Title: Umbral Moonshine and String Theory on K3

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Abstract: <p>The mathematical notion of moonshine relates the theory of finite groups with that of modular objects. The first example, 'Monstrous Moonshine', was clarified in the context of two dimensional conformal field theory in the 90's. In 2010, interest in moonshine in the physics community was reinvigorated when Eguchi et. al. observed representations of the finite group M24 appearing in the elliptic genus of nonlinear sigma models on K3. In 2013, Cheng, Duncan, and Harvey provided a uniform construction of 23 new examples of moonshine, called 'umbral moonshine', of which M24 moonshine is a special case. </p>

<p>In this talk, I will describe recent work studying the symmetries of certain Landau-Ginzburg orbifold theories that flow in the IR to c=6 N=(4, 4) superconformal field theories on the moduli space of K3 sigma models. We show that discrete symmetries of the UV theory implicate all 23 instances of umbral moonshine, not just M24 moonshine, in symmetries of K3 CFTs. I will then discuss a particular string theory compactification to three dimensions where we find a precise connection to all 23 umbral groups and type IIA string theory on K3.</p>

Umbral Moonshine and String Theory on K3

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April 22nd, 2016

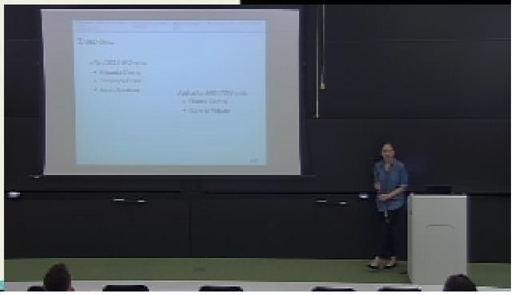
Based on...

arXiv:1512.04942 with:

- Miranda Cheng
- Francesca Ferrari
- Sarah Harrison

And arXiv:1603.07330 with:

- Shamit Kachru
- Roberto Volpato



Outline of this talk

"Lightning" review of moonshine
- Monstrous Moonshine
- Mathieu & Umbral Moonshine

Motivation for our work
- Umbral Moonshine and K3 Surfaces

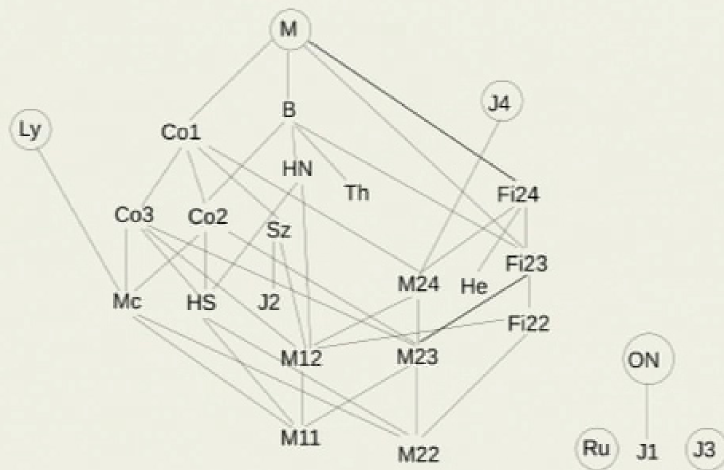
Models and results
- Cubic superpotentials
- Quartic superpotentials

Now playing: in 3D

Open questions

WHAT IS MOONSHINE?

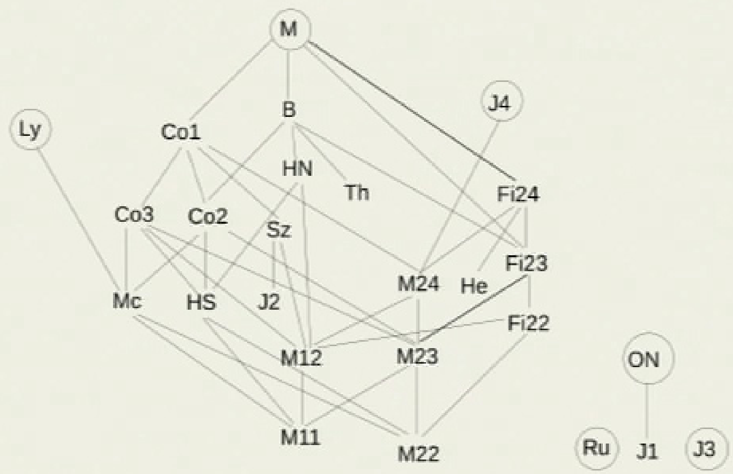
- **Moonshine**: finite groups ↔ modular forms



- Finite group often one of 26 simple sporadics.

What is moonshine?

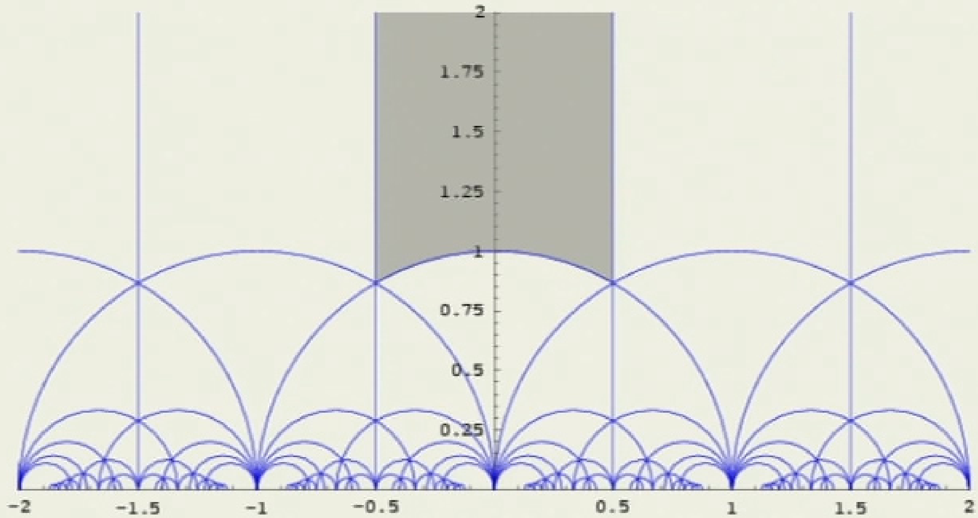
- **Moonshine**: finite groups ↔ modular forms

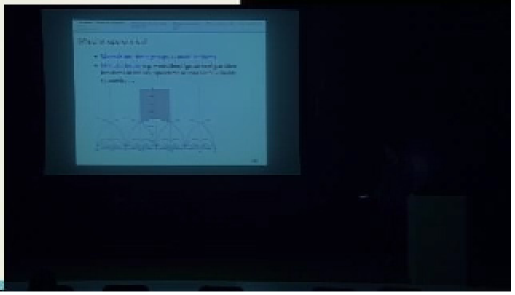


- Finite group often one of 26 simple sporadics.

WHAT IS MOONSHINE?

