \mathfrak{B}(\mathcal{H})^+_\star$	$\phi \in \mathcal{M}(\mathcal{N}) \subseteq L_1(\mathcal{N})^+ \cong \mathcal{N}^+_\star$
expectations of observables	$\mathfrak{B}(\mathcal{H}) = \mathfrak{G}_\infty(\mathcal{H}) \ni x \mapsto \mathrm{tr}(\rho x) \in \mathbb{C}$	$\mathcal{N} = L_\infty(\mathcal{N}) \ni x \mapsto \phi(x) \in \mathbb{C}$

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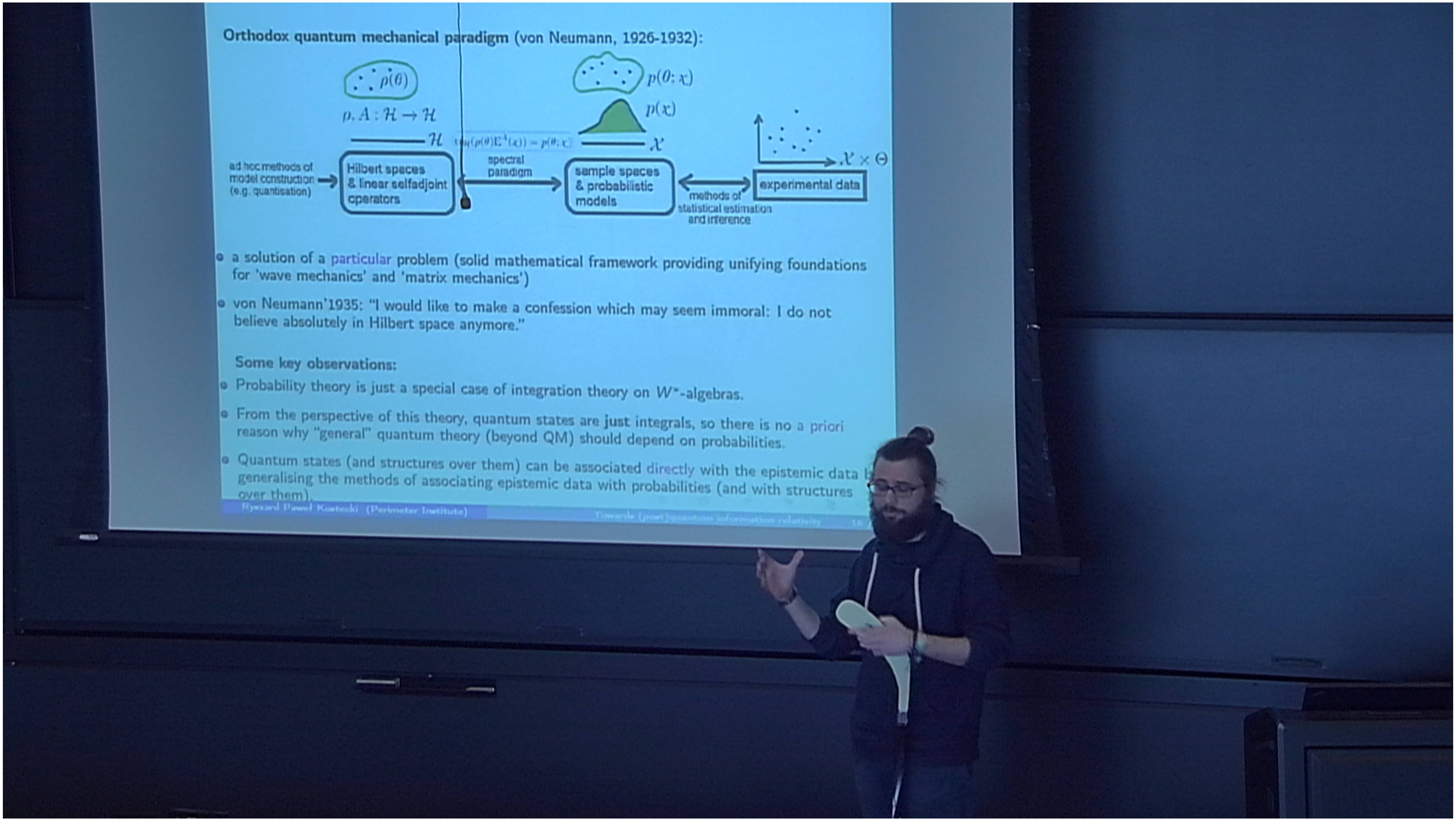
Orthodox quantum mechanical paradigm (von Neumann, 1926-1932):
ρ(θ)
ρ, A : H → H
H
p(θ; x)
p(x)
X
X × Θ
ad hoc methods of model construction (e.g. quantisation)
Hilbert spaces & linear selfadjoint operators
spectral paradigm
sample spaces & probabilistic models
methods of statistical estimation and inference
experimental data
