Title: Entanglement of spacetime

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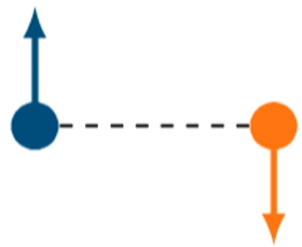
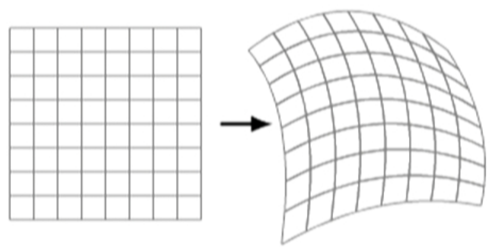
Abstract: <p>Entanglement is both a central feature of quantum mechanics and a powerful tool for studying quantum systems. Even empty spacetime is a highly entangled state, and this entanglement has the potential to explain puzzling thermodynamic properties of black holes. In order to apply the methods of quantum information theory to problems in gravity we have to confront a more fundamental question: what is a local subsystem, and what are its physical degrees of freedom? I will show that local subsystems in gravity come with new physical degrees of freedom living on the boundary, as well as new physical symmetries. These structures offer us new insight into how spacetime is entangled, and a new perspective on the problem of quantizing gravity.</p>

Quantum gravity

Quantum gravity must be consistent with principles of both quantum mechanics and general relativity.



Entanglement



General covariance

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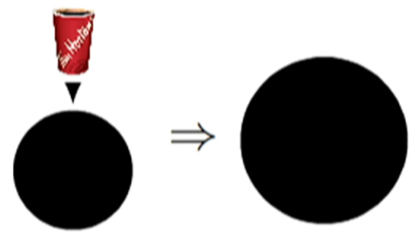
Black holes are thermodynamic systems

Thought experiment: Could we violate the second law of thermodynamics by throwing entropy down a black hole?
[Bekenstein 1972]

Black holes are thermodynamic systems, they radiate at the Hawking temperature
[Hawking 1975]



$$T_H = \frac{\hbar c^3}{8\pi G k_B} \frac{1}{M}.$$

When we throw things in, the black hole grows according to the first law of black hole thermodynamics:

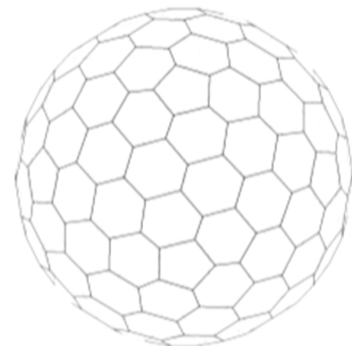
$$dE = T\, dS.$$

Where $E = Mc^2$.

Black holes have entropy

From the first law we infer that black holes have **Bekenstein-Hawking entropy**

$$S_{\mathsf{BH}} = \frac{A\,c^3}{4\,G\,\hbar}.$$



3×10^{69} bits/m^2

Usually entropy is the logarithm of phase space volume.

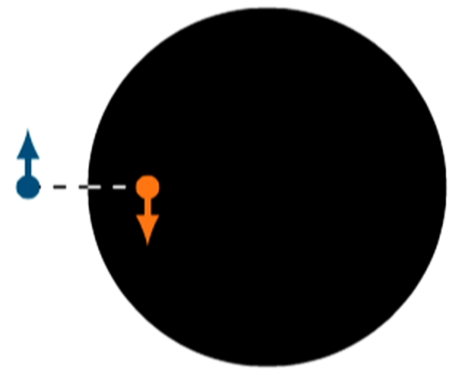
- What is the phase space?
- Why does S_{BH} scale like area, and not volume?

Entanglement

The inside of the black hole is entangled with its radiation.

This would naturally explain the area scaling of the entropy.

[Sorkin; Bombelli, Koul, Lee & Sorkin; Srednicki]



Entanglement plays many roles in theoretical physics:

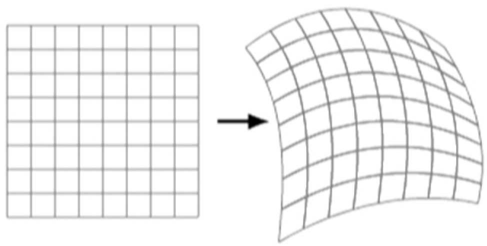
- A probe of bulk geometry in AdS/CFT. [Ryu & Takayanagi]
- A renormalization-group monotone. [Vidal, Latorre, Rico & Kitaev; Casini & Huerta]
- A probe of confinement in QCD. [Klebanov, Kutasov & Murugan; Lewkowycz]
- An order parameter for topological phases. [Levin & Wen; Kitaev & Preskill]

Entanglement & local symmetry

In all of these applications one encounters local symmetry.

- AdS/CFT — CFT is an $\mathrm{SU}(N)$ gauge theory.
- Renormalization — What can flow to the standard model?
- Confinement — QCD is an $\mathrm{SU}(3)$ gauge theory.
- Topological phases — Systems with emergent gauge symmetry.

Gravity also has a local symmetry: general covariance.



Problem: In theories with local symmetry the usual rules of entanglement don't apply.

A new approach to subsystems

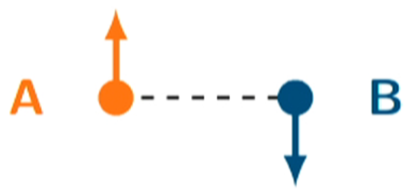
Local symmetries demand a new approach to subsystems.

Surface degrees of freedom
New states on the two-dimensional boundary of a subsystem.

Surface symmetry
A symmetry that determines how states are entangled.

Entanglement

Entanglement is information about a quantum system that is not reducible to information about its subsystems.



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\,|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle\, \right)$$

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Entanglement entropy

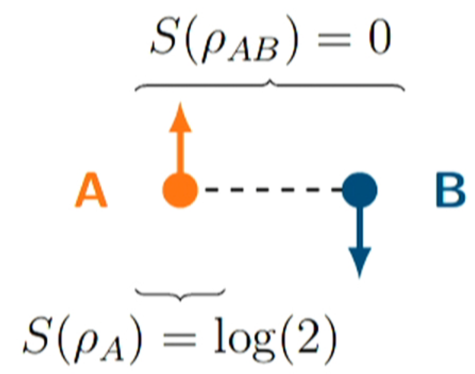
Lack of information is quantified with entropy.

For quantum states this is
the von Neumann entropy:

$$S(\rho) = -\operatorname{tr}(\rho \log \rho).$$

A pure state has no entropy,

$$S(|\psi\rangle\langle\psi|) = 0.$$



If AB is in a pure state, $S(\rho_A) = S(\rho_B) = S$.

S is called the **entanglement entropy**.

Subsystems can have more entropy than the whole system.

Symmetry determines entanglement

The singlet state $|\psi\rangle$ is invariant under rotations.

$$\frac{1}{\sqrt{2}}\left(\,|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle\, \right) = \frac{1}{\sqrt{2}}\left(\,|\nwarrow\rangle \otimes |\searrow\rangle - |\searrow\rangle \otimes |\nwarrow\rangle\, \right)$$

The reduced density matrices ρ_A and ρ_B are invariant.

This determines ρ_A and ρ_B, and hence $S = \log(2)$.

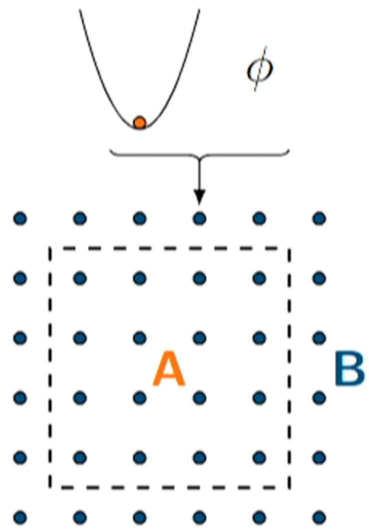
Entanglement of the vacuum

States in field theory — even the vacuum — are entangled.

On each lattice site we place
a harmonic oscillator ϕ.

The Hilbert space is a tensor product:

$$\mathcal{H} = \bigotimes_{\text{vertices } v} \mathcal{H}_v$$



The Hamiltonian includes interactions
that couple neighbouring oscillators.

The ground state exhibits vacuum fluctuations
that are correlated between regions A and B.

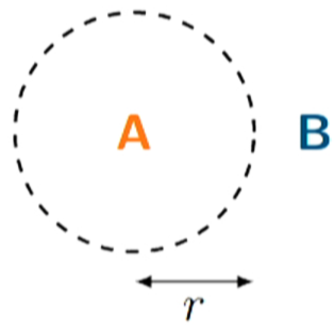
Entanglement scaling

Vacuum correlations are strongest at short distances.

Ground state entanglement naturally follows an area law.

In a conformal field theory,

$$S = \# \frac{r^2}{\epsilon^2} - 4a \log \frac{r}{\epsilon} + \dots$$



where a is a central charge.

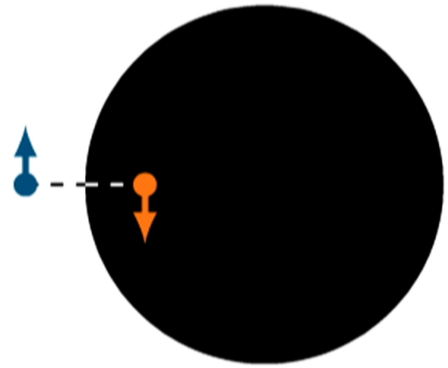
[Solodukhin; Casini, Huerta & Myers]

Entanglement entropy encodes universal information.

The area law

The area law term is divergent in field theory:

$$S = \# \frac{A}{\epsilon^2} + \dots$$

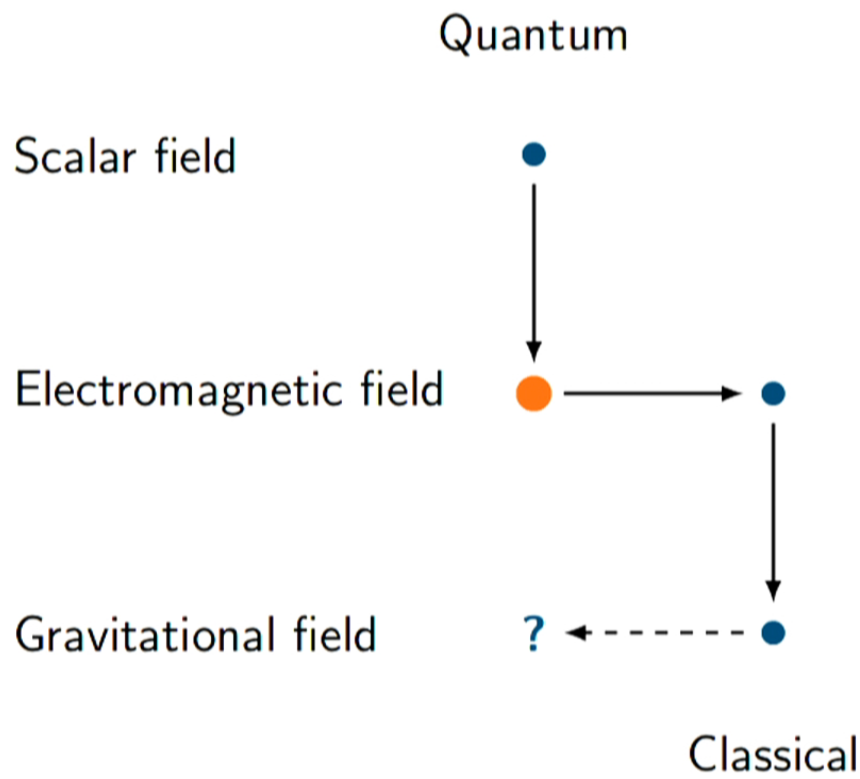


This term looks a lot like the Bekenstein-Hawking entropy:

$$S_{\mathsf{BH}} = \frac{Ac^3}{4G\hbar}.$$

The entanglement result is consistent with $G = 0$.

