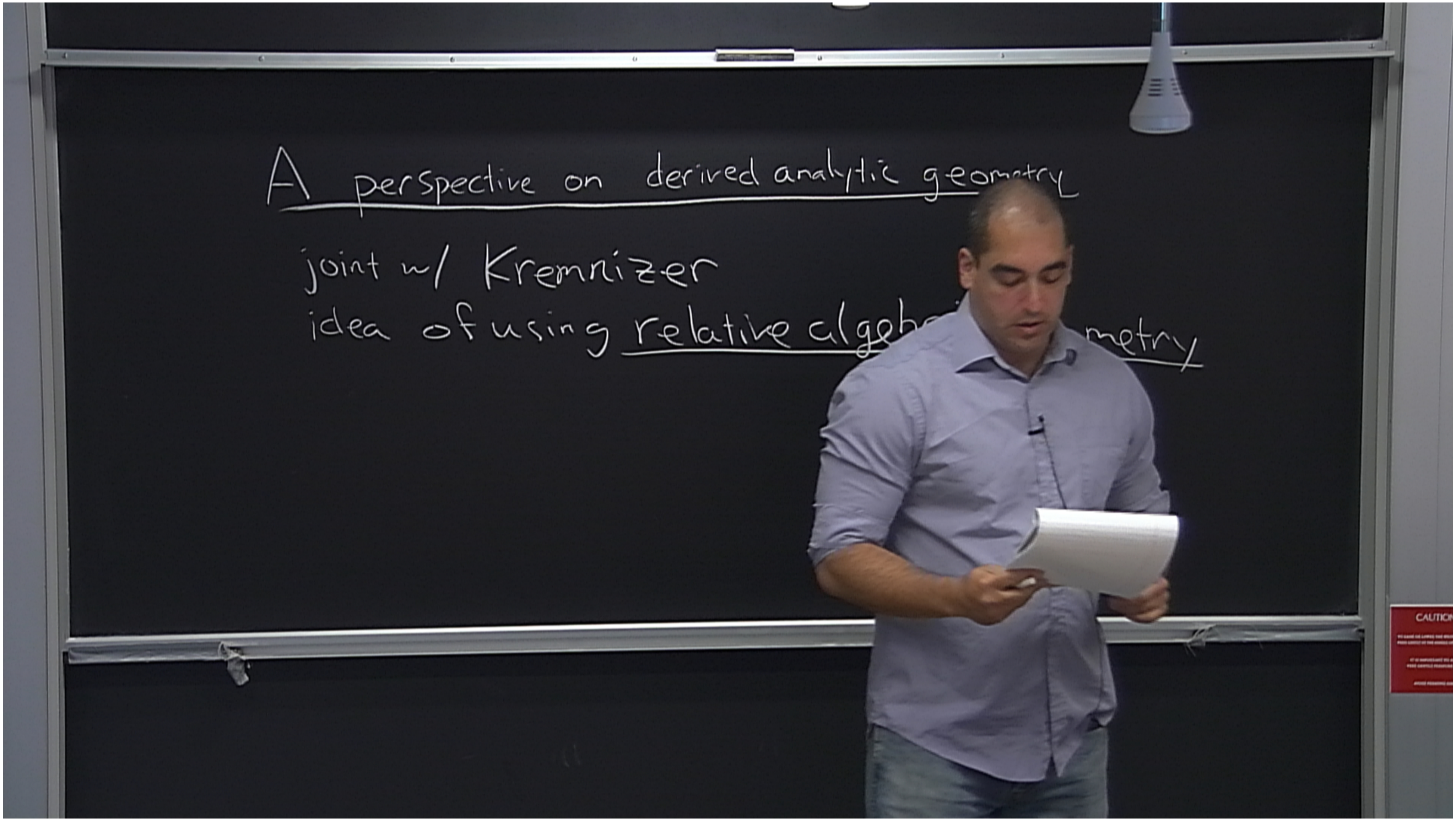
Title: A perspective on derived analytic geometry

Date: Apr 22, 2016 02:00 PM

URL: http://pirsa.org/16040087

Abstract: I will present a 'categorical' way of doing analytic geometry in which analytic geometry is seen as a precise analogue of algebraic geometry.
Our approach works for both complex analytic geometry and p-adic analytic geometry in a uniform way. I will focus on the idea of an 'open set' as used in various geometrical theories and how it is characterized
categorically. In order to do this, we need to study algebras and their modules in the category of Banach spaces. The categorical characterization that we need uses homological algebra in these 'quasi-abelian' categories which is work of Schneiders and Prosmans. In fact, we work with the larger category of Ind-Banach spaces for reasons I will explain. This gives us a way to establish foundations of derived analytic geometry (my joint project with Kobi Kremnizer). We compare this approach with standard standard notions such as the theory of affinoid algebras, Grosse-Klonne's theory of dagger algebras (over-convergent functions), the theory of Stein domains and others. I will formulate derived analytic geometry following the relative algebraic geometry approach of Toen, Vaquie and Vezzosi.

This talk involves various joint work with Federico Bambozzi and Kobi Kremnizer.


A perspective on derived analytic geom
joint w/ Kremnizer
idea of using relative algeb
metry

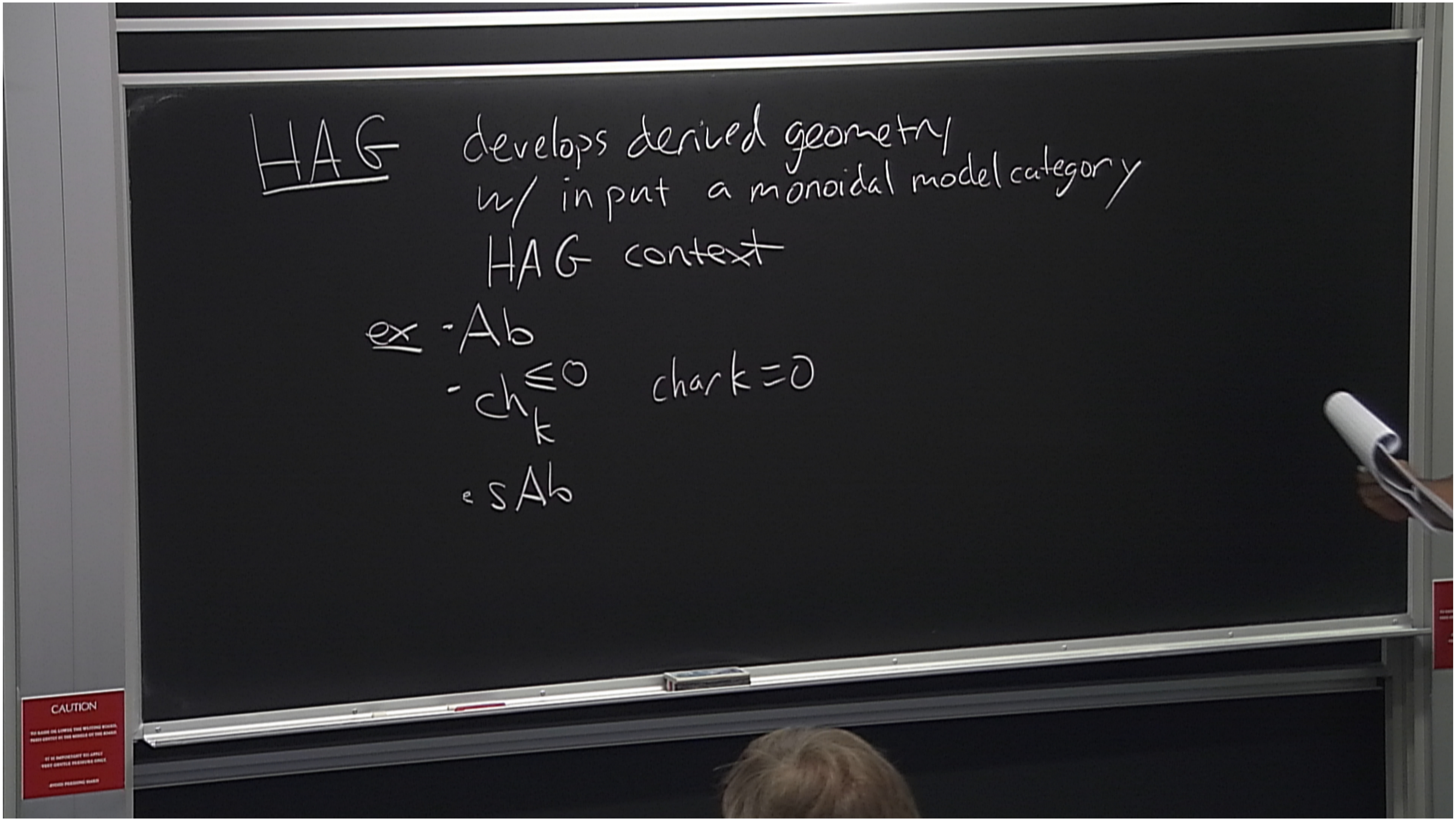
A perspective on derived analytic geometry

joint w/ Kremnizer

idea of using relative algebraic geometry

Under Spec $\mathbb{Z}$ (Toën-Vaquié)

HAG (Toën-Vezzosi)



HAG develops derived geometry
w/ input a monoidal model category
HAG context
ex - Ab
- $Ch_k^{\leq 0}$ char k = 0
- sAb
CAUTION

ex
- Ab
- $\mathrm{Ch}_k^{\leq 0}$ char $k = 0$
- sAb

suggestion $\mathcal{M} = s\mathrm{Ind}(\mathrm{Ban}_R)$ R a Banach ring

Thm (B, Kremnizer) $\mathcal{M}$ is a HAG context

Def A Banach ring is a ring R

equipped with $R \xrightarrow{|\,|} \mathbb{R}_{\geq 0}$

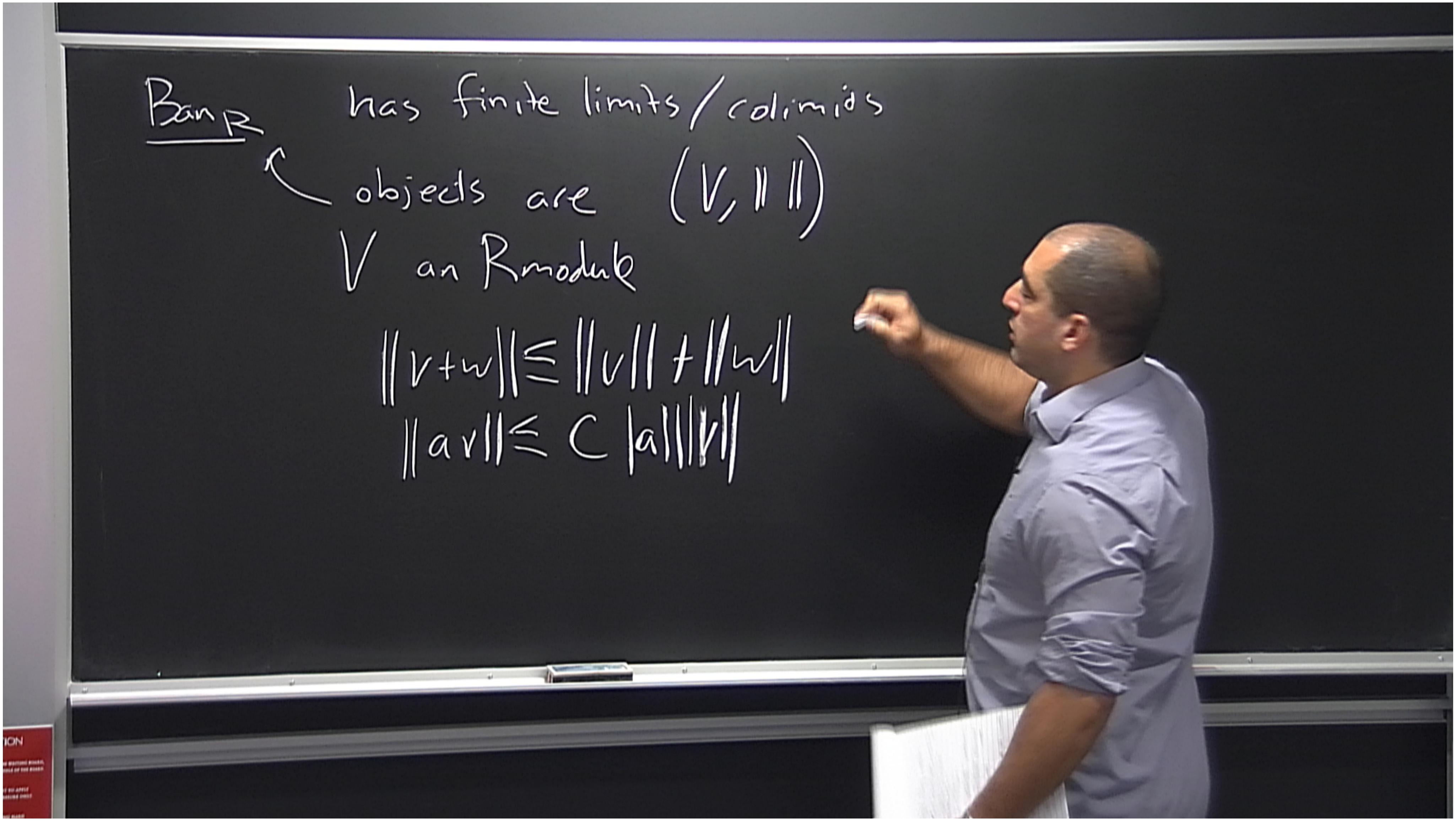
$$|a+b| \leq |a| + |b|$$

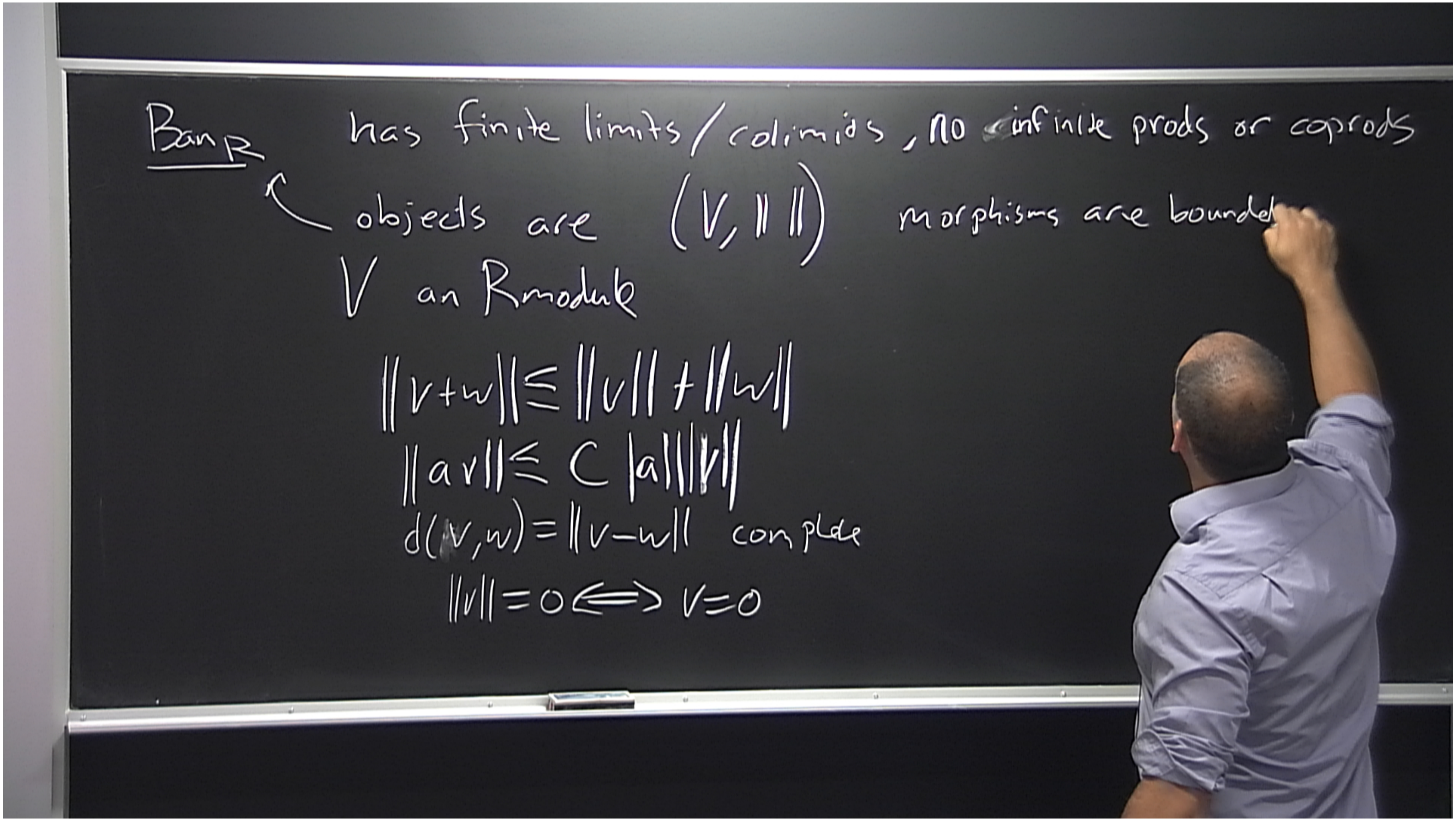
$$|ab| \leq C|a||b|$$

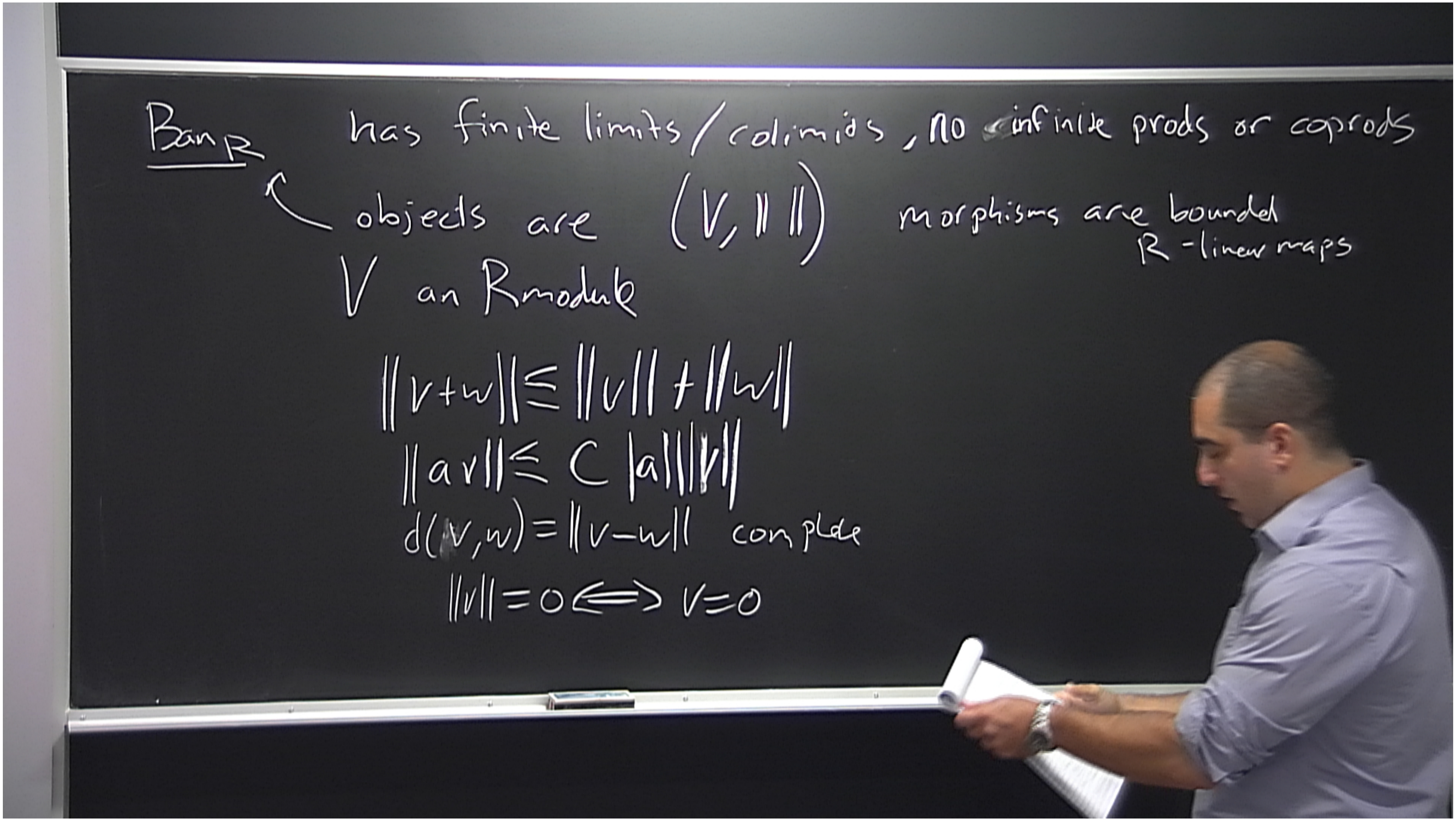
$$|a| = 0 \iff a = 0$$

$d(a,b) = |a-b|$ is complete

$$R = \mathbb{Z}, \mathbb{C}, \mathbb{R}, \underbrace{\mathbb{Q}_p, \mathbb{C}((t)), \text{Novikov}}$$


$\underline{Ban_R}$ has finite limits/colimits
objects are $(V, \| \ \|)$
V an Rmodule
$\|v+w\| \leq \|v\| + \|w\|$
$\|av\| \leq C\,|a|\,\|v\|$


Ban_R has finite limits / colimits, no infinite prods or coprods
objects are (V, || ||) morphisms are bounded
V an R module
||v+w|| ≤ ||v|| + ||w||
||av|| ≤ C |a| ||v||
d(v,w) = ||v−w|| complete
||v|| = 0 ⟺ v=0



$\underline{Ban_R}$ has finite limits / colimits, no infinite prods or coprods
objects are $(V, \| \|)$
morphisms are bounded
R-linear maps
V an Rmodule
$\|v+w\| \leq \|v\| + \|w\|$
$\|av\| \leq C |a| \|v\|$
$d(v,w) = \|v-w\|$ complete
$\|v\| = 0 \iff v = 0$