a solution of a particular problem (solid mathematical framework providing unifying foundations for 'wave mechanics' and 'matrix mechanics')
von Neumann'1935: "I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space anymore."
Some key observations:
Probability theory is just a special case of integration theory on W*-algebras.
From the perspective of this theory, quantum states are just integrals, so there is no a priori reason why "general" quantum theory (beyond QM) should depend on probabilities.
Quantum states (and structures over them) can be associated directly with the epistemic data by generalising the methods of associating epistemic data with probabilities (and with structures over them).
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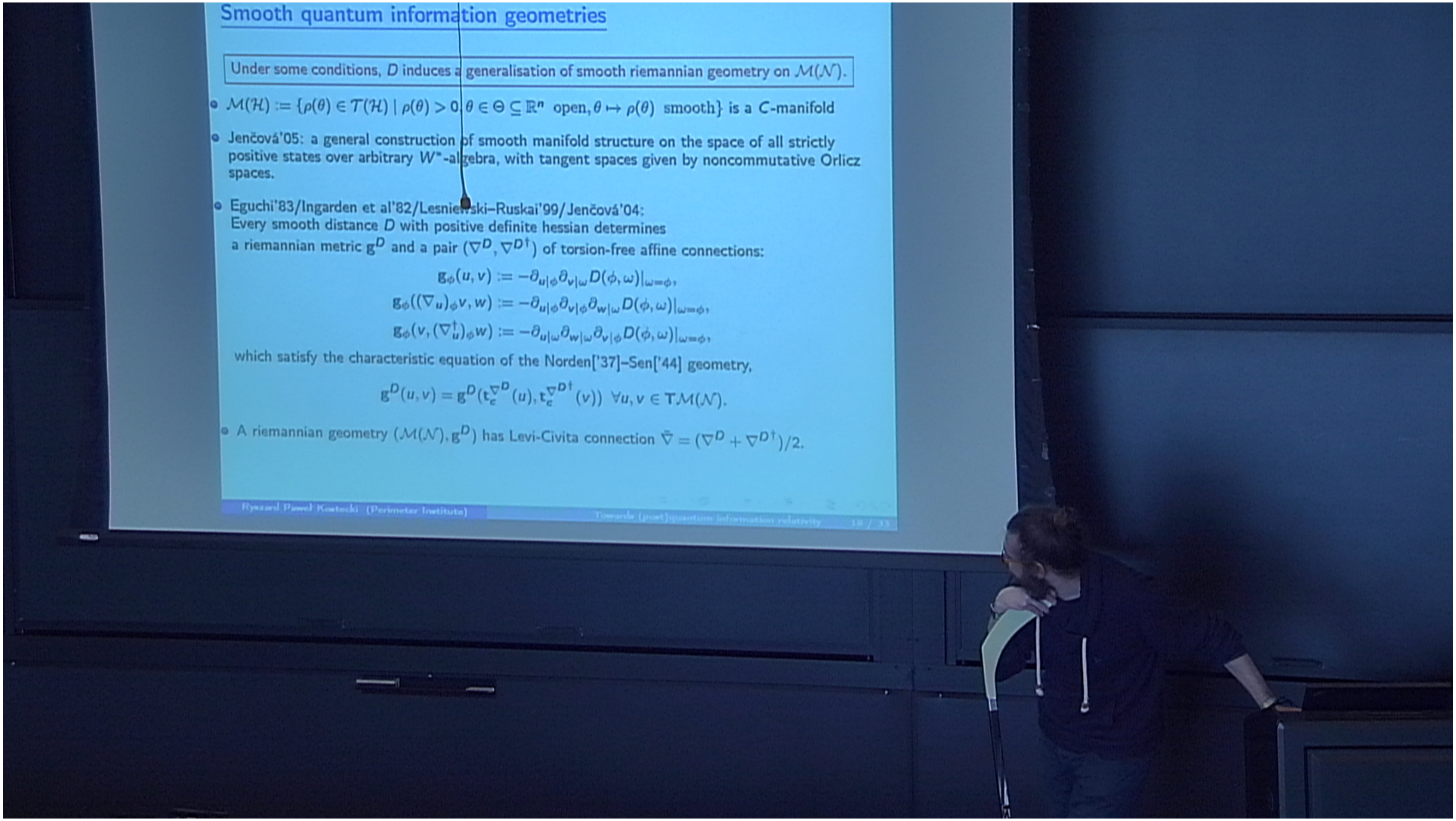
Towards (post)quantum information relativity


Smooth quantum information geometries
Under some conditions, D induces a generalisation of smooth riemannian geometry on $\mathcal{M}(\mathcal{N})$.
$\mathcal{M}(\mathcal{H}) := \{\rho(\theta) \in \mathcal{T}(\mathcal{H}) \mid \rho(\theta) > 0, \theta \in \Theta \subseteq \mathbb{R}^n \text{ open}, \theta \mapsto \rho(\theta) \text{ smooth}\}$ is a C-manifold
Jenčová'05: a general construction of smooth manifold structure on the space of all strictly positive states over arbitrary W^*-algebra, with tangent spaces given by noncommutative Orlicz spaces.
Eguchi'83/Ingarden et al'82/Lesniewski–Ruskai'99/Jenčová'04:
Every smooth distance D with positive definite hessian determines a riemannian metric $\mathbf{g}^D$ and a pair $(\nabla^D, \nabla^{D\dagger})$ of torsion-free affine connections:
$\mathbf{g}_\phi(u,v) := -\partial_{u|\phi}\partial_{v|\omega} D(\phi,\omega)|_{\omega=\phi}$,
$\mathbf{g}_\phi((\nabla_u)_\phi v, w) := -\partial_{u|\phi}\partial_{v|\phi}\partial_{w|\omega} D(\phi,\omega)|_{\omega=\phi}$,
$\mathbf{g}_\phi(v, (\nabla^\dagger_u)_\phi w) := -\partial_{u|\omega}\partial_{w|\omega}\partial_{v|\phi} D(\phi,\omega)|_{\omega=\phi}$,
which satisfy the characteristic equation of the Norden['37]–Sen['44] geometry,
$\mathbf{g}^D(u,v) = \mathbf{g}^D(\mathrm{t}_c^{\nabla^D}(u), \mathrm{t}_c^{\nabla^{D\dagger}}(v)) \ \forall u,v \in \mathrm{T}\mathcal{M}(\mathcal{N})$.
A riemannian geometry $(\mathcal{M}(\mathcal{N}), \mathbf{g}^D)$ has Levi-Civita connection $\bar{\nabla} = (\nabla^D + \nabla^{D\dagger})/2$.
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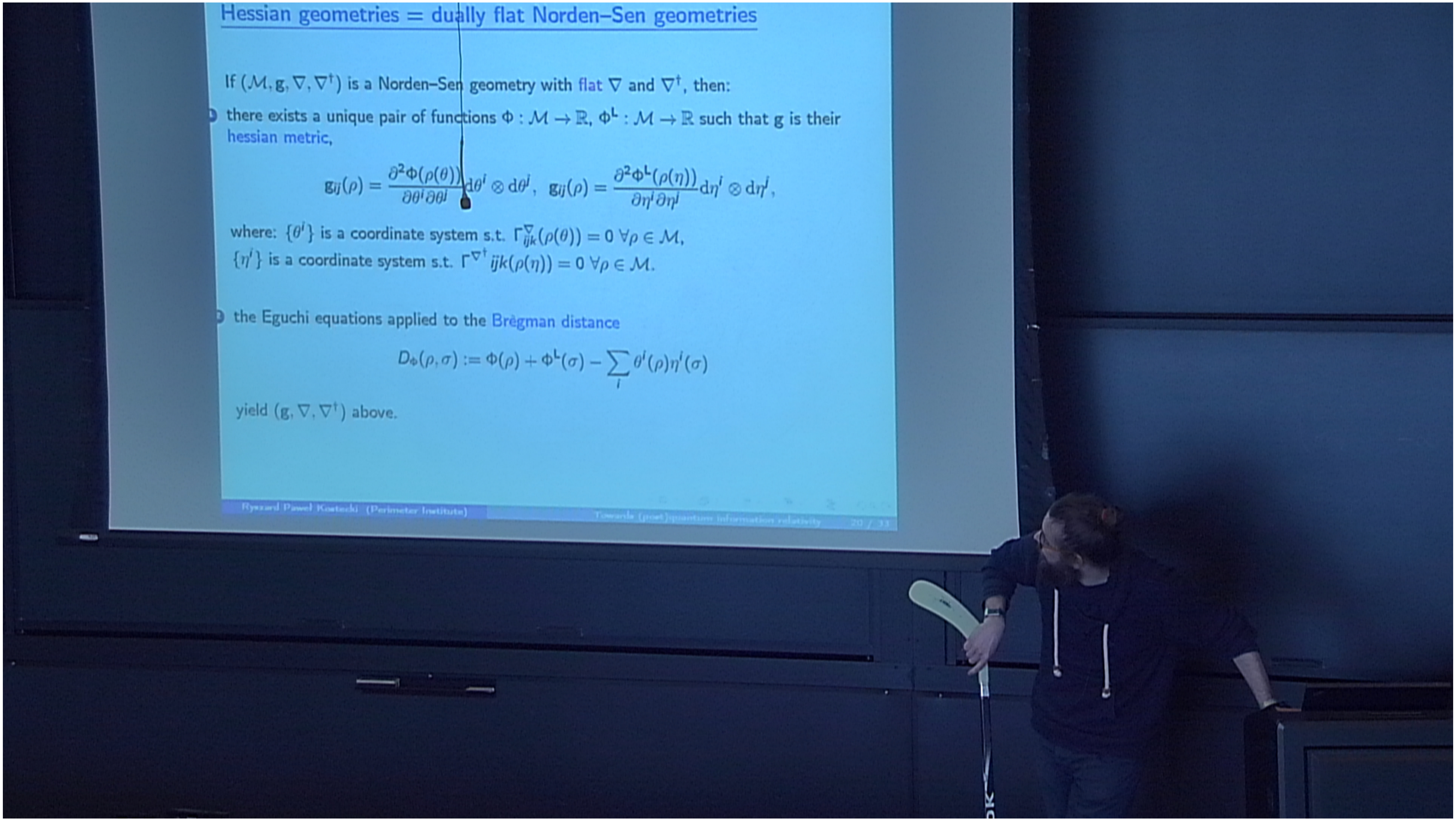
Hessian geometries = dually flat Norden–Sen geometries
If $(\mathcal{M}, g, \nabla, \nabla^\dagger)$ is a Norden–Sen geometry with flat ∇ and $\nabla^\dagger$, then:
there exists a unique pair of functions $\Phi : \mathcal{M} \to \mathbb{R}$, $\Phi^{\mathsf{L}} : \mathcal{M} \to \mathbb{R}$ such that g is their hessian metric,
$$g_{ij}(\rho) = \frac{\partial^2 \Phi(\rho(\theta))}{\partial \theta^i \partial \theta^j} \mathrm{d}\theta^i \otimes \mathrm{d}\theta^j, \quad g_{ij}(\rho) = \frac{\partial^2 \Phi^{\mathsf{L}}(\rho(\eta))}{\partial \eta^i \partial \eta^j} \mathrm{d}\eta^i \otimes \mathrm{d}\eta^j,$$
where: $\{\theta^i\}$ is a coordinate system s.t. $\Gamma^{\nabla}_{ijk}(\rho(\theta)) = 0 \;\forall \rho \in \mathcal{M}$,
$\{\eta^i\}$ is a coordinate system s.t. $\Gamma^{\nabla^\dagger} ijk(\rho(\eta)) = 0 \;\forall \rho \in \mathcal{M}$.
the Eguchi equations applied to the Brègman distance
$$D_\Phi(\rho, \sigma) := \Phi(\rho) + \Phi^{\mathsf{L}}(\sigma) - \sum_i \theta^i(\rho)\eta^i(\sigma)$$
yield $(g, \nabla, \nabla^\dagger)$ above.
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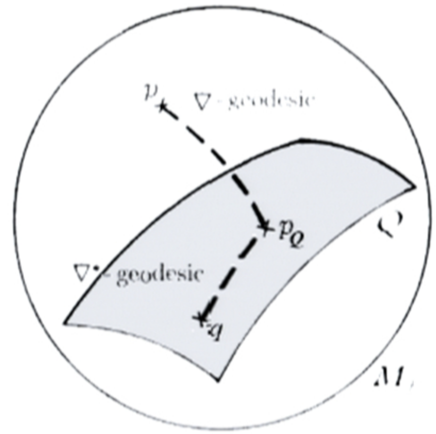
Smooth generalised pythagorean theorem

