Title: AKSZ quantization of shifted Poisson structures

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Abstract: One of the key constructions in the PTVV theory of shifted symplectic structures is the construction, via transgression, of a shifted symplectic structure on the derived mapping stack from an oriented manifold to a shifted symplectic stack vastly generalizing the AKSZ construction (which was formulated in the context of super manifolds). I will explain local-to-global approach to this construction, which also generalizes the construction to shifted Poisson structures and shows that the AKSZ/PTVV construction is compatible with quantization in a strong sense. One pleasant consequence is that every deformation quantization problem reduces to a version of BV-quantization. Time permitting, I will describe several geometric applications of the theory.

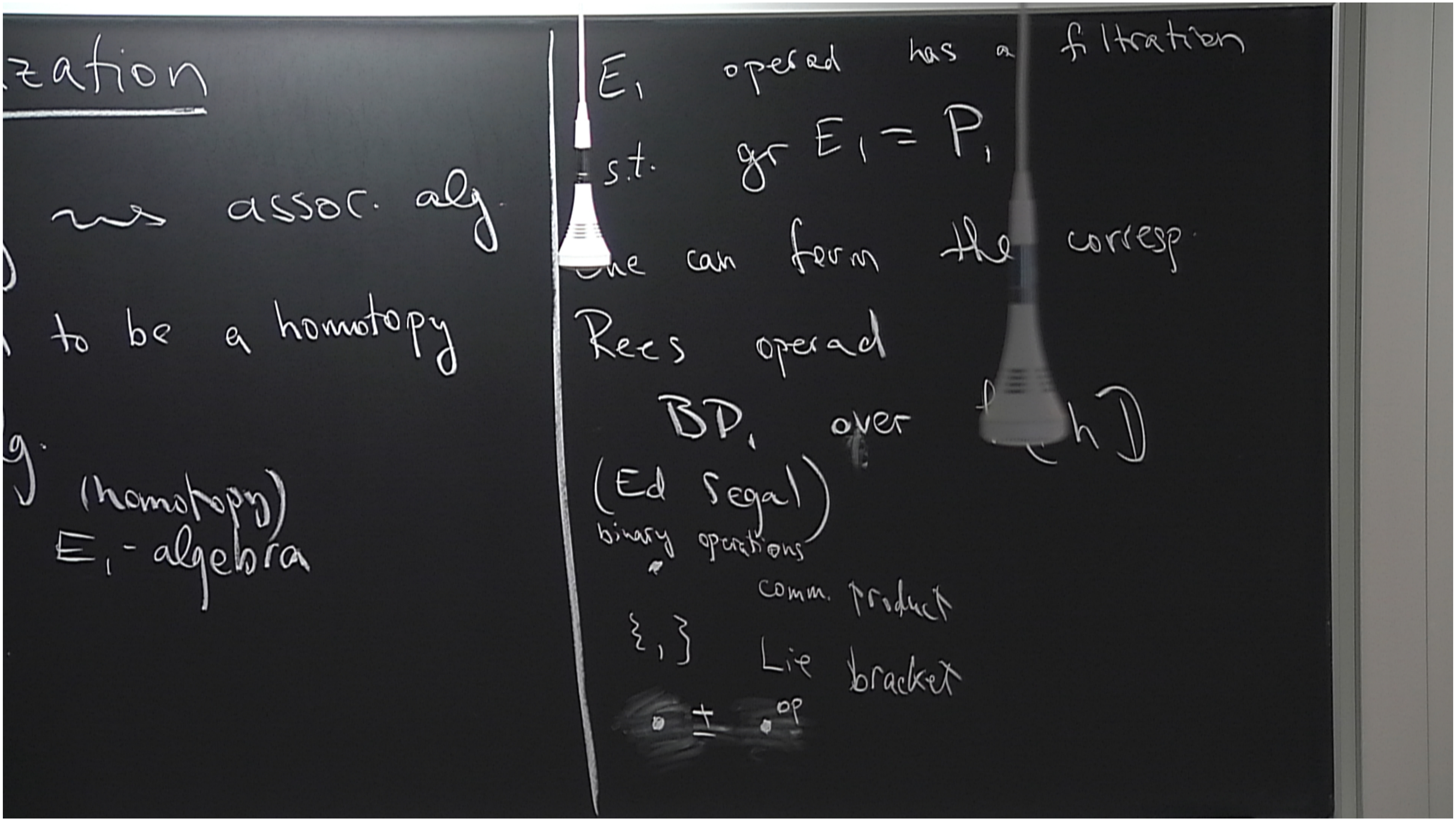
Deformation quantization

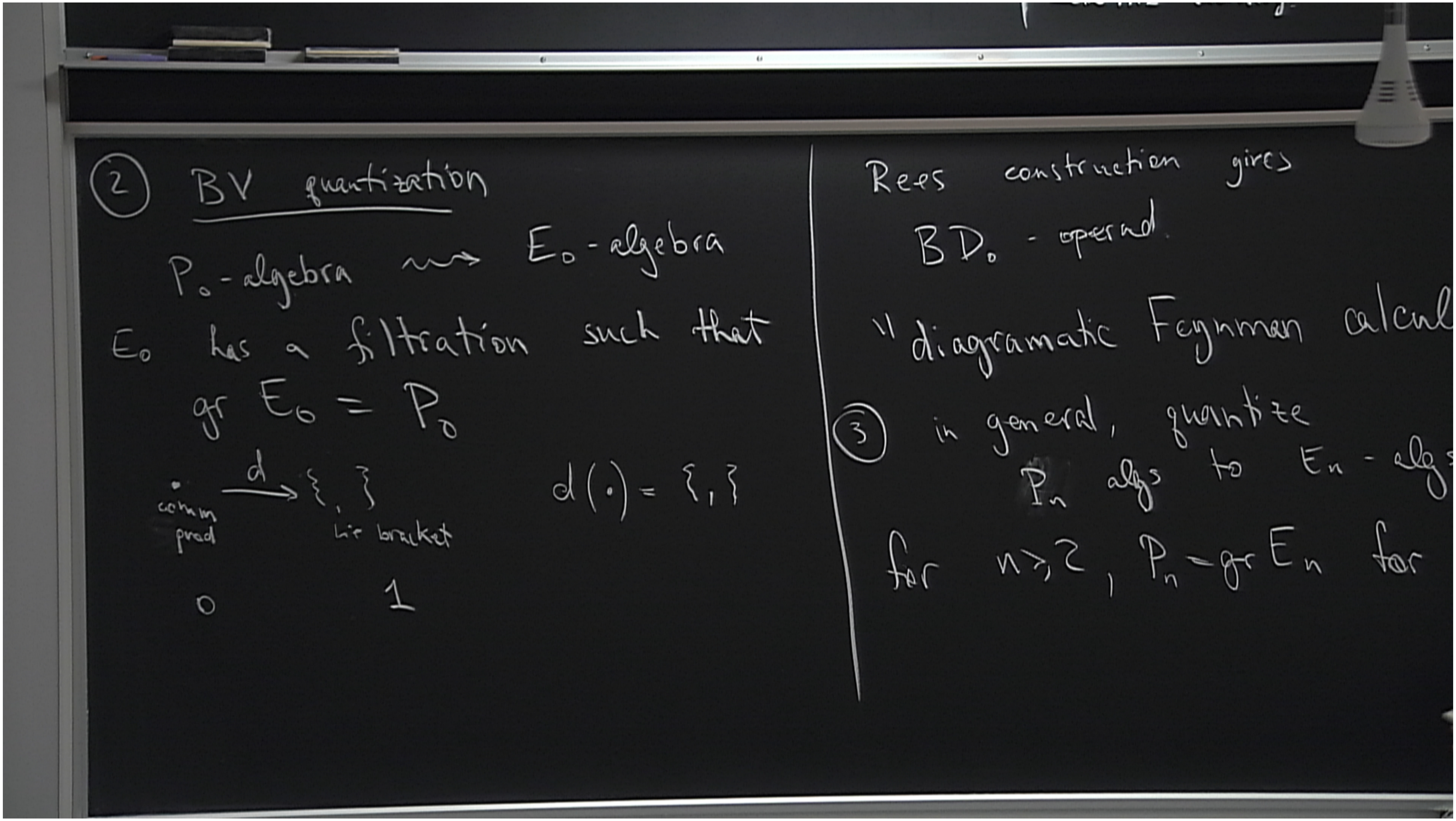
(1) A Poisson alg $\rightsquigarrow$ assoc. alg.

