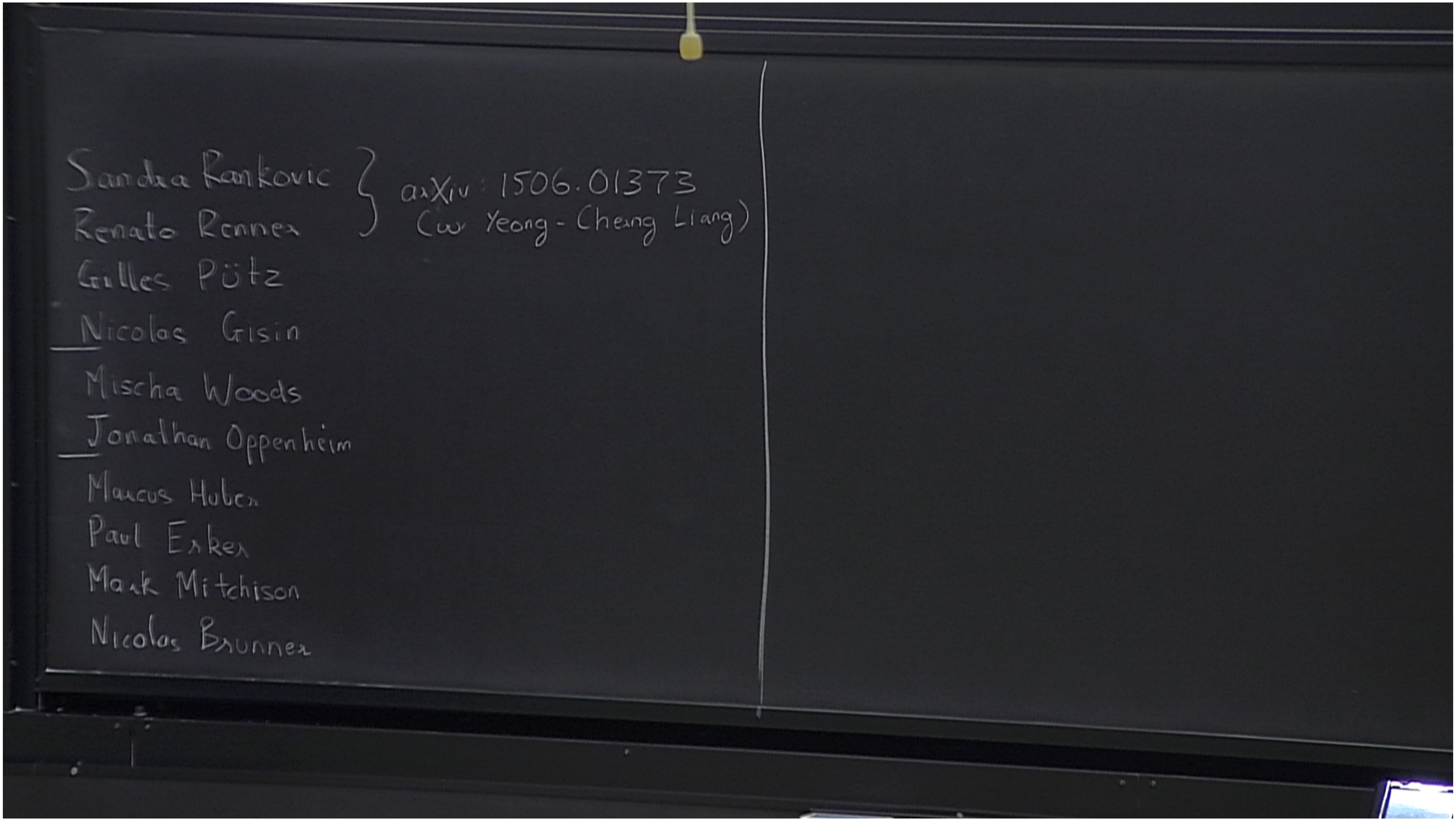
Title: The accuracy of finite quantum clocks: Fundamental constraints from dimension and thermodynamic considerations

Date: Apr 27, 2016 04:00 PM

URL: http://pirsa.org/16040107

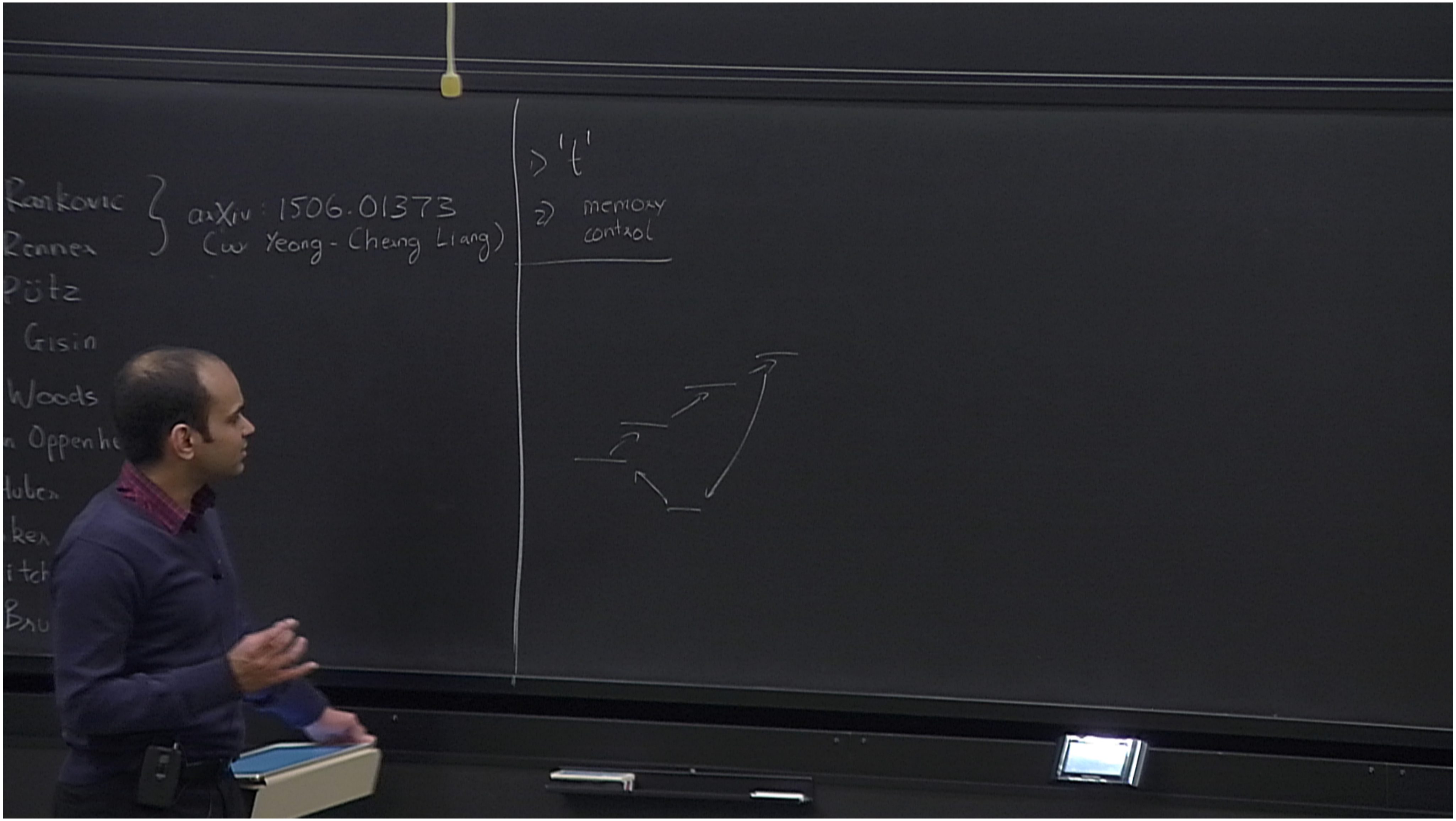
Abstract: <p>In this talk I will introduce recent research into quantum clocks of finite dimension, with the focus on their accuracy, as determined by their dimension, coherence, and power consumption.</p>

<p>I will present arguments to bound the synchronization time of any quantum clock as a function of its dimension. In addition, quantum coherence appears to be necessary to saturate these bounds, as the synchronization time of incoherent clocks is seen to have a worse bound. In addition, I will review simple proposals for autonomous clocks built out of thermal machines, and demonstrate that the power consumption of thermal clocks determines the limit of their accuracy. Finally, I will introduce an example of a finite quantum clock that is able to control any quantum operation up to a calculable accuracy, and discuss whether it represents a best case scenario for quantum clocks.</p>



Sandra Rankovic
Renato Renner
} arXiv: 1506.01373
(w/ Yeong-Cheng Liang)

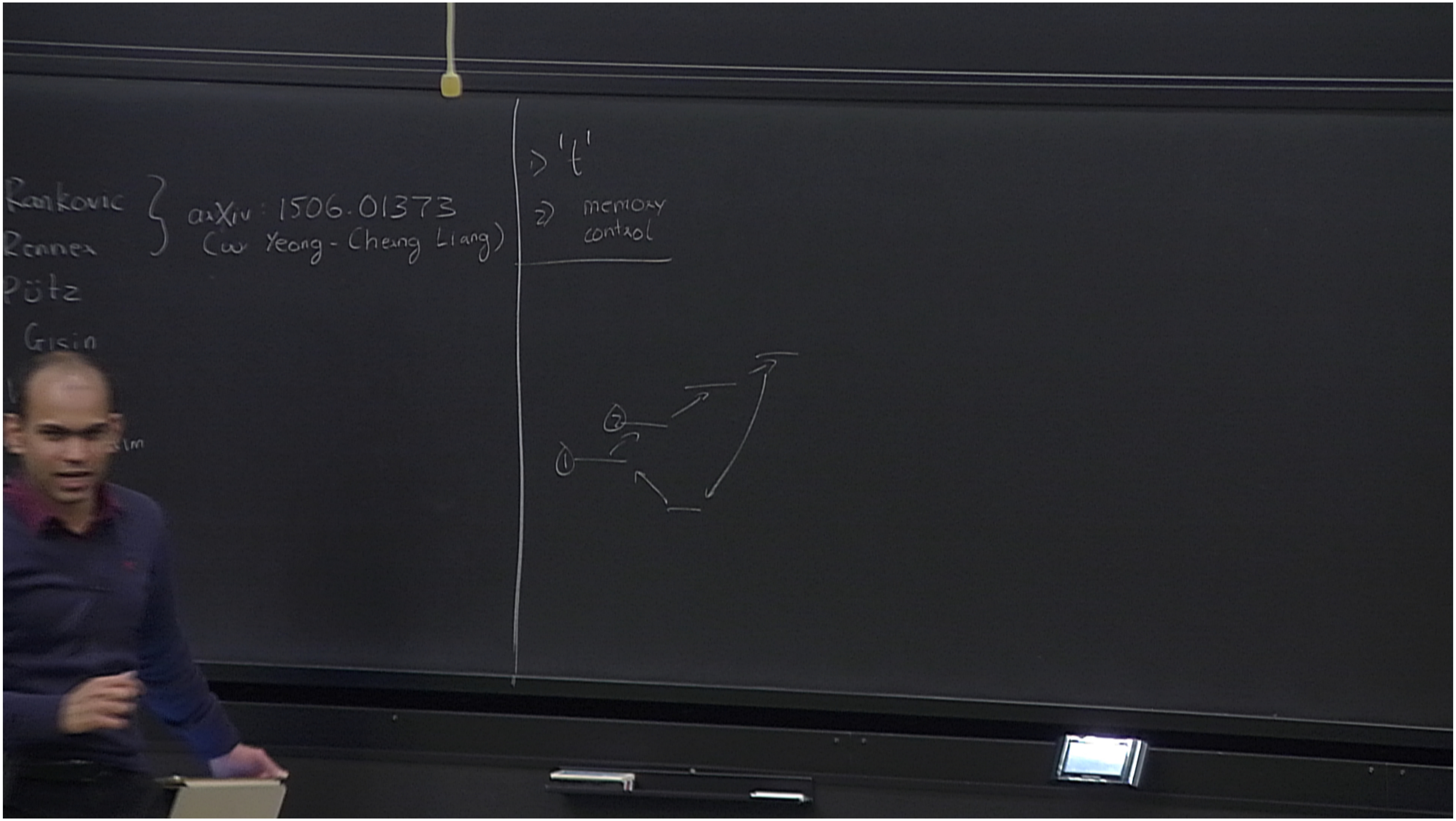
Gilles Pütz
Nicolas Gisin
Mischa Woods
Jonathan Oppenheim
Marcus Huber
Paul Erker
Mark Mitchison
Nicolas Brunner

