Title: P-adic Integers and Quantum Reality: Towards a realistic locally causal theory of fundamental physics.

Date: May 04, 2016 02:00 PM

URL: http://pirsa.org/16050016

Abstract: <div>Almost the first thing we learn as human beings is a sense of spatial awareness: the smaller the Euclidean distance between two objects, the closer they are. As adults, we apply this deeply held intuition to state space. In particular, as philosopher David Lewis made explicit in his seminal 1973 paper on Causation, we presume that one counterfactual world is closer to reality than another if this world resembles reality more than does the other. This intuition has guided the development of physical theory over the years. However, I will argue that our intuition is letting us down very badly. Motivated by results from nonlinear dynamical systems theory, I will argue that the so-called p-adic metric provides a much more physically meaningful measure of state-space distance than does the Euclidean metric, and moreover that the set of p-adic integers, for large p, provides the basis for constructing a realistic, locally causal description of quantum physics which is neither fine tuned nor violates experimenter free will, the Bell theorem notwithstanding. Indeed, using the p-adic metric in state space, I assert that experimenters (from Aspect onwards) are not actually testing the Bell inequalities at all - not even approximately! A description of cosmological state space based on the set of p-adic integers suggests a new geometric route to the unification of quantum and gravitational physics, consistent with general relativity.</div>

P-adic Integers and Quantum Reality:

Towards a Realistic Locally-Causal Theory of Fundamental Physics

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The p-adic "loophole"

violating realism, local causality or experimenter free will.

- Experimenters, from Aspect onwards, are not testing the Bell

- The reason why we have been fooled into believing otherwise is that we have misapplied a deeply held intuition about the notion of "closeness".

Our first skill – spatial awareness

Produces a deeply held intuition: that closeness is synonymous with smallness of Euclidean distance

From number theory, there is another inequivalent way to define distance.

Where this intuition is misapplied

- David Lewis – *Causation* (1974): “We may say that one world is closer to actuality than another if the first resembles our actual world more than the second does.”

- John Bell – *Free Variables and Local Causality (1995)*: The choice of possible setting of polarisers in an EPR expt depends on the oddness or evenness of the millionth decimal place of the input variable for a pseudo-random number generator: "But this peculiar piece of information is unlikely to be the vital piece for any distinctively different purpose, i.e. it is otherwise rather useless."

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PAPER • OPEN ACCESS

The lesson of causal discovery algorithms for quantum correlations: causal explanations of Bell-inequality violations require fine-tuning

Christopher J Wood[1,2] and Robert W Spekkens[3]

Published 3 March 2015 •
New Journal of Physics, Volume 17, March 2015

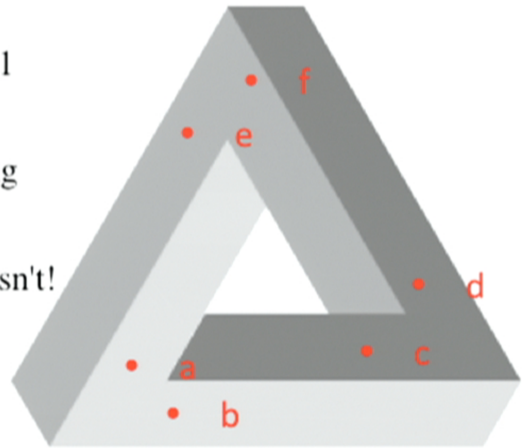
But the notion of fine-tuning is metric dependent. The Wood/Spekkens theorem holds for Euclidean but not p-adic metrics.

Analogy

If $D_{3D}(a,b) \ll 1$, $D_{3D}(c,d) \ll 1$

$\Rightarrow D_{3D}(e,f) \gg 1$

We were fooled into believing the distance between e and f was small, when in fact it wasn't!



Impossible to understand wrt a 2D Euclidean metric

Easy to understand wrt a 3D Euclidean metric.