Let $(\mathcal{M}, \mathbf{g}, \nabla, \nabla^\dagger)$ be a hessian geometry. Then for any $\mathcal{Q} \subseteq \mathcal{M}$ which is:

- $\nabla^\dagger$-autoparallel $:= \nabla^\dagger_u v \in \mathbf{T}\mathcal{Q} \; \forall u, v \in \mathbf{T}\mathcal{Q}$;
- $\nabla^\dagger$-convex $:= \forall \rho_1, \rho_2 \in \mathcal{Q} \; \exists! \; \nabla^\dagger$-geodesics in $\mathcal{Q}$ connecting ρ_1 and ρ_2;

there exists a unique projection

$$\mathcal{M} \ni \rho \mapsto \mathfrak{P}^{D_\Phi}_{\mathcal{Q}}(\rho) := \arg\inf_{\sigma \in \mathcal{Q}} \{ D_\Phi(\sigma, \rho) \} \in \mathcal{Q}.$$



- it is equal to a unique projection of ρ onto $\mathcal{Q}$ along a ∇-geodesic that is $\mathbf{g}$-orthogonal at $\mathcal{Q}$.
- it satisfies a generalised pythagorean equation

$$D_\Phi(\omega, \mathfrak{P}^{D_\Phi}_{\mathcal{Q}}(\rho)) + D_\Phi(\mathfrak{P}^{D_\Phi}_{\mathcal{Q}}(\rho), \rho) = D_\Phi(\omega, \rho) \quad \forall (\omega, \rho) \in \mathcal{Q} \times \mathcal{M}.$$

Hence, for Brègman distances D_Φ the local entropic projections are equivalent with geodesic projections.

Local effective dynamics

- One can combine locally the entropic projections with hamiltonian flows, by passing to the derived geodesic projections, and combining both in a single formula for effective dynamics.
- Given a hamiltonian observable h and a relative entropy D, the 1-form $dh(\phi) - d_{\nabla^D}(\phi)$ represents a local perturbation of causal dynamics by the information flow along entropic geodesics.
- In particular, $D_{1/2} = 2\left\|\sqrt{\rho} - \sqrt{\sigma}\right\|^2_{\mathcal{H}}$ gives Wigner–Yanase metric $\mathbf{g}^{1/2}$, with $d_{\mathbf{g}^{1/2}}(\rho, \sigma) = 2\arccos(\mathrm{tr}_{\mathcal{H}}(\sqrt{\rho}\sqrt{\sigma}))$. The free fall along the geodesics of Levi-Civita connection $\nabla^{1/2}$ encodes the continuous process of projective measurement.
- The resulting effective dynamics can be given mathematically exact form in terms of a continuous-time regularised path-integral

$$\lim_{\varepsilon \to +0} \int \mathcal{D}\phi(\cdot) \mathrm{e}^{\mathrm{i}\int_\gamma \mathrm{d}t \left\langle \Omega_{\phi(t)}, \mathbf{d}_{\nabla^{1/2}}(\phi(t)\Omega_{\phi(t)} \right\rangle_{\mathcal{H}_{\phi(t)}}} \cdot \tag{1}$$

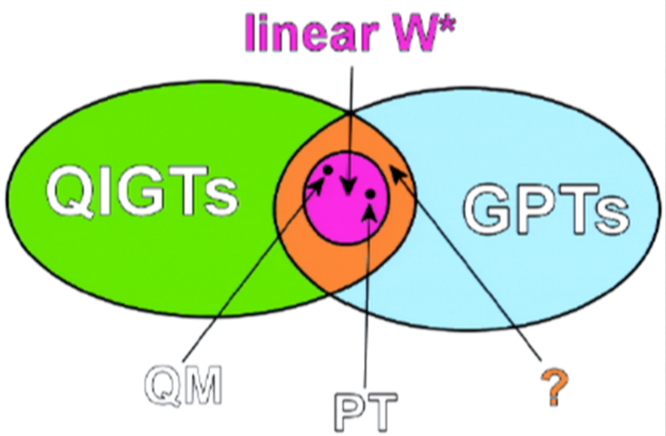
$$\cdot \mathrm{e}^{-\mathrm{i}\int_\gamma \mathrm{d}t \left\langle \Omega_{\phi(t)}, \pi_{\phi(t)}(\mathbf{d}h(\phi(t)))\Omega_{\phi(t)} \right\rangle} \mathrm{e}^{-\frac{\varepsilon}{2}\int_\gamma \mathrm{d}t \mathbf{g}^{1/2}_{ab}(\phi(t))\dot{\phi}^a \dot{\phi}^b}, \tag{2}$$