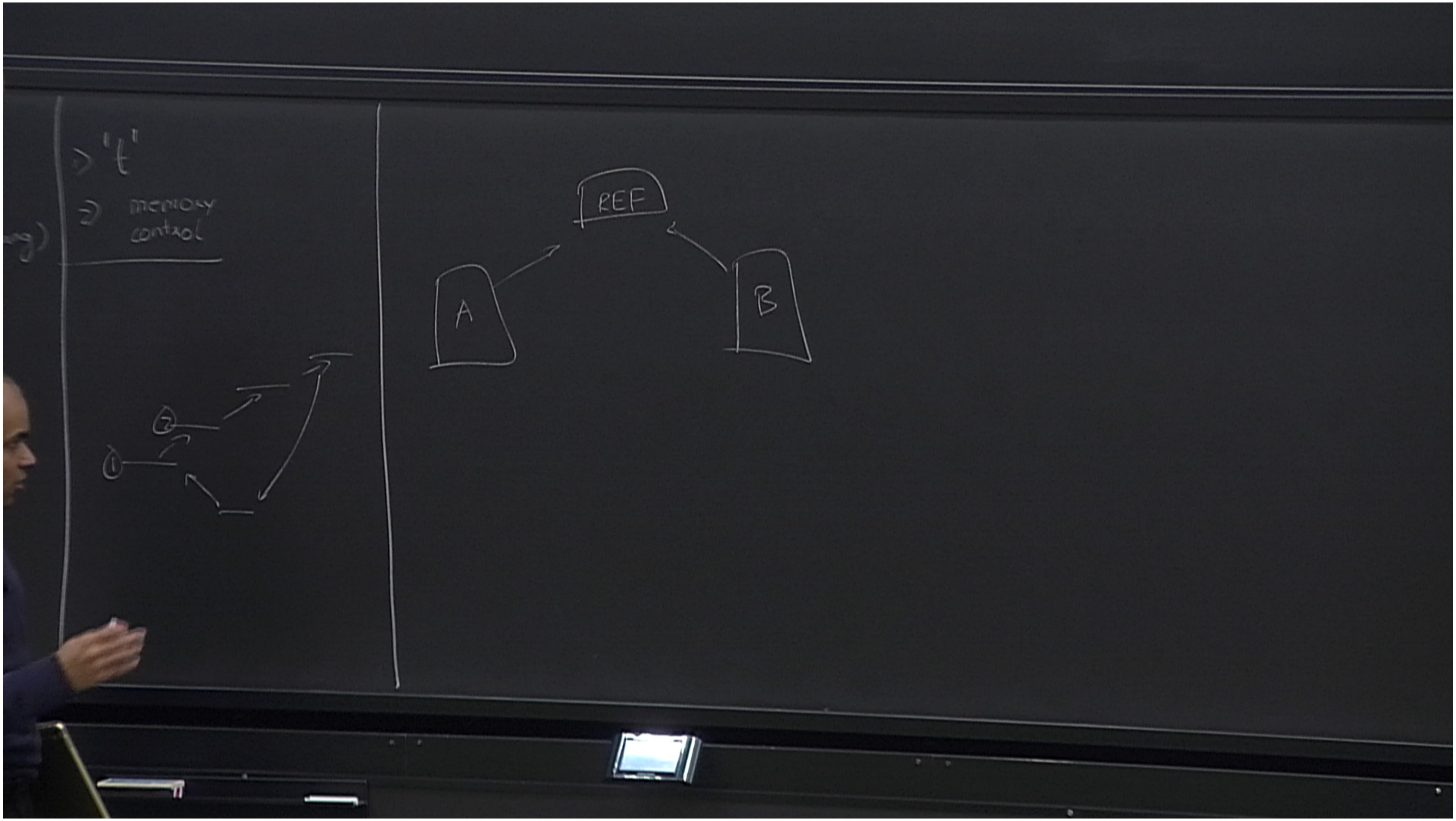


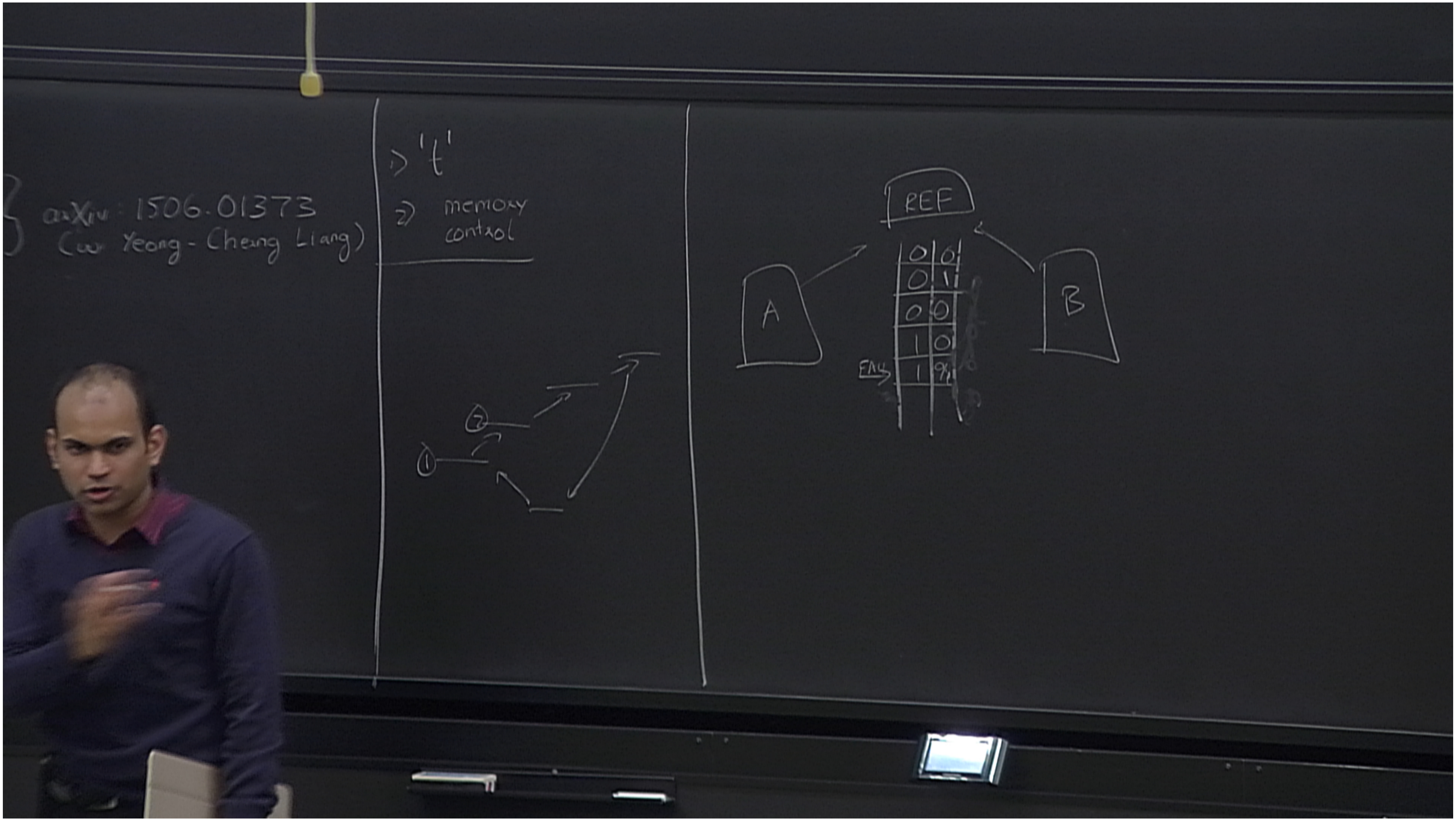
Rankovic
Renner
Pütz
Gisin
Woods
Oppenhe
Huber
ker
itch
Bru

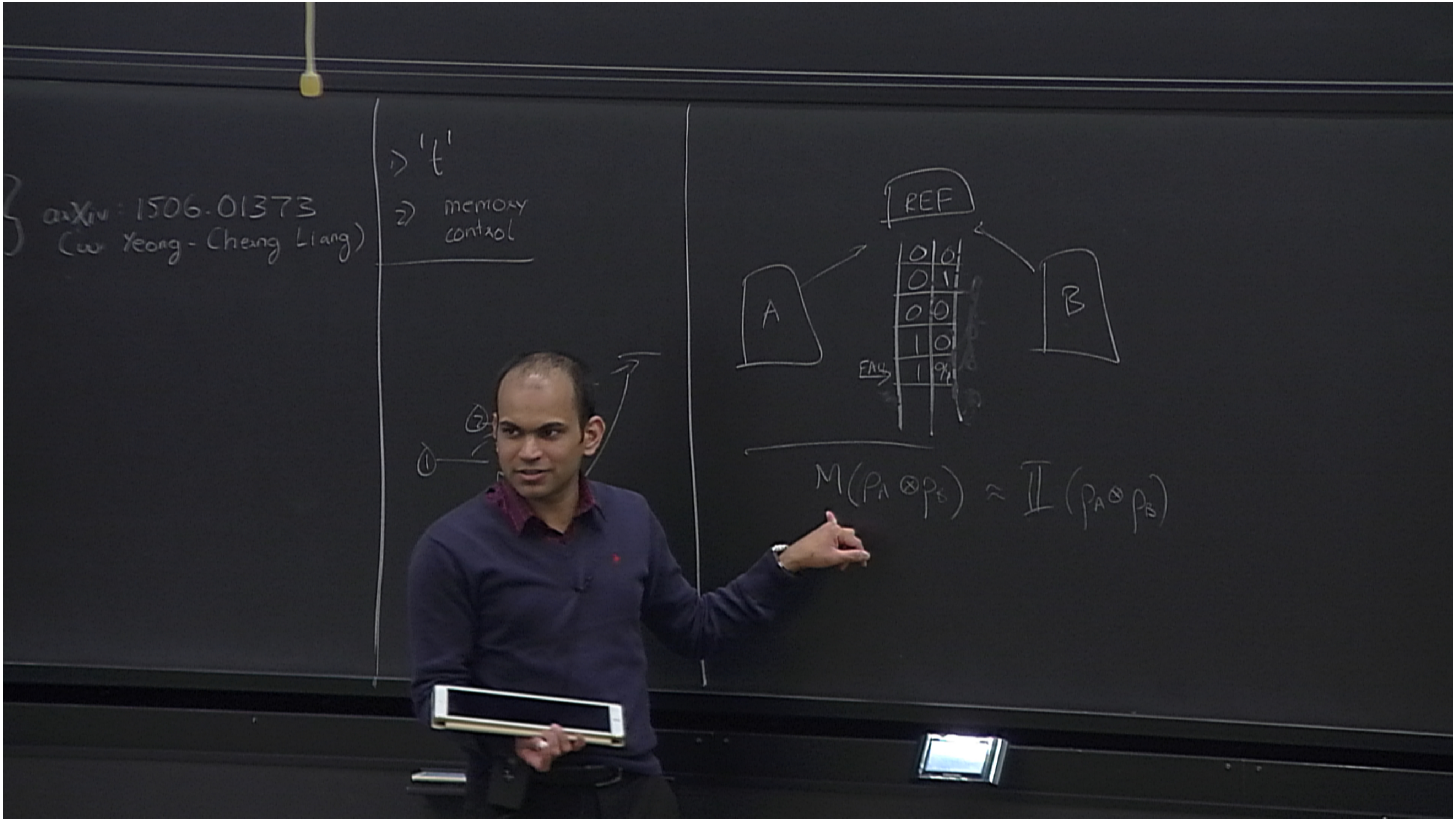
arXiv: 1506.01373
(w Yeong-Cheng Liang)

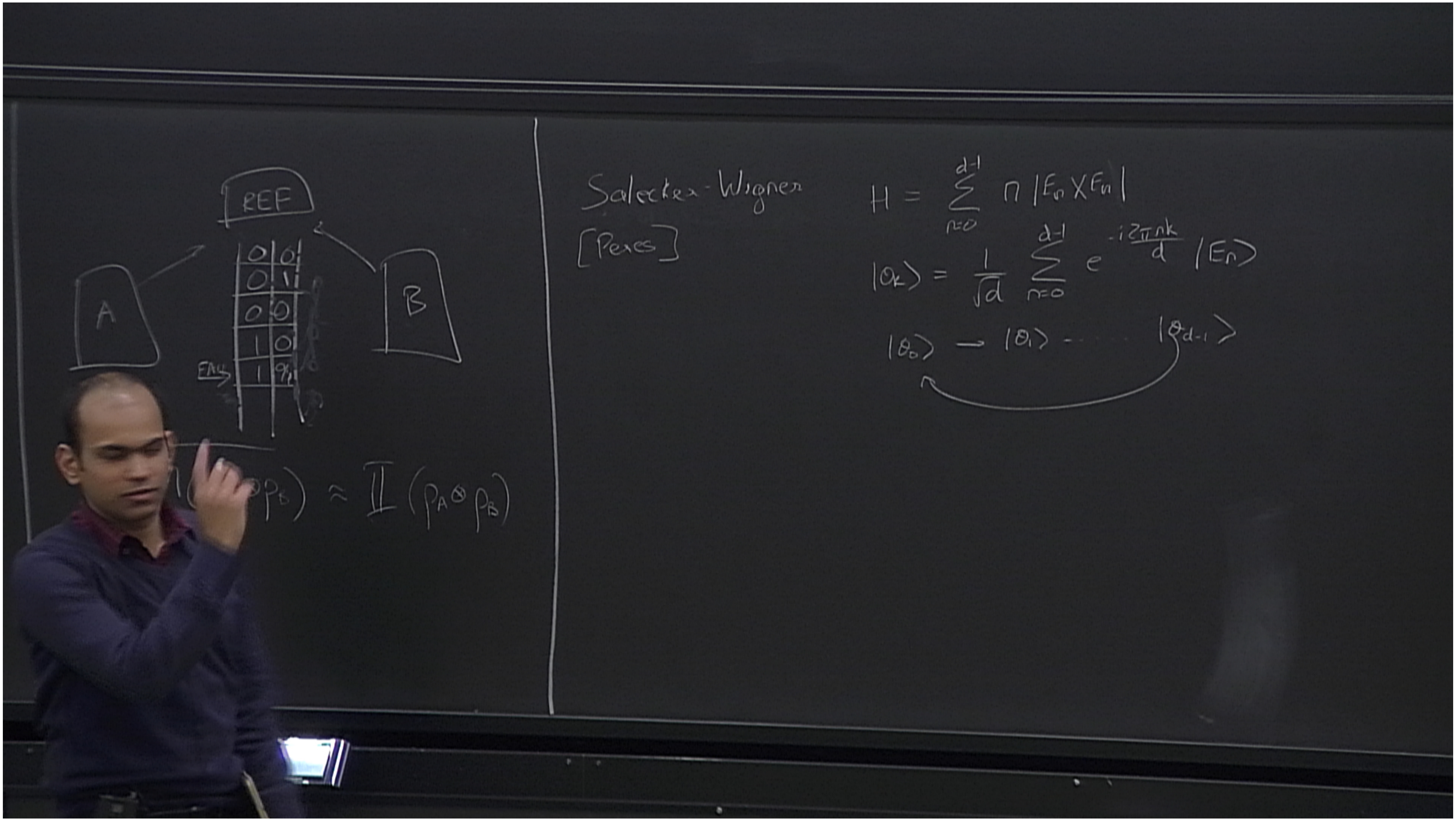
1) 't'
2) memory control


Rankovic
Renner
Pütz
Gisin
arXiv: 1506.01373
(w Yeong-Cherng Liang)
1) 't'
2) memory control
①
②


ng)
⇒ 't'
⇒ memory control
①
②
REF
A
B


arXiv: 1506.01373
(w/ Yeong-Cherng Liang)
1) 't'
2) memory control
①
②
REF
A
B
0 0
0 1
0 0
1 0
1 9/
FAIL


arXiv: 1506.01373
(w/ Yeong-Cherng Liang)
> 't'
2) memory control
REF
A
B
0 0
0 1
0 0
1 0
1 %
FAIL
①
②
M(ρ_A ⊗ ρ_B) ≈ I(ρ_A ⊗ ρ_B)



$$\ldots\rho_B) \approx \mathbb{1}(\rho_A \otimes \rho_B)$$

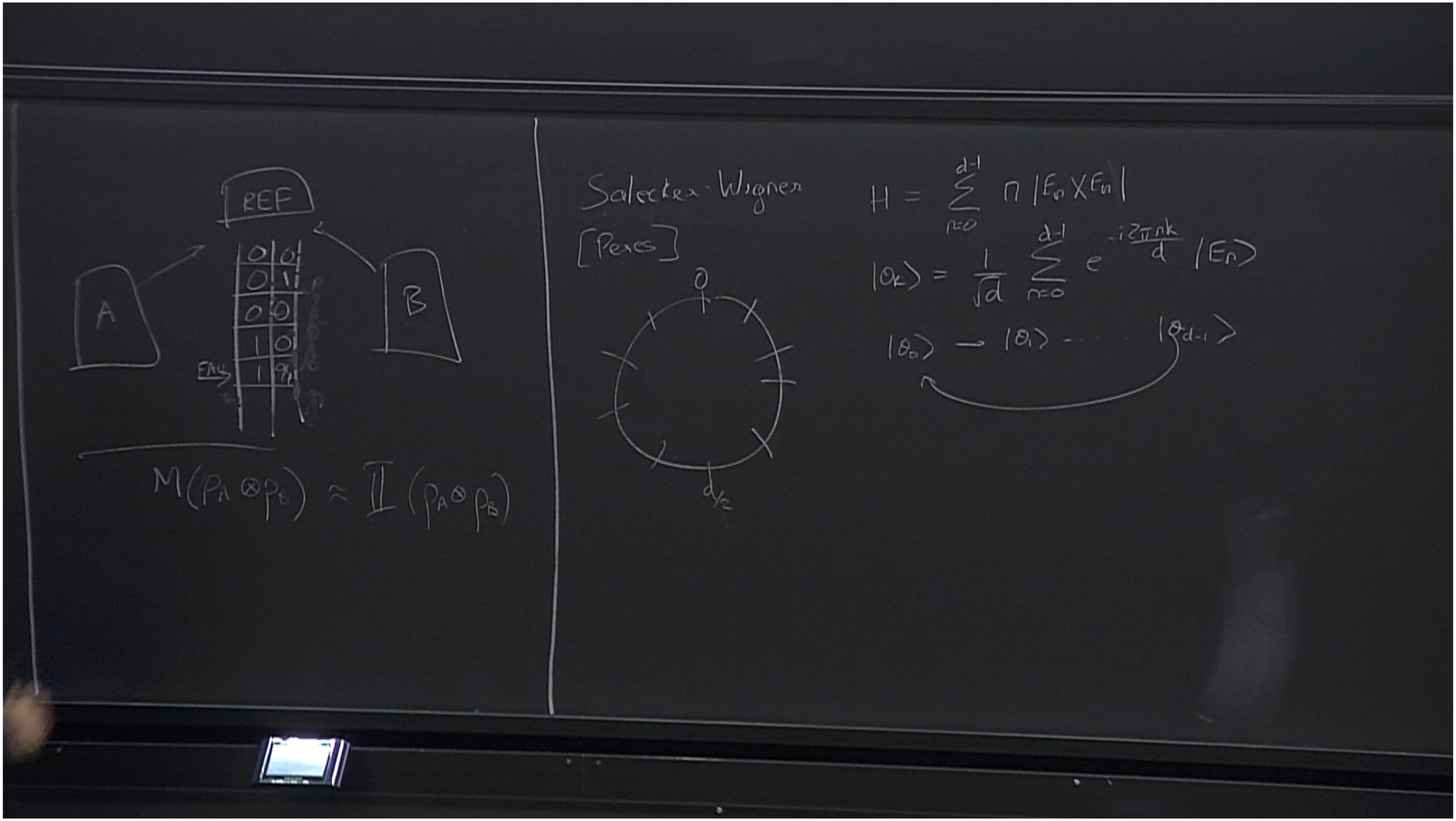
Salecker-Wigner

[Peres]

$$H = \sum_{n=0}^{d-1} n\,|E_n\rangle\langle E_n|$$

$$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-\frac{i2\pi nk}{d}} |E_n\rangle$$

$$|\theta_0\rangle \rightarrow |\theta_1\rangle \cdots\cdots |\theta_{d-1}\rangle$$



