

Title: Hilbert series for effective field theories

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Abstract: 

Effective field theories (EFT) are everywhere in particle physics. Given an EFT, the first question we ask is “what are all the operators consistent with the symmetries and degrees of freedom at a particular expansion order? In this talk I will show how this question can be attacked, and often answered, using an object called a Hilbert series.”

# Hilbert Series for Effective Field Theory

Adam Martin  
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based on 1503.07537, 1510.00372 AM, L. Lehman,  
also Henning et al 1512.03433, 1507.07240

Perimeter Institute, May 3<sup>rd</sup>, 2016

Wilsonian picture of field theory

$$\mathcal{L} = \int d^4x \sum_i c_i \mathcal{O}_i$$

take all degrees of freedom, form local operators  
of increasing dimension

all operators consistent with symmetries must be  
included

lowest mass dimension operators dominate IR physics

## SM is a poster child EFT: SMEFT

degrees of freedom are:  $Q$ ,  $u^c$ ,  $d^c$ ,  $L$ ,  $e^c$ ,  $H$ , gauge fields

symmetry is:  $\text{Lorentz} \otimes \text{SU}(3)_c \otimes \text{SU}(2)_w \otimes \text{U}(1)_Y$

low-dimension operators are easy, but quickly gets more complicated

dim  $\leq 4$ : Standard Model

dim 5: 1 operator (neutrino mass)

[Weinberg '79]

dim 6: 63 terms (neglecting flavor)

[Büchmüller, Wyler '86,  
Grzadkowski et al '10]

dim 7: 20 terms

[Lehman '14]

**dim 8: no complete set known (as of Oct. 2015)**

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## Can this be extended?

1.) to dimension-8?

2.) to all orders?

3.) to other EFTs?

} can we get the form (= field content) of operators in addition to total #?

higher dimension operators are complicated because there are more fields = number ways to contract indices grows rapidly!

**Yes, using algebraic technique known as  
Hilbert Series**

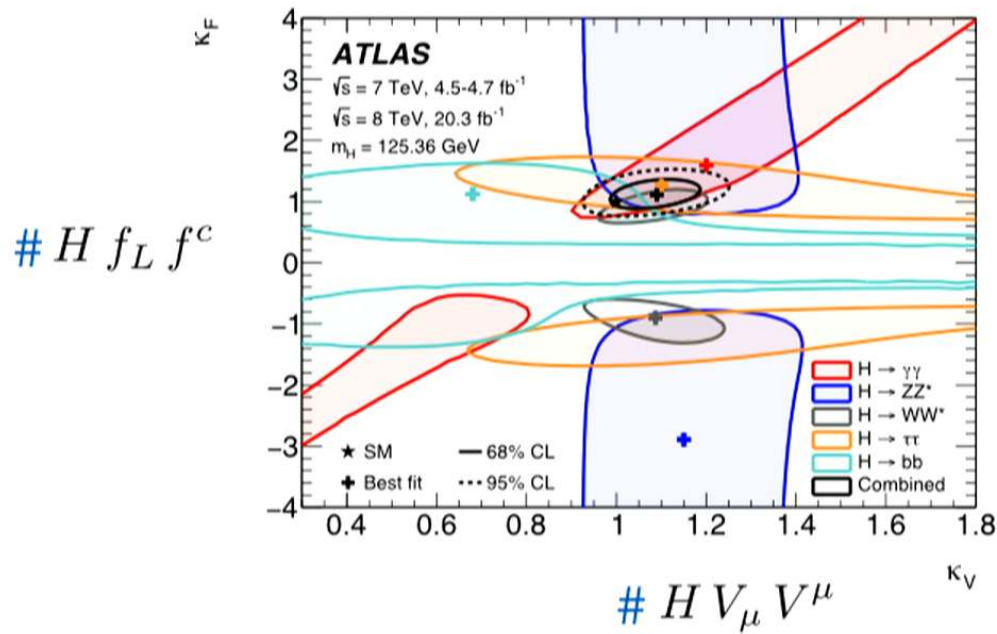
## Outline

- motivation for  $d > 6$  in the SMEFT
- introduction to Hilbert series, simple example
- towards full SMEFT, no derivatives
- adding derivatives: EOM and IBP troubles
- 'final' form:  $d = 8, 9, 10 \dots$  in SMEFT