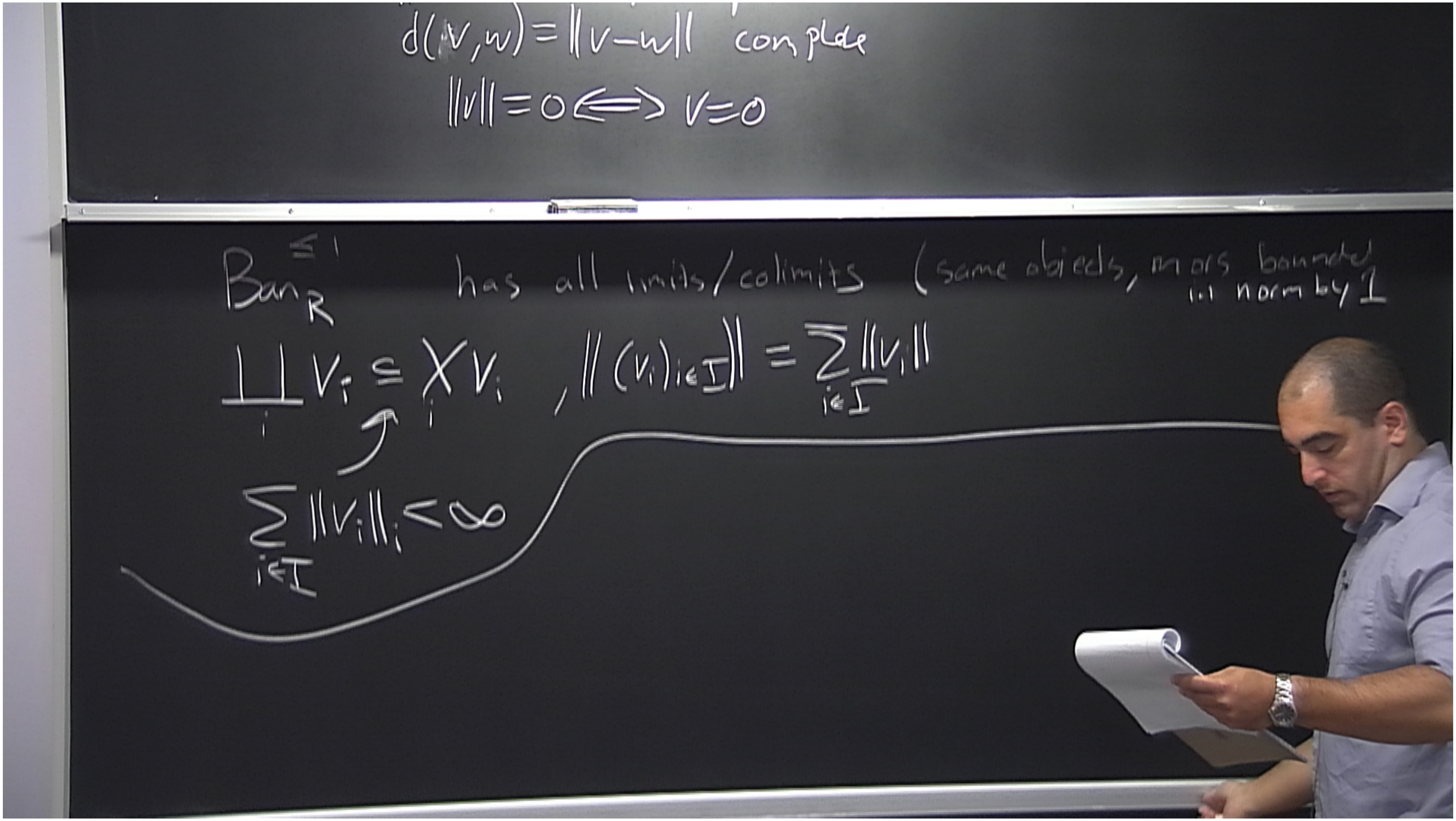
$d(v,w) = \|v-w\|$ complete

$\|v\| = 0 \Longleftrightarrow v = 0$

$\mathrm{Ban}_{\mathbb{R}}^{\leq 1}$ has all limits/colimits

$\coprod_i V_i \subseteq \bigtimes_i V_i$, $\|(v_i)_{i\in I}\| = \sum_{i\in I} \|v_i\|$

$\sum_{i\in I} \|v_i\|_i < \infty$


$d(v,w) = \|v-w\|$ complete
$\|v\| = 0 \Longleftrightarrow v = 0$
$\mathrm{Ban}_{\mathbb{R}}^{\leq 1}$ has all limits/colimits (same objects, mors bounded in norm by 1
$\coprod_i V_i \subseteq \prod_i V_i$, $\|(v_i)_{i\in I}\| = \sum_{i\in I} \|v_i\|$
$\sum_{i\in I} \|v_i\|_i < \infty$

$d(v,w) = \|v-w\|$ complete

$\|v\| = 0 \iff v = 0$

$\sum_{i \in I} \|v_i\|_i < \infty$

Ban_R is closed symmetric monoidal

$d(v,w) = \|v-w\|$ complete

$\|v\| = 0 \Longleftrightarrow v = 0$

$\coprod_i V_i \subseteq \bigtimes_i V_i$, $\|(v_i)_{i\in I}\| = \sum_{i\in I} \|v_i\|$

$\sum_{i\in I} \|v_i\|_i < \infty$

Ban_R is closed symmetric monoidal

$V \hat{\otimes}_R W \leftarrow$ completion of $V \otimes_R W$

w/rt $\|u\| = \inf \left\{ \sum_{i=1}^{n} \|v_i\| \|w_i\| \,\middle|\, u = \sum_{i=1}^{n} v_i \otimes w_i \right\}$

$$\coprod_i V_i \subseteq \bigtimes_i V_i \quad , \quad \|(v_i)_{i\in I}\| = \sum_{i\in I} \|v_i\|$$

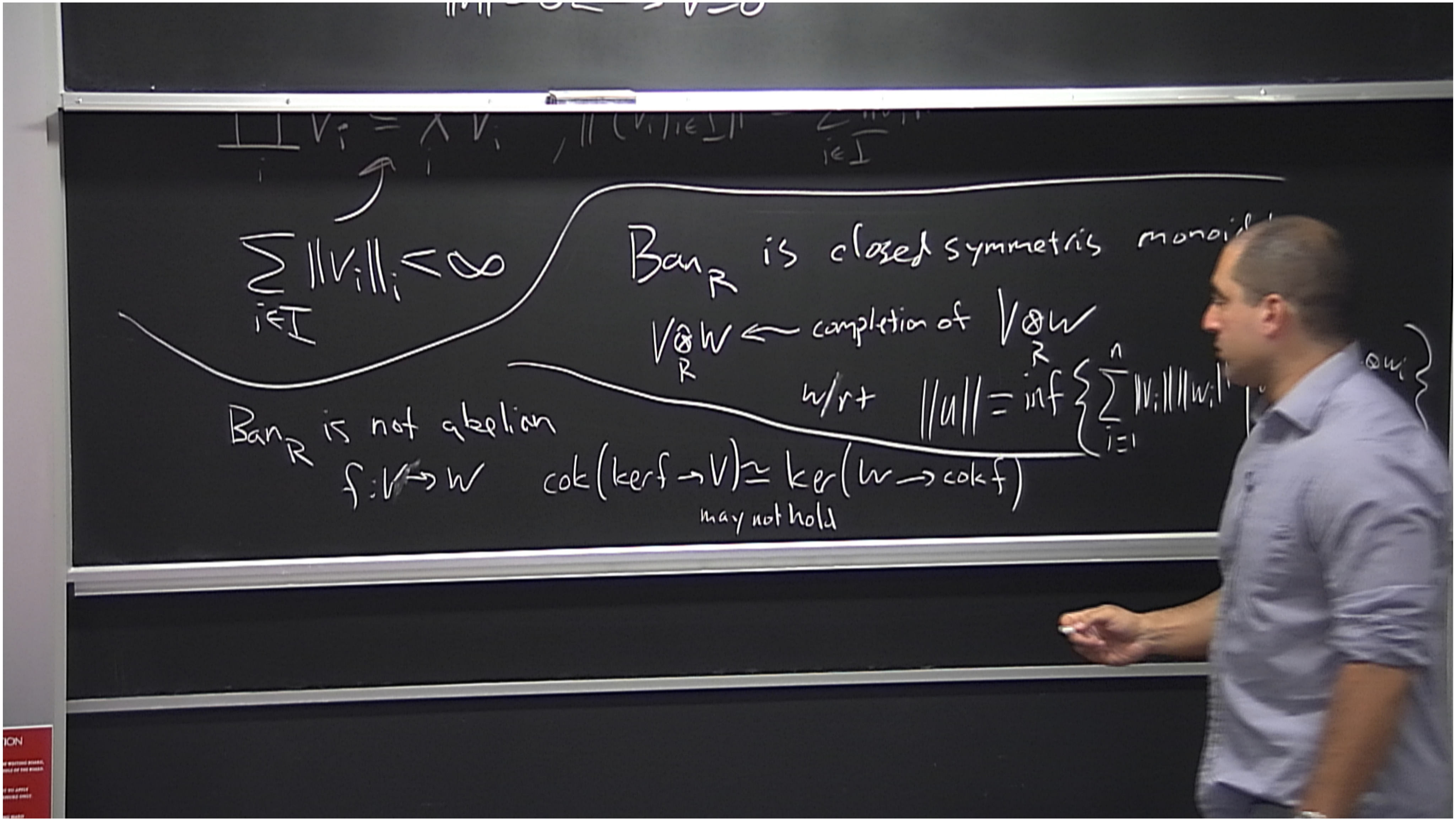
$$\sum_{i\in I} \|v_i\|_i < \infty$$

Ban_R is not abelian

Ban_R is closed symmetric monoidal

$V \hat{\otimes}_R W \leftarrow$ completion of $V \otimes_R W$

w/r+ $\|u\| = \inf \left\{ \sum_{i=1}^n \|v_i\| \|w_i\| \;\middle|\; u = \sum_{i=1}^n v_i \otimes w_i \right\}$


$\sum_{i \in I} \|v_i\|_i < \infty$
Ban_R is closed symmetric monoid
$V \hat{\otimes}_R W \leftarrow$ completion of $V \otimes_R W$
w/rt $\|u\| = \inf \{ \sum_{i=1}^{n} \|v_i\| \|w_i\|$
Ban_R is not abelian
$f : V \to W$ $\quad cok(\ker f \to V) \simeq \ker(W \to cok f)$
may not hold

Ban_R has all limits / colimits (same objects, mors bounded in norm by 1)

$$\coprod_i V_i \subseteq \bigtimes_i V_i \quad , \|(v_i)_{i\in I}\| = \sum_{i\in I} \|v_i\|$$

$$\sum_{i\in I} \|v_i\|_i < \infty$$

Ban_R is closed symmetric monoidal

$V \hat{\otimes}_R W \leftarrow$ completion of $V \otimes_R W$

w/rt $\|u\| = \inf \left\{ \sum_{i=1}^n \|v_i\| \|w_i\| \;\middle|\; u = \sum_{i=1}^n v_i \otimes w_i \right\}$

Ban_R is not abelian

$f: V \to W \quad \mathrm{cok}(\ker f \to V) \simeq \ker(W \to \mathrm{cok} f)$

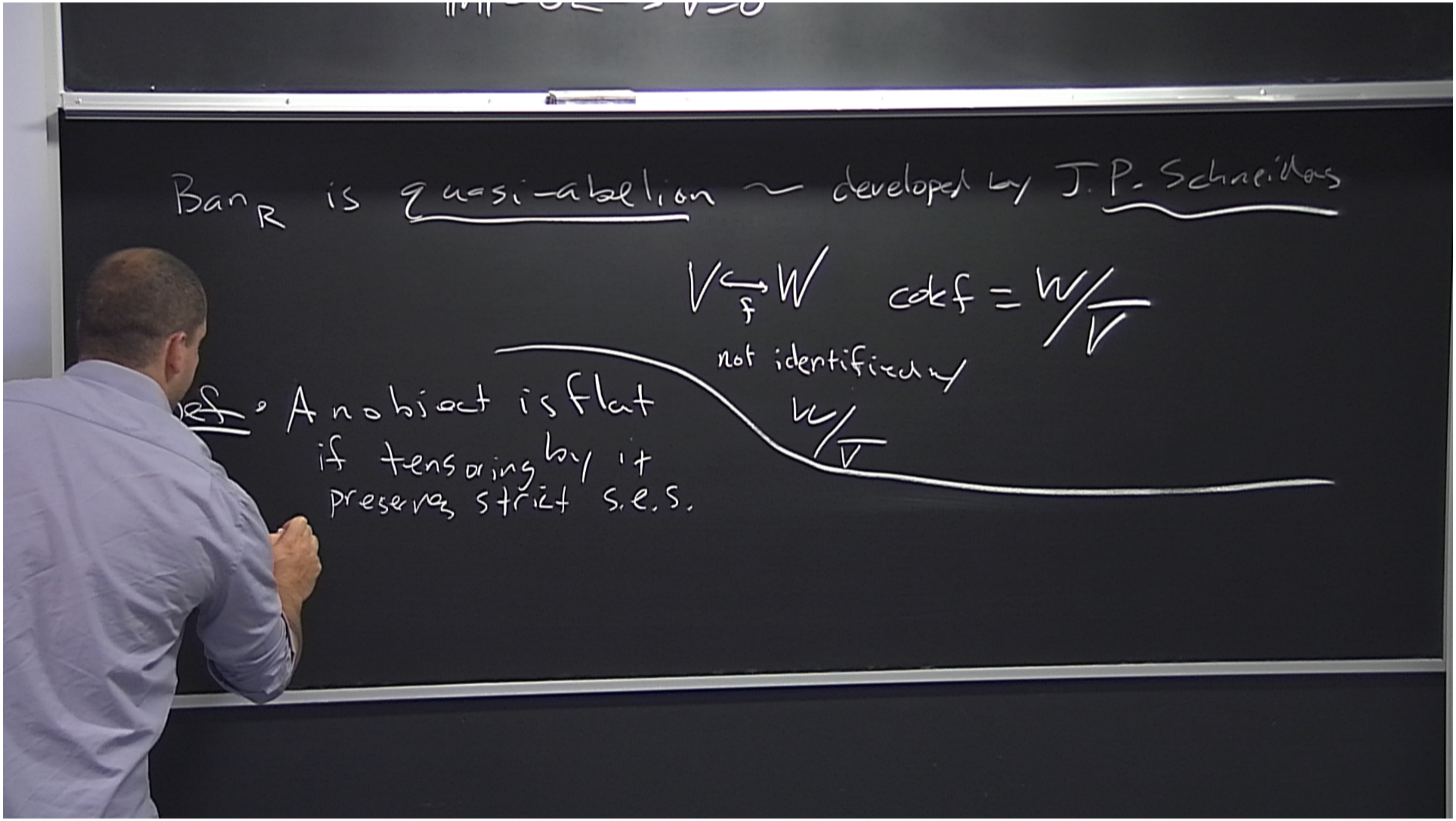
may not hold , f is strict if it holds

Ban_R is quasi-abelian ~ developed by J. P. Schneiders

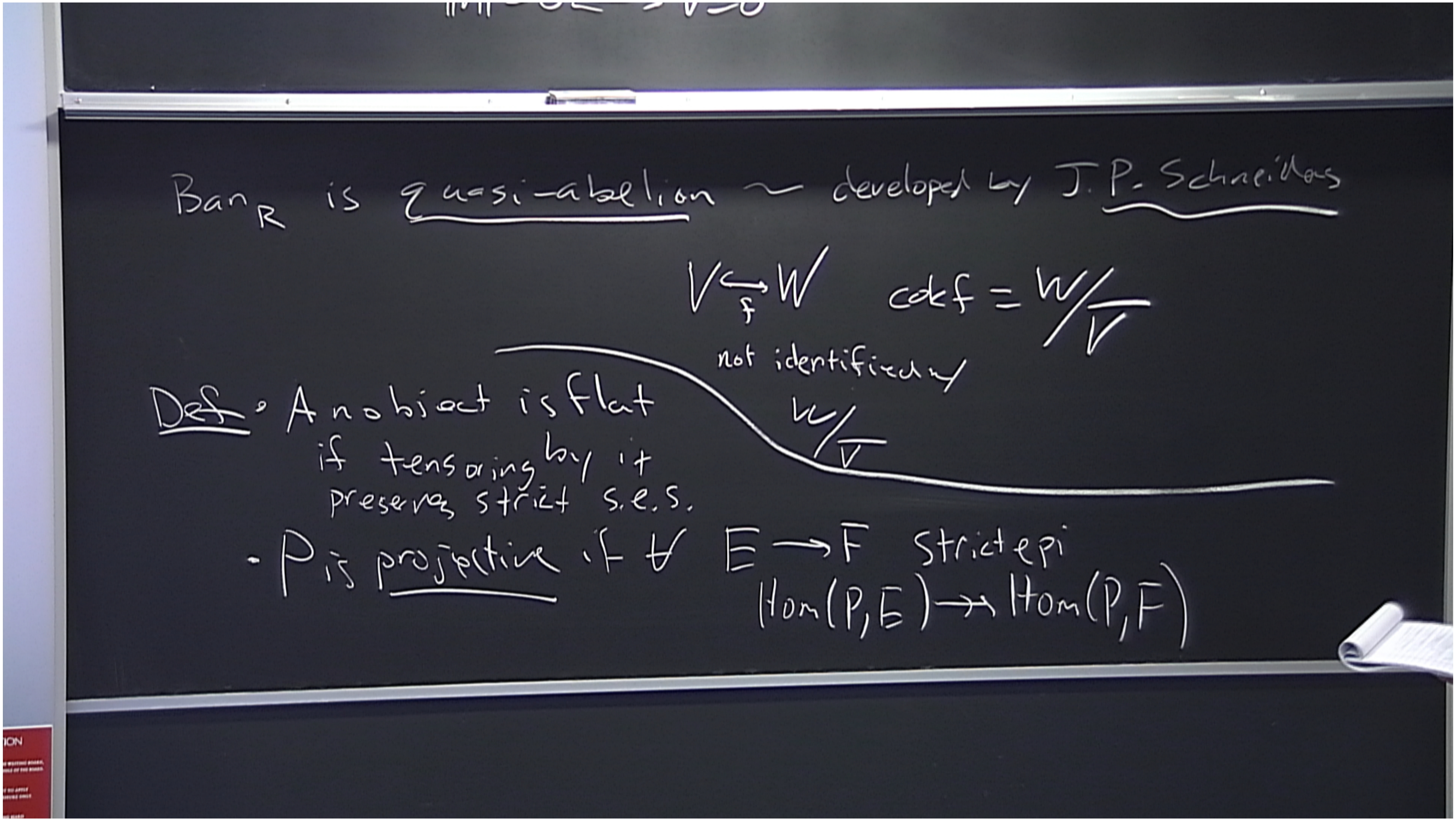
$$V \overset{f}{\hookrightarrow} W \qquad \mathrm{coker} f = W/\overline{V}$$

not identified w/

W/V



Ban_R is quasi-abelian ~ developed by J. P. Schneiders
V ↪ W
f
cok f = W/V̄
not identified w/
W/V
Def: An object is flat
if tensoring by it
preserves strict s.e.s.



Ban_R is quasi-abelian ~ developed by J. P. Schneiders
V ↪_f W   cok f = W/V
not identified w/ W/V
Def: · An object is flat if tensoring by it preserves strict s.e.s.
· P is projective if ∀ E → F strict epi
Hom(P,E) ↠ Hom(P,F)

Ban_R is quasi-abelian ~ developed by J. P. Schneiders

$V \overset{f}{\hookrightarrow} W$ $\quad \mathrm{cok} f = W/\overline{V}$

not identified w/ $W/\overline{V}$

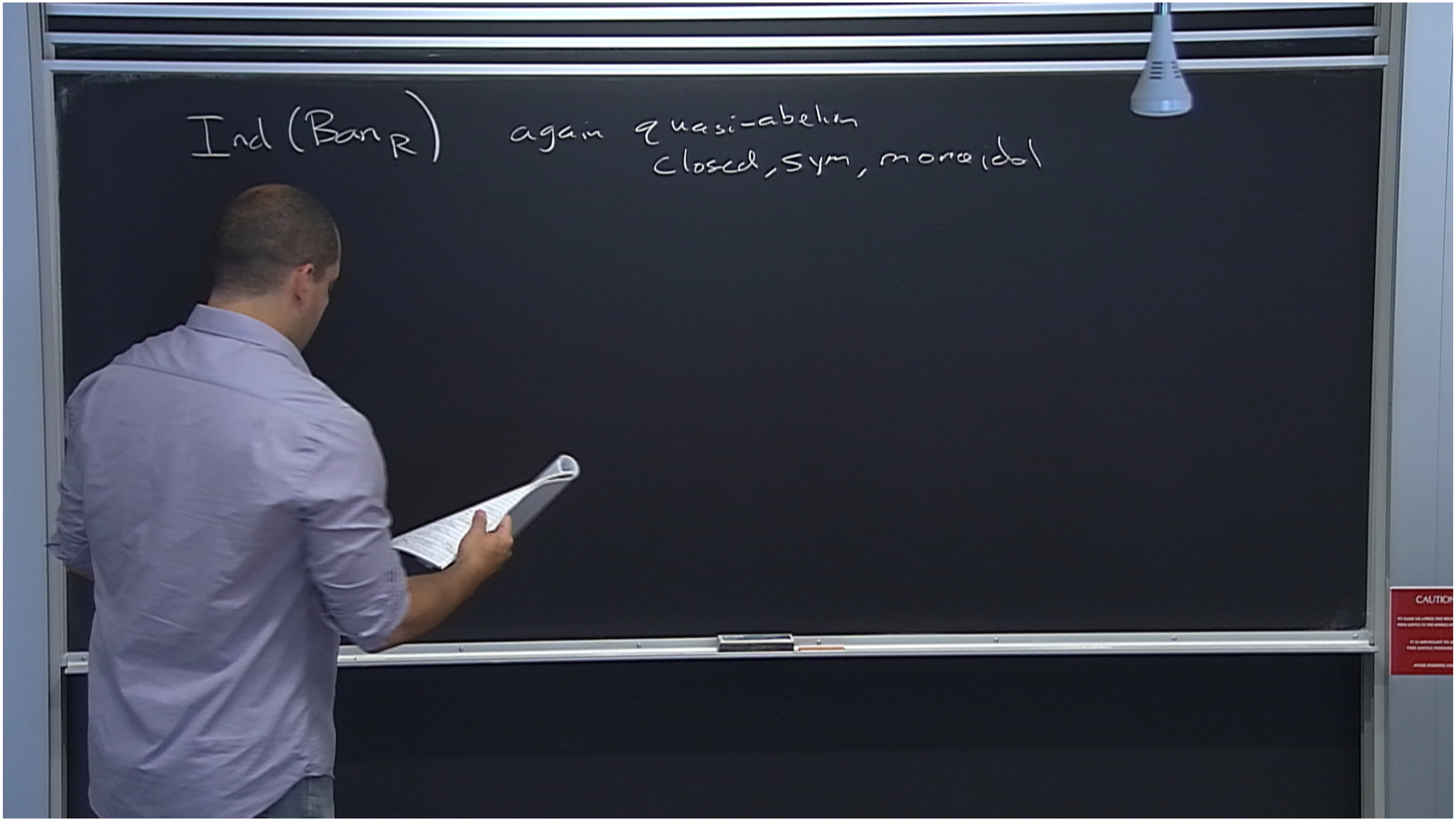
Def: · An object is flat if tensoring by it preserves strict s.e.s.