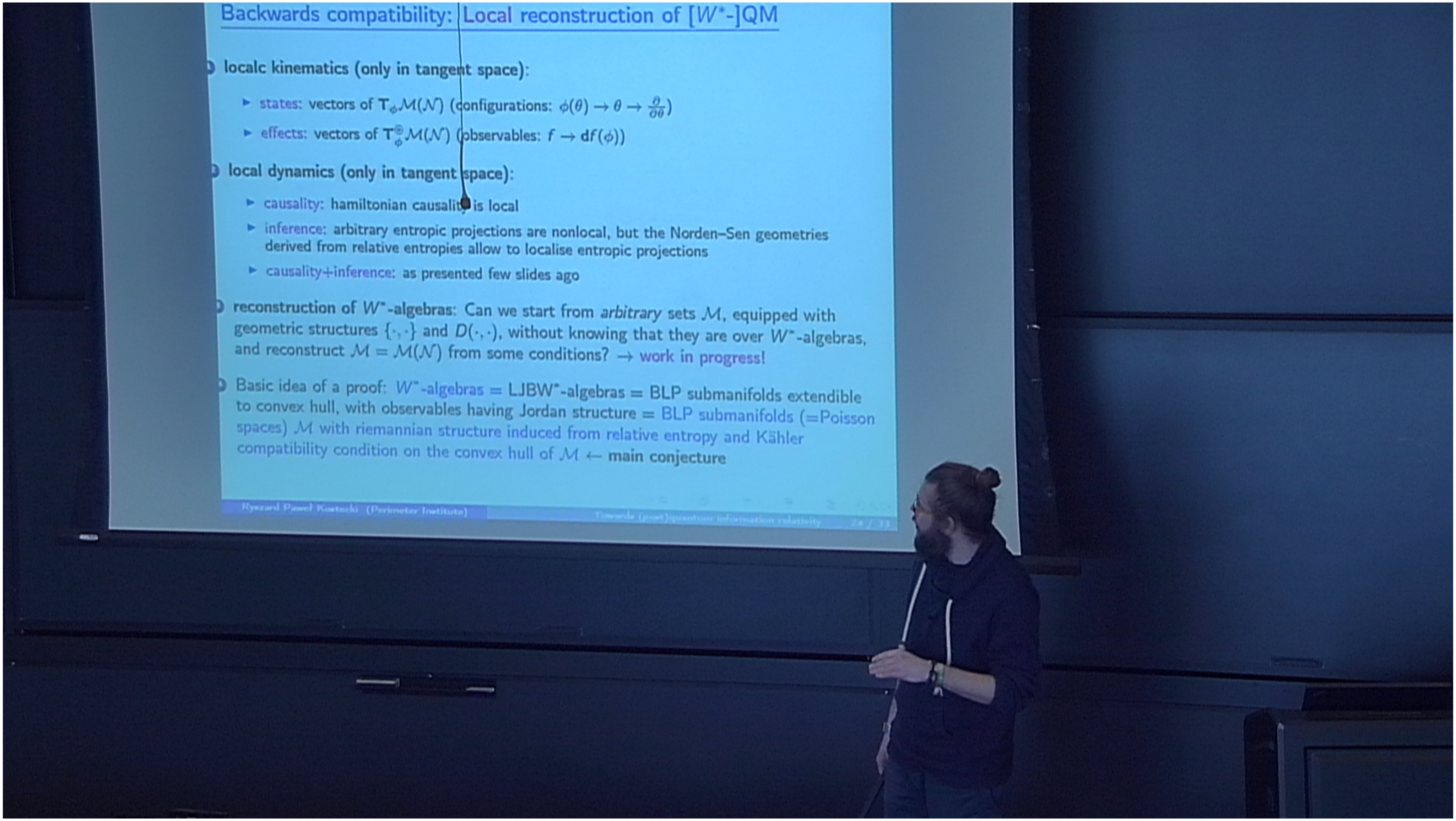
- If evaluated only on boundary pure states, and for $h(\phi) = \phi(\mathcal{H})$, it is known (Daubechies–Klauder'85, Anastopoulos–Savvidou'03) to be equal to $\left\langle \Omega(t = s), e^{-\mathrm{i}Hs}\Omega(t = 0) \right\rangle_{\mathcal{H}}$.

Backwards compatibility: Global reconstructions

1a. **(Global) Reconstruction of quantum mechanics:**
- $\mathcal{N}$: type I W^*-algebras
- $\mathcal{M}(\mathcal{N})$: normalised states
- D: $D_{1/2}$ or D_0
- $\{\cdot,\cdot\}$: generated by Banach Lie algebra $\mathcal{N}^{\mathrm{sa}}$
- observables: linear functions on $\mathcal{M}(\mathcal{N})$

2. **Reconstruction of probability theory:**
- $\mathcal{N}$: commutative algebras
- $\mathcal{M}(\mathcal{N})$: normalised states
- D: arbitrary (but for D_1 or D_0, and specific types of constraints, Bayes' and Jeffrey's rules are recovered)
- $\{\cdot,\cdot\}$: trivialises for commutative algebras
- observables: arbitrary or affine functions on $\mathcal{M}(\mathcal{N})$