What happens when we turn on gravity?


Quantum
Scalar field
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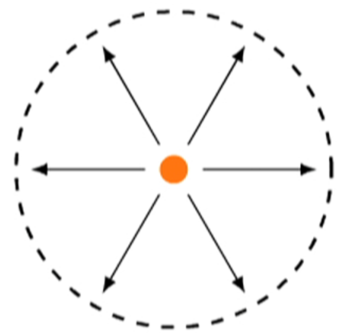
Electromagnetism

In a gauge theory, the degrees of freedom are non-local.

Electric field lines can only end on charges:

$$\nabla \cdot E = \rho.$$



A constraint equation.

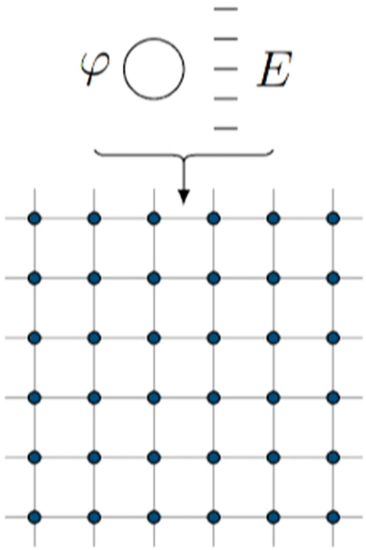
How do we divide these non-local degrees of freedom?

Lattice gauge theory

Electromagnetism has a more subtle structure than a scalar.

For each link, let φ be the phase picked up by a hopping electron:

$$\varphi \in \mathbb{C}, \qquad |\varphi| = 1.$$



Let E be the electric field,

$$E = n\,e, \qquad n \in \mathbb{Z}.$$

They are canonically conjugate

$$[\varphi, E] = \varphi.$$

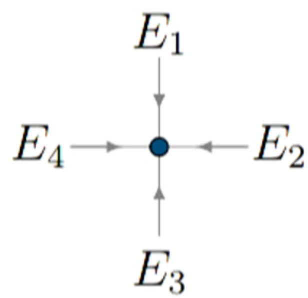
Each link of the lattice is a quantum system:
a free particle on a circle of radius $1/e$.

Gauss's law

Constraints restrict the Hilbert space.

In electrodynamics without charges, the constraint is Gauss's law:

$$\nabla \cdot E = 0.$$



Gauss's law generates a gauge transformation G_v at v.

On the lattice, we impose Gauss's law at each site:

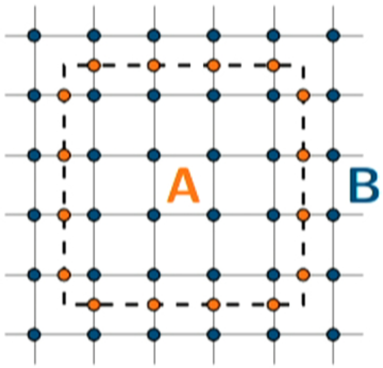
$$\mathcal{H} = \bigotimes_e \mathcal{H}_e \Big/ \bigotimes_v G_v \, .$$

Because of these constraints, $\mathcal{H}$ no longer factors.

Extended Hilbert space

How do we divide the degrees of freedom between two regions?

We split each boundary link in half, putting half in A and half in B.



Define the extended Hilbert space:
[WD 2011]

$$\mathcal{H}_A = \bigotimes_{e\in A} \mathcal{H}_e \Big/ \bigotimes_{v\in A} G_v \,.$$

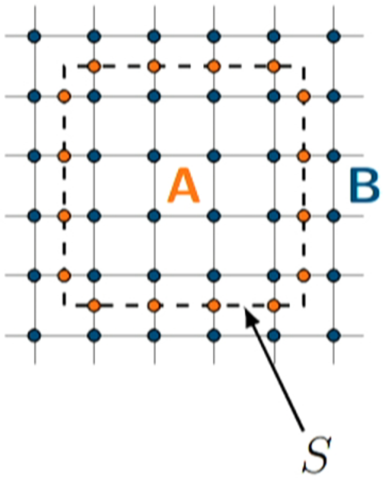
Where we impose Gauss's law at all interior vertices v.

$\mathcal{H}_A$ contains **surface degrees of freedom**: $(\varphi, E_\perp)$ at all points on the boundary.

Symmetry determines entanglement

The tensor product $\mathcal{H}_A \otimes \mathcal{H}_B$ contains unphysical states.

Gauss's law at points on S says that electric fields match on both sides.



The space of physical states is

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B / \mathcal{G}_S.$$

This amounts to matching $E_\perp$ at S.

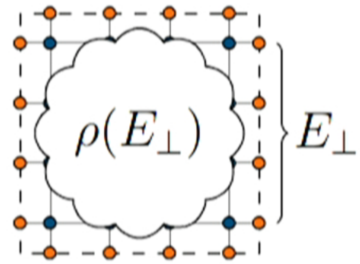
The symmetry tells us how to combine subsystems.

Reduced density matrix

States of $\mathcal{H}$ are invariant under the surface symmetry.

Hence ρ_A commutes with the electric fields normal to S.

