Title: Charting the AdS/CFT Landscape

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Abstract: <p>What are the bounds of the AdS/CFT correspondence? Which quantities in conformal field theory have simple descriptions in terms of classical anti-de Sitter spacetime geometry? These foundational questions in holography may be meaningfully addressed via the study of CFT correlation functions, which map to amplitudes in AdS. I will show that a basic building block in any CFT -- the conformal block -- is equivalent to an elegant geometric object in AdS, which moreover greatly streamlines and clarifies calculations of AdS amplitudes. By studying correlators with certain Lorentzian kinematics, one can constrain the space of consistent theories of AdS quantum gravity itself. In particular, by harnessing a recent bound on the rate of onset of chaos in thermal states, I will rule out the existence of certain classes of putative 2D CFTs and their 3D gravity duals, and argue that others exhibit signatures of Regge trajectories of string theory. This may be viewed as a novel, Lorentzian counterpart of the conformal bootstrap, relating dynamical constraints on the development of quantum chaos to the determination of the AdS/CFT landscape.</p>

Charting the AdS/CFT Landscape

Eric Perlmutter, Princeton University

Perimeter Institute for Theoretical Physics
Colloquium

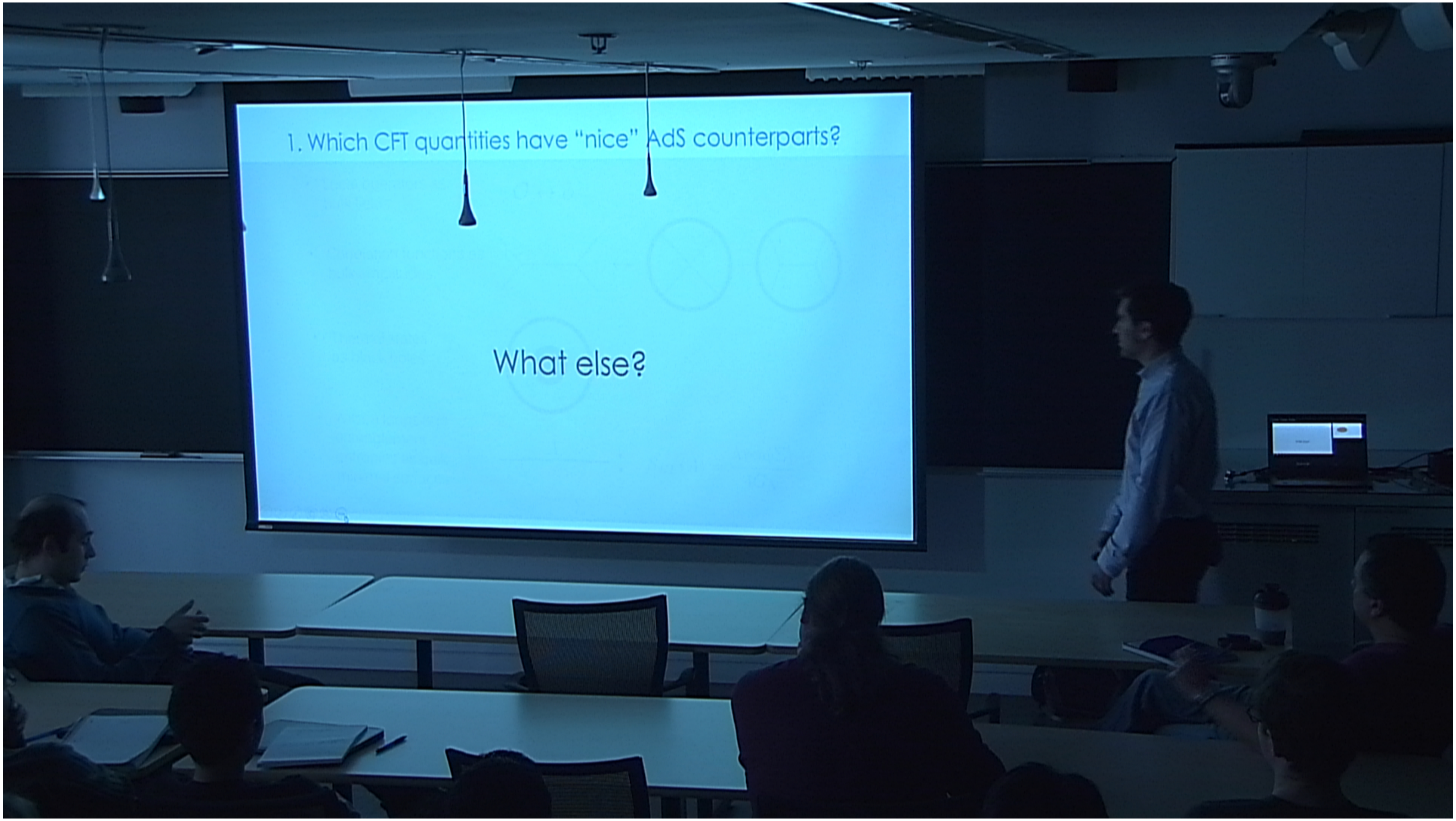
4/25/16

Three fundamental questions in AdS/CFT:

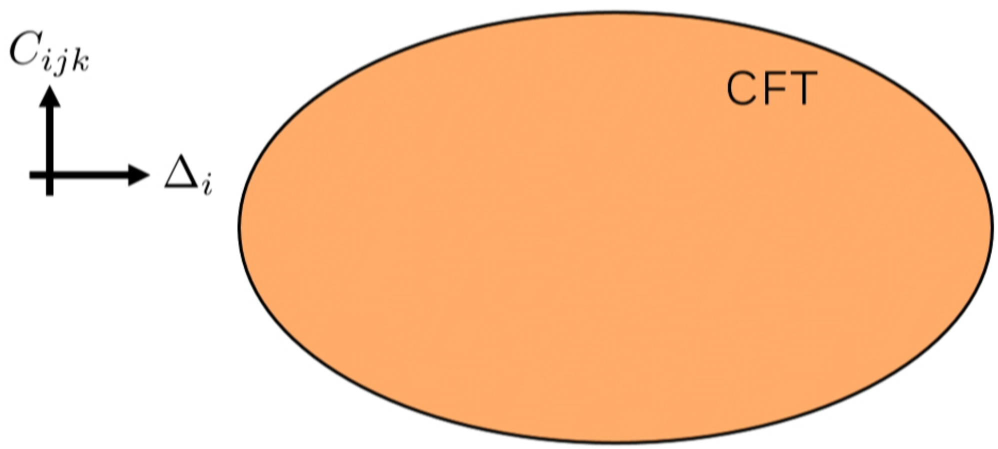
1. Which CFT quantities have “nice” AdS counterparts?

2. What is the space of CFTs, and of theories of AdS quantum gravity? Which CFTs give rise to emergent spacetime?

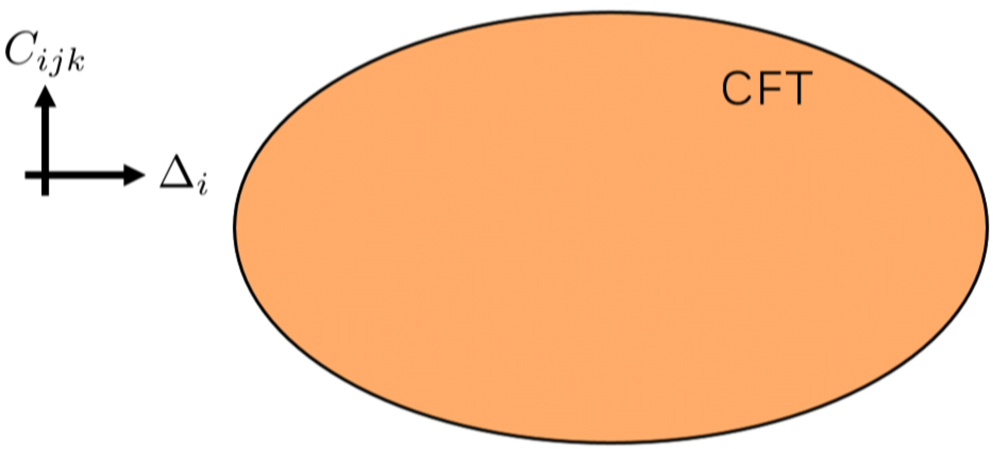
3. How is the structure of string/M-theory visible in CFT?

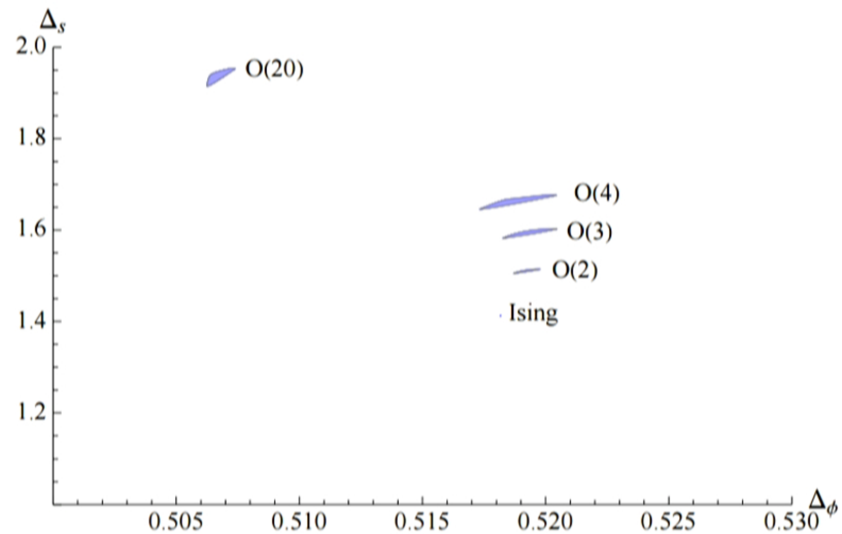

1. Which CFT quantities have "nice" AdS counterparts?
What else?

2. What is the space of CFTs, and of theories of AdS quantum gravity? Which CFTs give rise to emergent spacetime?

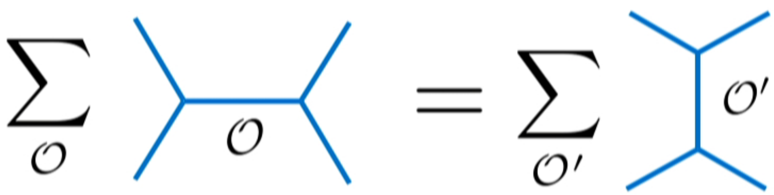


2. What is the space of CFTs, and of theories of AdS quantum gravity? Which CFTs give rise to emergent spacetime?



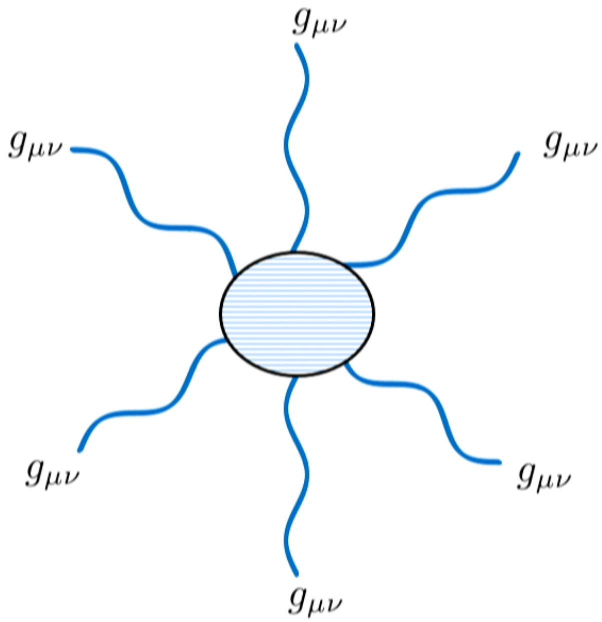


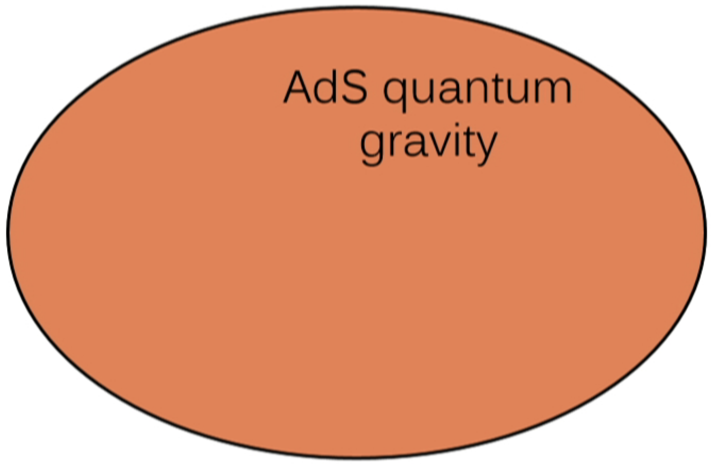
Constrained by e.g.
conformal bootstrap:



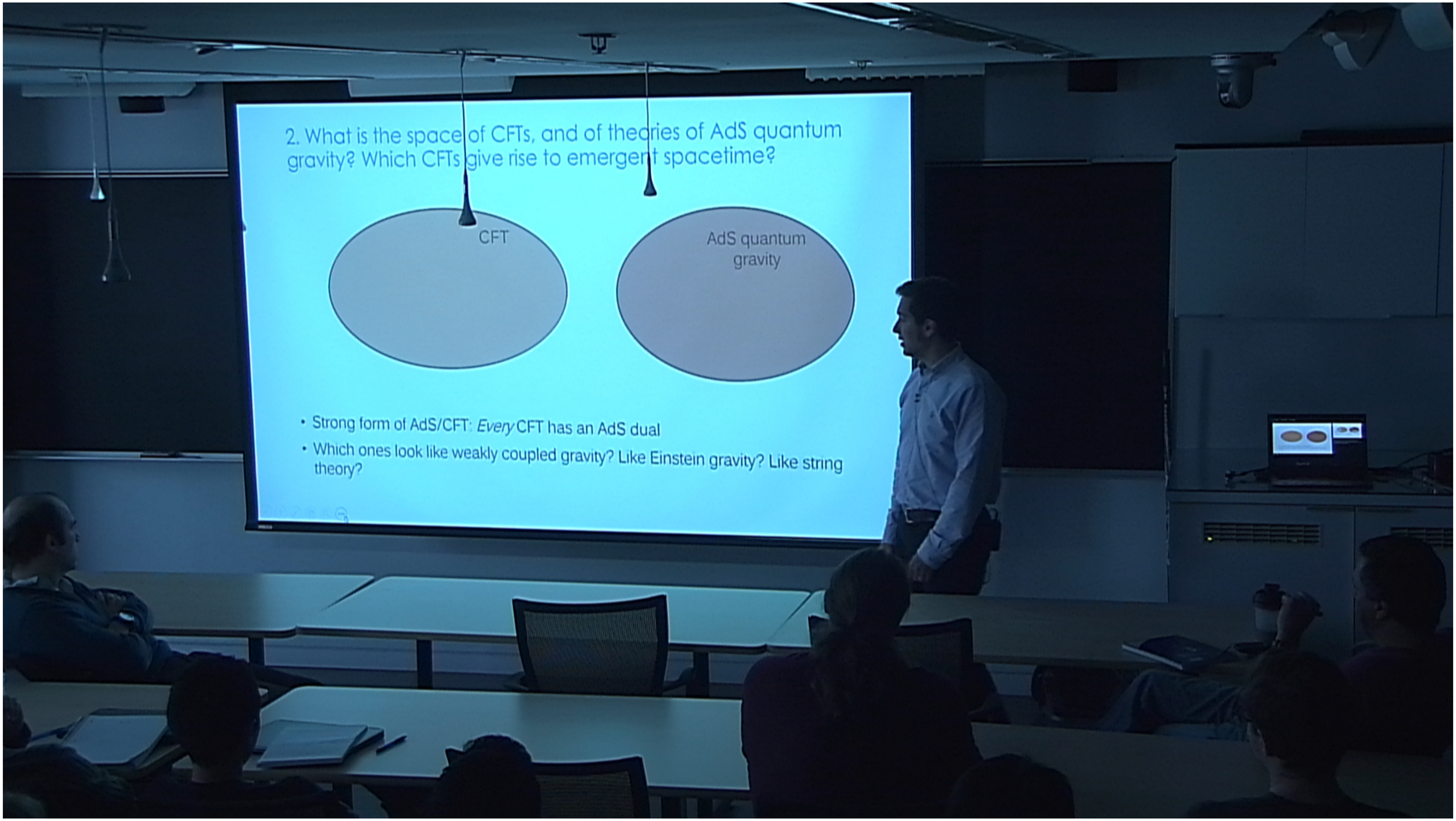
[Rattazzi, Rychkov, Vichi, Tonni;
Kos, Poland, Simmons-Duffin]

2. What is the space of CFTs, and of theories of AdS quantum gravity? Which CFTs give rise to emergent spacetime?





Constrained by high-energy behavior of scattering amplitudes, causality/unitarity, ...


2. What is the space of CFTs, and of theories of AdS quantum gravity? Which CFTs give rise to emergent spacetime?
CFT
AdS quantum gravity
• Strong form of AdS/CFT: *Every* CFT has an AdS dual
• Which ones look like weakly coupled gravity? Like Einstein gravity? Like string theory?


2. What is the space of CFTs, and of theories of AdS quantum gravity? Which CFTs give rise to emergent spacetime?
CFT
Large c CFT
"Sparse" large c CFT
AdS quantum gravity
Classical gravity
Einstein gravity
• Strong form of AdS/CFT: Every CFT has an AdS dual
• Which ones look like weakly coupled gravity? Like Einstein gravity? Like string theory?

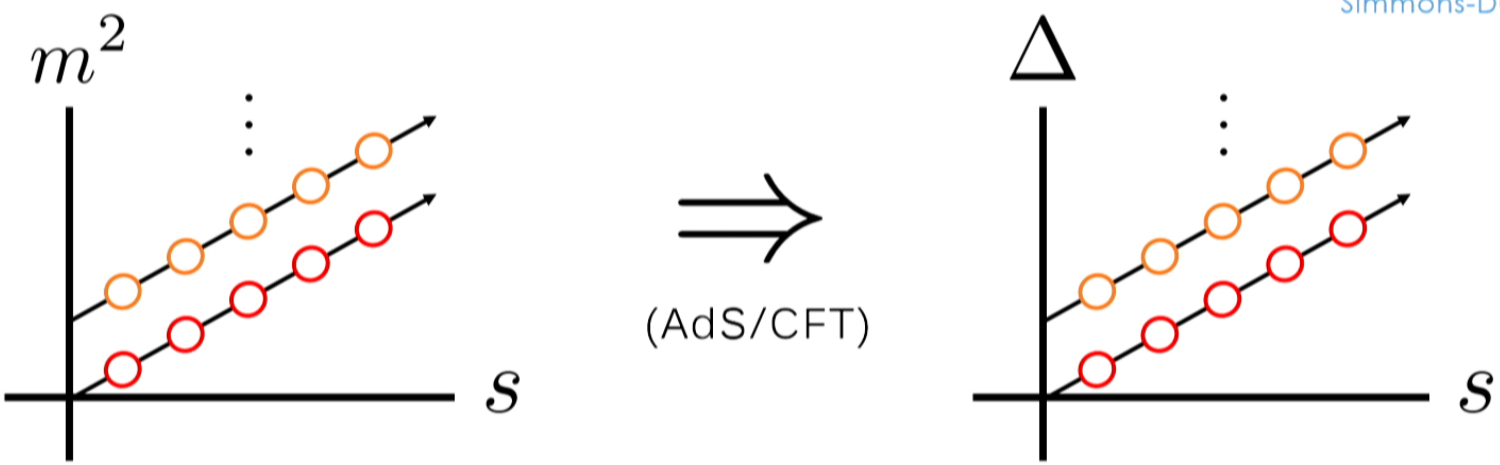
3. How is the structure of string/M-theory visible in CFT?

We know some, but not a lot, about the substructure of CFT spectra. Two examples:

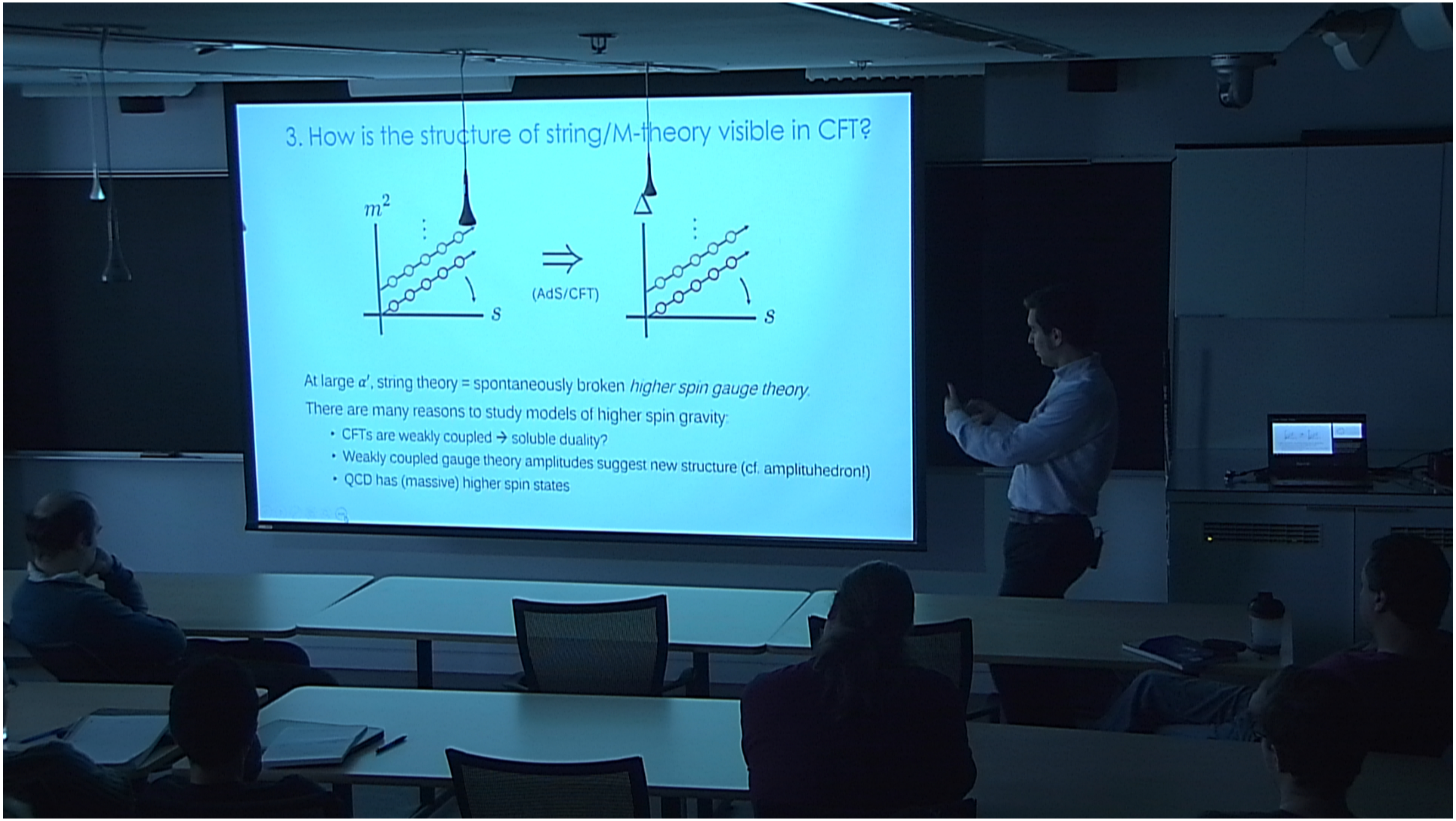
- Cardy formula in 2d CFT: $$d(\Delta \gg c) \sim e^{2\pi\sqrt{\frac{\Delta c}{6}}}$$
- Lightcone bootstrap in d>2 CFT: convex large spin spectrum of generalized free fields

$$\lim_{\ell\to\infty} \gamma(\mathcal{O}\partial^\ell\mathcal{O}) \approx -\frac{\alpha}{\ell^{\#}} + \ldots$$

[Komargodski, Zhiboedov; Fitzpatrick, Kaplan, Poland, Simmons-Duffin]



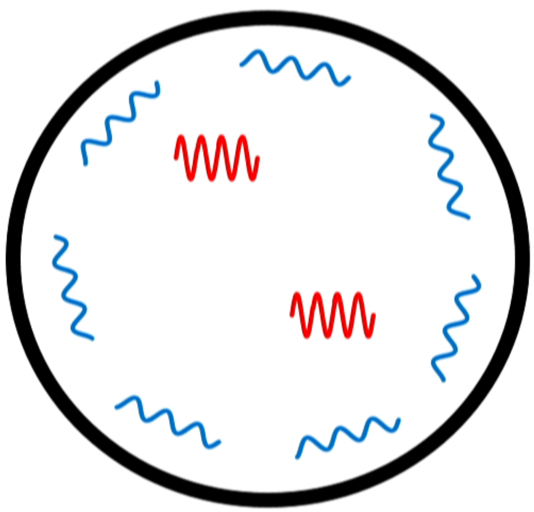
CFTs with string theory duals should have rich structure!



3. How is the structure of string/M-theory visible in CFT?
m^2
s
(AdS/CFT)
s
At large α', string theory = spontaneously broken *higher spin gauge theory*.
There are many reasons to study models of higher spin gravity:
• CFTs are weakly coupled → soluble duality?
• Weakly coupled gauge theory amplitudes suggest new structure (cf. amplituhedron!)
• QCD has (massive) higher spin states

3. How is the structure of string/M-theory visible in CFT?

In AdS_3/CFT_2, one can have a finite tower of higher spin gauge fields.



: HS fields ↔ HS currents (W-algebra)

: Matter ↔ Non-current primaries

- Contain HS black holes
- Reminiscent of singularity resolution in string theory/quantum gravity
- Vasiliev HS gravity couples *infinite* HS tower to a scalar field

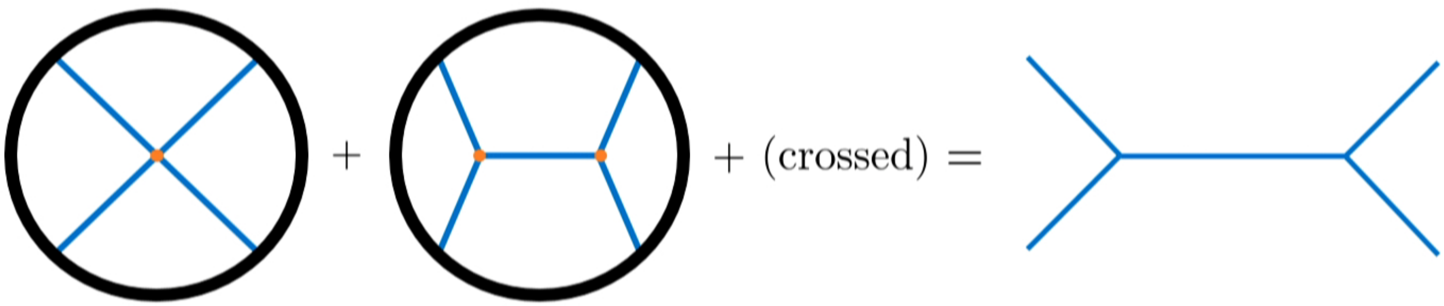
Are any of these theories consistent? If so, are they subsectors of string theory?

