

Title: Spin liquids on kagome lattice and symmetry protected topological phase

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Abstract: 

In my talk I will introduce the spin liquid phases that occur in kagome antiferromagnets, and discuss their physical origin that are closely related with the newly discovered symmetry protected topological phase (SPT). I will first present our numerical (DMRG) study on the kagome XXZ spin model that exhibits two distinct spin liquid phases, namely the chiral spin liquid and the kagome spin liquid (the groundstate of the nearest neighbor kagome Heisenberg model). Both phases extend from the extreme easy-axis limit, through

SU(2) symmetric point, to the pure easy-plane limit. The two phases are separated by a continuous phase transition. Motivated by these numerical results, I will then focus on the easy-axis kagome spin system, and reformulate it as a lattice gauge model. Such formulation enables us to achieve a controlled theoretical description for the spin liquid phases. We then show that the chiral spin liquid is indeed a gauged U(1) SPT phase. On the other hand, we also propose that the kagome spin liquid is a critical spin liquid phase, which can be considered as a gauged deconfined critical point between a SPT and a superfluid phase.

# Spin liquids on kagome lattice and Symmetry protected topological phase



MAX-PLANCK-GESELLSCHAFT

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(Apr 2016, Perimeter Institute)



DFG SFB  
1143

YCH, Fuji, and Bhattacharjee, arXiv:1512.05381 (2015).

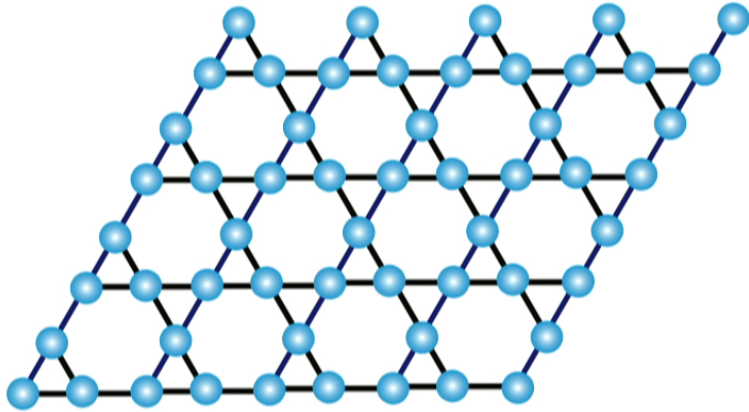
YCH, Bhattacharjee, Pollmann, and Moessner, PRL 115, 267209 (2015).

YCH, Bhattacharjee, Moessner, and Pollmann, PRL 115, 116803 (2015).

YCH and Chen, PRL 114, 037201 (2015).

YCH, Sheng and Chen, PRL 112, 137202 (2014).

# Spin liquids on kagome lattice



Kagome Heisenberg model

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j, \quad J > 0$$

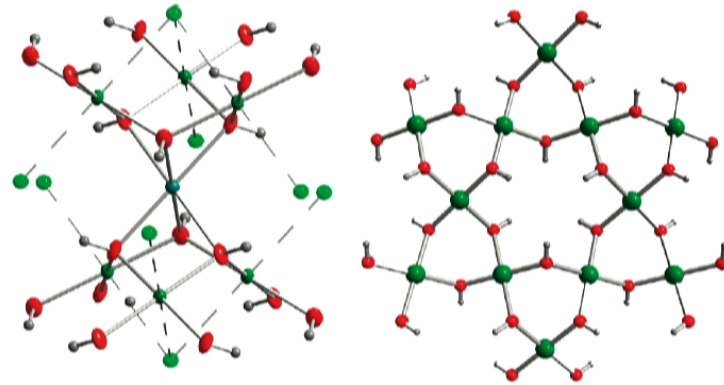
What is the ground state?

Yan, Huse, and White

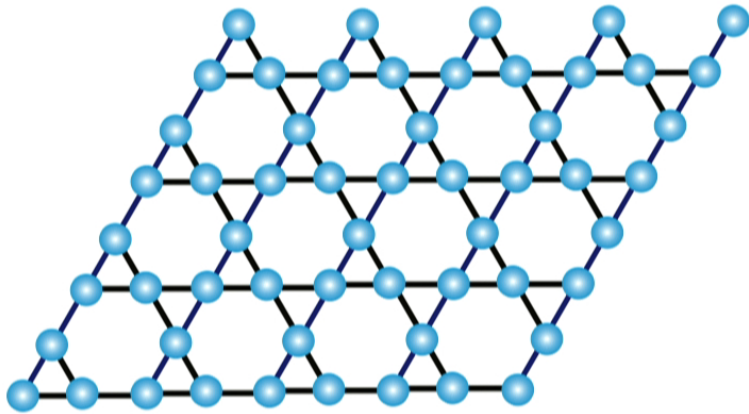
Spin liquid! But which one?

Herbersmithite  $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$

YS Lee group



# Spin liquids on kagome lattice



Kagome Heisenberg model

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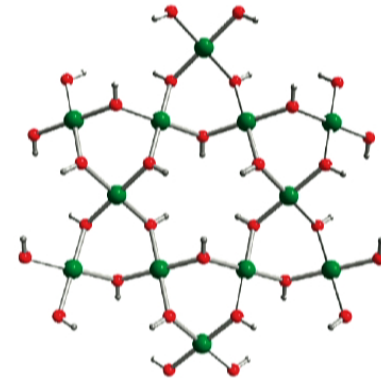
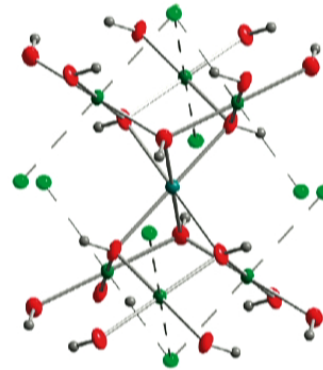
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YS Lee group





# Symmetry protected topological phase

interacting system

Chen, Gu, Liu, Wen

1D bosonic SPT: Haldane's spin-1 chain

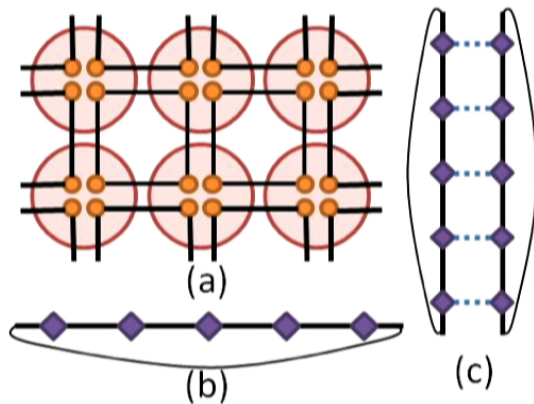


projective symmetry group

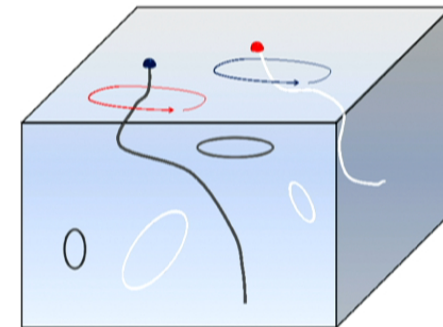
Pollmann, Turner, Berg, Oshikawa

beyond 1D: cohomology group

Chen, Gu, Liu, Wen



Chen, Liu, Wen

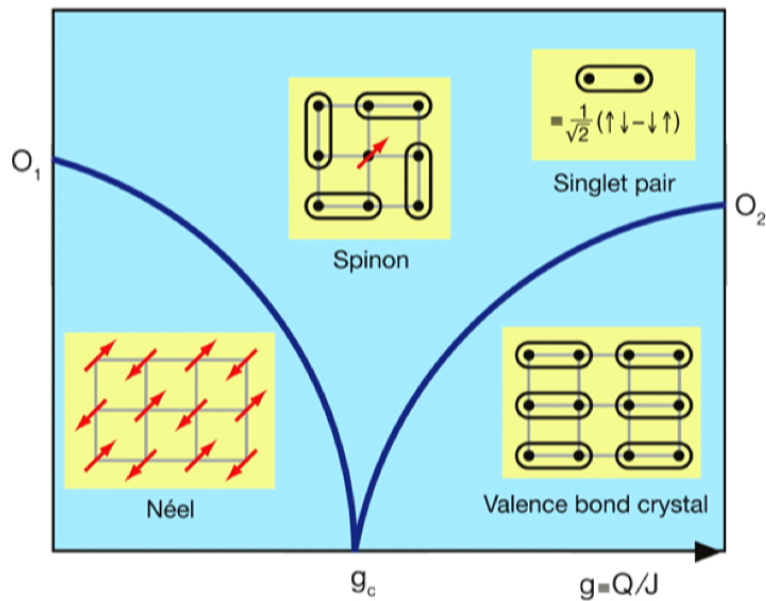


Vishwanath, Senthil

# Deconfined criticality

Senthil, Vishwanath, Balents, Sachdev, Fisher

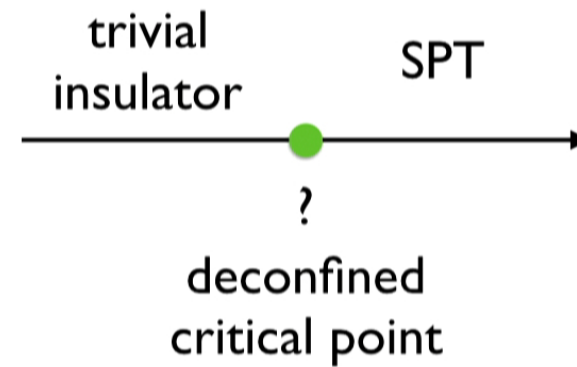
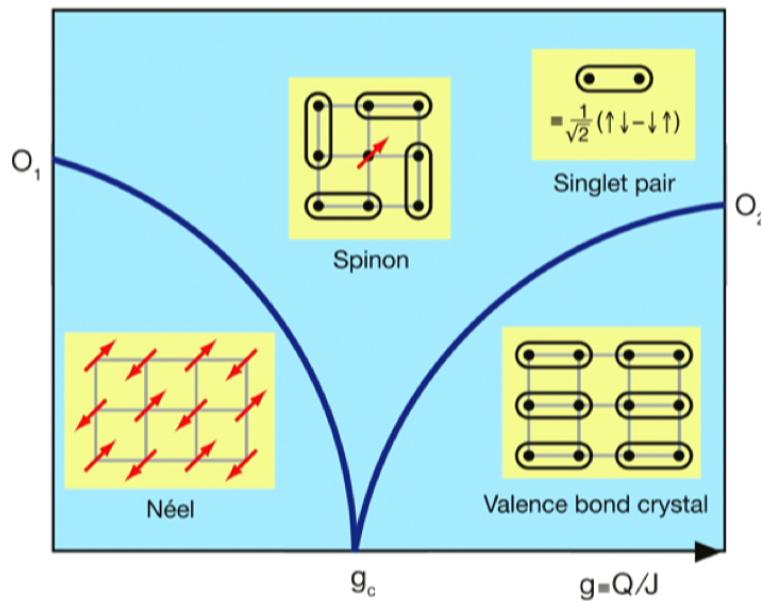
Neel to VBS



# Deconfined criticality

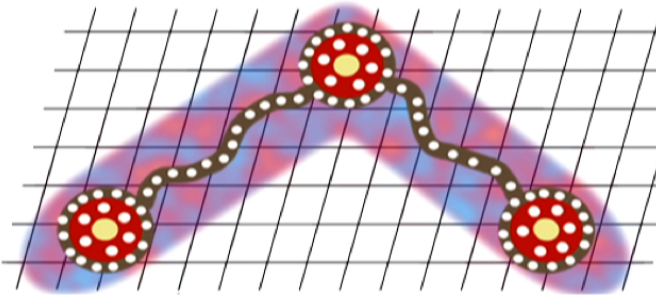
Senthil, Vishwanath, Balents, Sachdev, Fisher