We can simultaneously diagonalize $E_\perp$, and ρ_A is block diagonal in this basis:



$$\rho_A = \bigoplus_{E_\perp} p(E_\perp)\rho(E_\perp)$$

$p(E_\perp)$ is the probability distribution over different boundary conditions $E_\perp$.

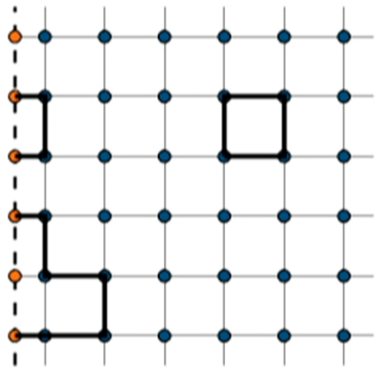
$\rho(E_\perp)$ is a density matrix with fixed boundary conditions.

Entanglement entropy

For gauge-invariant states the entropy simplifies: [WD 2011]

$$
\begin{aligned}
S &= -\operatorname{tr}(\rho_A \log \rho_A) \\
&= -\sum_{E_\perp} p(E_\perp) \log p(E_\perp) + \sum_{E_\perp} p(E_\perp) S(\rho(E_\perp)).
\end{aligned}
$$

The first term is the entropy of the surface degrees of freedom.



Surface degrees of freedom matter

Surface degrees of freedom make up $O(1)$ of the entropy.

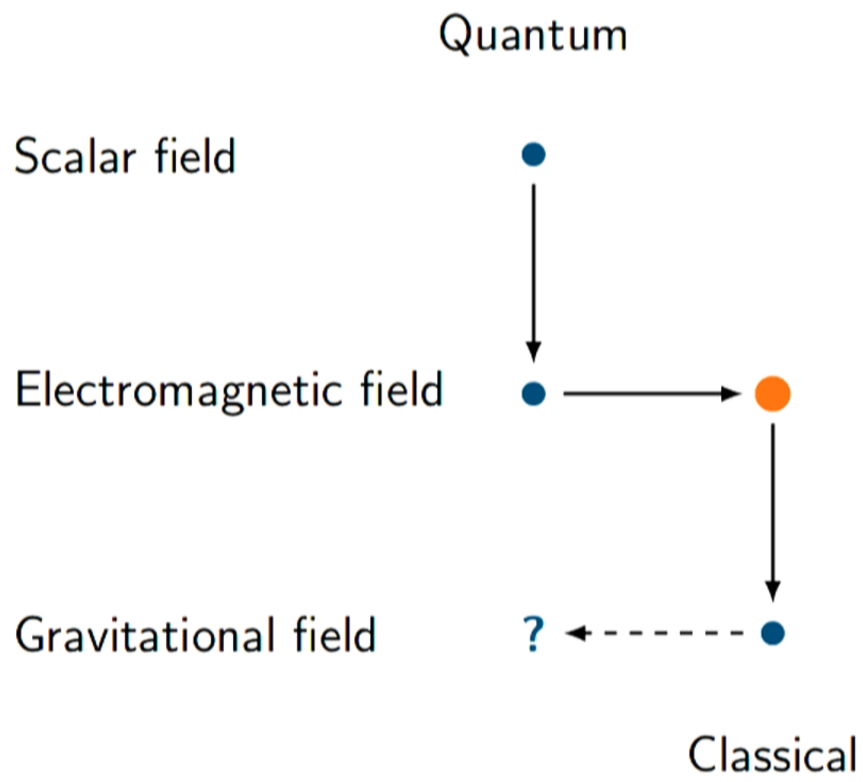
$$S = -\sum_{E_\perp} p(E_\perp) \log p(E_\perp) + \sum_{E_\perp} p(E_\perp) S(\rho(E_\perp)).$$

This result has been checked for many systems:

- The toric code [WD 2011]
- Electrodynamics in $2+1$ [Agon, Headrick, Jafferis, & Kasko 2013]
- Electrodynamics in $3+1$ [WD & Wall 2014]

The results generalizes to nonabelian theories [WD 2014]

- Quark-antiquark pairs [Lewkowycz & Maldacena 2013]
- Yang-Mills in $1+1$ [WD 2014, Gromov & Santos 2014]
- Yang-Mills in $2+1$ at weak coupling [Radičević 2015]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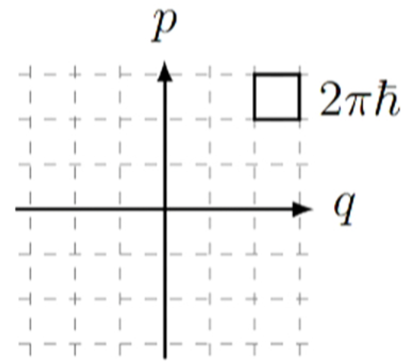
Phase space

The classical analog of a Hilbert space is a phase space.

The key structure is the **symplectic potential**

$$\Theta = p\,\delta q\,.$$



Θ encodes all the structure of classical mechanics:

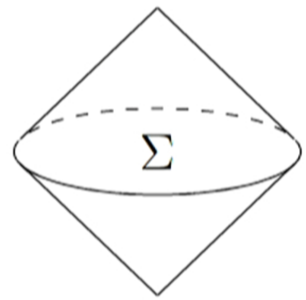
- Relates symmetries and conserved charges.
- Tells us what is physical and what is gauge.
- Determines the Poisson brackets ($\sim$ commutators).
- Gives the density of states in phase space.

Covariant phase space

We can associate a phase space to electrodynamics in a region.