In this talk, our primary tool will be CFT four-point functions.

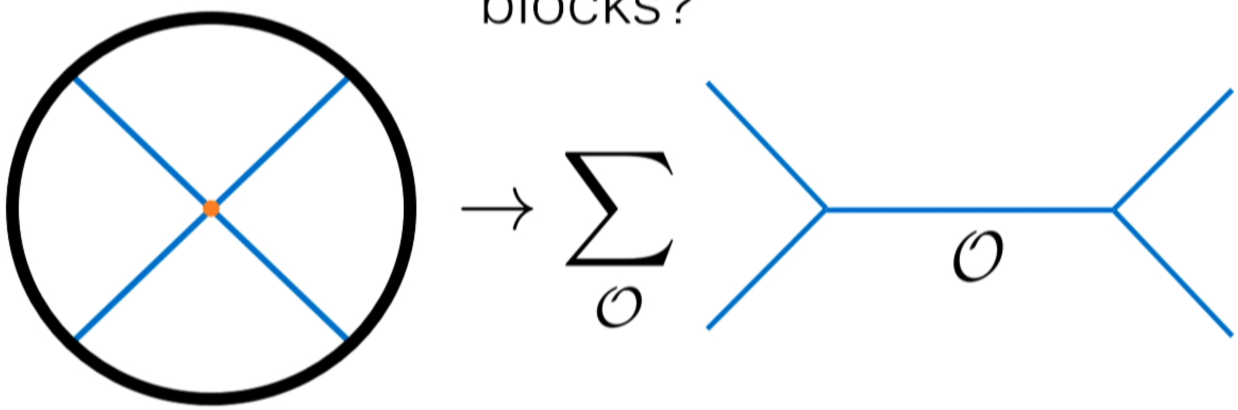
These depend on all OPE data, and admit interesting kinematics.

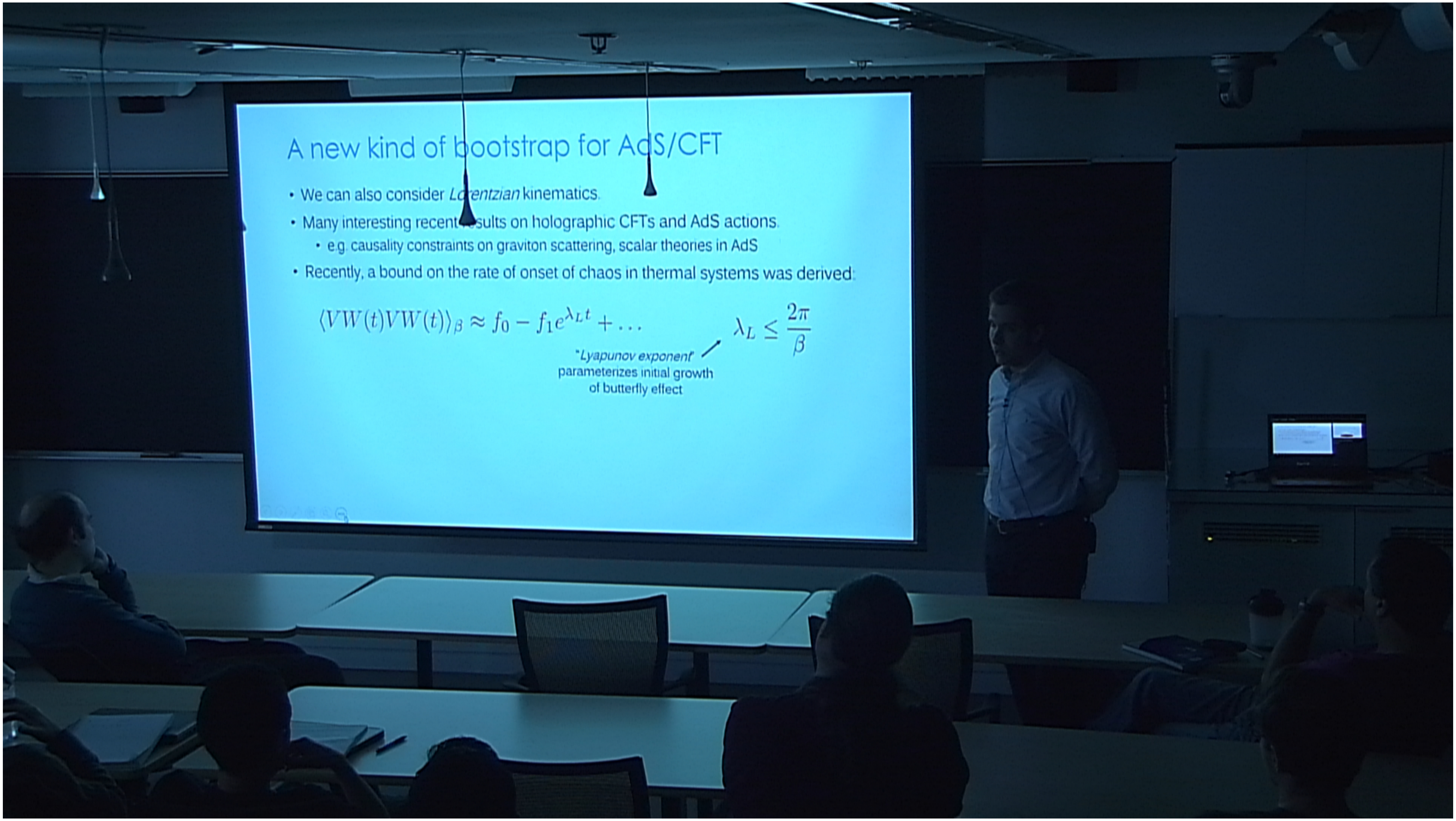
Studying these in different putative classes of AdS/CFT dual pairs will reveal pathologies in some cases, stringy substructure in others, and a new geometric structure of AdS amplitudes.

For large N CFTs with AdS duals, the conformal block decomposition is hidden in conventional bulk calculations.



How do we decompose Witten diagrams into conformal blocks?




A new kind of bootstrap for AdS/CFT
• We can also consider Lorentzian kinematics.
• Many interesting recent results on holographic CFTs and AdS actions.
• e.g. causality constraints on graviton scattering, scalar theories in AdS
• Recently, a bound on the rate of onset of chaos in thermal systems was derived:
$\langle VW(t)VW(t)\rangle_\beta \approx f_0 - f_1 e^{\lambda_L t} + \dots$
$\lambda_L \leq \frac{2\pi}{\beta}$
"Lyapunov exponent" parameterizes initial growth of butterfly effect

A new kind of bootstrap for AdS/CFT

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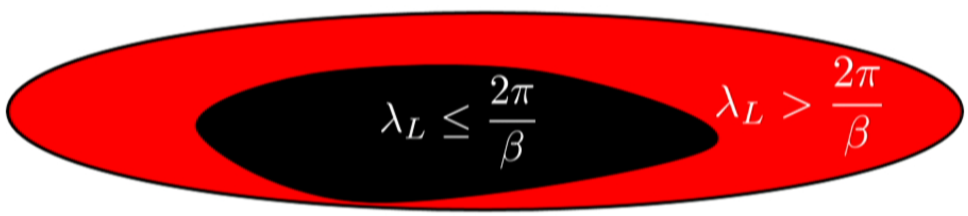
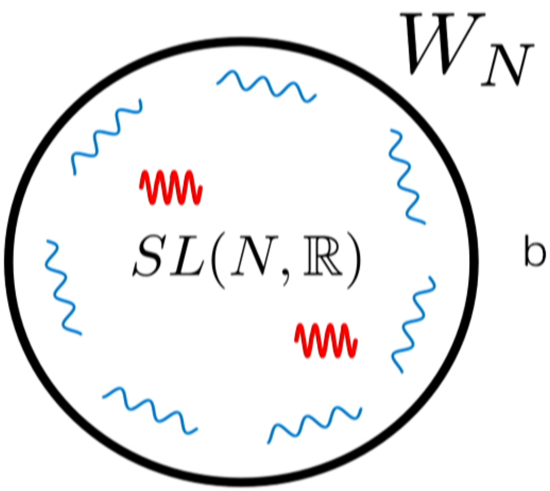
- Proposal: use as a "bootstrap" tool to narrow the landscape of AdS/CFT



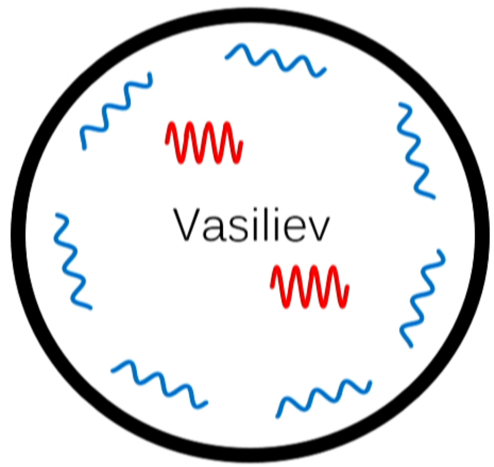
Bounding the space of higher spin CFTs with chaos

- We focus on higher spin theories in AdS_3/CFT_2.
- In d>2, *finite* towers of massive or massless higher spin fields either violate causality or symmetry, respectively.
- In d=2, use chaos bound to constrain four-point functions!
- Upshot: analogous result holds in AdS_3/CFT_2.

[Camanho, Edelstein, Maldacena, Zhiboedov; Maldacena, Zhiboedov]



Violate chaos bound, is acausal.



Obeys chaos bound: Regge-izes!

Summary of results

- "Geodesic Witten diagrams" as CFT conformal blocks.
- A new and simple approach to AdS amplitudes.
- Imposing the chaos bound on putative holographic CFTs constrains the AdS/CFT landscape.
- Higher spin AdS_3/CFT_2 must involve an infinite tower of higher spin fields, mimicking tensionless string theory.

Outline

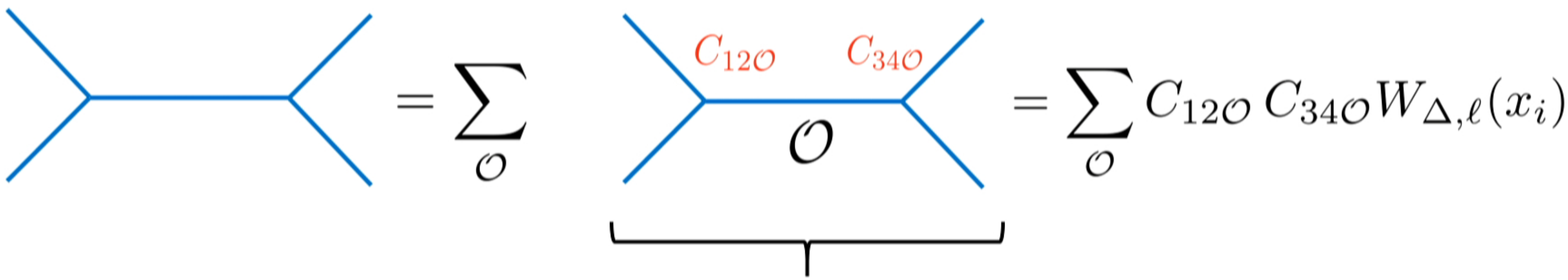
1. Euclidean correlators in AdS/CFT
   - Geodesic Witten diagrams as CFT_d conformal blocks
   - Decomposing AdS amplitudes
   - Virasoro blocks in AdS

2. Bounding the landscape of AdS_3 quantum gravity with chaos
   - A bound on chaos
   - Ruling out higher spin theories
   - Regge behavior in 3D Vasiliev theory

3. Future directions

In computing CFT correlation functions, as in life, we prefer to use symmetry as much as possible.

Vacuum correlation functions admit conformal partial wave decompositions:



Conformal partial wave: the contribution of an irreducible representation of the conformal group, SO(d+1,1).