Backwards compatibility: Local reconstruction of [W^*-]QM
localc kinematics (only in tangent space):
states: vectors of $\mathbf{T}_\phi\mathcal{M}(\mathcal{N})$ (configurations: $\phi(\theta) \to \theta \to \frac{\partial}{\partial\theta}$)
effects: vectors of $\mathbf{T}^{\circledast}_\phi\mathcal{M}(\mathcal{N})$ (observables: $f \to \mathbf{d}f(\phi)$)
local dynamics (only in tangent space):
causality: hamiltonian causality is local
inference: arbitrary entropic projections are nonlocal, but the Norden–Sen geometries derived from relative entropies allow to localise entropic projections
causality+inference: as presented few slides ago
reconstruction of W^*-algebras: Can we start from *arbitrary* sets $\mathcal{M}$, equipped with geometric structures $\{\cdot,\cdot\}$ and $D(\cdot,\cdot)$, without knowing that they are over W^*-algebras, and reconstruct $\mathcal{M} = \mathcal{M}(\mathcal{N})$ from some conditions? $\to$ work in progress!
Basic idea of a proof: W^*-algebras = LJBW*-algebras = BLP submanifolds extendible to convex hull, with observables having Jordan structure = BLP submanifolds (=Poisson spaces) $\mathcal{M}$ with riemannian structure induced from relative entropy and Kähler compatibility condition on the convex hull of $\mathcal{M}$ $\leftarrow$ main conjecture