· P is projective if $\forall$ $E \to F$ strict epi

$\mathrm{Hom}(P,E) \twoheadrightarrow \mathrm{Hom}(P,F)$

$P_V \to V$ strict epi

P_V flat + proj


$\mathrm{Ind}(\mathrm{Ban}_R)$ again quasi-abelian
closed, sym, monoidal

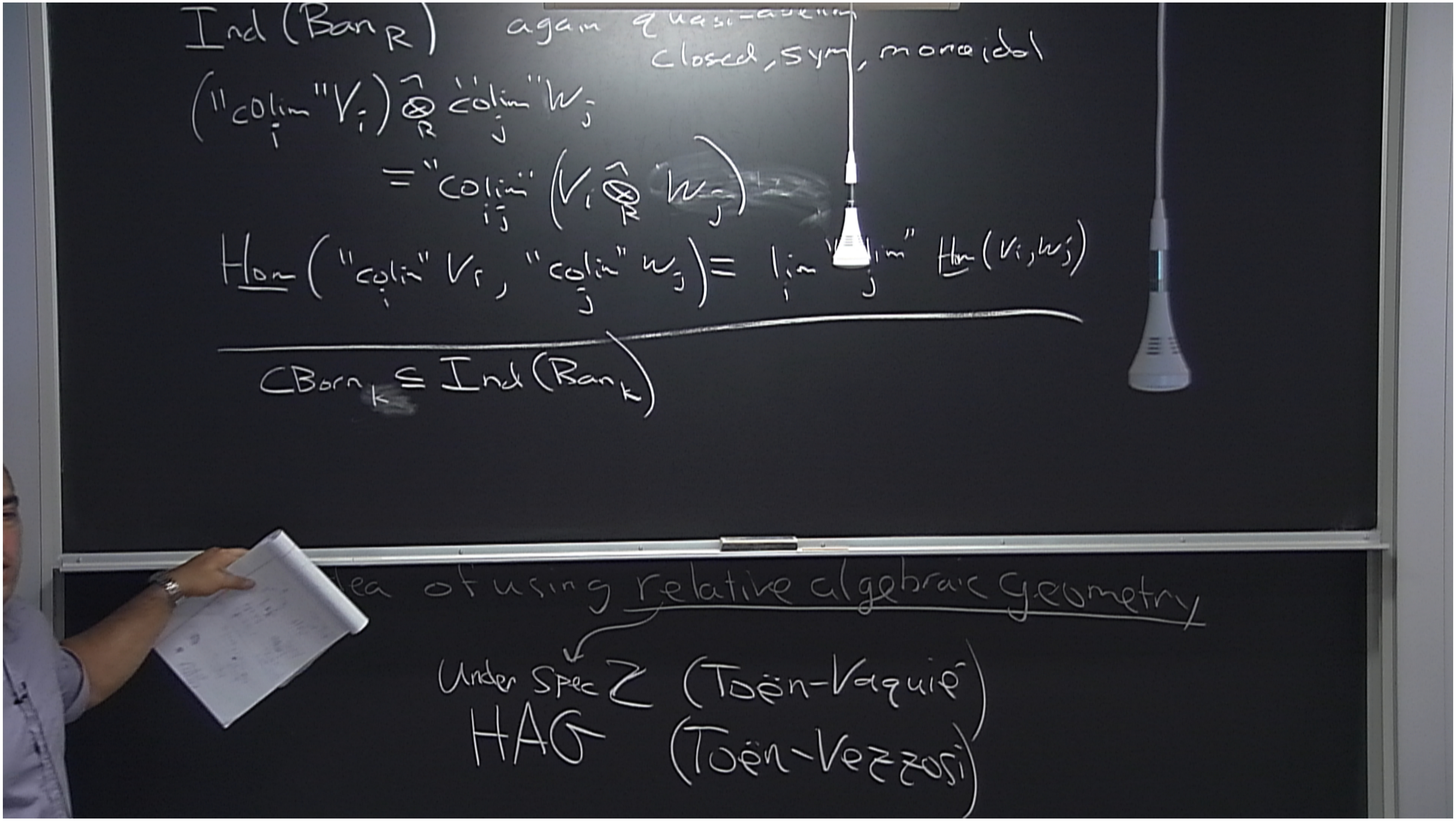
$\mathrm{Ind}(\mathrm{Ban}_R)$ again quasi-abelian

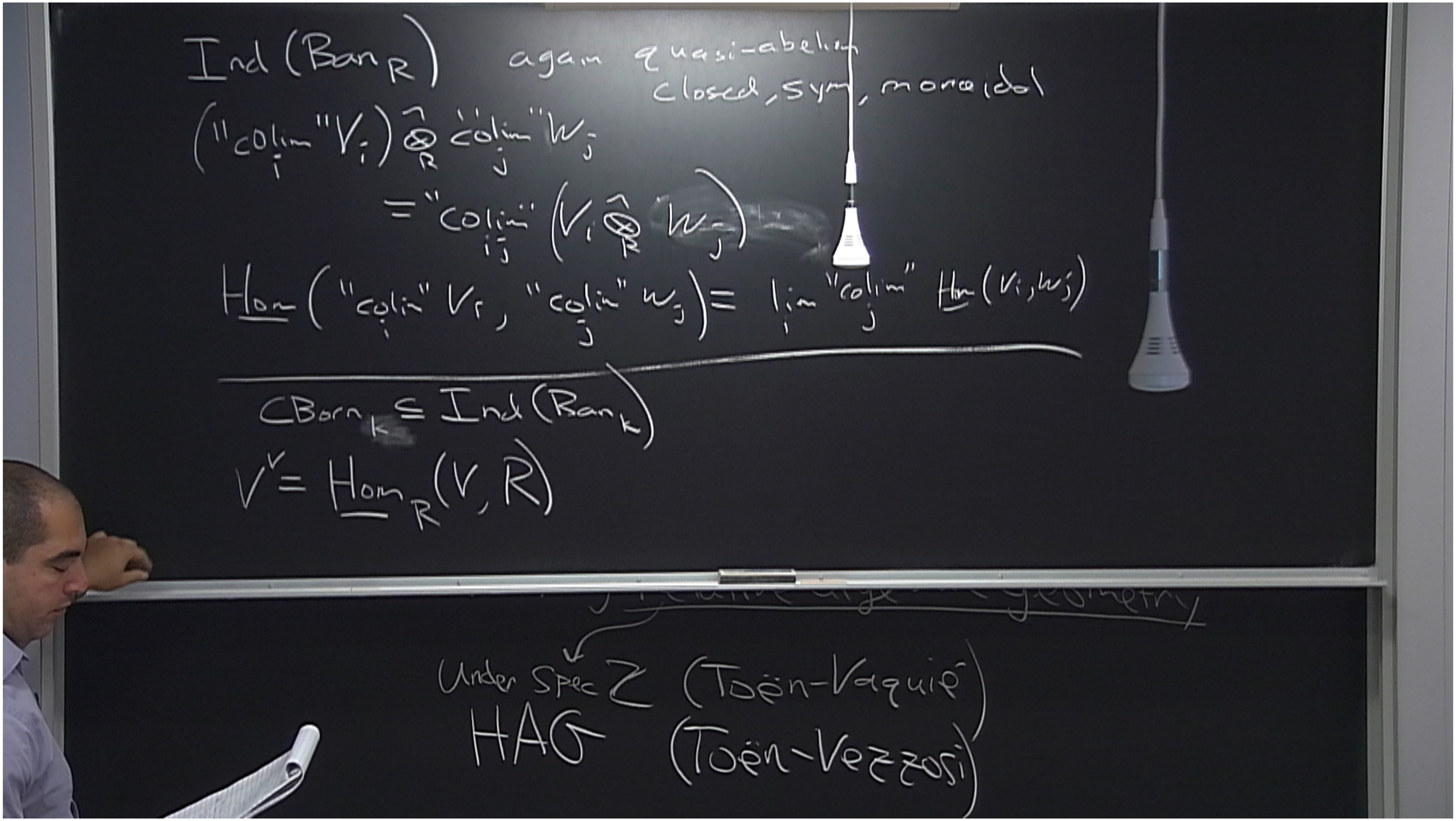
closed, sym. monoidal

$$\left(\text{"colim"}_i V_i\right) \hat{\otimes}_R \text{"colim"}_j W_j$$

$$= \text{"colim"}_{i,j} \left(V_i \hat{\otimes}_R W_j\right)$$

$$\underline{\mathrm{Hom}}\left(\text{"colim"}_i V_i, \text{"colim"}_j W_j\right) = \lim_i \text{"colim"}_j \underline{\mathrm{Hom}}(V_i, W_j)$$


$\mathrm{Ind}(\mathrm{Ban}_R)$ again quasi-abelian
closed, sym, monoidal
$(\text{"colim"}_i V_i) \hat{\otimes}_R \text{"colim"}_j W_j$
$= \text{"colim"}_{ij} (V_i \hat{\otimes}_R W_j)$
$\underline{\mathrm{Hom}}(\text{"colim"}_i V_i, \text{"colim"}_j W_j) = \lim_i \text{"lim"}_j \underline{\mathrm{Hom}}(V_i, W_j)$
$\mathrm{CBorn}_k \subseteq \mathrm{Ind}(\mathrm{Ban}_k)$
of using relative algebraic geometry
Under Spec $\mathbb{Z}$ (Toën-Vaquié)
HAG (Toën-Vezzosi)


$\mathrm{Ind}(\mathrm{Ban}_R)$ again quasi-abelian
closed, sym, monoidal
$(\text{"colim"}_i V_i) \hat{\otimes}_R \text{"colim"}_j W_j$
$= \text{"colim"}_{ij} (V_i \hat{\otimes}_R W_j)$
$\underline{\mathrm{Hom}}(\text{"colim"}_i V_i, \text{"colim"}_j W_j) = \lim_i \text{"colim"}_j \underline{\mathrm{Hom}}(V_i, W_j)$
$\mathrm{CBorn}_k \subseteq \mathrm{Ind}(\mathrm{Ban}_k)$
$V^\vee = \underline{\mathrm{Hom}}_R(V, R)$
Under Spec $\mathbb{Z}$ (Toën-Vaquié)
HAG (Toën-Vezzosi)

__Def__ $V \in \mathrm{Ind}(\mathrm{Ban}_R)$ is nuclear

if $\forall\, W \in \mathrm{Ban}_R$

$$\underline{\mathrm{Hom}}_R(W, V) \simeq W^\vee \hat{\otimes}_R V$$

Def $V \in \mathrm{Ind}(\mathrm{Ban}_R)$ is nuclear

if $\forall\, W \in \mathrm{Ban}_R$

$$\underline{\mathrm{Hom}}_R(W, V) \simeq W^\vee \hat{\otimes}_R V$$

$s\mathrm{Ind}(\mathrm{Ban}_R)$ has model structure

$X_\bullet \to Y_\bullet$ is a we/fib $\Leftrightarrow \forall\, P$ projective

Def $V \in \mathrm{Ind}(\mathrm{Ban}_R)$ is nuclear

if $\forall\, W \in \mathrm{Ban}_R$

$$\underline{\mathrm{Hom}}_R(W, V) \simeq W^\vee \hat{\otimes}_R V$$

$s\mathrm{Ind}(\mathrm{Ban}_R)$ has model structure

$X_\cdot \to Y_\cdot$ is a we/fib $\iff \forall\, P$ projective

$\mathrm{Hom}(P, X_\cdot) \to \mathrm{Hom}(P, Y_\cdot)$ is a we/fib

Def $V \in \mathrm{Ind}(\mathrm{Ban}_R)$ is nuclear

if $\forall\, W \in \mathrm{Ban}_R$

$$\underline{\mathrm{Hom}}_R(W, V) \simeq W^\vee \hat{\otimes}_R V$$

$s\mathrm{Ind}(\mathrm{Ban}_R)$ has model structure

$X_\bullet \to Y_\bullet$ is a we/fib $\iff \forall\, P$ projective

$\mathrm{Hom}(P, X_\bullet) \to \mathrm{Hom}(P, Y_\bullet)$ is a we/fib

Def $V \in \mathrm{Ind}(\mathrm{Ban}_R)$ is nuclear

if $\forall\, W \in \mathrm{Ban}_R$

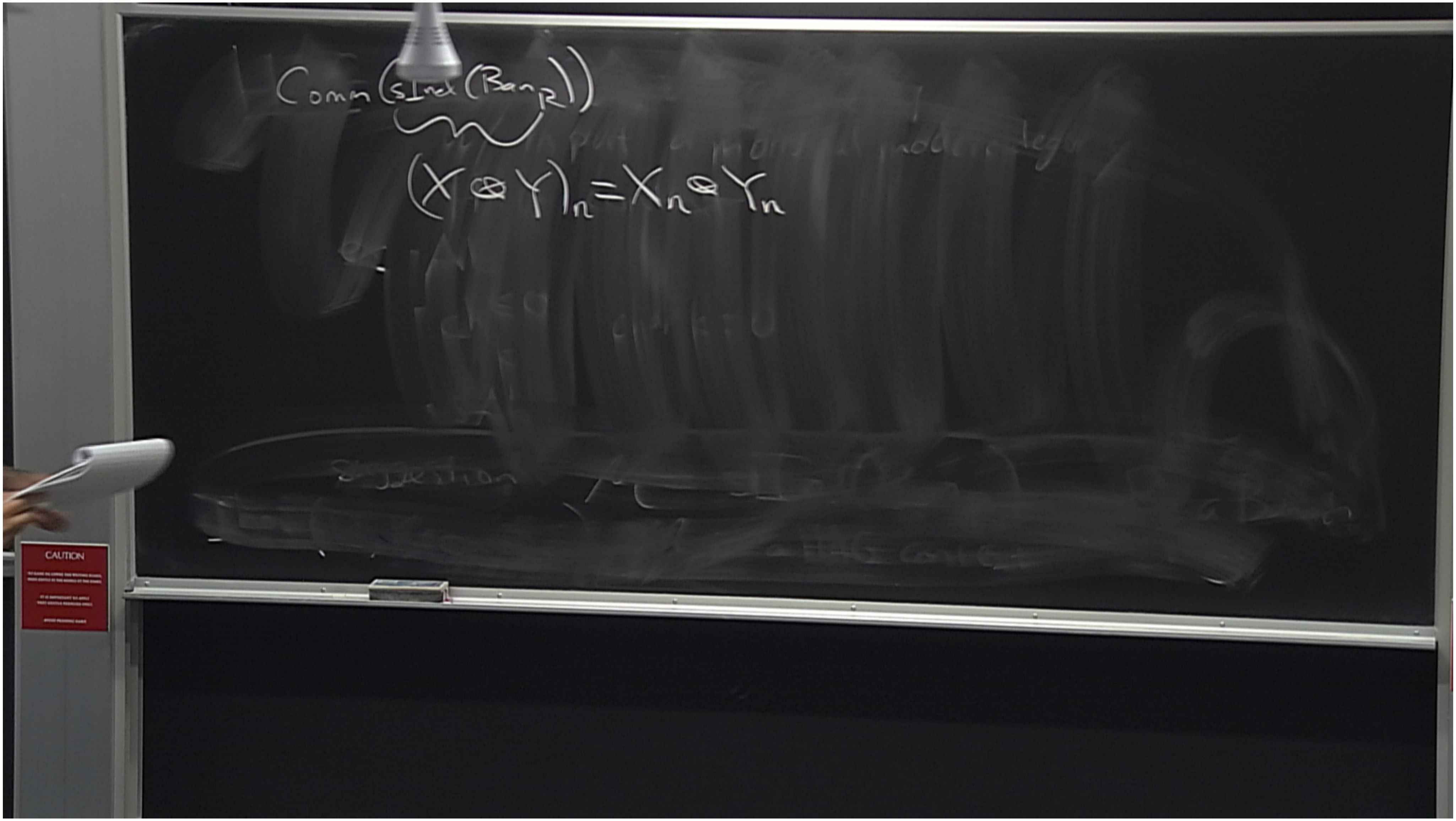
$$\underline{\mathrm{Hom}}_R(W, V) \simeq W^\vee \hat{\otimes}_R V$$

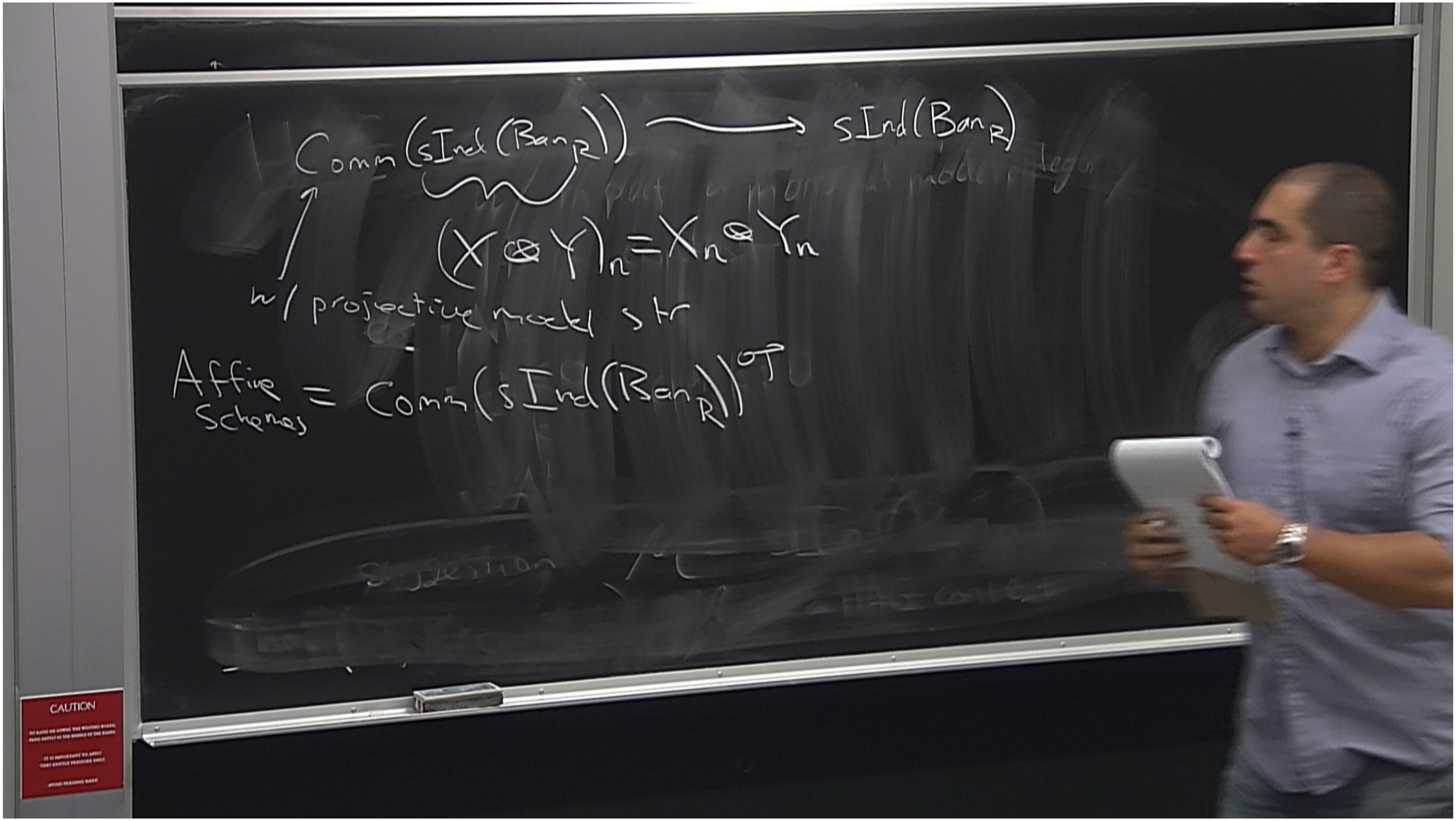
$s\mathrm{Ind}(\mathrm{Ban}_R)$ has model structure

$X_\bullet \to Y_\bullet$ is a we/fib $\iff \forall\, P$ projective

$\mathrm{Hom}(P, X_\bullet) \to \mathrm{Hom}(P, Y_\bullet)$ is a we/fib

gives a model cat str


$\mathrm{Comm}(\mathrm{sInd}(\mathrm{Ban}_R))$
$(X \otimes Y)_n = X_n \otimes Y_n$
CAUTION


Comm(sInd(Ban_R)) → sInd(Ban_R)
(X ⊗ Y)_n = X_n ⊗ Y_n
w/ projective model str
Affine Schemes = Comm(sInd(Ban_R))^op
CAUTION

model str

$$\text{Affine Schemes} = \mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))^{op}$$

$$A \in \mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))$$

$$\mathrm{Mod}(A)$$

model str

$$\text{Affine Schemes} = \mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))^{op}$$

$$A \in \mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))$$

$$\mathrm{Mod}(A)$$

$$A \otimes M \to M$$

$$\mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R)) \longrightarrow s\mathrm{Ind}(\mathrm{Ban}_R)$$

$$(X \otimes Y)_n = X_n \otimes Y_n$$

w/ projective model str

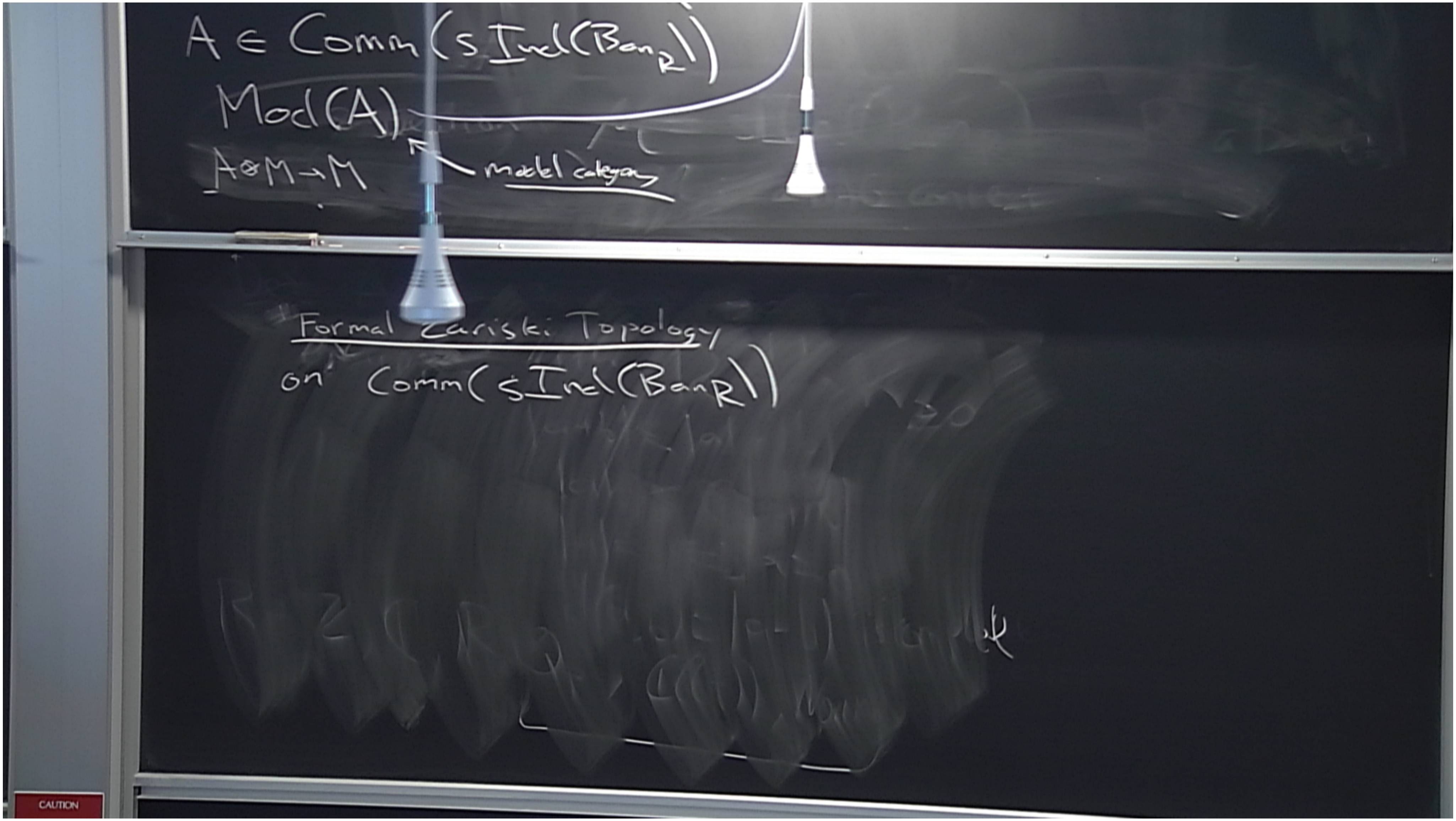
$$\text{Affine Schemes} = \mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))^{op}$$

