

Title: A perspective on derived analytic geometry

Date: Apr 22, 2016 02:00 PM

URL: <http://pirsa.org/16040087>

Abstract: I will present a 'categorical' way of doing analytic geometry in which analytic geometry is seen as a precise analogue of algebraic geometry.

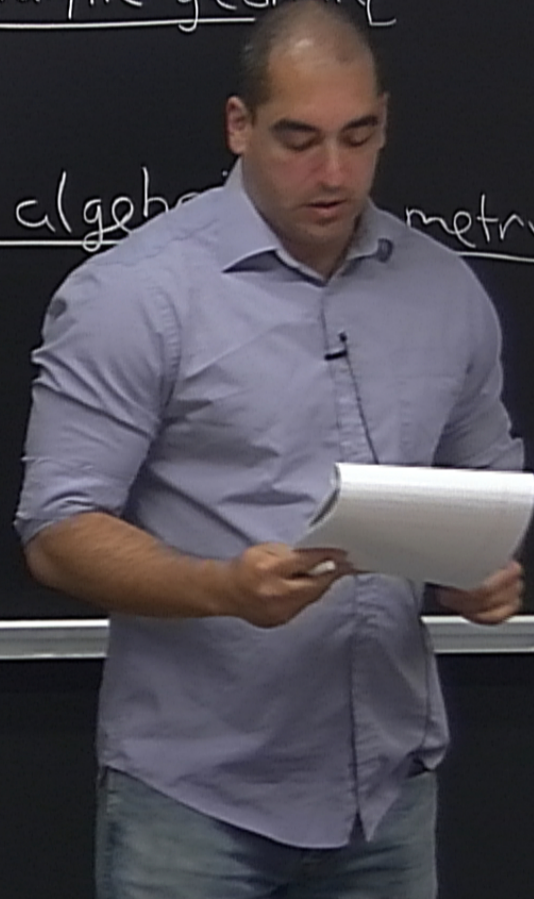
Our approach works for both complex analytic geometry and p-adic analytic geometry in a uniform way. I will focus on the idea of an 'open set' as used in various geometrical theories and how it is characterized categorically. In order to do this, we need to study algebras and their modules in the category of Banach spaces. The categorical characterization that we need uses homological algebra in these 'quasi-abelian' categories which is work of Schneiders and Prosmans. In fact, we work with the larger category of Ind-Banach spaces for reasons I will explain. This gives us a way to establish foundations of derived analytic geometry (my joint project with Kobi Kremnizer). We compare this approach with standard standard notions such as the theory of affinoid algebras, Grosse-Klonne's theory of dagger algebras (over-convergent functions), the theory of Stein domains and others. I will formulate derived analytic geometry following the relative algebraic geometry approach of Toën, Vaquié and Vezzosi.

This talk involves various joint work with Federico Bambozzi and Kobi Kremnizer.

A perspective on derived analytic geometry

joint w/ Kremnizer

idea of using relative algebraic geometry



A perspective on derived analytic geometry

joint w/ Kremnizer

idea of using relative algebraic geometry

Under $\text{Spec } \mathbb{Z}$ (Toën-Vaquité)
HAG (Toën-Vezzosi)

HAG

develops derived geometry
w/ input a monoidal model category

HAG context

ex - Ab

- $\text{ch}_k \leq 0$

$\text{char } k = 0$

$e \text{ SAb}$

CAUTION

DO NOT STAND ON LIMBS OR BRACKETS
PLEASE CONTACT US FOR ASSISTANCE AT THE BOTTOM
IF NECESSARY BY LIFT
DO NOT BRACKET BRACKETS
ATTEND PLEASE

ex - Ab
- $ch_k \leq 0$ $char k = 0$
= sAb

suggestion $\mathcal{M} = \text{siInd}(Ban_R)$ R a Banach ring
Thm (B, Krennizer) \mathcal{M} is a HAG context

CAUTION

NO BANG OR LINGER THE BOTTLE NECK.
PLEASE DRINK BY THE MOUTH OF THE BOTTLE.
IF NE NECESSARY DO APPLY
WASH BOTTLE NECESSARY ONCE
AVOID REUSING BOTTLE

Def A Banach ring is a ring R

equipped with $R \xrightarrow{\|\cdot\|} \mathbb{R}_{\geq 0}$

$$|a+b| \leq |a| + |b|$$

$$|ab| \leq |a||b|$$

$$|a| = 0 \Leftrightarrow a = 0$$

$R = \mathbb{Z}, \mathbb{C}, \mathbb{R}, \mathbb{Q}_p, \mathbb{C}((t)), \text{Novikov}$

$d(a,b) = |a-b|$ is complete

CAUTION

NO BANG OR LINGER WITH BLOWING WINDS.
PLEASE EXIT AT THE FRONT OF THE ROOM.
IF AN EMERGENCY DO APPLY
YOUR SEATBELT IMMEDIATELY.
AVOID READING WHILE

Ban_R has finite limits / colimits

objects are $(V, \|\cdot\|)$

V an R -module

$$\|v+w\| \leq \|v\| + \|w\|$$

$$\|av\| \leq C |a| \|v\|$$

Ban_R has finite limits / colimits, no infinite prods or coprods

objects are $(V, \|\cdot\|)$ morphisms are bounded
 V an R -module

$$\|v+w\| \leq \|v\| + \|w\|$$

$$\|av\| \leq |a| \|v\|$$

$$d(v,w) = \|v-w\| \text{ complete}$$

$$\|v\| = 0 \iff v = 0$$

Ban_R

has finite limits / colimits, no infinite prods or coprods

objects are $(V, \|\cdot\|)$
 V an R -module

morphisms are bounded
 R -linear maps

$$\|v+w\| \leq \|v\| + \|w\|$$

$$\|av\| \leq C |a| \|v\|$$

$$d(v,w) = \|v-w\| \text{ complete}$$

$$\|v\| = 0 \iff v = 0$$

$$d(v, w) = \|v - w\| \text{ complete}$$

$$\|v\| = 0 \iff v = 0$$

$\text{Ban}_R^{\leq 1}$ has all limits/colimits

$$\coprod_i v_i \subseteq \prod_i v_i, \quad \|(v_i)_{i \in I}\| = \sum_{i \in I} \|v_i\|$$

$$\sum_{i \in I} \|v_i\| < \infty$$

$$d(v, w) = \|v - w\| \text{ complete}$$

$$\|v\| = 0 \iff v = 0$$

Ban_R^{∞} has all limits/colimits (same objects, morphs bounded in norm by 1)

$$\prod_i v_i \cong \sum_i v_i, \quad \|(v_i)_{i \in I}\| = \sum_{i \in I} \|v_i\|$$

$$\sum_{i \in I} \|v_i\| < \infty$$

$$d(v, w) = \|v - w\| \text{ complete}$$

$$\|v\| = 0 \iff v = 0$$

$$\sum_{i \in I} \|v_i\| < \infty$$

Ban_R is closed symmetric monoidal

$$d(v, w) = \|v - w\| \text{ complete}$$

$$\|v\| = 0 \iff v = 0$$

$$\prod_i v_i \subseteq \sum_i v_i, \quad \|(v_i)_{i \in I}\| = \sum_{i \in I} \|v_i\|$$

$$\sum_{i \in I} \|v_i\| < \infty$$

Ban_R is closed symmetric monoidal

$V \otimes_R W \leftarrow$ completion of $V \otimes W$

$$\text{w/r.t. } \|u\| = \inf \left\{ \sum_{i=1}^n \|v_i\| \|w_i\| \mid u = \sum_{i=1}^n v_i \otimes w_i \right\}$$

$$\prod_i V_i \cong \prod_i V_i, \quad \|(v_i)_{i \in I}\| = \sum_{i \in I} \|v_i\|$$

$$\sum_{i \in I} \|v_i\| < \infty$$

Ban_R is not abelian

Ban_R is closed symmetric monoidal

$V \otimes_R W \leftarrow$ completion of $V \otimes_R W$

$$\text{w/r.t. } \|a\| = \inf \left\{ \sum_{i=1}^n \|v_i\| \|w_i\| \mid a = \sum_{i=1}^n v_i \otimes w_i \right\}$$