Generalize: allow A to be a homotopy Poisson alg.

homotopy assoc. alg = (homotopy) E_1-algebra

(homotopy) Poisson alg = (homotopy) P_1-alg.


zation
assoc. alg.
to be a homotopy
(homotopy)
E_1-algebra
E_1 operad has a filtration
s.t. gr $E_1 = P_1$
one can form the corresp.
Rees operad
BP_1 over
(Ed Segal)
binary operations
comm. product
$\{,\}$ Lie bracket
op


2 BV quantization
P0 - algebra ⇝ E0 - algebra
E0 has a filtration such that
gr E0 = P0
· comm prod —d→ { , } Lie bracket
d(·) = { , }
0 1
Rees construction gives
BD0 - operad.
" diagramatic Feynman calcul
3 in general, quantize
Pn algs to En - alg
for n ≥ 2, Pn = gr En for

(2) BV quantization

P_0-algebra $\rightsquigarrow$ E_0-algebra

E_0 has a filtration such that

$\mathrm{gr}\, E_0 = P_0$

$\underset{\text{comm prod}}{\cdot} \xrightarrow{d} \underset{\text{Lie bracket}}{\{\,,\,\}}$ $\qquad d(\cdot) = \{\,,\,\}$

0 $\qquad$ 1

Rees construction gives

BD_0 - operad.

"diagramatic Feynman calculus"

(3) in general, quantize P_n algs to E_n-algs.

for $n \geq 2$, $P_n = \mathrm{gr}\, E_n$ for the Postnikov filtration.

ebra

that

$= \{,\}$

Rees construction gives

BD_0 - operad.

"diagramatic Feynman calculus"

③ in general, quantize

P_n algs to E_n-algs.

for $n \geq 2$, $P_n = \mathrm{gr}\, E_n$ for the Postnikov filtration.

Remark: this is deformation quant. of $(n-1)$-shifted Poisson stacks.

④ Poisson vertex algebras

$\rightsquigarrow$ vertex algebras

Another bit of motivation:

X $(n-1)$-shifted symplectic stack

M m-dim'l compact oriented mfld

$\mathrm{Maps}(M, X)$ is $(n-m-1)$-shifted symplectic.

these mapping stacks should assemble into a top field theory

$$\mathrm{Bord}_n^{or} \longrightarrow \mathrm{Sympl}_n$$
$$* \longmapsto X$$

Cobordism hypothesis: field theory is determined by its value at X.

in particular, quantization of X should automatically give quant.s of $\mathrm{Maps}(M, X)$.

(4) Poisson vertex algebras
$\rightsquigarrow$ vertex algebras

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Cobordism hypothesis: field theory is determined by its value at $*$.

in particular, quantization of X should automatically give quant.s of $\mathrm{Maps}(M, X)$ for all M.

Thm (Dunn):

$$E_n \circledast E_m \simeq E_{n+m}$$

Thm (R.)

$$E_n \circledast BD_m \simeq BD_{n+m}.$$

Cor: $E_n \circledast P_m \simeq P_{n+m}.$

all $n \geq 0$, $m \in \mathbb{Z}$,

Ex: If A is a Poisson alg. we can regard A as an E_1 alg in P_0-algs.

$$\mathrm{Quant}(A) \simeq \left(\text{quant. of corr. } P_0\text{-alg } \underline{(\text{as an } E_1\text{-alg})} \right]$$

Bring in geometry using fact. algebras,

Thm (Beilinson-Drinfeld)

$$\left\{\begin{array}{c}\text{translation invariant fact} \\ \text{algs on } \mathbb{A}^1\end{array}\right\} \simeq \underline{\text{vertex algebras}}$$

Thm: (R.)

$$\left\{\begin{array}{c}\text{transl. inv't fact.} \\ P_0\text{-algebras on} \\ \mathbb{A}^1\end{array}\right\} \simeq \begin{array}{l}\text{Poisson} \\ \text{vertex} \\ \text{algebras.}\end{array}$$

Relation to mapping stacks

Thm (Beilinson-Drinfeld)

M n-dim'l mfld

$X = \mathrm{Spec}\, A$

Let $\mathcal{F}_A$ be the fact. alg on M corresp. to A. Then

$$\int_X \mathcal{F}_A := R\Gamma(\mathrm{Ran}(M), \mathcal{F}_A)$$

stacks

)

lg on M

(M), $\mathcal{F}_A$)

$$\int_M \mathcal{F}_A \simeq$$ space of functions on the derived mapping stack $\mathrm{Maps}(M, X)$.

If X is $(n-1)$ shifted Poisson then $\mathcal{F}_A$ is a fact. alg of P_{n-m}-algebras, $m = \dim M$.

alg. $\tilde{A}$

$_m$ - alg.

Rmk. X sta

$\mathrm{Quant}^{qs}(T^*[-1]X)$

$\simeq$ (projective) hol. volume forms on X.

These are often interesting, e.g. can get

- Todd genus
- Witten genus
- B-model operations

etc.

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Cor: