

Title: Particle-hole symmetry without particle-hole symmetry in the quantum Hall effect at $\hat{\nu} = 5/2$

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Abstract: Numerical results suggest that the quantum Hall effect at $\nu = 5/2$ is described by the Pfaffian or anti-Pfaffian state in the absence of disorder and Landau level mixing. Those states are incompatible with the observed transport properties of GaAs heterostructures, where disorder and Landau level mixing are strong. We show that the recent proposal of a PH-Pfaffian topological order by Son is consistent with all experiments. The absence of the particle-hole symmetry at $\nu =$

$5/2$ is not an obstacle to the existence of the PH-Pfaffian order since the order is robust to symmetry breaking.

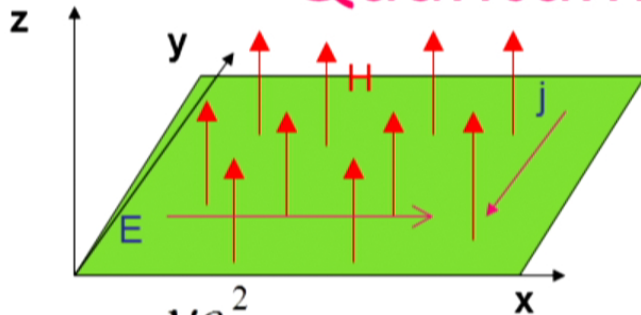
Particle-hole symmetry without particle-hole symmetry in QHE at $\nu=5/2$

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P. T. Zucker and D. E. Feldman, arXiv:1603.03754



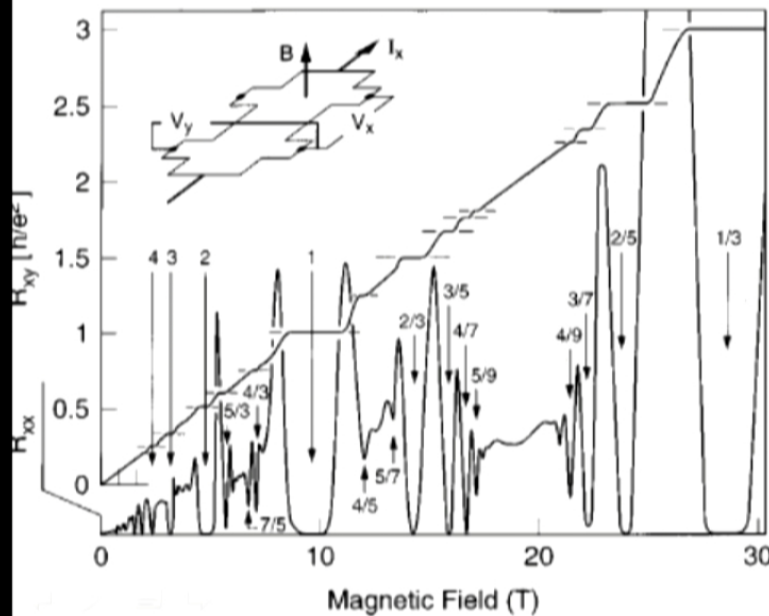
Quantum Hall Effect



$$I_x = \sigma_{xx} V = 0$$

$$I_y = \sigma_{xy} V$$

$$I = \frac{\nu e^2}{h} V; \quad \nu - \text{rational number (filling factor)}$$



$$\frac{e^2}{h} = (25.8k\Omega)^{-1}$$

$$= 1 \text{ Klitzing}$$



H.L. Stormer *et al.*, RMP S298 (1999)

Laughlin State

$$\nu = \frac{1}{2n+1}; \Psi = \prod (z_i - z_j)^{2n+1} \exp(-\sum |z_i|^2 / 4l^2); z_k = x_k + iy_k$$

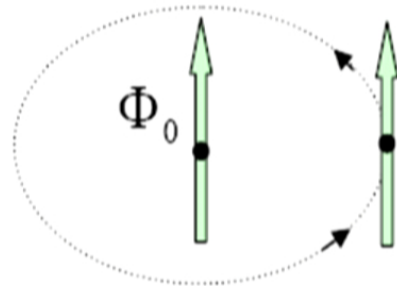
Quasiparticle charge (Laughlin)

$$q = \nu e$$

Quasiparticle statistics

(Arovas, Schrieffer, Wilczek)