$$M(\rho_A \otimes \rho_B) \approx \mathbb{I}(\rho_A \otimes \rho_B)$$

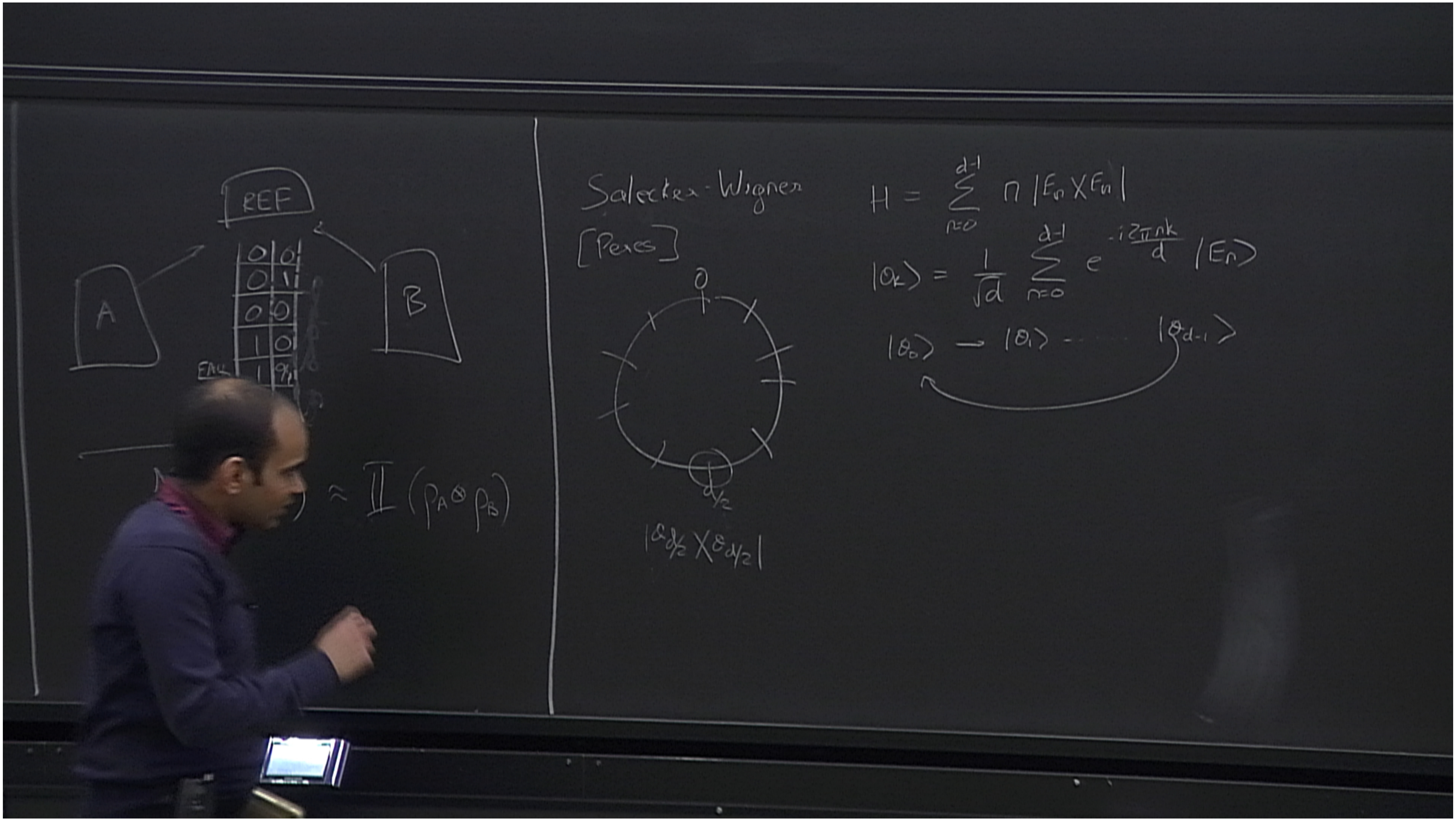
Salecker-Wigner

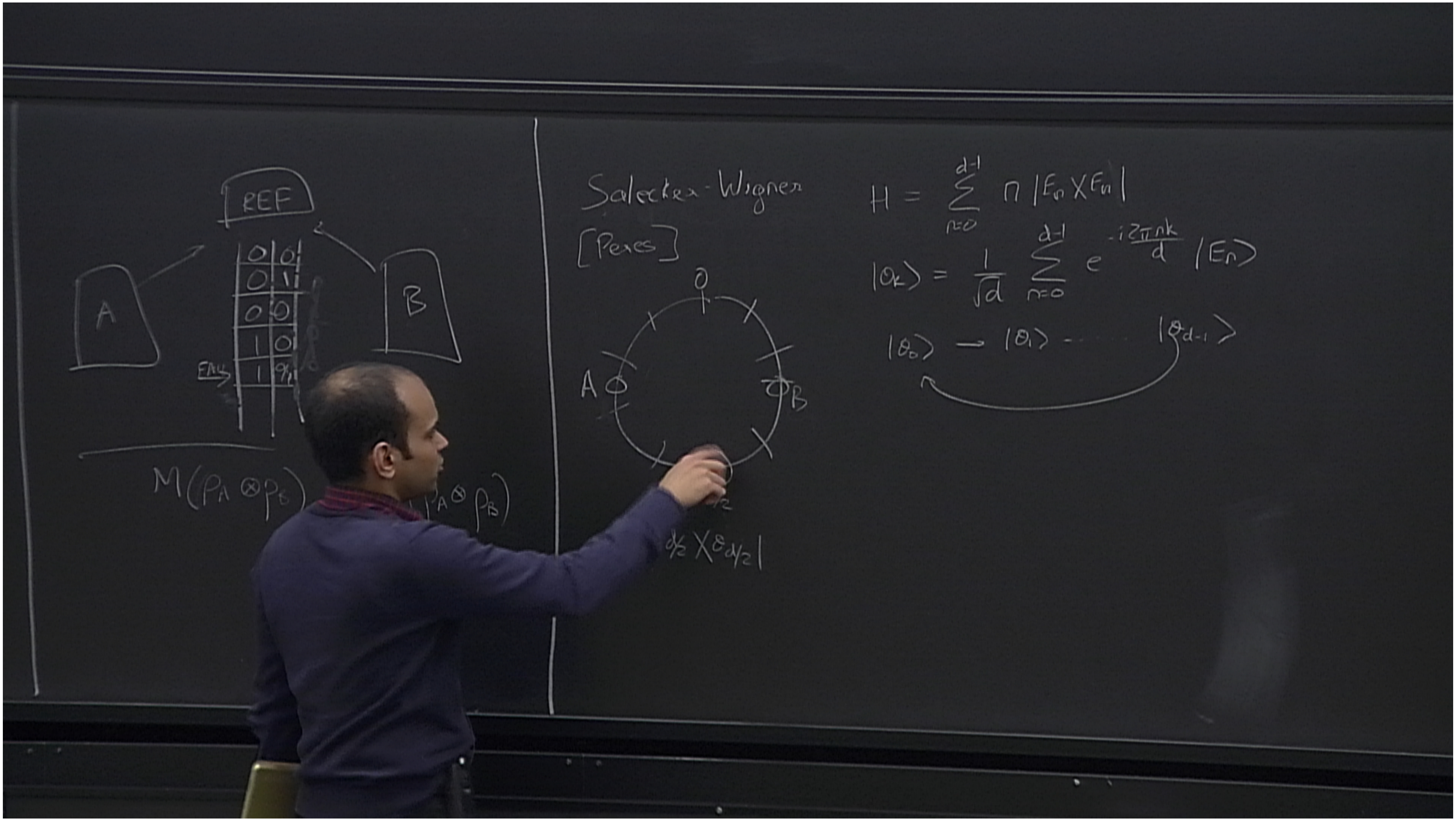
[Peres]

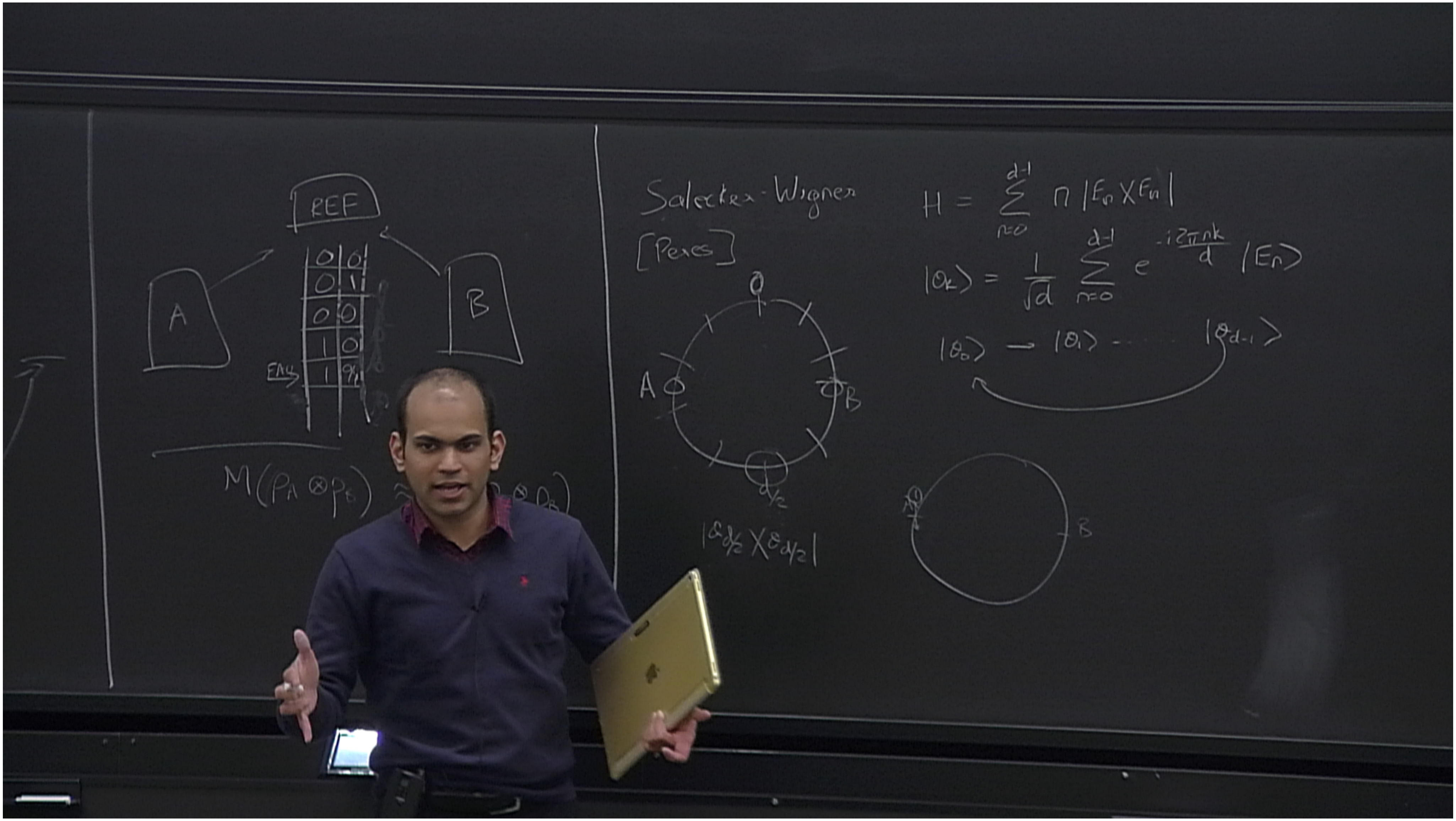
$$H = \sum_{n=0}^{d-1} n |E_n\rangle\langle E_n|$$

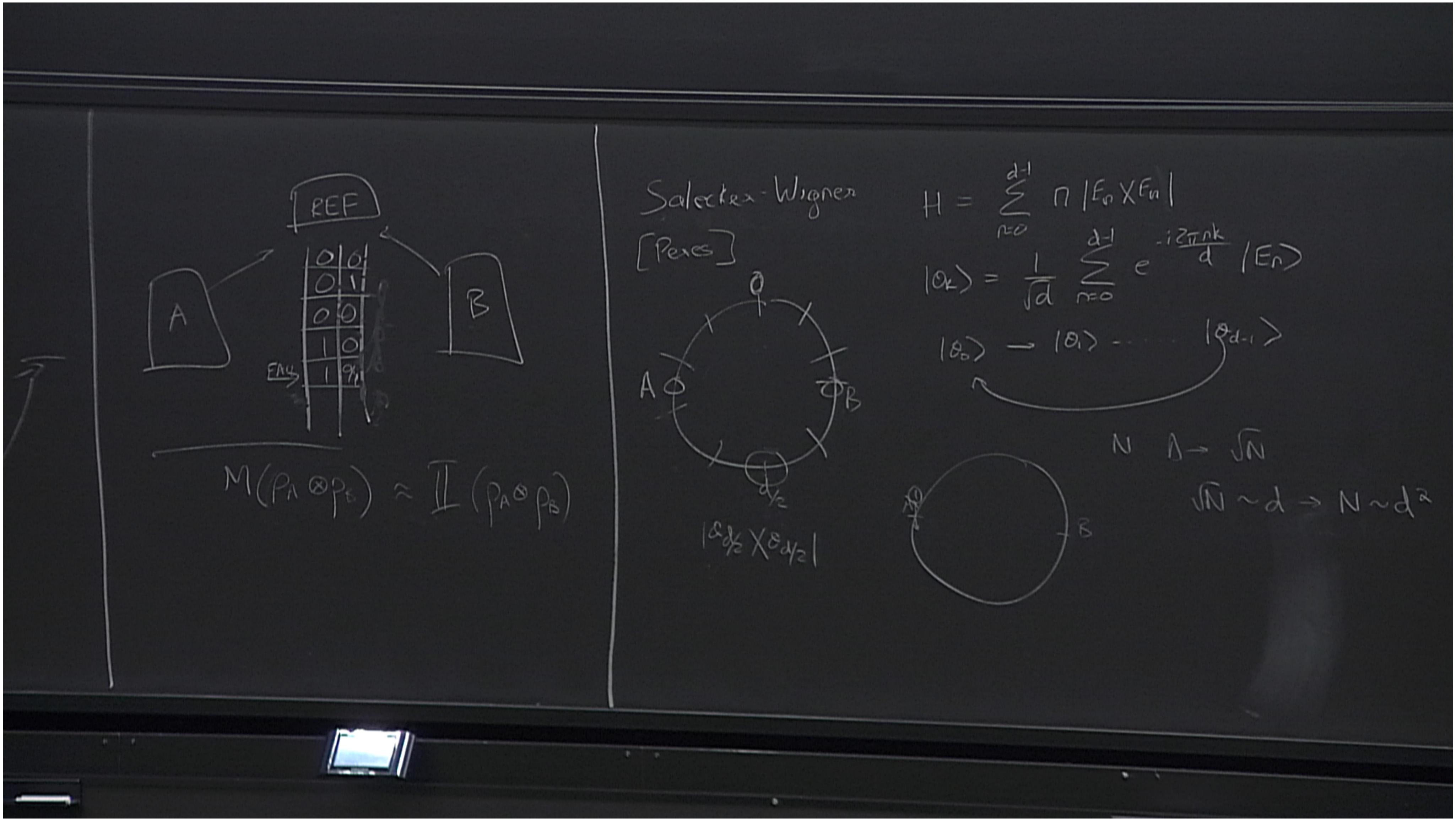
$$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i\frac{2\pi nk}{d}} |E_n\rangle$$

$$\begin{tikzcd} |\theta_0\rangle \arrow[r] & |\theta_1\rangle \cdots & |\theta_{d-1}\rangle \arrow[ll, bend left] \end{tikzcd}$$


REF
A
B
0 0
0 1
0 0
1 0
FAIL 1 $\%_2$
$\approx \mathbb{I}(\rho_A \otimes \rho_B)$
Salecker-Wigner
[Peres]
0
$d/2$
$|\theta_{d/2}\rangle\langle\theta_{d/2}|$
$H = \sum_{n=0}^{d-1} n |E_n\rangle\langle E_n|$
$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i\frac{2\pi nk}{d}} |E_n\rangle$
$|\theta_0\rangle \rightarrow |\theta_1\rangle \cdots\cdots |\theta_{d-1}\rangle$


REF
A
B
Salecker-Wigner
[Peres]
A
B
$M(\rho_A \otimes \rho_B)$
$H = \sum_{n=0}^{d-1} n |E_n\rangle\langle E_n|$
$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i\frac{2\pi nk}{d}} |E_n\rangle$
$|\theta_0\rangle \to |\theta_1\rangle \cdots |\theta_{d-1}\rangle$
$\theta_{d/2}\rangle\langle\theta_{d/2}|$