$$\| \sum_{i \in I} v_i \| = \sum_{i \in I} \| v_i \|, \quad \| (v_i)_{i \in I} \| = \sum_{i \in I} \| v_i \|$$

$$\sum_{i \in I} \| v_i \| < \infty$$

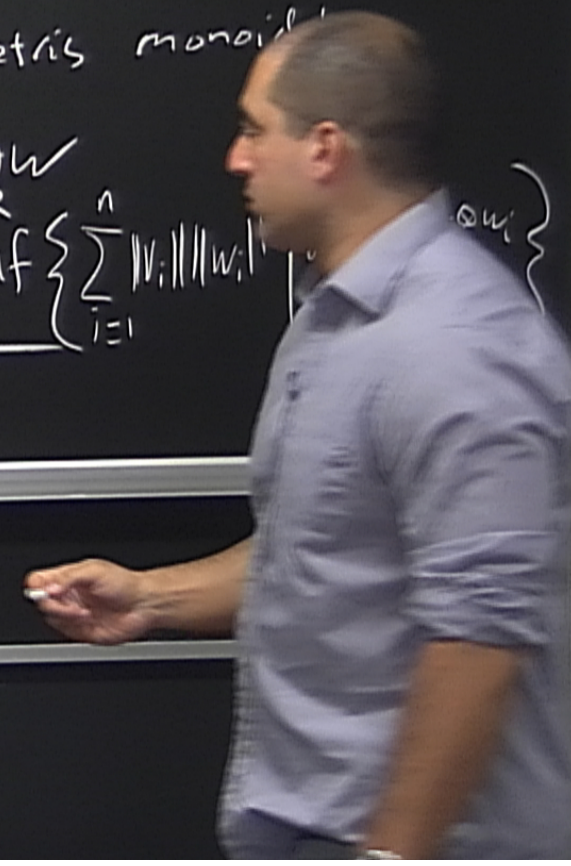
Ban_R is closed symmetric monoidal

$V \otimes_R W \leftarrow$ completion of $V \otimes W$

w/r/t $\| u \| = \inf \left\{ \sum_{i=1}^n \| v_i \| \| w_i \| \right\}$

Ban_R is not abelian
 $f: V \rightarrow W$

$\text{cok}(\ker f \rightarrow V) \cong \ker(W \rightarrow \text{cok} f)$
 may not hold



Ban_R has all limits/colimits (same objects, morphs bounded in norm by 1)

$$\coprod_i V_i \cong \prod_i V_i, \quad \|(v_i)_{i \in I}\| = \sum_{i \in I} \|v_i\|$$

$$\sum_{i \in I} \|v_i\| < \infty$$

Ban_R is closed symmetric monoidal

$V \otimes_R W \leftarrow$ completion of $V \otimes W$

$$\text{w/r/t } \|a\| = \inf \left\{ \sum_{i=1}^n \|v_i\| \|w_i\| \mid a = \sum_{i=1}^n v_i \otimes w_i \right\}$$

Ban_R is not abelian
 $f: V \rightarrow W$

$\text{cok}(ker f \rightarrow V) \cong ker(W \rightarrow \text{cok} f)$
 may not hold, f is strict if it holds

Ban_R is quasi-abelian ~ developed by J. P. Schaefer

$$V \xrightarrow{f} W \quad \text{cdf} = W/V$$

not identified by
 W/V

Ban_R is quasi-abelian ~ developed by J.P. Schneider

$$V \xrightarrow{f} W \quad \text{coker} f = W/V$$

not identified w/

$$W/V$$

def • An object is flat if tensoring by it preserves strict s.e.s.

Ban_R is quasi-abelian ~ developed by J.P. Schneider

$$V \xrightarrow{f} W \quad \text{coker} = W/V$$

not identified w/

$$W/V$$

Def • An object is flat if tensoring by it preserves strict s.e.s.

• P is projective if $\forall E \rightarrow F$ strict epi
 $\text{Hom}(P, E) \rightarrow \text{Hom}(P, F)$

Ban_R is quasi-abelian ~ developed by J.P. Schneider

$$V \xrightarrow{f} W \quad \text{coker} = W/V$$

not identified w/

$$W/V$$

Def. An object is flat if tensoring by it preserves strict s.e.s.

- P is projective if \forall

$$E \rightarrow F \text{ strict epi}$$

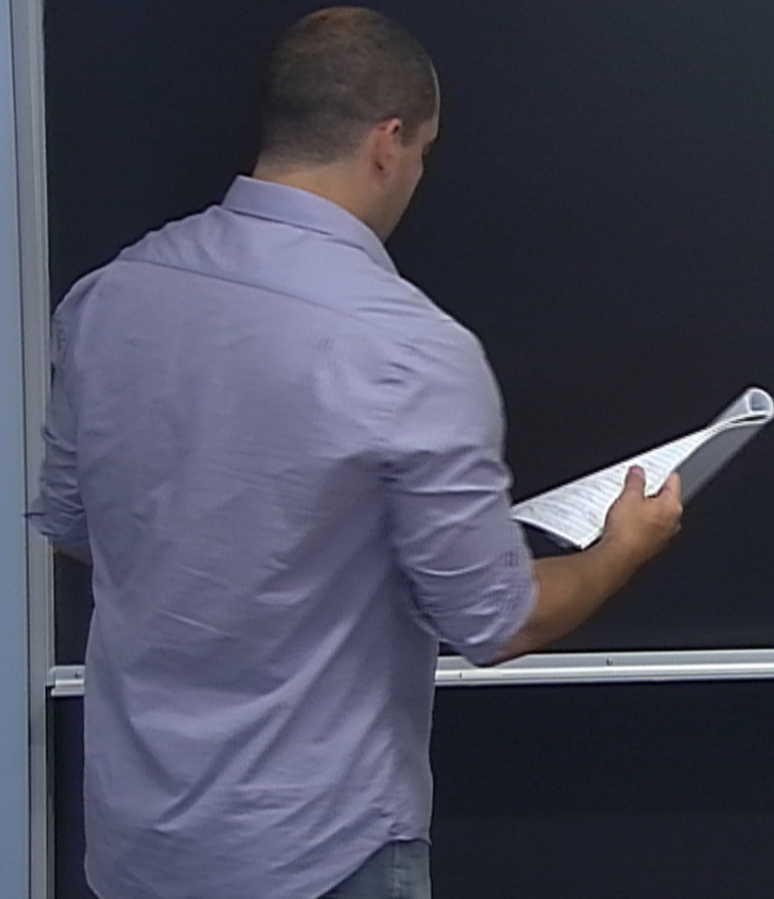
$$\text{Hom}(P, E) \rightarrow \text{Hom}(P, F)$$

$$P_V \rightarrow V \text{ strict epi}$$

P_V flat + proj

$\text{Ind}(\text{Ban}_R)$

again quasi-abelian
closed, sym, monoidal



$\text{Ind}(\text{Ban}_R)$ again quasi-abelian
closed, symmetric monoidal

$$(\text{"colim"}_i V_i) \otimes_R \text{"colim"}_j W_j$$

$$= \text{"colim"}_{i,j} (V_i \otimes_R W_j)$$

$$\lim_i (\text{"colim"}_j V_i, \text{"colim"}_j W_j) = \lim_i \text{"colim"}_j \lim_i (V_i, W_j)$$

(150222V-1901)

Ind (Ban_R) again quasi-abelian
closed, sym, monoidal

$$\begin{aligned} & \left(\text{"colim"} V_i \right) \hat{\otimes}_R \text{"colim"} W_j \\ &= \text{"colim"} (V_i \hat{\otimes}_R W_j) \end{aligned}$$

$$\underline{\text{Hom}} \left(\text{"colim"} V_i, \text{"colim"} W_j \right) = \varinjlim_j \underline{\text{Hom}}(V_i, W_j)$$

$$\text{CBorn}_k \subseteq \text{Ind}(\text{Ban}_k)$$

Idea of using relative algebraic geometry

Under Spec \mathbb{Z} (Toën-Vaquié)
HAG (Toën-Vezzosi)

Ind (Ban_R) again quasi-abelian
closed, sym, monoidal

$$(\text{"colim"} V_i) \hat{\otimes}_R \text{"colim"} W_j \\ = \text{"colim"} (V_i \hat{\otimes}_R W_j)$$

$$\underline{\text{Hom}}(\text{"colim"} V_i, \text{"colim"} W_j) = \varinjlim \text{"colim"} \underline{\text{Hom}}(V_i, W_j)$$

$$\text{CBorn}_K \subseteq \text{Ind}(\text{Ban}_K)$$

$$V^\vee = \underline{\text{Hom}}_R(V, R)$$

Under Spec Z (Toën-Vaquié)
HAG (Toën-Vezzosi)

Def $V \in \text{Ind}(\text{Ban}_R)$ is nuclear
if $\forall W \in \text{Ban}_R$
 $\text{Hom}_R(W, V) \cong W^V \otimes_R V$

Def $V \in \text{Ind}(\text{Ban}_R)$ is nuclear
if $\forall W \in \text{Ban}_R$
 $\text{Hom}_R(W, V) \cong W^V \otimes_R V$

$\text{SInd}(\text{Ban}_R)$ has model structure
 $X \rightarrow Y$ is a cofib $\iff \forall P$ projective

Def $V \in \text{Ind}(\text{Ban}_R)$ is nuclear
if $\forall W \in \text{Ban}_R$
$$\underline{\text{Hom}}_R(W, V) \cong W^V \otimes_R V$$

$S\text{Ind}(\text{Ban}_R)$ has model structure

$X_\bullet \rightarrow Y_\bullet$ is a wf/fib $\iff \forall P$ projective

$\text{Hom}(P, X_\bullet) \rightarrow \text{Hom}(P, Y_\bullet)$ is a wf/fib

Def $V \in \text{Ind}(\text{Ban}_R)$ is nuclear
if $\forall W \in \text{Ban}_R$
$$\underline{\text{Hom}}_R(W, V) \cong W^V \otimes_R V$$

$S\text{Ind}(\text{Ban}_R)$ has model structure

$X_\bullet \rightarrow Y_\bullet$ is a $wf/fib \iff \forall P$ projective

$\text{Hom}(P, X_\bullet) \rightarrow \text{Hom}(P, Y_\bullet)$ is a wf/fib

Def $V \in \text{Ind}(\text{Ban}_R)$ is nuclear
 if $\forall W \in \text{Ban}_R$

$$\underline{\text{Hom}}_R(W, V) \cong W^V \otimes_R V$$

$S\text{Ind}(\text{Ban}_R)$ has model structure

$X_\bullet \rightarrow Y_\bullet$ is a $wf/fib \iff \forall P$ projective

$\text{Hom}(P, X_\bullet) \rightarrow \text{Hom}(P, Y_\bullet)$ is a wf/fib
 gives a model cat str

Comm (SInd (Ban_p))

$$(X \otimes Y)_n = X_n \otimes Y_n$$

Question

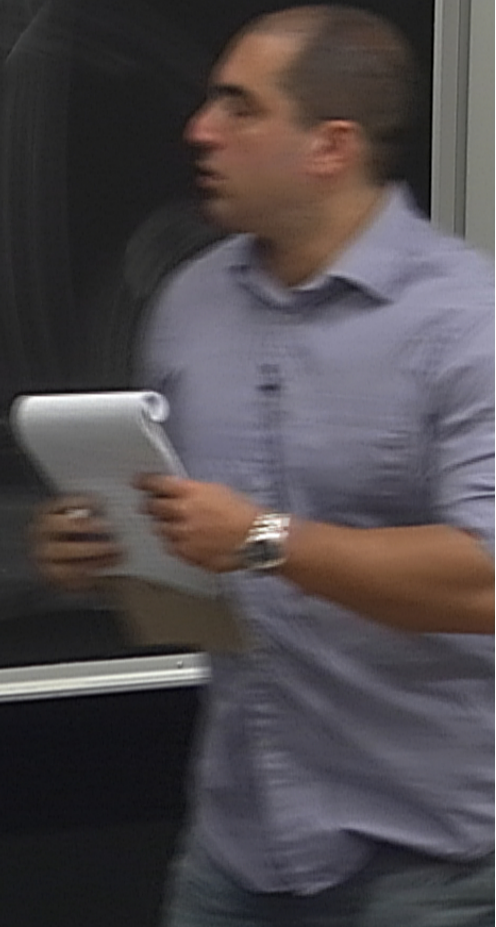
CAUTION

NO SHARP OR POINTY OBJECTS SHOULD BE USED ON THE BOARD OR THE BOARDER.
IF AN EMERGENCY OCCURS, PLEASE CONTACT THE BOARDER.
BOARDER PHONE NUMBER: 1-800-555-1234

$$\text{Comm}(s\text{Ind}(\text{Ban}_R)) \xrightarrow{\sim} s\text{Ind}(\text{Ban}_R)$$

$(X \otimes Y)_n = X_n \otimes Y_n$
 w/ projective moduli str

Affine Schemes = $\text{Comm}(s\text{Ind}(\text{Ban}_R))^{\text{op}}$



CAUTION
 DO NOT STAND ON CHALKBOARD SURFACE.
 PLEASE RESPECT THE PERSONNEL OF THE BOARD.
 SE SI SAPPREGIATE IL SUPERFICIE
 DELLA CHALKBOARD. GRAZIE.
 ATTENTI PERCHÉ NON CADATE.

meet str
Affine Schemes = $\text{Cons}(s\text{Incl}(\mathbb{A}^n_R))^{op}$
 $A \in \text{Comm}(s\text{Incl}(\mathbb{A}^n_R))$
 $\text{Mod}(A)$

CAUTION
DO NOT TOUCH THE BOARD
IF NECESSARY BY THE BOARD
DO NOT TOUCH THE BOARD

meet str
Affine Schemes = $\text{Cons}(s\text{-Ind}(\text{Ban}_R))^{op}$

$A \in \text{Comm}(s\text{-Ind}(\text{Ban}_R))$

$\text{Mod}(A)$

$A \otimes M \rightarrow M$

CAUTION
NO SMOKING OR OPENING OF MATCHES
OR OTHER FLAMMABLE MATERIALS
IN OR NEARBY THIS AREA
SMOKING PROHIBITED

$\text{Comm}(s\text{Ind}(\text{Ban}_R)) \xrightarrow{\quad} s\text{Ind}(\text{Ban}_R)$

$(X \otimes Y)_n = X_n \otimes Y_n$
w/ projection maps str

Affine Schemes = $\text{Comm}(s\text{Ind}(\text{Ban}_R))_{\text{op}}$

$A \in \text{Comm}(s\text{Ind}(\text{Ban}_R))$

$\text{Mod}(A)$

$A \otimes M \rightarrow M$ modl category

$A \in \text{Comm}(s\text{-Incl}(\text{Ban}_R))$

$\text{Mod}(A)$

$A \otimes M \rightarrow M$

model category

Formal Caristi Topology
on $\text{Comm}(s\text{-Incl}(\text{Ban}_R))$

$A \in \text{Comm}(S\text{-Incl}(B \text{ on } R))$

$\text{Mod}(A)$

$A \otimes M \rightarrow M$

model category

Formal Zariski Topology
on $\text{Comm}(S\text{-Incl}(B \text{ on } R))$

$\{A \rightarrow B_i\}_{i \in I}$ is a cover

$A \in \text{Comm}(\leq \text{Incl}(\mathbb{B} \text{ on } \mathbb{R}))$

$\text{Mod}(A)$

$A \otimes M \rightarrow M$

model category

Formal Zariski Topology

on $\text{Comm}(\leq \text{Incl}(\mathbb{B} \text{ on } \mathbb{R}))$

$\{A \rightarrow B_i\}_{i \in I}$ is a cover
if $J \subseteq I$ finite

$A \in \text{Comm}(s\text{-Incl}(\text{Ban}_R))$

$\text{Mod}(A)$

$A \otimes M \rightarrow M$

model category

Formal Zariski Topology

on $\text{Comm}(s\text{-Incl}(\text{Ban}_R))$

$\{A \rightarrow B_i\}_{i \in I}$ is a cover

if $J \subseteq I$ finite st.

1) $H_0(\text{Mod}(B_i)) \rightarrow H_0(\text{Mod}(A))$ is ff. $\forall i \in J$

2) $M \rightarrow N$ in $H_0(\text{Mod}(A))$ is isom \Leftrightarrow

$A \in \text{Comm}(s\text{-Incl}(\text{Ban}_R))$

$\text{Mod}(A)$

$A \otimes M \rightarrow M$

model category

Formal Zariski Topology
on $\text{Comm}(s\text{-Incl}(\text{Ban}_R))$

$\{A \rightarrow B_i\}_{i \in I}$ is a cover

if $J \subseteq I$ finite st.