This phase space is defined by the symplectic potential:

$$\Theta = \int_\Sigma d\Sigma_\mu \, \theta^\mu .$$



In electrodynamics we have

$$\theta^\mu = F^{\mu\nu} \, \delta A_\nu .$$

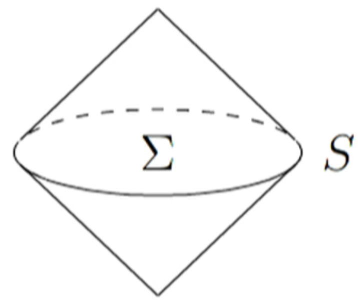
The vector potential is conjugate to the electric field.

Covariant phase space

The symplectic potential Θ must be gauge invariant.

Under a gauge transformation $A_\mu \to A_\mu + \partial_\mu \alpha$.

$$\theta^\mu = F^{\mu\nu}\,\delta A_\nu .$$
$$\theta^\mu \to \theta^\mu + \partial_\nu \left(F^{\mu\nu}\,\delta\alpha\right).$$



This is a total derivative,
hence a surface integral

$$\Theta \to \Theta + \oint_S dS\, E_\perp \delta\alpha .$$

We have to eliminate this anomalous term.

We **must not** set $E_\perp = 0$: we would undercount the entropy.

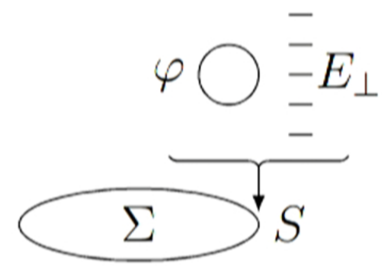
Extended phase space

Key idea: extend the phase space. [WD & Freidel 2016]

We introduce the $\varphi \in \mathrm{U}(1)$ to the phase space.

This allows us to define an extended symplectic potential:

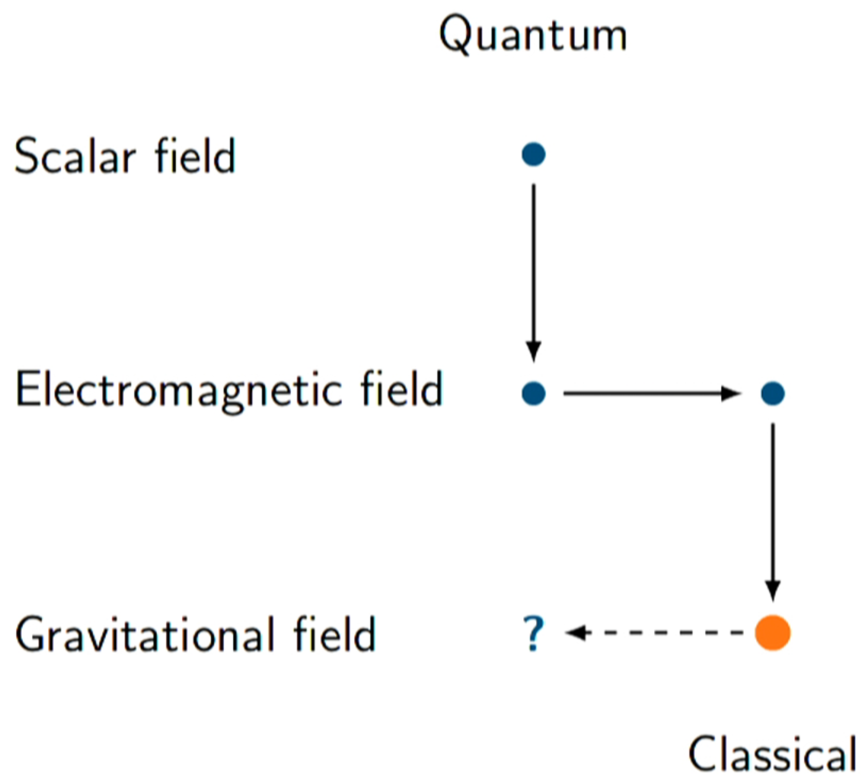
$$\Theta = \int_\Sigma d\Sigma_\mu \, \theta^\mu + \oint_S dS \; E_\perp \varphi^{-1} \delta\varphi.$$



Θ is determined by gauge invariance:
the boundary term cancels out
the anomalous term.

The **surface degrees of freedom** $(\varphi, E_\perp)$ are physical.

$E_\perp$ generates rotations of φ — the **surface symmetry**.



Quantum
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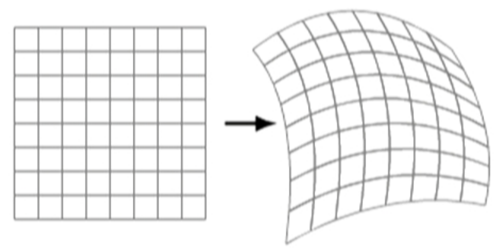
Gravity

Gravity is a theory with a local symmetry, general covariance.

This symmetry leads to constraint equations:
equations of motion with no time derivatives.

The constraints are responsible
for the force of gravity:

$$\nabla^2\Phi = 4\pi G\,\rho.$$



Regions of space are not independent subsystems.

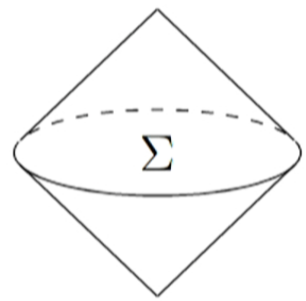
We can treat them the same way as in electrodynamics.

General relativity

The covariant canonical formalism extends naturally to gravity.

In general relativity the fundamental field is the metric $g_{\mu\nu}$.