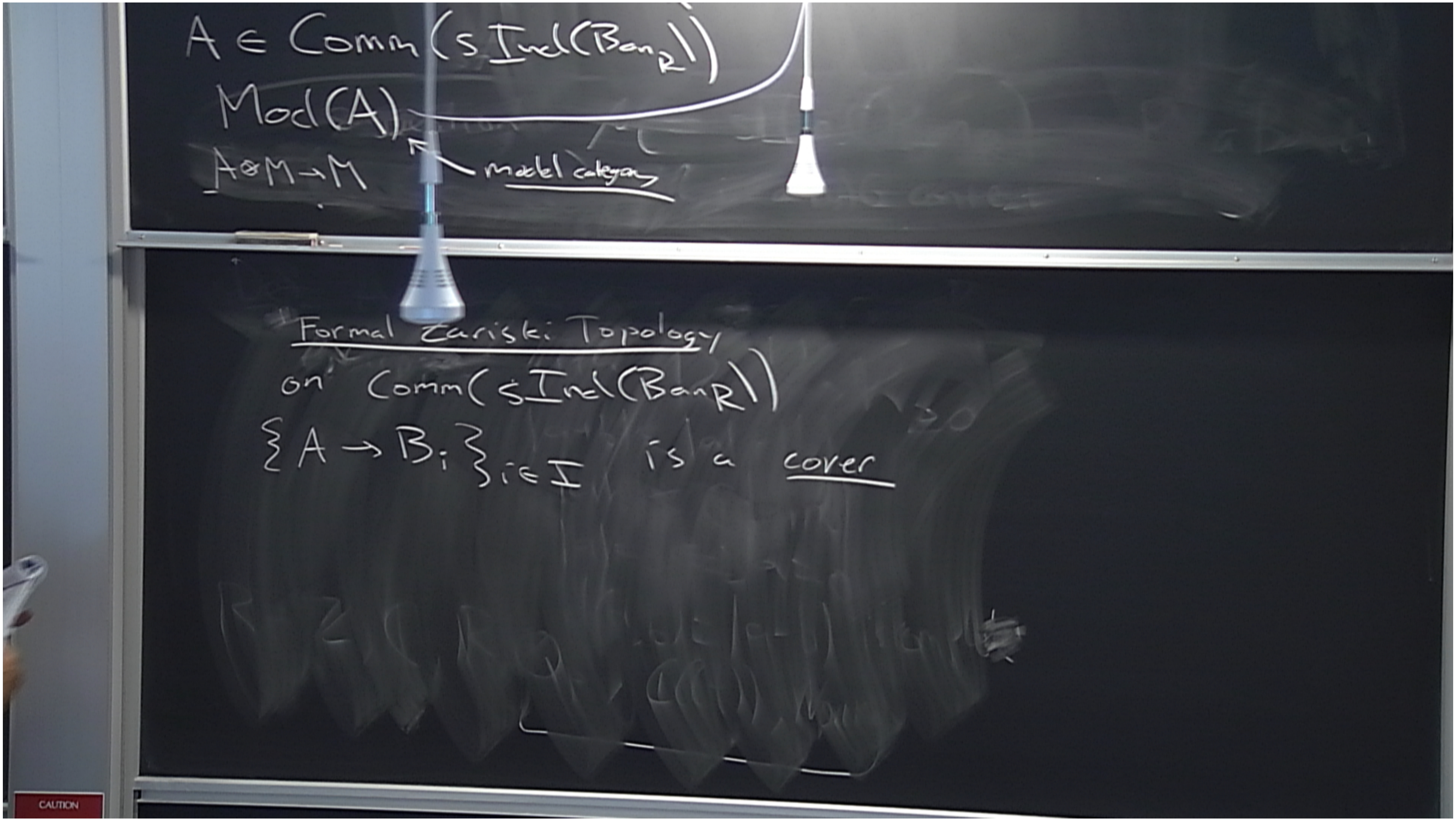
$$A \in \mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))$$

$$\mathrm{Mod}(A) \longrightarrow s\mathrm{Ind}(\mathrm{Ban}_R)$$

$$A \otimes M \to M$$

model category


A ∈ Comm(sInd(Ban_R))
Mod(A)
A⊗M→M
model category
Formal Zariski Topology
on Comm(sInd(Ban_R))



$A \in \mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))$
$\mathrm{Mod}(A)$
$A \otimes M \to M$
model category
Formal Zariski Topology
on $\mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))$
$\{A \to B_i\}_{i \in I}$ is a cover

$A \in \mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))$

$\mathrm{Mod}(A)$

$A \otimes M \to M$

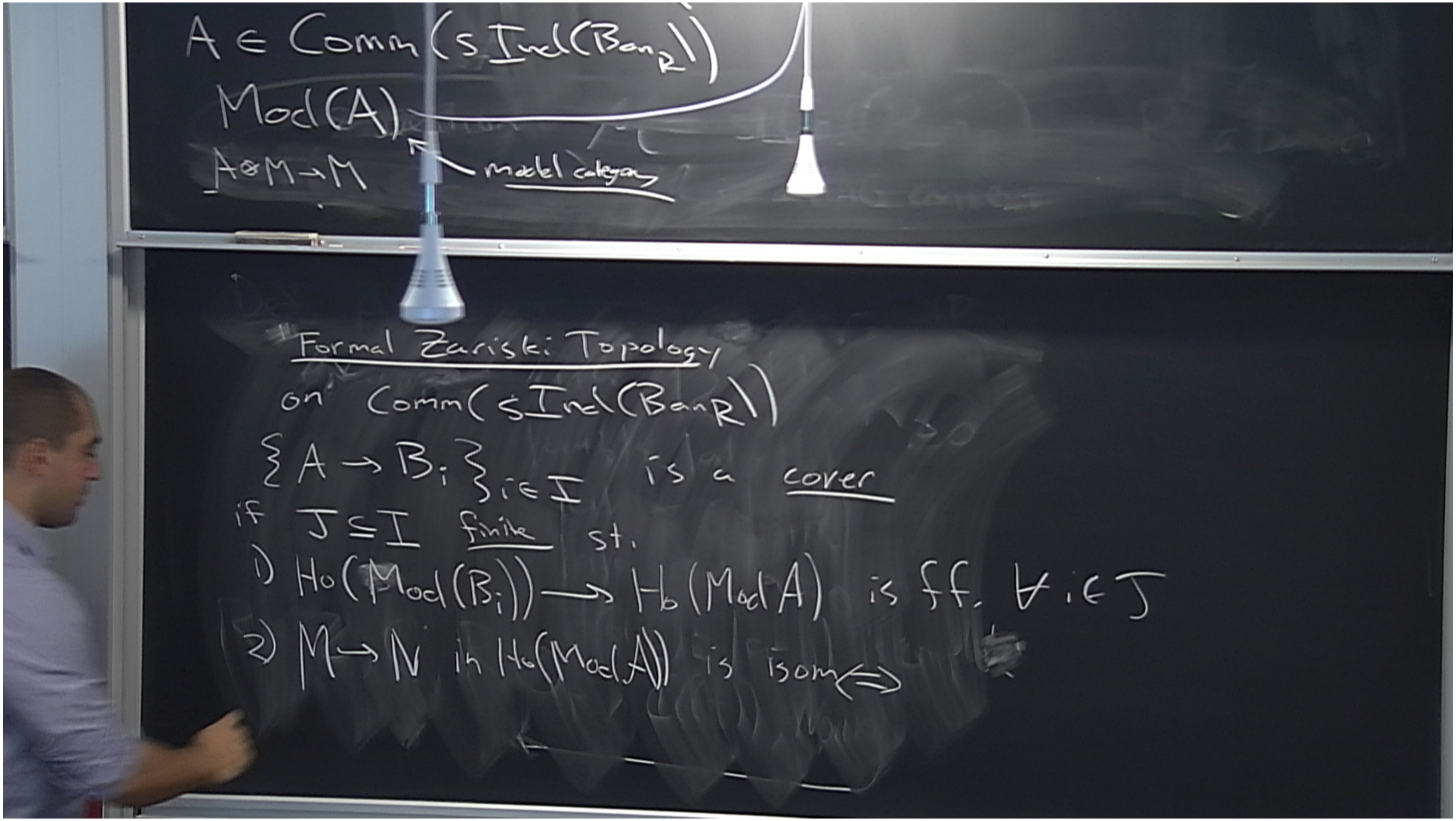
model category

Formal Zariski Topology

on $\mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))$

$\{A \to B_i\}_{i \in I}$ is a cover

if $J \subseteq I$ finite


A ∈ Comm(sInd(Ban_R))
Mod(A)
A⊗M→M
model category
Formal Zariski Topology
on Comm(sInd(Ban_R))
{A → B_i}_{i∈I} is a cover
if J ⊆ I finite st.
1) Ho(Mod(B_i)) → Ho(Mod A) is ff. ∀ i ∈ J
2) M → N in Ho(Mod(A)) is isom ⟺

$A \in \mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))$

$\mathrm{Mod}(A) \leftarrow$ model category

$A \otimes M \to M$

Formal Zariski Topology

on $\mathrm{Comm}(s\mathrm{Ind}(\mathrm{Ban}_R))$

$\{A \to B_i\}_{i \in I}$ is a cover

if $J \subseteq I$ finite s.t.

1) $\mathrm{Ho}(\mathrm{Mod}(B_i)) \longrightarrow \mathrm{Ho}(\mathrm{Mod}\,A)$ is ff. $\forall\, i \in J$

2) $M \to N$ in $\mathrm{Ho}(\mathrm{Mod}(A))$ is isom $\Longleftrightarrow$

$M \bar{\otimes}_A B_i \to N \bar{\otimes}_A B_i$ is isom $\forall\, i \in J$

Formal Zariski Topology

on $\mathrm{Comm}(\underbrace{\leq \mathrm{Ind}(\mathrm{Ban}_{\mathbb{R}})}_{M})$

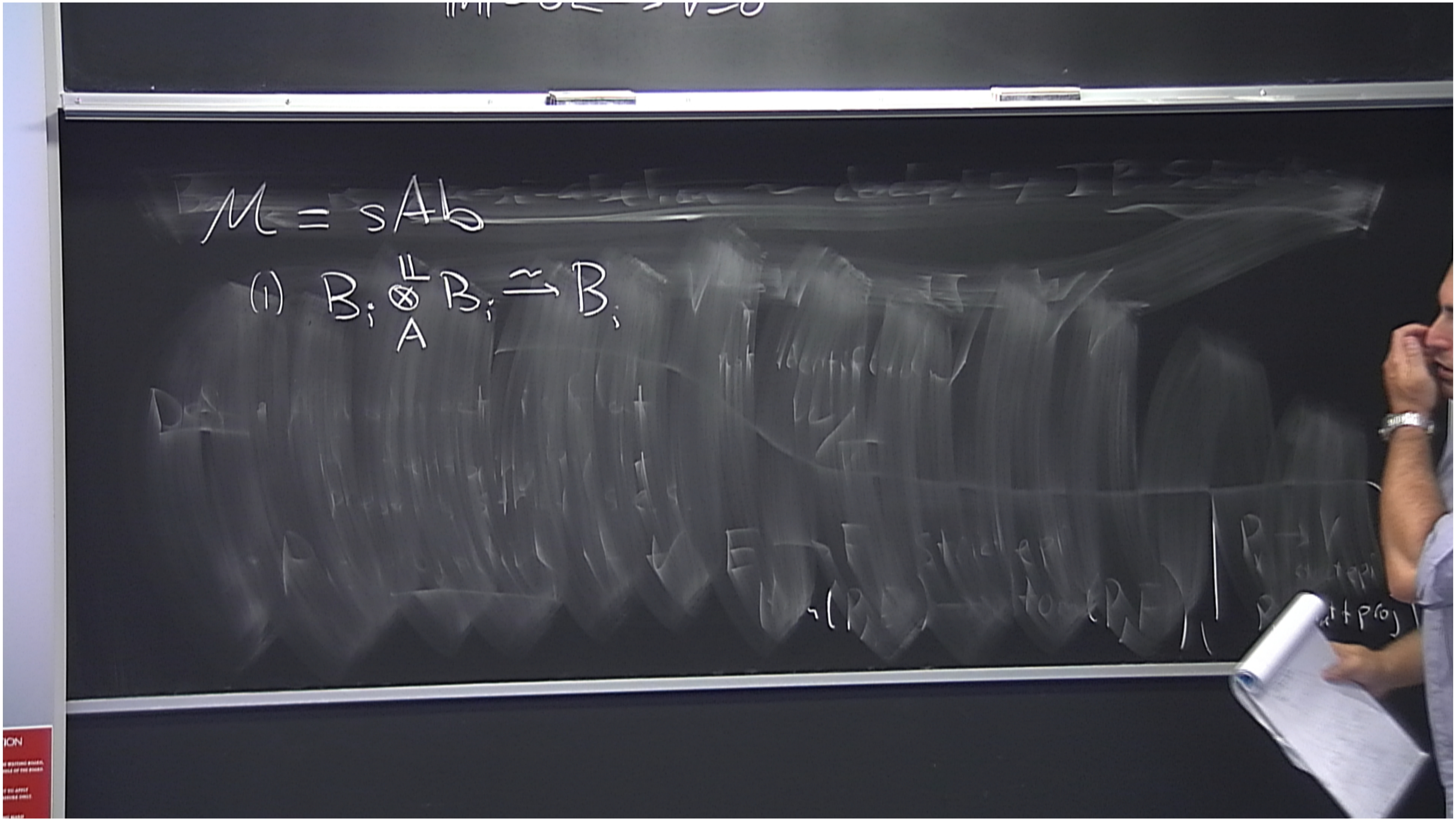
$\{A \to B_i\}_{i \in I}$ is a cover

if $J \subseteq I$ finite st.

1) $\mathrm{Ho}(\mathrm{Mod}(B_i)) \longrightarrow \mathrm{Ho}(\mathrm{Mod}\,A)$ is ff. $\forall i \in J$

2) $M \to N$ in $\mathrm{Ho}(\mathrm{Mod}(A))$ is isom $\Leftrightarrow$

$M \bar{\otimes}_A B_i \to N \bar{\otimes}_A B_i$ is isom $\forall i \in J$


$\mathcal{M} = sAb$
(1) $B_i \overset{\mathbb{L}}{\underset{A}{\otimes}} B_i \xrightarrow{\simeq} B_i$

$\mathcal{M} = sAb$

(1) $B_i \overset{\mathbb{L}}{\underset{A}{\otimes}} B_i \xrightarrow{\simeq} B_i$

homotopy epi $\Longleftarrow$ being a flat epi

P_v flat proj

$\mathcal{M} = sAb$

(1) $B_i \overset{\mathbb{L}}{\underset{A}{\otimes}} B_i \xrightarrow{\sim} B_i$

homotopy epi $\Longleftarrow$ being a flat epi

$\boxed{\begin{array}{l} A \to B_i \text{ is of f.p.} \\ \text{then flat epi} = \text{ho epi} \end{array}}$

$A \to A_p$

$$\mathcal{M} = sAb$$

(1) $B_i \overset{\mathbb{L}}{\underset{A}{\otimes}} B_i \xrightarrow{\sim} B_i$

$$\mathcal{B} \overset{\mathbb{L}}{\underset{A}{\hat{\otimes}}} \mathcal{B} \xrightarrow{\sim} \mathcal{B}$$

homotopy epi $\Longleftarrow$ being a flat epi

homotopy epi $\not\Longrightarrow$ being a flat epi

$$\boxed{\begin{array}{l} A \to B_i \text{ is of f.p.} \\ \text{then flat epi} = \text{ho epi} \end{array}}$$

$$A \to A_p$$

P_v flat + proj

$\mathcal{M} = sAb$

$B \hat{\otimes}^{\mathbb{L}}_{A} B \xrightarrow{\simeq} B$

(1) $B_i \otimes^{\mathbb{L}}_{A} B_i \xrightarrow{\simeq} B_i$

homotopy epi $\Longleftarrow$ being a flat epi

homotopy epi $\not\Longrightarrow$ being a flat epi

$\boxed{\begin{array}{l} A \to B_i \text{ is of f.p.} \\ \text{then flat epi} = \text{ho epi} \\ = \{\operatorname{Spec} B \subseteq \operatorname{Spec} A \ \text{Zar open}\} \end{array}}$

$A \to A_p$

P_v flat proj

$\mathcal{M} = sAb$

$B \hat{\otimes}^{\mathbb{L}}_{A} B \xrightarrow{\simeq} B$

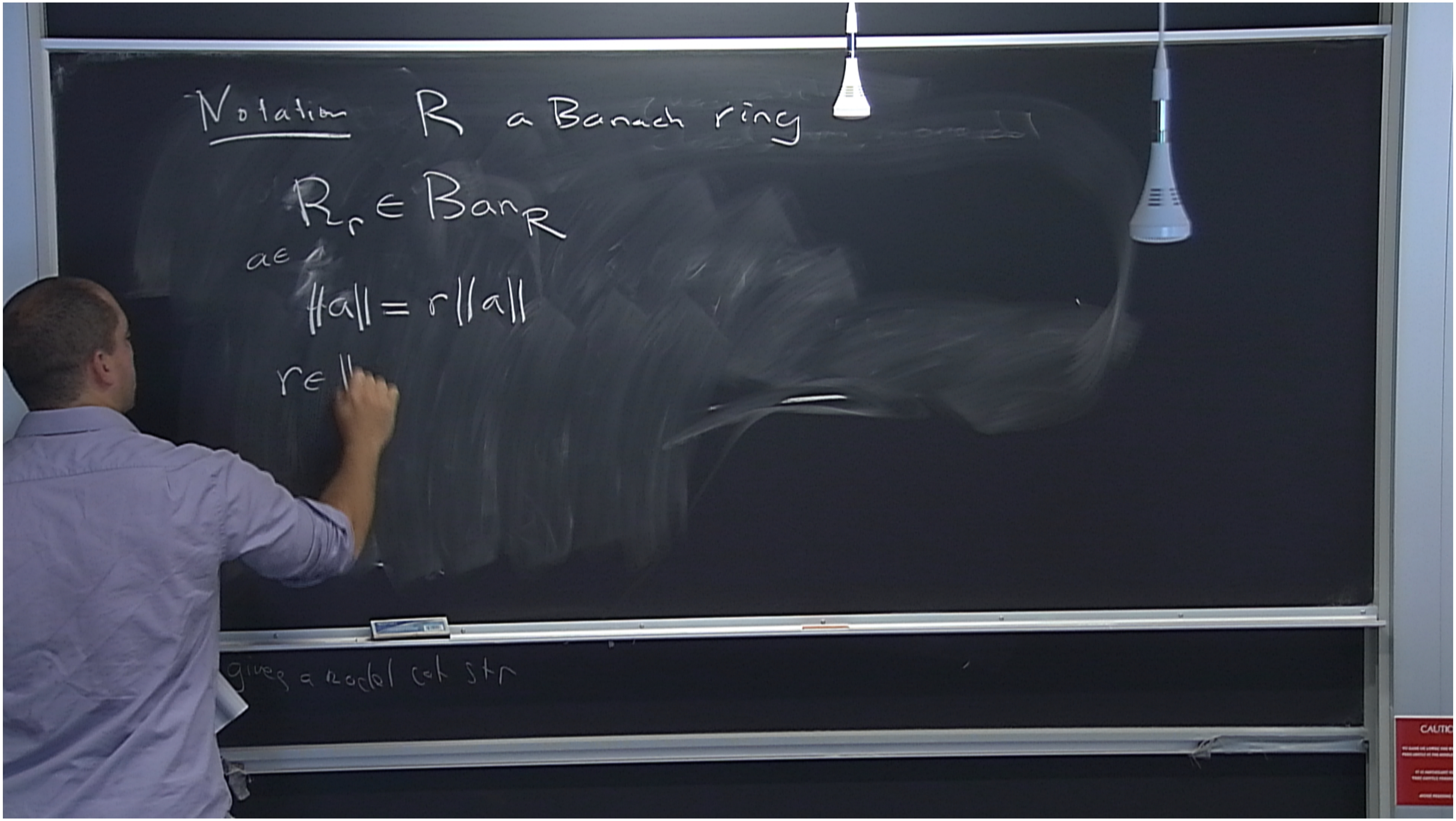
(1) $B_i \otimes^{\mathbb{L}}_{A} B_i \xrightarrow{\simeq} B_i$

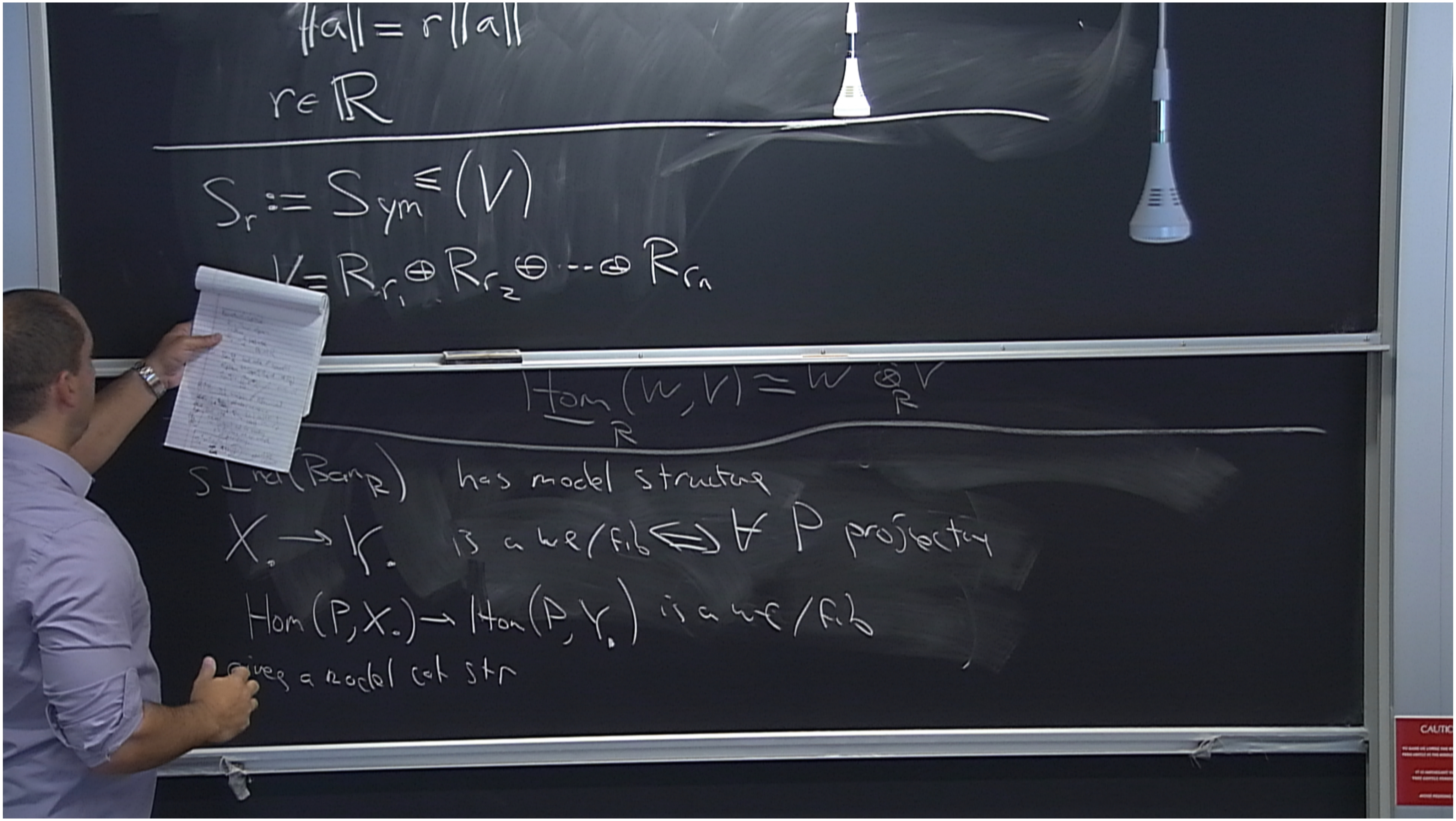
homotopy epi $\Longleftarrow$ being a flat epi

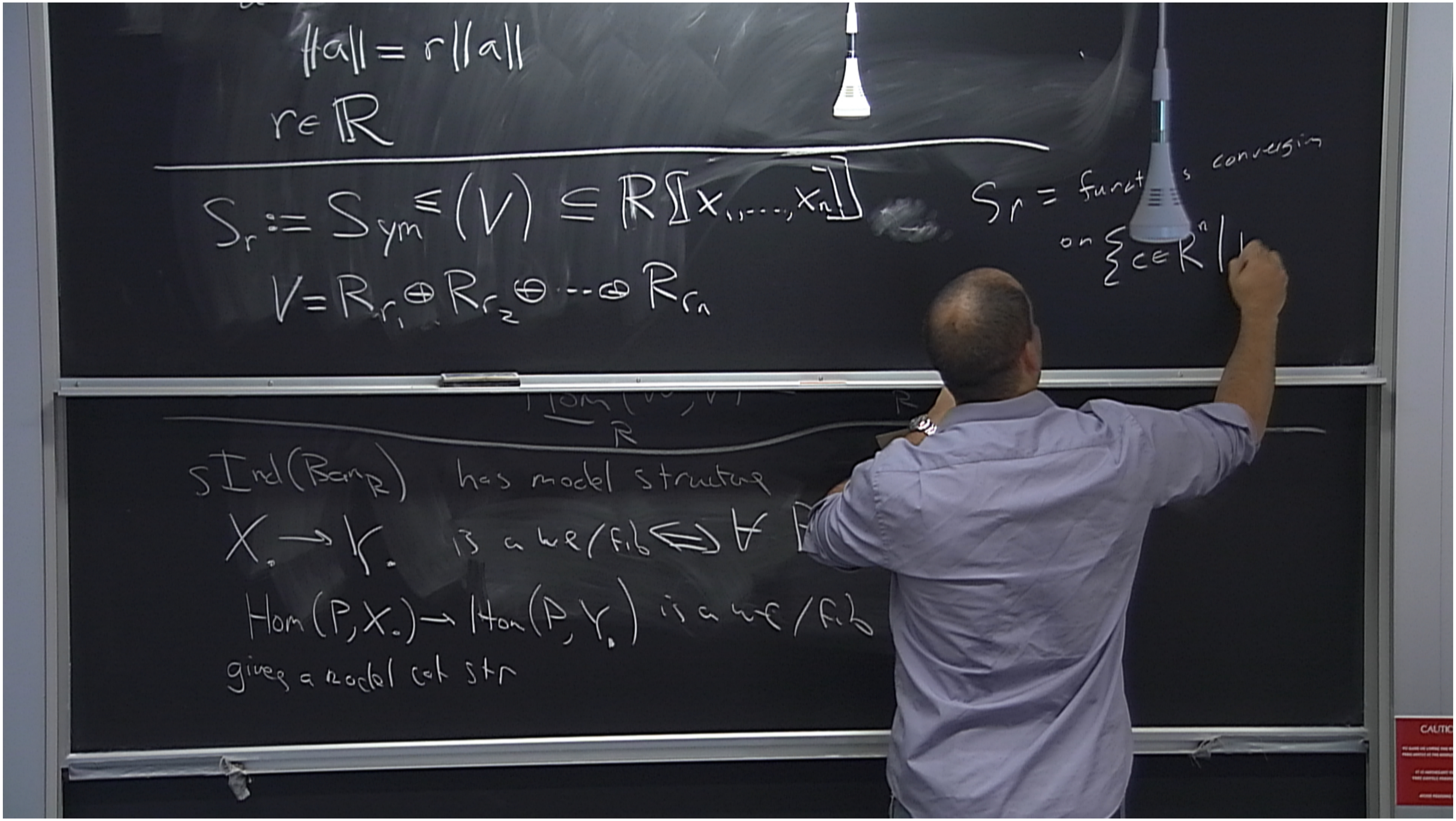
$\boxed{\begin{array}{l} A \to B_i \text{ is of f.p.} \\ \text{then flat epi} = \text{ho epi} \\ = \{ \operatorname{Spec} B \subseteq \operatorname{Spec} A \text{ Zar open} \} \end{array}}$

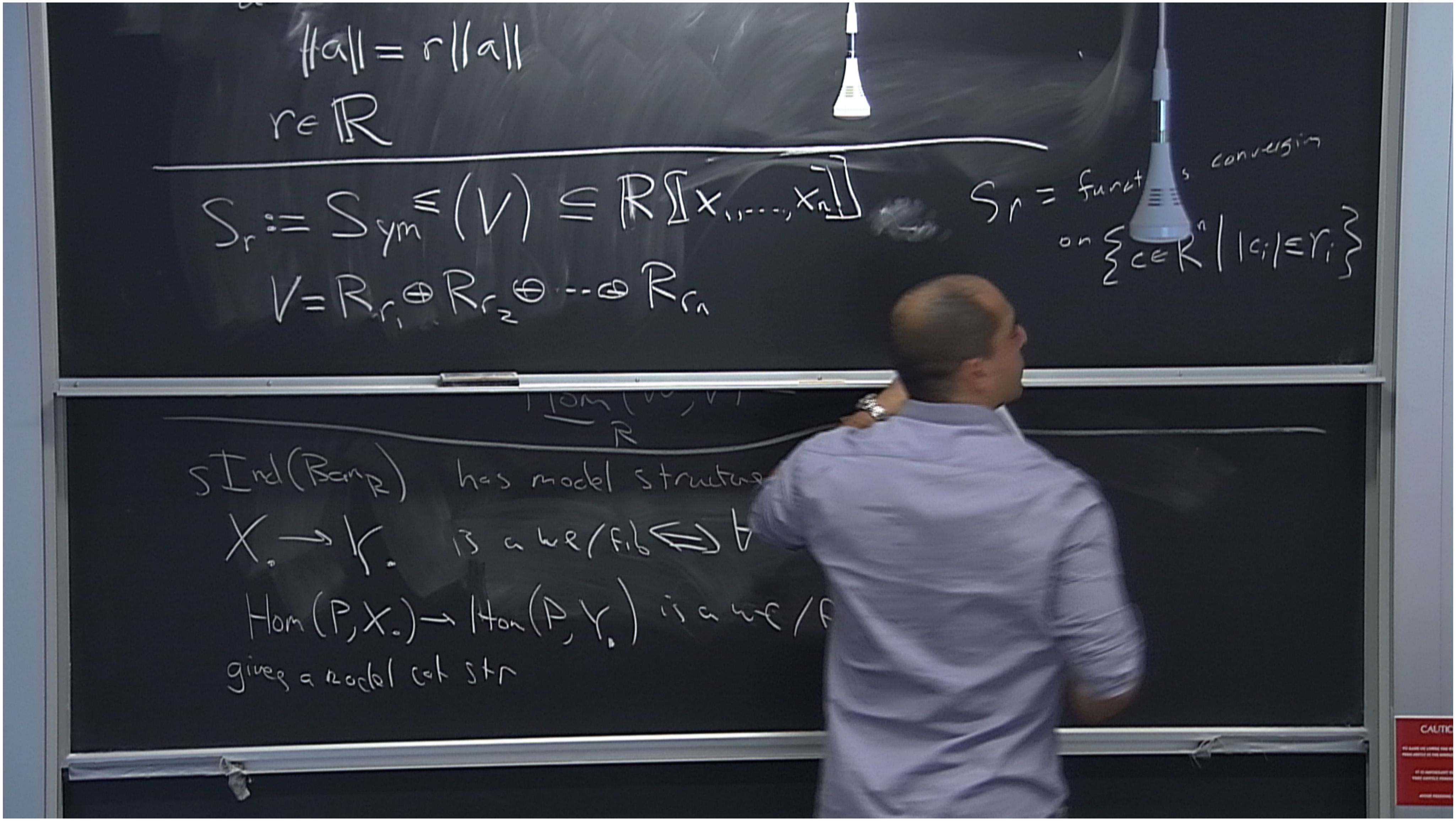
$A \to A_p$

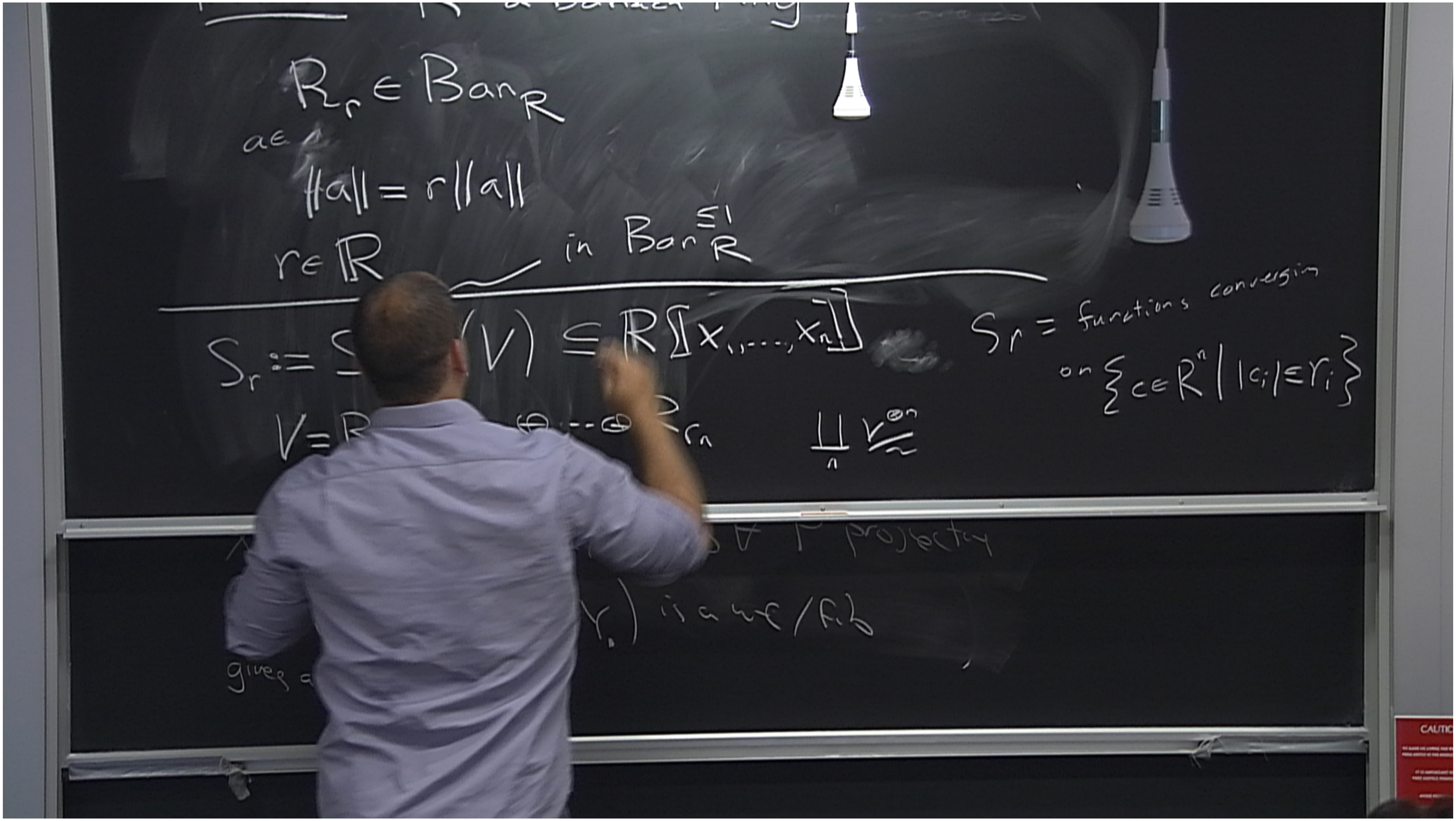
P_v flat + proj


Notation R a Banach ring
R_r ∈ Ban_R
a ∈
||a|| = r||a||
r ∈
gives a model cat str


||a|| = r||a||
r ∈ ℝ
S_r := Sym^≤(V)
V = R_{r_1} ⊕ R_{r_2} ⊕ ··· ⊕ R_{r_n}
Hom_R(W, V) ≃ W ⊗_R V
sInd(Ban_R) has model structure
X. → Y. is a we/fib ⟺ ∀ P projective
Hom(P, X.) → Hom(P, Y.) is a we/fib
gives a model cat str


$\|ra\| = r\|a\|$
$r \in \mathbb{R}$
$S_r := \mathrm{Sym}^{\leq}(V) \subseteq \mathbb{R}[[x_1, \dots, x_n]]$
$V = \mathbb{R}_{r_1} \oplus \mathbb{R}_{r_2} \oplus \cdots \oplus \mathbb{R}_{r_n}$
S_r = funct s convergin
on $\{c \in \mathbb{R}^n \mid$
sInd(Ban$_{\mathbb{R}}$) has model structure
$X_\bullet \to Y_\bullet$ is a we/fib $\Leftrightarrow \forall$
Hom$(P, X_\bullet) \to$ Hom$(P, Y_\bullet)$ is a we/fib
gives a model cat str


‖a‖ = r‖a‖
r ∈ ℝ
S_r := Sym^≤(V) ⊆ ℝ[[x_1, ..., x_n]]
V = ℝ_{r_1} ⊕ ℝ_{r_2} ⊕ ··· ⊕ ℝ_{r_n}
S_r = functions converging
on {c ∈ ℝ^n | |c_i| ≤ r_i}
sInd(Ban_ℝ) has model structure
X. → Y. is a we/fib ⟺ ∀
Hom(P, X.) → Hom(P, Y.) is a we/f
gives a model cat str

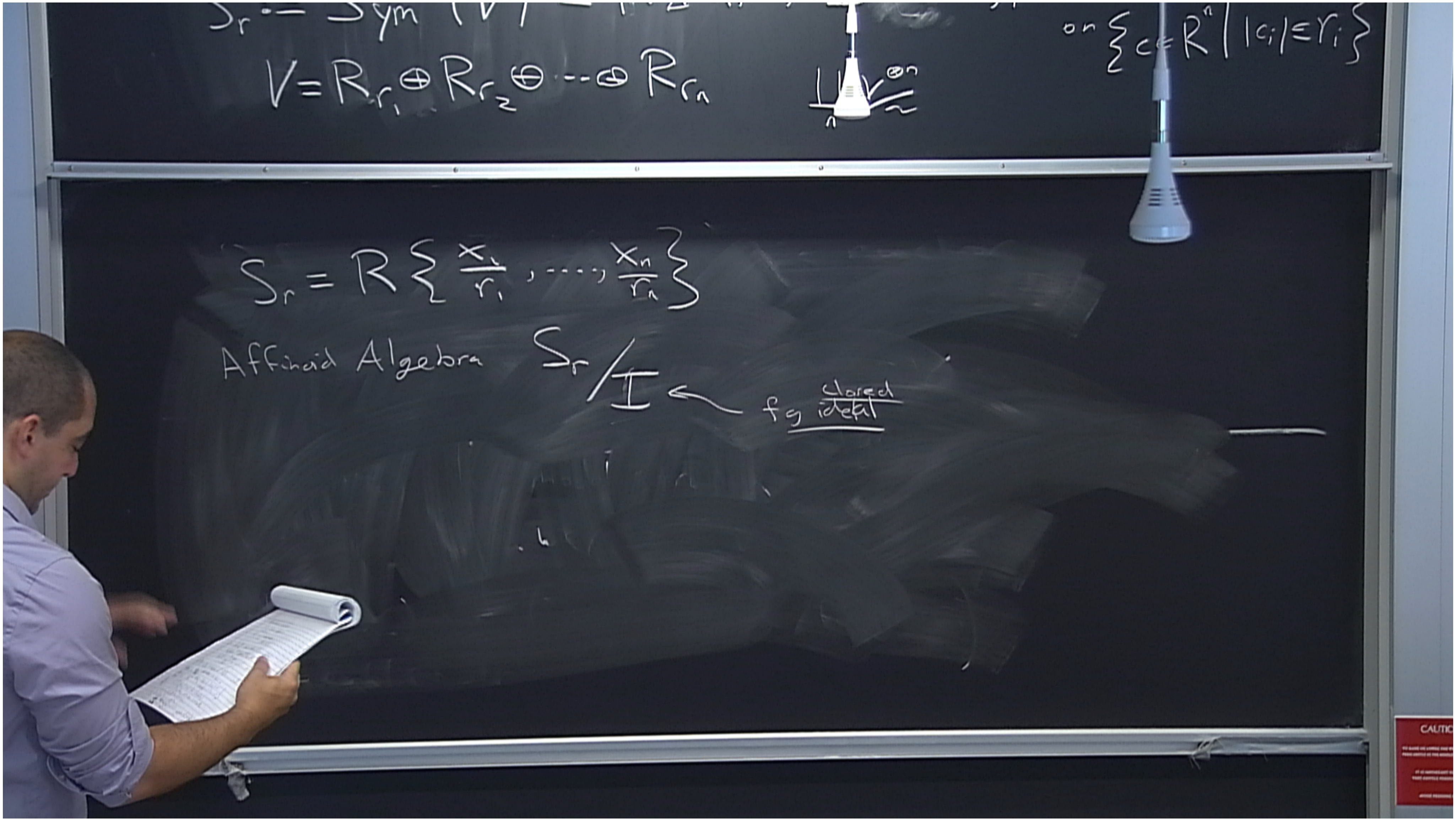

R_r ∈ Ban_R
a ∈
||a|| = r||a||
r ∈ ℝ
in Ban_R^{≤1}
S_r = functions convergin
on {c ∈ ℝ^n | |c_i| ≤ r_i}
projectiv
is a we/fib
give a

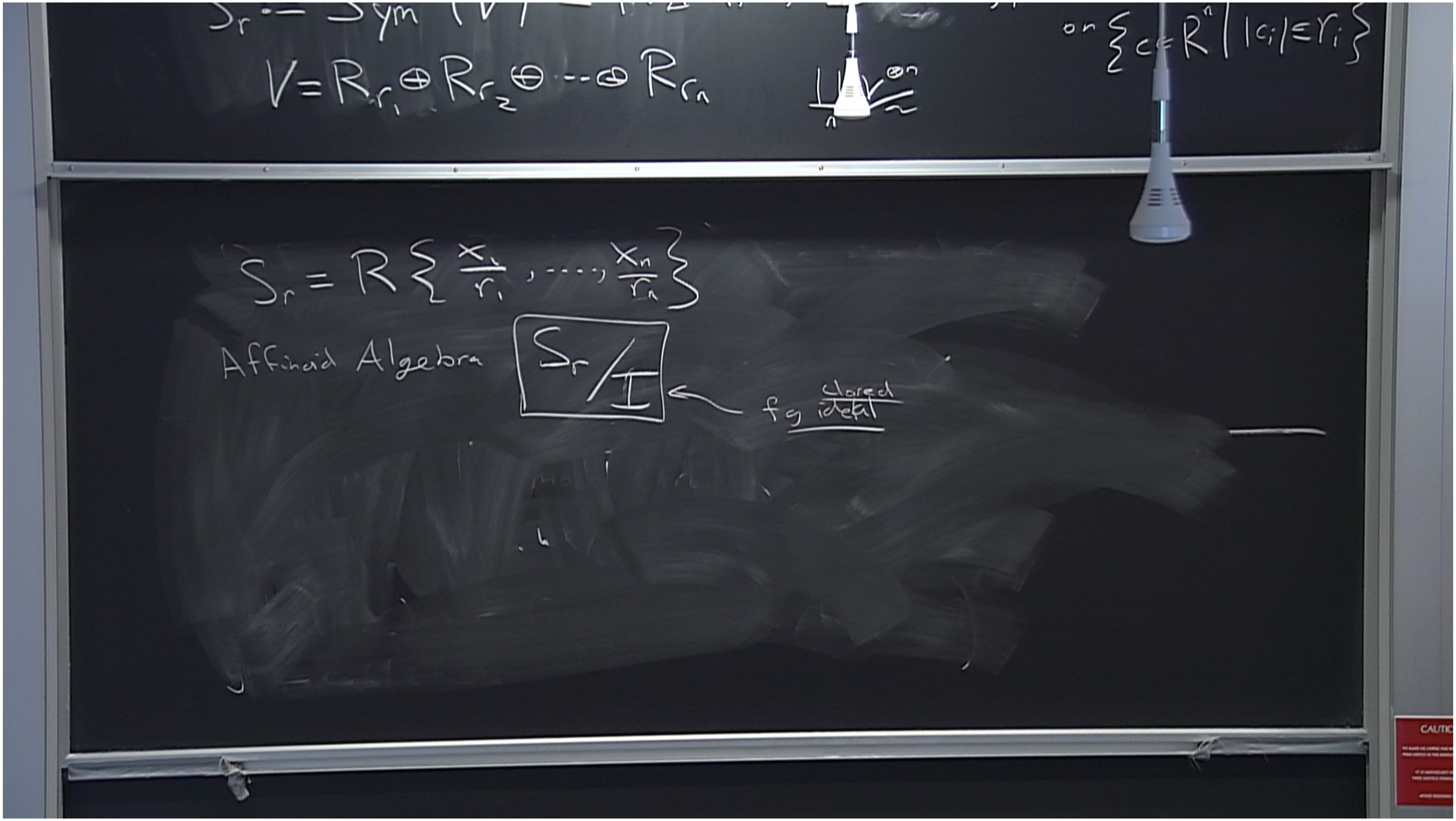
$S_r := \mathrm{Sym}(V)$ ... on $\{c \in \mathbb{R}^n \mid |c_i| \in r_i\}$

$V = \mathbb{R}_{r_1} \oplus \mathbb{R}_{r_2} \oplus \cdots \oplus \mathbb{R}_{r_n}$

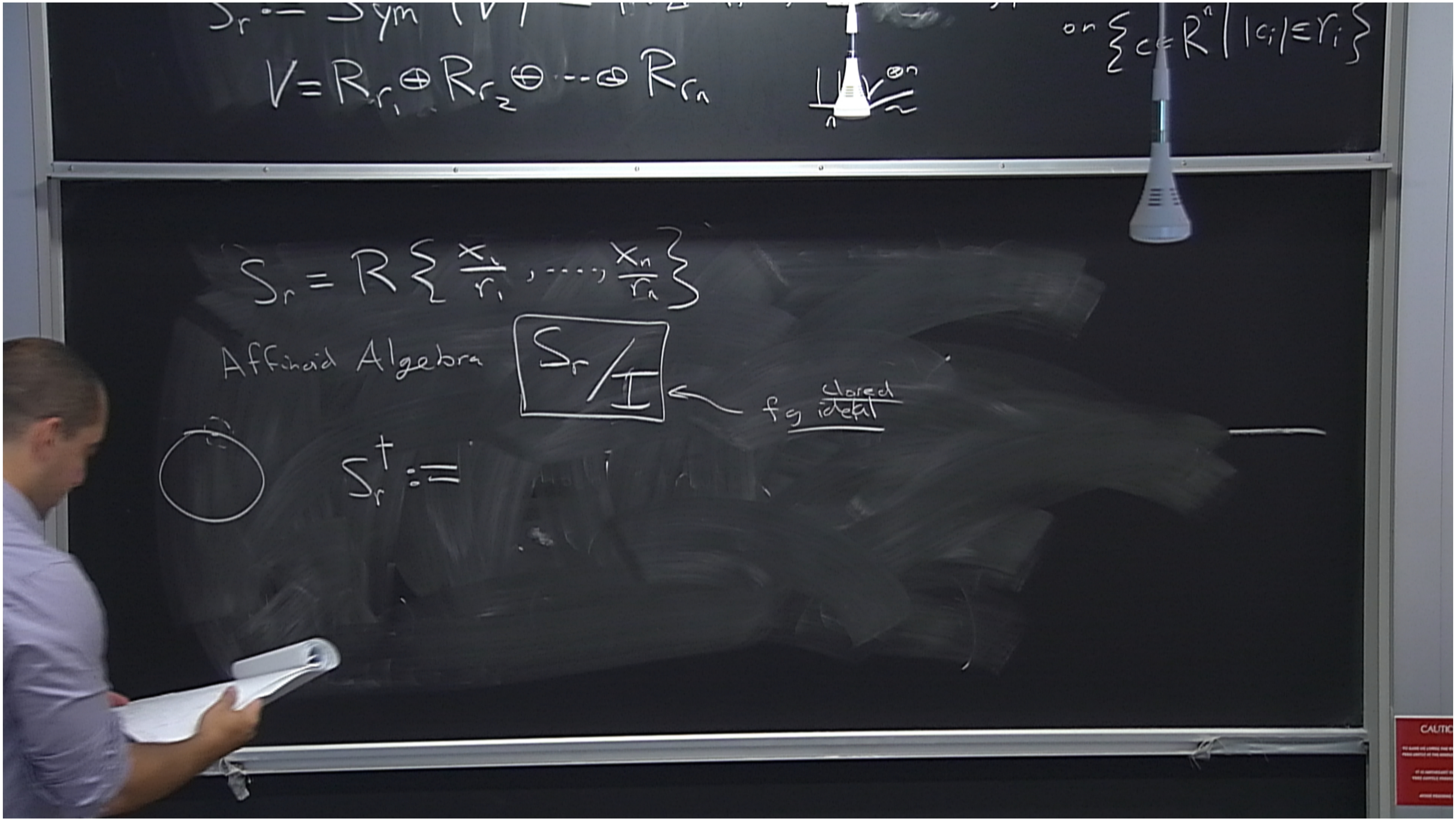
$\coprod_n V^{\otimes n}$

$S_r = \mathbb{R}\left\{\frac{x_1}{r_1}, \ldots, \frac{x_n}{r_n}\right\}$


$V = R_{r_1} \oplus R_{r_2} \oplus \cdots \oplus R_{r_n}$
on $\{ c \in \mathbb{R}^n \mid |c_i| \leq r_i \}$
$S_r = R\left\{ \frac{x_1}{r_1}, \ldots, \frac{x_n}{r_n} \right\}$
Affinoid Algebra S_r / I
fg closed ideal



on {c ∈ R^n | |c_i| ≤ r_i}
V = R_{r_1} ⊕ R_{r_2} ⊕ ··· ⊕ R_{r_n}
S_r = R{x_1/r_1, ..., x_n/r_n}
Affinoid Algebra
S_r/I
fg closed ideal