Neel to VBS



# Outline

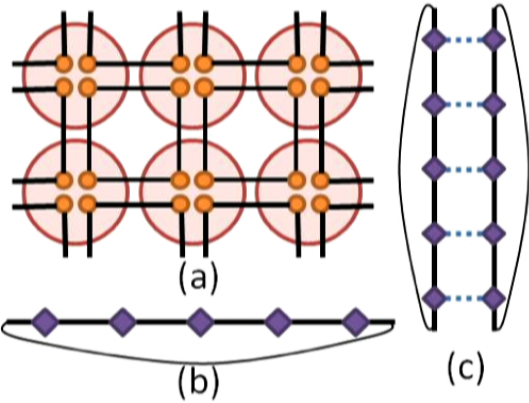
Pompidou Metz



1. spin liquids on kagome lattice

2. lattice gauge theory

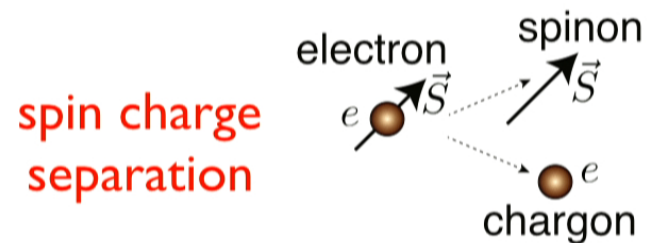
3. symmetry protected topological phase (SPT) and its deconfined criticality



Chen, Gu, Liu, Wen

# Spin liquid: more than absence of order

- Fractionalization in 2D/3D



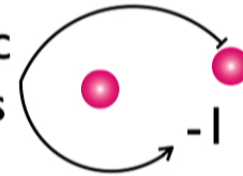
- Emergent gauge field: U(1), Z<sub>2</sub>.....
- Fractional quasiparticles (anyon)
- Parent state of a superconductor
- ...

# Examples of spin liquid

- Chiral spin liquid, gapped Kalmeyer & Laughlin 1987 PRL

spinon ● 1/2 spin

Semionic statistics

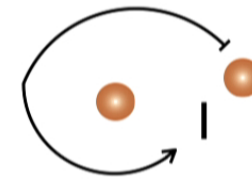
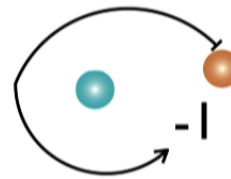
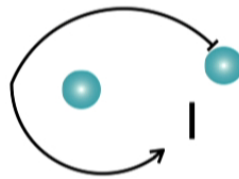


- Z2 spin liquid, gapped

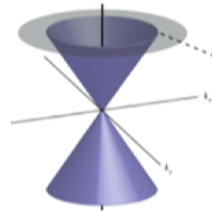
Read & Sachdev PRL 1991; Moessner & Sondhi PRL 2001...

spinon ●

vison ●



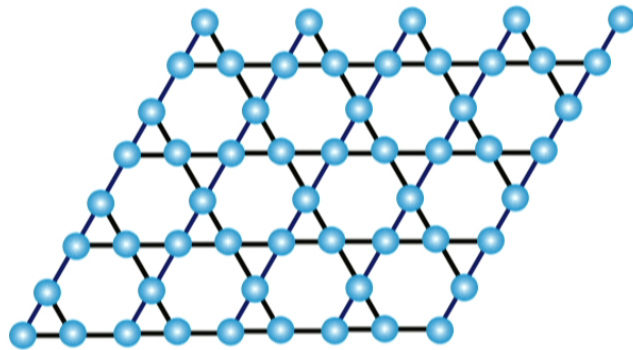
- Dirac spin liquid, gapless



$$\mathcal{L} = \sum_{i=1}^4 \bar{\psi}_i i \gamma_\mu (\partial_\mu + i A_\mu) \psi + \frac{1}{2e^2} F_{\mu\nu} F_{\mu\nu}$$

Hastings PRB 2000; Ran, Hermele, Lee & Wen PRL 2007

# Kagome Heisenberg model



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j, \quad J > 0$$

What is the ground state?

every possible candidate  
has been proposed?!

"kagome spin liquid"

Partial list...

Read & Sachdev (1991)

Marston & Zeng (1991)

Chalker, Holdsworth, Shender (1992)

Yang, Warman & Girvin (1993)

Hastings (2000)

Wang & Vishwanath (2006)

Ran, Hermele, Lee & Wen (2007)

Singh & Huse (2007)

Evenbly & Vidal (2010)

Yan, Huse & White (2011)

Lauchli, Sudan, Sorensen (2011)

Iqbal, Becca & Poilblanc (2011)

Depenbrock, McCulloch & Schollwöck (2012)

Jiang, Wang & Balents (2012)

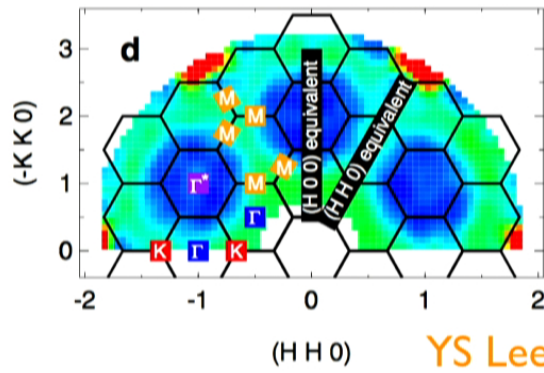
Xie, et. al., Xiang (2014)

YCH, Sheng, & Chen (2014)

....



# Current status



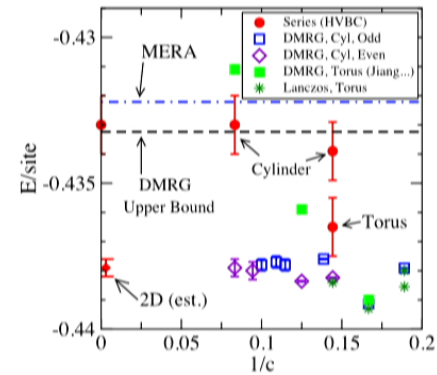
YS Lee group

Experiments

no order

spinon detected

gapless (or gapped?)



Numerics

S.White group

no order

spinon detected

gapped (or gapless?)

Spin liquid, but which one?

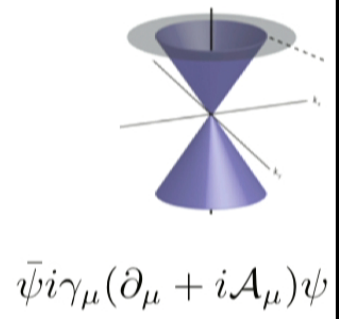
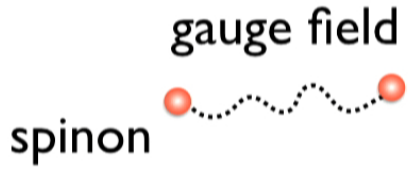
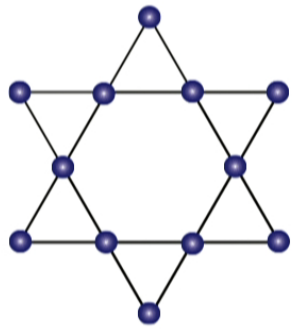
"kagome spin liquid"



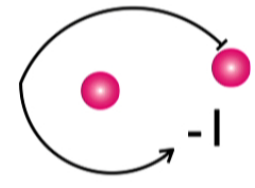
# How to solve?

$$J \sum \vec{S}_i \cdot \vec{S}_j$$

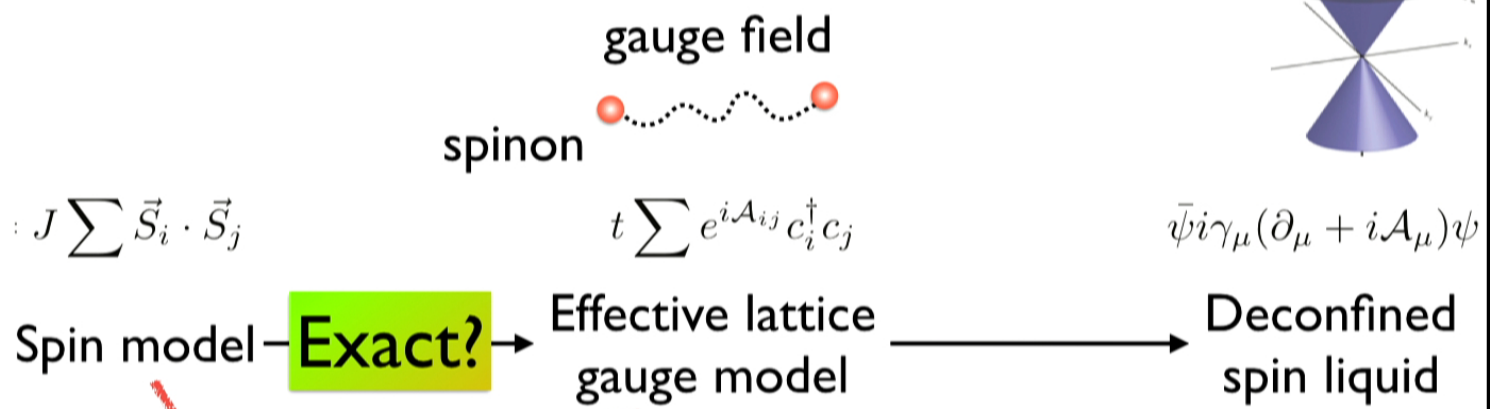
Spin model



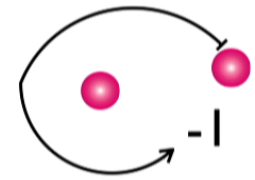
Deconfined spin liquid



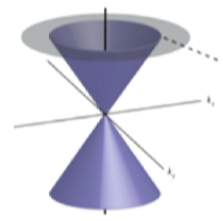
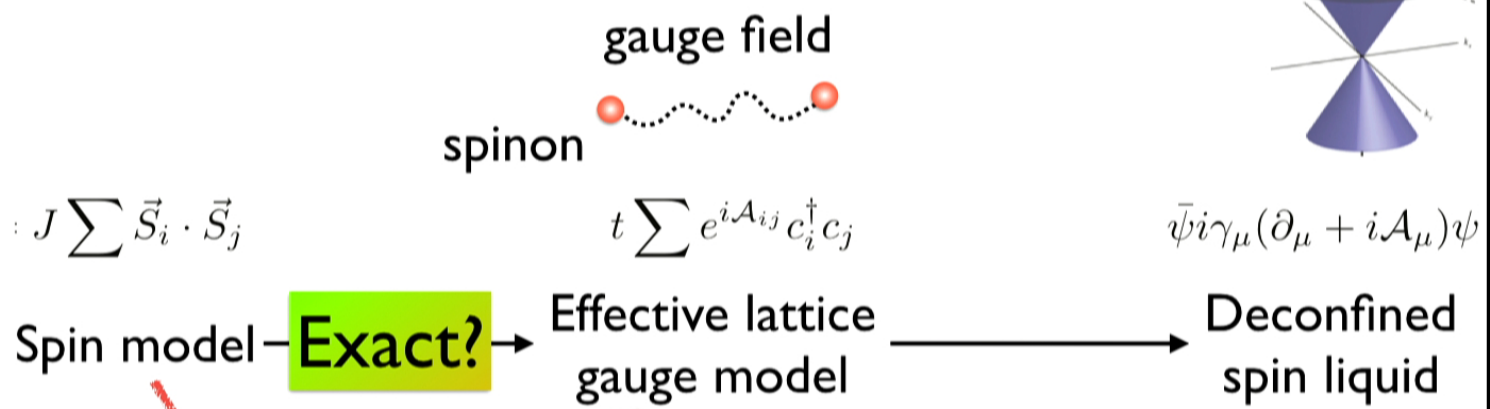
# How to solve?



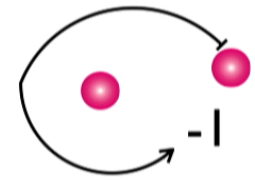
~~uncontrolled mean field  
 slave particle  
 $\vec{S}_i = c_{i,s}^\dagger \vec{\sigma}_{s,s'} c_{i,s'}$   
 $\langle c_i^\dagger c_j \rangle \neq 0$~~



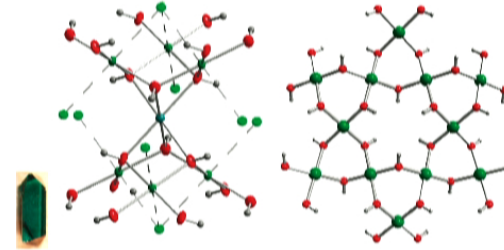
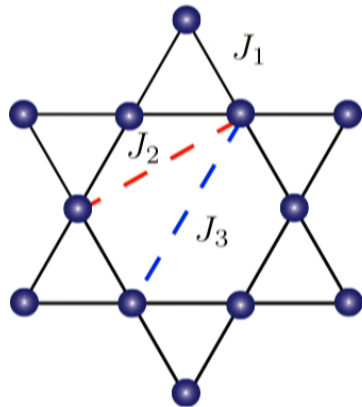
# How to solve?



~~uncontrolled mean field  
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 $\langle c_i^\dagger c_j \rangle \neq 0$~~



# Make it more general



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j, \quad J > 0$$

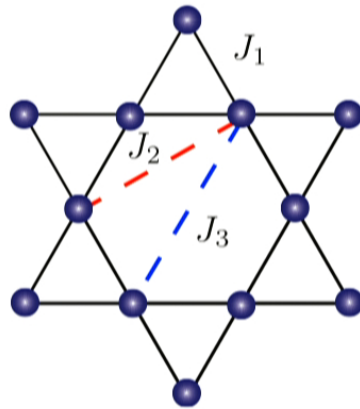
XXZ anisotropy  
second neighbor  
third neighbor  
DM interaction  
interlayer coupling  
impurity  
....

# Extended kagome model

YCH & Chen, PRL 2015

$$H_{XXZ} = J_1^z \sum_{\langle pq \rangle} S_p^z S_q^z + \frac{J_1^{xy}}{2} \sum_{\langle pq \rangle} (S_p^+ S_q^- + h.c.) \quad \text{1st XXZ}$$

$$+ \frac{J_{23}^{xy}}{2} \left( \sum_{\langle\langle pq \rangle\rangle} + \sum_{\langle\langle\langle pq \rangle\rangle\rangle} \right) (S_p^+ S_q^- + h.c.) \quad \text{2nd, 3rd XY}$$

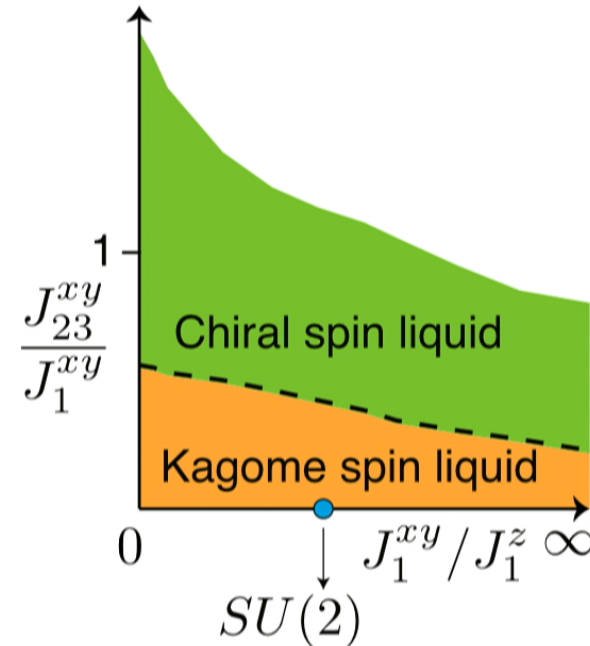


DMRG results

independent of  
XXZ anisotropy

also see ED calculation:

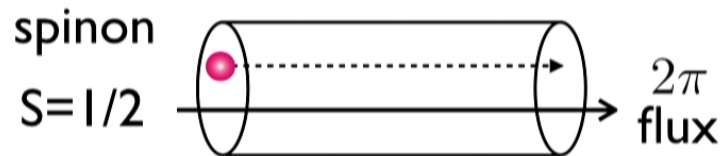
Lauchli & Moessner, arXiv (2015)



# Numerics for the chiral spin liquid

Hall conductance

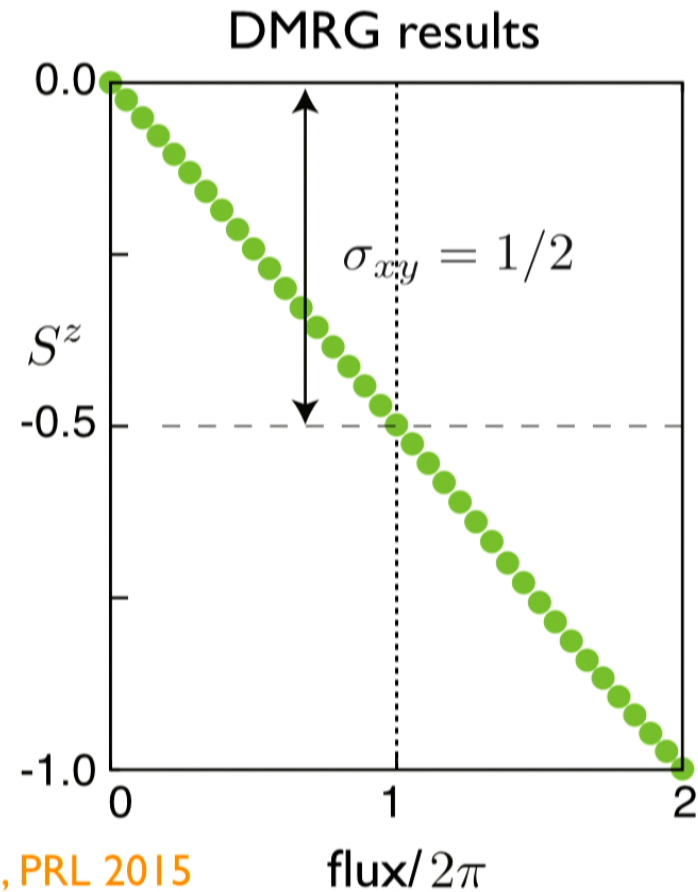
Flux insertion Laughlin, PRB 1981



$2\pi$  flux pumps one spinon

DMRG: YCH, Sheng, Chen PRB 2014

YCH, Sheng & Chen, PRL 2014;  
Gong, Zhu & Sheng, Sci. Rep 2014;  
Bauer, et al. Nat. Comm. 2014; YCH & Chen, PRL 2015



# Fractional statistics from DMRG

Wen, Int. J. Mod. Phys. B, 1990

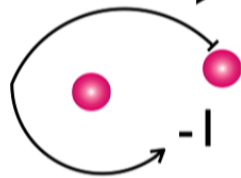
Zhang, Grover, Turner, Oshikawa & Vishwanath, PRB 2012

Cincio & Vidal, PRL 2013

Modular Matrix  $V_{ij} = \langle \psi_i | R_{2\pi/3} | \psi_j \rangle \sim \mathcal{U}\mathcal{S}$

$$\mathcal{S} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\mathcal{U} = e^{-i(2\pi/24)} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

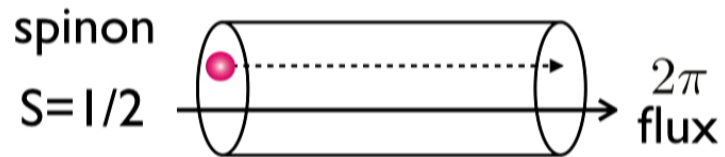


YCH, Sheng & Chen, PRL 2014; Gong, Zhu & Sheng, Sci. Rep 2014;  
Bauer, et al. Nat. Comm. 2014; YCH & Chen, PRL 2015

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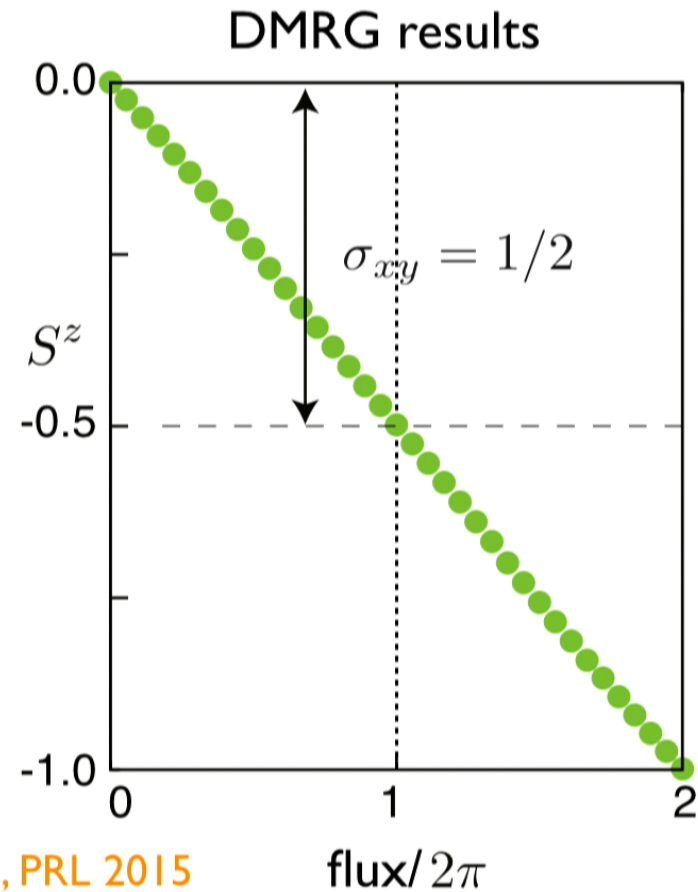
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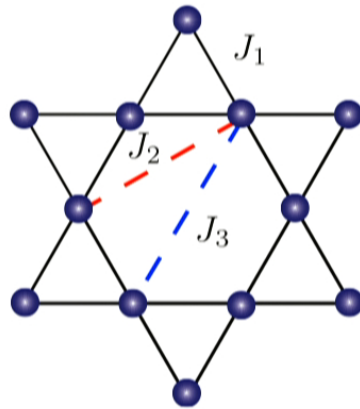


# Extended kagome model

YCH & Chen, PRL 2015

$$H_{XXZ} = J_1^z \sum_{\langle pq \rangle} S_p^z S_q^z + \frac{J_1^{xy}}{2} \sum_{\langle pq \rangle} (S_p^+ S_q^- + h.c.) \quad \text{1st XXZ}$$

$$+ \frac{J_{23}^{xy}}{2} \left( \sum_{\langle\langle pq \rangle\rangle} + \sum_{\langle\langle\langle pq \rangle\rangle\rangle} \right) (S_p^+ S_q^- + h.c.) \quad \text{2nd, 3rd XY}$$

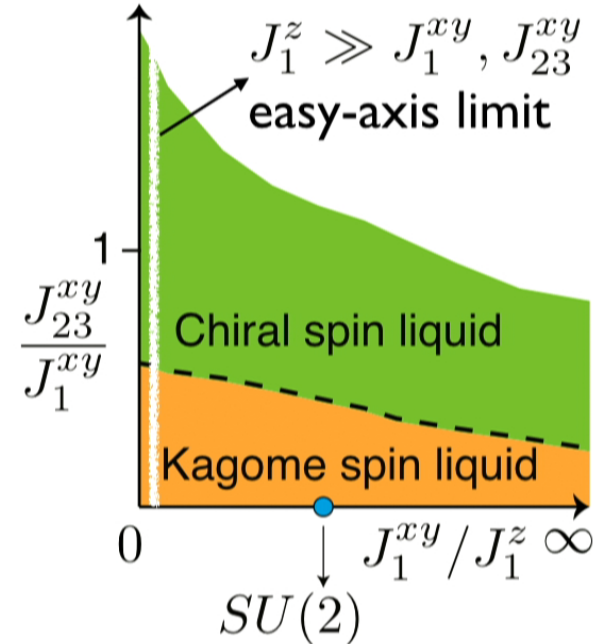


DMRG results

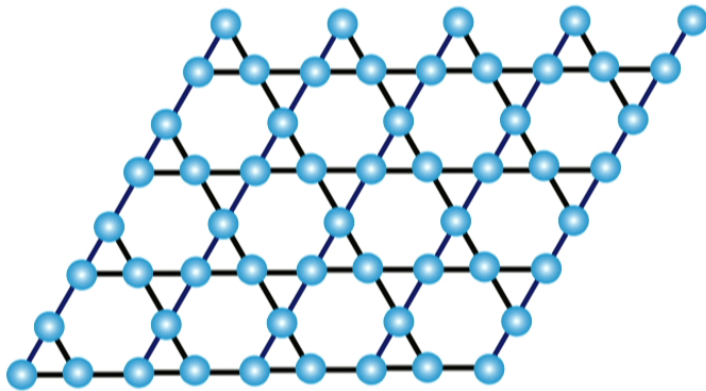
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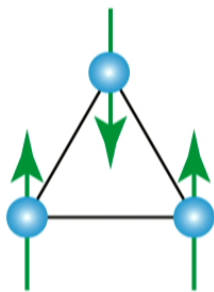


# Easy axis kagome

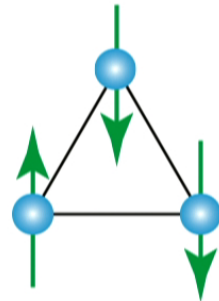


$$H = J_1^z \sum_{\text{1st}} S_i^z S_j^z + \lambda H_1$$

$$J_z \gg \lambda > 0$$



$$\sum S_i^z = \frac{1}{2}$$



$$\sum S_i^z = -\frac{1}{2}$$

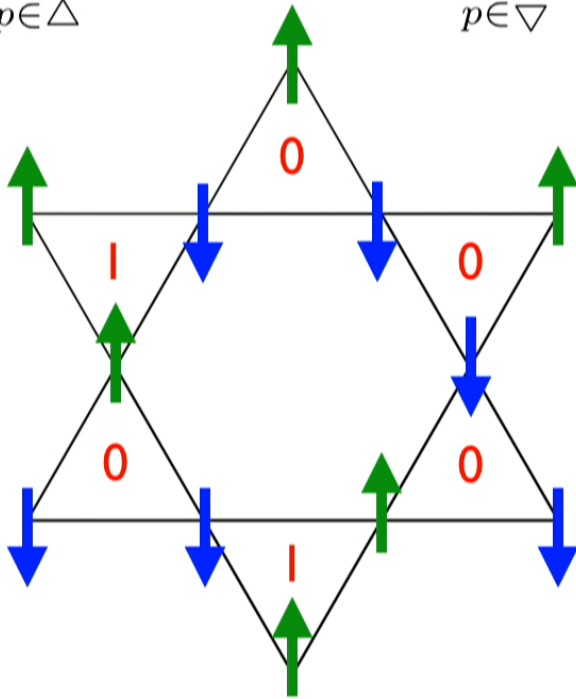
extensive classical degeneracy

$H_1$  lifts the classical degeneracy

# Exact lattice gauge mapping

$$\sum_{p \in \triangle} S_p^z = a^\dagger a - \frac{1}{2} \quad \sum_{p \in \nabla} S_p^z = b^\dagger b - \frac{1}{2}$$

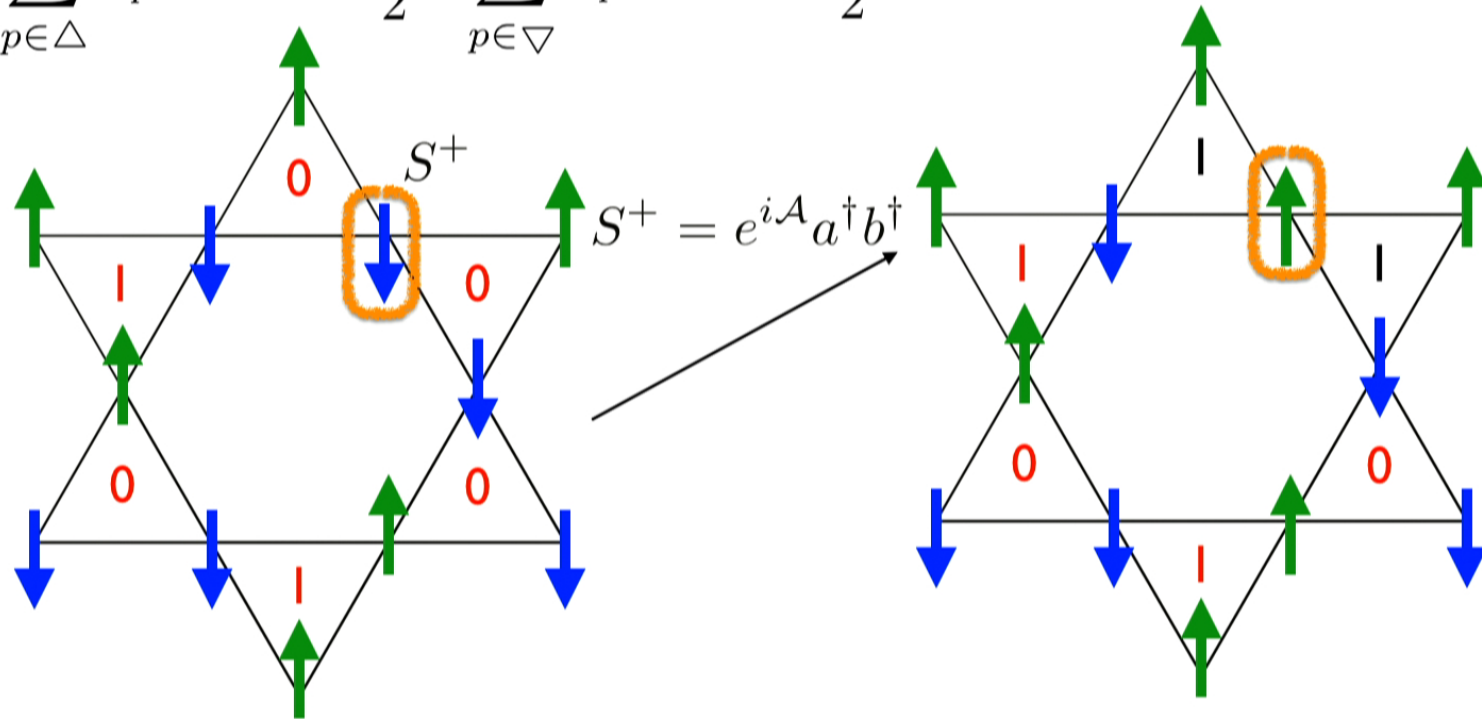
Nikolic & Senthil 2005 PRB



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Nikolic & Senthil 2005 PRB



similar system: quantum dimer model, pyrochlore lattice

Fradkin & Kivelson, 1990

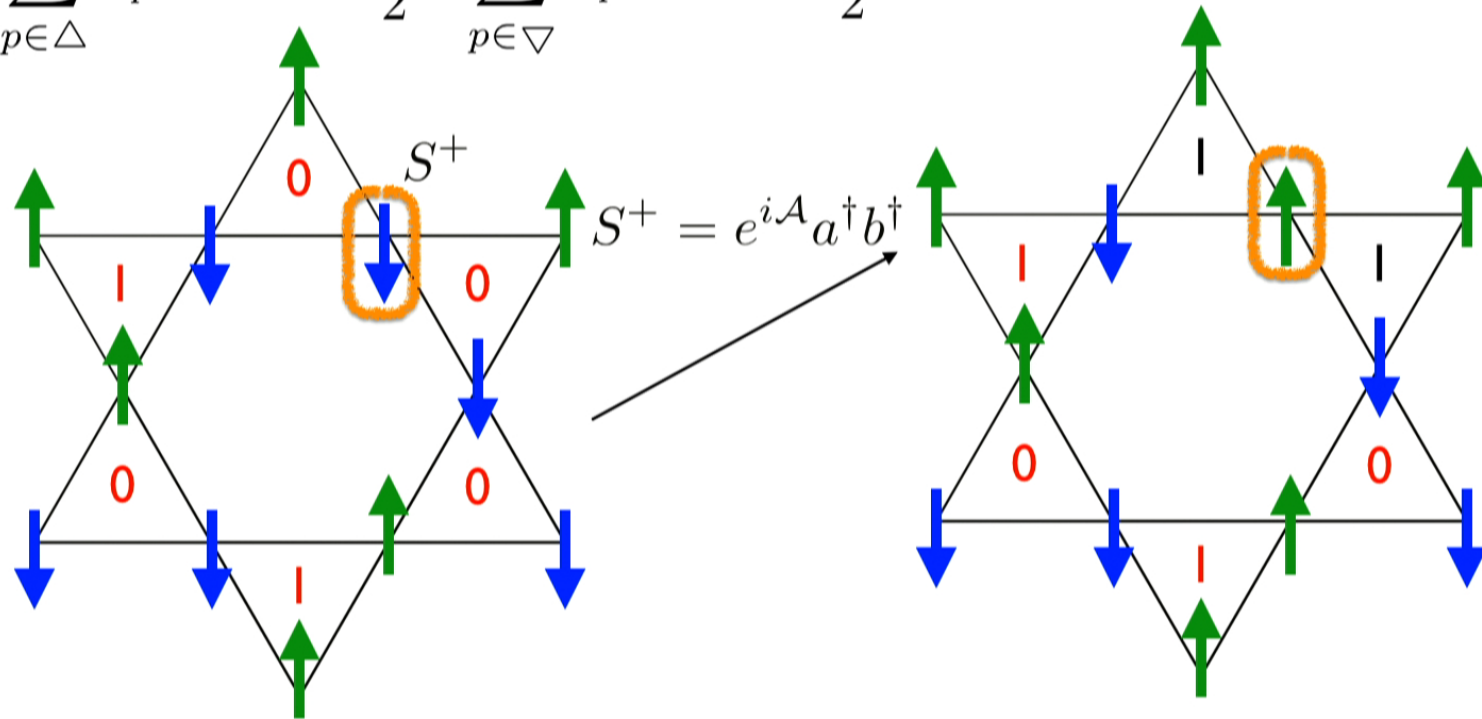
Hermele, Fisher & Balents 2004

Castelnovo, Moessner & Sondhi 2008

# Exact lattice gauge mapping

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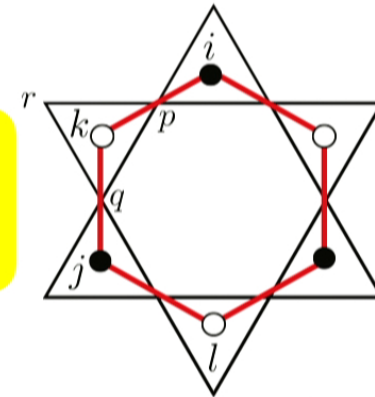
Castelnovo, Moessner & Sondhi 2008

# Lattice gauge mapping: XXZ kagome

$$\begin{aligned}
 H = & J_1^z \sum_{\langle pq \rangle} S_p^z S_q^z + \frac{J_1^{xy}}{2} \sum_{\langle pq \rangle} (S_p^+ S_q^- + h.c.) \\
 & + \frac{J_{23}^{xy}}{2} \sum_{\langle\langle pq \rangle\rangle} (S_p^+ S_q^- + h.c.) + \frac{J_{23}^{xy}}{2} \sum_{\langle\langle\langle pq \rangle\rangle\rangle} (S_p^+ S_q^- + h.c.)
 \end{aligned}$$

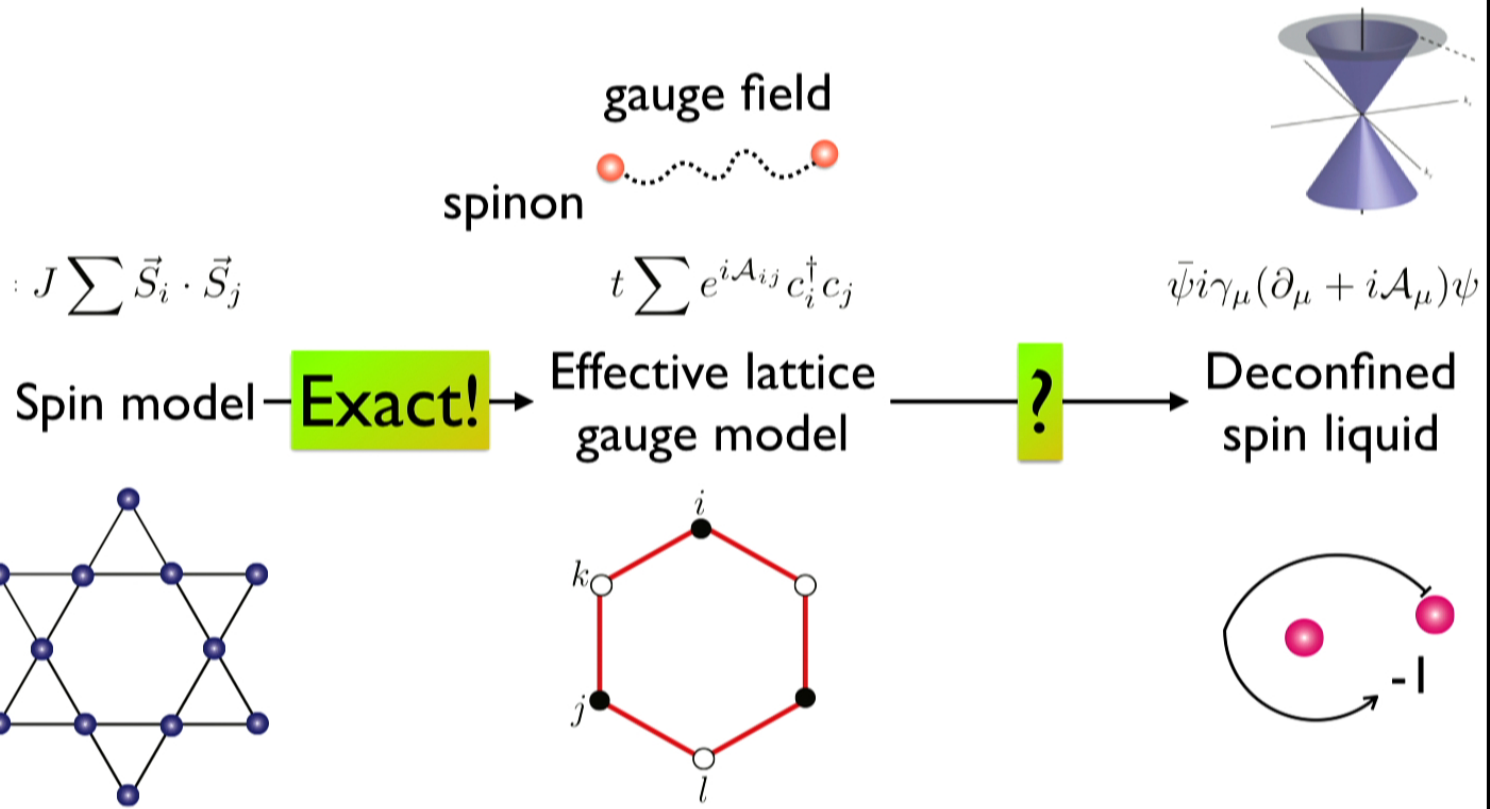
**Exact!**

$S_p^+ = e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger$   
lattice gauge mapping



$$\begin{aligned}
 H^{\text{LGT}} = & J_1^{xy} \left[ \sum_{\langle\langle ij \rangle\rangle} e^{i\mathcal{A}_{ij}} a_i^\dagger a_j + \sum_{\langle\langle kl \rangle\rangle} e^{i\mathcal{A}_{lk}} b_k^\dagger b_l + h.c. \right] \\
 & + J_{23}^{xy} \sum_{\langle ik \rangle, \langle jl \rangle \in \square} \left[ (e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger) (e^{i\mathcal{A}_{lj}} b_l a_j) + h.c. \right] \\
 & + \kappa \sum E_{ik}^2 + 1/\kappa \sum \cos(\sum \mathcal{A}_{ik}) \quad \kappa \sim \kappa_{\text{SL}}
 \end{aligned}$$

# Solving the kagome spin liquid phase



# No "free" spin liquid

$U(1)$  compact gauge field + Dynamical bosonic spinons



confinement in 2+1D

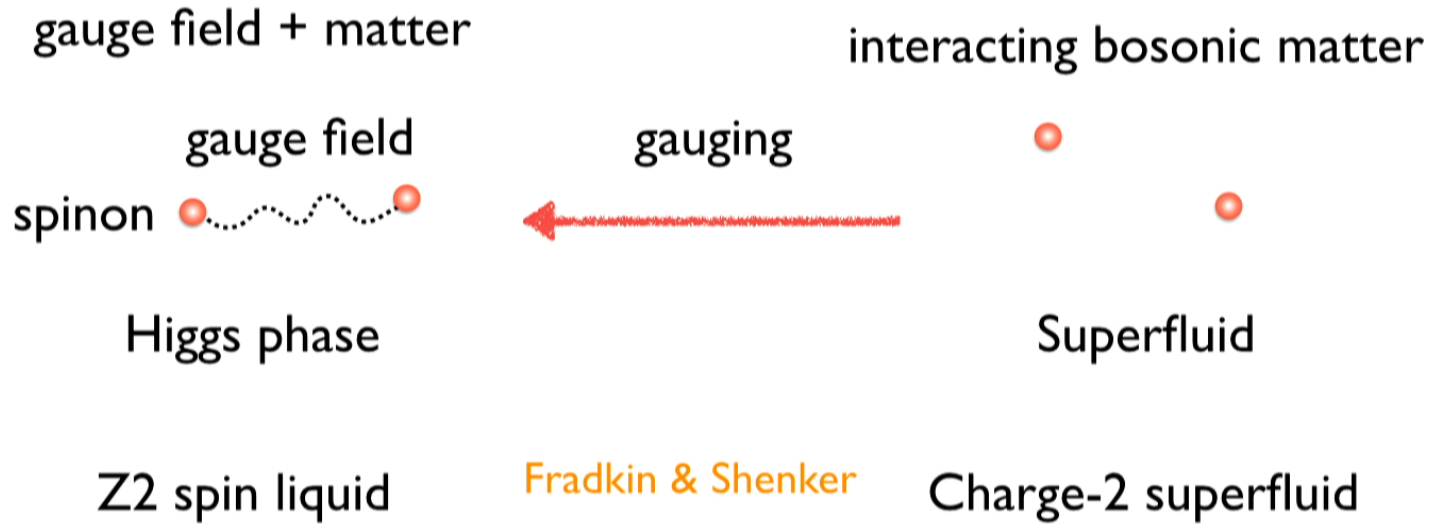
Polyakov



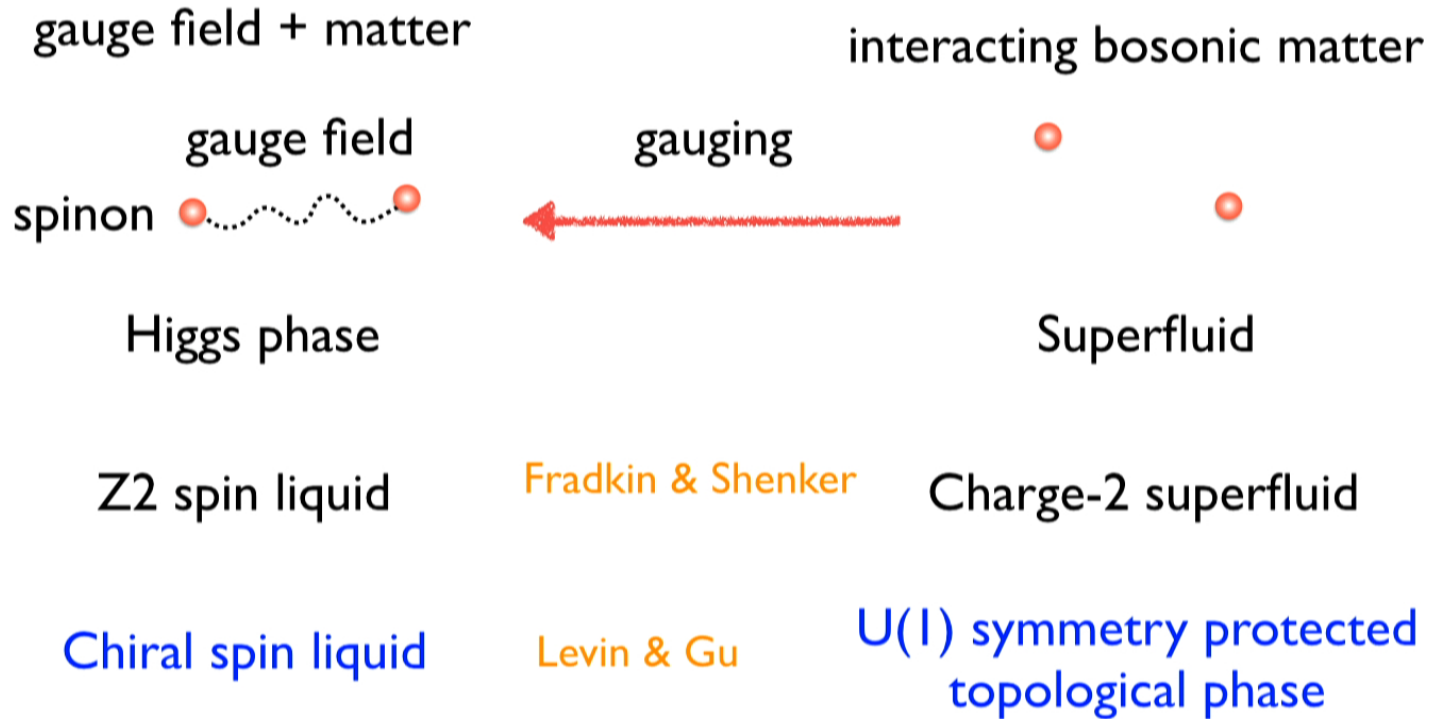
# Phases in a lattice gauge model



# Phases in a lattice gauge model



# Phases in a lattice gauge model



# Symmetry protected topological phase!

Classified by cohomology group  $H^{d+1}[G, U(1)]$

Chen, Gu, Liu & Wen, PRB 2012

SPT protected by U(1) charge conservation



Bosonic integer quantum Hall

Senthil & Levin, PRL 2013

Lu & Vishwanath, PRB 2012

# Symmetry protected topological phase!

Classified by cohomology group  $H^{d+1}[G, U(1)]$

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SPT protected by U(1) charge conservation



Senthil & Levin, PRL 2013

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Bosonic integer quantum Hall

gauging



$$\frac{2}{4\pi} \varepsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

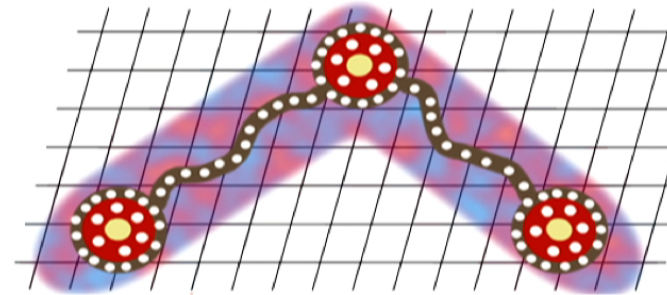
chiral spin liquid

See also:

YCH, Bhattacharjee, Pollmann, and Moessner, PRL 2015    Barkeshli, arxiv 2013

# Outline

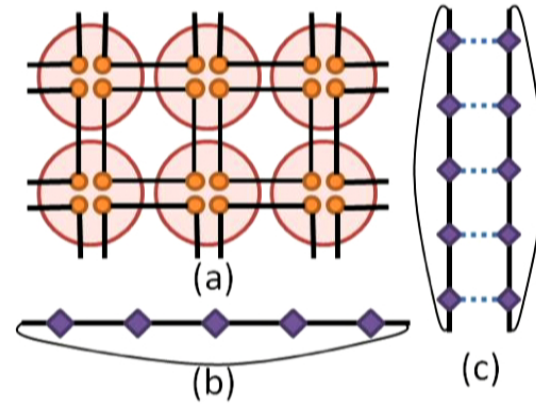
Pompidou Metz



1. spin liquids on kagome lattice

2. lattice gauge theory

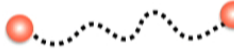
3. symmetry protected topological phase (SPT)

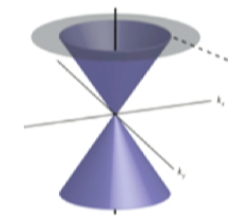


Chen, Gu, Liu, Wen

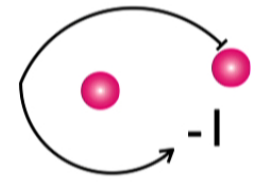
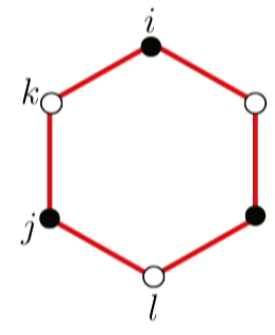
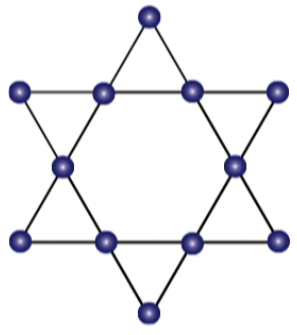
# Solving the kagome spin liquid phase

$J \sum \vec{S}_i \cdot \vec{S}_j$       gauge field       $t \sum e^{iA_{ij}} c_i^\dagger c_j$        $\bar{\psi} i \gamma_\mu (\partial_\mu + iA_\mu) \psi$

spinon      



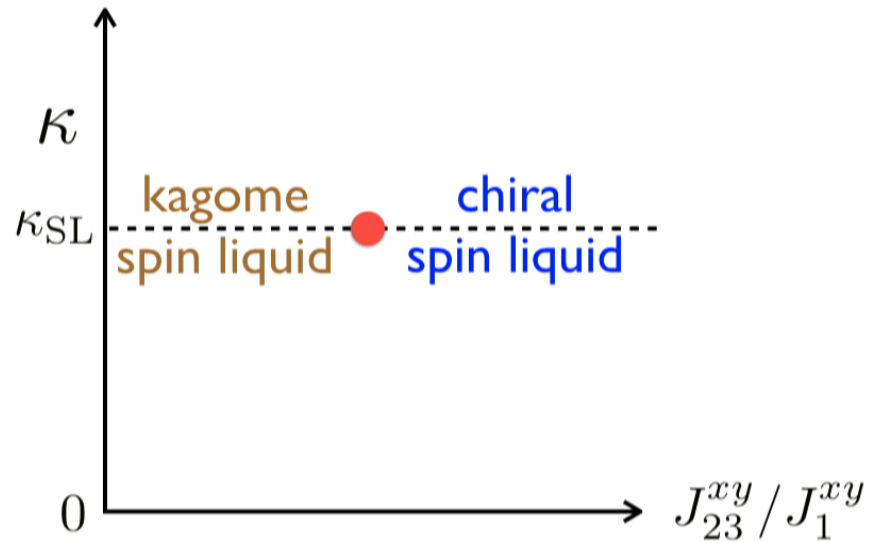
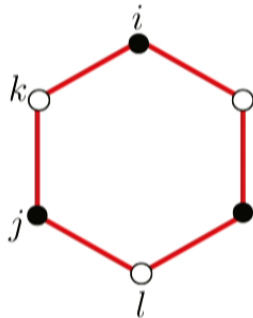
Spin model **Exact!** → Effective lattice gauge model **?** → Deconfined spin liquid



# Lattice gauge model

YCH, Bhattacharjee, Pollmann, and Moessner, PRL 2015

$$\begin{aligned}
 H^{\text{LGT}} = & J_1^{xy} \left[ \sum_{\langle\langle ij \rangle\rangle} e^{i\mathcal{A}_{ij}} a_i^\dagger a_j + \sum_{\langle\langle kl \rangle\rangle} e^{i\mathcal{A}_{lk}} b_k^\dagger b_l + h.c. \right] \\
 & + J_{23}^{xy} \sum_{\langle ik \rangle, \langle jl \rangle \in \square} \left[ (e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger) (e^{i\mathcal{A}_{lj}} b_l a_j) + h.c. \right] \\
 & + \kappa \sum E_{ik}^2 + 1/\kappa \sum \cos(\sum \mathcal{A}_{ik})
 \end{aligned}$$

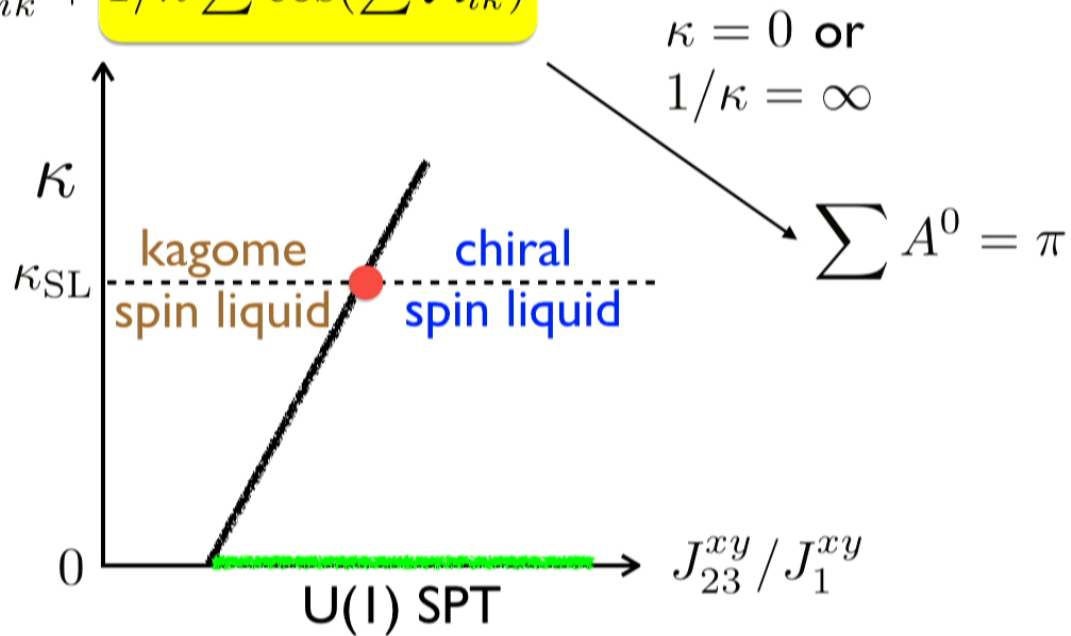
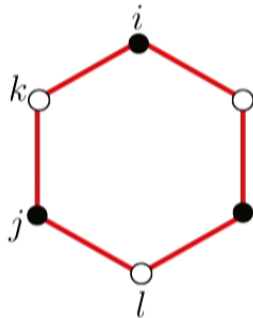




# Lattice gauge model

YCH, Bhattacharjee, Pollmann, and Moessner, PRL 2015

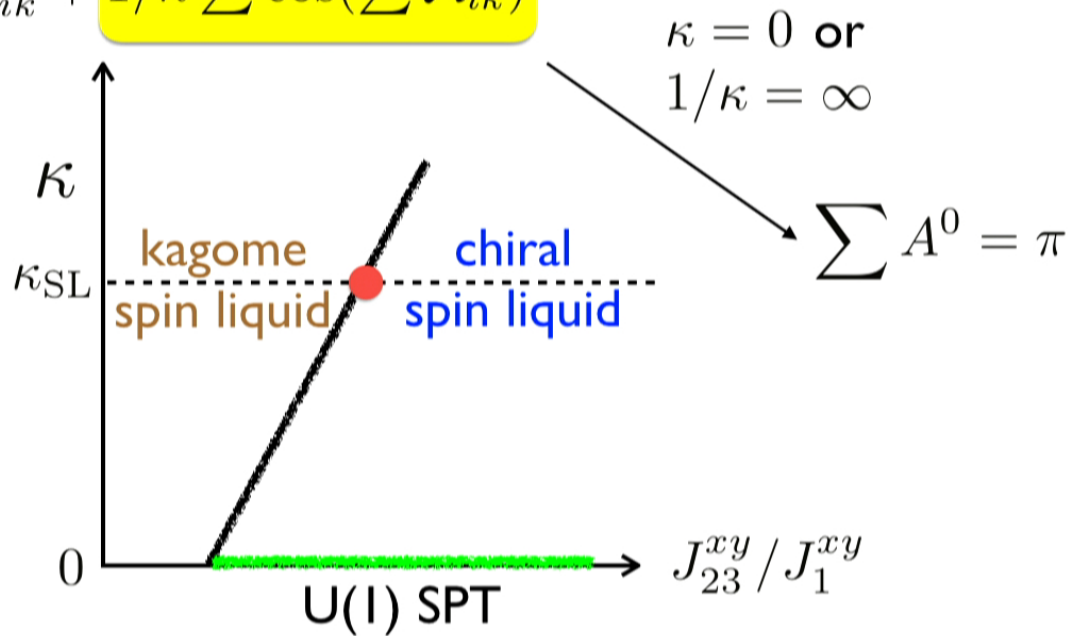
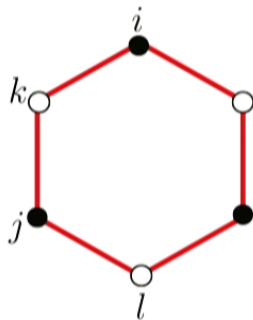
$$\begin{aligned}
 H^{\text{LGT}} = & J_1^{xy} \left[ \sum_{\langle\langle ij \rangle\rangle} e^{i\mathcal{A}_{ij}} a_i^\dagger a_j + \sum_{\langle\langle kl \rangle\rangle} e^{i\mathcal{A}_{lk}} b_k^\dagger b_l + h.c. \right] \\
 & + J_{23}^{xy} \sum_{\langle ik \rangle, \langle jl \rangle \in \square} \left[ (e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger) (e^{i\mathcal{A}_{lj}} b_l a_j) + h.c. \right] \\
 & + \kappa \sum E_{ik}^2 + \frac{1}{\kappa} \sum \cos(\sum \mathcal{A}_{ik})
 \end{aligned}$$



# Lattice gauge model

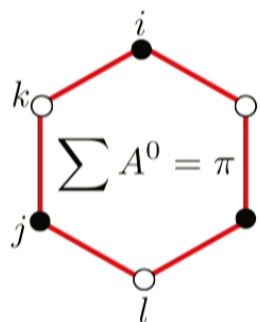
YCH, Bhattacharjee, Pollmann, and Moessner, PRL 2015

$$\begin{aligned}
 H^{\text{LGT}} = & J_1^{xy} \left[ \sum_{\langle\langle ij \rangle\rangle} e^{i\mathcal{A}_{ij}} a_i^\dagger a_j + \sum_{\langle\langle kl \rangle\rangle} e^{i\mathcal{A}_{lk}} b_k^\dagger b_l + h.c. \right] \\
 & + J_{23}^{xy} \sum_{\langle ik \rangle, \langle jl \rangle \in \square} \left[ (e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger) (e^{i\mathcal{A}_{lj}} b_l a_j) + h.c. \right] \\
 & + \kappa \sum E_{ik}^2 + \boxed{1/\kappa \sum \cos(\sum \mathcal{A}_{ik})}
 \end{aligned}$$



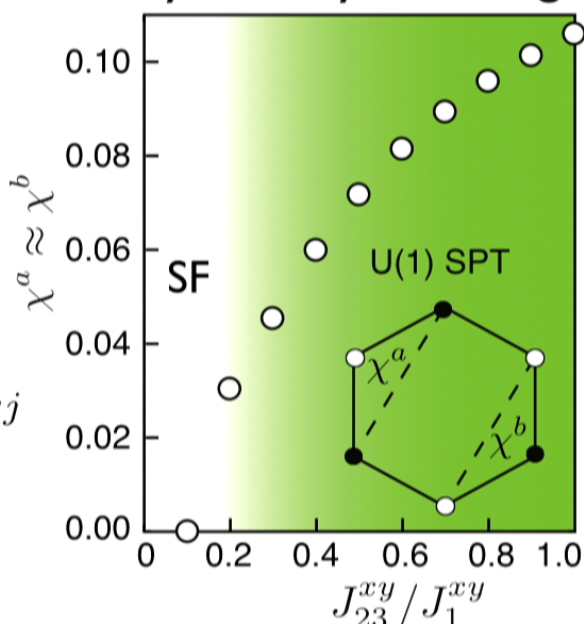
# U(1) SPT phase

$$\begin{aligned} \tilde{H} = & J_1^{xy} \left[ \sum_{\langle\langle ij \rangle\rangle} e^{iA_{ij}^0} a_i^\dagger a_j + \sum_{\langle\langle kl \rangle\rangle} e^{iA_{lk}^0} b_k^\dagger b_l + h.c. \right] \\ & + J_{23}^{xy} \sum_{\langle ik \rangle, \langle jl \rangle \in \square} \left[ (e^{iA_{ik}^0} a_i^\dagger b_k^\dagger) (e^{iA_{lj}^0} b_l a_j) + h.c. \right] \end{aligned}$$



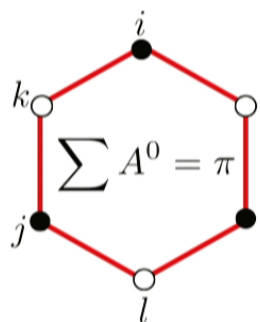
$$\chi^a = ie^{i\pi n_k^b} a_i^\dagger a_j$$

spontaneous time-reversal  
symmetry breaking



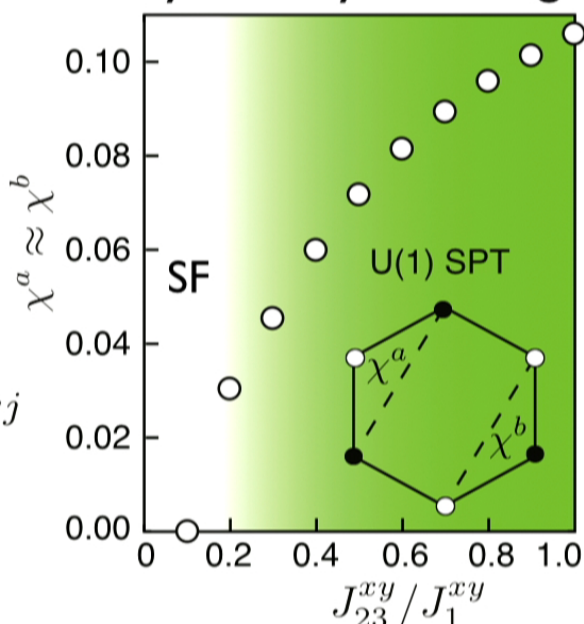
# U(1) SPT phase

$$\begin{aligned} \tilde{H} = & J_1^{xy} \left[ \sum_{\langle\langle ij \rangle\rangle} e^{iA_{ij}^0} a_i^\dagger a_j + \sum_{\langle\langle kl \rangle\rangle} e^{iA_{lk}^0} b_k^\dagger b_l + h.c. \right] \\ & + J_{23}^{xy} \sum_{\langle ik \rangle, \langle jl \rangle \in \square} \left[ (e^{iA_{ik}^0} a_i^\dagger b_k^\dagger) (e^{iA_{lj}^0} b_l a_j) + h.c. \right] \end{aligned}$$



$$\chi^a = ie^{i\pi n_k^b} a_i^\dagger a_j$$

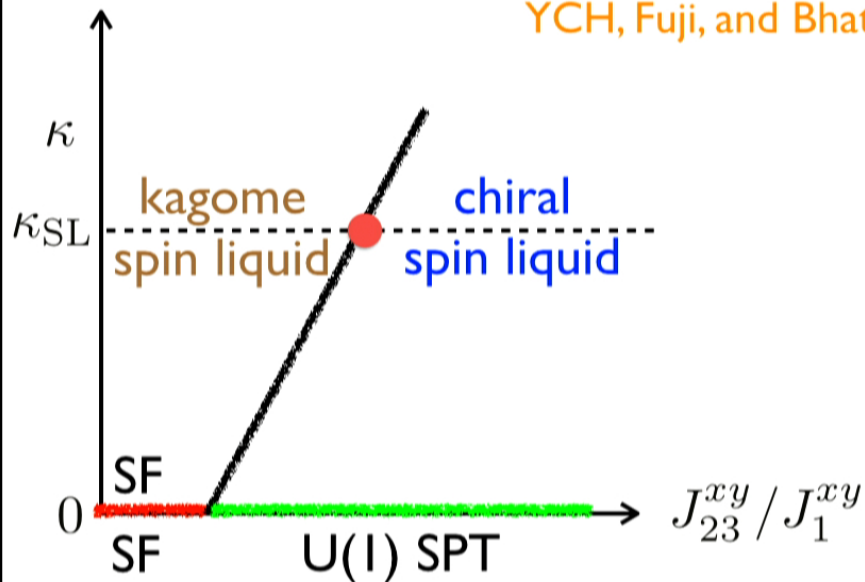
spontaneous time-reversal  
symmetry breaking



# Whole phase diagram

$$\begin{aligned}
 H^{\text{LGT}} = & J_1^{xy} \left[ \sum_{\langle\langle ij \rangle\rangle} e^{i\mathcal{A}_{ij}} a_i^\dagger a_j + \sum_{\langle\langle kl \rangle\rangle} e^{i\mathcal{A}_{lk}} b_k^\dagger b_l + h.c. \right] \\
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 & + \kappa \sum E_{ik}^2 + 1/\kappa \sum \cos(\sum \mathcal{A}_{ik})
 \end{aligned}$$

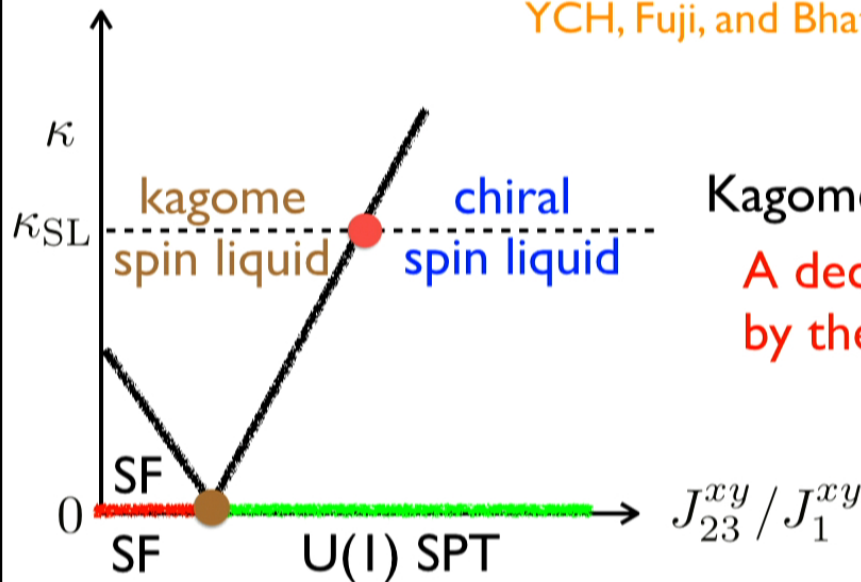
YCH, Fuji, and Bhattacharjee, arXiv:1512.05381 (2015).



# Whole phase diagram

$$\begin{aligned}
 H^{\text{LGT}} = & J_1^{xy} \left[ \sum_{\langle\langle ij \rangle\rangle} e^{i\mathcal{A}_{ij}} a_i^\dagger a_j + \sum_{\langle\langle kl \rangle\rangle} e^{i\mathcal{A}_{lk}} b_k^\dagger b_l + h.c. \right] \\
 & + J_{23}^{xy} \sum_{\langle ik \rangle, \langle jl \rangle \in \square} \left[ (e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger) (e^{i\mathcal{A}_{lj}} b_l a_j) + h.c. \right] \\
 & + \kappa \sum E_{ik}^2 + 1/\kappa \sum \cos(\sum \mathcal{A}_{ik})
 \end{aligned}$$

YCH, Fuji, and Bhattacharjee, arXiv:1512.05381 (2015).



Kagome spin liquid:

A deconfined phase driven  
by the U(1) gauge fluctuation!

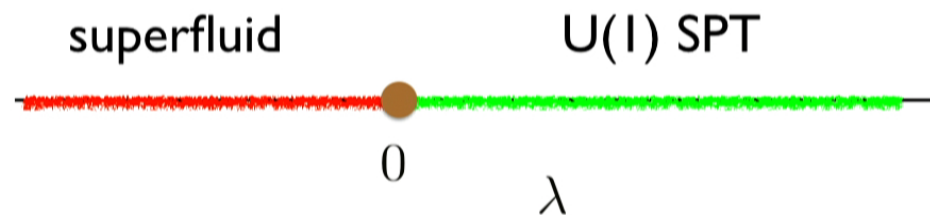
"gauged" deconfined  
critical point

# Field theory for zero gauge fluctuation

$$\begin{aligned} \mathcal{L} = & \sum_{t=\pm} \bar{f}_t [i\gamma^\mu (\partial_\mu - ia_\mu^f - itA_\mu^c)] f_t - \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} A_\mu^s \partial_\nu a_\rho^f \\ & + \sum_{t=\pm} \bar{g}_t [i\gamma^\mu (\partial_\mu - ia_\mu^g - itA_\mu^c)] g_t + \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} A_\mu^s \partial_\nu a_\rho^g \\ & + \sum_{t=\pm} \phi (\bar{f}_t f_t + \bar{g}_t g_t) - 2\lambda\phi^2 + u\phi^4 + \dots \end{aligned}$$