Cauchy Sequences

$\{1, 1.4, 1.41, 1.414, 1.4142, 1.41421\ldots\}$

is a Cauchy sequence relative to the Euclidean metric

$|a-b|,\ a,b \in \mathbb{Q}$

$\{1,\ 1+2,\ 1+2+2^2,\ 1+2+2^2+2^3,\ 1+2+2^2+2^3+2^4\ldots\}$

is a Cauchy sequence relative to the metric $|a-b|_2$ where for $a-b \neq 0$

$|a-b|_p = p^{-\mathrm{ord}_p(a-b)}$

and

$\mathrm{ord}_p x =$ the highest power of p that divides x, if $x \in \mathbb{Z}$ (generalises for $x \in \mathbb{Q}$)

E.g.

$|(1+2+2^2) - (1+2)|_2 = 2^{-2} = 1/4,$

$|(1+2+2^2+2^3) - (1+2+2^2)|_2 = 2^{-3} = 1/8$

These metrics provide two inequivalent ways of completing the field of rational numbers $\mathbb{Q}$:

$|a-b|\ : \mathbb{Q} \Rightarrow \mathbb{R}$ consistent with and applicable to Euclidean Geometry

$|a-b|_p\ : \mathbb{Q} \Rightarrow \mathbb{Q}_p$ consistent with and applicable to Fractal Geometry

In base 2

$a = \ldots 1011101.$

is a 2-adic integer ($\in \mathbb{Z}_2$), whilst

$b = \ldots 1011101.000001$

is a 2-adic number ($\in \mathbb{Q}_2$).

$$|b - a|_2 = |0.000001|_2 = 2^6$$

Can add and multiply on $\mathbb{Z}_p$ (an integral domain);

and can do calculus, Lie group theory and Fourier analysis

in $\mathbb{Q}_p$ (a field) if p is prime.

P-adic Integers and Cantor Sets

$$C_2 = \bigcap_{k \in \mathbb{N}} C_2(k)$$

Now

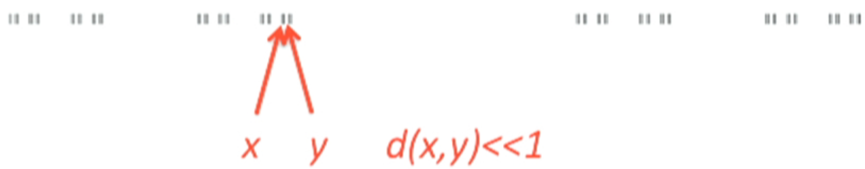
$$F \;:\; \sum_{k=0}^{\infty} a_k 2^k \leftrightarrow \sum_{k=0}^{\infty} \frac{2a_k}{3^{k+1}} \quad a_k \in \{0,1\}$$

is a bijection between $\mathbb{Z}_2 \subset \mathbb{Q}_2$ and $\mathrm{C}_2 \subset [0,1]$.

More generally, $F : \mathbb{Z}_p \leftrightarrow \mathrm{C}_p$. For C_p, divide the unit interval into $2p-1$ equal subintervals and remove every second open subinterval leaving p pieces.

The metric $|a-b|_p$ on $\mathbb{Z}_p \leftrightarrow d_p(x,y)$ on C_p.

The Key Physical Point



- If $x, y \in C_p$ and $d(x,y) \ll 1$, then $d_p(x,y) \ll 1$

- However, if $x \in C_p$ and $y \in \mathbb{Q} \notin C_p$ then $d_p(x,y) \geq p$.

- If $x \in C_p$ and $y \notin C_p$ then from the p-adic perspective, x **is not close to** y even if $d(x,y) \ll 1$

P-adics and Fundamental Physics
Why?

5 Reasons.

1. Andrew Wiles personal communication 2015.

“We [number theorists] tend to work almost as much p-adically as with the reals or complexes nowadays, and in fact it is usually best to consider all at once.”

If number theorists, why not physicists?