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Towards (post)quantum information relativity
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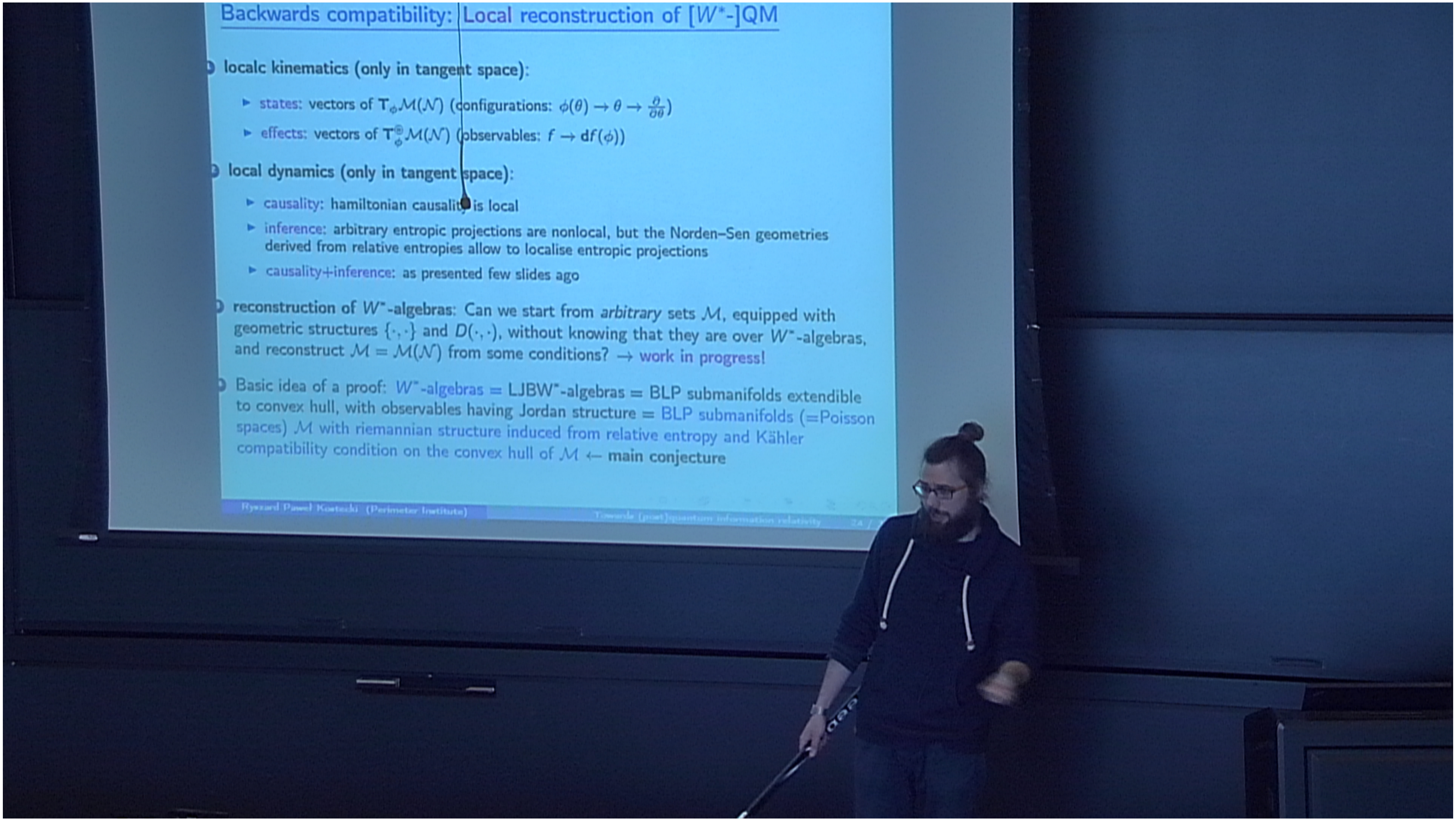
Backwards compatibility: Local reconstruction of [W*-]QM
localc kinematics (only in tangent space):
states: vectors of T_φM(N) (configurations: φ(θ) → θ → ∂/∂θ)
effects: vectors of T^⊛_φM(N) (observables: f → df(φ))
local dynamics (only in tangent space):
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causality+inference: as presented few slides ago
reconstruction of W*-algebras: Can we start from arbitrary sets M, equipped with geometric structures {·,·} and D(·,·), without knowing that they are over W*-algebras, and reconstruct M = M(N) from some conditions? → work in progress!
Basic idea of a proof: W*-algebras = LJBW*-algebras = BLP submanifolds extendible to convex hull, with observables having Jordan structure = BLP submanifolds (=Poisson spaces) M with riemannian structure induced from relative entropy and Kähler compatibility condition on the convex hull of M ← main conjecture
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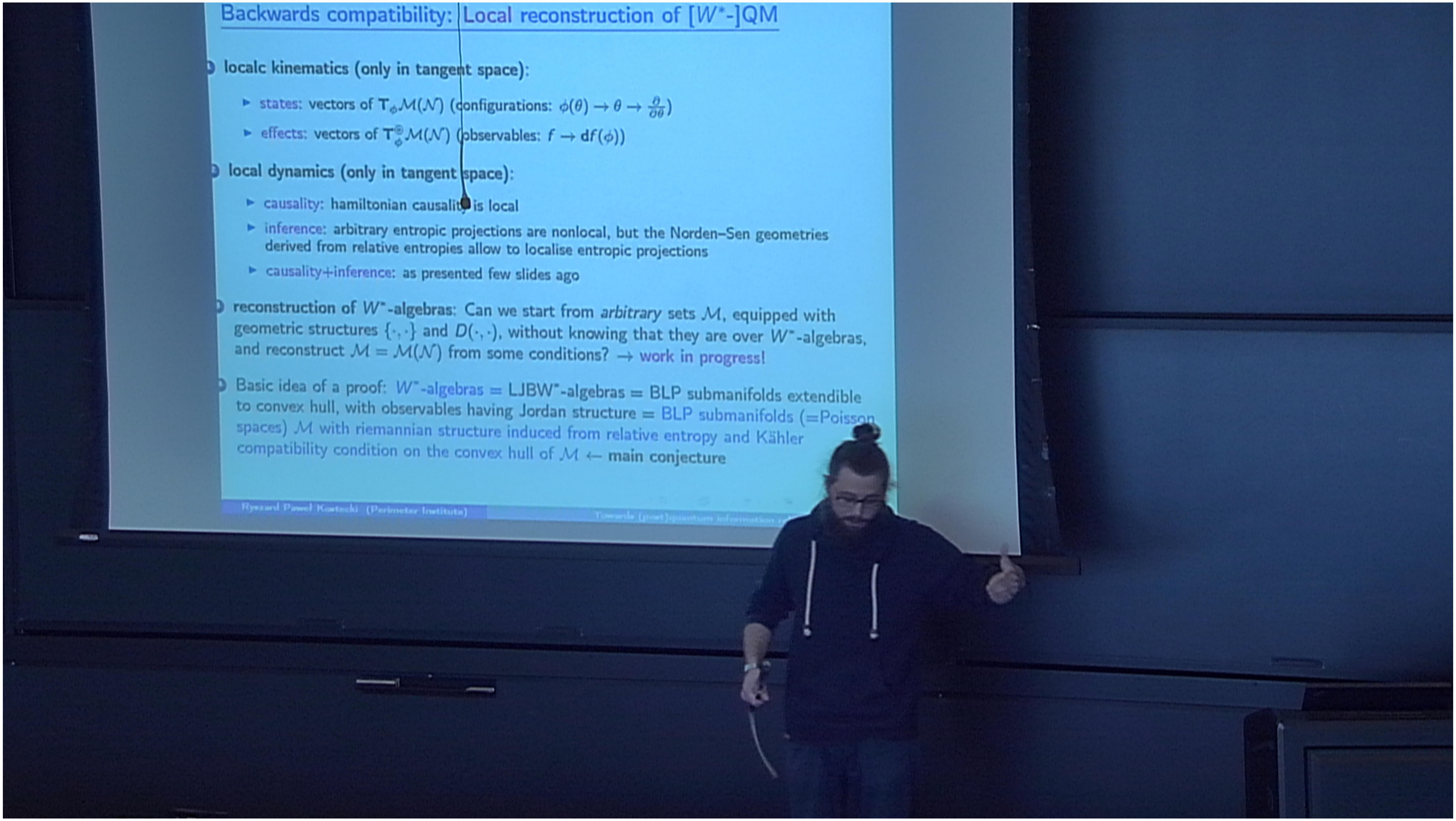
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Plan