$$\theta = 2\pi\nu$$



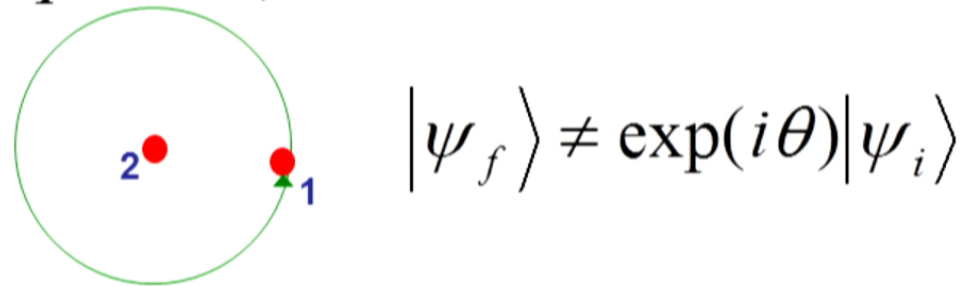
$$\theta = \frac{q}{\hbar c} \oint \vec{A} d\vec{r}$$

Many other odd-denominator states, e.g., $\nu = p / (2pn + 1)$

Moore-Read (Pfaffian) state

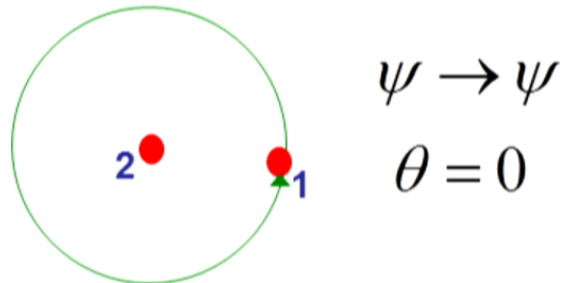
$$\nu = 5/2$$

$q = e/4$; non - Abelian statistics

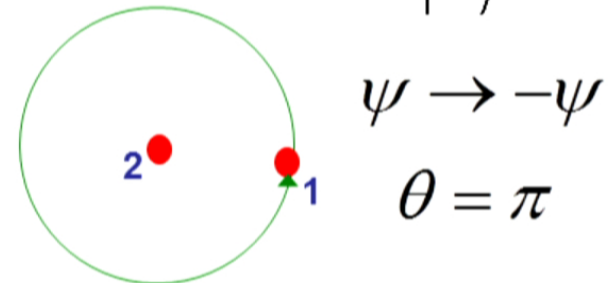


Several states at given quasiparticle positions

Vacuum superselection sector $|1\rangle$



Fermion sector $|\varepsilon\rangle$



$$\alpha|1\rangle + \beta|\varepsilon\rangle \rightarrow \alpha|1\rangle - \beta|\varepsilon\rangle$$

Theoretical proposals at $\nu=5/2$

**FQHE of bosons naturally corresponds to even denominators.
Cooper pairing?**

Experimental confirmation: charge $e/4$.

Numerous ways to build Cooper pairs:

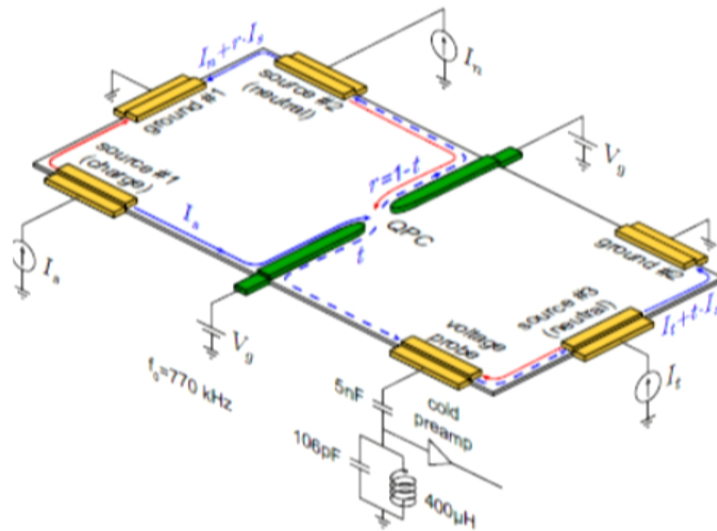
- Pfaffian state
- 331 state
- K=8 state
- $SU(2)_2$ state
- anti-Pfaffian state
- anti-331 state
- anti- $SU(2)_2$ state

Numerics favors Pfaffian and anti-Pfaffian

Pfaffian and anti-Pfaffian states

- numerics supports Pfaffian and anti-Pfaffian states in the absence of disorder and Landau level mixing
- poor results for the energy gap
 - strong disorder
 - LLM parameter ~ 1.3
- Small energy differences for proposed states
[J. Biddle *et al.*, *Phys. Rev. B* **87**, 235134 (2013)]
- no QHE at realistic LLM in numerics
[K. Pakrouski *et al.*, *Phys. Rev. X* **5**, 021004 (2015)]