- **Moonshine**: finite groups ↔ modular forms
- Modular forms: e.g. worldsheet (genus one) partition functions or indices, spacetime actions with S-duality symmetry …





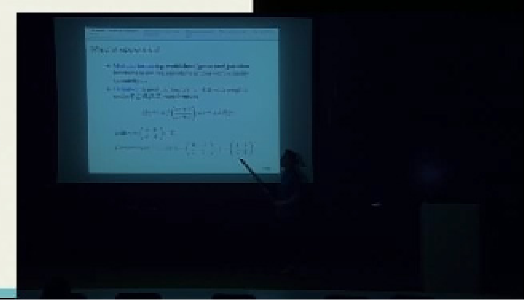
WHAT IS MOONSHINE?

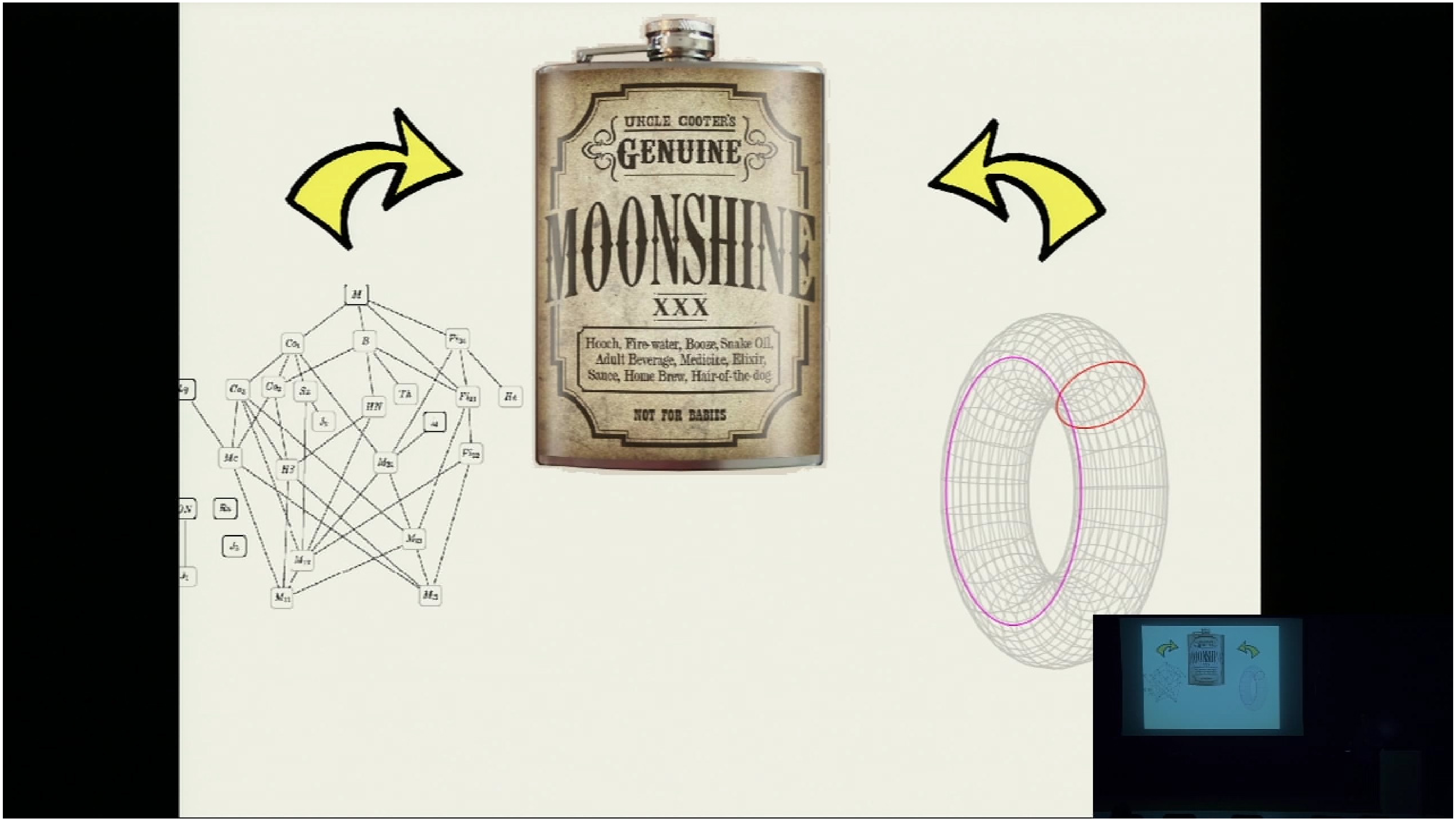
- Modular forms: e.g. worldsheet (genus one) partition functions or indices, spacetime actions with S-duality symmetry …
- Definition: A modular form $f(\tau), \tau \in \mathbb{H}$ with weight k under $\Gamma \subseteq SL(2,\mathbb{Z})$ transforms as

$$f(\gamma.\tau) := f\left(\frac{a\tau+b}{c\tau+d}\right) = (c\tau+d)^k f(\tau)$$

with $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$.

Generators of $SL(2,\mathbb{Z})$: $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$




UNCLE COOTER'S
GENUINE
MOONSHINE
XXX
Hooch, Fire-water, Booze, Snake Oil, Adult Beverage, Medicine, Elixir, Sauce, Home Brew, Hair-of-the-dog
NOT FOR BABIES

The Monster Group & The J-Function

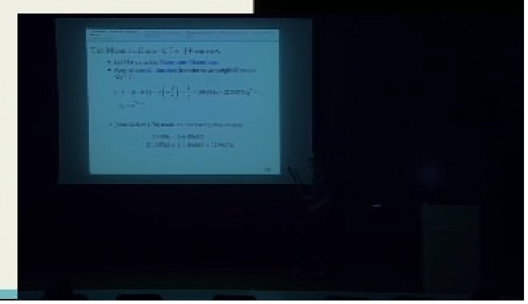
- Let's be concrete: Monstrous Moonshine.
- Story of a mod. function (transforms as weight 0) under $SL(2,\mathbb{Z})$:

$$j(\tau) = j(\tau+1) = j\left(-\frac{1}{\tau}\right) = \frac{1}{q} + 196884q + 21493760q^2 + \ldots$$

$$(q = e^{2\pi i \tau})$$

- John McKay ('78) made an interesting observation:

$$196884 = 1 + 196883$$
$$21493760 = 1 + 196883 + 21296876$$
$$\vdots$$



The Monster Group & The J-Function

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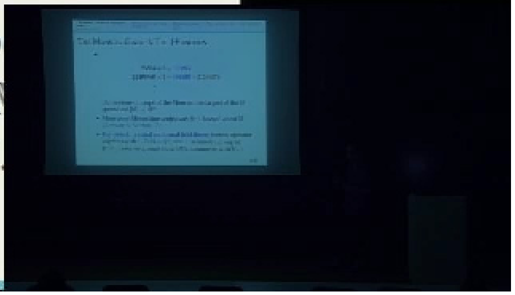
$$196884 = 1 + 196883$$
$$21493760 = 1 + 196883 + 21296876$$
$$\vdots$$

  Dimensions of irreps of the Monster, the largest of the 26 sporadics! $|\mathbb{M}| \sim 10^{54}$
- Monstrous Moonshine conjecture: $j(\tau)$ 'knows' about $\mathbb{M}$ (Conway & Norton, '79).
- Key object: a chiral conformal field theory (vertex op algebra) with 1. $Z(\tau) = j(\tau)$ and 2. symmetry group M (that preserves ground state, OPE, commutes with H.