1) $\text{Ho}(\text{Mod}(B_i)) \rightarrow \text{Ho}(\text{Mod}(A))$ is ff, $\forall i \in J$

2) $M \rightarrow N$ in $\text{Ho}(\text{Mod}(A))$ is isom \Leftrightarrow

$M \otimes_{A_i} B_i \rightarrow N \otimes_{A_i} B_i$ is isom $\forall i \in J$

Formal Zariski Topology

on $\text{Comm}(\underbrace{\text{S-Incl}}_M(\text{BanR}))$

$\{A \rightarrow B_i\}_{i \in I}$ is a cover

if $J \subseteq I$ finite st.

1) $\text{Ho}(\text{Mod}(B_i)) \rightarrow \text{Ho}(\text{Mod } A)$ is ff, $\forall i \in J$

2) $M \rightarrow N$ in $\text{Ho}(\text{Mod } A)$ is isom \Leftrightarrow

$M \otimes_A B_i \rightarrow N \otimes_A B_i$ is isom $\forall i \in J$

CAUTION

TO AVOID THE RISK OF PERSONAL INJURY, PLEASE READ THE INSTRUCTIONS CAREFULLY.

IF YOU ARE UNABLE TO READ THE INSTRUCTIONS, PLEASE CONTACT THE SUPPORT DEPARTMENT.

PLEASE READING CAREFULLY

$$M = SAb$$

$$(i) B_i \underset{A}{\otimes} B_j \cong B_k$$

$$M = sAb$$

$$(1) B_i \otimes_A B_j \xrightarrow{\sim} B_k$$

homotopy epi \leftarrow being a flat epi

\mathcal{P}_V flat + proj

$$M = SA_b$$

$$(i) B_i \otimes_A B_i \cong B_i$$

homotopy epi \iff being a flat epi

$A \rightarrow B_i$ is of f.p.
then flat epi = ho epi

$$A \rightarrow A_p$$

$P \rightarrow P$
 P flat + PG

$$\mathcal{M} = \text{SAb}$$

$$(1) B_i \otimes_A B_i \cong B_i$$

homotopy epi \iff being a flat epi

$A \rightarrow B$ is of f.p.
then flat epi = ho epi

$$A \rightarrow A_p$$

$P \rightarrow B$ flat epi
 $P \rightarrow P \oplus P$ flat epi

$$B \otimes_A \hat{B} \cong B$$

$$\mathcal{M} = \text{SAb}$$

$$(1) B_i \otimes_A B_j \cong B_k$$

homotopy epi \iff being a flat epi

$A \rightarrow B$ is of f.p.
 then flat epi = ho epi
 $\iff \text{Spec } B \subseteq \text{Spec } A$
 Zariski open

$$A \rightarrow A_p$$

$P \rightarrow A_p$ flat + p.g.j

$$B \otimes_A \hat{B} \cong B$$

$$M = SA_b$$

$$B \hat{\otimes}_A B \xrightarrow{\sim} B$$

$$(1) B_i \hat{\otimes}_A B_i \xrightarrow{\sim} B_i$$

homotopy epi \iff being a flat epi

$A \rightarrow B$ is of f.p.
 then flat epi = ho epi
 $\iff \text{Spec } B \subseteq \text{Spec } A$
 Zar open

$$A \rightarrow A_p$$

$P \rightarrow A_p$ flat epi
 $P \rightarrow A_p$ flat + proj

Notation R a Banach ring

$R_r \in \text{Ban}_R$
as

$$\|ra\| = r\|a\|$$

$re\|$

gives a local cot str

$$\|ra\| = r\|a\|$$

$$r \in \mathbb{R}$$

$$S_r := S_{\text{ym}}^{\leq}(V)$$

$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$$\underline{\text{Hom}}_{\mathbb{R}}(W, V) \cong W \otimes_{\mathbb{R}} V$$

$S\text{Inn}(B_{\text{comp}})$ has model structure

$X_* \rightarrow Y_*$ is a wf/fib $\Leftrightarrow \forall P$ projective

$\text{Hom}(P, X_*) \rightarrow \text{Hom}(P, Y_*)$ is a wf/fib

gives a model cat str

$$\|ra\| = r\|a\|$$

$$r \in \mathbb{R}$$

$$S_r := S_{\text{sym}}(V) \subseteq \mathbb{R}[x_1, \dots, x_n]$$

$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$S_r =$ functions converging
on $\sum c_i \mathbb{R}^n$

$\text{Ind}(\text{Comp}_R)$ has model structure

$X_0 \rightarrow Y_0$ is a cofibration $\iff \forall P$

$\text{Hom}(P, X_0) \rightarrow \text{Hom}(P, Y_0)$ is a cofibration
gives a model cat str

$$\|a\| = r \|a\|$$

$$r \in \mathbb{R}$$

$$S_r := S_{\text{sym}}(V) \subseteq \mathbb{R}[x_1, \dots, x_n]$$

$$V = \mathbb{R}_{r_1} \oplus \mathbb{R}_{r_2} \oplus \dots \oplus \mathbb{R}_{r_n}$$

$S_r =$ functions converging
on $\{c \in \mathbb{R}^n \mid |c_i| \leq r_i\}$

$\text{Hom}(V, V)$
 \mathbb{R}
 $S\text{Ind}(\text{Comp}_{\mathbb{R}})$ has model structure

$X_* \rightarrow Y_*$ is a cof/fib $\iff b$

$\text{Hom}(P, X_*) \rightarrow \text{Hom}(P, Y_*)$ is a cof/fib
gives a model cat str

$$R_r \in \text{Ban}_R$$

$$\|r\| = r/\|a\|$$

$$r \in \mathbb{R} \quad \text{in } \text{Ban}^{\mathbb{R}}$$

$$S_r := S(V) \subseteq \mathbb{R}[x_1, \dots, x_n]$$

$$V = \mathbb{R} \oplus \dots \oplus \mathbb{R} \quad \prod_{i=1}^n \mathbb{R}$$

$S_r =$ functions converging on $\{c \in \mathbb{R}^n \mid |c_i| \leq r_i\}$

gives a Δ property
 (x_i) is a w.e./f.b.

$$S_r = \text{Sym}(V) \quad \Delta$$
$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$$\text{on } \left\{ c \in \mathbb{R}^n \mid |c_i| = r_i \right\}$$

$$S_r = \mathbb{R} \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

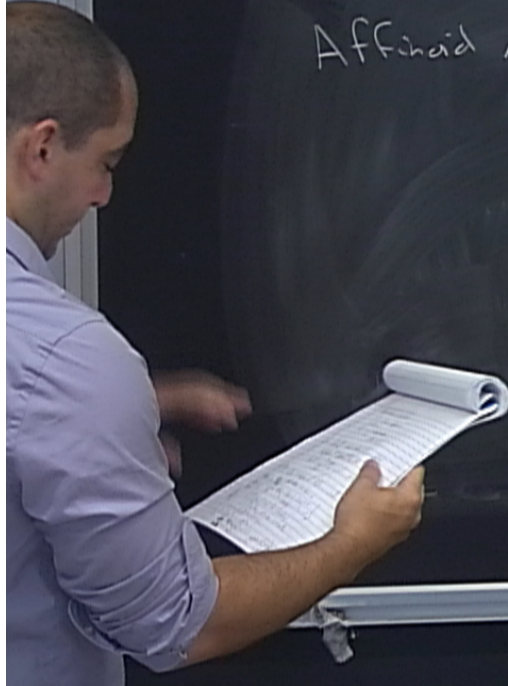
$$S_r = \text{Sym}(V) = \bigoplus_{i=0}^{\infty} S^i(V)$$

$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$$\text{on } \left\{ c \in R^n \mid |c_i| = r_i \right\}$$

$$S_r = R \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

Affine Algebra $S_r / I \leftarrow$ fg ideal



$$S_r = \text{Sym}(V) = \bigoplus_{i=0}^{\infty} S^i(V)$$

$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$$\text{on } \left\{ c \in R^n \mid |c_i| = r_i \right\}$$

$$S_r = R \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

Affine Algebra $\boxed{S_r / I}$ ← fg closed ideal

$$S_r = \text{Sym}(V) \cong \mathbb{R}[x_1, \dots, x_n]$$

$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$$\text{on } \left\{ c \in \mathbb{R}^n \mid |c_i| = r_i \right\}$$

$$S_r = \mathbb{R} \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

Affine Algebra

$$\boxed{S_r / I}$$

fg closed ideal

$$S_r^+ :=$$



$$S_r = \text{Sym}(V) \quad \text{on } \sum_{i=1}^n c_i e_i$$

$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$$\text{on } \left\{ c \in \mathbb{R}^n \mid |c_i| = r_i \right\}$$

$$S_r = \mathbb{R} \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

Affine Algebra $\boxed{S_r / I}$ ← fg closed ideal



$S_r^+ := \text{"colim"}$

colim $\text{Incl}(R_{\text{an}} \mathbb{R})$

$$S_r = \text{Sym}(V) \quad \text{on } \sum_{i=1}^n c_i e_i$$

$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$$\text{on } \left\{ c \in R^n \mid |c_i| = r_i \right\}$$

$$S_r = R \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

Affine Algebra

$$\boxed{S_r / I}$$

fg ideal



$$S_r^t := \text{"colim"}_n S_{r+t_n}$$

$$\text{colim} \text{ Incl } (R_{a_n} \rightarrow R)$$

$$S_r = \text{Sym}(V) \text{ ...}$$

$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$$\text{on } \left\{ c \in R^n \mid |c_i| = r_i \right\}$$

$$S_r = R \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

Affine Algebra $\boxed{S_r / I}$

fg closed ideal



$$S_r^t := \text{"colim"}_n S_{r+t_n}$$

colim $\text{Incl}(R_{a_n}, R)$

$$\underline{S_{r_1} \rightarrow S_{r_2} \text{ is nuclear}}$$

$$\underline{r_1 > r_2}$$

$$S_r = \text{Sym}(V) \text{ ...}$$

$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$$\text{on } \left\{ c \in R^n \mid |c_i| = r_i \right\}$$

$$S_r = R \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

Affine Algebra

$$\boxed{S_r / I}$$

fg closed ideal



$$S_r^+ := \text{"colim"}_n S_{r+\frac{1}{n}}$$

colim Incl $(\mathbb{R}_{>0}, \mathbb{R})$

$$A = S_r^+ / I \quad A^{vv} = A$$

$$\underline{S_{r_1} \rightarrow S_{r_2} \text{ is nuclear}} \\ \underline{r_1 > r_2}$$

$$S_r \text{ -- Sym } (V) \text{ -- } \dots$$

$$V = R_{r_1} \oplus R_{r_2} \oplus \dots \oplus R_{r_n}$$

$$\text{on } \{c \in R^n \mid |c_i| = r_i\}$$

$$S_r = R \{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \}$$

Affine Algebra $\boxed{S_r / I}$ ← fg closed ideal

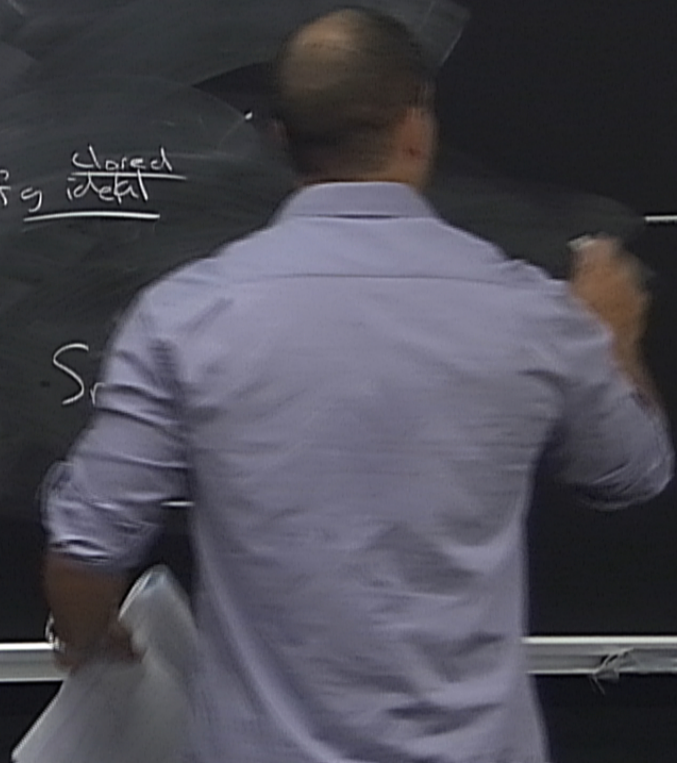


$$S_r^+ := \text{"colim"}_n S_{r+\frac{1}{n}}$$

colim Incl $(\mathbb{R}_{>0})$

$$A = S_r^+ / I \quad A^{vv} = A$$

Dagger Affine Algebra



$$S_r = \text{Sym}(V) \text{ on } \sum_{i=1}^n c_i \mathbb{R}^n \text{ with } |c_i| = r_i$$

$$V = \mathbb{R}_{r_1} \oplus \mathbb{R}_{r_2} \oplus \dots \oplus \mathbb{R}_{r_n}$$

$$\text{on } \left\{ c \in \mathbb{R}^n \mid |c_i| = r_i \right\}$$

$$S_r = \mathbb{R} \left\langle \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\rangle$$

Affine Algebra $\boxed{S_r / I}$

fg closed ideal

two full cats
of
 $\text{Comm}(\text{Ind}(\text{Ban}_R))$



$$S_r^+ := \text{"colim"}_n S_{r+\frac{1}{n}}$$

colim $\text{Ind}(\text{Ban}_R)$

$$A = S_{r_1}^+ / I \quad A^{vv} = A$$

zer Affine Algebra

$$\frac{S_{r_1} \rightarrow S_{r_2} \text{ is nuclear}}{r_1 > r_2}$$

Introduce certain $A \rightarrow B$ A, B affinoid
degree affinoid

$$B = A_V$$

CAUTION
DO NOT TOUCH THE BOARD SURFACE
OR COVER BY THE BOARD OR THE BOARD
IT IS SUPPORTED BY SPACERS
DO NOT REMOVE SPACERS

Introduce certain $A \rightarrow B$ A, B affinoid
classical affinoid

example

① Laurent $A_v = A \left\{ \frac{x}{s} \right\} / (yf-1) \quad f \in A$

CAUTION

BE CAREFUL TO AVOID THE FOLLOWING DANGER
IF YOU ARE NOT SURE OF THE CORRECT USE OF THIS EQUIPMENT
PLEASE CONTACT THE SUPPORT DEPARTMENT

Introduce certain $A \rightarrow B$ A, B affinoid
dagger affinoid

example

$$B = A_V$$
$$\textcircled{1} \text{ Laurent } A_V = A \left\{ \frac{y}{s} \right\} / (yf - 1) \quad f \in A$$
$$V \in M(A)$$

CAUTION
DO NOT USE GUMMA AND RUBBER BANDS
AS THEY CAN DAMAGE THE BOARD OR THE BOARD
IT IS RECOMMENDED TO USE
THE SAFETY STRAPS AND BUCKLES
WHEN MOUNTING BOARD

Introduce certain $A \rightarrow B$ A, B affinoid
degenerate affinoid

example

① Laurent $A_V = A \left\{ \frac{x}{s} \right\} / (yf - 1) \quad f \in A$

$V \subseteq M(A)$

$\left\{ x \mid |f(x)| \geq \frac{1}{s} \right\}$

A

CAUTION

DO NOT USE GUMMA TAP OR OTHER OBJECTS TO REMOVE CHALK FROM THE BOARD.
IT IS PROHIBITED TO WRITE WITH GUMMA TAP OR OTHER OBJECTS ON THE BOARD.
OTHER WARNING SIGNS

example

① Laurent

$$A_v = A \left\{ \frac{x}{s} \right\} / (yf-1)$$

$$f \in A$$

$$V \subseteq M(A)$$

$$\left\{ x \mid |f(x)| \geq \frac{1}{s} \right\}$$

$$A \hookrightarrow A_v$$

closed

$$B = A_v$$

closed affine

example

$$B = A_V$$

$$\textcircled{1} \text{ Laurent } A_V = A \left\{ \frac{x}{s} \right\} / (yf-1) \quad f \in A$$

$$V \in M(A)$$

$$A \hookrightarrow A_V$$

closed

$$\left\{ x \mid |f(x)| \geq \frac{1}{s} \right\}$$

$\textcircled{2}$ Weierstrass

$$A_V = A \left\{ \frac{x}{s} \right\} / (1-g)$$

$$g \in A$$

$$\left\{ x \mid |g(x)| \leq s \right\}$$

0

CAUTION
DO NOT USE LAMP OR REMOVE BOARD
WITH CARE AT THE END OF THE HOUR
IT IS PROHIBITED TO OPEN
THE BOARD WITHOUT ASKING
OTHER BOARDERS

degree affinen

examples

$$B = A_V$$

① Laurent $A_V = A \left\{ \frac{x}{s} \right\} / (yf-1) \quad f \in A$

$$V \subseteq M(A)$$

$$A \hookrightarrow A_V$$

closed

$$\left\{ x \mid |f(x)| \geq \frac{1}{s} \right\}$$

② Weierstrass

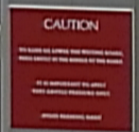
$$A_V = A \left\{ \frac{1}{s} \right\} / (1-g)$$

$$g \in A$$

$$\left\{ x \mid |g(x)| \leq s \right\}$$

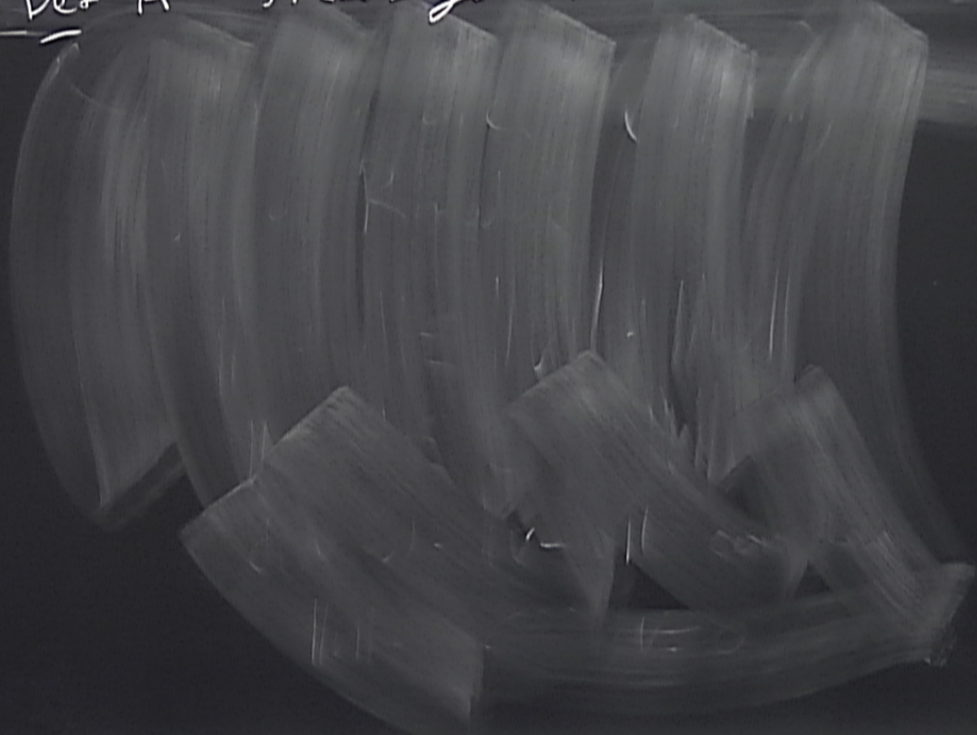
nuclear

$$A \hookrightarrow A_V \text{ dense}$$



$f: V \rightarrow W$ $\text{cok}(ker f \rightarrow V) \cong ker(W \rightarrow \text{cok } f)$
may not hold, f is strict if it holds

Def A Stein algebra



$f: V \rightarrow W$ $\text{cok}(\ker f \rightarrow V) \cong \ker(W \rightarrow \text{cok} f)$
may not hold, f is strict if it holds

Def A Stein algebra in $\text{Comm}(\text{Ind}(\text{Ban}_\mathbb{R}))$

is a filtered limit

$$A = \varprojlim (\dots \rightarrow A_3 \rightarrow A_2 \rightarrow A_1)$$



$f: V \rightarrow W$ $\text{cok}(ker f \rightarrow V) \cong ker(W \rightarrow \text{cok} f)$
 may not hold, f is strict if it holds

Def A Stein algebra in $\text{Comm}(\text{Ind}(\text{Ban}_\mathbb{R}))$

is a filtered limit

$$A = \varprojlim (\dots \rightarrow A_3 \rightarrow A_2 \rightarrow A_1)$$

each $A_{i+1} \rightarrow A_i$ is Weierstrass



$f: V \rightarrow W$ $\text{cok}(\ker f \rightarrow V) \cong \ker(W \rightarrow \text{cok} f)$
may not hold, f is strict if it holds

Def A Stein algebra in $\text{Comm}(\text{Ind}(\text{Ban}_\mathbb{R}))$

is a filtered limit

$$A = \varprojlim (\cdots \rightarrow A_3 \rightarrow A_2 \rightarrow A_1)$$

• each $A_{i+1} \rightarrow A_i$ is unimodular

$$\bullet M(A_i) \subseteq \text{int} M(A_{i+1})$$

$f: V \rightarrow W$ $\text{cok}(ker f \rightarrow V) \cong ker(W \rightarrow \text{cok} f)$
 may not hold, f is strict if it holds

Def A Stein algebra in $\text{Comm}(\text{Ind}(\text{Ban}_\mathbb{R}))$

is a filtered limit

$$A = \varinjlim (\dots \rightarrow A_3 \rightarrow A_2 \rightarrow A_1)$$

• each $A_{i+1} \rightarrow A_i$ is weirstrass A_i

$$\bullet M(A_i) \subseteq \text{int} M(A_{i+1})$$

reflexive and nuclear



Exan

is a filtered limit
 $A = \lim(\dots \rightarrow A_3 \rightarrow A_2 \rightarrow A_1)$

• each $A_{i+1} \rightarrow A_i$ is *weirstrass* A_i

• $M(A_i) \subseteq \text{int} M(A_{i+1})$



reflexive and nuclear

example $\mathbb{R}\{\frac{x}{r}\}^+$ is dual to
 closed disc of $\frac{1}{r}$

$\lim_{S = \frac{1}{r} - \frac{1}{n}} \mathbb{R}\{\frac{x}{3}\}$

open radius $\frac{1}{r}$
 disc

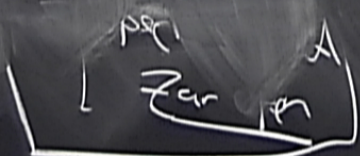
$\mathcal{O}(A'_R) =$

$\text{sp}(B) \subseteq \text{sp}(A)$
 $\text{For } \mathcal{O}(A)$

$\text{flit } P_{\mathcal{O}(j)}$

open factors
disi

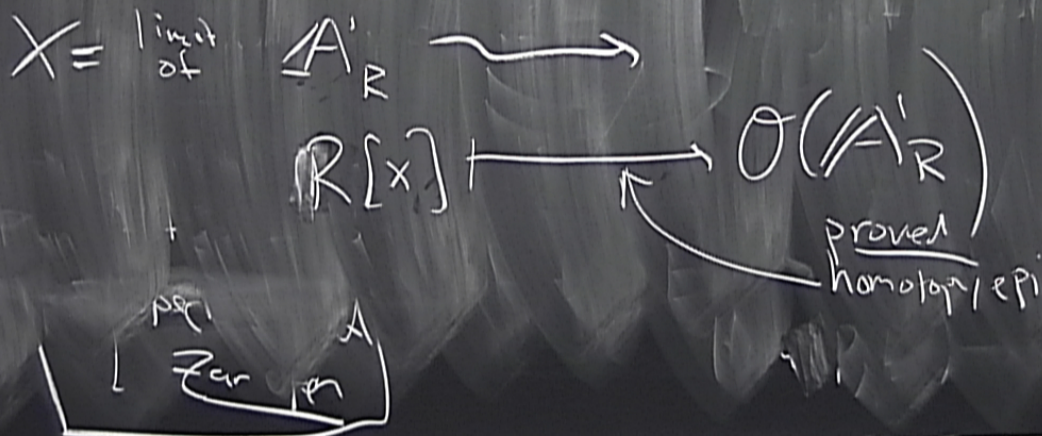
$$\theta(A_R^n) = \lim_{r \rightarrow \infty} R \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$



open factors
disi

$$\mathcal{O}(A_R^n) = \lim_{r \rightarrow \infty} R \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