REF
A
B
Env
Salecker-Wigner
[Peres]
$H = \sum_{n=0}^{d-1} n |E_n\rangle\langle E_n|$
$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i2\pi nk/d} |E_n\rangle$
$|\theta_0\rangle \to |\theta_1\rangle \cdots |\theta_{d-1}\rangle$
0
d/2
$|\theta_{d/2}\rangle\langle\theta_{d/2}|$
$M(\rho_A \otimes \rho_B)$



$$M(\rho_A \otimes \rho_B) \approx \mathbb{I}(\rho_A \otimes \rho_B)$$

Salecker-Wigner
[Peres]

$$|\theta_{d/2}\rangle\langle\theta_{d/2}|$$

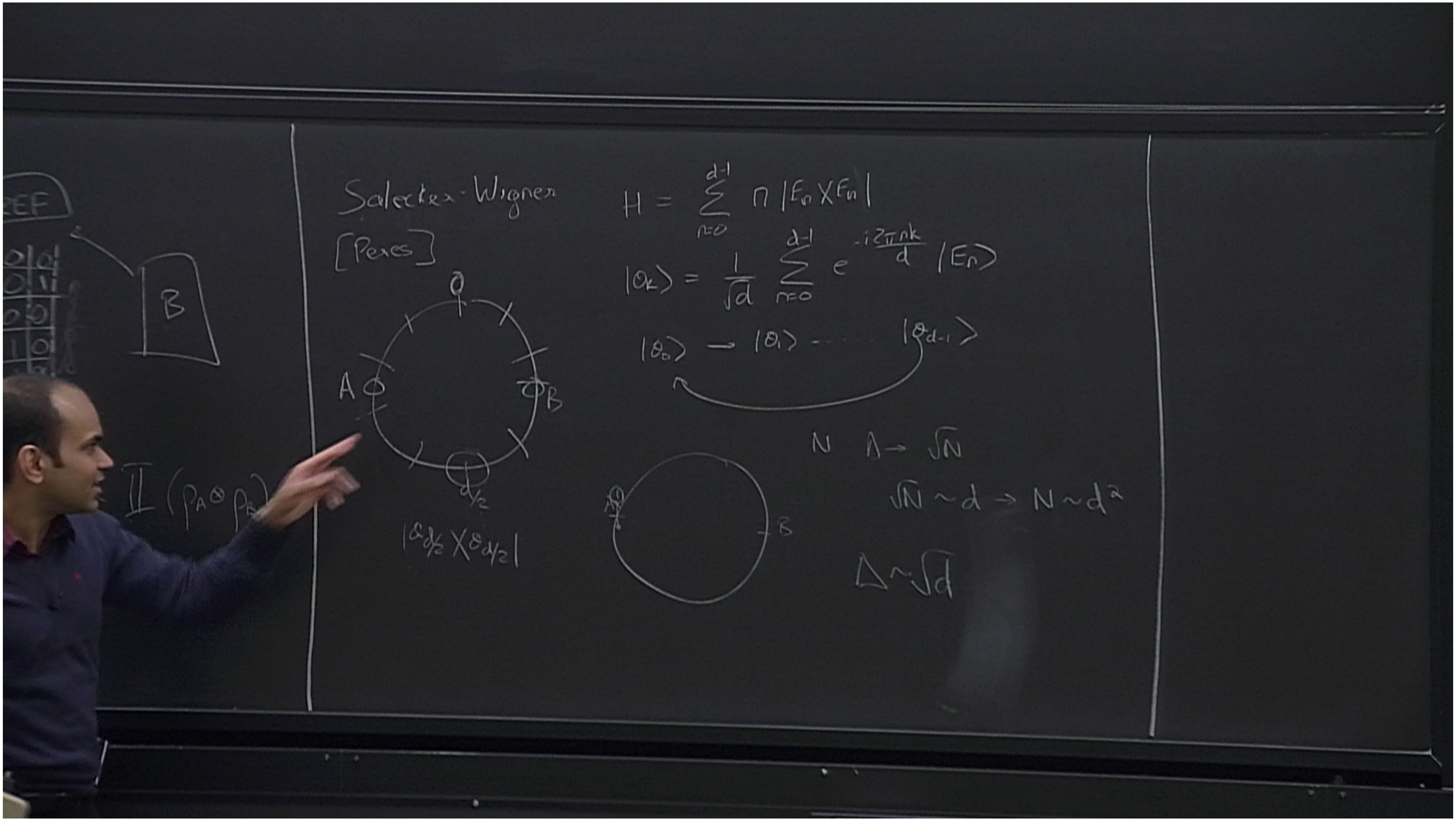
$$H = \sum_{n=0}^{d-1} n\,|E_n\rangle\langle E_n|$$

$$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-\frac{i2\pi nk}{d}} |E_n\rangle$$

$$|\theta_0\rangle \to |\theta_1\rangle \cdots\cdots |\theta_{d-1}\rangle$$

$N \quad \Delta \to \sqrt{N}$

$\sqrt{N} \sim d \to N \sim d^2$



Salecker-Wigner

[Peres]

$$\mathbb{1}(\rho_A \otimes \rho_B)$$

$$|\theta_{d/2}\rangle\langle\theta_{d/2}|$$

$$H = \sum_{n=0}^{d-1} n\,|E_n\rangle\langle E_n|$$

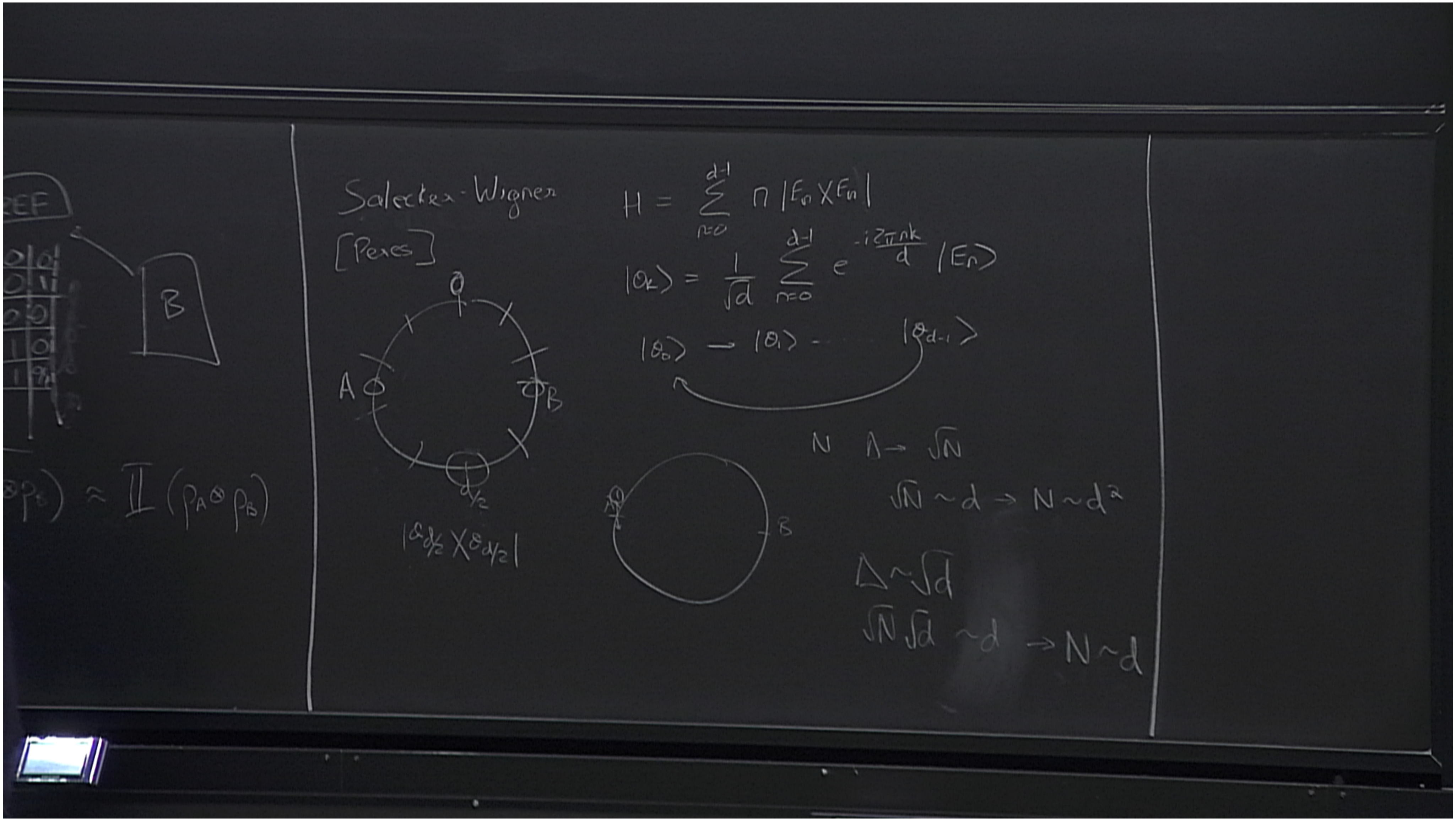
$$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i\frac{2\pi n k}{d}} |E_n\rangle$$

$$|\theta_0\rangle \rightarrow |\theta_1\rangle \cdots\cdots |\theta_{d-1}\rangle$$

$N \quad \Delta \rightarrow \sqrt{N}$

$\sqrt{N} \sim d \rightarrow N \sim d^2$

$\Delta \sim \sqrt{d}$



$\otimes\rho_B) \approx \mathbb{I}(\rho_A \otimes \rho_B)$

Salecker-Wigner

[Peres]

$|\theta_{d/2}\rangle\langle\theta_{d/2}|$

$$H = \sum_{n=0}^{d-1} n |E_n\rangle\langle E_n|$$

$$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i\frac{2\pi nk}{d}} |E_n\rangle$$

$$|\theta_0\rangle \rightarrow |\theta_1\rangle \cdots \quad |\theta_{d-1}\rangle$$

$N \quad \Delta \rightarrow \sqrt{N}$

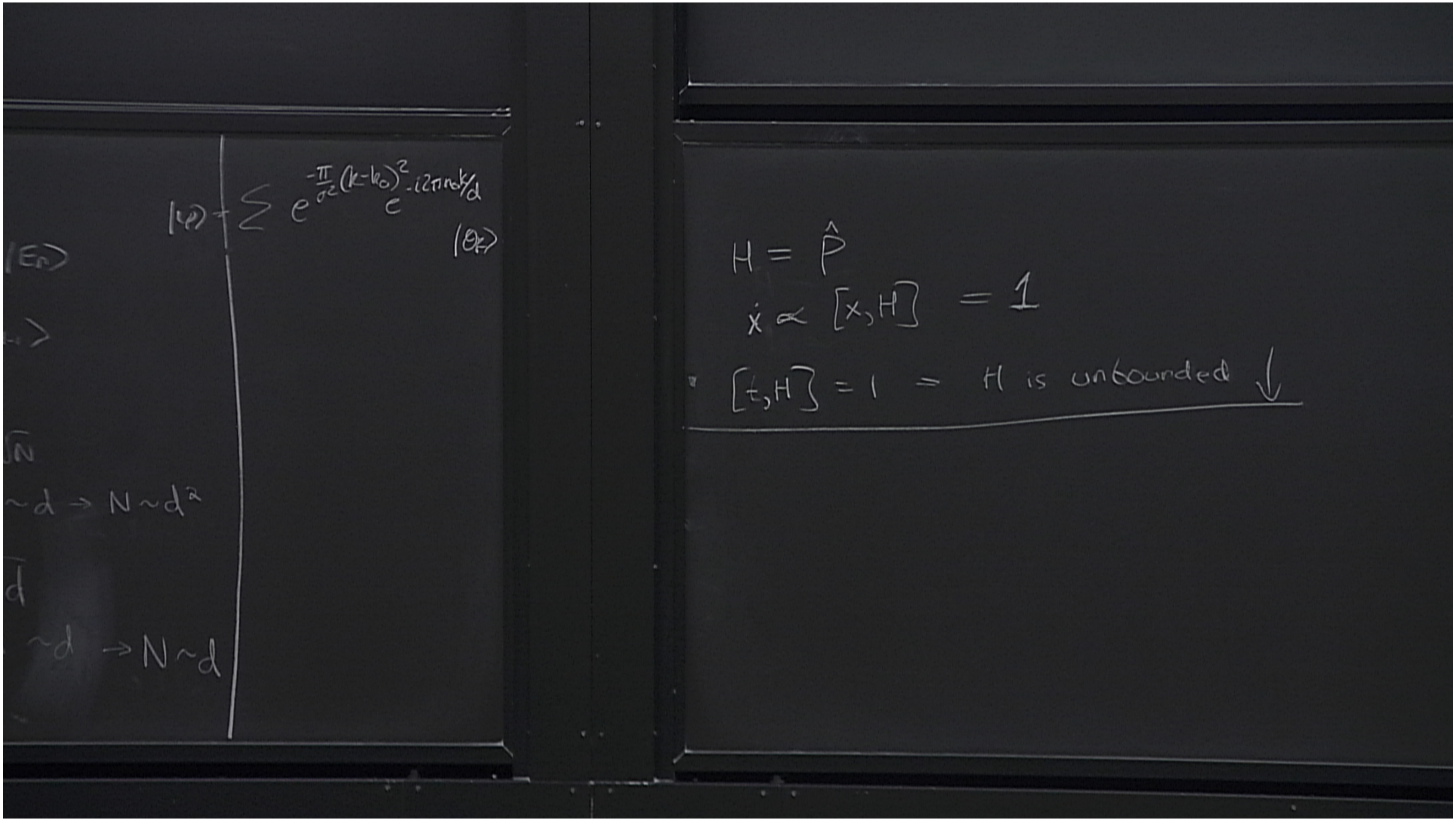
$\sqrt{N} \sim d \rightarrow N \sim d^2$

$\Delta \sim \sqrt{d}$

$\sqrt{N}\sqrt{d} \sim d \rightarrow N \sim d$

$$H = \hat{P}$$

$$\dot{x} \propto [x, H] = 1$$



$|E_n\rangle$

$\rangle$

$\sqrt{N}$

$\sim d \rightarrow N \sim d^2$

d

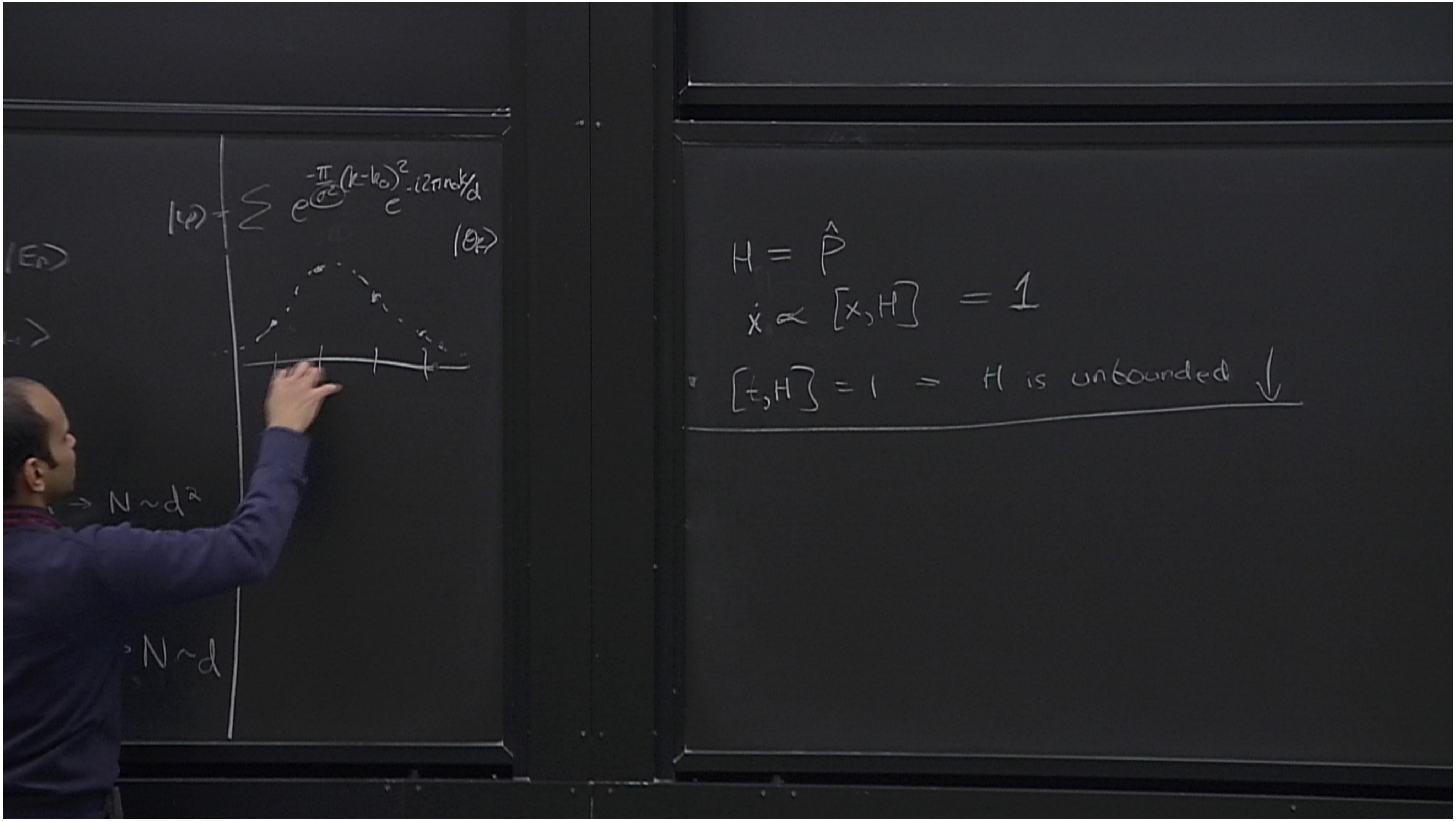
$\sim d \rightarrow N \sim d$

$$|\psi\rangle = \sum e^{-\frac{\pi}{\sigma^2}(k-k_0)^2} e^{-i2\pi n_0 k/d} |\theta_k\rangle$$

$$H = \hat{P}$$

$$\dot{x} \propto [x, H] = 1$$

$[t, H] = 1$ = H is unbounded ↓



$|\psi\rangle = \sum e^{-\frac{\pi}{\sigma^2}(k-k_0)^2} e^{-i2\pi n_0 k/d} |\theta_k\rangle$