U(1) charge  $A^c$

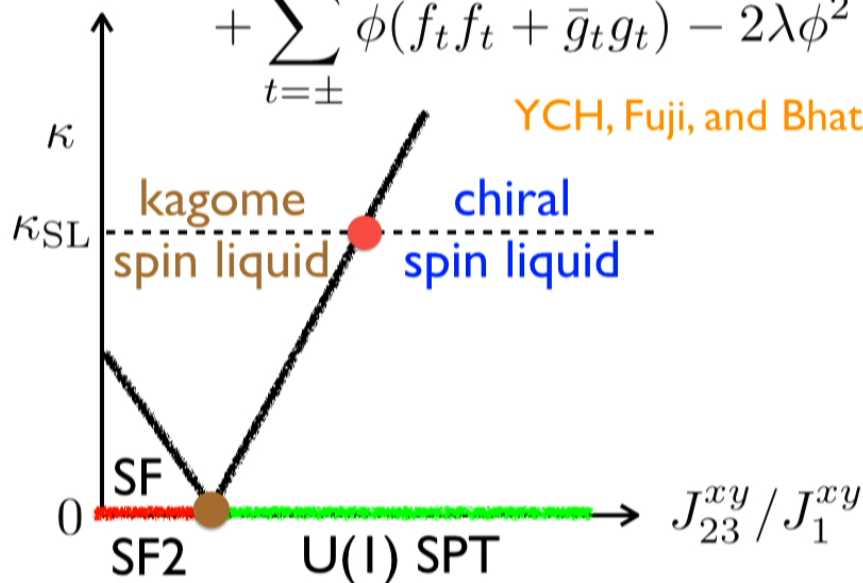
U(1) pseudospin  $A^s$



# Field theory for finite gauge fluctuation

$$\begin{aligned} \mathcal{L} = & \sum_{t=\pm} \bar{f}_t [i\gamma^\mu (\partial_\mu - ia_\mu^f - itA_\mu^c)] f_t - \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} A_\mu^s \partial_\nu a_\rho^f \\ & + \sum_{t=\pm} \bar{g}_t [i\gamma^\mu (\partial_\mu - ia_\mu^g - itA_\mu^c)] g_t + \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} A_\mu^s \partial_\nu a_\rho^g \\ & + \sum_{t=\pm} \phi (\bar{f}_t f_t + \bar{g}_t g_t) - 2\lambda\phi^2 + u\phi^4 + \dots \end{aligned}$$

YCH, Fuji, and Bhattacharjee, arXiv:1512.05381 (2015)



$$A^c \longrightarrow \frac{A^{S^z}}{2}$$

$$A^s \longrightarrow A$$



# Fate of DCP under gauge fluctuation

$$\mathcal{L} = \sum_{t=\pm} \bar{f}_t [i\gamma^\mu (\partial_\mu - ia_\mu^f - itA_\mu^c)] f_t - \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} A_\mu^s \partial_\nu a_\rho^f$$

$$+ \sum_{t=\pm} \bar{g}_t [i\gamma^\mu (\partial_\mu - ia_\mu^g - itA_\mu^c)] g_t + \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} A_\mu^s \partial_\nu a_\rho^g$$

Nf=2 QED3

$$A^c \longrightarrow \frac{A^{S^z}}{2} \quad A^s \longrightarrow \mathcal{A}$$

Nf=4 QED3!!!!

$$\mathcal{L}_D = \sum_{t=\pm} \bar{f}_t [i\gamma^\mu (\partial_\mu - ia_\mu^f - it\frac{A_\mu^{S^z}}{2})] f_t - \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} \mathcal{A}_\mu \partial_\nu a_\rho^f$$

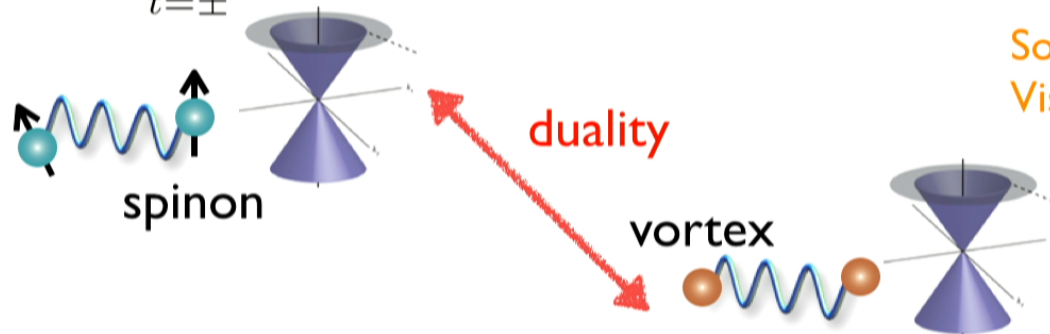
$$+ \sum_{t=\pm} \bar{g}_t [i\gamma^\mu (\partial_\mu - ia_\mu^g - it\frac{A_\mu^{S^z}}{2})] g_t + \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} \mathcal{A}_\mu \partial_\nu a_\rho^g$$

Integrate out dynamical gauge field  $\mathcal{A}$        $a^f = a^g = a$

# Dirac spin liquid, and more!

$$\mathcal{L}_D = \sum_{t=\pm} \bar{f}_t \left[ i\gamma^\mu (\partial_\mu - ia_\mu^f - it \frac{A_\mu^{S^z}}{2}) \right] f_t - \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} \mathcal{A}_\mu \partial_\nu a_\rho^f$$

$$+ \sum_{t=\pm} \bar{g}_t \left[ i\gamma^\mu (\partial_\mu - ia_\mu^g - it \frac{A_\mu^{S^z}}{2}) \right] g_t + \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} \mathcal{A}_\mu \partial_\nu a_\rho^g$$



Son, Metlitski, Senthil  
Vishwanath, Wang, ...

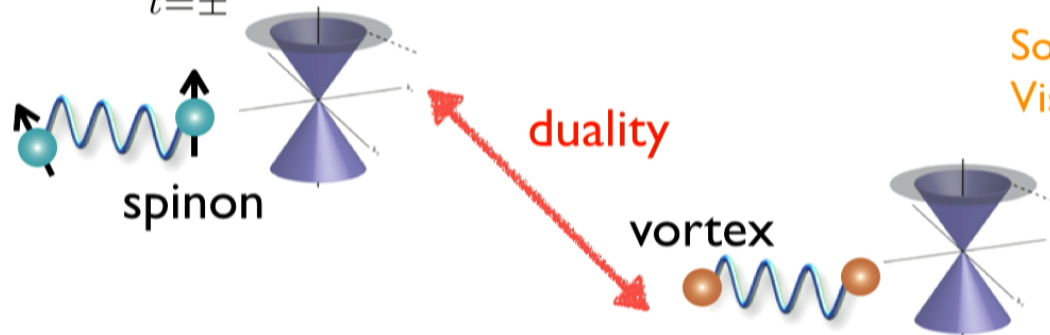
$$\mathcal{L}_D = \sum_{t=\pm} \bar{\tilde{f}}_t \left[ i\gamma^\mu (\partial_\mu - i\tilde{a}_\mu^f - it \mathcal{A}_\mu) \right] \tilde{f}_t - \frac{1}{4\pi} \varepsilon_{\mu\nu\rho} A_\mu^{S^z} \partial_\nu \tilde{a}_\rho^f$$

$$+ \sum_{t=\pm} \bar{\tilde{g}}_t \left[ i\gamma^\mu (\partial_\mu - i\tilde{a}_\mu^g - it \mathcal{A}_\mu) \right] \tilde{g}_t + \frac{1}{4\pi} \varepsilon_{\mu\nu\rho} A_\mu^{S^z} \partial_\nu \tilde{a}_\rho^g$$

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$$\mathcal{L}_D = \sum_{t=\pm} \bar{f}_t \left[ i\gamma^\mu (\partial_\mu - ia_\mu^f - it \frac{A_\mu^{S^z}}{2}) \right] f_t - \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} \mathcal{A}_\mu \partial_\nu a_\rho^f$$

$$+ \sum_{t=\pm} \bar{g}_t \left[ i\gamma^\mu (\partial_\mu - ia_\mu^g - it \frac{A_\mu^{S^z}}{2}) \right] g_t + \frac{1}{2\pi} \varepsilon_{\mu\nu\rho} \mathcal{A}_\mu \partial_\nu a_\rho^g$$

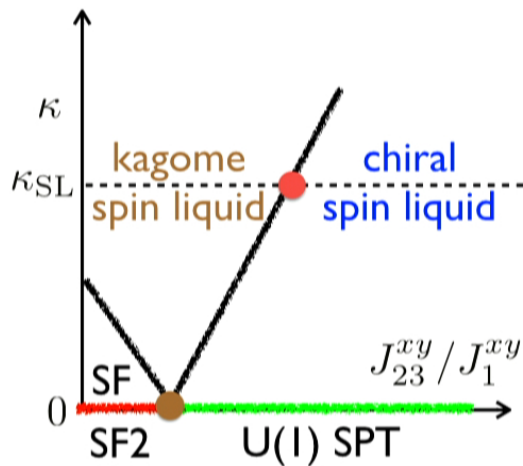
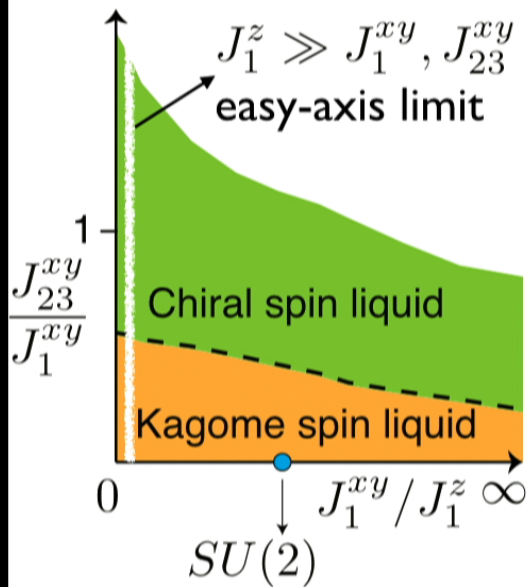
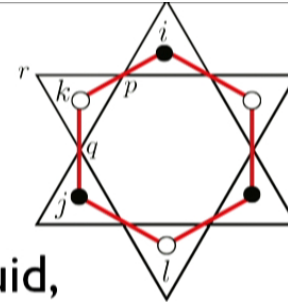


Son, Metlitski, Senthil  
Vishwanath, Wang, ...

$$\mathcal{L}_D = \sum_{t=\pm} \bar{\tilde{f}}_t \left[ i\gamma^\mu (\partial_\mu - i\tilde{a}_\mu^f - it \mathcal{A}_\mu) \right] \tilde{f}_t - \frac{1}{4\pi} \varepsilon_{\mu\nu\rho} A_\mu^{S^z} \partial_\nu \tilde{a}_\rho^f$$

$$+ \sum_{t=\pm} \bar{\tilde{g}}_t \left[ i\gamma^\mu (\partial_\mu - i\tilde{a}_\mu^g - it \mathcal{A}_\mu) \right] \tilde{g}_t + \frac{1}{4\pi} \varepsilon_{\mu\nu\rho} A_\mu^{S^z} \partial_\nu \tilde{a}_\rho^g$$

# Summary



1. Besides the kagome spin liquid, we discover a chiral spin liquid.
2. Spin liquids on kagome lattice are independent of the XXZ anisotropy.
3. A controlled theoretical analysis for the chiral spin liquid.
4. Kagome spin liquid is a deconfined phase driven by gauge fluctuation (gauged DCP)?
5. Make a concrete connection between topological order, critical spin liquid, SPT phase, deconfined criticality.

# Thanks for your attention!



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ICTS, Bangalore



Yohei Fuji  
MPI-PKS, Dresden



Yan Chen  
Fudan, Shanghai  
also thanks D. N. Sheng (CSUN)



Frank Pollmann  
MPI-PKS, Dresden



Roderich Moessner  
MPI-PKS, Dresden



YCH, Fuji, and Bhattacharjee, arXiv:1512.05381 (2015).

YCH, Bhattacharjee, Pollmann, and Moessner, PRL 115, 267209 (2015).

YCH, Bhattacharjee, Moessner, and Pollmann, PRL 115, 116803 (2015).

YCH and Chen, PRL 114, 037201 (2015).

YCH, Sheng and Chen, PRL 112, 137202 (2014).

# Numerical results on the spin gap