V=R_r1 ⊕ R_r2 ⊕ ··· ⊕ R_rn
on {c∈R^n | |c_i| ≤ r_i}
S_r = R{x_1/r_1, ..., x_n/r_n}
Affinoid Algebra
S_r/I
fg closed ideal
S_r^† :=

$S_r := \mathrm{Sym}(V) \ldots$ on $\{c \in R^n \mid |c_i| \le r_i\}$

$V = R_{r_1} \oplus R_{r_2} \oplus \cdots \oplus R_{r_n}$

$S_r = R\left\{\frac{x_1}{r_1}, \ldots, \frac{x_n}{r_n}\right\}$

Affinoid Algebra $\boxed{S_r / I}$ ← f.g. closed ideal

$S_r^\dagger :=$ "colim"

colim $\mathrm{Ind}(\mathrm{Ban}_R)$

$S_r := \mathrm{Sym}(V^\vee)$ ... on $\{c \in \mathbb{R}^n \mid |c_i| \le r_i\}$

$V = \mathbb{R}_{r_1} \oplus \mathbb{R}_{r_2} \oplus \cdots \oplus \mathbb{R}_{r_n}$ $\quad \coprod_n V^{\otimes n} / \sim$

$$S_r = \mathbb{R}\left\{\frac{x_1}{r_1}, \dots, \frac{x_n}{r_n}\right\}$$

Affinoid Algebra $\boxed{S_r / I}$ ← f.g. closed ideal

$$S_r^\dagger := \text{"}\mathrm{colim}_n\text{"}\, S_{r+\frac{1}{n}}$$

$\underline{\mathrm{colim}}$ $\mathrm{Ind}(\mathrm{Ban}_R)$

$S_r := \mathrm{Sym}(V) \ldots$ on $\{c \in \mathbb{R}^n \mid |c_i| \leq r_i\}$

$V = \mathbb{R}_{r_1} \oplus \mathbb{R}_{r_2} \oplus \cdots \oplus \mathbb{R}_{r_n}$

$$S_r = \mathbb{R}\left\{\frac{x_1}{r_1}, \ldots, \frac{x_n}{r_n}\right\}$$

Affinoid Algebra $\boxed{S_r / I}$ f.g. closed ideal

$$S_r^\dagger := \text{"}\mathrm{colim}_n\text{"}\, S_{r+\frac{1}{n}}$$

colim $\mathrm{Ind}(\mathrm{Ban}_{\mathbb{R}})$

$S_{r_1} \to S_{r_2}$ is nuclear

$r_1 > r_2$

$S_r := \mathrm{Sym}(V)$ ... on $\{c \in \mathbb{R}^n \mid |c_i| \leq r_i\}$

$V = R_{r_1} \oplus R_{r_2} \oplus \cdots \oplus R_{r_n}$

$$S_r = R\left\{ \frac{x_1}{r_1}, \ldots, \frac{x_n}{r_n} \right\}$$

Affinoid Algebra $\boxed{S_r / I}$ ← f.g. closed ideal

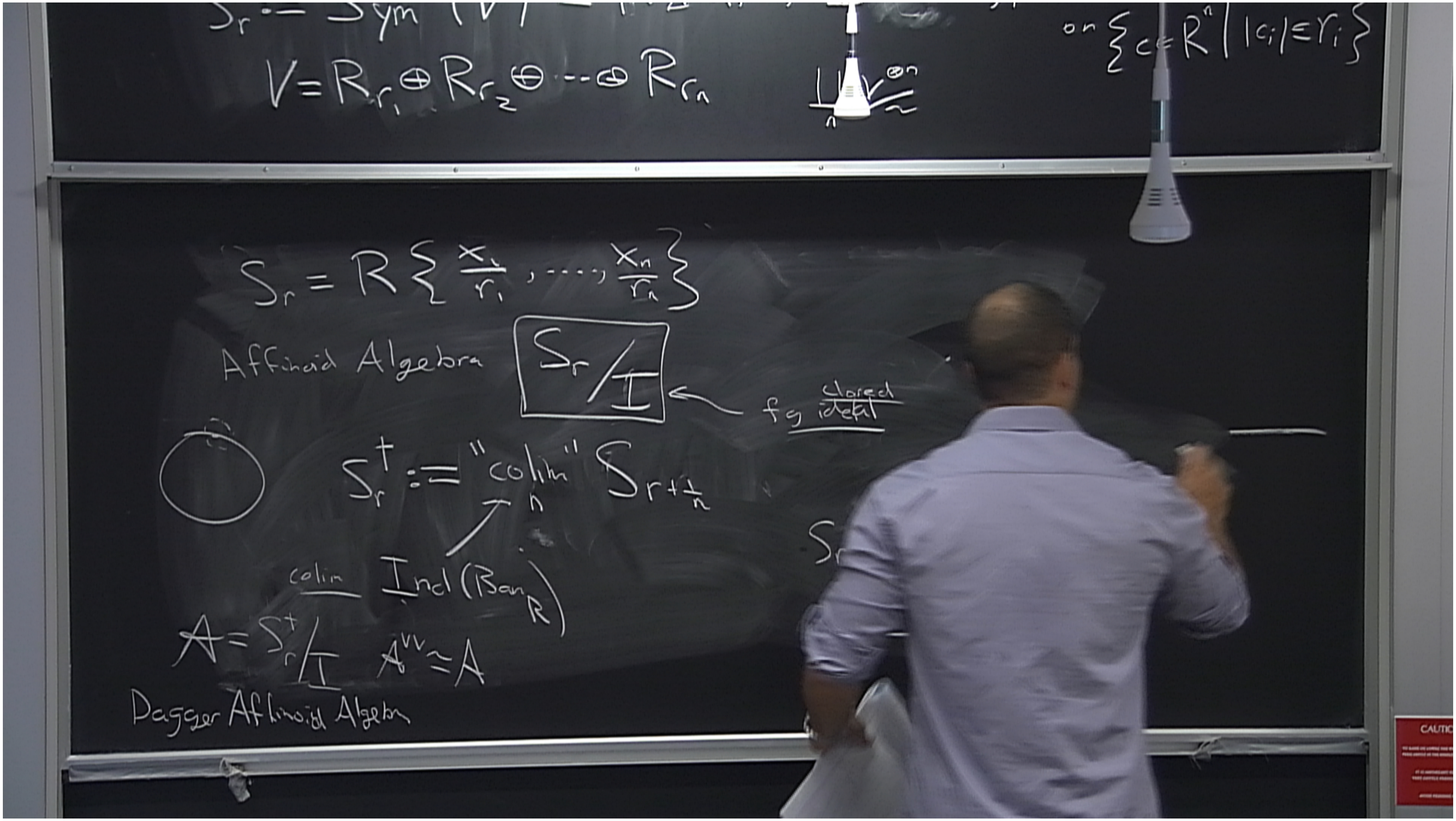
$$S_r^{\dagger} := \text{"}\underset{n}{\mathrm{colim}}\text{"}\, S_{r+\frac{1}{n}}$$

colim Ind(Ban_R)

$A = S_r^{\dagger}/I \quad A^{\vee\vee} \simeq A$

$S_{r_1} \to S_{r_2}$ is nuclear

$r_1 > r_2$


V=R_{r_1} ⊕ R_{r_2} ⊕ ··· ⊕ R_{r_n}
on {c ∈ R^n | |c_i| ≤ r_i}
S_r = R{x_1/r_1, ..., x_n/r_n}
Affinoid Algebra
S_r/I
fg closed ideal
S_r^† := "colim" S_{r+1/n}
colim Ind(Ban_R)
A = S_r^†/I
A^{vv} ≃ A
Dagger Affinoid Algebra

$S_r := \mathrm{Sym}(V) \ldots$ on $\{c \in \mathbb{R}^n \mid |c_i| \leq r_i\}$

$V = R_{r_1} \oplus R_{r_2} \oplus \cdots \oplus R_{r_n}$ $\quad \coprod_n V^{\otimes n}$

$S_r = R\left\{\frac{x_1}{r_1}, \ldots, \frac{x_n}{r_n}\right\}$

Affinoid Algebra $\boxed{S_r / I}$ ← f.g. closed ideal

$S_r^\dagger := \text{"colim"}_n \, S_{r+\frac{1}{n}}$

colim $\mathrm{Ind}(\mathrm{Ban}_R)$

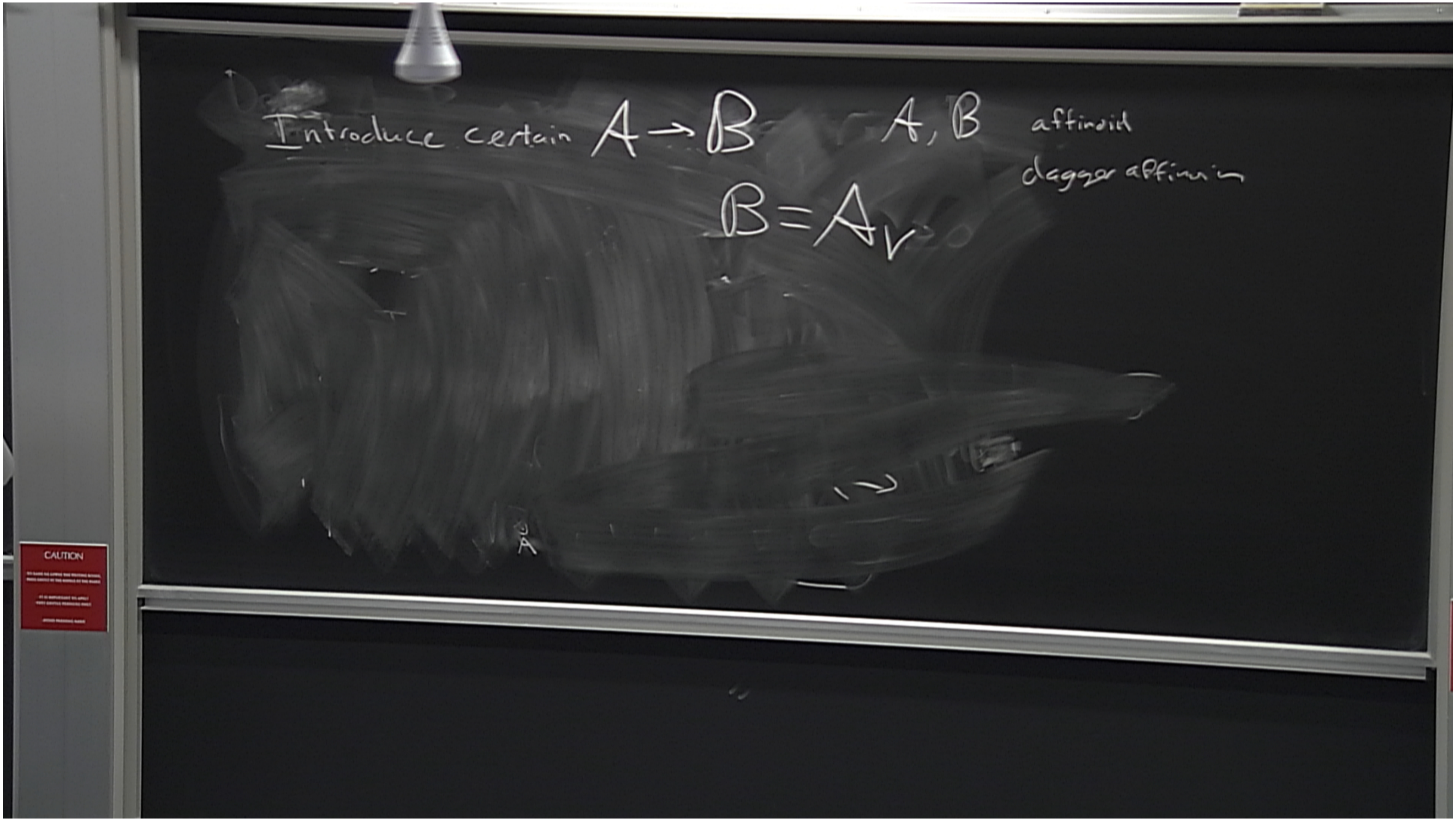
$A = S_r^\dagger / I \quad A^{\vee\vee} \simeq A$

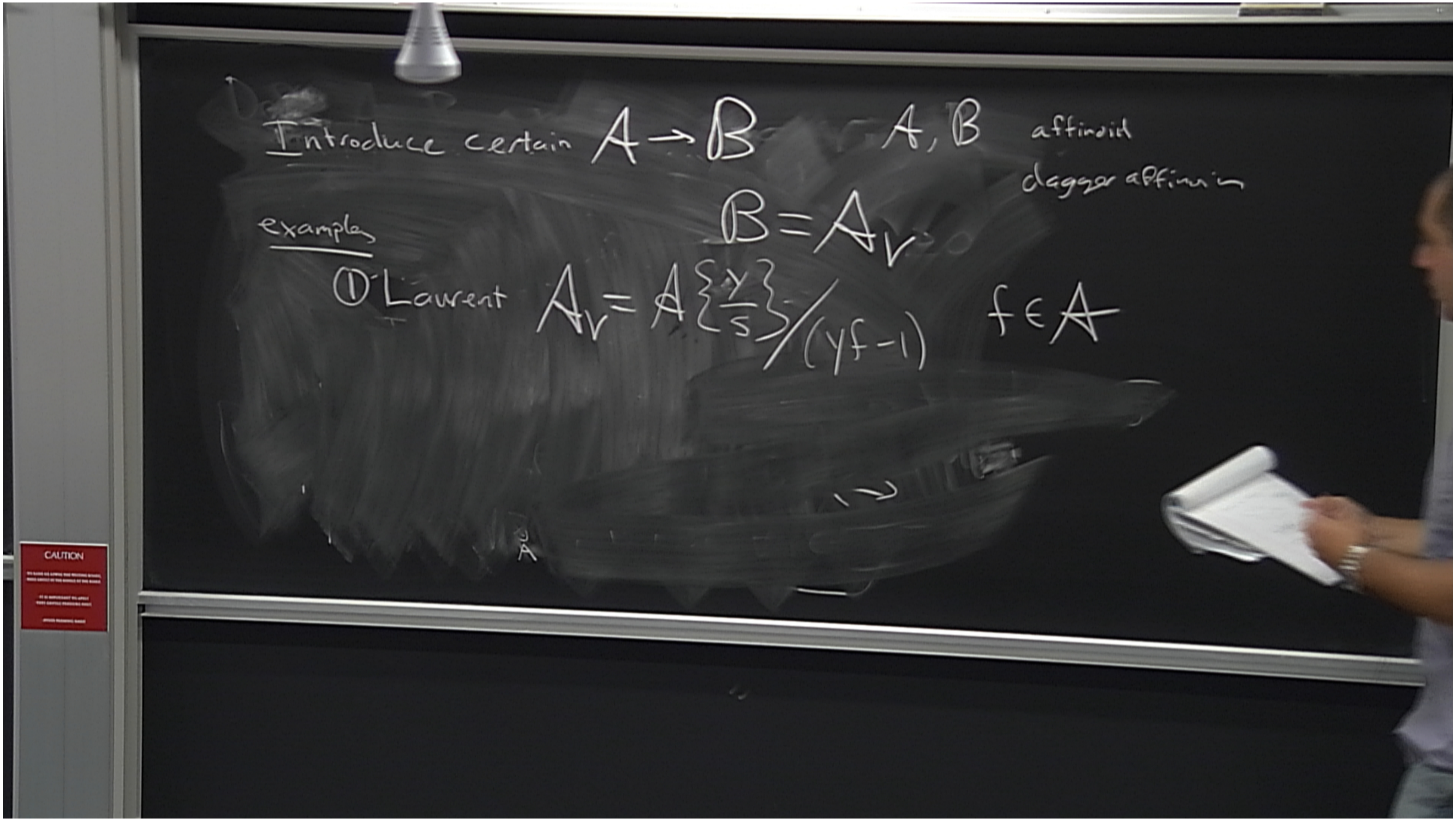
...ger Affinoid Algebra

two full cats of $\mathrm{Comm}(\mathrm{Ind}(\mathrm{Ban}_R))$

$S_{r_1} \to S_{r_2}$ is nuclear

$r_1 > r_2$


Introduce certain A → B
A, B affinoid
dagger affinoid
B = A_V


Introduce certain A → B
A, B affinoid
dagger affinoid
B = A_V
examples
① Laurent A_V = A{y/s}/(yf−1)
f ∈ A

Introduce certain $\mathbb{A} \to \mathbb{B}$ $\mathbb{A}, \mathbb{B}$ affinoid

dagger affinoid

$\mathbb{B} = \mathbb{A}_V$

examples

① Laurent $\mathbb{A}_V = \mathbb{A}\left\{\frac{y}{s}\right\} / (yf-1)$ $f \in \mathbb{A}$

$V \subseteq M(\mathbb{A})$

Introduce certain $\mathcal{A} \to \mathcal{B}$ $\mathcal{A}, \mathcal{B}$ affinoid

dagger affinoid

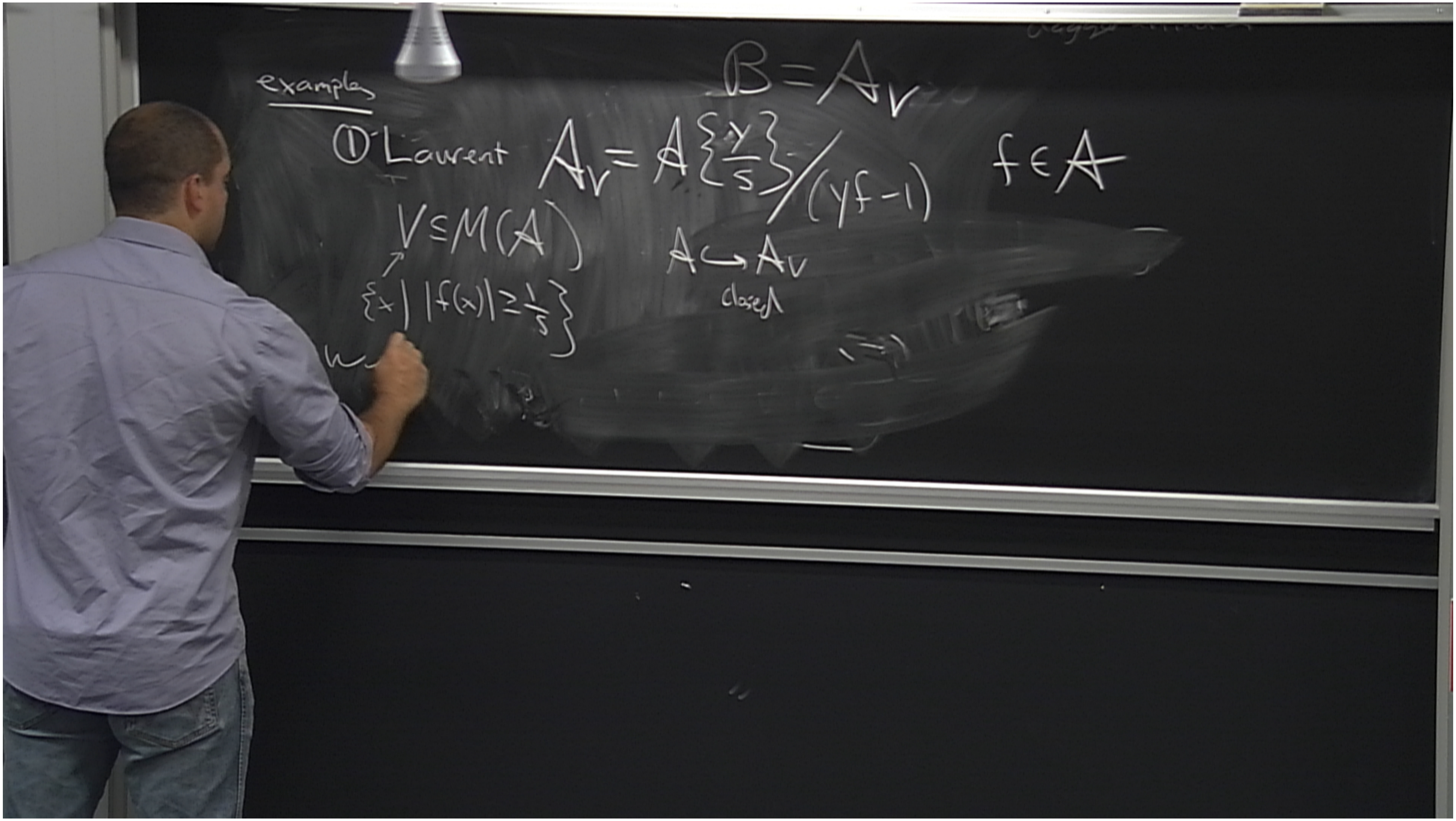
$\mathcal{B} = \mathcal{A}_V$

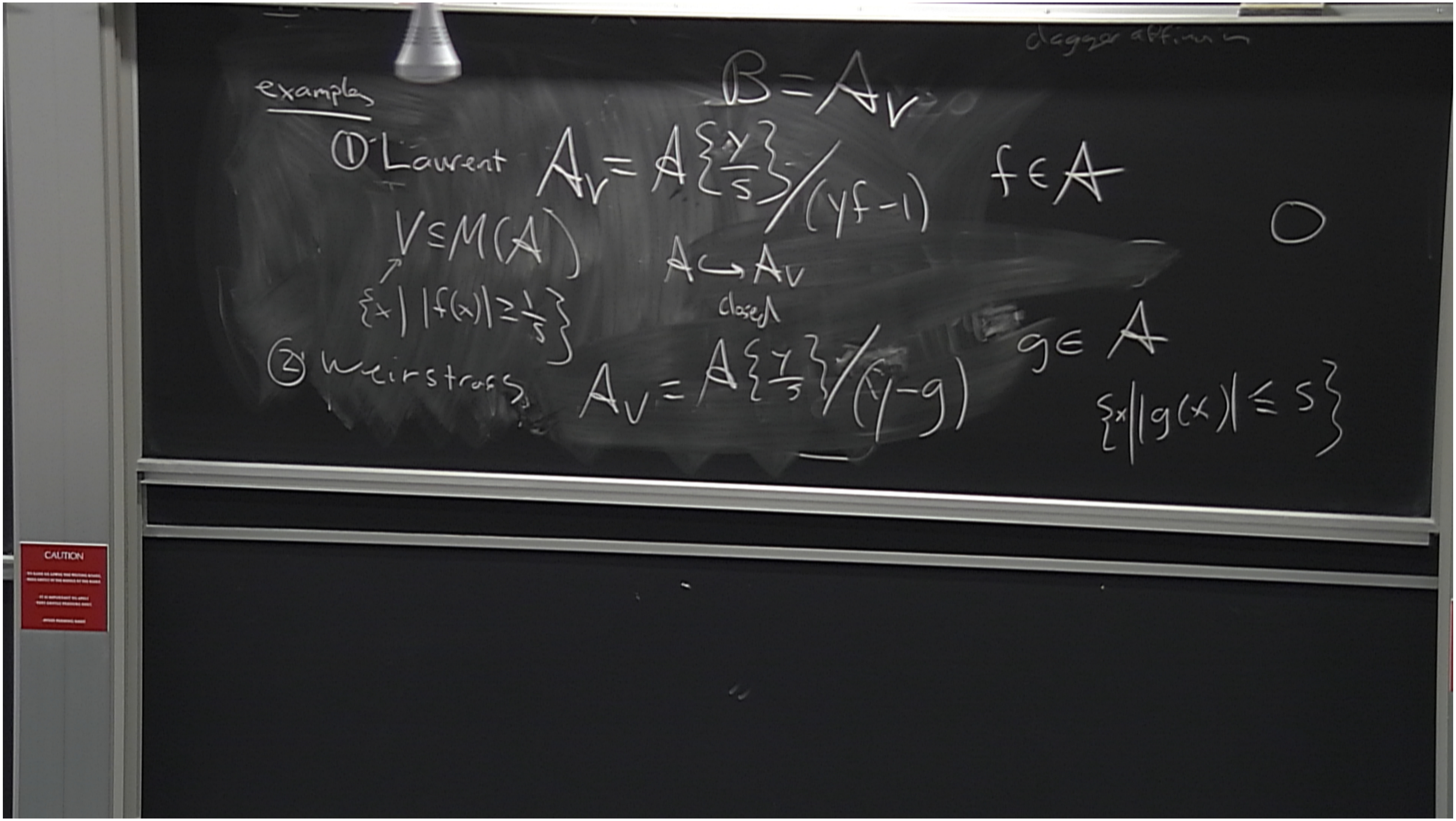
examples

① Laurent $\mathcal{A}_V = \mathcal{A}\{\frac{y}{s}\}/(yf-1)$ $f \in \mathcal{A}$