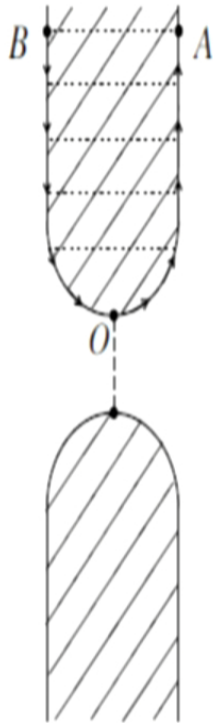
Upstream neutral modes



Observation of a topologically protected upstream neutral mode [A. Bid *et al.*, *Nature* **466**, 585 (2010)]. Compatible with **anti-Pfaffian**. Incompatible with **Pfaffian**.

The observed physics at $\nu=5/2$ is similar to $\nu=2/3$. It differs from the filling factors where no topologically protected upstream modes are present but edge reconstruction is possible [H. Inoue *et al.*, *Nature Comm.* **5**, 4067 (2014).]

Tunneling



Theory: $G \sim T^{2g-2}$

- Pfaffian: $g = \frac{1}{4}$
- Anti-Pfaffian: $g = \frac{1}{2}$

Experiment: $g_{\text{exp}} > g_{\text{theor}}$

Experiment gives an upper bound on g
The upper bound of 0.4 is consistent with
Pfaffian and excludes **anti-Pfaffian**

X. Lin *et al.*, *Phys. Rev. B* **85**, 165321 (2012)

S. Baer *et al.*, *Phys. Rev. B* **90**, 075403 (2014)

Filling factor $1/2$

[D. T. Son, *Phys. Rev. X* **5**, 031027 (2015)]

The existing theory breaks the particle-hole symmetry between filling factors f and $1-f$.

The experiment is compatible with such symmetry.

Son reconciled theory and experiment by proposing that the low-energy theory is a theory of Dirac composite fermions.

The fermion doubling theorem is not violated because the particle-hole symmetry acts non-locally and electrons interact.

PH-Pfaffian state

[D. T. Son, *Phys. Rev. X* **5**, 031027 (2015)]
s-pairing of Dirac fermions

Particle-hole symmetry:



$$G = \frac{e^2}{2h}; \quad k = \pi^2 T / 6h$$

Edge theory:

$$-\frac{2}{4\pi} [\partial_t \varphi \partial_x \varphi + v_c \partial_x \varphi \partial_x \varphi] + i\psi(\partial_t - v_n \partial_x)\psi$$

$$\psi = \psi^+$$

Wave function:

$$\int \{d^2 s_i\} \text{Pf} \left\{ \frac{1}{\bar{s}_i - \bar{s}_j} \right\} \prod (s_i - s_j)^2 \exp[-|s_i|^2 + 2\bar{s}_i z_i - |z_i|^2]$$

PH-Pfaffian state

[D. T. Son, *Phys. Rev. X* **5**, 031027 (2015)]

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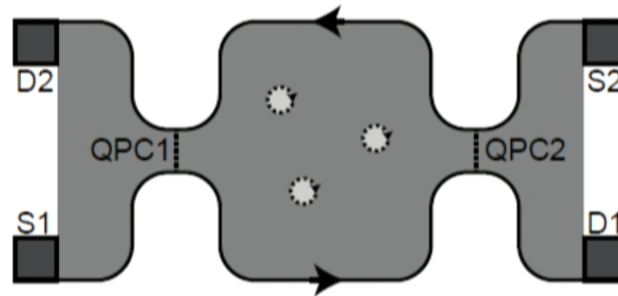
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Fusion and braiding rules

- 6 quasiparticle types:
 - topological charge σ and electric charges $\pm e/4$;
 - topological charges 1 and ψ and electric charges 0 and $e/2$
- **Fusion rules** $\psi \times \psi = 1$; $\psi \times \sigma = \sigma$; $\sigma \times \sigma = 1 + \psi$
- **Braiding rules** determine the phase accumulated by a quasiparticle moving around another quasiparticle. The phase depends on the fusion channel.