In units where $16\pi G = 1$,
the symplectic potential is



$$\Theta = \int_\Sigma d\Sigma_\mu \, \theta^\mu.$$

Where the vector field θ is given by:
[Ashtekar; Crnković; Witten; Wald;...]

$$\theta^\mu = (g^{\mu\alpha} g^{\nu\beta} - g^{\mu\nu} g^{\alpha\beta}) \nabla_\nu \delta g_{\alpha\beta}.$$

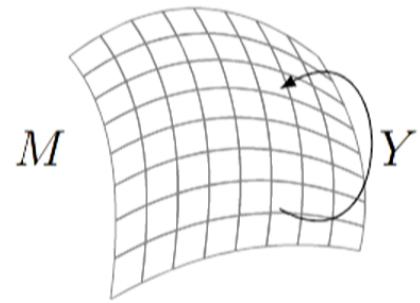
General covariance

The symplectic potential Θ is not diffeomorphism invariant.

Let $Y : M \to M$ be a diffeomorphism.

Under this symmetry, θ transforms as:

$$\theta^\mu \to Y^*(\theta^\mu + \partial_\nu Q^{\mu\nu}[\delta_Y]).$$



Q is Wald's Noether charge: [Wald 1993]

$$Q_{\mu\nu}[\xi] = \partial_\mu \xi_\nu - \partial_\nu \xi_\mu.$$

There are two sources of non-invariance:

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- There is a boundary term that depends on δ_Y.

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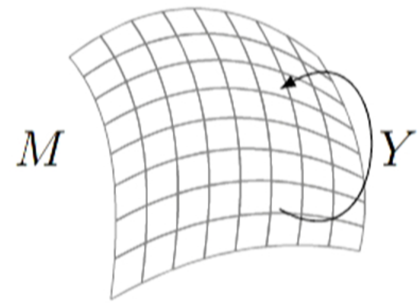

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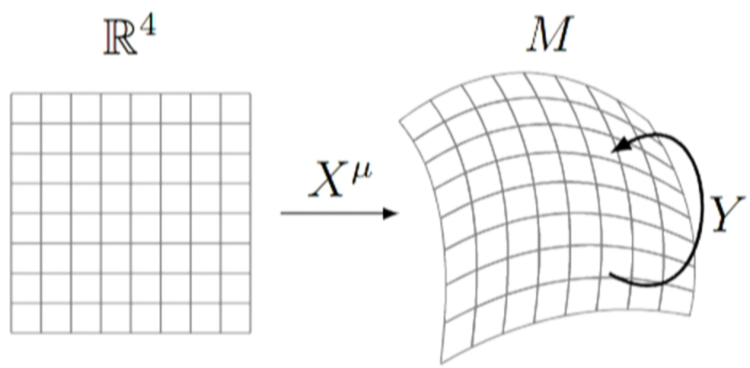
Extended phase space

In gravity, a choice of gauge is a choice of coordinates.

Introduce $X^\mu : \mathbb{R}^4 \to M$ to the phase space [WD & Freidel 2016]



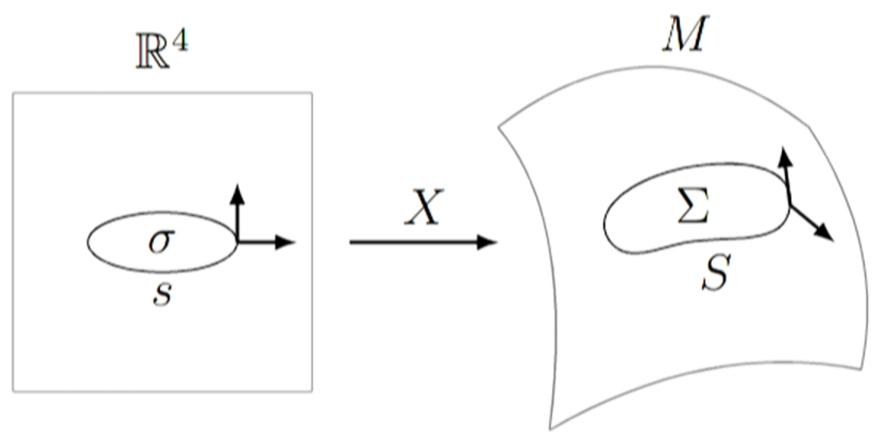
X^μ is the gravitational analog of φ.

Extended phase space

Using X we define the extended symplectic potential:

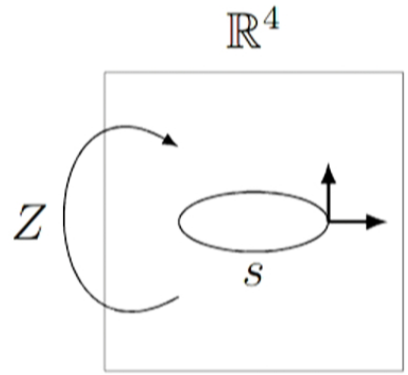
$$\Theta = \int_{X(\sigma)} d\Sigma_\mu \, \theta^\mu + \oint_{X(s)} dS_{\mu\nu} \, Q^{\mu\nu}[\delta_X].$$

Θ is determined by diffeomorphism invariance.



Surface symmetry

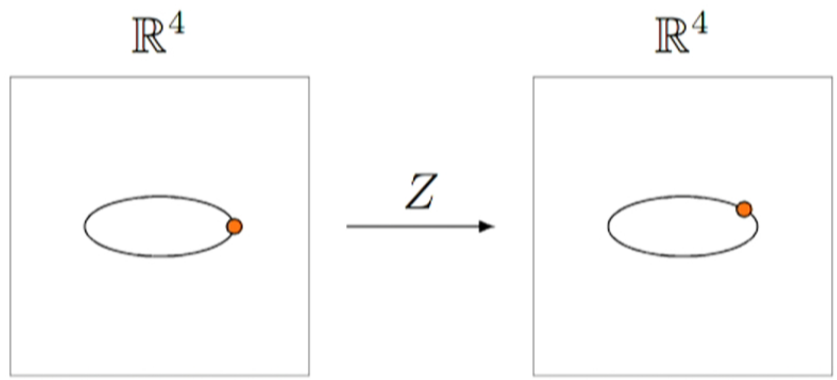
The **surface symmetries** are diffeomorphisms $Z : \mathbb{R}^4 \to \mathbb{R}^4$ that change s and its frame.