$|E_n\rangle$

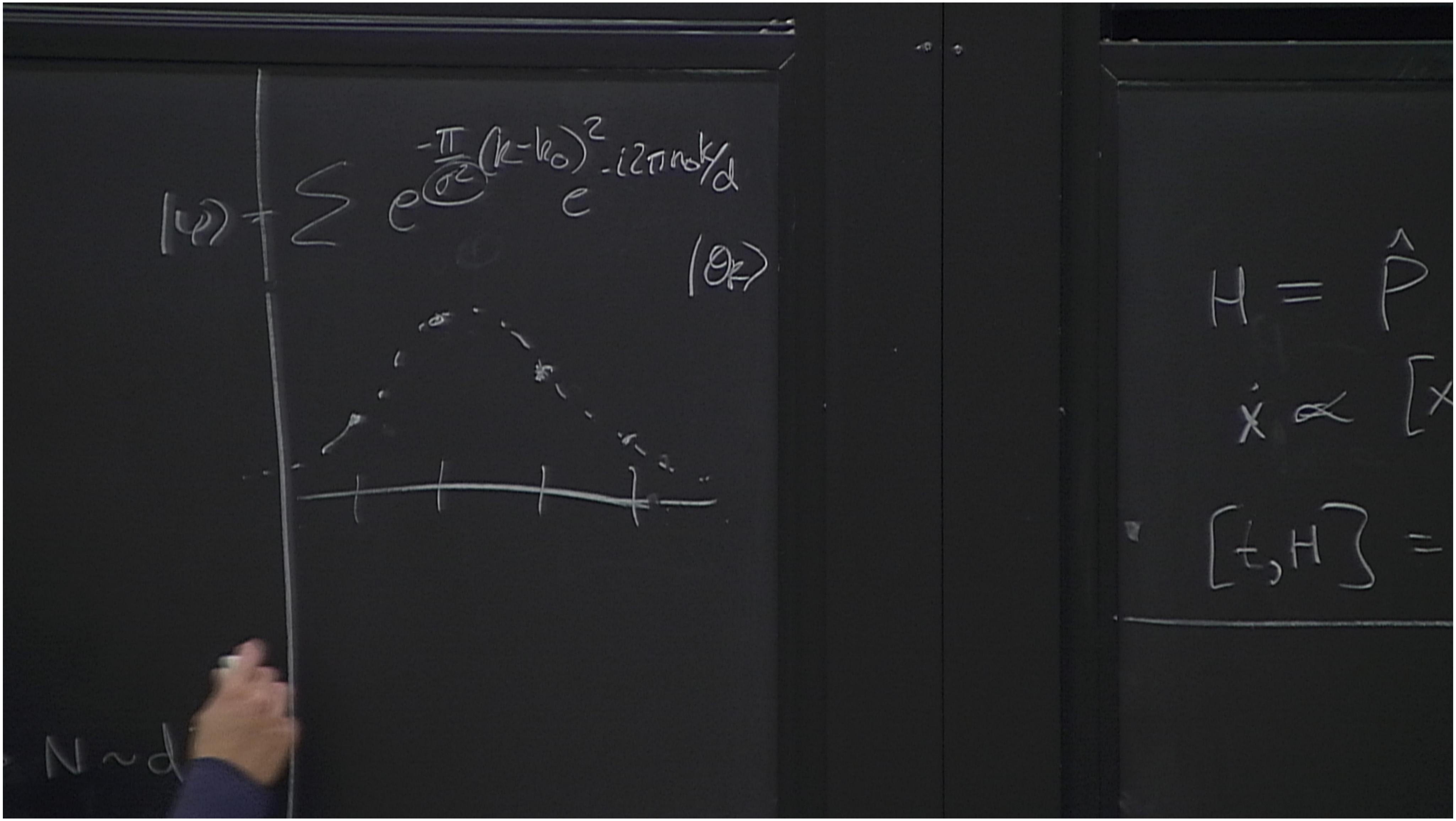
$\rightarrow N \sim d^2$

$N \sim d$

$H = \hat{P}$

$\dot{x} \propto [x, H] = 1$

$[t, H] = 1 \Rightarrow H$ is unbounded $\downarrow$



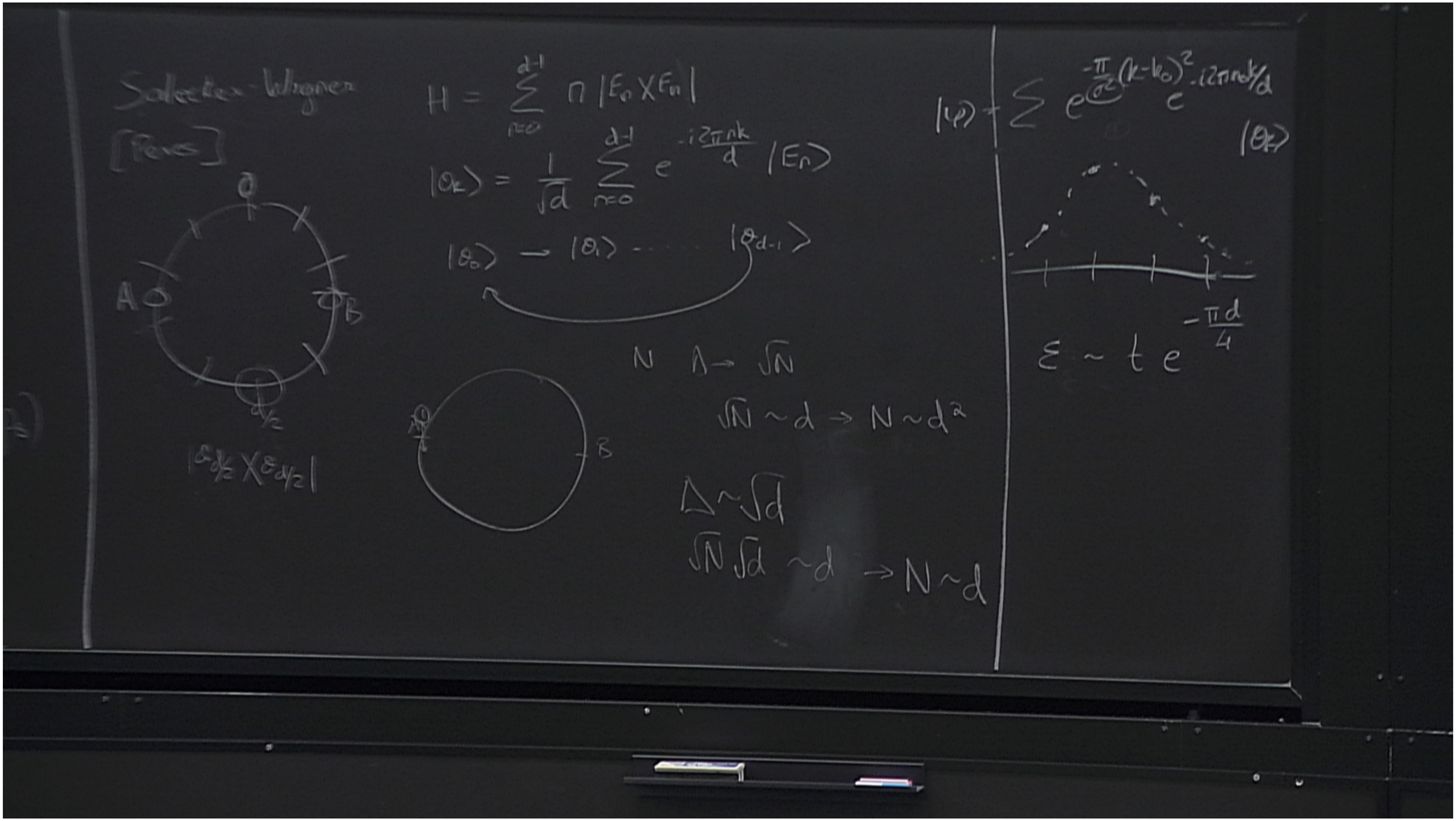
$$|\psi\rangle = \sum e^{-\frac{\pi}{\boxed{\sigma^2}}(k-k_0)^2} e^{-i2\pi n_0 k/d} \, |\theta_k\rangle$$

$N \sim d$

$H = \hat{P}$

$\dot{x} \propto [x$

$[t, H] =$



Salecker-Wigner

[Peres]

$|E_{d/2}\rangle\langle E_{d/2}|$

$$H = \sum_{n=0}^{d-1} n\,|E_n\rangle\langle E_n|$$

$$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i\frac{2\pi nk}{d}}\,|E_n\rangle$$

$$|\theta_0\rangle \longrightarrow |\theta_1\rangle \cdots\cdots |\theta_{d-1}\rangle$$

$N \quad \Delta \to \sqrt{N}$

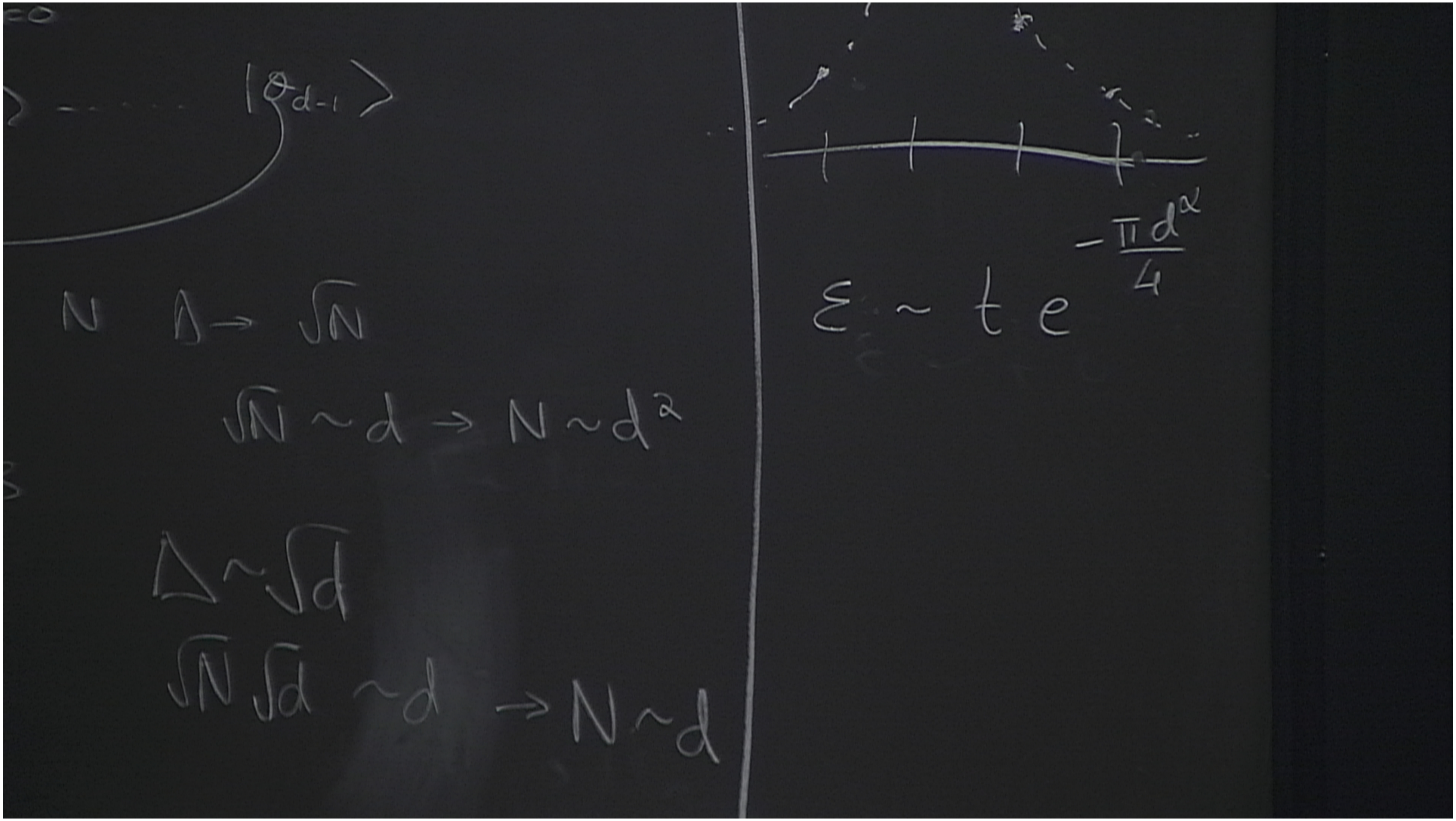
$\sqrt{N} \sim d \to N \sim d^2$

$\Delta \sim \sqrt{d}$

$\sqrt{N}\sqrt{d} \sim d \to N \sim d$

$$|\psi\rangle = \sum e^{-\frac{\pi}{\sigma^2}(k-k_0)^2} e^{-i2\pi n k_0/d}\,|\theta_k\rangle$$