# Why?

**precision:** LHC, HL-LHC, etc. will soon test SM to unprecedented precision = sensitivity to effects from even higher dimension

1507.04548v1



## Why?

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**new effects:** lower dim. operators have accidental symmetries (i.e. baryon #, lepton #). Higher dim. operators are the first place violation of these symmetries occurs

## How?

Consider a simple setup:  $\phi, \phi^*$  with charge +1, -1

all invariants are of the form  $(\phi \phi^*)^n$ , and for each  $n$   
there is only one invariant

**Hilbert series** is defined as:  $h = \sum_n \kappa_n t^n$



*number of invariants of degree  $n$*       *invariants*

for us:

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(call it a `spurion')... then we can formally sum series

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rewrite

$$h_\phi = \frac{1}{2\pi} \int_0^{2\pi} \frac{d\theta}{(1 - \phi e^{i\theta})(1 - \phi^* e^{-i\theta})}$$

change to  $z = e^{i\theta}$

$$h_\phi = \frac{1}{2\pi i} \oint_{|z|=1} \frac{dz}{z} \frac{1}{(1 - \phi z)(1 - \frac{\phi^*}{z})}$$

overly complicated for simple example, but will be generalizable to more fields, symmetries

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only one invariant at each order:  $\kappa_i = 1$

$$\begin{aligned}
\frac{1}{(1 - \phi z)(1 - \frac{\phi^*}{z})} &= 1 + (\phi\phi^*) + (\phi\phi^*)^2 + (\phi\phi^*)^3 + \dots \\
&\quad + z(\phi + \phi(\phi\phi^*) + \phi(\phi\phi^*)^2 + \phi(\phi\phi^*)^3 + \dots) \\
&\quad + \frac{1}{z}(\phi^* + \phi^*(\phi\phi^*) + \phi^*(\phi\phi^*)^2 + \phi^*(\phi\phi^*)^3 + \dots) \\
&\quad + \dots
\end{aligned}$$

generates **all** possible combinations of  $\phi$ ,  $\phi^*$ . Combinations can be grouped according to their charge

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manipulate further

$$\begin{aligned} \frac{1}{(1 - \phi z)(1 - \frac{\phi^*}{z})} &= \exp \left( -\log(1 - \phi z) - \log(1 - \frac{\phi^*}{z}) \right) \\ &= \exp \left( \sum_{r=1}^{\infty} \left\{ \frac{(\phi z)^r}{r} + \frac{1}{r} \left( \frac{\phi^*}{z} \right)^r \right\} \right) \end{aligned}$$

this will be the most useful (= generalizable) form

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objects                      'charge'

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Plethystic exponential



Hilbert series

$$h = \int d(\text{measure}) \left( \text{dinosaur} \right)$$

**more complicated example:**

$$\phi_1, \phi_1^*, \phi_2, \phi_2^*$$

$$\text{charge: } +1, -1, +2, -2$$

now there are four invariants

$$(\phi_1\phi_1^*), (\phi_2\phi_2^*), (\phi_1^2\phi_2^*), (\phi_1^{*2}\phi_2)$$

based on last example, may guess that

$$h_{\phi_1\phi_2} = \frac{1}{(1 - (\phi_1\phi_1^*))(1 - (\phi_2\phi_2^*))(1 - (\phi_1^2\phi_2^*))(1 - (\phi_1^{*2}\phi_2))}$$

generates all invariants

not correct! misses relations among invariants:

$$(\phi_1^2 \phi_2^*)(\phi_1^{*2} \phi_2) = (\phi_1 \phi_1^*)^2 (\phi_2 \phi_2^*)$$

correct series is

$$h_{\phi_1 \phi_2} = \frac{1 - \phi_1^2 \phi_1^{*2} \phi_2 \phi_2^*}{(1 - (\phi_1 \phi_1^*)) (1 - (\phi_2 \phi_2^*)) (1 - (\phi_1^2 \phi_2^*)) (1 - (\phi_1^{*2} \phi_2))}$$