2. Ostrowsky's Theorem

In number theory, every non-trivial absolute value on the rational numbers is equivalent to either the usual real absolute value or to a p-adic absolute value.

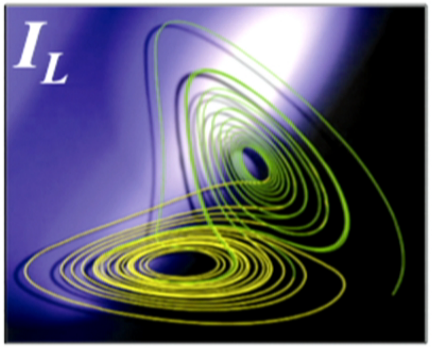
Why should Nature only make use of one of these, and not the other?

3. Fractal Invariant Sets are Generic in Nonlinear Dynamical Systems Theory (Classical Physics)

E.g.

$$\dot{X} = -\sigma X + \sigma Y$$
$$\dot{Y} = -XZ + rX - Y$$
$$\dot{Z} = XY - bZ$$

Non-computable



Locally

$$I_L = \mathbb{R} \times (\text{Cantor Set})$$

The Cosmological Invariant Set Postulate

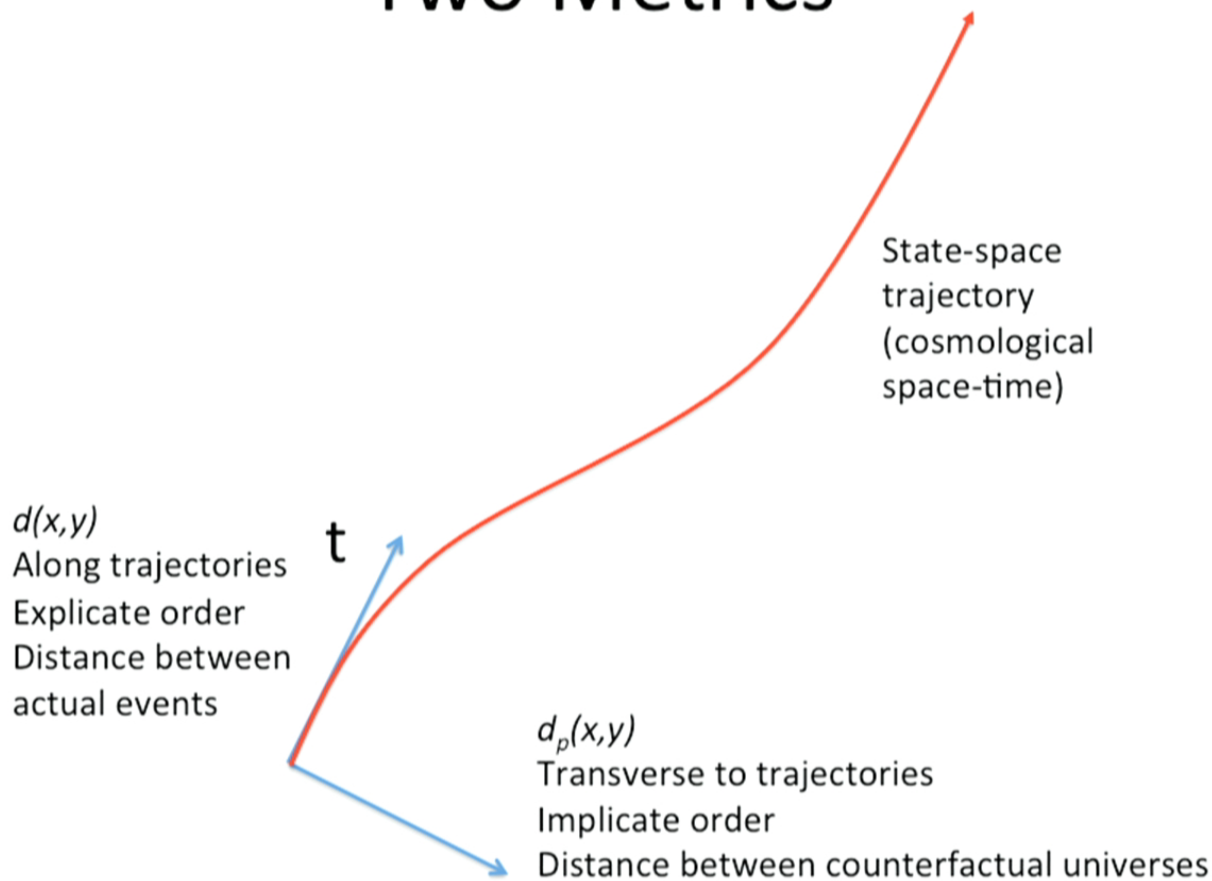
Contemporary Physics (Dennis Sciama Memorial Lecture), 2014
http://arxiv.org/abs/1605.01051