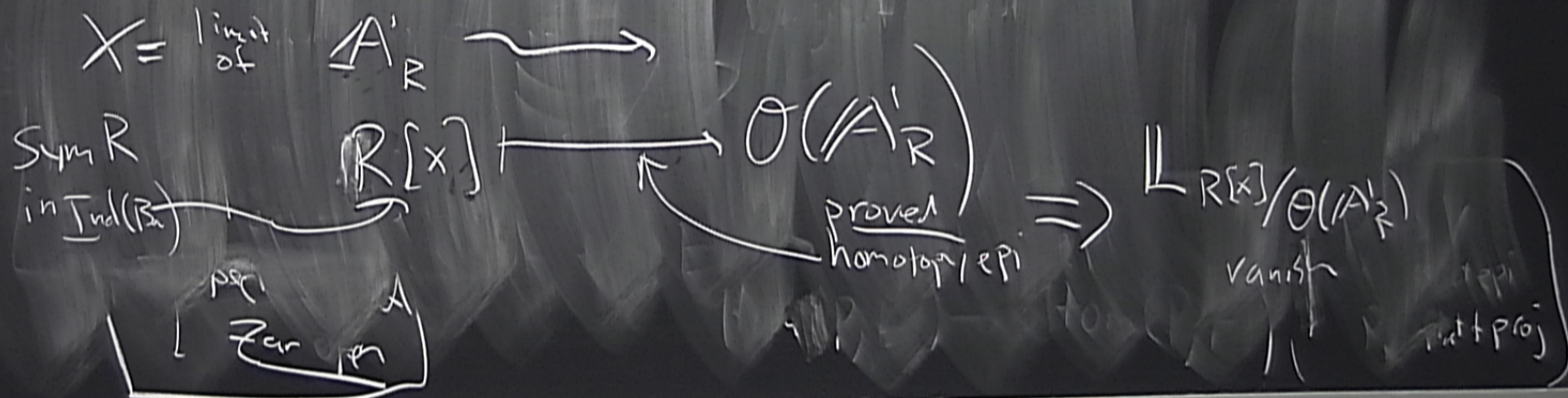
analytification



open factors
disi

$$\theta(A_R^n) = \lim_{n \rightarrow \infty} R \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

analytification

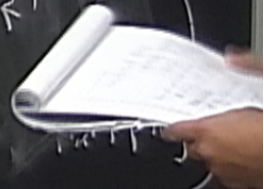
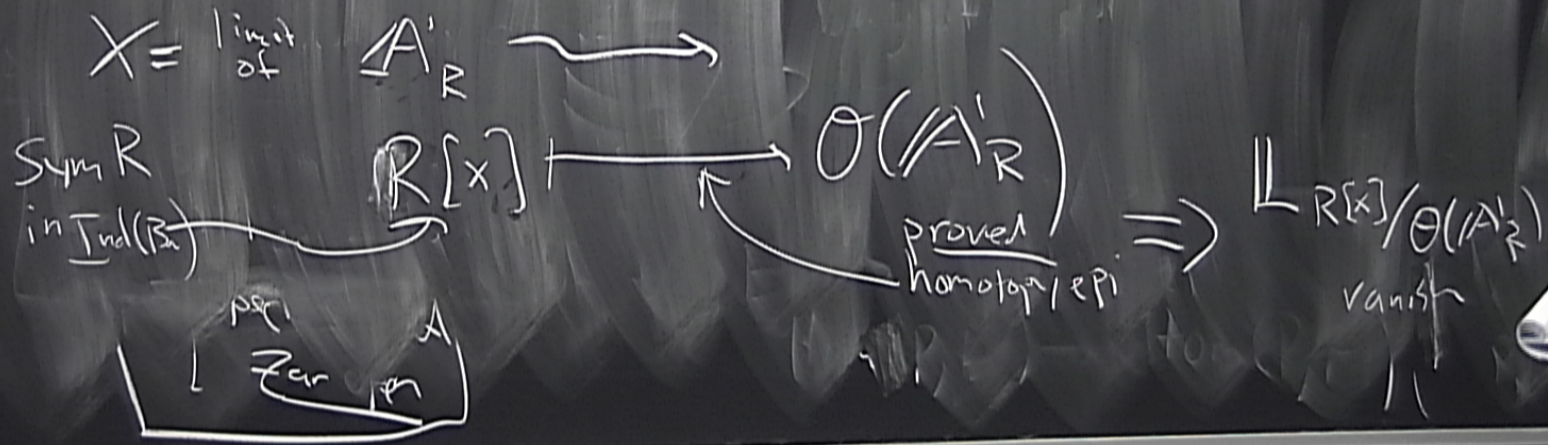


open factors
disi

$$\theta(A_R^n) = \lim_{n \rightarrow \infty} R \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

analytification

$$\theta(A^n) = \theta(A') \overset{H}{\otimes} \overset{H}{R} \dots \overset{H}{\otimes} \overset{H}{R} \theta(A')$$

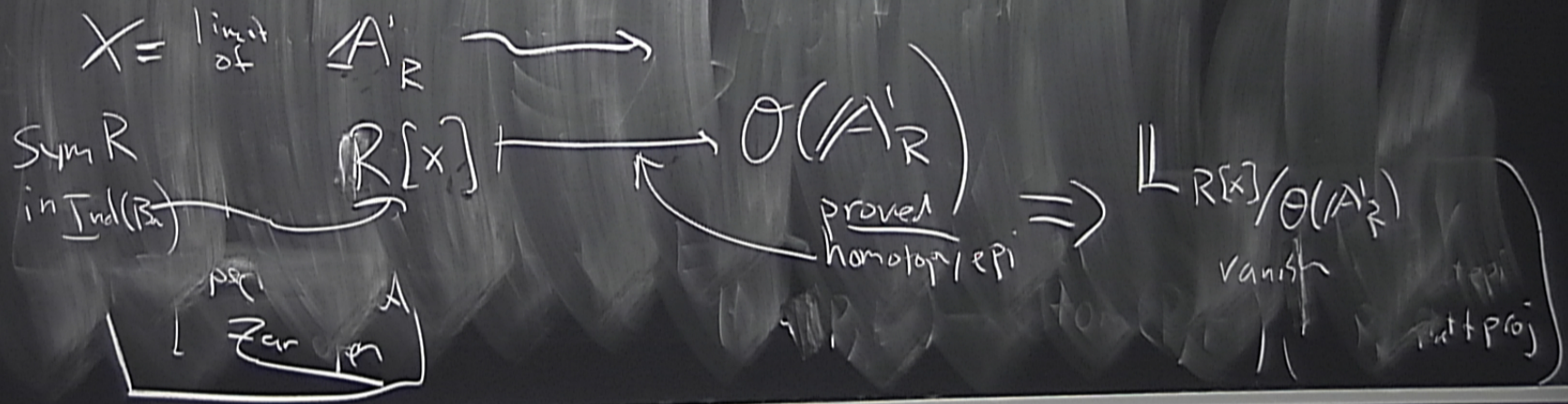


open factors
disi

$$\theta(A_R^n) = \lim_{n \rightarrow \infty} R \left\{ \frac{x_1}{r_1}, \dots, \frac{x_n}{r_n} \right\}$$

analytification

$$\theta(A^n) = \theta(A') \overset{H}{\otimes} \overset{H}{R} \dots \overset{H}{\otimes} \overset{H}{R} \theta(A')$$



Projects

1) w/ Krennitzer showed weak G-top
on aff: $\mathbb{A}^n = \text{formal } \mathbb{A}^n$