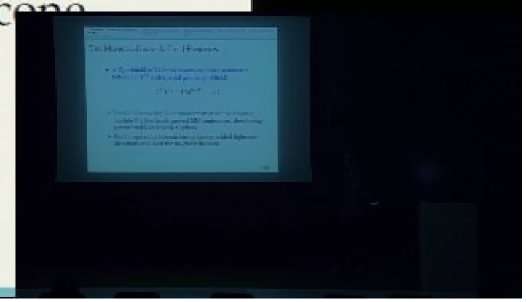


The Monster Group & The J-Function

- A $\mathbb{Z}_2$ orbifold of 24 chiral bosons on a very symmetric lattice (c.f. T^{24} with special geometry, B-field)

$$Z^{V^\natural}(\tau) = Tr(q^{L_0 - 1}) = j(\tau)$$

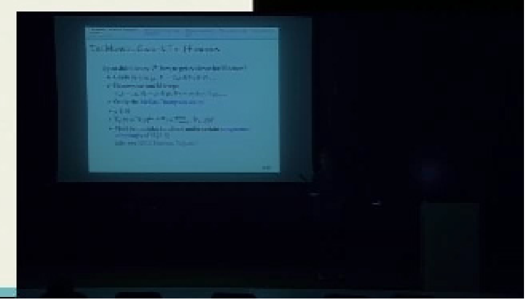
- Frenkel/Lepowsky/Meurman constructed the Monster module $V^\natural$; Borcherds proved MM conjectures, developing generalized Kac-Moody algebras.
- Proof inspired by bosonic string theory: added lightcone directions and used the no-ghost theorem.



THE MONSTER GROUP & THE J-FUNCTION

If you didn't know $V^\natural$, how to get evidence for $\mathbb{M}$ action?

- Grade by energy: $V = V_{-1} \oplus V_1 \oplus V_2 \ldots$
- Decompose into $\mathbb{M}$ irreps:
  $V_{-1} = \rho_0, V_1 = \rho_1 \oplus \rho_0, V_2 = \rho_2 \oplus \rho_1 \oplus \rho_0, \ldots$
- Study the McKay-Thompson series:
- $g \in \mathbb{M}$
- $T_g(\tau) = \mathrm{Tr}(g q^{L_0 - c/24}) = \sum_{i=-1}^{\infty} Tr_{V_i}(g) q^i$.
- Must be modular functions under certain congruence subgroups of $SL(2, \mathbb{Z})$
  (also see NMP, Persson, Volpato)

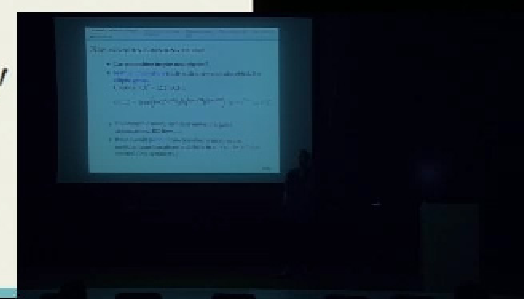


New moonshines in string theory

- Can moonshine inspire new physics?
- Mathieu moonshine starts with a new modular object, the elliptic genus.
  Consider a $\mathcal{N} = (2,2)$ SCFT:

$$\phi(\tau, z) = \mathrm{Tr}_{RR}\left((-1)^{F_L+F_R} y^{J_0} q^{L_0 - c/24} \bar{q}^{\bar{L}_0 - c/24}\right) \quad (y = e^{2\pi i z}), z \in \mathbb{C}$$

- Holomorphic index, invariant under marginal deformations, RG-flow, ...
- It is a (weak) Jacobi form– transforms nicely under modular transformations and shifts in $z \to z + l\tau + l'$ spectral flow symmetry).



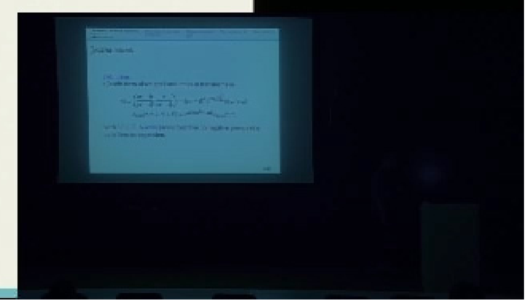
Jacobi forms

Definition:
a Jacobi form of weight k and index m transforms as

$$\phi_{k,m}\left(\frac{a\tau+b}{c\tau+d}, \frac{z}{c\tau+d}\right) = (c\tau+d)^k e^{2\pi i \frac{mcz^2}{c\tau+d}} \phi_{k,m}(\tau, z)$$

$$\phi_{k,m}(\tau, z + l\tau + l') = e^{2\pi i m(l^2\tau + 2lz)} \phi_{k,m}(\tau, z)$$

with $l, l' \in \mathbb{Z}$. A weak Jacobi form has no negative powers of q in its Fourier expansion.



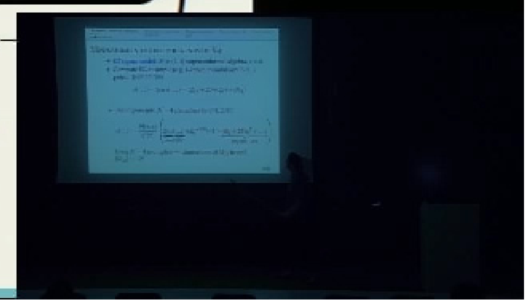
Moonshine in the elliptic genus of K3

- K3 sigma model: $\mathcal{N} = (4,4)$ superconformal algebra, $c = 6$.
- Compute EG at simple (e.g. Gepner, toroidal orbifold...) point: (E-O-T-Y '89)

$$\phi(\tau, z) = 2\phi_{0,1}(\tau, z) = (2/y + 20 + 2y) + O(q)$$

- Decompose into $\mathcal{N} = 4$ characters (E-O-T, 2010)

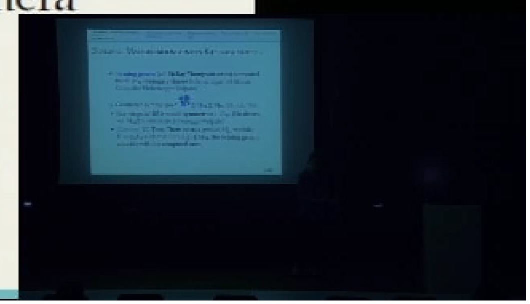
$$\phi(\tau, z) = \frac{\theta_1^2(\tau, z)}{\eta^3(\tau)} \left(\underbrace{24\mu(\tau, z)}_{short, BPS} + 2q^{-1/8}(-1 + \underbrace{45q + 231q^2 + \ldots}_{long, non-}) \right)$$



Long $\mathcal{N} = 4$ multiplets $\leftrightarrow$ dimensions of M_{24} irreps!
$|M_{24}| \sim 10^8$