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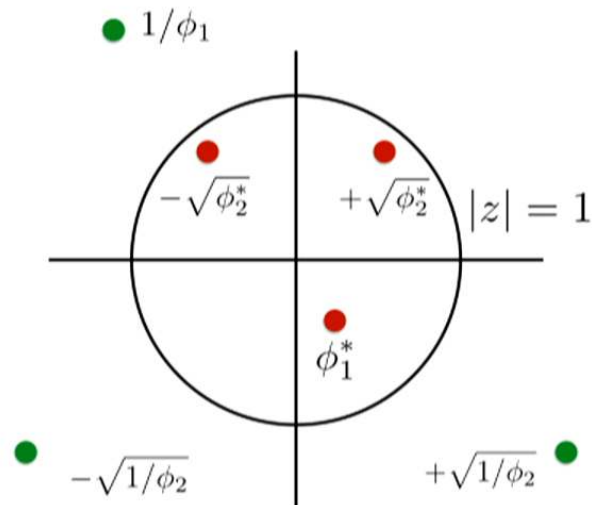
however, if we work with the PE, we get this **automatically**.

extend

$$\exp \left( \sum_{r=1}^{\infty} \left\{ \frac{(\phi_1 z)^r}{r} + \frac{1}{r} \left( \frac{\phi_1^*}{z} \right)^r + \frac{(\phi_2 z^2)^r}{r} + \frac{1}{r} \left( \frac{\phi_2^*}{z^2} \right)^r \right\} \right)$$

$$\frac{1}{2\pi i} \oint_{|z|=1} \frac{dz}{z} \frac{1}{(1 - \phi_1 z)(1 - \frac{\phi_1^*}{z})(1 - \phi_2 z^2)(1 - \frac{\phi_2^*}{z^2})}$$

multiple poles, but not all reside in  $|z| < 1$  ( $\phi_1, \phi_2$  are also mod  $< 1$ )



**Molien form = PE**

developed to capture invariants correctly

[Melia]

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## Molien/Hilbert

developed these tools to study invariant polynomials  
related to mathematics of **rings, ideals**

Use in formal field theory (counting BPS states, moduli space)

[Benvenuti, Feng, Hanany, He '06] [Feng, Hanany, He '07]

[Dolan '07] [Gray, Hanany, He, Jejjala, Mekareeya '08]

[Hanany, Mekareeya, Torri '08] ...many more

Some use in counting flavor invariants

[Jenkins, Manohar '09], [Hanany, Jenkins, Manohar, Torri '10],  
[Merle, Zwicky '11]

all invariants, keeping track of redundancies captured by the PE approach. We want to use this to generate all EFT operators;  $\phi \rightarrow Q, u^c, d^c, H, F_{\mu\nu}$ , etc.

**Need to:**

- 1.) expand to other larger groups
- 2.) deal with anticommuting objects
- 3.) incorporate derivatives ; brings difficulty of equations of motion (EOM) and integration by parts (IBP)

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for a 'field' in a representation R of a group G,

$z \rightarrow \chi_R(z_i)$ , the **character** of the representation R

character?

if, under G  $\phi_i \rightarrow D_{R,ij} \phi_j$  then  $\chi_R = \text{tr}(D_R)$

$\chi_R$  are functions of **j** complex numbers, **j = rank of G**

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## Other groups:

$$\frac{1}{2\pi i} \oint \frac{dz}{z} \rightarrow \int d\mu, \text{ Haar measure}$$

Haar measure: volume of compact group expressed as an integral over the  $j$  complex variables = Cartan subalgebra variables

$$\text{SU}(2): \quad \int d\mu_{\text{SU}(2)} = \frac{1}{2\pi i} \oint dz \frac{(z^2 - 1)}{z}$$


$$\text{SU}(3): \quad \int d\mu_{\text{SU}(3)} = \frac{1}{(2\pi i)^2} \oint dz_1 dz_2 \frac{(1 - z_1 z_2)}{z_1 z_2} \left(1 - \frac{z_1^2}{z_2}\right) \left(1 - \frac{z_2^2}{z_1}\right)$$

**Peter-Weyl theorem:** characters of compact Lie groups form an orthonormal basis set for functions of the  $j$  complex variables

$$\int_G d\mu \chi_M(z_i) \chi_N^*(z_i) = \delta_{MN}$$

and we can expand any function of  $z_i$  as a linear combination of  $\chi_M(z_i)$

$$F(z_i) = \sum_M A_M \chi_M(z_i)$$

  
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So, for some field content in irreps under a SU(2):