Dagger Affinoid Algebras

Projects

1) w/ Kremnitzer showed weak G-top
on affinoids = formal Zar Affinoids

2) w/

Page

A

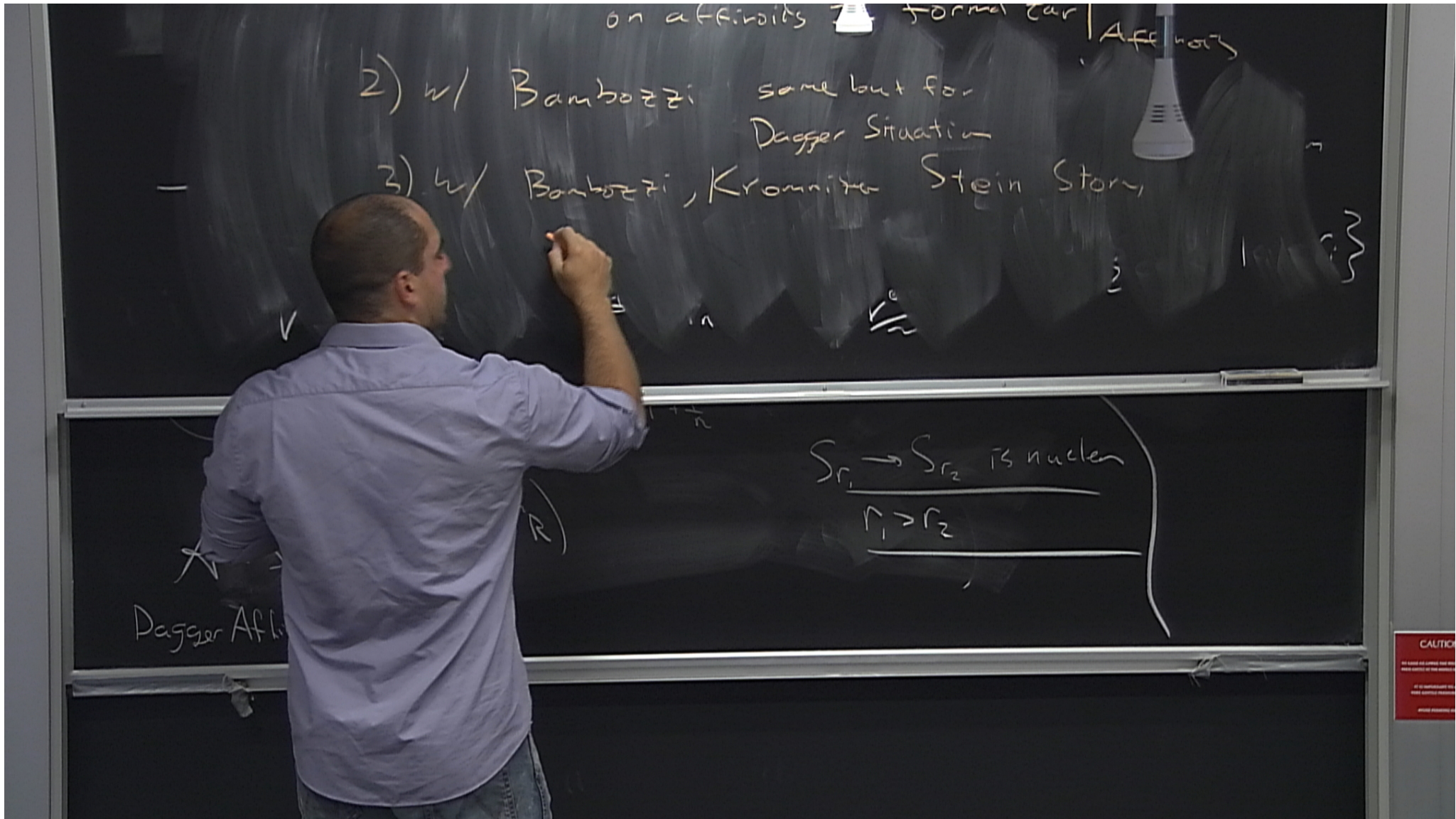
CAUTION
Do not touch the...
If it...
Please...
© 2008...

Projects

1) w/ Kremnitzer showed weak G-top
on affairs = formal Zar Affinity

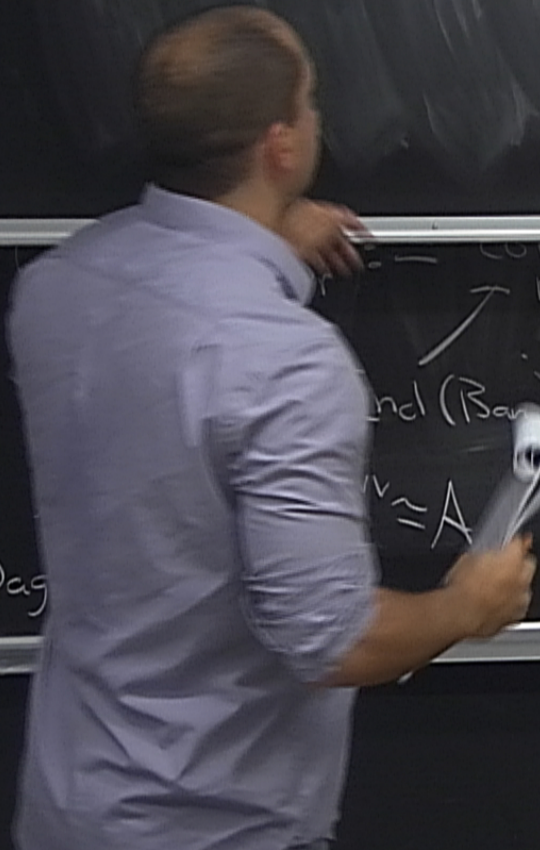
2) w/ Bambozzi same but for
Dagger Situation

3) w/



2) w/ Banbozzi same but for
Dagger Situation

3) w/ Banbozzi, Kronitzer Stein Story,
recover \mathbb{C} -analytic topology



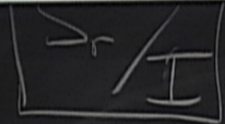
$\text{colim}_n S_{r+\frac{1}{n}}$
 $\text{incl}(\text{Ban})$
 $\cong A$

$S_{r_1} \rightarrow S_{r_2}$ is nuclear
 $r_1 > r_2$

3) w/ Banaschewski, Krombholz Stein Stone,
 recover \mathbb{C} -analytic topology
 \mathbb{Q}_p

4) w/ K. Ardakov

Final Alg



fg closed ideal

$\text{Comm}(\text{Ind}(B_{an} R))$

"colim" $S_{r+\frac{1}{n}}$

$S_{r_1} \rightarrow S_{r_2}$ is nuclear
 $r_1 > r_2$

$\text{Ind}(B_{an} R)$

$A^{vv} = A$

obs

Dagger Situation

3) w/ Banaschewski, Krombholz Stein Story
recover \mathbb{C} -analytic topology

\mathbb{Q}_p

4) w/ Kapranov $\mathcal{J} = \text{End}_R(\mathcal{O}, \mathcal{O})$ step of ind Banalg
 $R = \overline{\mathbb{Q}_p}$

$S_r^+ := \text{"colin"} S_{r+\frac{1}{n}}$

$\text{colin } \text{Incl}(\text{Ban}_R)$

$A = S_r^+ / I \quad A^{vv} \cong A$

Dagger Affinoid Algebras

$S_{r_1} \rightarrow S_{r_2}$ is nuclear
 $r_1 > r_2$

Dagge Situation

3) w/ Bambi, Kronitzer Stein Story,
recover \mathbb{C} -analytic topology

\mathbb{Q}_p

4) w/ K. Ardakov $\mathbb{J} = \text{End}_R(\mathcal{O}, \mathcal{O})$... step of ind Ban algs
 $R = \overline{\mathbb{Q}_p}$ agrees w/ \mathbb{J}



fg closed ideal

R1

$$S_r^+ := \text{"colim"} S_{r+\frac{1}{n}}$$

colim $\text{Incl}(\text{Ban}_R)$

$$A = S_r^+ / I \quad A^{vv} \cong A$$

$S_{r_1} \rightarrow S_{r_2}$ is nuclear
 $r_1 > r_2$

Dagge Affinoid Algebras