Comparison with the experiment

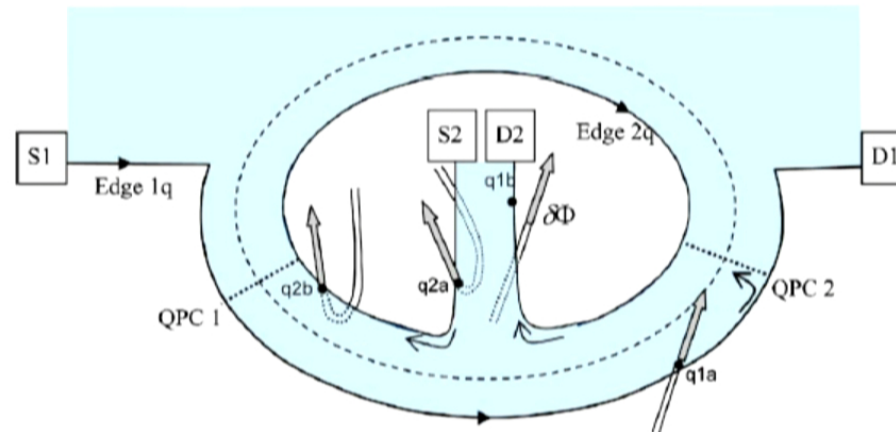
- An upstream neutral mode
- Tunneling exponent $g = \frac{1}{4}$
- Topological even-odd effect



New experimental signatures

Thermal Hall conductance $\frac{\pi^2 k^2 T^2}{6h}$

Mach-Zehnder interferometry

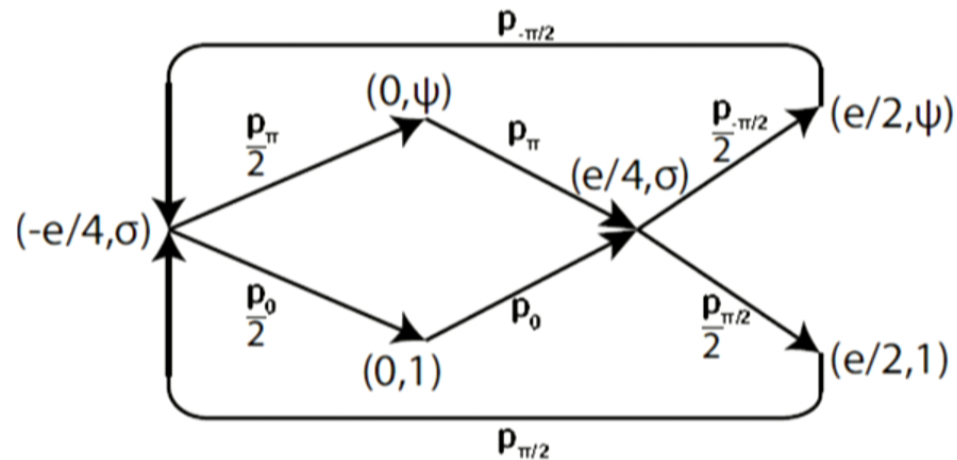


No magnetic field dependence of the current.
Shot noise diverges at some magnetic fields.

Mach-Zehnder interferometry

Tunneling probability depends on the accumulated topological charge

$$P \sim |\Gamma_1|^2 + |\Gamma_2|^2 + 2u|\Gamma_1\Gamma_2| \cos(\varphi_{AB} + \varphi_{stat} + \alpha)$$



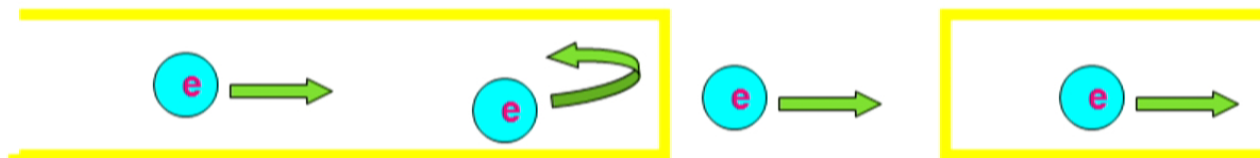
$$I = \text{const}[|\Gamma_1|^2 + |\Gamma_2|^2]$$

Shot Noise

$$S = \int [\langle I(0)I(t) \rangle - \langle I \rangle^2] dt = q \langle I \rangle$$



Walter Shottky



L. Saminadayar et al., R. de Picciotto et al. (1997): $q=e/3$

Shot noise in MZ interferometer

$$S = q^* \langle I \rangle$$

$$q^* = \frac{e}{64} \sum p_i \sum \frac{1}{p_i} \text{ diverges at some fields}$$

Closing Argument

- PH-Pfaffian topological order is consistent with all experiments
- Numerics with the particle-hole symmetric Hamiltonians supports states that break the particle-hole symmetry
- Realistic Hamiltonians have no symmetry
- The ground state is not symmetric, yet the topological order is compatible with the particle-hole symmetry