1.) form the PE:  $PE[\phi_1(\chi(z)) + \phi_2(\chi'(z)) + \dots]$

2.) PE is a function of the complex variables parameterizing the groups,  $z$ . Can be expanded in terms of characters

$$PE = \sum_M A_M \chi_M(z) \quad (\text{singlet, doublet, triplet, etc..})$$

3.) Integrate over Haar measure

$$\int d\mu_{SU(2)} \sum_M A_M \chi_M(z)$$

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## Generalizes to multiple symmetry groups

1.) form the PE:  $PE[\phi_1(\chi_1(z_1), \chi_2(z_2)\dots) + \phi_2(\chi_1'(z_1), \chi_2'(z_2)) + \dots]$

2.) PE is a function of the complex variables parameterizing the groups,  $z$ , can be expanded in terms of characters

$$PE = \prod_G \left( \sum_M A_M^G \chi_M^G(z_i) \right) \text{ (combo of all reps of all groups)}$$

3.) Integrate over Haar measure

$$\int \prod_G d\mu_G \prod_G \left( \sum_M A_M^G \chi_M^G(z_i) \right) = \prod_G A_0^G$$

only piece that survives is  $A_0$ , coefficient of **overall** singlet/  
invariant irrep

Ex: doublet scalar with Higgs charges under  $SU(2)_w \otimes U(1)_Y$

$$PE[H(0, \frac{1}{2}, -\frac{1}{2}) + H^\dagger(0, \frac{1}{2}, \frac{1}{2})]$$



$$PE[H(z + \frac{1}{z})u^{-1/2} + H^\dagger(z + \frac{1}{z})u^{1/2}]$$



$$\frac{1}{(2\pi i)^2} \oint_u \frac{du}{u} \oint_z dz \frac{(z^2 - 1)}{z} PE[H, H^\dagger]$$

## Fermions:

Ex: doublet scalar with Higgs charges under  $SU(2)_w \otimes U(1)_Y$  asymmetric, plus they transform under Lorentz group

$$PE[H(0, \frac{1}{2}, -\frac{1}{2}) + H^\dagger(0, \frac{1}{2}, \frac{1}{2})]$$

Asymmetry:

Plethystic Exponential (PE)  $[H(z)u^{-1/2} + H^\dagger(z + \frac{1}{z})u^{1/2}]$  [Hanany '14]  
 $\rightarrow$  Fermionic Plethystic Exponential (PEF)

$$PEF[\psi] = \frac{1}{(2\pi i)^2} \int_u \frac{du}{u} \left\{ \sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{(z^2 - 1)^r} (\psi \chi(z_i))^r \right\}$$

character

Lorentz group:

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BUT: orthonormality of characters only applies to compact Lie groups;  $SO(3,1)$  not compact

However: we don't care about dynamics, just counting invariants.

Work in Euclidean space!

$$SO(3,1) \rightarrow SO(4) \cong SU(2)_R \otimes SU(2)_L$$

Lorentz group just looks like two more symmetry groups

use LH fermions only for simplicity:  $Q, u^c, d^c, \text{ etc } \sim (0, 1/2)$

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Field strengths:

$$X_{\mu\nu}^{\pm} = X_{\mu\nu} \pm i\tilde{X}_{\mu\nu} \quad \text{in } (1,0) \text{ or } (0,1) \text{ irrep.}$$

**put the pieces together:**

$$\mathcal{H}_{0,SM} = \int \prod_G d\mu_{Gi} PE[H, F^+, W^+, G^+ + c.c.] \times PEF[Q, u^c, d^c, L, e^c + c.c.]$$


$$[SU(2)_L \times SU(2)_R] \times SU(3)_c \times SU(2)_W \times U(1)_Y$$

generates **all** invariants (with one flavor of QUDLE) with no derivatives

## comments:

1.)  $N_F > 1$  easy:

$$PEF[N_f \psi] = \exp \left\{ \sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r} (N_f \psi \chi(z_i))^r \right\}$$

2.) finding all poles (= invariants to all order) of the giant fraction is hard. Instead, weight by mass dimension

$$H \rightarrow \epsilon H, Q \rightarrow \epsilon^{3/2} Q, \text{ etc}$$

expand to desired order in  $\epsilon$ . Now all poles are at  $z_i = 0$ ; much easier