1. The universe U is evolving precisely on a fractal invariant subset I_U in U's state space.

2. I_U is locally of the form $I_U = \mathbb{R} \times C_p$, $p=2^N+1$, and where C_p is homeomorphic to $\mathbb{Z}_p$.

3. The most primitive expression of the laws of physics is a description of the geometry of I_U.

Two Metrics


State-space
trajectory
(cosmological
space-time)
t
$d(x,y)$
Along trajectories
Explicate order
Distance between
actual events
$d_p(x,y)$
Transverse to trajectories
Implicate order
Distance between counterfactual universes

4.Use of the p-adic metric in state space provides a locally causal realistic resolution of the Bell Theorem which is not fine tuned.

5. Use of the p-adic metric in state space provides a completely new perspective on how quantum and gravitational physics might be synthesised.

A locally causal realistic resolution of the Bell Theorem (without fine tuning) in 11 steps

1. Bell Inequality

The Bell inequality

(1) $|C(\theta)-C(\phi)|-C(\theta-\phi)\leq 1$

where $C(\theta)$ denotes a correlation between entangled particle pairs.

In demonstrating that a standard hidden-variable model $Sp(\lambda;\theta_A)$ satisfies the Bell inequality we $\therefore$ consider the three sets of pairs for given λ:

- $Sp(\lambda;\ \theta_A),\ Sp(\lambda;\ \theta_B),\ \theta=\theta_A-\theta_B \Rightarrow C(\theta)$
- $Sp(\lambda;\ \phi_A),\ Sp(\lambda;\ \phi_B),\ \phi=\phi_A-\phi_B \Rightarrow C(\phi)$
- $Sp(\lambda;\ \theta_A-\phi_A),\ Sp(\lambda;\ \theta_B-\phi_B) \Rightarrow C(\theta-\phi)$

(1) and the CHSH generalisation is seemingly violated experimentally for a range of values θ,ϕ.