The many faces of a conformal block

- Conformal partial waves and conformal blocks are proportional:

$$W_{\Delta,\ell}(x_i) \equiv \left(\frac{x_{24}^2}{x_{14}^2}\right)^{\frac{1}{2}\Delta_{12}} \left(\frac{x_{14}^2}{x_{13}^2}\right)^{\frac{1}{2}\Delta_{34}} \frac{G_{\Delta,\ell}(u,v)}{(x_{12}^2)^{\frac{1}{2}(\Delta_1+\Delta_2)}(x_{34}^2)^{\frac{1}{2}(\Delta_3+\Delta_4)}}$$

Conformal partial wave

Conformal block

- Partial waves with external scalars:
  - Have series and integral representations

$$G_{\Delta,\ell}(u,v) \approx u^{\frac{\Delta-\ell}{2}}(1+\ldots)$$

  - Are hypergeometric in even d
  - Are eigenfunctions of an SO(d+1,1) Casimir,

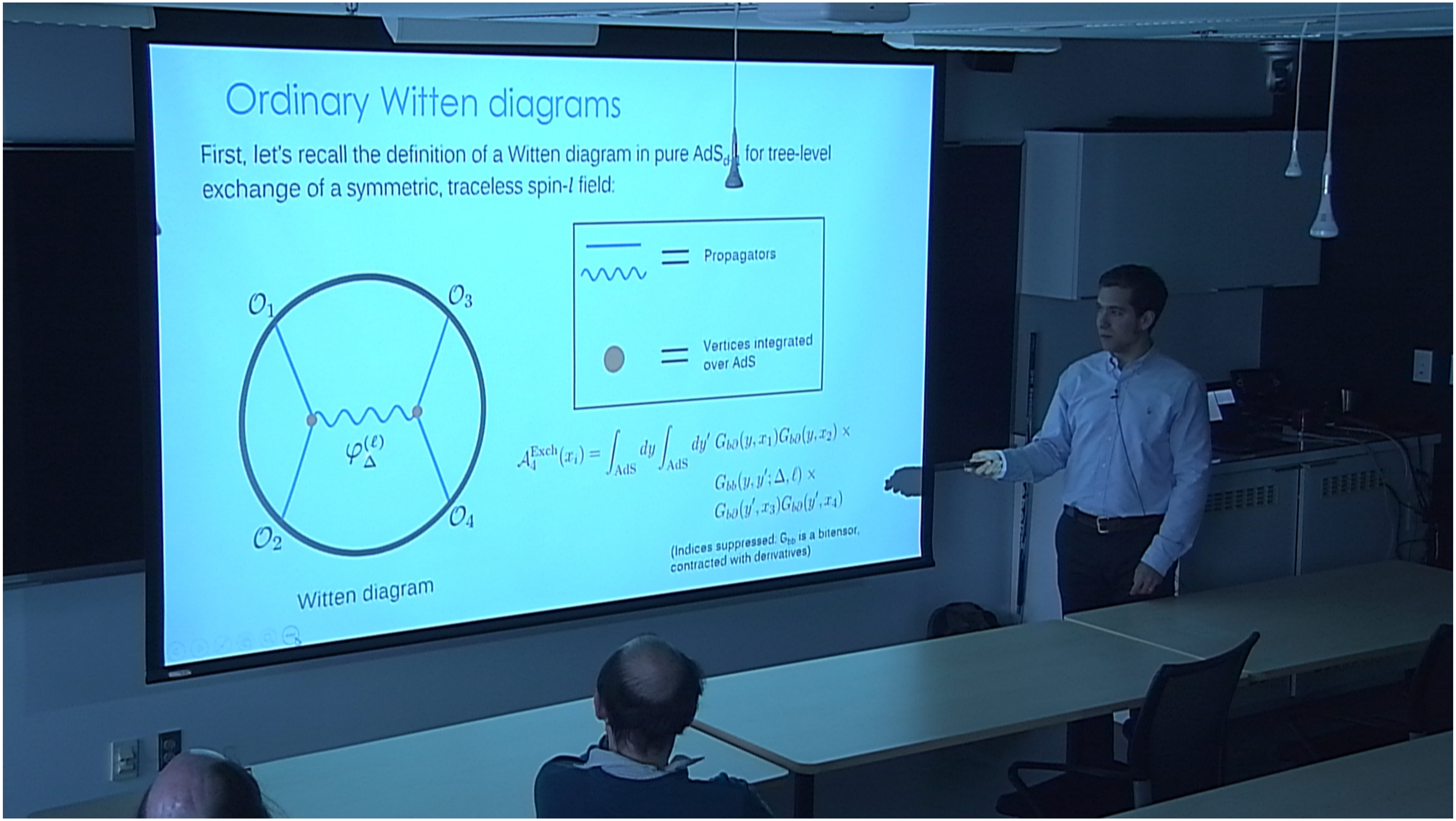
$$(L_{AB}^1 + L_{AB}^2)^2 W_{\Delta,\ell}(x_i) = C(\Delta,\ell) W_{\Delta,\ell}(x_i)$$

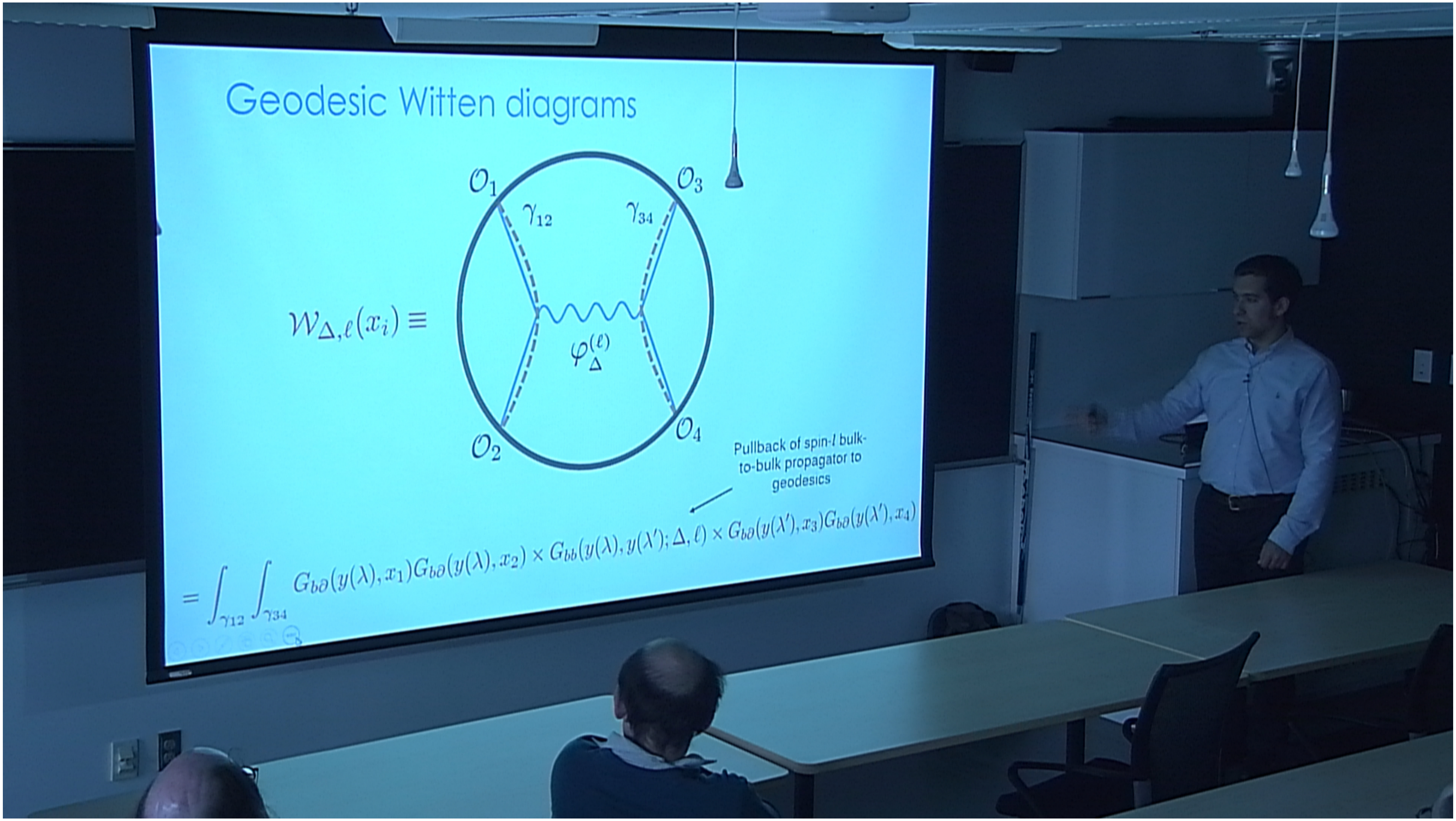
Definitions:

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

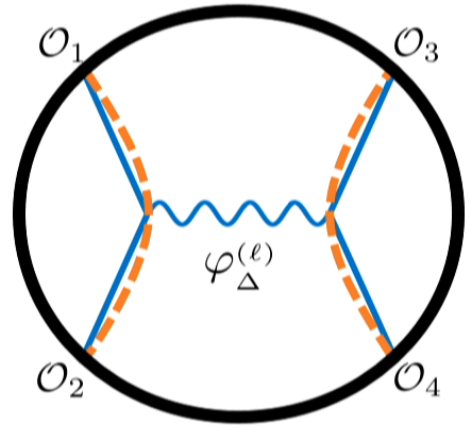
$$\Delta_{ij} = \Delta_i - \Delta_j, \quad x_{ij} = x_i - x_j$$

[Ferrara, Gatto, Grillo; BPZ; Zamolodchikov; Dolan, Osborn; Kos, Poland, Simmons-Duffin; Costa, Penedones, Poland, Rychkov; ...]


Ordinary Witten diagrams
First, let's recall the definition of a Witten diagram in pure AdS for tree-level exchange of a symmetric, traceless spin-l field:
Propagators
Vertices integrated over AdS
Witten diagram
(Indices suppressed: G_{bb} is a bitensor, contracted with derivatives)


Geodesic Witten diagrams
$\mathcal{W}_{\Delta,\ell}(x_i) \equiv$
$\mathcal{O}_1$
$\mathcal{O}_2$
$\mathcal{O}_3$
$\mathcal{O}_4$
γ_{12}
γ_{34}
$\varphi_{\Delta}^{(\ell)}$
Pullback of spin-l bulk-to-bulk propagator to geodesics
$= \int_{\gamma_{12}} \int_{\gamma_{34}} G_{b\partial}(y(\lambda), x_1) G_{b\partial}(y(\lambda), x_2) \times G_{bb}(y(\lambda), y(\lambda'); \Delta, \ell) \times G_{b\partial}(y(\lambda'), x_3) G_{b\partial}(y(\lambda'), x_4)$

Two Proofs



1. Direct computation: e.g. for l=0 exchange,

$$\mathcal{W}_{\Delta,0}(x_i) \propto u^{\frac{\Delta-\Delta_3-\Delta_4}{2}} \int_0^1 d\sigma\, \sigma^{\frac{\Delta+\Delta_{34}-2}{2}}(1-\sigma)^{\frac{\Delta-\Delta_{34}-2}{2}}(1-(1-v)\sigma)^{\frac{-\Delta+\Delta_{12}}{2}}$$
$$\times\, {}_2F_1\left(\frac{\Delta+\Delta_{12}}{2}, \frac{\Delta-\Delta_{12}}{2}; \Delta-\frac{d-2}{2}; \frac{u\sigma(1-\sigma)}{1-(1-v)\sigma}\right)$$

[Ferrara, Gatto, Grillo, Parisi '72]!

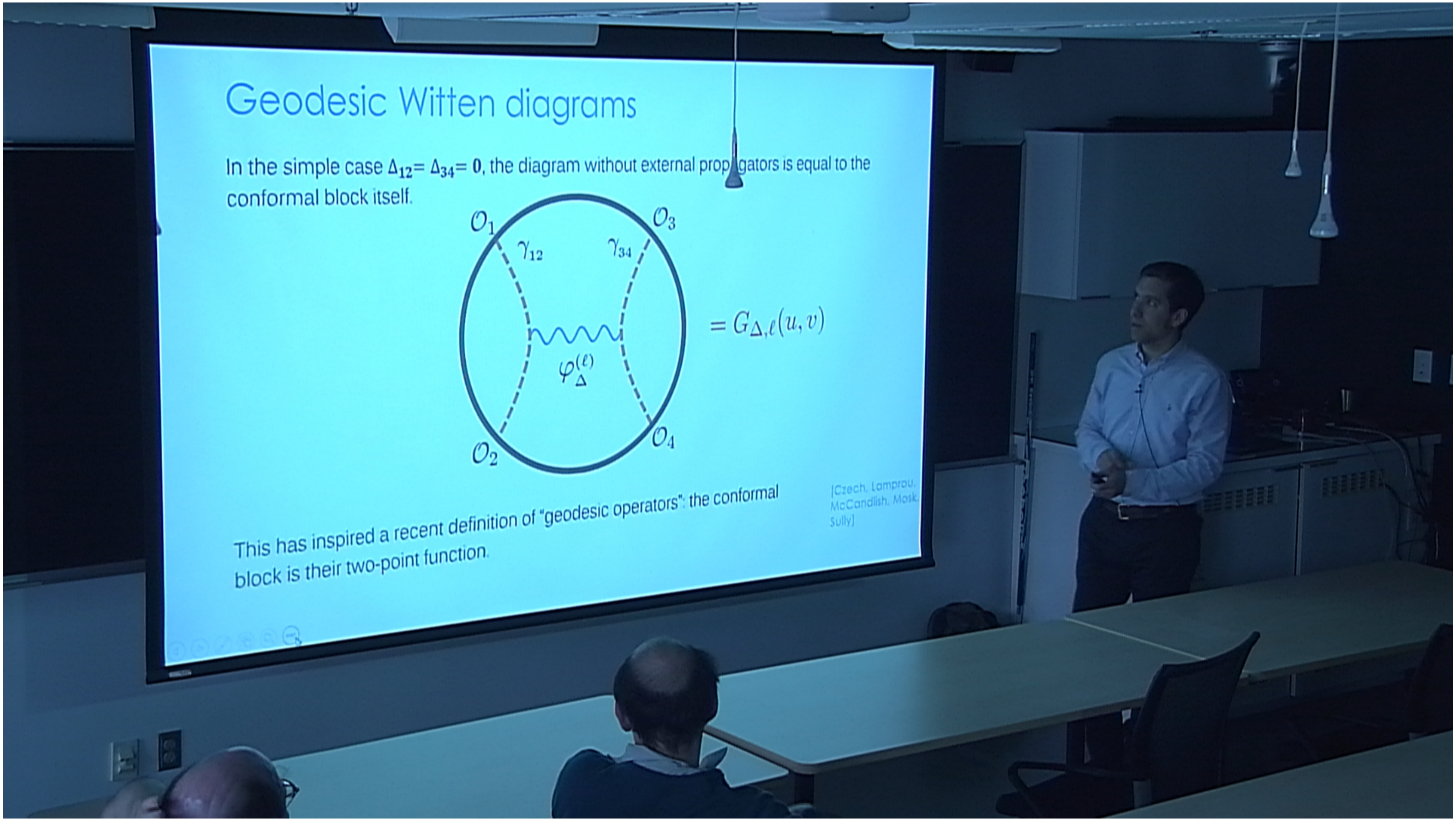
2. Satisfies the conformal Casimir equation (with correct boundary condition):

$$(L^1_{AB}+L^2_{AB})^2\mathcal{W}_{\Delta,\ell} = (\nabla^2_\ell + \ell(\ell+d-1))\mathcal{W}_{\Delta,\ell} = C(\Delta,\ell)\mathcal{W}_{\Delta,\ell}$$

- This (partially) explains "why" this works: e.g. for l=0 exchange,

$$\nabla^2 G_{bb}(y,y';\Delta) = (-\Delta(\Delta-d)+\delta(y-y'))G_{bb}(y,y';\Delta)$$

No support on geodesic diagram!