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example: QQQL operators,  $N_f = 3$

$$PEF[3Q(0, 1/2; 3, 2, 1/6) + 3L(0, 1/2; 1, 2, -1/2)]$$

$x, y$  for  $SU(2)_R \times SU(2)_L$ ;  $(w_1, w_2)$  for  $SU(3)$ ,  $z$  for  $SU(2)_W$ ,  $u$  for  $U(1)_Y$

$$PEF[3Q\left(y + \frac{1}{y}\right)\left(z + \frac{1}{z}\right)\left(w_1 + \frac{w_2}{w_1} + \frac{1}{w_2}\right)u^{1/6} \\ + 3L\left(y + \frac{1}{y}\right)\left(z + \frac{1}{z}\right)u^{-1/2}]$$

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## derivatives:

general EFT expansion can have derivatives on fields  
as well as fields

$$\mathcal{L} \supset \phi^n, (\partial_\mu \phi)^n \phi^m, \text{ etc}$$

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$\partial_\mu \sim (1/2, 1/2)$  of Lorentz, so doesn't look too terrible

but even at  $\partial^2$  there are two possibilities:

$$\partial_{\{\mu, \nu\}} \phi, \quad \square \phi$$

$$(1, 1), \quad (0, 0)$$

but any polynomial containing any  $\square\phi$  formed by the PE

i.e.  $\phi^m \square\phi$

always reduces via the EOM

$$\square\phi = m^2\phi^2 + \lambda\phi^3 \quad (\text{for } \phi^4 \text{ theory})$$

form of RHS of EOM is not important. We only care that  $\square\phi$  can **always be replaced** by terms with fewer derivatives

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(all polynomials in  $\phi$  and  $\partial^2\phi =$  all polynomials in  $\phi$ )

by same logic, at higher derivative order, only keep the fully symmetric term

$$PE[\phi] \rightarrow PE[\phi(0,0) + D\phi(1/2,1/2) + D^2\phi(1,1) + \dots]$$

similar story for fermions:

derivative on LH field:

$$\partial_\mu \begin{matrix} \text{Q} \\ (\frac{1}{2}, \frac{1}{2}) \end{matrix} \otimes (0, \frac{1}{2}) = (\frac{1}{2}, 0) \oplus (\frac{1}{2}, 1)$$

$$\text{EOM: } \not{D}Q = y_u H u^{c\dagger} + y_d H^* d^{c\dagger}$$

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EOM tells us  $(1/2,0)$  part of  $\partial_\mu Q$  is redundant with terms w/ fewer derivatives. Only add  $(1/2,1)$  term to PEF..

$$PEF[\psi] = PEF[\psi(0, \frac{1}{2}) + D\psi(\frac{1}{2}, 1) + D^2\psi(1, \frac{3}{2}) + \dots]$$

& analogous story for field strengths

we can now 'derivative-extend' the SM PE.

$$H \rightarrow H\left(0, 0; \frac{1}{2}, -\frac{1}{2}\right) + DH\left(\frac{1}{2}, \frac{1}{2}; \frac{1}{2}, -\frac{1}{2}\right) + D^2H\left(1, 1; \frac{1}{2}, \frac{1}{2}\right) \dots$$

$SU(2)_R, SU(2)_L$        $SU(2)_W, U(1)_Y$       + same for all other fields

Generates **all** invariant polynomials of fields & their derivatives, **no EOM redundancy**

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& analogous story for field strengths

## comments:

- 1.) for each derivative, we need a new entry in the PE/PEF.  
Can sum derivatives to all orders:

$$\frac{(1 - D^2)}{(1 - D x y)(1 - D/(xy))(1 - D x/y)(1 - D y/x)} \quad (\text{scalars})$$

$\{x, y\}$  parametrize  $SU(2)_L \times SU(2)_R$

- 2.)  $\mathbf{D}$  is a separate spurion; i.e.  $\mathbf{D}\phi$  is two separate objects, not one. This allows us to keep track of the total # derivatives in an invariant

- 3.) At a given mass dimension, don't need all orders (at dim=8, know <3 derivs)

## Integration by parts (IBP)

derivative-extended PE still contains redundancy from IBP:

ex.)