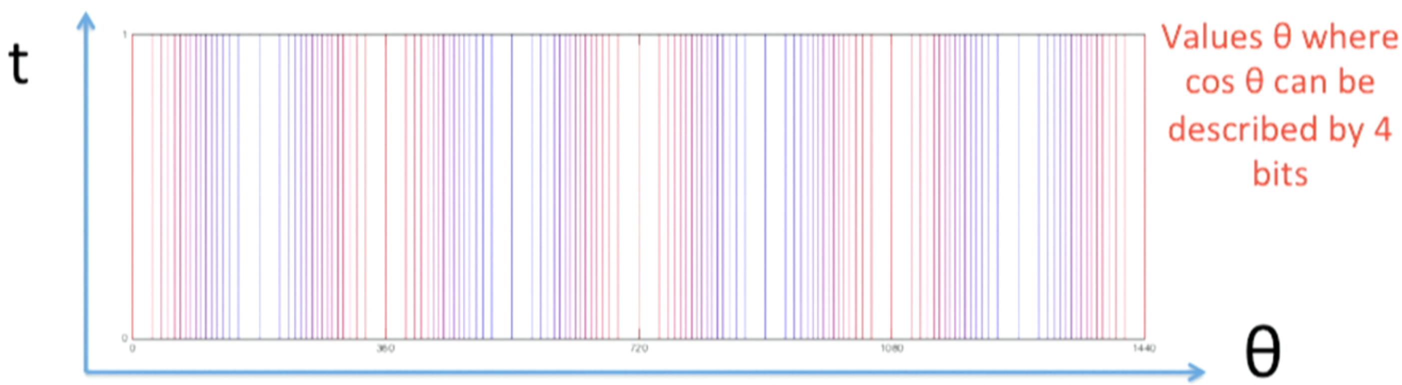
2. Correlation

- In Quantum Theory, $C(\theta) = -\cos\theta$.
- In Invariant Set Theory (IST) $C(\theta) = -\cos\theta$

providing $\cos\theta$ is describable by finite N bits, otherwise $C(\theta)$ is undefined.

In IST, $I_U \sim \mathbb{R} \times C_p$ where $p = 2^N + 1$.

Below we will assume $N \ggg 1$



3. Finite Precision

Alice and Bob's apparatuses have finite precision.

In IST, Alice and Bob can only control the leading M bits of θ_A, θ_B, respectively. M can be as big as you like as long as N is bigger still.

E.g. $\cos\theta = .1011010100100100\ldots0101$

$O(M)$

N

4. Holistic but not Nonlocal

The constraint that $\cos\theta$ is describable by finite N bits is not encodable in the formula $Sp(\lambda;\theta_A)$ for a local hidden variable theory, but does not violate local causality. Why?

Since Alice and Bob's choices only determine the leading $O(M)$ bits of $\cos\theta$, then the event

- E_A : Alice sets the orientation of her apparatus

can still allow Bob complete freedom to set his $O(M)$ bits without violating the condition that $\cos\theta$ is describable by N bits. Hence E_A is independent of the event

- E_B : Bob sets the orientation of his apparatus

and *vice versa*, when E_A and E_B are spacelike separated.
Alice and Bob can be treated as independent free agents.
IST is holistic **but not nonlocal**.

5. Number Theory

$\cos(\theta-\phi)=\cos\theta\cos\phi+\sin\theta\sin\phi$

Let $\cos\theta=\pm n_1/2^{N-1}, n_1\in\mathbb{Z}, 0\le n_1<2^{N-1}$.

Is $\sin\theta=\pm n_2/2^{N-1}, n_2\in\mathbb{Z}, 0\le n_2<2^{N-1}$?

Suppose yes. Since $\cos^2\theta+\sin^2\theta=1$ then

$${n_1}^2+n_2^2=2^{2N-2}$$

However, by the Euclidean theory of Pythagorean triples, there are no non-zero solutions to $n_1^2+n_2^2=2^{2N-2}$.

Hence in general $\sin\theta$ is not describable by finite N bits.

Similarly, if $\cos\phi$ is describable by N bits, in general $\sin\phi$ is not.

Hence, in general, for independent θ and ϕ, $\cos(\theta-\phi)$ is not describable by N bits

6. Fine Tuned?

Hence, by number theoretic properties of the (co)sine function, if
$U_\theta \in I_U$ and $U_\phi \in I_U$ then $U_{\theta-\phi} \notin I_U$.
Hence IST violates $|C(\theta)-C(\phi)|-C(\theta-\phi) \le 1$ because one of the correlation functions is always undefined.

However, $\exists$ an angle η where $\cos\eta$ is describable by N bits and agrees with $\cos(\theta-\phi)$ to N bits. Hence, by intuition, there should exist a world U_η very close to $U_{\theta-\phi}$ such that

$$|C(\theta)-C(\phi)|-C(\eta) \le 1$$

If so, then consistent with Wood and Spekkens, IST is very fine tuned and hence an unappealing and unsatisfactory approach to resolve the Bell Theorem.

NO!