1. **Nonlinear generalisations of quantum dynamics:**
   - Geometric structures on quantum state spaces $\rightarrow$ relative entropies & Poisson brackets
   - Lüders' rules $\rightarrow$ constrained relative entropy maximisations
   - Unitary evolution $\rightarrow$ nonlinear hamiltonian flows
2. **Geometric (post)quantum information foundations:**
   - Mathematical and physical principles
   - Global and local dynamics
   - Global and local reconstruction of QM
3. **Category-theoretic operational semantics:**
   - Adjointness in foundations, functorial localisation
   - Resource theories a la LdR–LK–RR
   - Beyond adjointness: local monad–comonad systems
4. **Towards [(post)quantum] local information relativity:**
   - From equlibrium to nonequilibrium space-time thermodynamics
   - Two-dimensional surfaces (θ, σ) and geometry in Klauder–Daubechies quantisation
   - Quantum dynamics of (θ, σ)-spaces

What is the predictive content of semantics of probability theory?

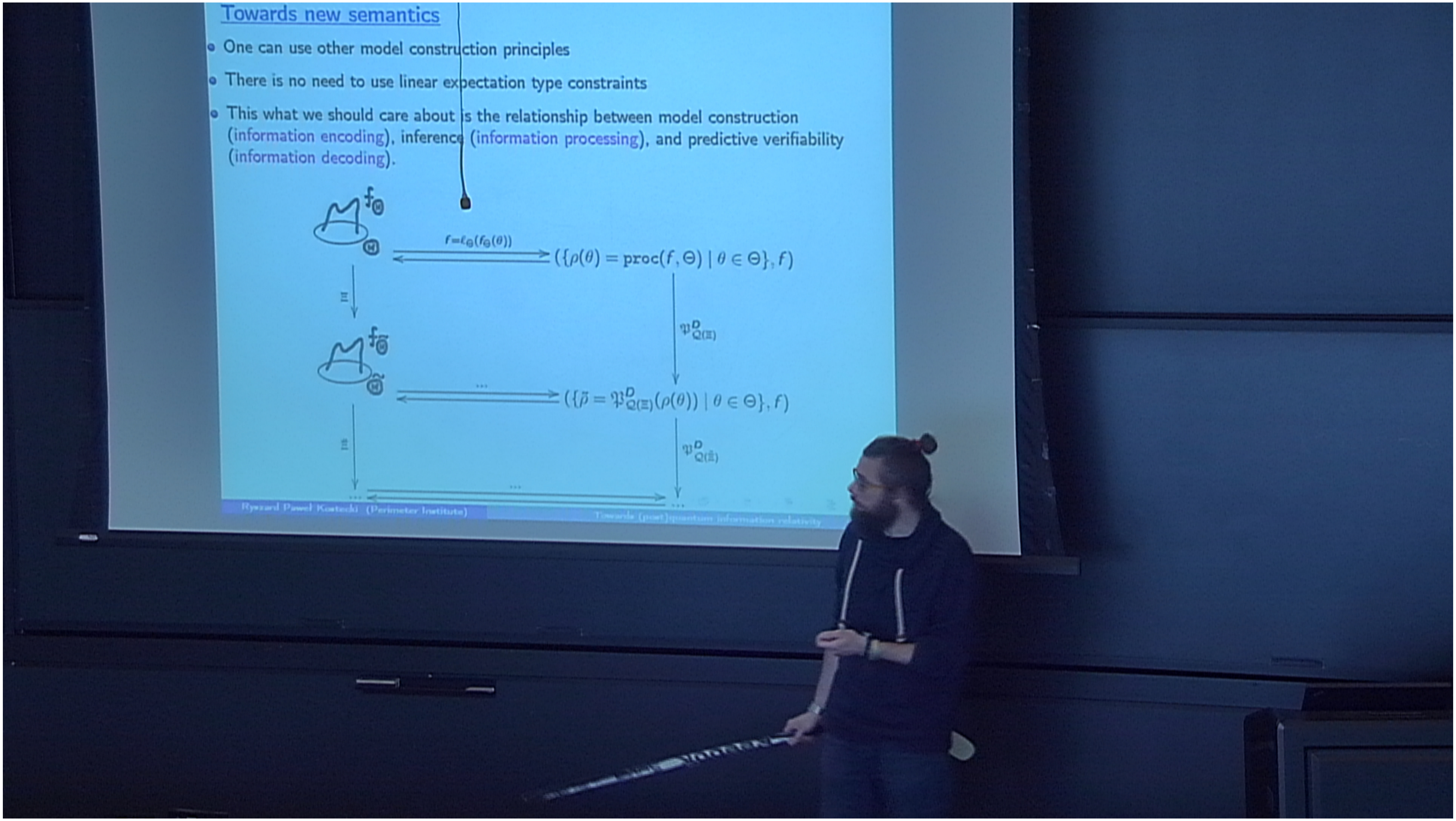
Consider:

- Θ: a space of possible configurations
- $\Theta \ni \theta = (\theta_1, \ldots, \theta_n)$: average values of n types of experimental configuration variables
- Ξ: a space of registrations

Example: **MaxEnt + entropic projections (or Bayes' rule) + prediction:**

$$\text{configurations \& registrations} \underset{\text{predictive verification}}{\overset{\text{model construction}}{\rightleftarrows}} \text{beliefs \& updatings}$$

$$\begin{tikzcd}
\Theta \arrow[r, shift left, "\theta_i = \mathrm{tr}_{\mathcal{H}}(\rho h_i)"] \arrow[d, "\Xi"'] & \{\rho(\theta) = \mathrm{e}^{-\sum_i \beta(\theta_i) h_i} \mid \theta \in \Theta\} \arrow[l, shift left] \arrow[d, "\mathfrak{P}^{D_1}_{\mathfrak{Q}(\Xi)}"] \\
\tilde{\Theta} \arrow[r, shift left, "\tilde{\theta}_i = \mathrm{tr}_{\mathcal{H}}(\tilde{\rho} h_i)"] \arrow[d, "\tilde{\Xi}"'] & \{\tilde{\rho} = \mathfrak{P}^{D_1}_{\mathfrak{Q}(\Xi)}(\rho(\theta)) \mid \theta \in \Theta\} \arrow[l, shift left] \arrow[d, "\mathfrak{P}^{D_1}_{\mathfrak{Q}(\tilde{\Xi})}"] \\
\hat{\Theta} \arrow[r, shift left, "\hat{\theta}_i = \mathrm{tr}_{\mathcal{H}}(\hat{\rho} h_i)"] & \{\hat{\rho} \mid \ldots\} \arrow[l, shift left]
\end{tikzcd}$$


Towards new semantics
One can use other model construction principles
There is no need to use linear expectation type constraints
This what we should care about is the relationship between model construction (information encoding), inference (information processing), and predictive verifiability (information decoding).
$(\{\rho(\theta) = \mathrm{proc}(f, \Theta) \mid \theta \in \Theta\}, f)$
$(\{\tilde{\rho} = \mathfrak{P}^{D}_{\mathcal{Q}(\Xi)}(\rho(\theta)) \mid \theta \in \Theta\}, f)$
Ryszard Paweł Kostecki (Perimeter Institute)