$$D_\mu H D^\mu H H^{\dagger 2}, \quad D_\mu H^\dagger D^\mu H^\dagger H^2, \quad D_\mu H D^\mu H^\dagger (H^\dagger H)$$

are not all independent

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$$D_{\{\mu,\nu\}} H^\dagger D^{\{\mu\nu\}} H \quad \text{completely reduces by IBP + EOM}$$

options:

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options:

explore using a toy theory of a single real scalar

$$h_\phi = \int d\mu_{SU(2)_L} d\mu_{SU(2)_R} PE[\phi(1 + D(\frac{1}{2}, \frac{1}{2}) + D^2(1, 1) + \dots)]$$

we can group terms in  $h_\phi$  by the number of  $D$  and  $\phi$

$$\mathcal{O}(D^m \phi^n) = m \text{ derivs, } n \text{ scalars}$$

**idea:**

all  $\mathcal{O}(D^m \phi^n)$  must come from  $D \times \mathcal{O}(D^{m-1} \phi^n)$

if we can count the number of  $\mathcal{O}(D^{m-1} \phi^n)$ , that's a set of constraints on the  $\mathcal{O}(D^m \phi^n)$

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total # of invariants, including IBP redundancy

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to get  $\# \mathcal{O}(D^{m-1} \phi^n)$ :

recall PE generates **all** combinations, not just invariants.  
we just need to pick out the part of the PE in the (1/2, 1/2)  
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$$PE[\phi] = \sum_R A_{\phi,R} \chi_R(z_i)$$

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$$h_{D\phi} = \int d\mu_{SU(2)_L} d\mu_{SU(2)_R} \left(\frac{1}{2}, \frac{1}{2}\right) PE[\phi] = A_{(1/2,1/2)}$$

picks out just the term we want

conjecture:

**number of invariants from a set of fields in linear representations of symmetry groups, including EOM and IBP redundancy**

$$h_{\phi_i, \psi_i} - D h_{D, \phi_i, \psi_i}$$

Hilbert series, projecting out invariants

gets dimension correct

Hilbert series, projecting out  $(1/2, 1/2)$

in gory detail: parametrize  $SU(2)_L \times SU(2)_R$  with  $\{x, y\}$

$$\underbrace{d\mu_G}_{\substack{\frac{1}{(2\pi i)^2} \int dx \frac{(1-x^2)}{x} \int dy \frac{(1-y^2)}{y} \times \left(1 - D\left(\frac{1}{x} + x\right)\left(\frac{1}{y} + y\right)\right) \\ \times PE \left[ \frac{(1-D^2)\phi}{(1-Dxy)(1-D/(xy))(1-Dx/y)(1-Dy/x)} \right]}}$$

some dim-8, according to this algorithm:

$(d_c^\dagger d_c)(e_c^\dagger e_c)F^L$	$(u_c^\dagger u_c)(e_c^\dagger e_c)F^L$	$2(d_c^\dagger d_c)(u_c^\dagger u_c)F^L$	$(d_c^\dagger d_c)(L^\dagger L)F^L$	
$(u_c^\dagger u_c)(L^\dagger L)F^L$	$(e_c^\dagger e_c)(L^\dagger L)F^L$	$(e_c^\dagger e_c)(Q^\dagger Q)F^L$	$(d_c Q)(e_c^\dagger L^\dagger)F^L$	
$(d_c Q)(e_c^\dagger L^\dagger)F^R$	$2(L^\dagger L)(Q^\dagger Q)F^L$	$2(d_c^\dagger d_c)(Q^\dagger Q)F^L$	$2(u_c^\dagger u_c)(Q^\dagger Q)F^L$	
$3(e_c L)(u_c Q)F^L$	$3(u_c d_c)Q^2 F^L$	$(d_c^\dagger d_c)(L^\dagger L)W^L$	$(e_c^\dagger e_c)(L^\dagger L)W^L$	
$(e_c^\dagger e_c)(Q^\dagger Q)W^L$	$(u_c^\dagger u_c)(L^\dagger L)W^L$	$(L^\dagger L)^2 W^L$	$(e_c^\dagger L^\dagger)(d_c Q)W^L$	
$(e_c L)(d_c^\dagger Q^\dagger)W^L$	$2(d_c^\dagger d_c)(Q^\dagger Q)W^L$	$2(u_c^\dagger u_c)(Q^\dagger Q)W^L$	$3(L^\dagger L)(Q^\dagger Q)W^L$	
$2(Q^\dagger Q)^2 W^L$	$3(e_c L)(u_c Q)W^L$	$3(u_c d_c)Q^2 W^L$	$(d_c^\dagger)^2 d_c^2 G^L$	
$(u_c^\dagger)^2 u_c^2 G^L$	$(d_c^\dagger d_c)(e_c^\dagger e_c)G^L$	$(u_c^\dagger u_c)(e_c^\dagger e_c)G^L$	$4(d_c^\dagger d_c)(u_c^\dagger u_c)G^L$	
$(Q^\dagger Q)(e_c^\dagger e_c)G^L$	$(d_c^\dagger d_c)(L^\dagger L)G^L$	$(u_c^\dagger u_c)(L^\dagger L)G^L$	$2(Q^\dagger Q)(L^\dagger L)G^L$	
$4(d_c^\dagger d_c)(Q^\dagger Q)G^L$	$4(u_c^\dagger u_c)(Q^\dagger Q)G^L$	$2(Q^\dagger)^2 Q^2 G^L$	$(d_c Q)(e_c^\dagger L^\dagger)G^L$	
$(d_c Q)(e_c^\dagger L^\dagger)G^R$	$3(e_c L)(u_c Q)G^L$	$6(d_c u_c)Q^2 G^L$		