7. Enter the p-adic metric

However, to measure the distance between the actual and counterfactual worlds U_η and $U_{\theta-\phi}$, we instead use the (transverse) p-adic metric $d_p(a,b)$ with $p = 2^N + 1$.

$U_\eta \in I_U$, $U_{\theta-\phi} \notin I_U \Rightarrow$

There is no U_η **p – adically close to** $U_{\theta-\phi}$ even though $|\eta - (\theta - \phi)| \ll 1$.

8. Bell Inequalities have not been tested experimentally

In testing whether the Bell inequality is violated experimentally, an experimenter is actually testing (according to IST)

(1) $$|C(\theta)-C(\phi)|-C(\eta)\leq 1$$

where $\cos\theta$, $\cos\phi$ and $\cos\eta$ are all describable by N bits and where $\cos\eta$ and $\cos(\theta-\phi)$ agree in their leading M bits. By relying on our intuition about distance in state space we have been fooling ourselves that experimenters have been testing the Bell inequality.

(2) $$|C(\theta)-C(\phi)|-C(\theta-\phi)\leq 1$$

Insofar as U_η isn't close to $U_{\theta-\phi}$, **they have not - not even approximately!**
In IST (1) is violated $\because$ correlations based on three separate samples.
(2) is violated simply $\because$ the equation necessarily has ill-defined terms.

9. Robust to Noise

In IST, violation of (1) and (2) is completely robust to p-adic noise i.e. noise which has small amplitude wrt the p-adic metric. Such noise could never perturb a state U of the universe from one where $\cos\theta$ was describable by N bits, to one where $\cos\theta$ was not describable by N bits.

10. CHSH

Alice can choose $\mathbf{a}$ or $\mathbf{a}'$; Bob can choose $\mathbf{b}$ or $\mathbf{b}'$

$$-2 \le Corr(\mathbf{a},\mathbf{b}) - Corr(\mathbf{a},\mathbf{b}') + Corr(\mathbf{a}',\mathbf{b}) + Corr(\mathbf{a}',\mathbf{b}') | \le 2$$

$\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, $\mathbf{b}'$ represent points on $\mathbb{S}^2$ and $Corr(\mathbf{a}, \mathbf{b}) = -\cos\theta_{ab}$

As before, not possible to satisfy inequality if we require all cosines to be described by finite N bits.

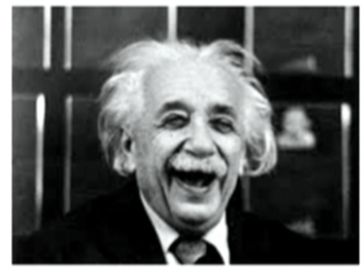
As before, this is not what is tested experimentally - not even approximately. Instead what is tested is:

$$-2 \le Corr(\mathbf{a},\mathbf{b}) - Corr(\tilde{\mathbf{a}},\mathbf{b}') + Corr(\tilde{\mathbf{a}}',\mathbf{b}) + Corr(\mathbf{a}',\mathbf{b}') | \le 2$$

Although $|\theta_{ab'} - \theta_{\tilde{a}b'}| \ll 1$, $d_p(U_{\theta_{ab'}}, U_{\theta_{\tilde{a}b'}}) \gg 1$

11. So what is this telling us?

$\Rightarrow$



This result may imply a limitation to traditional reductionist approaches to fundamental theories of physics.

May 10th Seminar

"We [number theorists] tend to work almost as much p-adically as with the reals or complexes nowadays, and in fact it is usually best to consider all at once."

- A new approach to the role of complex numbers (and spinors) in quantum physics
- Quantum Interferometry from number theory
- The Dirac equation as a singular limit of IST at p=infinity
- A novel realistic causal geometric approach to quantum gravity:
  - P-adic generalisation of GR
  - The dark universe (nothing to do with particle physics)
  - Elimination of space-time singularities
  - Resolution of the Black hole information paradox

- Analogies with biology