$V \subseteq M(\mathcal{A})$

$\{x \mid |f(x)| \geq \frac{1}{s}\}$


examples
① Laurent
$A_V = A\{\frac{y}{s}\}/(yf-1)$
$f \in A$
$B = A_V$
$V \subseteq M(A)$
$\{x \mid |f(x)| \geq \frac{1}{s}\}$
$A \hookrightarrow A_V$
closed



dagger affin
examples
$\mathcal{B} = A_V$
① Laurent $A_V = A\left\{\frac{y}{s}\right\} / (yf-1)$
$f \in A$
$V \subseteq M(A)$
$A \hookrightarrow A_V$
$\{x \mid |f(x)| \geq \frac{1}{s}\}$
closed
② Weirstrass $A_V = A\left\{\frac{y}{s}\right\} / (y-g)$
$g \in A$
$\{x \mid |g(x)| \leq s\}$
CAUTION

dagger affin

$\mathcal{B} = A_V$

examples

① Laurent $A_V = A\left\{\frac{y}{s}\right\}/(yf-1)$ $\quad f \in A$

$V \subseteq M(A)$

$\{x \mid |f(x)| \geq \frac{1}{s}\}$

$A \hookrightarrow A_V$ closed

② Weierstrass $A_V = A\left\{\frac{y}{s}\right\}/(y-g)$ $\quad g \in A$

$\{x \mid |g(x)| \leq s\}$

nuclear $A \hookrightarrow A_V$ dense

$f: V \to W \quad \operatorname{cok}(\ker f \to V) \simeq \ker(W \to \operatorname{cok} f)$

may not hold, f is strict if it holds

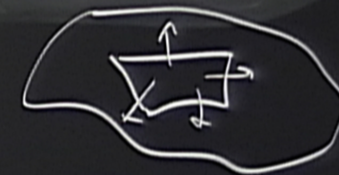
Def A Stein algebra

$f: V \to W \quad \mathrm{cok}(\ker f \to V) \simeq \ker(W \to \mathrm{cok} f)$

may not hold, f is strict if it holds

Def A Stein algebra in $\mathrm{Comm}(\mathrm{Ind}(\mathrm{Ban}_R))$ is a filtered limit

$$A = \lim(\cdots \to A_3 \to A_2 \to A_1)$$



$f: V \to W$ $\quad \mathrm{cok}(\ker f \to V) \simeq \ker(W \to \mathrm{cok} f)$

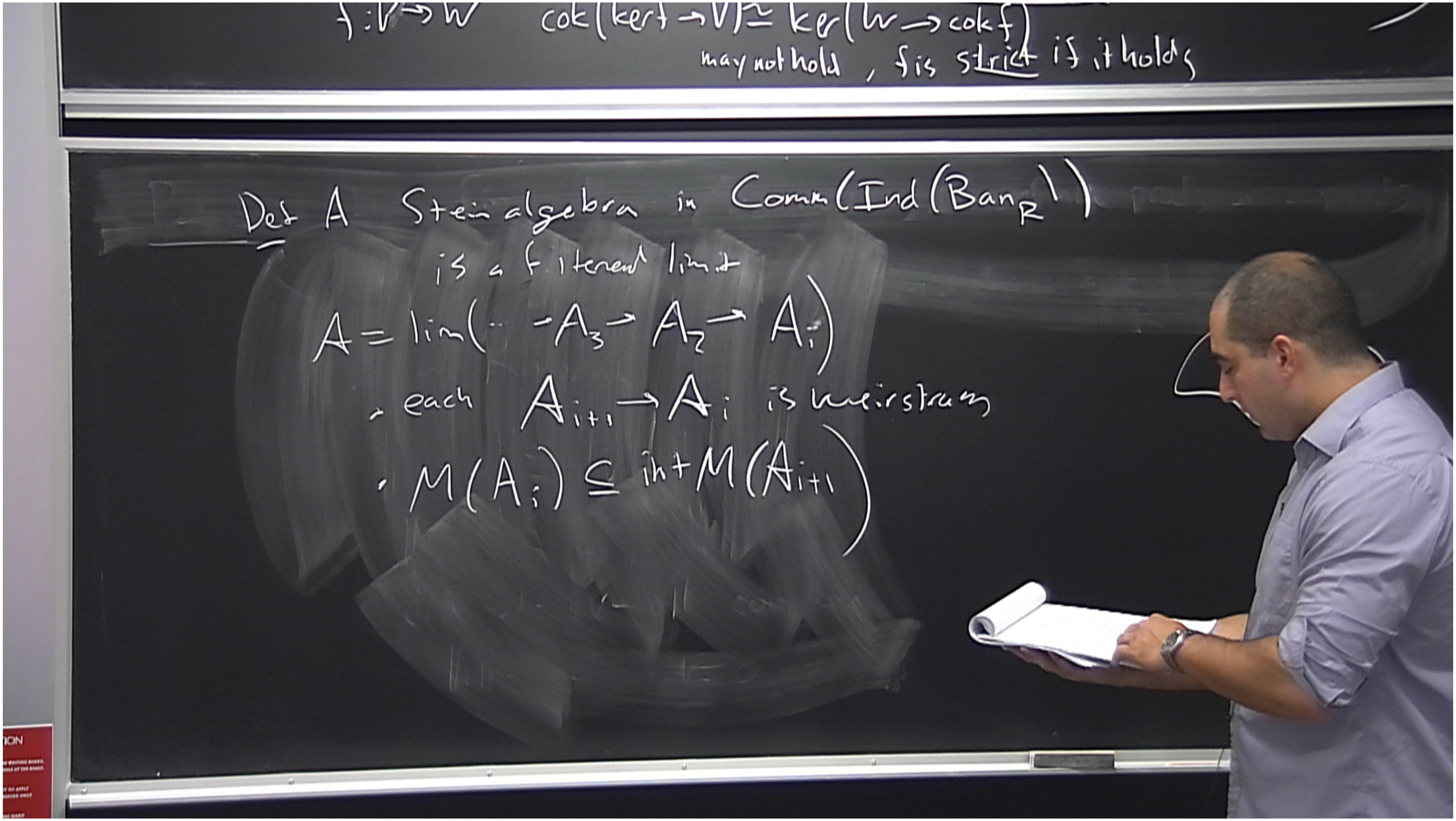
may not hold, f is <u>strict</u> if it holds

<u>Def</u> A Stein algebra in $\mathrm{Comm}(\mathrm{Ind}(\mathrm{Ban}_R))$

is a filtered limit

$$A = \lim(\cdots \to A_3 \to A_2 \to A_1)$$

each $A_{i+1} \to A_i$ is weierstrass


f: V → W   cok(ker f → V) ≅ ker(W → cok f)
may not hold, f is strict if it holds
Def A Stein algebra in Comm(Ind(Ban_R))
is a filtered limit
A = lim(··· → A_3 → A_2 → A_1)
· each A_{i+1} → A_i is weierstrass
· M(A_i) ⊆ int M(A_{i+1})

$f: V \to W \quad \mathrm{cok}(\ker f \to V) \simeq \ker(W \to \mathrm{cok} f)$

may not hold, f is strict if it holds

Def A Stein algebra in $\mathrm{Comm}(\mathrm{Ind}(\mathrm{Ban}_R))$

is a filtered limit

$$A = \lim(\cdots \to A_3 \to A_2 \to A_1)$$

- each $A_{i+1} \to A_i$ is Weierstrass
- $M(A_i) \subseteq \mathrm{int}\, M(A_{i+1})$

A is reflexive and nuclear

exam

is a filtered limit

$$A = \lim(\cdots \to A_3 \to A_2 \to A_1)$$

- each $A_{i+1} \to A_i$ is weierstrass
- $M(A_i) \subseteq \operatorname{int} M(A_{i+1})$

A is reflexive and nuclear

example $R\{\frac{x}{r}\}^{\dagger}$ is dual to $\lim\limits_{s = \frac{1}{r} - \frac{1}{n}} R\{\frac{x}{s}\}$

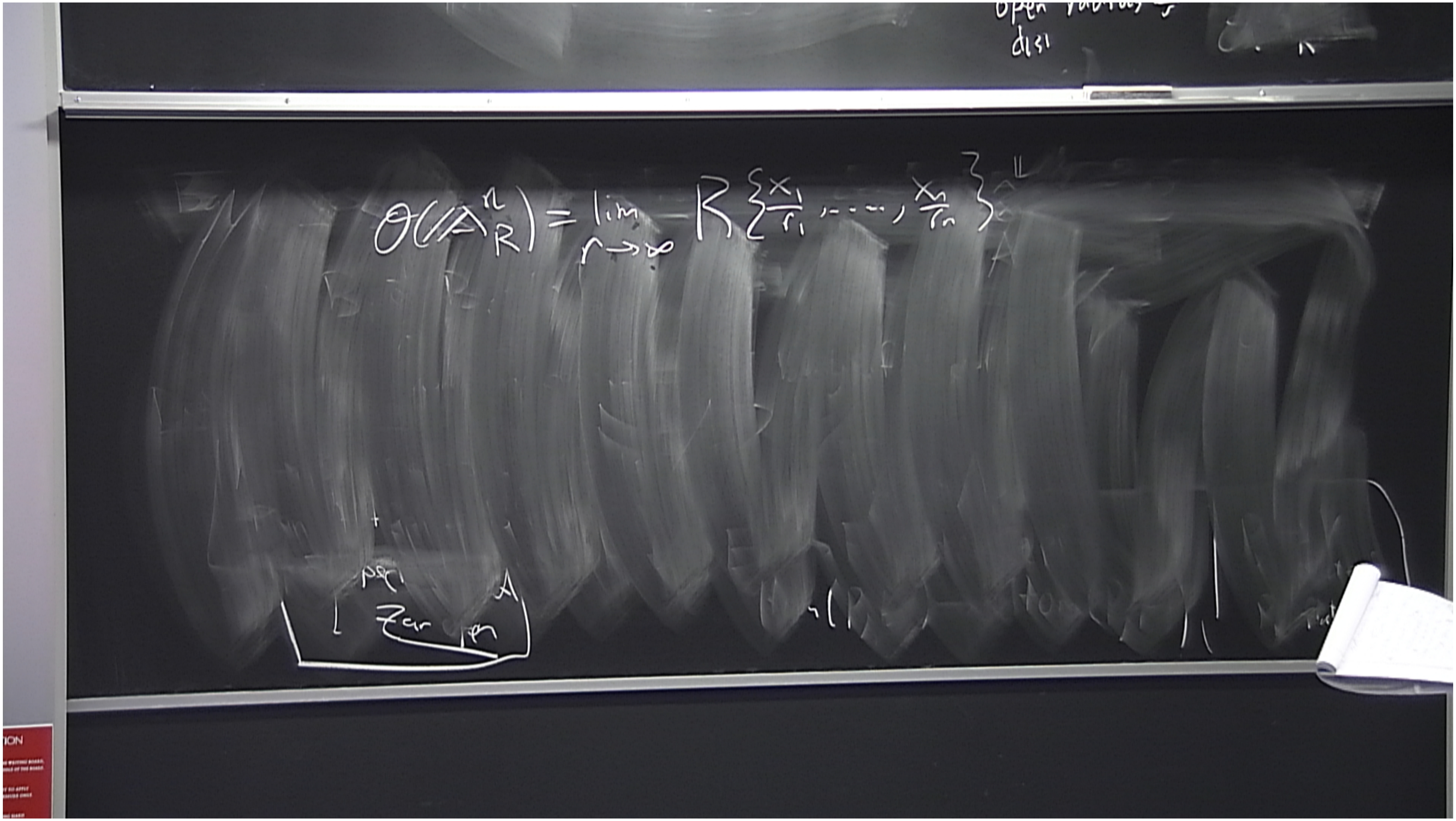
closed disk of $\frac{1}{r}$

open radius $\frac{1}{r}$ disk

$\mathcal{O}(\mathbb{A}^1_R) :=$

$\operatorname{Spec} B \subseteq \operatorname{Spec} A$ Zar Open

P_r flat + proj


$\mathcal{O}(\mathbb{A}^n_R) = \lim_{r \to \infty} R\left\{\frac{x_1}{r_1}, \dots, \frac{x_n}{r_n}\right\}^\wedge$

open ...
disc

$$\mathcal{O}(\mathbb{A}^n_R) = \lim_{r\to\infty} R\left\{\frac{x_1}{r_1},\dots,\frac{x_n}{r_n}\right\}$$

analytification

$X =$ limit of $\mathbb{A}^1_R \rightsquigarrow$

$$R[x] \longmapsto \mathcal{O}(\mathbb{A}^1_R)$$

proved homotopy epi

Zar

$$\mathcal{O}(\mathbb{A}^n_R) = \lim_{r\to\infty} R\left\{\frac{x_1}{r_1}, \dots, \frac{x_n}{r_n}\right\}^{\mathbb{L}}$$

analytification

$X =$ limit of $\mathbb{A}^1_R$ $\rightsquigarrow$

Sym R in Ind(Ban)

$R[x] \longmapsto \mathcal{O}(\mathbb{A}^1_R)$

proved homotopy epi $\Longrightarrow$ $\mathbb{L}_{R[x]/\mathcal{O}(\mathbb{A}^1_R)}$ vanish

Zar open

open ... disk

$$\mathcal{O}(\mathbb{A}^n_R) = \lim_{r\to\infty} R\left\{\frac{x_1}{r_1}, \dots, \frac{x_n}{r_n}\right\}^{\mathbb{L}}$$

analytification

$$\mathcal{O}(\mathbb{A}^n) = \mathcal{O}(\mathbb{A}^1)\,\hat{\otimes}^{\mathbb{L}}_R \cdots \hat{\otimes}^{\mathbb{L}}_R\,\mathcal{O}(\mathbb{A}^1)$$

$X =$ limit of $\mathbb{A}^1_R \rightsquigarrow$

Sym R in Ind(Ban) $\longrightarrow$ $R[x] \longmapsto \mathcal{O}(\mathbb{A}^1_R)$

proved homotopy epi $\Longrightarrow$ $\mathbb{L}_{R[x]/\mathcal{O}(\mathbb{A}^1_R)}$ vanish

Zar open

open ...
disk

$$\mathcal{O}(\mathbb{A}^n_R) = \lim_{r\to\infty} R\left\{\frac{x_1}{r_1}, \dots, \frac{x_n}{r_n}\right\}^{\mathbb{L}}$$

analytification

$$\mathcal{O}(\mathbb{A}^n) = \mathcal{O}(\mathbb{A}^1) \otimes^{\mathbb{L}}_R \cdots \otimes^{\mathbb{L}}_R \mathcal{O}(\mathbb{A}^1)$$

$X =$ limit of $\mathbb{A}^1_R \rightsquigarrow$

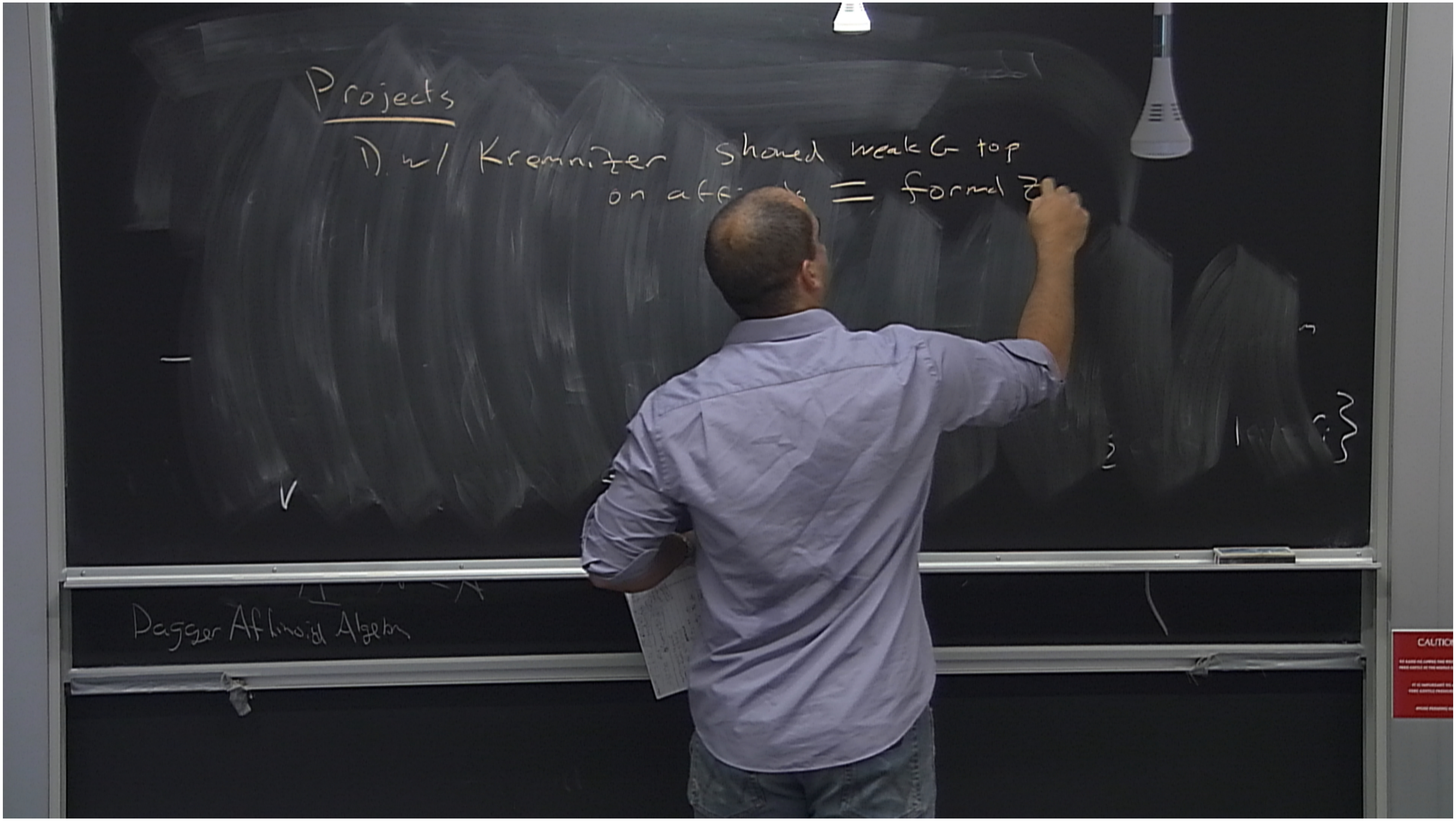
Sym R in Ind(Ban) $\quad R[x] \longmapsto \mathcal{O}(\mathbb{A}^1_R)$

proved homotopy epi $\Longrightarrow \mathbb{L}_{R[x]/\mathcal{O}(\mathbb{A}^1_R)}$ vanish