There are three classes of symmetries.

Surface diffeomorphisms

Surface diffeomorphisms leave the entangling surface invariant, but move the points around.

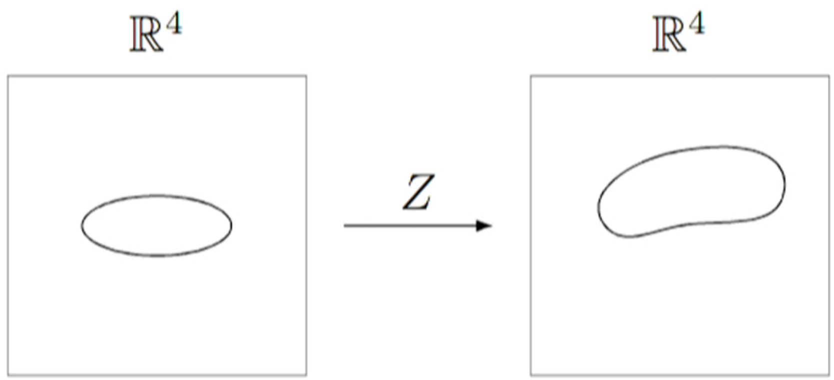


Infinitesimal surface diffeomorphisms are vector fields such that

$$W_\perp|_S = 0 \qquad \text{and} \qquad W_\parallel|_S \neq 0.$$

Surface translations

Surface translations move the entangling surface.



Infinitesimal surface translations are vector fields such that

$$W_\perp|_S \neq 0.$$

They are not canonical symmetries of the phase space; there can be flux of states through the boundary.

Surface symmetry algebra

The generators are precisely encoded in the geometry of S:

- Surface diffeomorphisms are generated by the curvature of the normal connection:

$$\mathsf{Diff}(S).$$

- Surface boosts are generated by the normal metric:

$$\mathrm{SL}(2,\mathbb{R}).$$

The Casimir is the area element.

These are the **surface degrees of freedom** analogous to $E_\perp$.

They generate a nonabelian algebra: [WD & Freidel 2016]

$$\mathsf{Diff}(S) \ltimes \mathsf{SL}(2,\mathbb{R})^S.$$

The geometry of the boundary is noncommutative.

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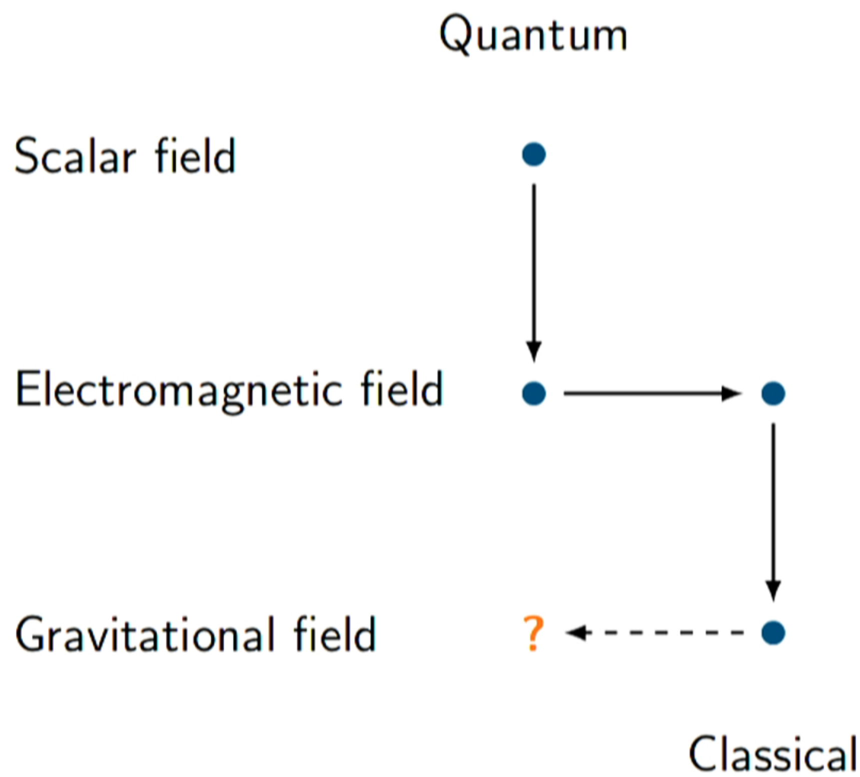
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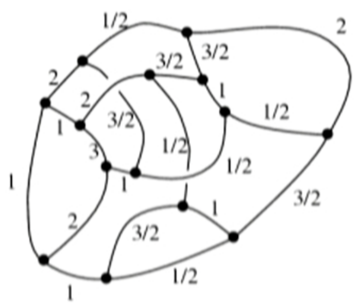
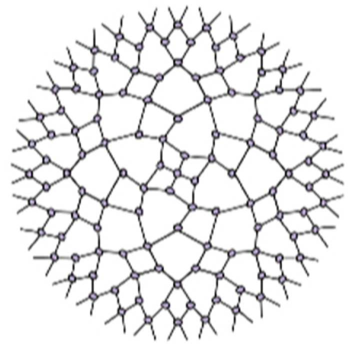
Building blocks of space

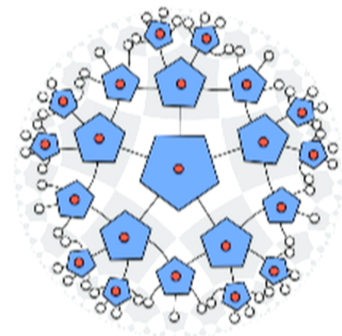
Space is a network of entangled subsystems.