Geodesic Witten diagrams
In the simple case $\Delta_{12} = \Delta_{34} = 0$, the diagram without external propagators is equal to the conformal block itself.
$\mathcal{O}_1$
$\mathcal{O}_2$
$\mathcal{O}_3$
$\mathcal{O}_4$
γ_{12}
γ_{34}
$\varphi_\Delta^{(\ell)}$
$= G_{\Delta,\ell}(u,v)$
This has inspired a recent definition of "geodesic operators": the conformal block is their two-point function.
[Czech, Lamprou, McCandlish, Mosk, Sully]

Holographic CFT spectra

- Large c CFTs with local gravity duals have the following primaries:
  - A finite density of light ($\Delta \sim c^0$), single-trace operators of spin s ≤ 2 — $\mathcal{O}_i$
  - Their multi-trace composites — $[\mathcal{O}_i\mathcal{O}_j]_{n,\ell} \equiv \mathcal{O}_i \partial^{2n} \partial_{\mu_1} \cdots \partial_{\mu_\ell} \mathcal{O}_j$
  - Heavy operators (Δ grows with c, or λ) — $\vdots$

- Light operators $\Longleftrightarrow$ Perturbative fields + bound states
- Heavy operators $\Longleftrightarrow$ Geometries (e.g. black holes) or string states

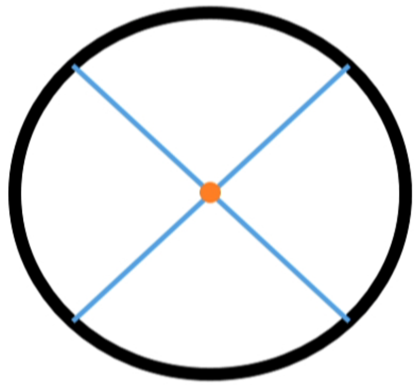
- Single-trace operators in holographic CFTs are *generalized free fields*:

$$\Delta^{(ij)}(n,\ell) = \Delta_i + \Delta_j + 2n + \ell + \gamma^{(ij)}(n,\ell)$$

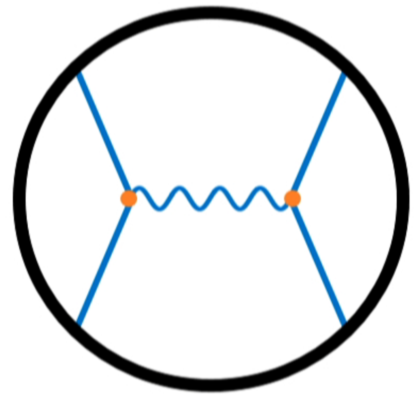
[Heemskerk, Penedones, Polchinski, Sully; El-Showk, Papadodimas; Hartman, Keller, Stoica]

Witten diagrams primer

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle \big|_{1/c} =$$



$$+ \sum$$



$$+ \text{(crossed)}$$

What has been computed?

- 1998-2002: Scalar contact; l=0,1,2 exchange between external scalars.
  - Computed in double-OPE expansion...Where are the blocks?

- Recent: higher spin exchanges; Mellin amplitudes
  - Technically involved, not in position space, or both.

[GKPW; Liu; D'Hoker, Freedman, Mathur, Matusis, Rastelli; Liu, Tseytlin; Arutyunov, Frolov, Petkou; Dolan, Osborn; Costa, Goncalves, Penedones; Bekaert, Erdmenger, Ponomarev, Sleight]

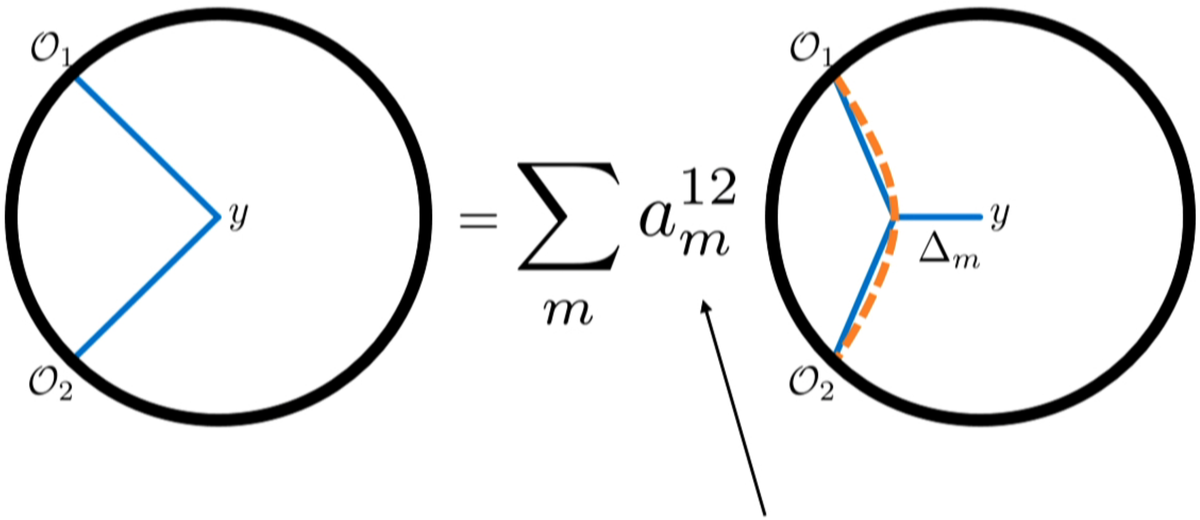
Goal:

Decompose ordinary Witten diagrams into geodesic Witten diagrams, i.e. conformal partial waves.

0. A geodesic identity in AdS
I. Scalar contact
II. Scalar exchange

A geodesic identity in AdS

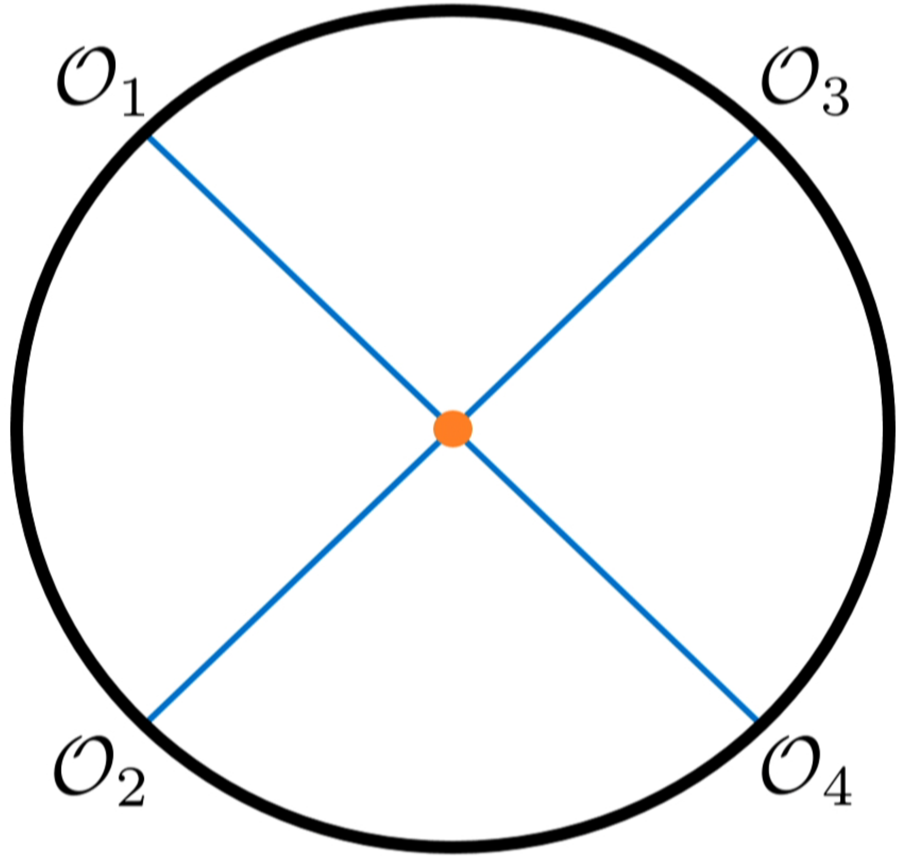
- To proceed, we introduce one more identity:



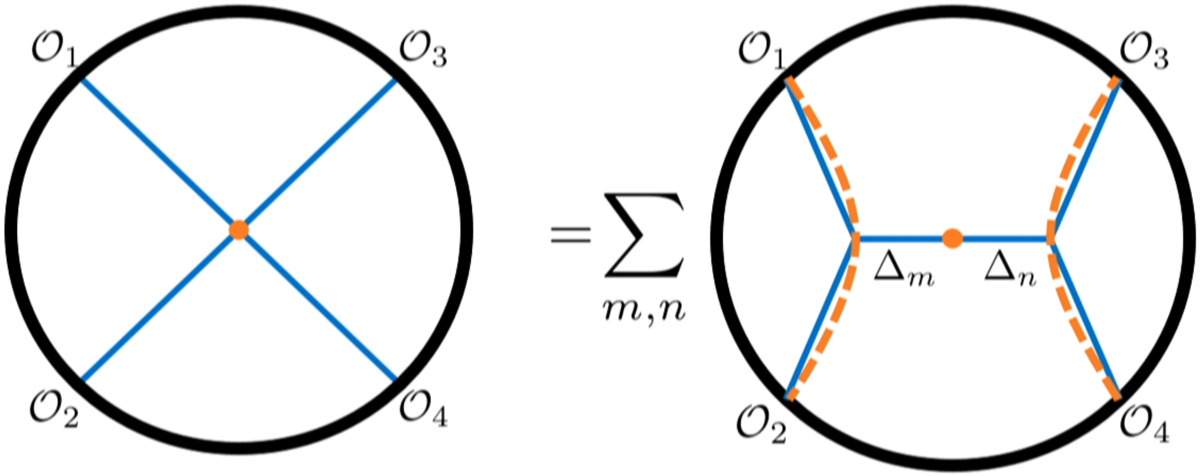
where $\Delta_m = \Delta_1 + \Delta_2 + 2m$

Known coefficients

I. Scalar contact diagram



I. Scalar contact

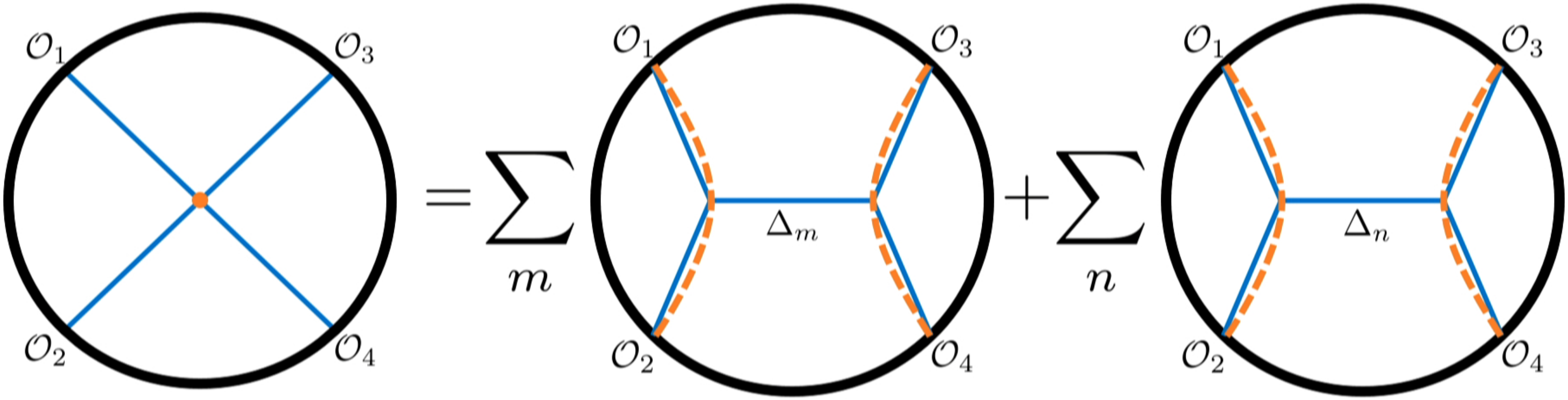


(Apply geodesic identity twice)