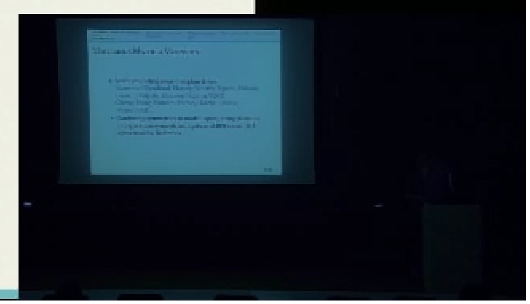
Status of Mathieu moonshine in K3 sigma models

- Twining genera (c.f. McKay-Thompson series) computed for all M_{24} conjugacy classes (Cheng, Eguchi-Hikami, Gaberdiel-Hohenegger-Volpato)
- Geometric symmetries?  $\subset M_{23} \subset M_{24}$ (Mukai, '80s).
- Non-singular K3 σ-model symmetries $\subset Co_0$. (No theory w/ M_{24}!) (Gaberdiel-Hohenegger-Volpato)
- (Gannon '12) Thm: There exists a graded M_{24} module $V = \oplus_n V_n$ such that for all $[g] \in M_{24}$, the twining genera coincide with the computed ones.



Mysterious Mathieu Moonshine

- Many interesting ideas to explain it: see Taormina/Wendland, Harvey/Murthy, Eguchi/Hikami, Persson/Volpato, Harrison/Kachru/NMP, Cheng/Dong/Duncan/Harvey/Kachru/Wrase, Wrase/NMP...
- Combining symmetries in moduli space, string dualities and spacetime symmetries, algebras of BPS states, $(0,4)$ sigma models, fivebranes...



Mock Modular Forms in Moonshine

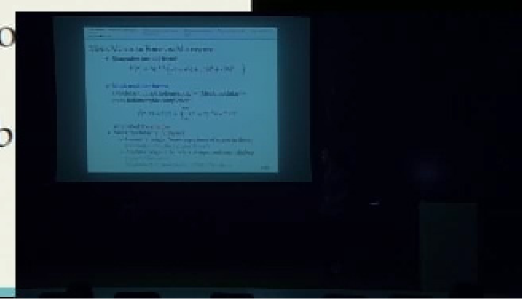
- Remember our old friend:

$$H(\tau) = 2q^{-1/8}\left(-1 + 45q + 231q^2 + 770q^3 \dots\right)$$

- Mock modular forms:
  (Modular but not holomorphic) = (Mock modular) + (non-holomorphic completion)

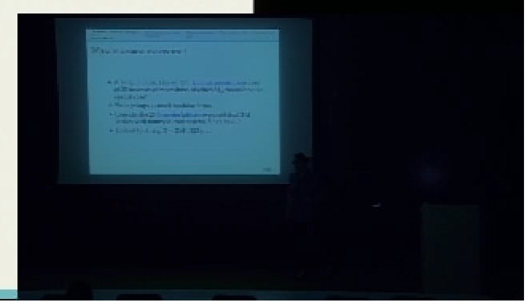
$$\hat{f}(\tau, \bar{\tau}) = f(\tau) + \int_{-\bar{\tau}}^{\infty} (\tau' + \tau)^{-k} \bar{s}(-\bar{\tau}') d\tau'$$

  $s(\tau)$ called the shadow
- Mock modularity in physics:
  - Geometric origin: Non-compactness of spaces in theo
    (Dabholkar/Murthy/Zagier, Troost)
  - Algebraic origin: Characters of superconformal algeb
    (Eguchi/Taormina, Benjamin/Harrison/Kachru/NMP/Whalen)

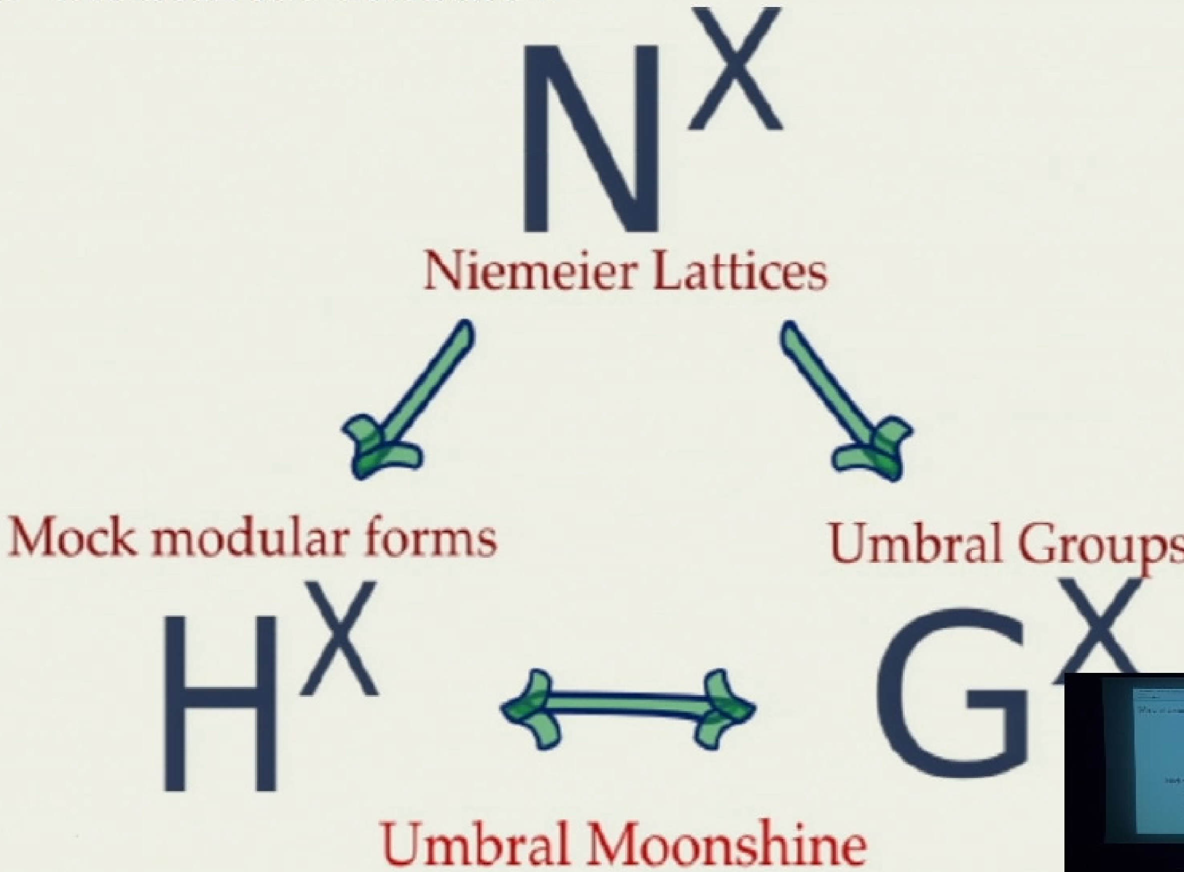


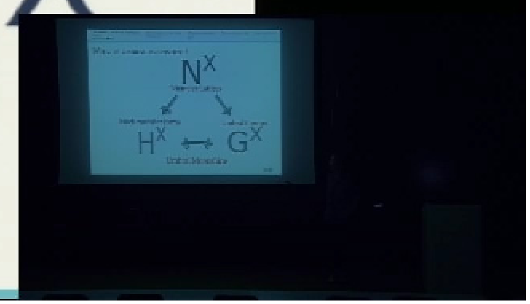
What is umbral moonshine?

- (Cheng, Duncan, Harvey '13): Umbral moonshine is a set of 23 instances of moonshine, of which M_{24} moonshine is a special case!
- Finite groups $\leftrightarrow$ mock modular forms
- Consider the 23 Niemeier lattices: even, self-dual 24d lattices with nontrivial root systems $X : \langle x, x \rangle = 2$.
- Labeled by X: e.g. $X = 24A_1, 12A_2, \ldots$



What is umbral moonshine?



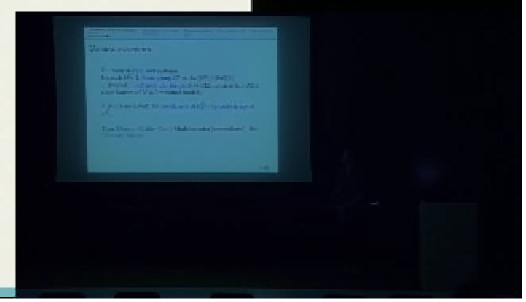


Umbral moonshine

X: union of ADE root systems
For each N^X: 1. finite group $G^X = Aut(N^X)/Weyl(X)$
2. (twined) mock modular forms: $X \leftrightarrow$ CIZ matrices (c.f. ADE classification of $\mathcal{N} = 2$ minimal models).

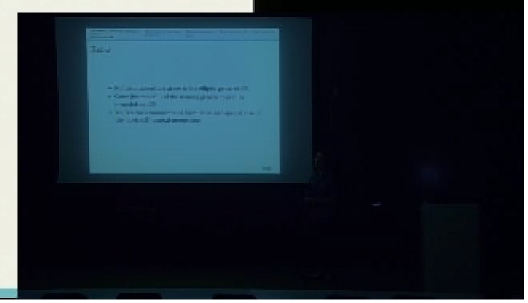
A priori unrelated! Yet coefficients of $H_g^X(\tau)$ encode irreps of G^X.

Thm (Duncan-Griffin-Ono): Modules exist (somewhere)!– See Duncan-Harvey



Recap

- Mathieu moonshine arises in the elliptic genus of $K3$.
- Some (but not all!) of the twining genera appear in σ-models on $K3$.
- Yet, Mathieu moonshine is formulated as a special case of the (abstract!) umbral moonshine.



Overview

"Lightning" review of moonshine

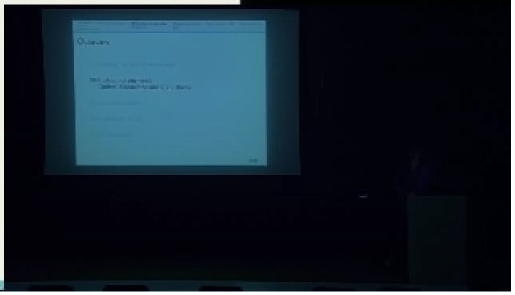
Motivation for our work

- Umbral Moonshine and K3 Surfaces

Models and results

Now playing: in 3D

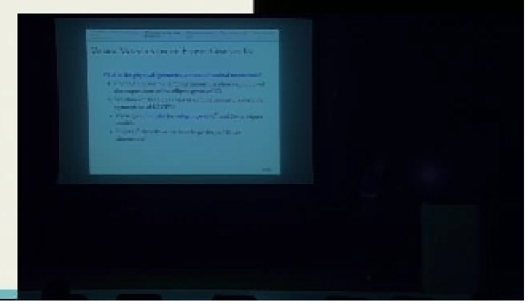
Open questions



Umbral Moonshine in the Elliptic Genus of K3

What is the physical/geometric context of umbral moonshine?

- Cheng-Harrison '14: Umbral moonshine shows up in novel decompositions of the elliptic genus of K3.
- We showed that all 23 cases of umbral moonshine describe symmetries of K3 CFTs!
- These give modules for subgroups of G^X and Co_0 in sigma models.
- To get G^X directly $\rightarrow$ we have to go deeper! (three dimensions)

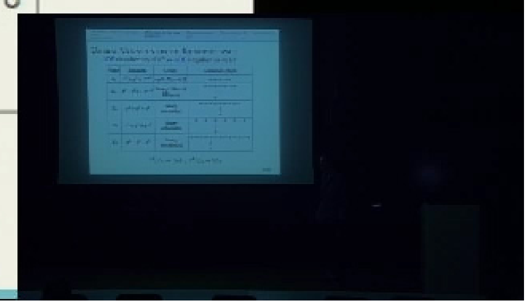


Umbral Moonshine in the K3 moduli space

ADE classification of $N^X \leftrightarrow$ ADE singularities on K3

Name	Equation	Group	Resolution graph
A_n	$x^2+y^2+z^{n+1}$	cyclic $\mathbb{Z}/(n+1)$	o — o ··· o
D_n	$x^2+y^2z+z^{n-1}$	binary dihedral $\mathrm{BD}_{4(n-2)}$	o — o — o ··· o, with one node attached below the second node
E_6	$x^2+y^3+z^4$	binary tetrahedral	o — o — o — o — o, with one node attached below the third node
E_7	$x^2+y^3+yz^3$	binary octahedral	o — o — o — o — o — o, with one node attached below the third node
E_8	$x^2+y^3+z^5$	binary icosahedral	o — o — o — o — o — o — o, with one node attached below the third node

$$T^4/\mathbb{Z}_2 \to 16A_1,\ T^4/\mathbb{Z}_3 \to 9A_2$$



Umbral Moonshine in the Elliptic Genus of K3

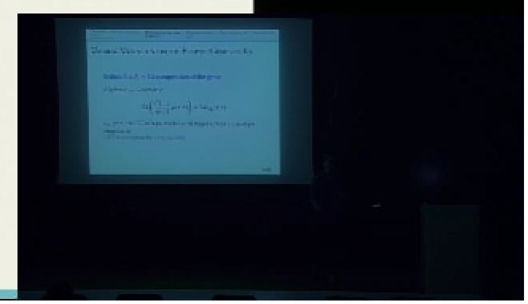
Rethink the $\mathcal{N} = 4$ decomposition of the genus

Algebraic $\rightarrow$ Geometric

$$24\left(\frac{\theta_1^2(\tau,z)}{\eta^3(\tau)}\mu(\tau,z)\right) = 24\phi_{A_1}(\tau,z)$$

$\phi_{A_1}(\tau,z)$ the EG of a (σ-model with target space) an A_1-type singularity.

CFT description by Ooguri-Vafa



Umbral Moonshine in the Elliptic Genus of K3

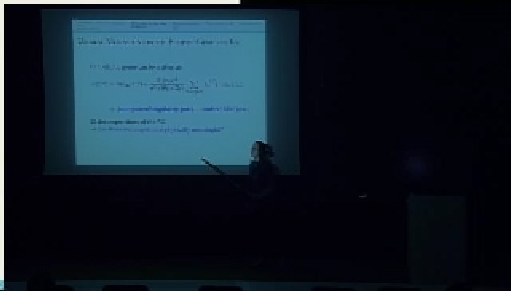
Total elliptic genus can be written as:

$$\phi(\tau,z) = 24\phi_{A_1}(\tau,z) - \frac{i\theta_1(\tau,z)^2}{\eta^3(\tau)\theta_1(\tau,2z)} \sum_{r\in\mathbb{Z}/(4\mathbb{Z})} H_r^{A_1^{24}}(\tau)\theta_{2,r}(\tau,z)$$

$$= (\textbf{root system/singularity part}) + (\textbf{umbral MM part})$$

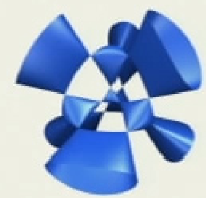
23 decompositions of the EG

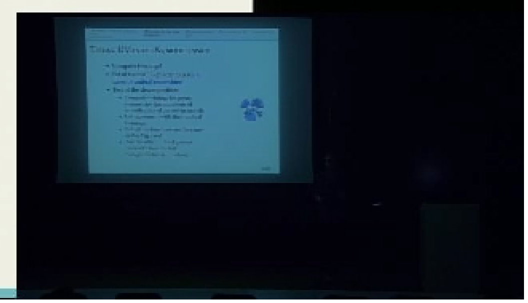
→ Are these decompositions physically meaningful?



Testing UM in the K3 moduli space

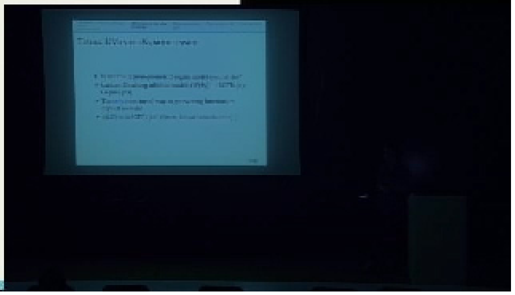
- Compute twinings!
- Set of twined EG predictions for all cases of umbral moonshine
- Test of the decomposition:
  - Compute twinings for geom. symmetries (permutations of smooth rational curves) in models.
  - Get agreement with their umbral twinings.
  - But all the functions are the same as the M_{24} case!
  - Are the other umbral groups relevant? Need to test non-geometric symmetries.





Testing UM in the K3 moduli space

- What about (non-geometric) sigma model symmetries?
- Landau-Ginzburg orbifold models ($W(\Phi_i)$) $\rightarrow$ SCFTs (e.g. Gepner pts)
- The only (non-torus) way to get twining functions in explicit models!
- $\phi(LG) = \phi(SCFT)$ (c.f. Witten, Kawai-Yamada-Yang...)



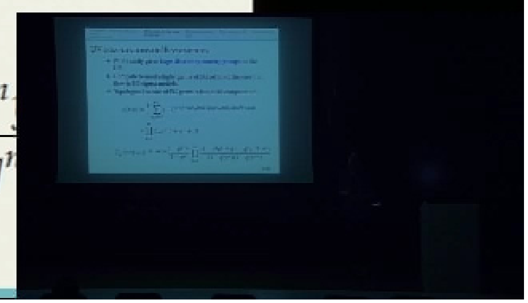
UV computations of IR symmetries

- $W(\Phi)$ easily gives large discrete symmetry groups in the UV.
- Compute twined elliptic genus of LG orbifold theories that flow to K3 sigma models.
- Topological nature of EG permits free-field computation:

$$\phi(\tau,z) = \frac{1}{h}\sum_{\alpha,\beta=0}^{h-1} (-1)^{\alpha+\beta+\alpha\beta} e^{2\pi i(\hat{c}/2)\alpha\beta} e^{2\pi i(\hat{c}/2)(\alpha^2\tau+2\alpha z)}$$

$$\times \prod_{i=1}^{N} Z_{\omega_i}(\tau, z+\alpha\tau+\beta)$$

$$Z_\omega(\tau,z) = y^{-(1-2\omega)/2}\frac{1-y^{1-\omega}}{1-y^\omega}\prod_{n=1}^{\infty}\frac{(1-q^n y^{1-\omega})(1-q^n}{(1-q^n y^\omega)(1-q^n}$$

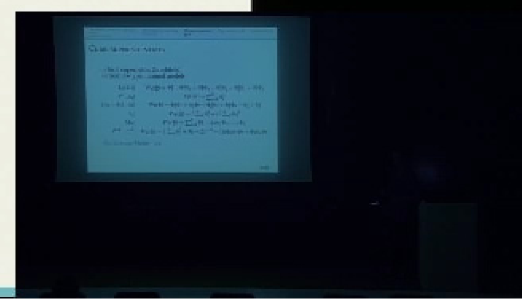


Cubic superpotentials

6 chiral superfields, $\mathbb{Z}_3$ orbifold
Fermat: A_2-type minimal models

$$
\begin{aligned}
L_2(11)| \quad & \mathcal{W}_1(\underline{\Phi}) = \Phi_0^3 + \Phi_1^2\Phi_5 + \Phi_2^2\Phi_4 + \Phi_3^2\Phi_2 + \Phi_4^2\Phi_1 + \Phi_5^2\Phi_3 \\
3^4 : A_6| \quad & \mathcal{W}_2(\underline{\Phi}) = \textstyle\sum_{i=0}^{5} \Phi_i^3 \\
(\mathbb{Z}_3 \times A_5) : \mathbb{Z}_2| \quad & \mathcal{W}_3(\underline{\Phi}) = \Phi_0^2\Phi_1 + \Phi_1^2\Phi_2 + \Phi_2^2\Phi_3 + \Phi_3^2\Phi_0 + \Phi_4^3 + \Phi_5^3 \\
A_7| \quad & \mathcal{W}_4(\underline{\Phi}) = \textstyle\sum_{i=0}^{5} \Phi_i^3 - (\sum_{i=0}^{5} \Phi_i)^3 \\
M_{10}| \quad & \mathcal{W}_5(\underline{\Phi}) = \textstyle\sum_{i=0}^{5} \Phi_i^3 + \lambda.\sigma_3(\Phi_0, \ldots, \Phi_5) \\
3^{1+4} : 2.2^2| \quad & \mathcal{W}_6(\underline{\Phi}) = \textstyle\sum_{i=0}^{5} \Phi_i^3 + 3(i - 2e^{\pi i/6} - 1)(\Phi_0\Phi_1\Phi_2 + \Phi_3\Phi_4\Phi_5)
\end{aligned}
$$

(See Mason/Hohn '14)