[Rovelli & Smolin]



[Vidal]



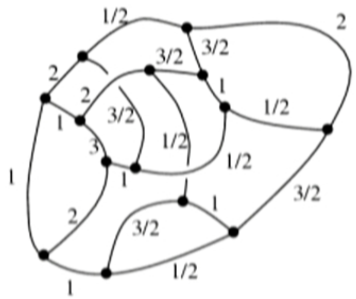
[Pastawski, Yoshida, Harlow, & Preskill]

What are the subsystems for gravity?

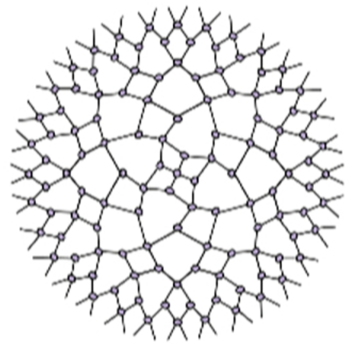
How are they entangled?

Building blocks of space

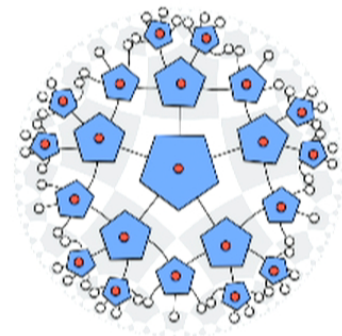
Space is a network of entangled subsystems.



[Rovelli & Smolin]



[Vidal]



[Pastawski, Yoshida, Harlow, & Preskill]

What are the subsystems for gravity?

How are they entangled?

Quantization

We have constructed classical subsystems, the next step is to quantize them.

To quantize a system, first study representations of its symmetries.

We have a huge symmetry group

$$\mathsf{Diff}(S) \ltimes \mathsf{SL}(2,\mathbb{R})^S.$$

What can we say about its representations?

Quantum surface symmetries

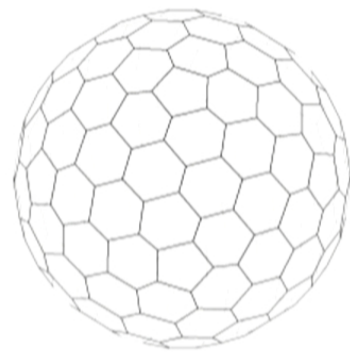
The surface symmetry is a semidirect product, like Poincaré.
[Wigner 1939]

Poincaré	Surface symmetry
$\mathrm{SO}(1,3) \ltimes \mathbb{R}^4$	Diff$(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S$
Translations	Surface boosts
4-momentum	Area element
Mass	Total area
Little group $\mathrm{SO}(3)$	**A**rea-**P**reserving **D**iffeomorphisms APD(S)
Spin	Irreducible representation of APD(S)

Representations labelled by **area** and transform under **APD**.

Quantum phase space

What are the area-preserving diffeomorphisms?



APD(S^2) is the symmetry group of a classical phase space: a sphere!

Representations are quantizations of this phase space.

The size of a Planckian cell is determined by Θ: the Planck area $\sim G\hbar/c^3$.

Is the horizon quantized just like a phase space?

Conclusions

- Entanglement is everywhere — even empty space.
- Local symmetries demand a new approach to subsystems:
  Surface degrees of freedom
  Surface symmetry
- Surface symmetry determines how states are entangled.
- Surface symmetry is present in classical gravity, and offers:
  A definition of local subsystems,
  A way to understand entropy of black holes, and
  A new perspective on quantizing gravity.

Thank you!

Surface symmetry generators

The generators of surface-preserving symmetries are geometric.

In coordinates around S the metric takes the form

$$\mathrm{d}s^2 = h_{ij}\mathrm{d}x^i\mathrm{d}x^j + q_{\mu\nu}\left(\mathrm{d}\sigma^\mu - A_i^\mu \mathrm{d}x^i\right)\left(\mathrm{d}\sigma^\nu - A_j^\nu \mathrm{d}x^j\right)$$

The generators are given by

$$H_w = \int_S \left(H_i{}^j(\partial_j W_\perp^i) + W_\parallel^\mu F_\mu\right)$$

The boost generator is the conformal normal metric:

$$H_i{}^j = \frac{\sqrt{q}}{\sqrt{|h|}} h_{ik}\epsilon^{kj}$$

The diffeomorphism generator is the curvature of A_i^μ:

$$F^\mu = \frac{\sqrt{q}}{\sqrt{|h|}}\left(\partial_0 A_1^\mu - \partial_1 A_0^\mu + [A_0, A_1]^\mu\right)$$

Surface symmetry algebra

The relation $\{H_v, H_w\} = H_{[v,w]}$ gives the Poisson brackets:

$$\begin{aligned}
\{H_i{}^j(\sigma), H_k{}^l(\sigma')\} &= (\delta_i^l H_k{}^j - H_i{}^l \delta_k^j)(\sigma)\delta^{(D-2)}(\sigma,\sigma'),\\
\{F_\mu(\sigma), F_\nu(\sigma')\} &= F_\mu(\sigma')\partial_\nu \delta^{(D-2)}(\sigma,\sigma') - F_\nu(\sigma)\partial'_\mu \delta^{(D-2)}(\sigma,\sigma'),\\
\{H_i{}^j(\sigma), F_\mu(\sigma')\} &= H_i{}^j(\sigma')\partial_\mu \delta^{(D-2)}(\sigma,\sigma').
\end{aligned}$$

These realize the surface symmetry group

$$\mathcal{G} = \text{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S$$

The $\mathrm{SL}(2,\mathbb{R})$ Casimir is $\det H = -\det q$.

The normal metric breaks $\mathrm{SL}(2,\mathbb{R})$ to a boost subgroup
(*c.f.* Carlip & Teitelboim)