$\Delta_m = \Delta_1 + \Delta_2 + 2m$

$\Delta_n = \Delta_3 + \Delta_4 + 2n$

I. Scalar contact

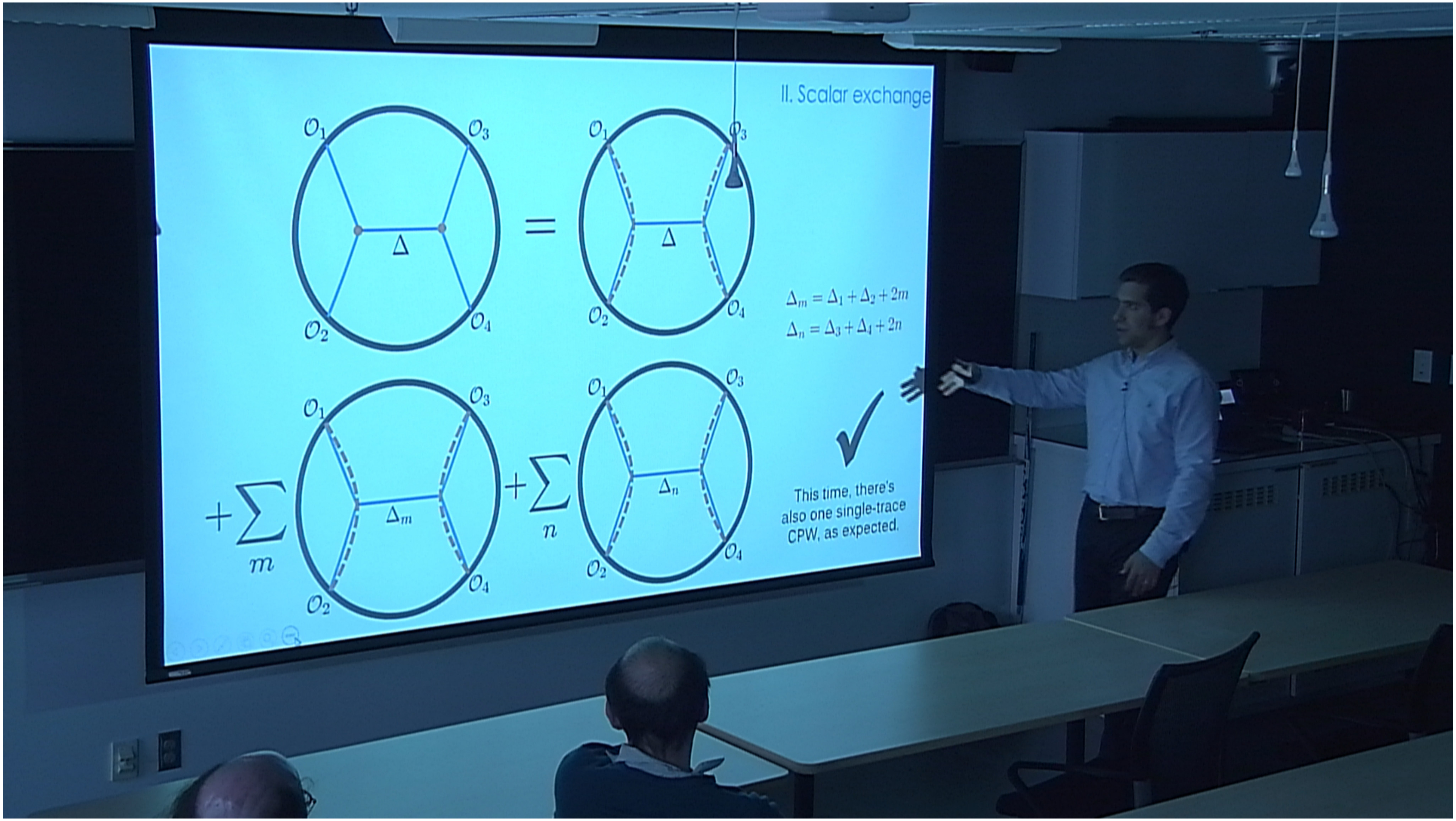


This is the final result:

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4}(x_i) = \sum_m P_1^{(12)}(m,0)\, \mathcal{W}_{\Delta_m,0}(x_i) + \sum_n P_1^{(34)}(n,0)\, \mathcal{W}_{\Delta_n,0}(x_i)$$

with squared "OPE coefficients"

$$P_1^{(12)}(m,0) = a_m^{12}\left(\sum_n \frac{a_n^{34}}{m_m^2 - m_n^2}\right) \ , \quad P_1^{(34)}(n,0) = a_n^{34}\left(\sum_m \frac{a_m^{12}}{m_n^2 - m_m^2}\right)$$

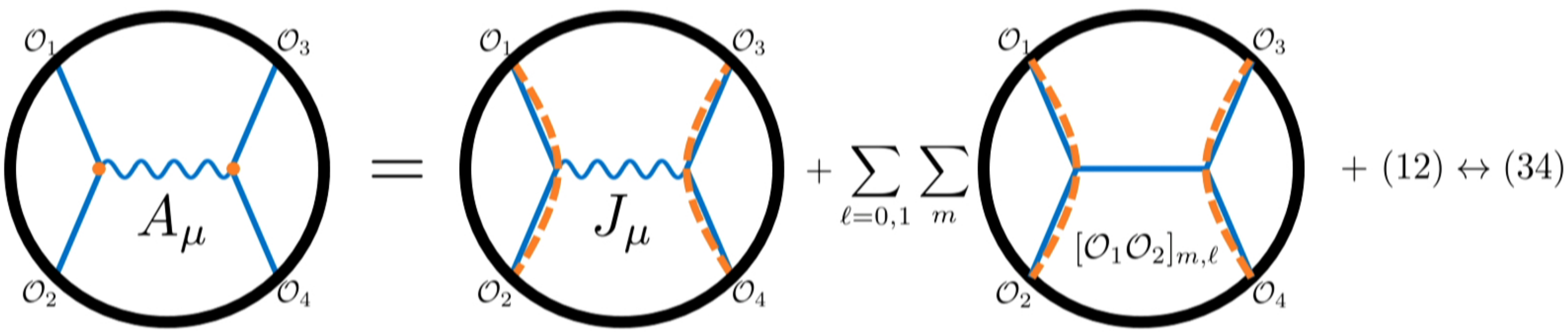

II. Scalar exchange
$\mathcal{O}_1$
$\mathcal{O}_2$
$\mathcal{O}_3$
$\mathcal{O}_4$
Δ
$=$
$+\sum_m$
Δ_m
$+\sum_n$
Δ_n
$\Delta_m = \Delta_1 + \Delta_2 + 2m$
$\Delta_n = \Delta_3 + \Delta_4 + 2n$
This time, there's also one single-trace CPW, as expected.

Comments

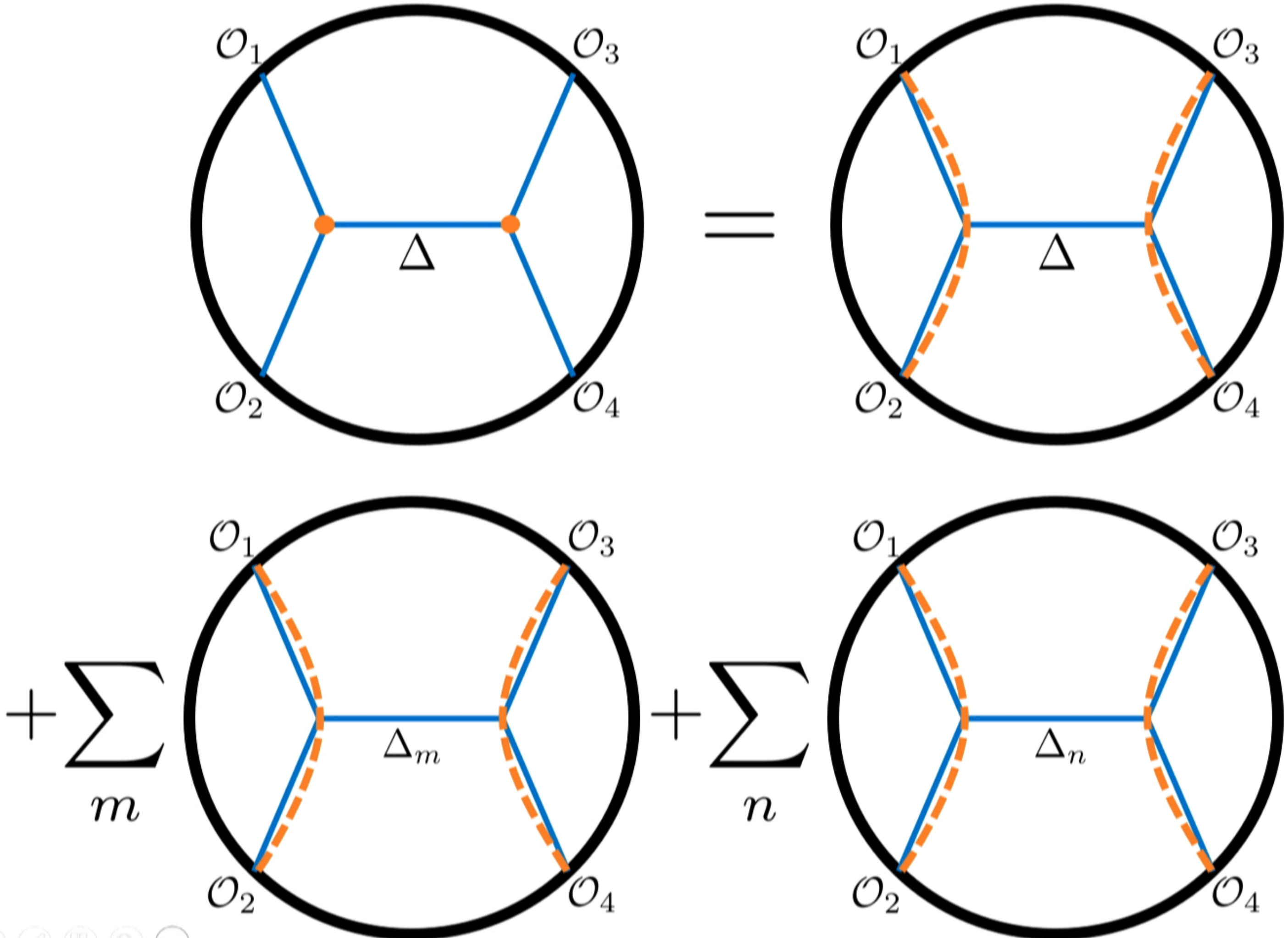
- No integration needed!
- Logarithmic singularities (anomalous dimensions) appear trivially.

$$\frac{\partial}{\partial \gamma} G_{\Delta_0+\gamma,\ell}(u,v) \approx \frac{u^{\frac{\Delta_0-\ell}{2}}}{2} \log u + \dots$$

- Vector exchange Witten diagram can be similarly decomposed:



II. Scalar exchange



$$\Delta_m = \Delta_1 + \Delta_2 + 2m$$
$$\Delta_n = \Delta_3 + \Delta_4 + 2n$$

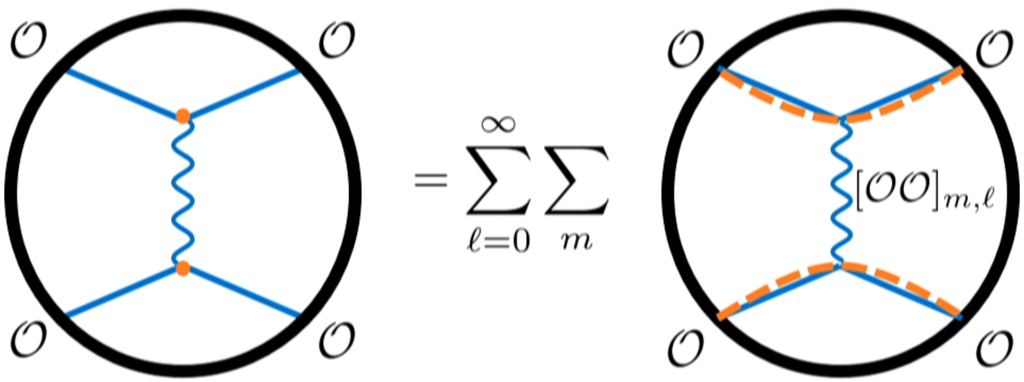
✔

This time, there's also one single-trace CPW, as expected.

To do

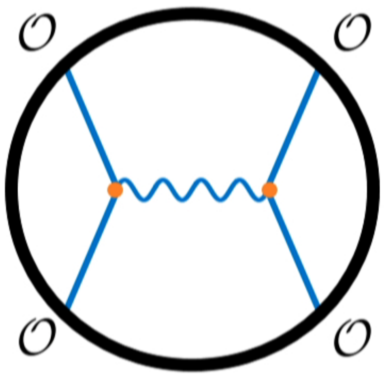
- Add external spin
- n-point diagrams

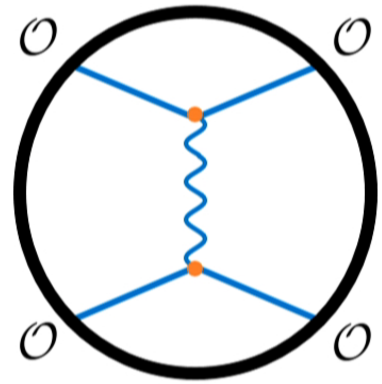
- What about crossed diagrams?
  - Note: crossed channel decomposition involves sum over double-trace operators of unbounded spin.



On crossing symmetry

- A full amplitude may involve a sum over exchanges in multiple channels.
- A magical feature of AdS/CFT is the manifest crossing symmetry in the bulk:

$\mathcal{A}_4 =$ 

$+$ 

$+ \ldots$

- Our method naturally decomposes the correlator into blocks of *different* channels.
- This is a feature, not a bug! A manifestly crossing-symmetric expansion in conformal blocks of multiple channels simultaneously, in terms of the *same* set of $l \leq 2$ single- and double-trace operators.

Black holes, defects and Virasoro blocks

- In d=2, conformal symmetry is enhanced:

$$SO(2,2) \simeq SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \subset \text{Vir} \times \text{Vir}$$

- Virasoro blocks contain contributions of *local* conformal families:

$$\langle \mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\mathcal{O}_3(z_3,\bar{z}_3)\mathcal{O}_4(z_4,\bar{z}_4)\rangle = \sum_p C_{12p}C^p{}_{34}|\mathcal{F}(c,h_i,h_p,z)|^2$$

Global descendants

$$\begin{cases} \mathcal{O}_p(z,\bar{z}) \\ \partial\mathcal{O}_p(z,\bar{z}) \\ \partial^2\mathcal{O}_p(z,\bar{z})\ , \quad :T(z)\mathcal{O}_p(z,\bar{z}): \\ \partial^3\mathcal{O}(z,\bar{z})\ , \quad :\partial T(z)\mathcal{O}_p(z,\bar{z}): \end{cases}$$

$$\vdots \qquad\qquad \vdots$$

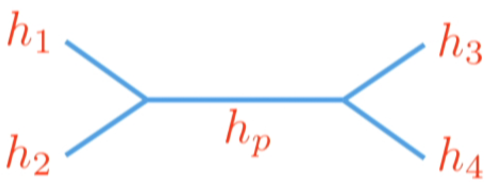
$$L_0|\mathcal{O}_i\rangle = h_i|\mathcal{O}_i\rangle$$

- Unlike global SL(2,R) blocks, these depend on the central charge, c.