Twinings, exceptional and umbral

$$Z_{orb,g} = \frac{1}{3}\sum_{a,b=0}^{2} q^{a^2} y^{2a} \prod_{i=1}^{6} \frac{\theta_1\left(q, \lambda_i y^{-\frac{2}{3}} q^{-\frac{2a}{3}} e^{-\frac{4\pi i b}{3}}\right)}{\theta_1\left(q, \lambda_i y^{\frac{1}{3}} q^{\frac{a}{3}} e^{\frac{2\pi i b}{3}}\right)}$$

$$(\lambda_n = e^{2\pi i/n})$$

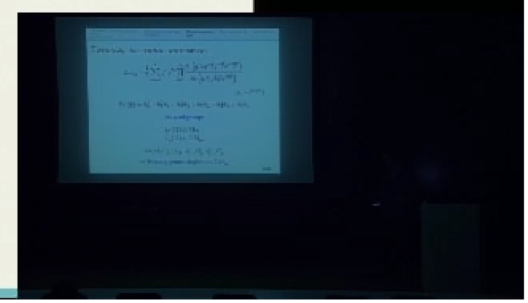
$$\mathcal{W}_1(\underline{\Phi}) = \Phi_0^3 + \Phi_1^2\Phi_5 + \Phi_2^2\Phi_4 + \Phi_3^2\Phi_2 + \Phi_4^2\Phi_1 + \Phi_5^2\Phi_3$$

As a subgroup:

$$L_2(11) \subset M_{24}$$
$$L_2(11) \subset 2.M_{12}$$

$$ord(11) : (1, \lambda_{11}, \lambda_{11}^3, \lambda_{11}^4, \lambda_{11}^5, \lambda_{11}^9)$$

$\rightarrow$ Twining genus singles out $2.M_{12}$!



Twinings, exceptional and umbral

$$Z_{orb,g} = \frac{1}{3} \sum_{a,b=0}^{2} q^{a^2} y^{2a} \prod_{i=1}^{6} \frac{\theta_1\left(q, \lambda_i y^{-\frac{2}{3}} q^{-\frac{2a}{3}} e^{-\frac{4\pi i b}{3}}\right)}{\theta_1\left(q, \lambda_i y^{\frac{1}{3}} q^{\frac{a}{3}} e^{\frac{2\pi i b}{3}}\right)}$$

$$L_2(11) \subset 2.M_{12} \to ord(11) : (1, \lambda_{11}, \lambda_{11}^3, \lambda_{11}^4, \lambda_{11}^5, \lambda_{11}^9)$$

$$3^4 : A_6 \subset Co_0$$

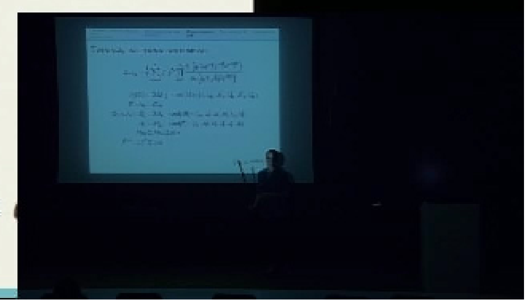
$$(\mathbb{Z}_3 \times A_5) : \mathbb{Z}_2 \subset 3.S_6 \to ord(15) : (\lambda_5, \lambda_5^3, \lambda_5^4, \lambda_5^2, \lambda_3, \lambda_3^2)$$

$$A_7 \subset M_{24} \to ord(7) : (\lambda_7, \lambda_7^2, \lambda_7^3, \lambda_7^4, \lambda_7^5, \lambda_7^6)$$

$$M_{10} \subset M_{24}, 2.M_{12}$$

$$3^{1+4} : 2.2^2 \subset Co_0$$

$(\lambda_n =$



Quartic superpotentials

4 chiral superfields, $\mathbb{Z}_4$ orbifold
Fermat: A_3-type minimal models

$$(\mathbb{Z}_2 \times \mathbb{Z}_4^2) \rtimes S_4 | \quad \mathcal{W}_1(\underline{\Phi}) = \Phi_1^4 + \Phi_2^4 + \Phi_3^4 + \Phi_4^4$$

$$\mathbb{Z}_2 \times L_2(7) | \quad \mathcal{W}_2(\underline{\Phi}) = \Phi_1^3\Phi_2 + \Phi_2^3\Phi_3 + \Phi_3^3\Phi_1 + \Phi_4^4$$

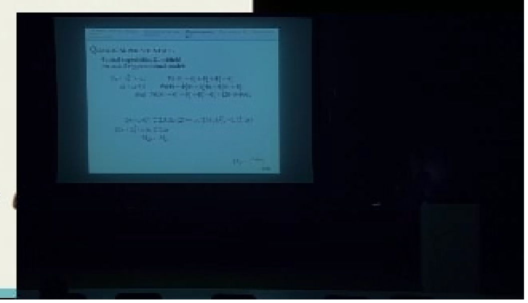
$$M_{20} | \quad \mathcal{W}_3(\underline{\Phi}) = \Phi_1^4 + \Phi_2^4 + \Phi_3^4 + \Phi_4^4 + 12\Phi_1\Phi_2\Phi_3\Phi_4$$

$$\mathbb{Z}_2 \times L_2(7) \subset 2.AGL_3(2) \to ord(14) : (\lambda_7^2, -1, \lambda_7^4, \lambda_7)$$

$$(\mathbb{Z}_2 \times \mathbb{Z}_4^2) \rtimes S_4 \subset Co_0$$

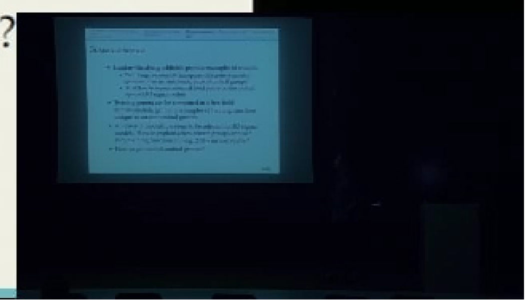
$$M_{20} \subset M_{24}$$

$(\lambda_n =$



Summary so far

- Landau-Ginzburg orbifolds provide examples of models
  - With large, reasonably transparent discrete symmetry groups (often maximal subgroups of umbral groups)
  - That flow to superconformal fixed points on the moduli space of K3 sigma models
- Twining genera can be computed in a free-field approximation; get many examples of twining functions unique to certain umbral groups.
- All cases of moonshine seem to be relevant for K3 sigma models. How to explain when certain groups appear? Why twining functions for, e.g. $2.M_{12}$ instead of M_{24}?
- How to get the full umbral groups?



Where are the G^X?