112 at  $O(D^0)$

$3D(d_c^\dagger d_c)(LH e_c)$	$D(e_c^\dagger e_c)(LH e_c)$	$3D(L^\dagger L)(LH e_c)$	$3D(d_c^\dagger d_c)(QH d_c)$
$3D(e_c^\dagger e_c)(QH d_c)$	$6D(L^\dagger L)(QH d_c)$	$6D(Q^\dagger Q)(LH e_c)$	$6D(Q^\dagger Q)(QH d_c)$
$3D(d_c^\dagger u_c)(LH^\dagger e_c)$	$6D(d_c^\dagger d_c)(QH^\dagger u_c)$	$3D(e_c^\dagger e_c)(QH^\dagger u_c)$	$6D(L^\dagger L)(QH^\dagger u_c)$
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181 at  $O(D)$

535 total (931 counting +h.c. separately)

## issues:

method breaks down if #  $\mathcal{O}(D^{m-1}\phi^n)$  terms  
are not independent..

problems arise when there are multiple ways to partition  
derivatives on fields

m=6, n=4:

$$(\partial^2\phi)^2(\partial\phi)^2$$

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$$P = \left(1 - D(1/2, 1/2) + D^2((1, 0) + (0, 1)) - D^3(1/2, 1/2) + D^4\right)^{-1}$$

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**works**, free of issues

- extend d=8 SMEFT set to 992 (+62 from Lehman, AM)
- count d=9,10,11,12 SMEFT operators (560, 15456, 11962..)
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operators in conformal theory: primary  $\mathcal{O}$ , descendants  $\partial_\mu \mathcal{O}$

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operators in conformal theory: primary  $\mathcal{O}$ , descendants  $\partial_\mu \mathcal{O}$

removing IBP redundancy = eliminating all operators that are  
descendants of other ops.

accomplished by keeping only the **highest conformal weight** of operator products

integrate over  $SO(4,2)/SO(3,1)$  (dilations, conformal trans)

results in **P** prefactor

## What now?

- knowing all dim-8 SMEFT, we can study which operators have an impact at LHC. Specifically, dim-8 important to understand uncertainty on dim-6

$$|\mathcal{A}_{SM} + A_6 + A_8|^2 \supset |A_{SM}|^2 + 2 \operatorname{Re}(A_{SM} A_6) + |A_6|^2 + 2 \operatorname{Re}(A_{SM} A_8) \cdots$$

[pp  $\rightarrow$  hV, Lehman, AM in progress]

- analytic properties?
- application to EFT with nonlinear fields?

...

## conclusions:

given symmetry  
group  $\mathbf{G}$ ,  
fields  $\phi_i, \psi_i, X_{iL,R}$



# and form of **all**  
invariant (Lorentz &  
gauge) operators,  
accounts for IBP,  
EOM

- generates all possible combinations of operators, uses **character orthonormality** to pick out invariants
- derivatives tricky, but issues recently overcome

**lots of interesting directions to explore!**