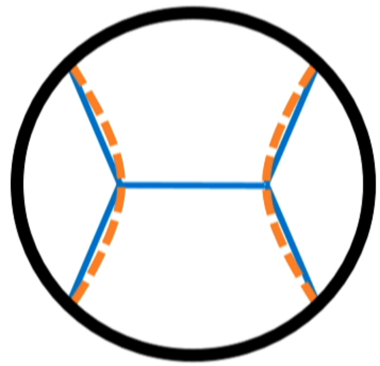
- At large c, we can construct their holographic duals using classical AdS geometry!

Black holes, defects and Virasoro blocks

- Large c Virasoro blocks = geodesic Witten diagrams in locally AdS_3 spacetimes.



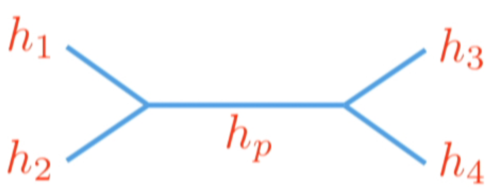
h_i fixed :



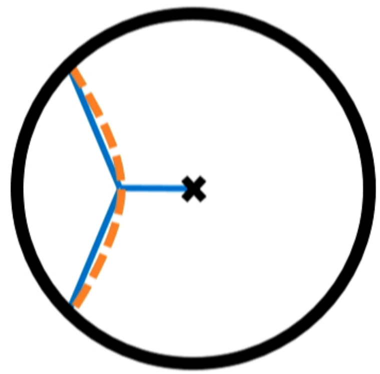
Geodesic Witten diagram in AdS_3

One pair of operators becomes heavy…

One source backreacts!



$$h_{3,4} \to \infty, \quad \frac{h_{3,4}}{c} \text{ fixed :}$$



Geodesic Witten diagram in conical defect/Euclidean BTZ geometry

Outline

1. Euclidean correlators in AdS/CFT
   - Geodesic Witten diagrams as CFT_d conformal blocks
   - Decomposing AdS amplitudes
   - Virasoro blocks in AdS

2. Bounding the landscape of AdS_3 quantum gravity with chaos
   - A bound on chaos
   - Ruling out higher spin theories
   - Regge behavior in 3D Vasiliev theory

3. Future directions

Quantum chaos in CFT

- Consider two local operators V and W, in a thermal state with inverse temperature β, separated in space by a distance x and in real time by t.
- Their squared commutator has been proposed as a diagnostic of chaos in CFT:

$$\langle [V, W(t)]^2 \rangle_\beta \qquad \text{where} \qquad W(t) = e^{-iHt} W e^{iHt}$$

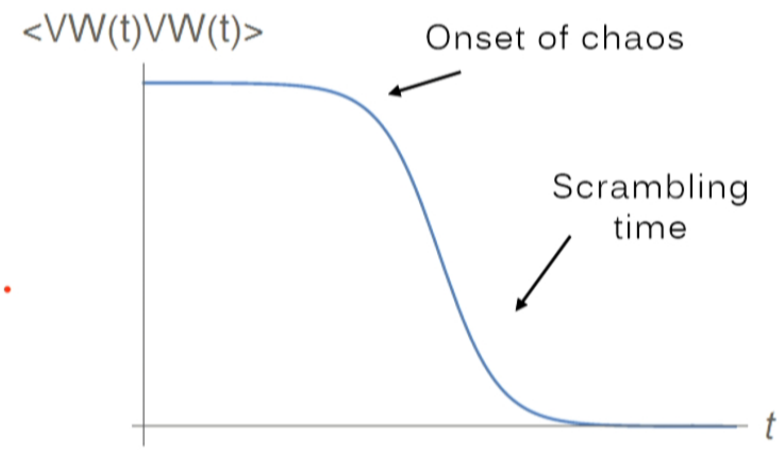
- At t=0, the operators commute. In a chaotic system, W(t) becomes increasingly non-local in time, eventually saturating the system size. This spreading is the butterfly effect:

$$\langle [V, W(t)]^2 \rangle_\beta \propto e^{2\lambda_L t} \text{ for } t \gg \beta$$

- Equivalently, the out-of-time-order (OTO) correlator decreases exponentially in time:

$$\langle V W(t) V W(t) \rangle_\beta \approx f_0 - f_1 e^{\lambda_L t} + \ldots$$

[Shenker, Stanford; Roberts, Stanford]



A bound on chaos

The *Lyapunov exponent*, λ_L, characterizes the rate of onset of chaos.

In large c CFTs,

$$\frac{\langle VW(t)VW(t)\rangle_\beta}{\langle VV\rangle_\beta \langle W(t)W(t)\rangle_\beta} \approx 1 - \frac{e^{\lambda_L t}}{c} f(x) + \ldots$$

(Subleading in large t, and in 1/c)

All unitary, causal CFTs (obeying reasonable assumptions) satisfy a bound:

$$\boxed{\lambda_L \leq \frac{2\pi}{\beta}}$$

[Maldacena, Shenker, Stanford]

This refines the "fast scrambling conjecture":

$$t_* \geq \frac{\beta}{2\pi} \log c$$

[Sekino, Susskind; Hayden, Preskill]

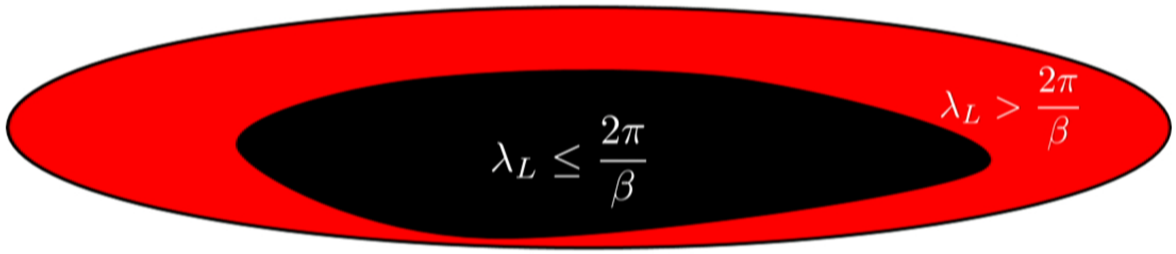
Also closely related to *causality* of the CFT:

$$\langle W|[V(x_1), V(x_2)]|W\rangle = 0 \quad \text{for } (x_1 - x_2)^2 > 0$$

[Hartman, Jain, Kundu]

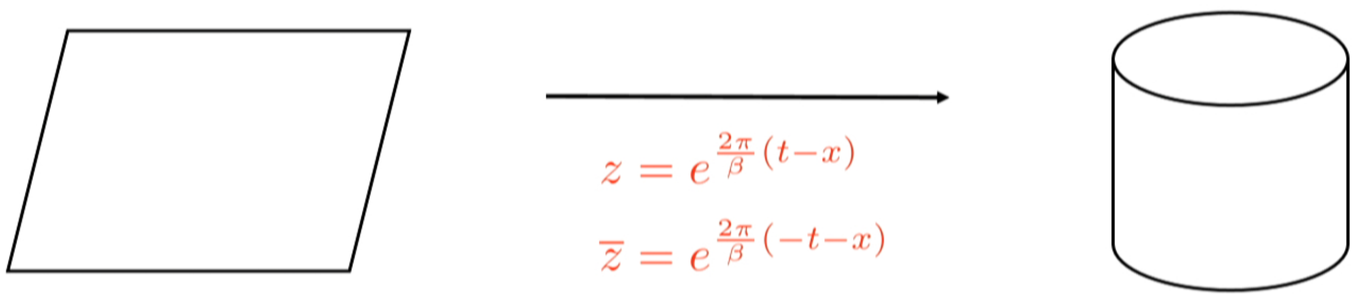
A bound on chaos

- This suggests a new kind of bootstrap: not all putative CFTs will obey this bound!



- Morally similar to an "entanglement bootstrap," where imposing laws of entanglement (e.g. strong subadditivity, positivity of relative entropy) constrains properties of CFTs.

- Strategy: compute thermal correlators by conformal transformation from the plane.

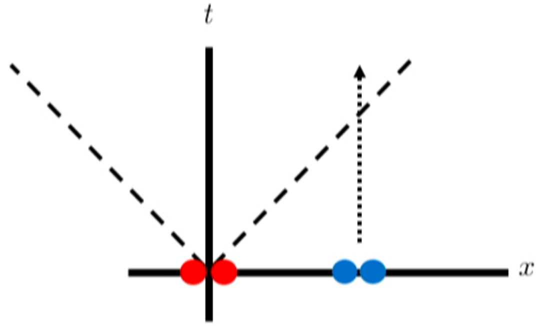


Chaotic kinematics

- We want Lorentzian kinematics, $t > x > 0$, with $t \gg \beta$. Starting from Euclidean correlators,

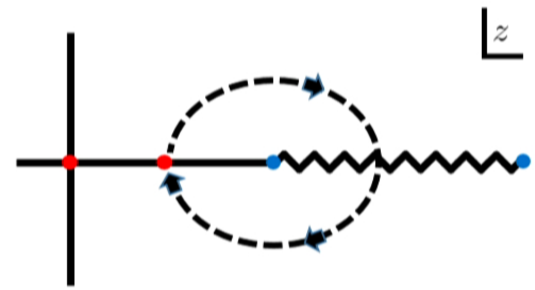
$$\frac{\langle V^\dagger(z_1,\bar{z}_1)V(z_2,\bar{z}_2)W^\dagger(z_3,\bar{z}_3)W(z_4,\bar{z}_4)\rangle}{\langle V^\dagger(z_1,\bar{z}_1)V(z_2,\bar{z}_2)\rangle\langle W^\dagger(z_3,\bar{z}_3)W(z_4,\bar{z}_4)\rangle} = \mathcal{A}(z,\bar{z})$$

we analytically continue:



$$(1-z) \to e^{-2\pi i}(1-z)$$

$$z \to 0\ , \quad \bar{z} \to 0\ , \quad \frac{\bar{z}}{z} \equiv \eta \text{ fixed}$$

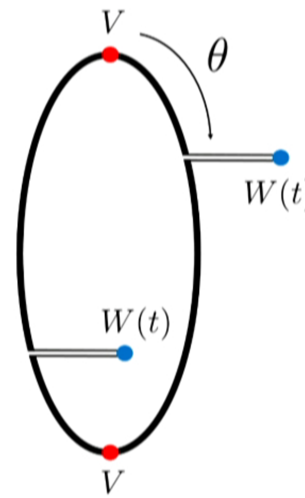


- This is known as the Regge limit:

$$\frac{\langle V^\dagger W^\dagger(t) V W(t)\rangle_\beta}{\langle V^\dagger V\rangle_\beta \langle W^\dagger(t) W(t)\rangle_\beta} = \mathcal{A}^{\text{Regge}}(z,\eta)$$

From whence comes the bound?

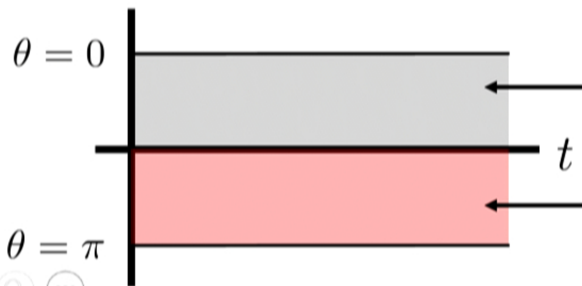
The bound is derived upon assuming analyticity and boundedness of OTO correlator in a half-strip in the complex time plane:



The following test function violates the chaos bound for any n>1:

$$f(\theta, t) = 1 + \frac{i\epsilon}{z^n} + \dots$$

e.g. for n=2,



$$|f| \approx 1 - e^{\frac{4\pi}{\beta} t}$$

$$|f| \approx 1 + e^{\frac{4\pi}{\beta} t}$$

$$z = -4 e^{i\theta} e^{\frac{2\pi}{\beta}(x-t)}\,,$$

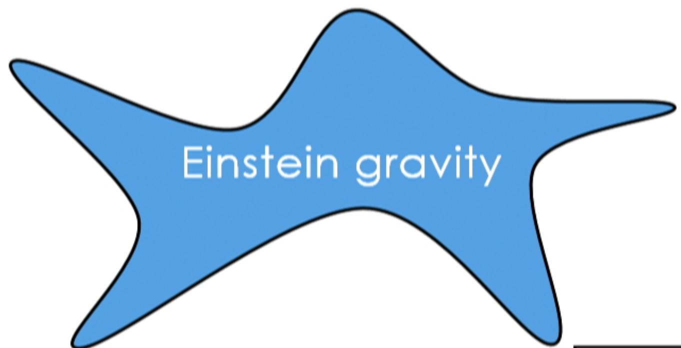
$$\eta = e^{-\frac{4\pi}{\beta} x}$$

Holographic CFTs with Einstein gravity duals

- For CFT$_d$'s with Einstein gravity duals, bulk calculations *saturate* the bound.

$$\left.\frac{F(\lambda \to \infty)}{F(\lambda \to 0)}\right|_{\mathcal{N}=4 \text{ SYM}} = \frac{3}{4}$$

$$a = c$$



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

$$S_{EE} = \frac{\text{Area}}{4G_N}$$

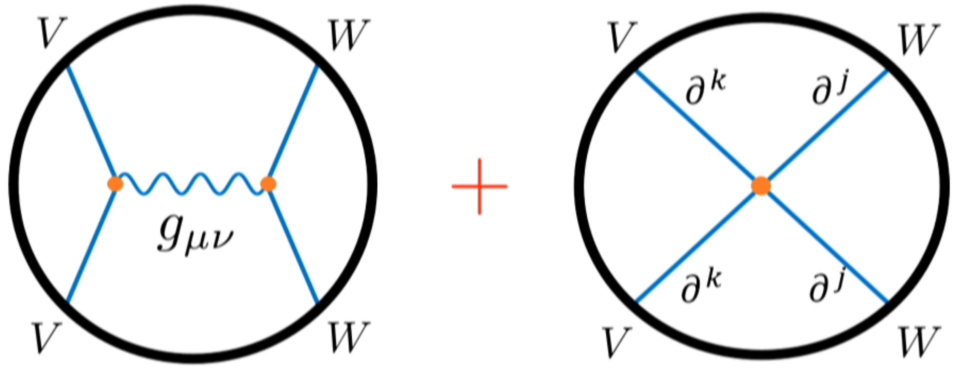
$$\boxed{\lambda_L = \frac{2\pi}{\beta}}$$

[Shenker, Stanford; Roberts, Stanford, Susskind]

$\lambda_L = \frac{2\pi}{\beta}$ from CFT

- In a somewhat simplified model of holography, only s ≤ 2 exchanges in VV-WW channel.
  - Equivalently, the only Witten diagrams are

$$\mathcal{A}(z,\bar{z})\big|_{1/c} =$$



Note: causality limits j,k = 0 or 1!