$$\varepsilon \sim t\,e^{-\frac{\pi d}{4}}$$



$\rangle \cdots\cdots |\theta_{d-1}\rangle$

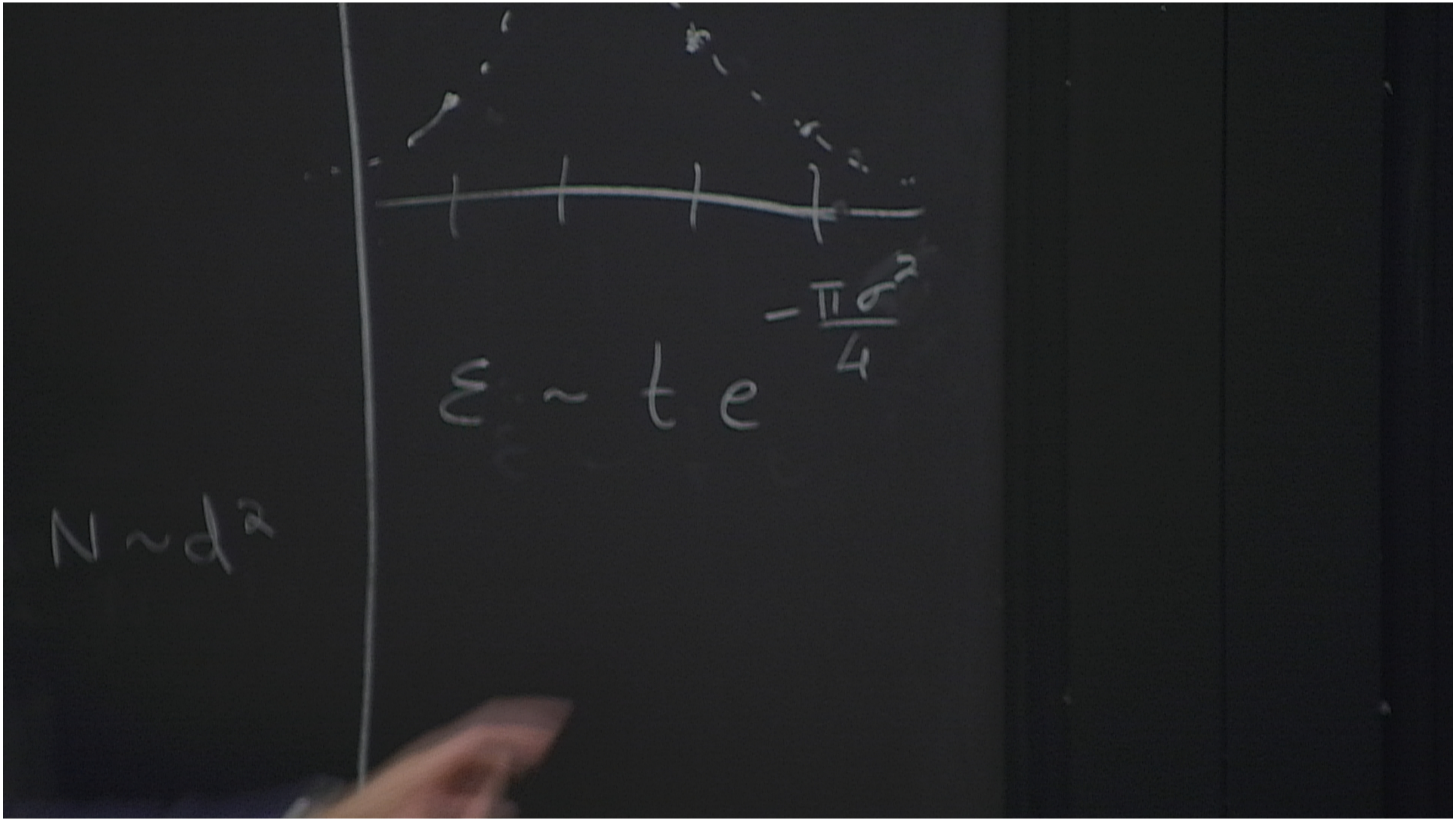
$N \quad \Delta \to \sqrt{N}$

$\sqrt{N} \sim d \to N \sim d^2$

$\Delta \sim \sqrt{d}$

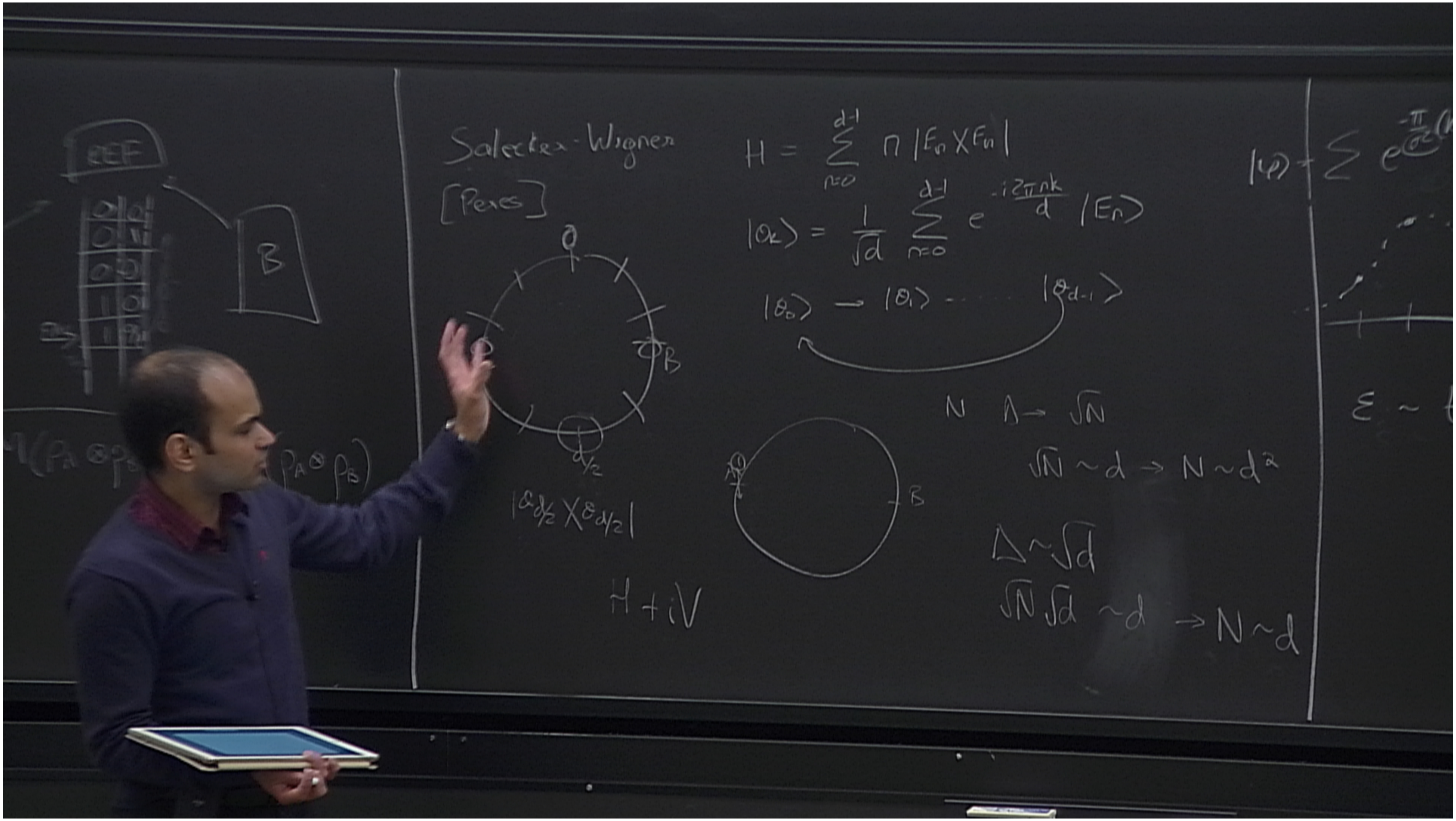
$\sqrt{N}\sqrt{d} \sim d \to N \sim d$

$\varepsilon \sim t\, e^{-\frac{\pi d^{\alpha}}{4}}$



$$\varepsilon \sim t\, e^{-\frac{\pi \sigma^2}{4}}$$

$$N \sim d^2$$



Salecker-Wigner
[Peres]

$|\theta_{d/2}\rangle\langle\theta_{d/2}|$

$H + iV$

$$H = \sum_{n=0}^{d-1} n\,|E_n\rangle\langle E_n|$$

$$|\theta_k\rangle = \frac{1}{\sqrt{d}}\sum_{n=0}^{d-1} e^{-\frac{i2\pi nk}{d}}|E_n\rangle$$

$$|\theta_0\rangle \rightarrow |\theta_1\rangle \cdots\cdots |\theta_{d-1}\rangle$$

$N \quad \Delta \rightarrow \sqrt{N}$

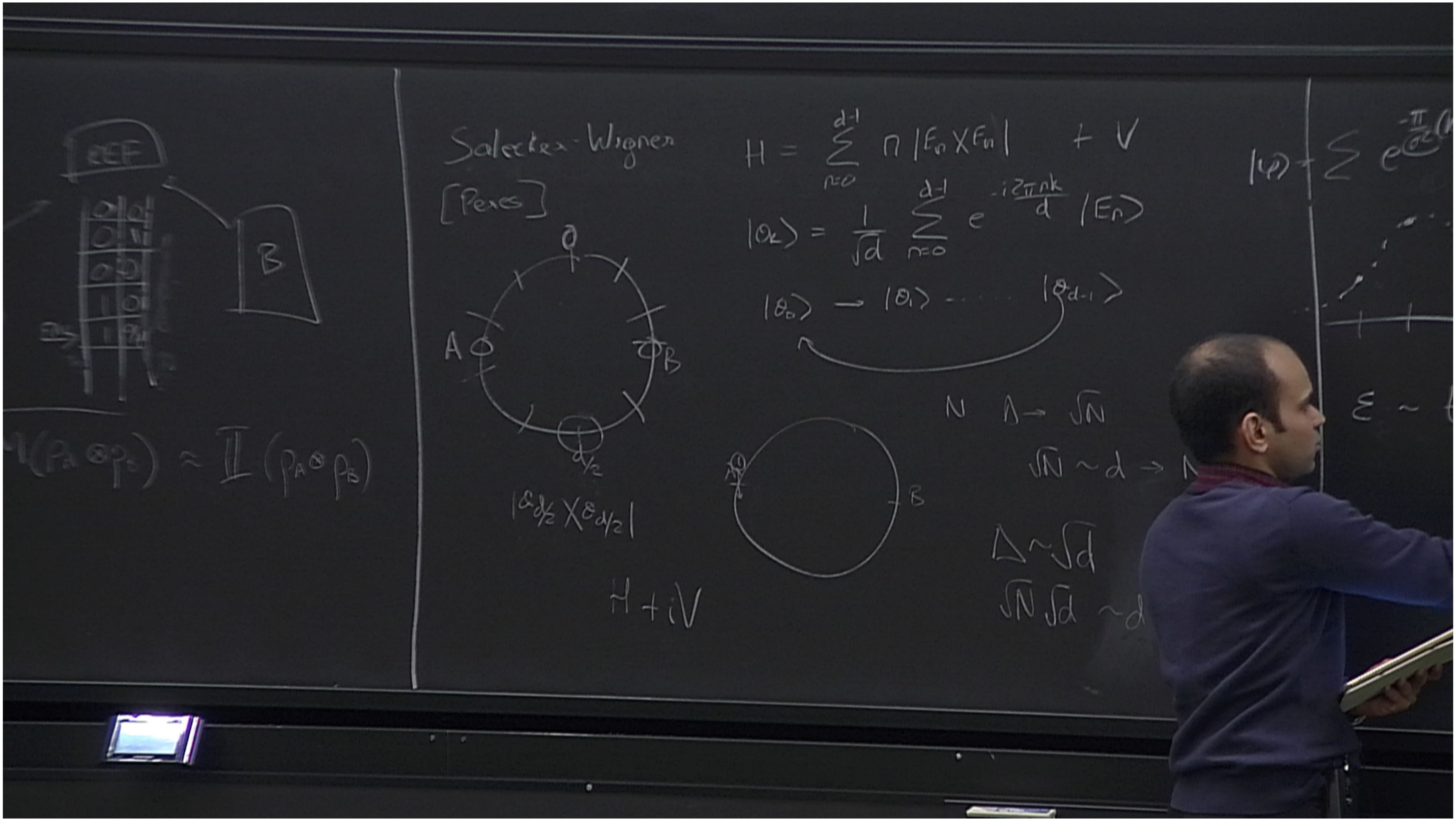
$\sqrt{N} \sim d \rightarrow N \sim d^2$

$\Delta \sim \sqrt{d}$

$\sqrt{N}\sqrt{d} \sim d \rightarrow N \sim d$

$|\psi\rangle = \sum e^{-\frac{\pi}{\sigma^2}(}$

$\varepsilon \sim$



$\Lambda(\rho_A \otimes \rho_B) \approx \mathbb{I}(\rho_A \otimes \rho_B)$

Salecker-Wigner

[Peres]

$|\theta_{d/2}\rangle\langle\theta_{d/2}|$

$H + iV$

$$H = \sum_{n=0}^{d-1} n\,|E_n\rangle\langle E_n| + V$$

$$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i\frac{2\pi nk}{d}} |E_n\rangle$$

$$|\theta_0\rangle \to |\theta_1\rangle \cdots\cdots |\theta_{d-1}\rangle$$

$N \quad \Delta \to \sqrt{N}$

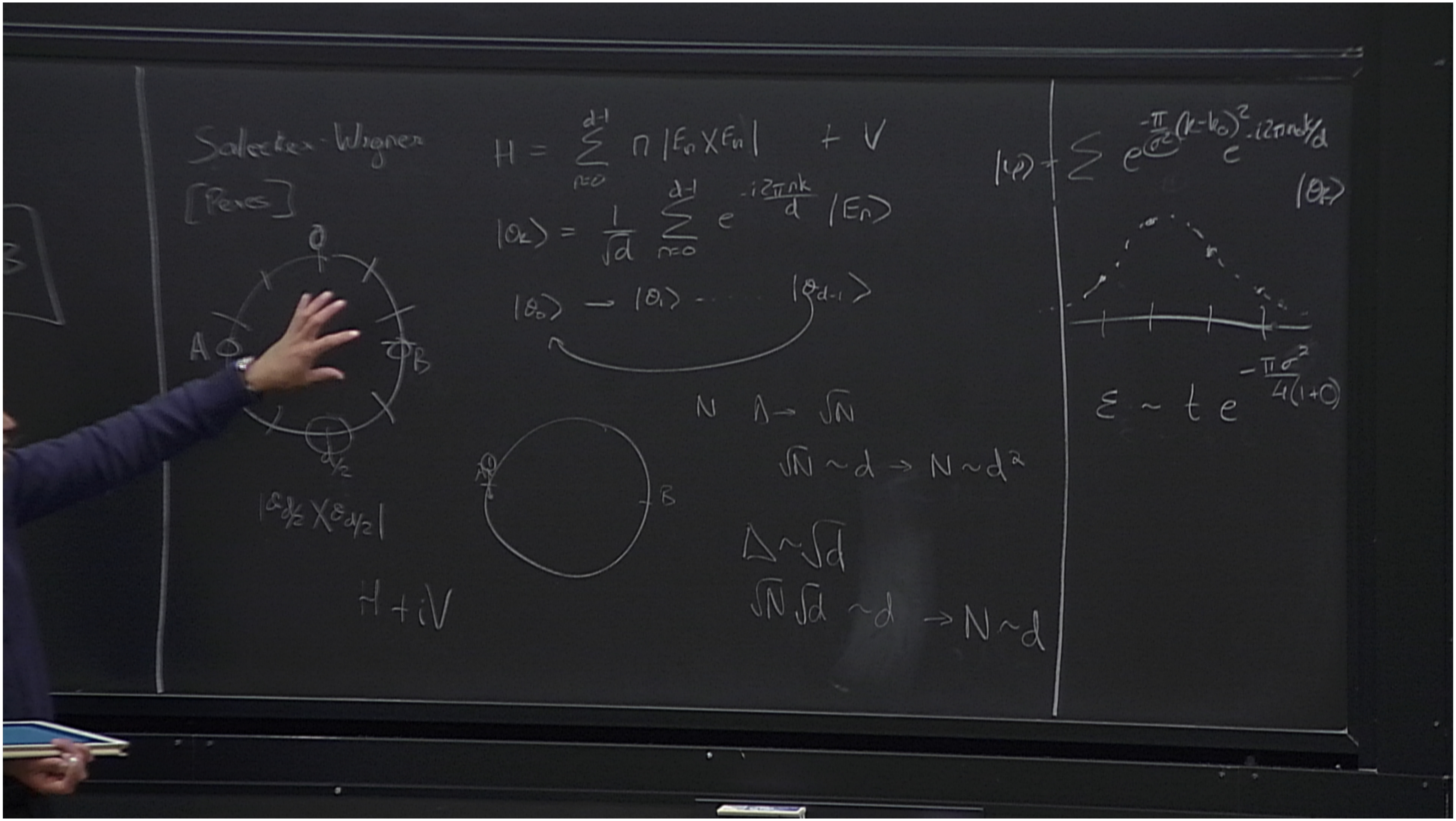
$\sqrt{N} \sim d \to N$

$\Delta \sim \sqrt{d}$

$\sqrt{N}\sqrt{d} \sim d$

$|\psi\rangle = \sum e^{\frac{-\pi}{\sigma^2}(}$

$\varepsilon \sim$



Salecker-Wigner

[Peres]