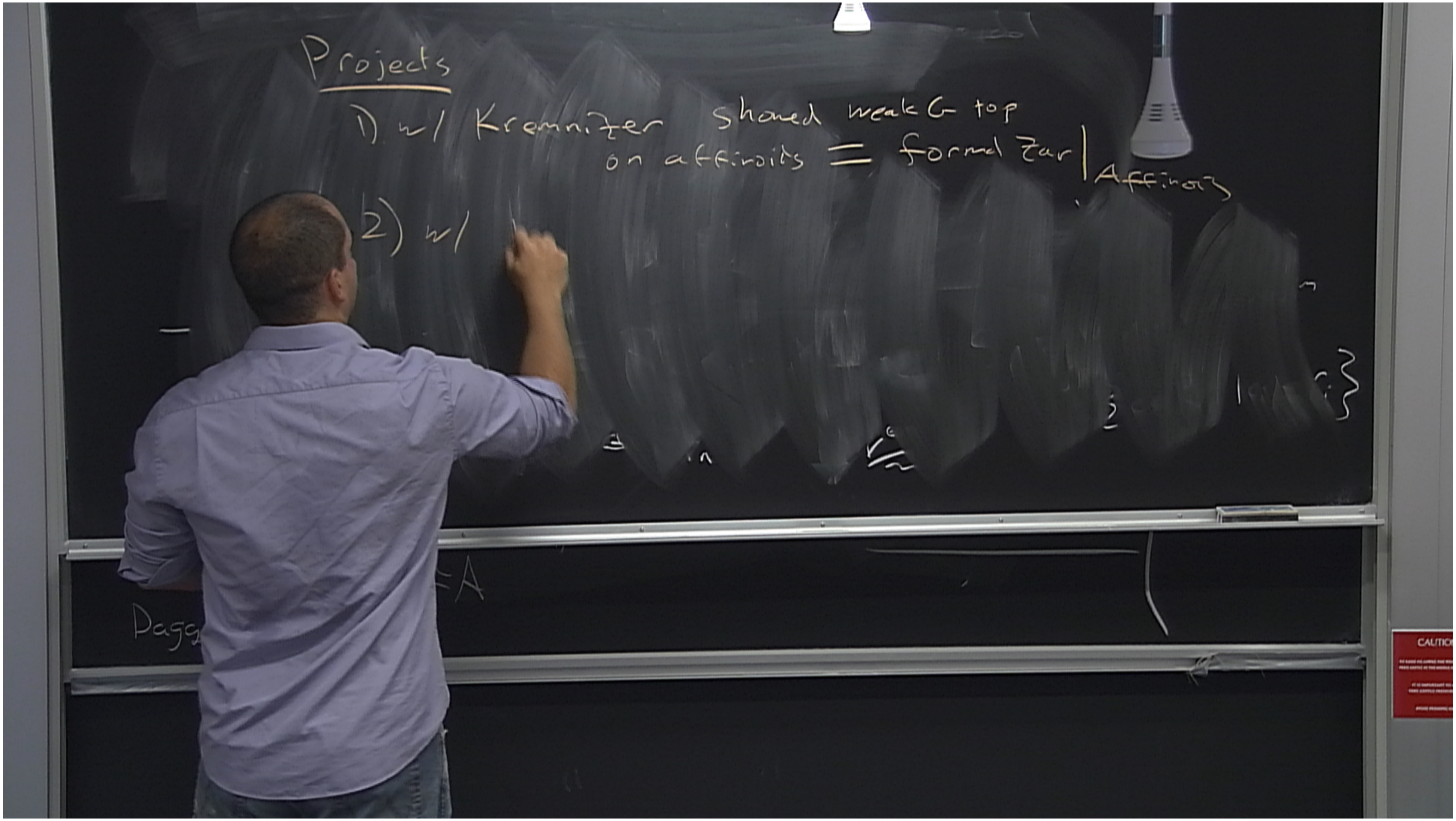
Zar open

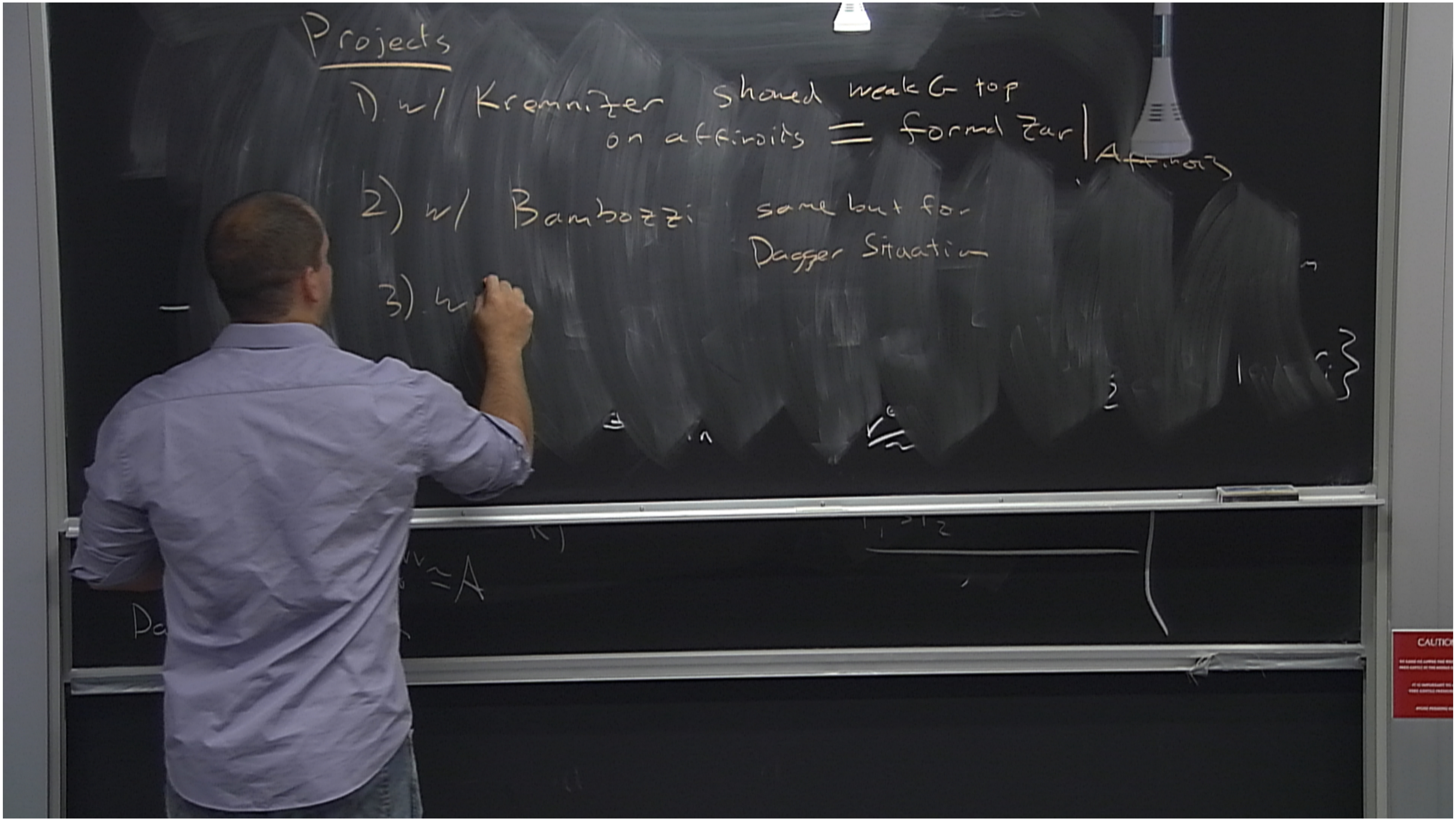
epi

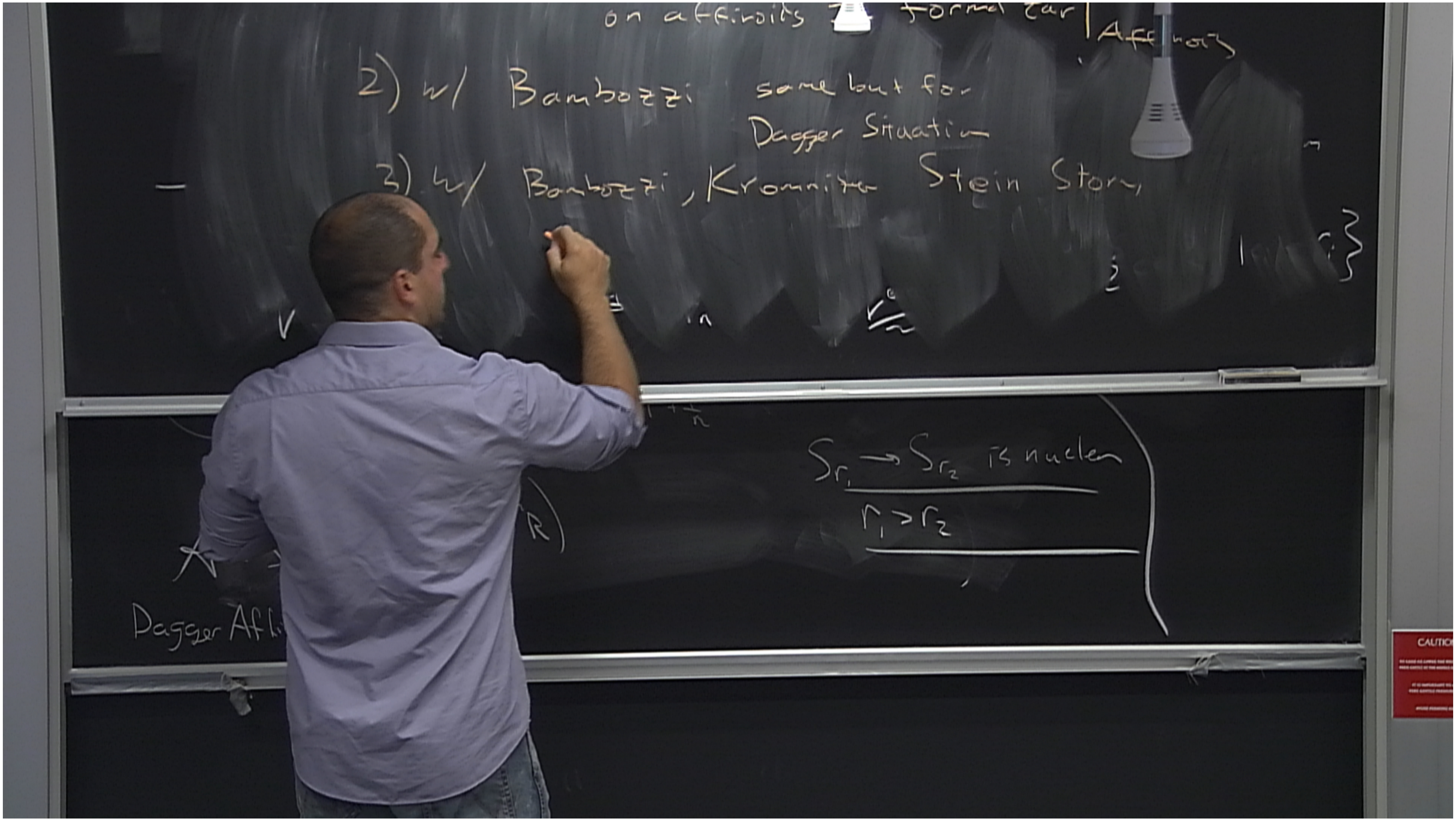
flat proj



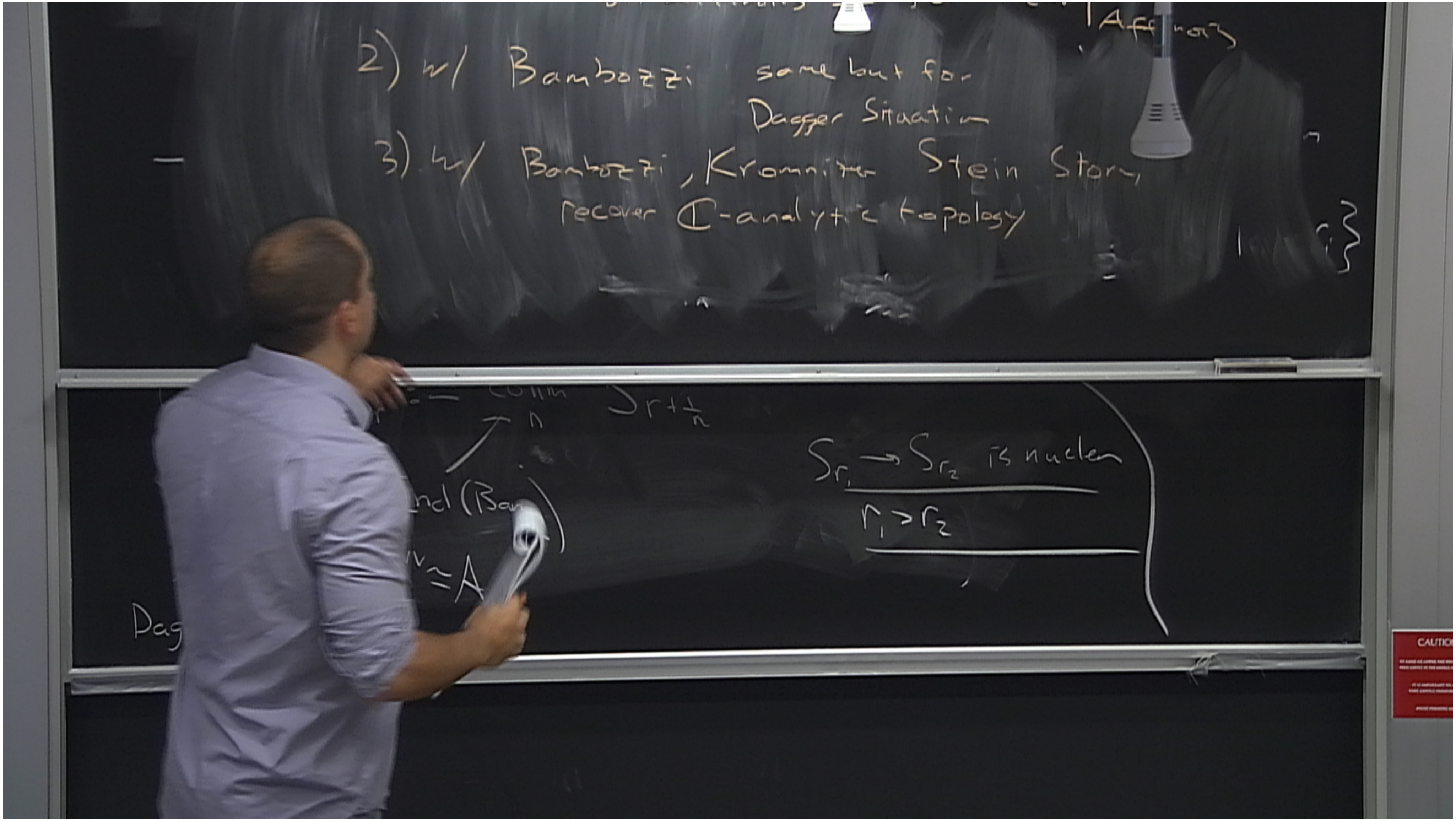
Projects
1) w/ Kremnizer showed weak G top
on aff... = formal Z
Dagger Affinoid Algebra

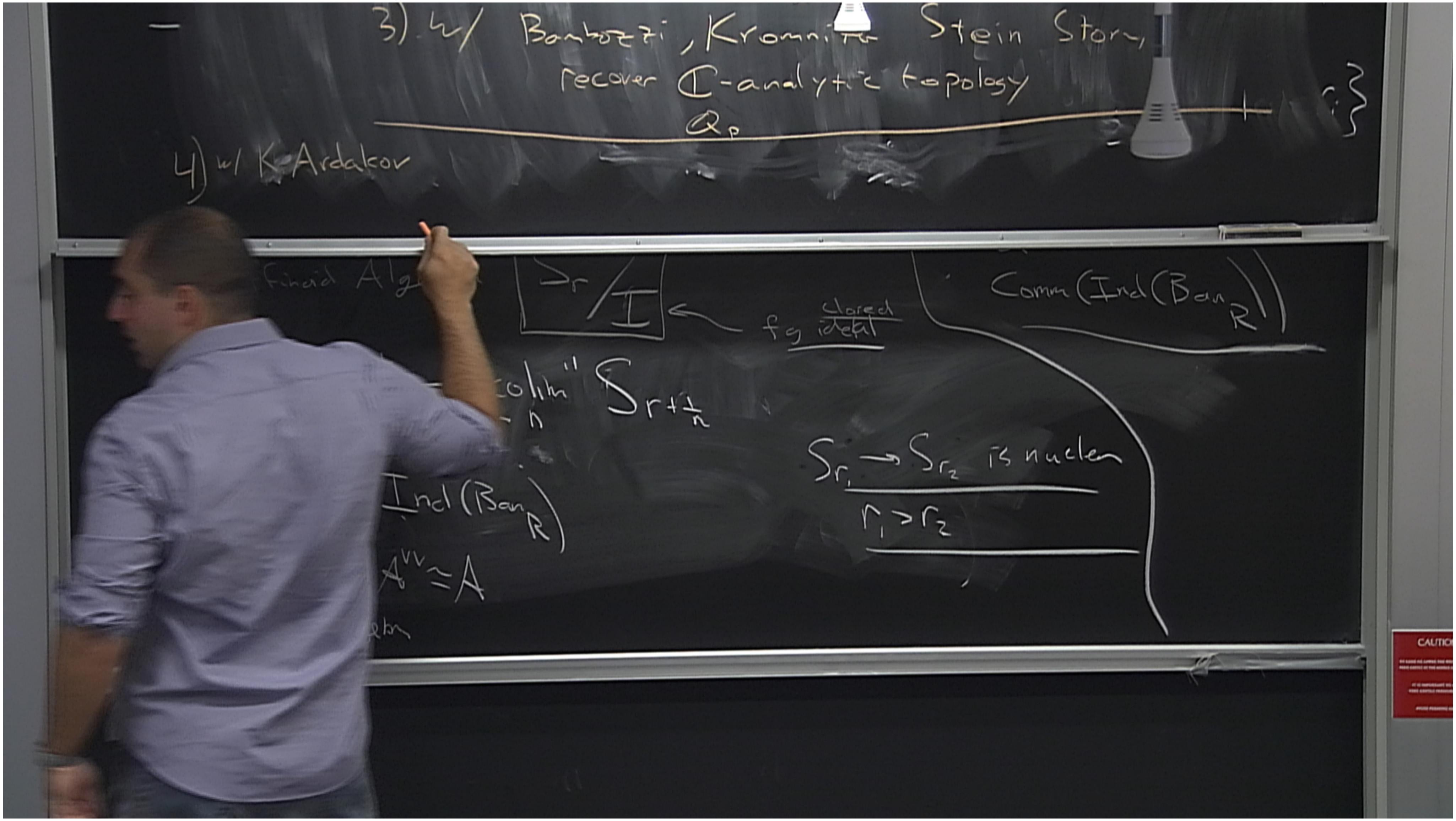

Projects
1) w/ Kremnizer showed weak G top
on affinoids = formal Zar | Affinoids
2) w/


Projects
1) w/ Kremnizer showed weak G top
on affinoids = formal Zar | Affinoids
2) w/ Bambozzi same but for
Dagger Situation
3) w

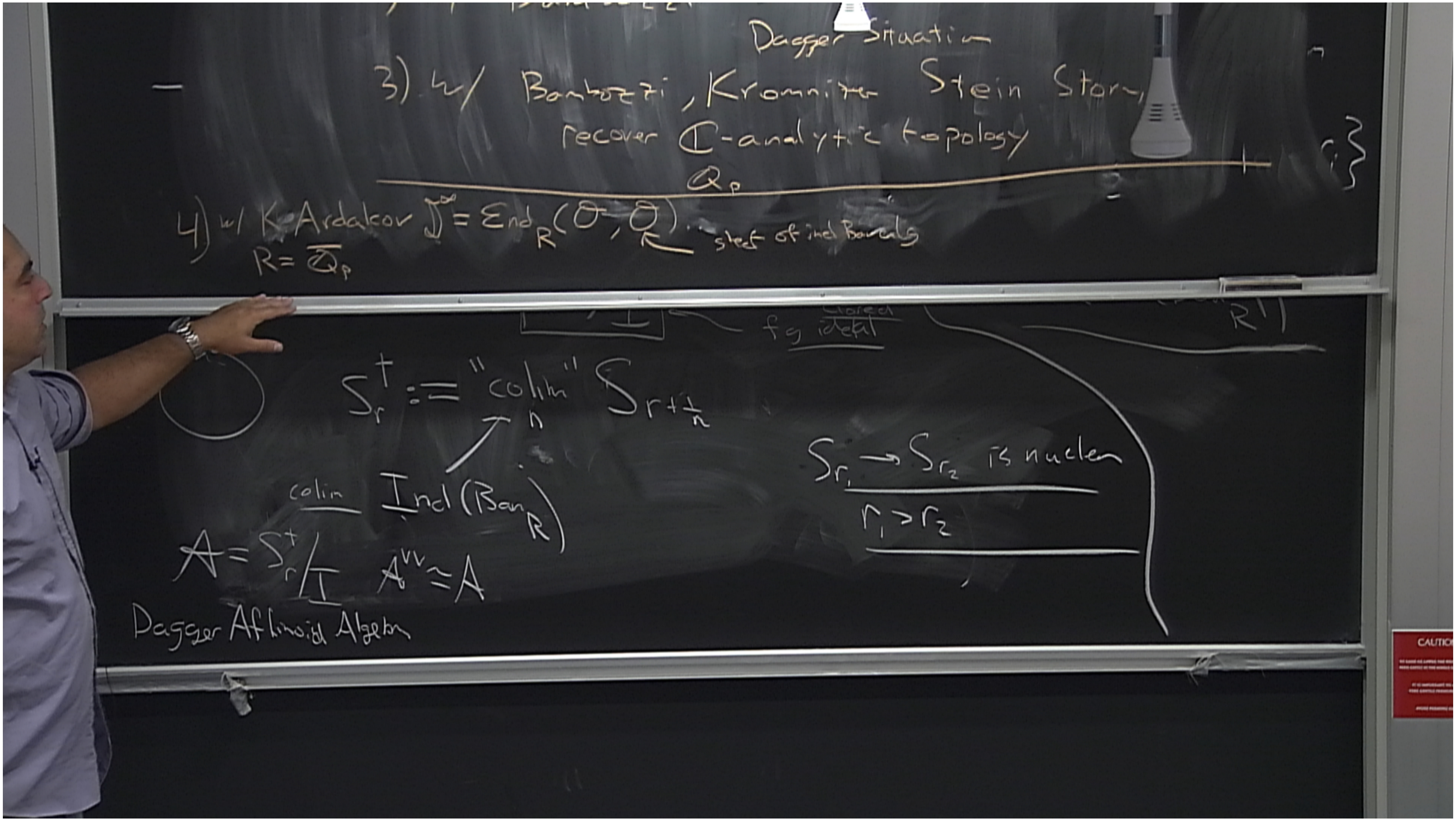


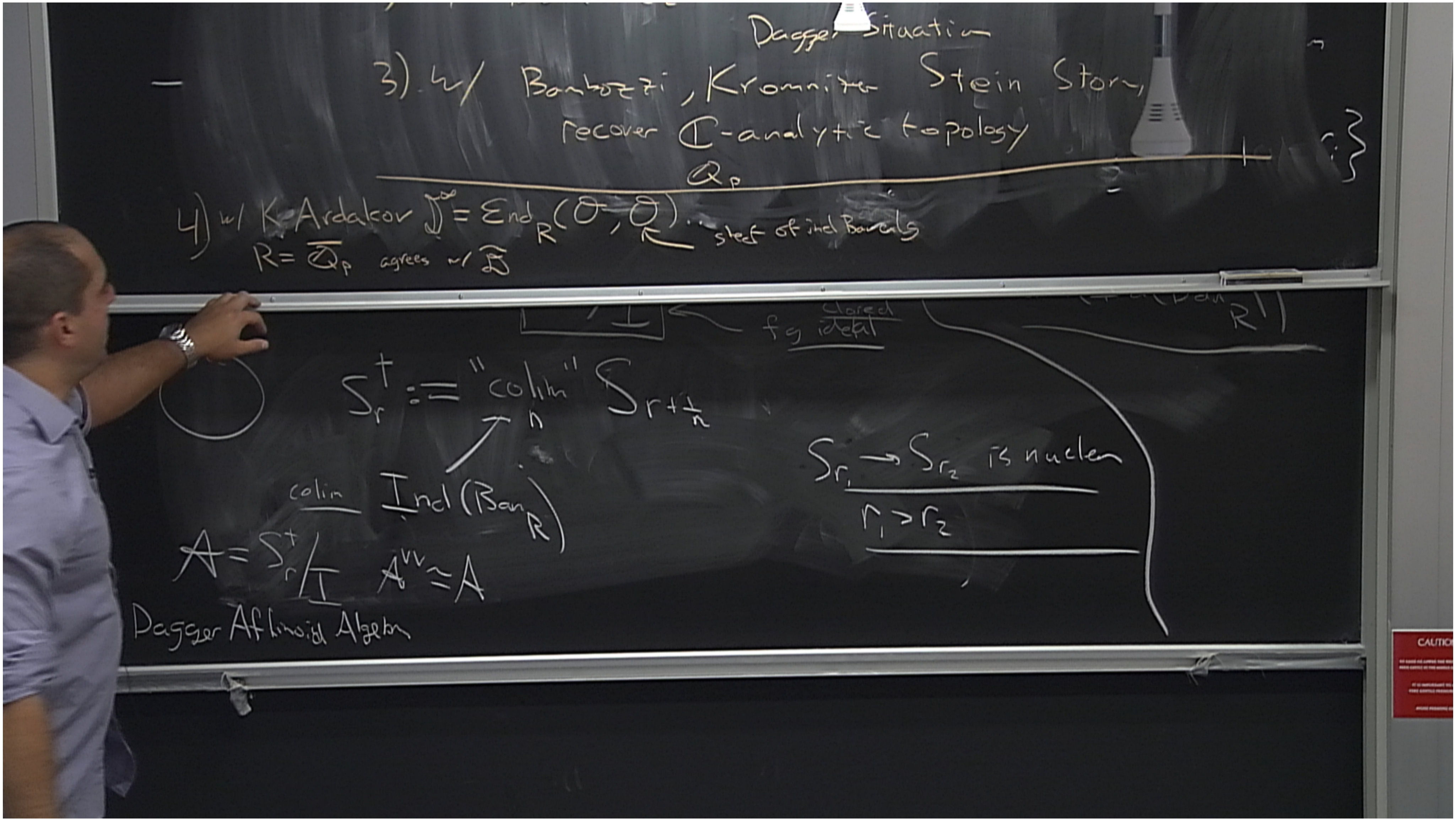
2) w/ Bambozzi
same but for
Dagger Situation
3) w/ Bambozzi, Kremnizer
Stein Story
Sr1 → Sr2 is nuclear
r1 > r2


2) w/ Bambozzi same but for
Dagger Situation
3) w/ Bambozzi, Kremnizer Stein Story,
recover ℂ-analytic topology
$S_{r_1} \to S_{r_2}$ is nuclear
$r_1 > r_2$



3) w/ Bambozzi,
Stein Story,
recover ℂ-analytic topology
Q_p
4) w/ K. Ardakov
fg closed ideal
Comm(Ind(Ban_R))
"colim" S_{r+1/n}
S_{r_1} → S_{r_2} is nuclear
r_1 > r_2
Ind(Ban_R)
A^{vv} ≅ A


Dagger Situation
3) w/ Bambozzi, Kremnitzer Stein Story
recover ℂ-analytic topology
$\mathbb{Q}_p$
4) w/ K. Ardakov $\mathcal{D}^\infty = \mathrm{End}_R(\mathcal{O}, \mathcal{O})$ stalk of ind Banach
$R = \overline{\mathbb{Q}_p}$
closed
f.g. ideal
$S_r^\dagger := \text{"colim"}_n\, S_{r+\frac{1}{n}}$
colim
$\mathrm{Ind}(\mathrm{Ban}_R)$
$S_{r_1} \to S_{r_2}$ is nuclear
$r_1 > r_2$
$A = S_r^\dagger / I \quad A^{\vee\vee} \simeq A$
Dagger Affinoid Algebra
CAUTION


Dagger Situation
3) w/ Bambozzi, Kremnizer Stein Story,
recover ℂ-analytic topology
ℚp
4) w/ K. Ardakov D^∞ = End_R(O, O) … sheaf of ind Banachs
R = ℚ̄p agrees w/ D
closed
fg ideal
S_r^† := "colim" S_{r+1/n}
n
colim Ind(Ban_R)
S_{r_1} → S_{r_2} is nuclear
r_1 > r_2
A = S_r^† / I   A^∨∨ ≅ A
Dagger Affinoid Algebra