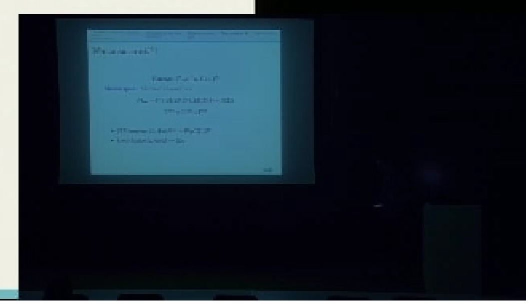
$$\text{Heterotic}, T^7 \leftrightarrow \text{IIA}, K3 \times T^3$$

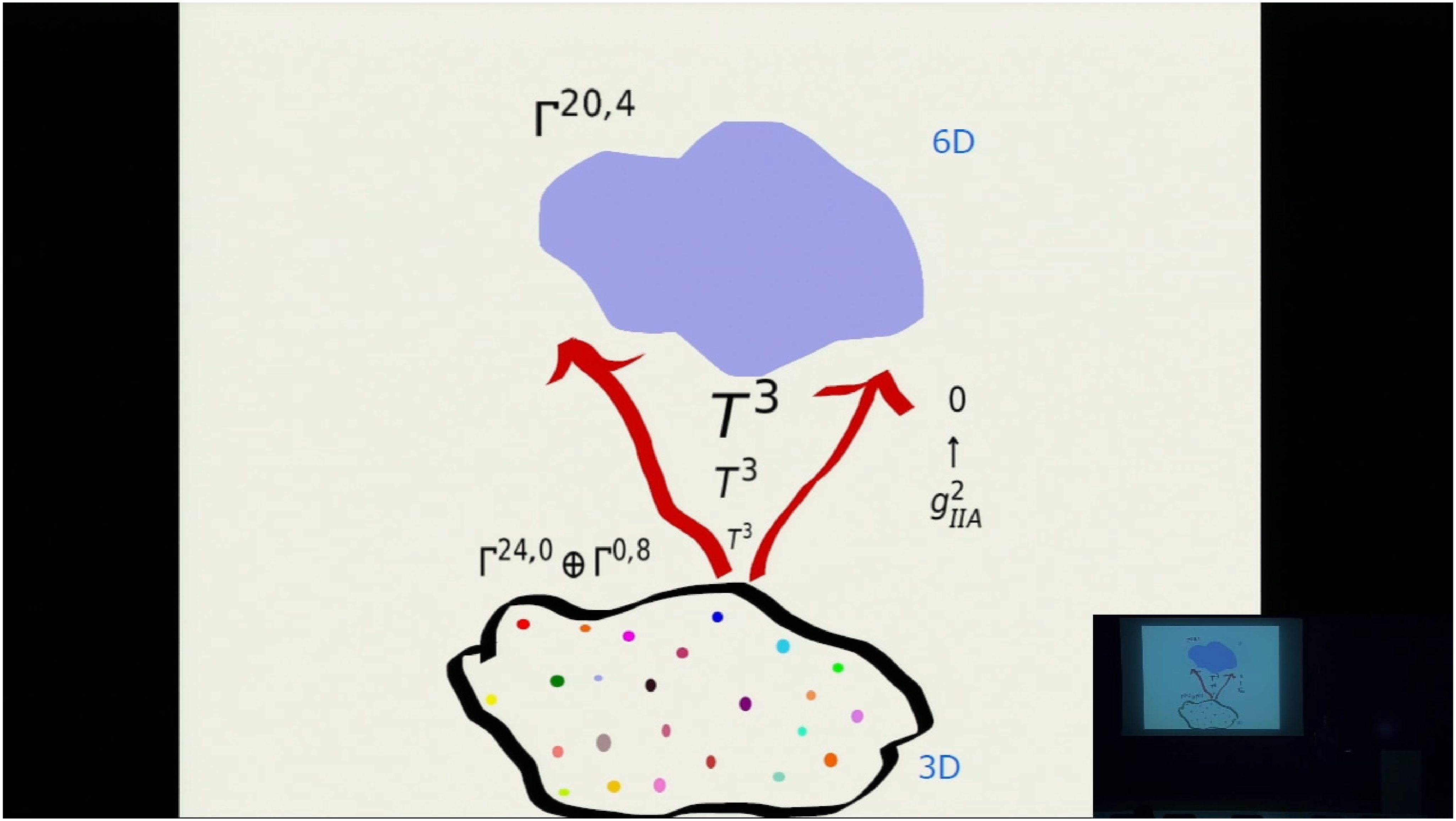
Moduli space: Marcus/Schwarz, Sen

$$\mathcal{M}_{vac} = O(8,24;\mathbb{Z})\backslash O(8,24)/O(8) \times O(24)$$

$$\Gamma^{24,8} = \Gamma^{24,0} \oplus \Gamma^{0,8}$$

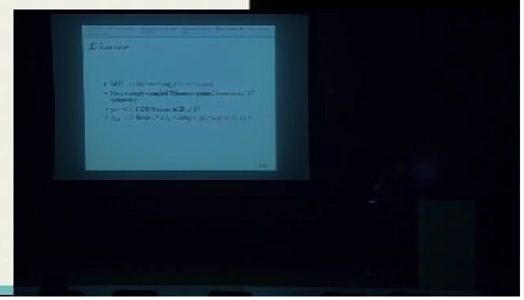
- 23 Niemeiers L^X, $Aut(N^X) = Weyl(X).G^X$
- Leech lattice Λ, $Aut(\Lambda) = Co_0$




$\Gamma^{20,4}$
6D
T^3
T^3
T^3
0
$\uparrow$
g^2_{IIA}
$\Gamma^{24,0} \oplus \Gamma^{0,8}$
3D

3D Summary

- 3d $F^4 \to (\partial\phi)^4$ coupling (Obers/Pioline)
- Near strongly-coupled 'Niemeier points', theories w/ G^X symmetry
- $g_{3H} \to 0$: 1/2-BPS states in **23** of G^X
- $g_{IIA} \to 0$: Break G^X, Co_0 to subgroups predicted by GHV



3D SUMMARY

- 3d $F^4 \to (\partial\phi)^4$ coupling (Obers/Pioline)
- Near strongly-coupled 'Niemeier points', theories w/ G^X symmetry
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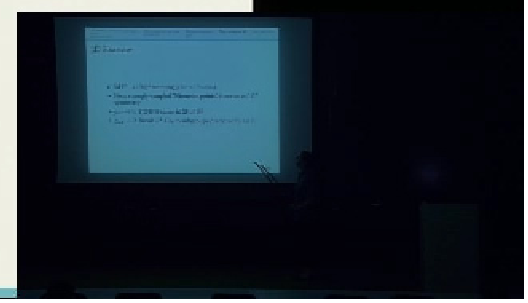
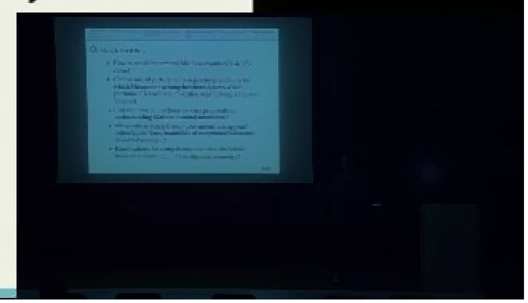


Open questions

- How to get all the umbral MMFs as counts of 1/4-BPS states?
- Can we use 3d picture to make precise predictions for which Niemeiers/twining functions govern which portions of K3 σ-model moduli space? (Cheng, Harrison, Volpato)
- Can we make connections to other proposals for understanding Mathieu/umbral moonshine?
- Where else in string theory does moonshine appear? (other Calabi-Yaus/manifolds of exceptional holonomy, EGs of 6d strings...?)
- Ramifications for string theory dualities, black hole microstate counting, …? For algebraic geometry?



Thank you for your attention!