$|E_{d/2}\rangle\langle E_{d/2}|$

$H + iV$

$$H = \sum_{n=0}^{d-1} n |E_n\rangle\langle E_n| \quad + V$$

$$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i\frac{2\pi nk}{d}} |E_n\rangle$$

$$|\theta_0\rangle \to |\theta_1\rangle \cdots\cdots |\theta_{d-1}\rangle$$

$N \quad \Delta \to \sqrt{N}$

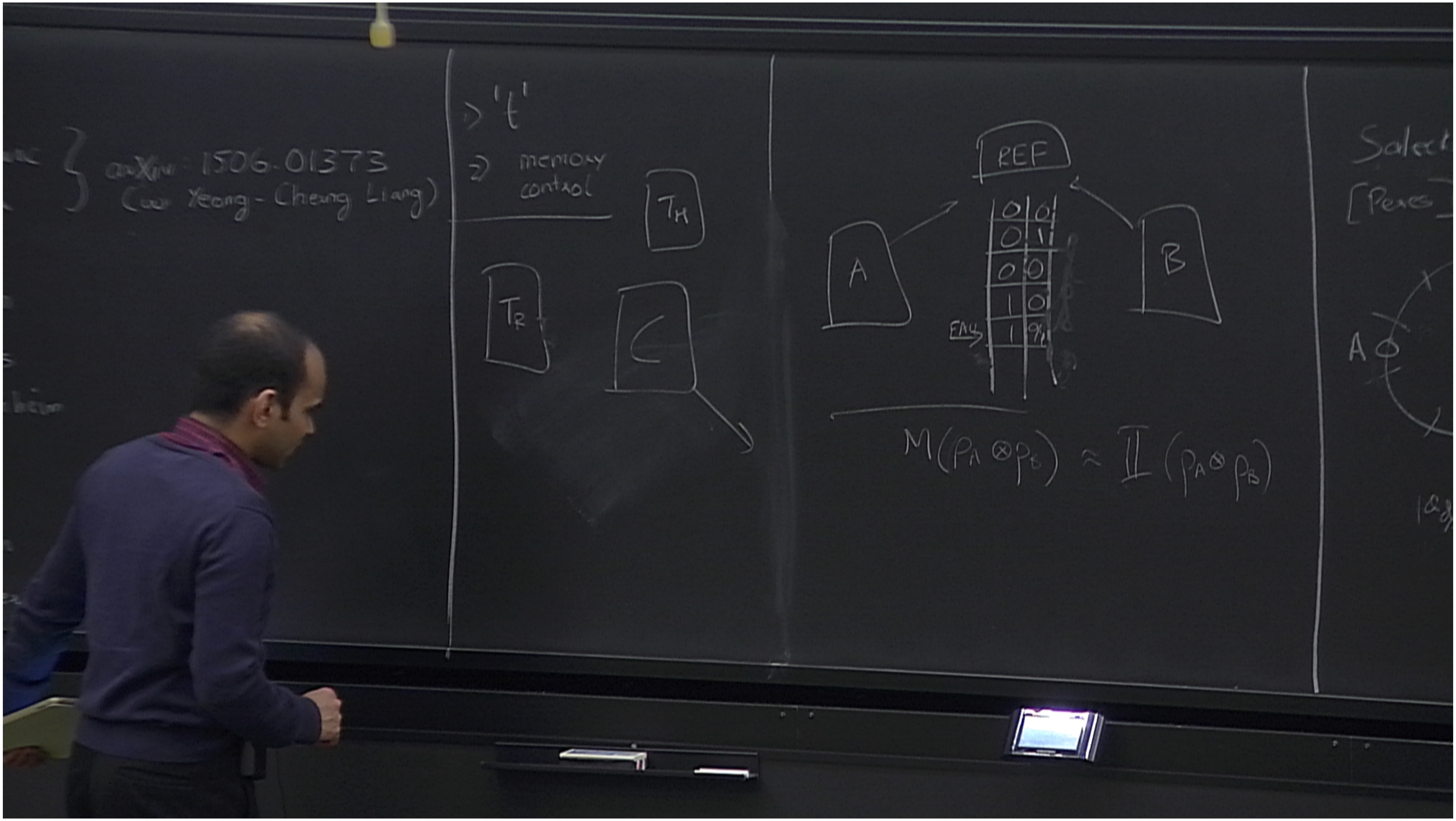
$\sqrt{N} \sim d \to N \sim d^2$

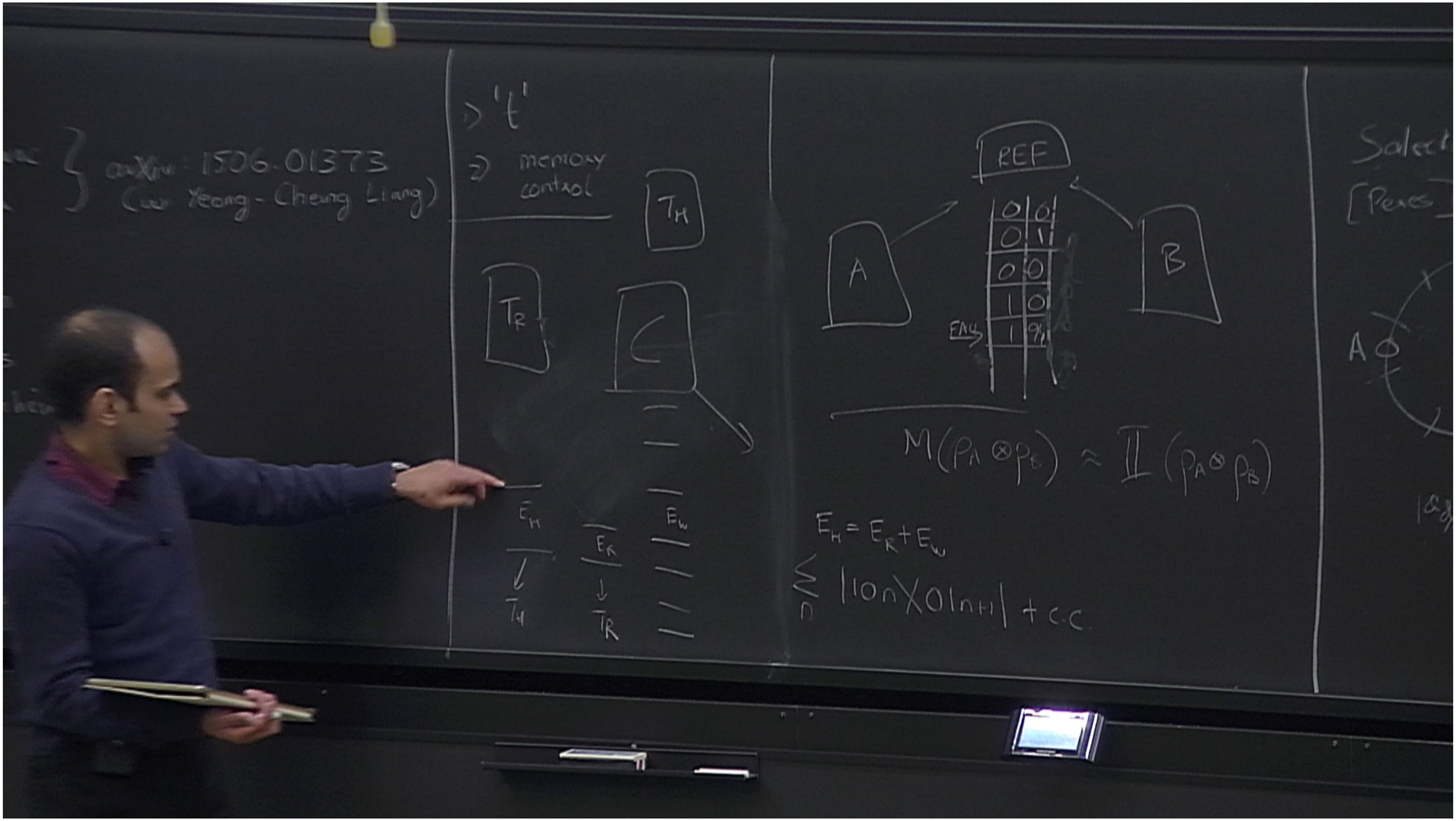
$\Delta \sim \sqrt{d}$

$\sqrt{N}\sqrt{d} \sim d \to N \sim d$

$$|\psi\rangle = \sum e^{-\frac{\pi}{\sigma^2}(k-k_0)^2} e^{-i2\pi n_0 k/d} |\theta_k\rangle$$

$$\varepsilon \sim t\, e^{-\frac{\pi\sigma^2}{4(1+O)}}$$


arXiv: 1506.01373
(w/ Yeong-Cherng Liang)
> 't'
⇒ memory control
T_H
T_R
C
REF
A
B
0 0
0 1
0 0
1 0
1 φ_1
$M(\rho_A \otimes \rho_B) \approx \mathbb{I}(\rho_A \otimes \rho_B)$
Select
[Peres
A