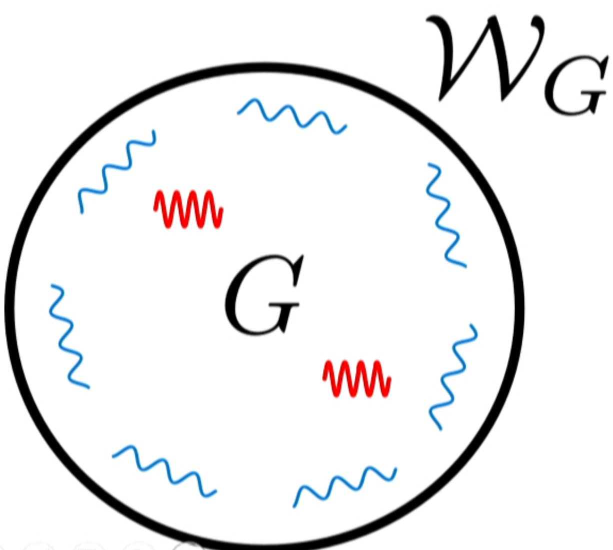
- The Regge limit of spin-s conformal block behaves like spin-s exchange, so spin-2 exchanges dominate:

$$\mathcal{A}^{\text{Regge}}(z,\eta) \sim 1 + \frac{i}{cz} f(\eta) + \ldots$$

[EP]

Chaotic destruction of higher spin theories

- 2d CFTs can have higher spin currents, $\{J_s(z)\}$, of spins s=2,3,...,N. These generate a W-algebra.
- Primaries carry spin-s charges: $J_{s,0}|W\rangle = q_w^{(s)}|W\rangle \ , \quad J_{s,0}|V\rangle = q_v^{(s)}|V\rangle$
- "3d higher spin gravity" = matter coupled to G x G Chern-Simons theory.



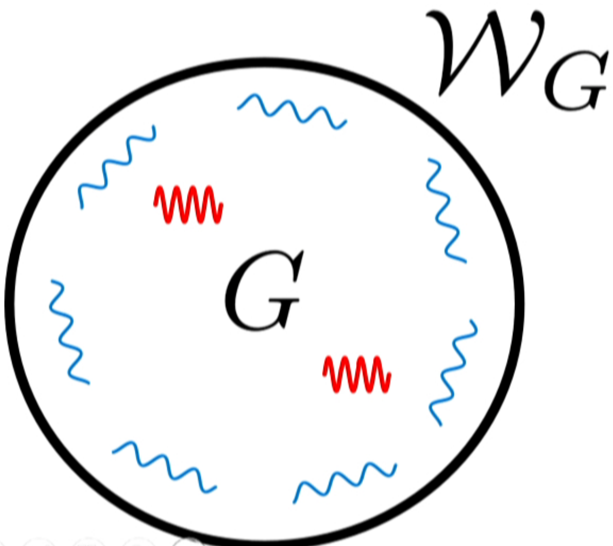
Chaotic destruction of higher spin theories

- Previous argument generalizes to higher spin CFTs, for W-algebra with currents of spin s ≤ N for finite N.

Result: $$\lambda_L = \frac{2\pi}{\beta}(N-1)$$

This violates the bound!

→ Unitary, causal holographic CFTs with a finite tower of higher spin currents do not exist!



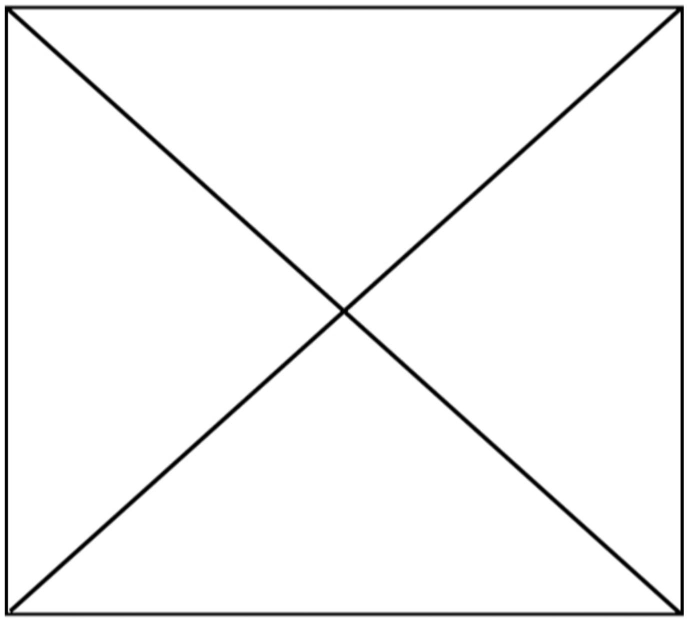
$$t_* = \frac{\beta}{2\pi(N-1)} \log c$$

Higher spin CFTs are too-fast scramblers

[EP]

Chaotic destruction of higher spin theories

→ Weakly coupled higher spin gravities with finite towers of higher spin gauge fields are inconsistent.



Start with eternal BTZ black hole, dual to thermofield double state

Aside: SL(N) vs. Gauss-Bonnet

- Both theories are acausal, require infinite towers of higher spin fields for completion
- Gauss-Bonnet (like all other higher derivative theories in $AdS_{d>3}$) has a coupling which sets the scale of new massive higher spin fields:

[Camanho, Edelstein, Maldacena, Zhiboedov]

$$\mathcal{L} = \mathcal{L}_{\rm EH} + \alpha R^2 : \quad \alpha \sim M_{HS}^{-2}$$

- SL(N) theories are in even worse shape: all fields are massless, only one scale in the problem: L_{AdS}
- This requires infinite tower of *massless* higher spin fields!

Lesson so far: in holographic CFTs, λ_L may be extracted from the vacuum block alone.

What happens for an infinite tower of HS currents?

To W_∞ (and beyond?)

- There exists a (probably unique) classical W-algebra with one current at each spin s=2,3,...

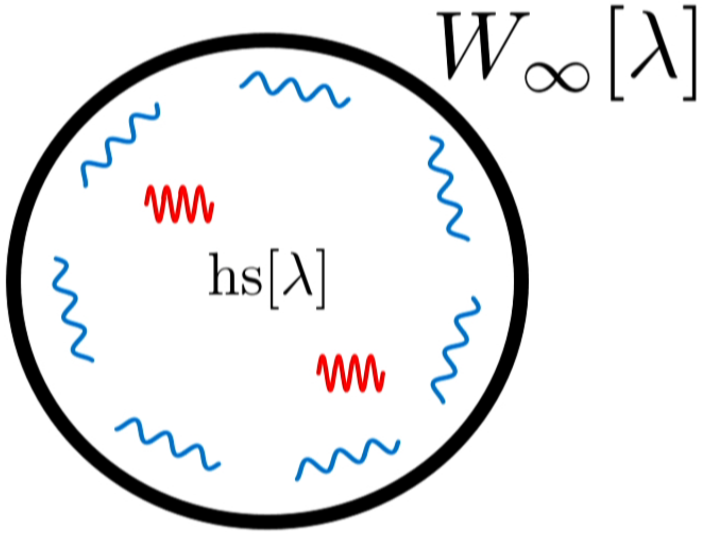
$$\mathcal{W}_\infty[\lambda]$$

  - CFT realizations: free bosons (λ=1) and fermions (λ=0); Gaberdiel-Gopakumar limit of W_N minimal models ($0 \leq \lambda \leq 1$).

- This is the asymptotic symmetry of 3d Vasiliev higher spin gravity.

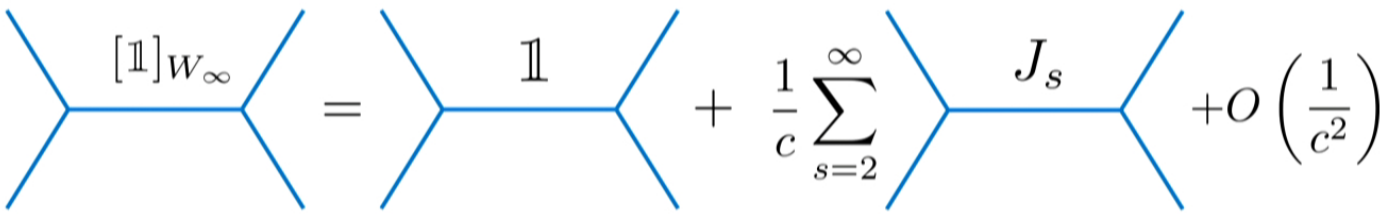
- Evidence that a string theory (D1-D5) embedding of SUSY-Vasiliev exists. What about non-SUSY?

  - *Regge-ization as proxy for stringiness.*



To W_∞ (and beyond?)

- With an infinite tower of currents, re-summation is possible.

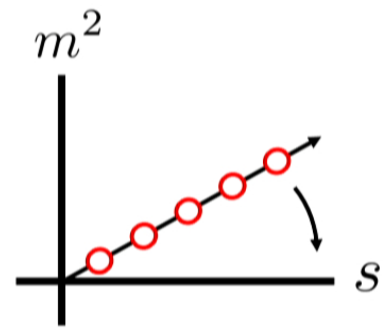


- Take V and W both in the "minimal" representation.
  - Result:

$$\mathcal{F}_{\mathrm{vac},\infty}(z|\lambda) = 1 + \frac{1}{c}(1-\lambda^2)\big(z\,{}_2F_1(1,1,1-\lambda,z) + \log(1-z)\big) + O\left(\frac{1}{c^2}\right)$$

$$\downarrow$$

$$\lambda_L = 0$$



→ Encourages a tensionless string theory interpretation of the 3D Vasiliev theory

Classification of higher spin CFTs

- The CFTs we ruled out are sparse, large c higher spin CFTs.
- Related arguments from causality allow us to drop the sparseness assumption:

Unitary CFTs with large c, a finite tower of higher spin currents and light primary operators do not exist.

- This suggests the following.

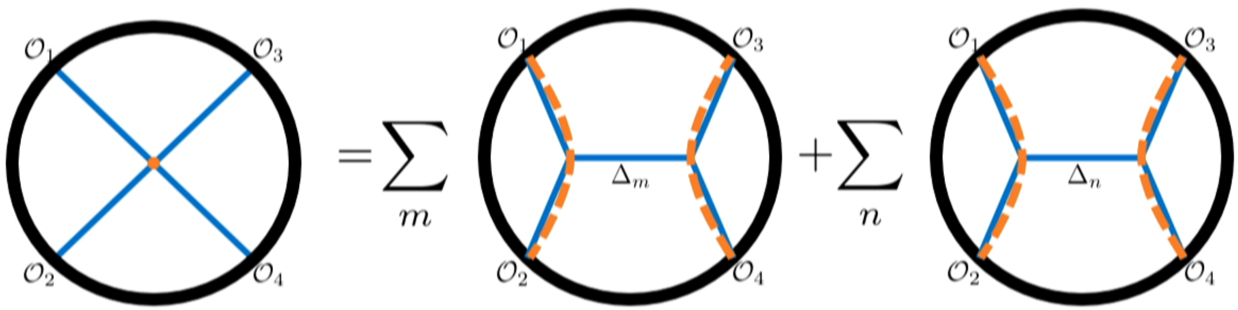
Conjecture (?): All finitely-generated, compact higher spin CFTs are rational.

  - d=2 version of Maldacena-Zhiboedov no-go theorem on higher spin theories.

[T. Hartman, A. Maloney, EP, work in progress]

Summary of results

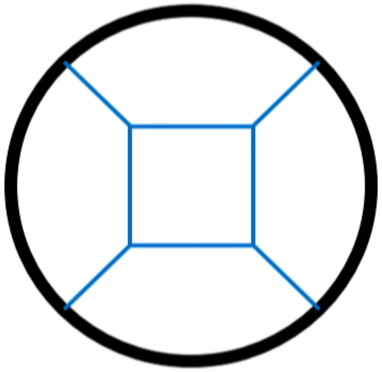
- Geodesic Witten diagrams geometrize CFT conformal blocks, and provide a simple way to decompose AdS amplitudes.



- Theories of AdS_3 gravity, and large c CFT_2's, with finite towers of higher spin fields are inconsistent theories: they violate the chaos bound.

- 3D Vasiliev theory exhibits Regge-ization, mimicking tensionless string theory.

Future directions



- Toward AdS *quantum* gravity using amplitudes
  - Loops in AdS using geodesic Witten diagrams?
  - Can we learn from flat space to compute loops in AdS, e.g. on-shell methods, scattering equations?
- <TTTT> in Einstein gravity vs. the conformal bootstrap at large central charge.
  - (An old question: does pure AdS_3 gravity exist?)
- Relating chaos to other observables in CFT
- New diagnostics of strings in CFT and emergent spacetime?