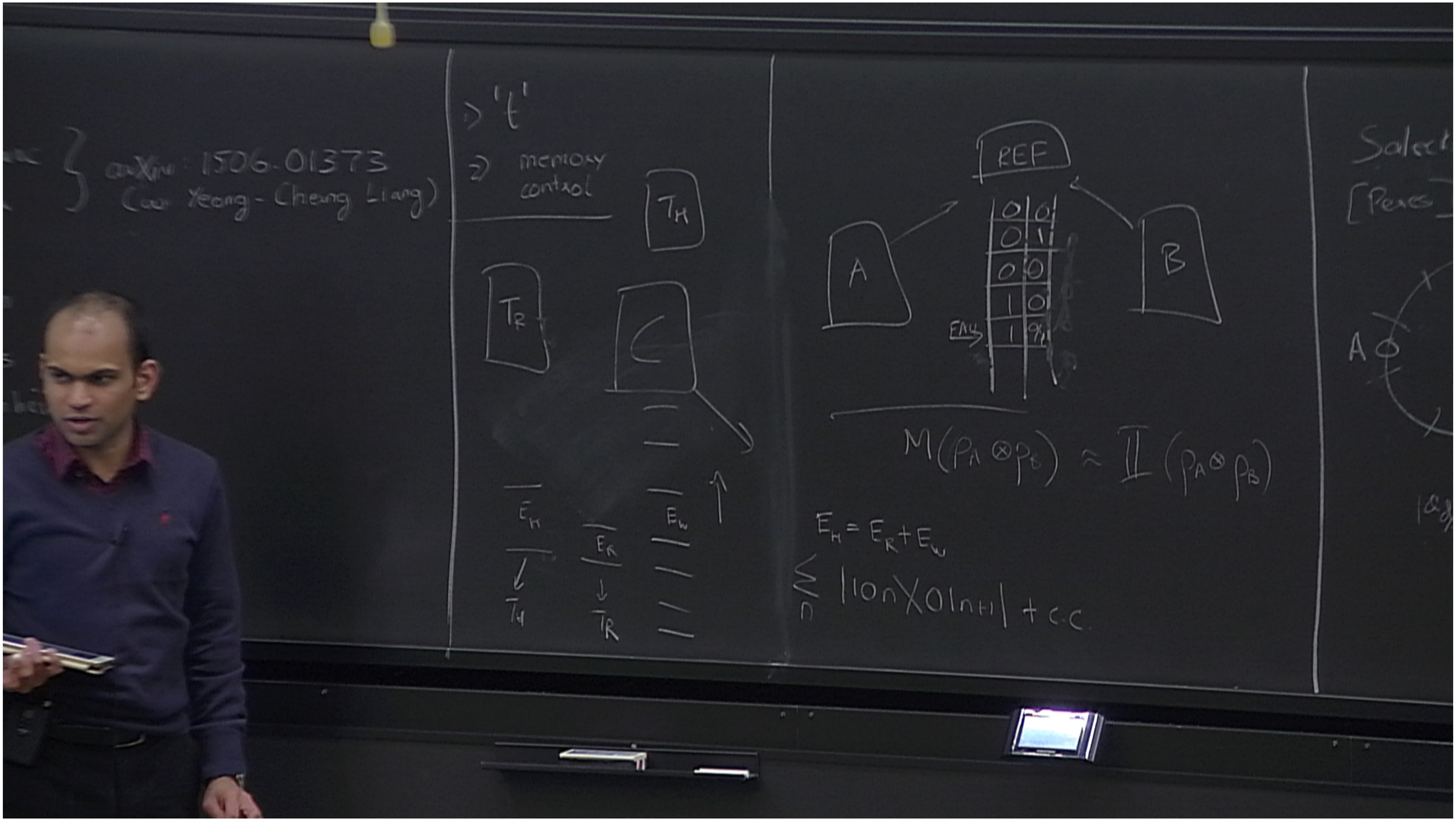


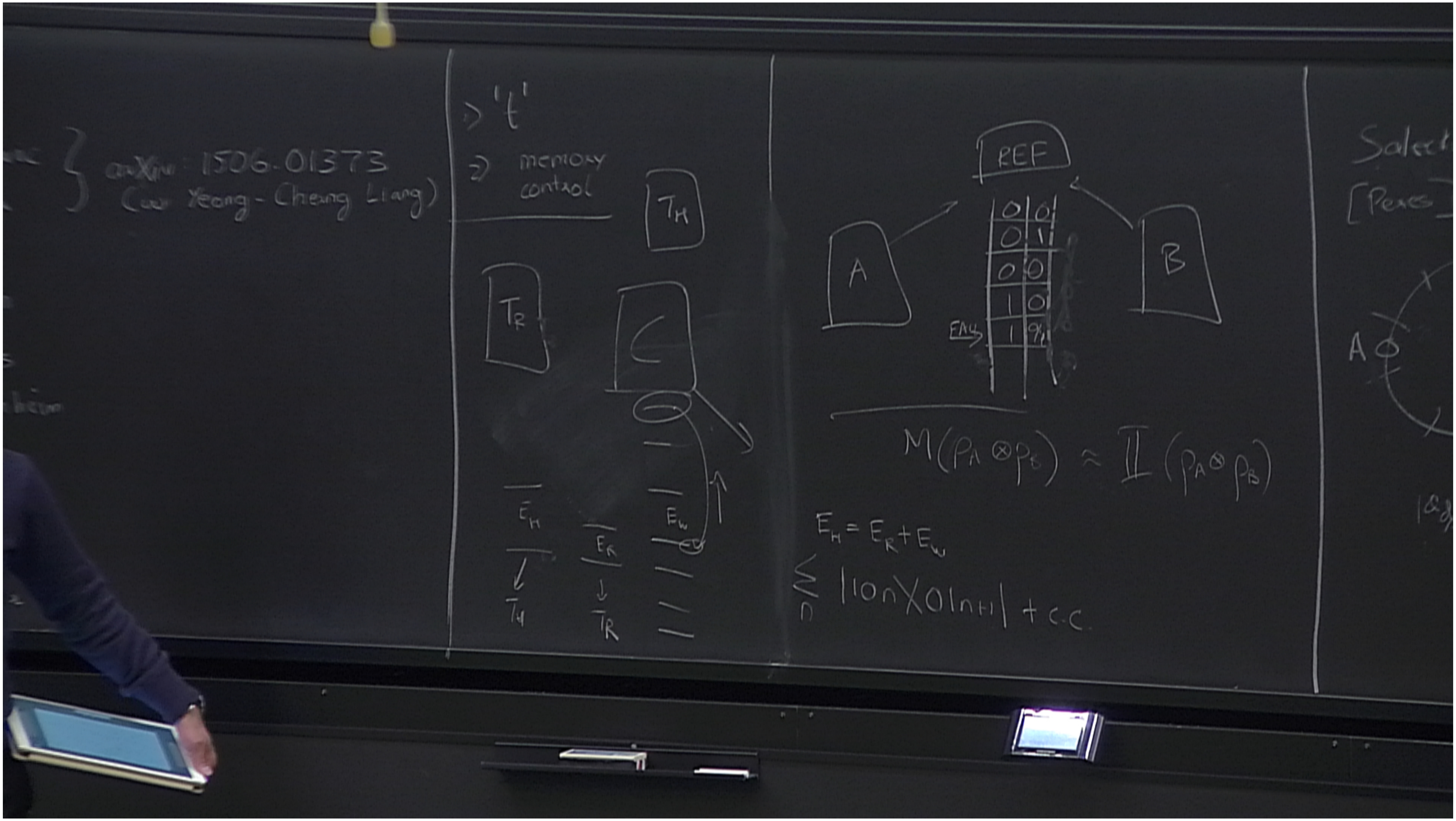
0	0
0	1
0	0
1	0
1	0/1

$$M(\rho_A \otimes \rho_B) \approx \mathbb{I}(\rho_A \otimes \rho_B)$$

$$E_H = E_R + E_W$$

$$\sum_n |1 0_n\rangle\langle 0 1_{n+1}| + c.c.$$

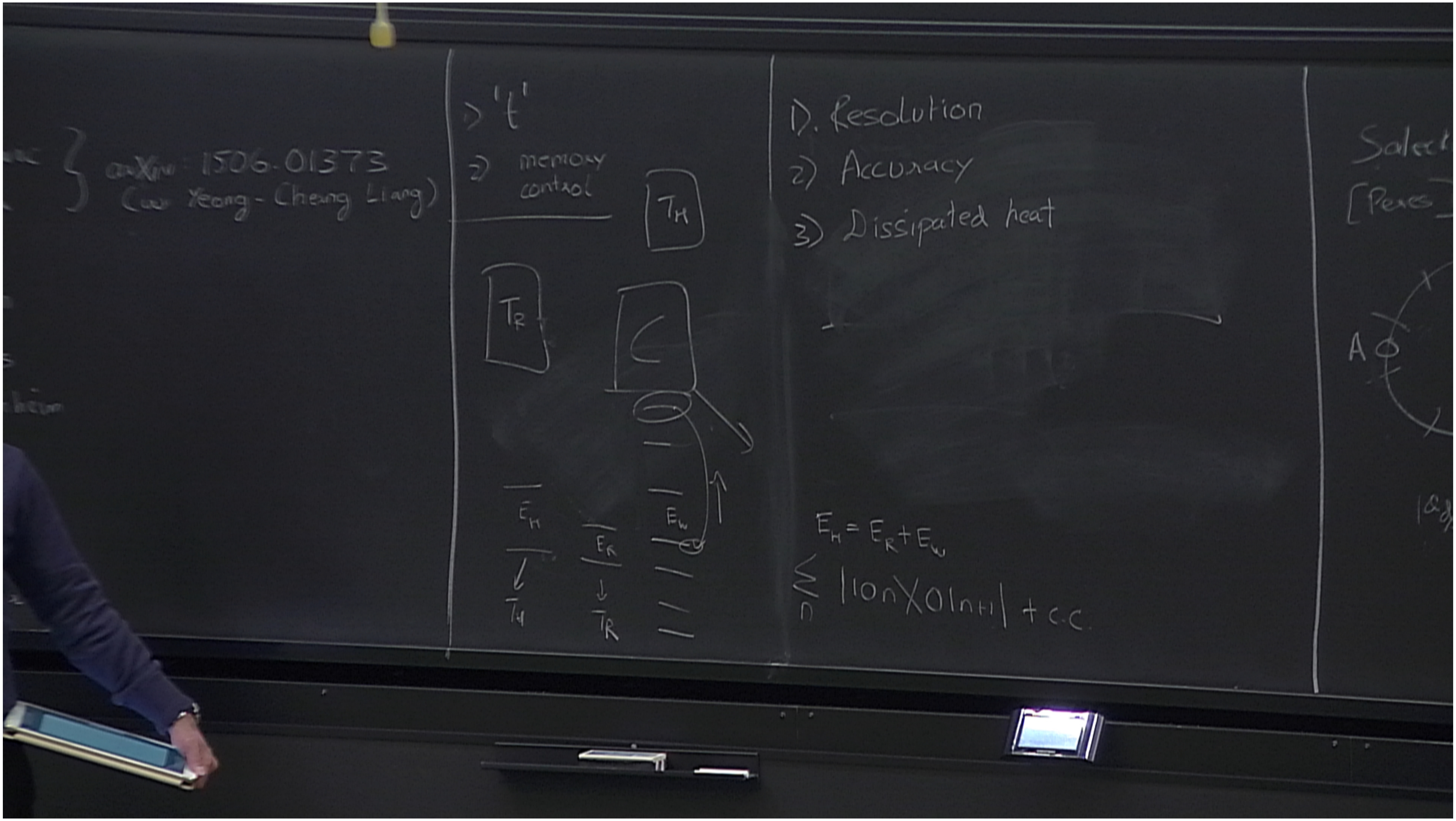

arXiv: 1506.01373
(w/ Yeong-Cheng Liang)
> 't'
⇒ memory control
T_H
T_R
E_H
E_R
E_w
REF
A
B
$M(\rho_A \otimes \rho_B) \approx \mathbb{1}(\rho_A \otimes \rho_B)$
$E_H = E_R + E_w$
$\sum_n |1 0_n\rangle\langle 0 1_{n+1}| + c.c.$
Select
[Peres
A



$$M(\rho_A \otimes \rho_B) \approx \mathbb{I}(\rho_A \otimes \rho_B)$$

$$E_H = E_R + E_W$$

$$\sum_n |1 0_n\rangle\langle 0 1_{n+1}| + c.c.$$



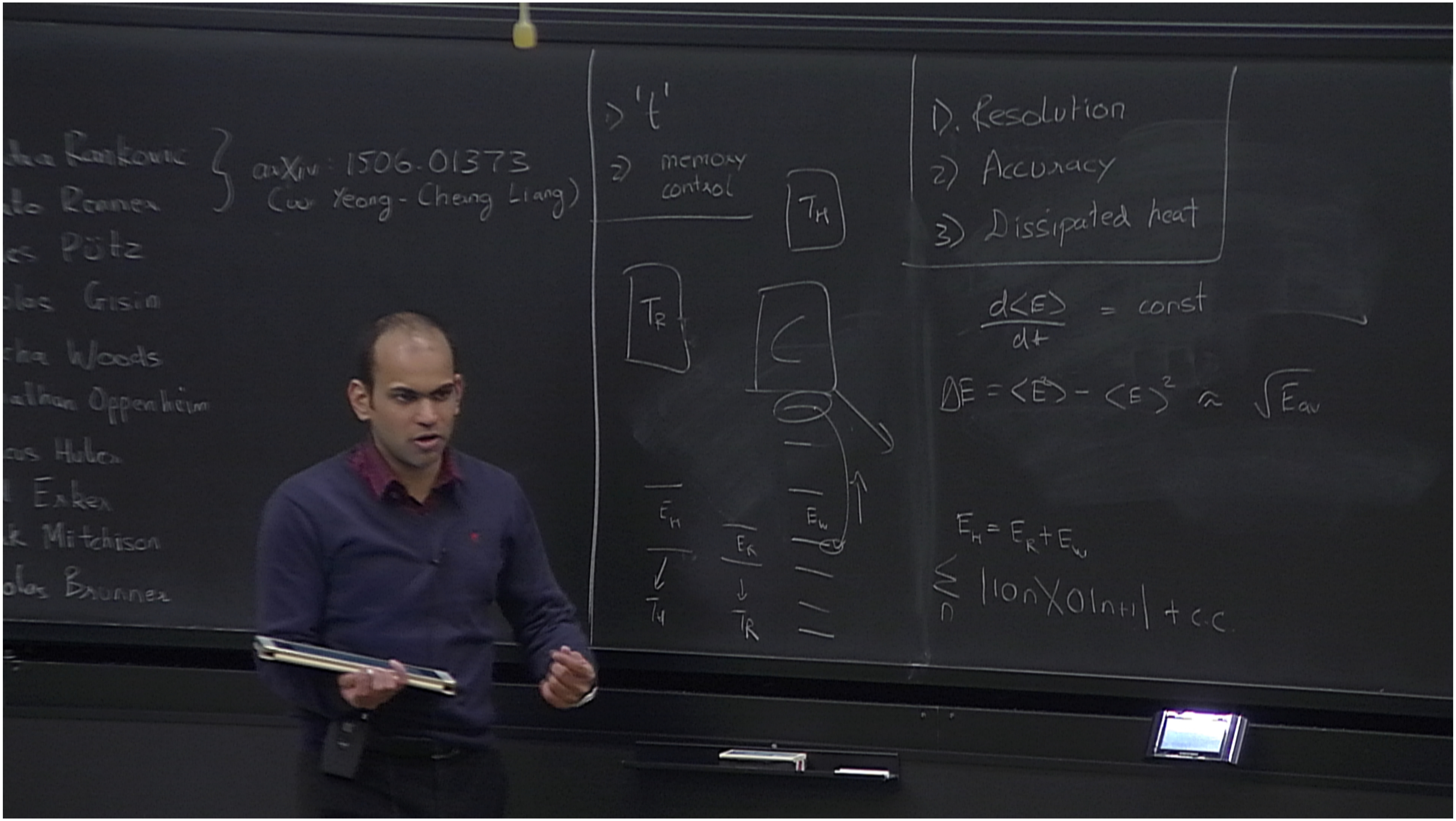
1). Resolution

2) Accuracy

3) Dissipated heat

$$E_H = E_R + E_W$$

$$\sum_n |1 0 n\rangle\langle 0 1 n{+}1| + c.c.$$



ka Rankovic
ato Renner
es Pütz
olas Gisin
cha Woods
athan Oppenheim
cus Huber
l Exner
k Mitchison
olas Brunner

} arXiv: 1506.01373
(w/ Yeong-Cheng Liang)

1) 't'
2) memory control

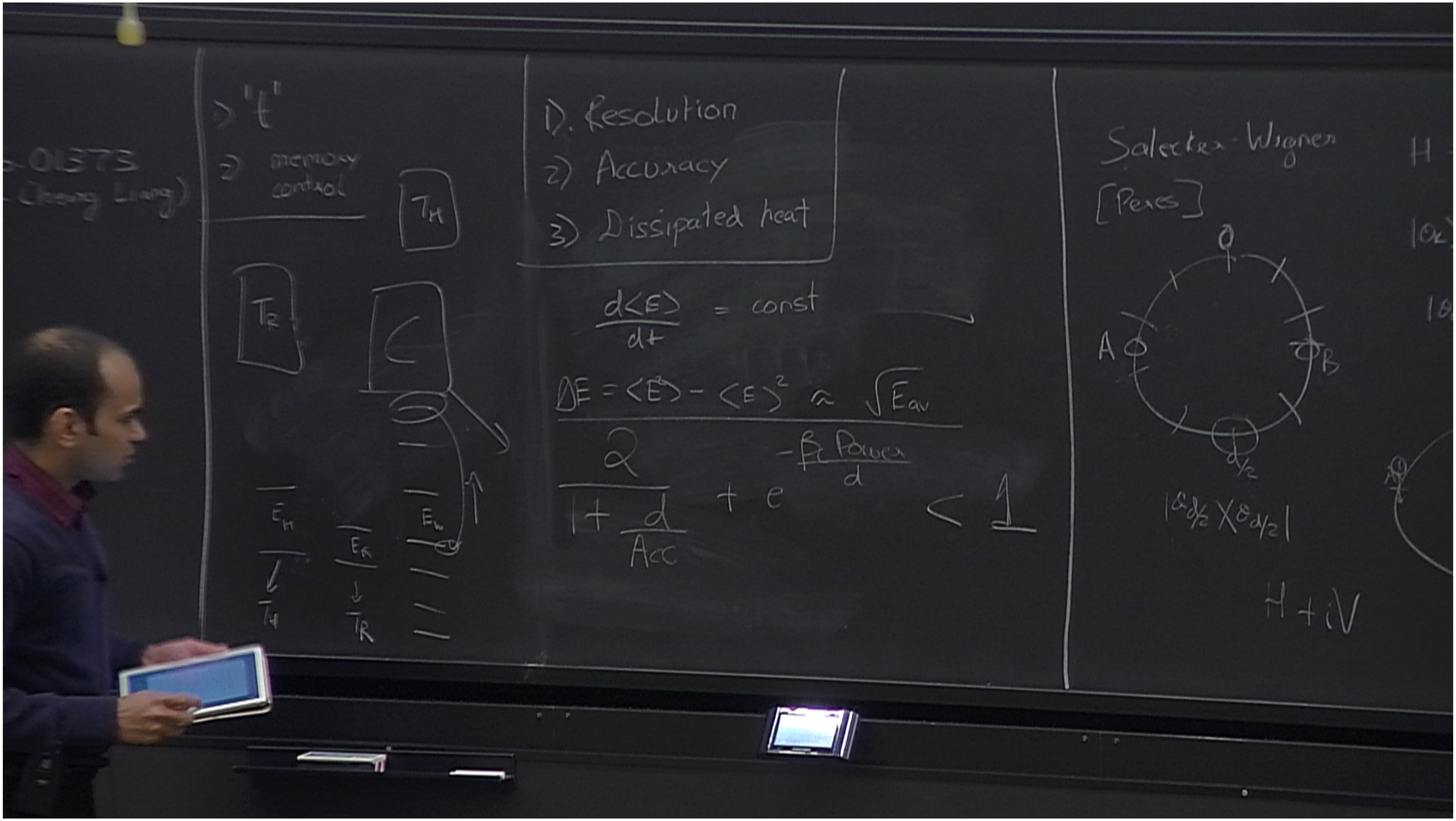
1). Resolution
2) Accuracy
3) Dissipated heat

$$\frac{d\langle E\rangle}{dt} = \text{const}$$

$$\Delta E = \langle E^2\rangle - \langle E\rangle^2 \approx \sqrt{E_{av}}$$

$$E_H = E_R + E_w$$

$$\sum_n |1 0_n\rangle\langle 0 1_{n+1}| + c.c.$$



1). Resolution
2) Accuracy
3) Dissipated heat

$$\frac{d\langle E\rangle}{dt} = \text{const}$$

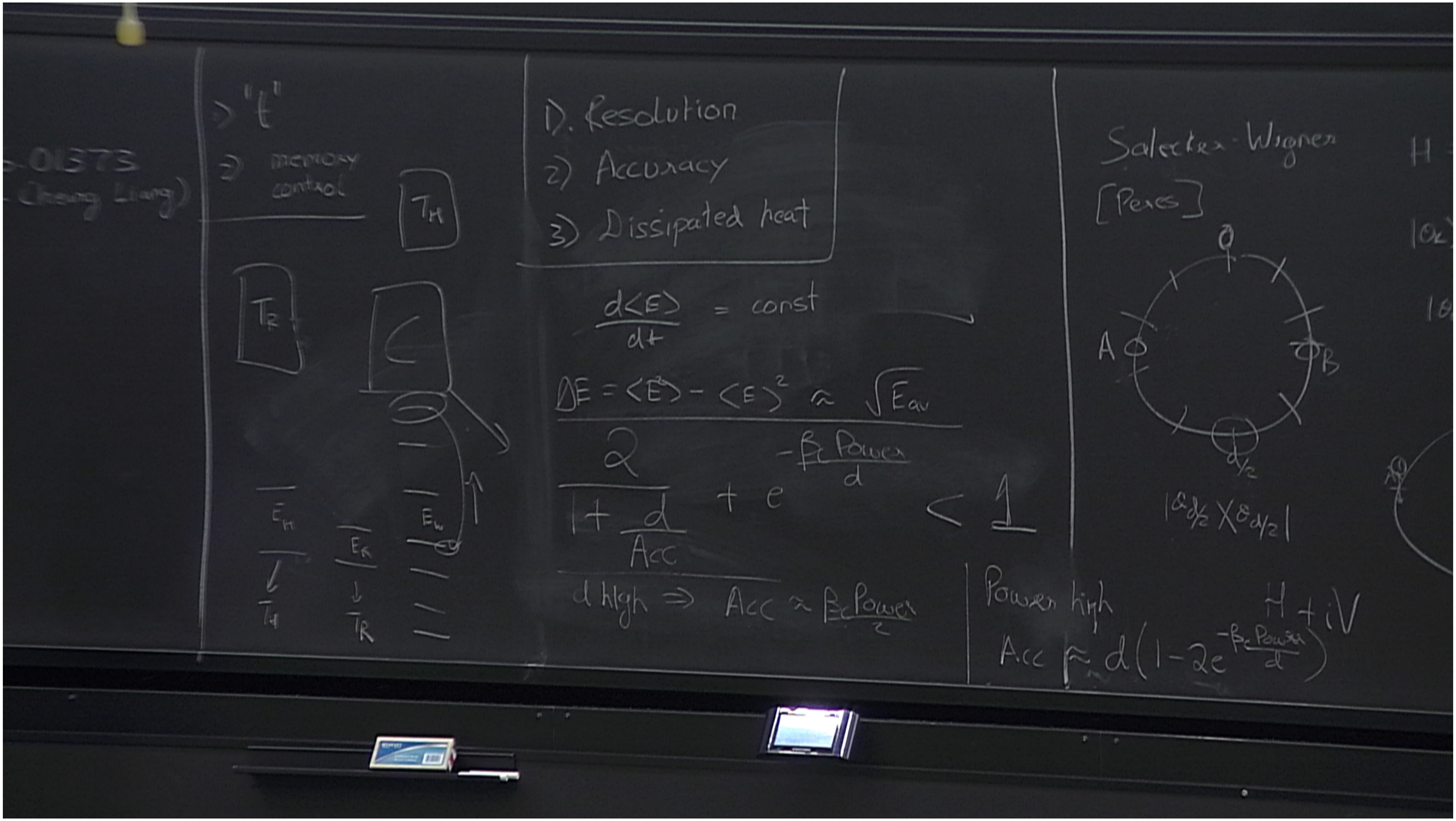
$$\Delta E = \langle E^2\rangle - \langle E\rangle^2 \approx \sqrt{E_{av}}$$

$$\frac{2}{1 + \frac{d}{Acc}} + e^{-\frac{\beta_c \text{Power}}{d}} < 1$$

Salecker-Wigner
[Peres]

$$|\theta_{d/2}\rangle\langle\theta_{d/2}|$$

$$H + iV$$



-01373

(Cheung Liang)

> "t"

⇒ memory control

T_H

T_R

E_H E_R E_w

T_H T_R

1). Resolution

2) Accuracy

3) Dissipated heat

$$\frac{d\langle E\rangle}{dt} = \text{const}$$

$$\Delta E = \langle E^2\rangle - \langle E\rangle^2 \approx \sqrt{E_{av}}$$

$$\frac{2}{1+\frac{d}{Acc}} + e^{-\frac{\beta_c Power}{d}} < 1$$

d high ⇒ $Acc \approx \frac{\beta_c Power}{2}$

Power high

$$Acc \approx d\left(1-2e^{-\frac{\beta_c Power}{d}}\right)$$

$H + iV$

Salecker-Wigner

[Peres]

$$|\theta_{d/2}\rangle\langle\theta_{d/2}|$$

H

$|\theta_k$

$|\theta$