Title: Electron viscosity, current vortices and negative nonlocal resistance in graphene

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Abstract: <p>Quantum-critical strongly correlated electron systems are predicted to feature universal collision-dominated transport resembling that of viscous fluids. Investigation of these phenomena has been hampered by the lack of known macroscopic signatures of electron viscosity. Here we identify vorticity as such a signature and link it with a readily verifiable striking macroscopic DC transport behavior. Produced by the viscous flow, vorticity can drive electric current against an applied field, resulting in a negative nonlocal voltage. The latter may play the same role for the viscous regime as zero electrical resistance does for superconductivity. Besides offering a diagnostic which distinguishes viscous transport from ohmic currents, the sign-changing electrical response affords a robust tool for directly measuring the viscosity-to-resistivity ratio. Strongly interacting electron-hole plasma in high-mobility graphene affords a unique link between quantum-critical electron transport and the wealth of fluid mechanics phenomena.</p>

<p>Levitov and Falkovich, Nature Physics, 22 Feb 2016</p>

Why solid people need fluid mechanics?

Levitov and Falkovich
MIT WIS

April 26, 2016
Perimeter Institute

Is hydrodynamics ever relevant in metals?

- In one-component fluid or gas a hydrodynamic approach works because one has local conservation of energy and momentum. Macroscopic hydrodynamic equations describe propagation of conserved quantities in space.

- Electron fluid in a solid can exchange energy and momentum with the lattice. Hydrodynamics not relevant?

- High-mobility electron systems (GaAs 2DES, graphene).

Non-Fermi liquids, high-Tc superconductors, strange metals

Critical electron fluid in graphene

- Interactions enhanced in 2D, strong near DP
- Vanishing DOS but long-range interactions, strong coupling
- Fast p-conserving collisions, shear viscosity
- Low viscosity-to-entropy ratio h/s (near-perfect fluid)
- Comparable to universal low bound (AdS CFT, black holes)

VOLUME 56, NUMBER 14 1

Nonzero-temperature transport near quantum critical points

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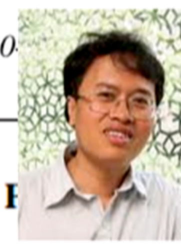


PRL **94,** 111601 (2005) PHYSICAL REVIEW LETTERS

Viscosity in Strongly Interacting Quantum Field Theories from Black Hole P

P. K. Kovtun,[1] D. T. Son,[2] and A. O. Starinets[3]

Carrier-carrier scattering versus disorder scattering

$\gamma_{ee} \sim (k_B T)^2 / E_F$ in the degenerate limit

Near charge neutrality, the rate γ_{ee} grows

$\gamma_{ee} \approx A\alpha^2 k_B T / \hbar$, where α is the interaction strength.

Fritz, L., Schmalian, J., Müller, M. & Sachdev, S. Quantum critical transport in clean graphene. Phys. Rev. B **78**, 085416 (2008).
Kashuba, A. B. Conductivity of defectless graphene. Phys. Rev. B 78, 085415 (2008).

$$\gamma_{ee}^{-1} \approx 80\,\mathrm{fs}$$

Disorder scattering can be estimated from
mean free path values, which reach a few microns at large doping

$\gamma_p \propto n^{-1/2}$ $\qquad$ $n \lesssim 10^{10}\,\mathrm{cm}^{-2}$

$$\gamma_p^{-1} \sim 0.5\,\mathrm{ps}$$

Hydrodynamic description of transport

$$\gamma_{\mathrm{p}} \ll \gamma_{\mathrm{ee}}$$

Navier-Stokes equation

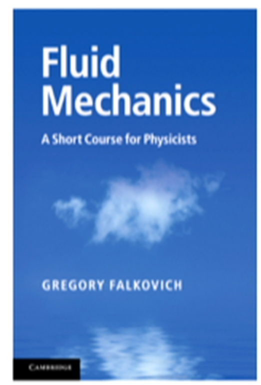
$$\partial_t v + (v\nabla)v - \nu\nabla^2 v = -\nabla P/mn$$

$$\nu \approx (1/2)v_{\mathrm{F}}^2\gamma_{\mathrm{ee}}^{-1}$$

$$P = e\int_{n_0}^{n}\Phi(n')\mathrm{d}n'$$

$E_{\mathrm{F}} \gg k_{\mathrm{B}}T$

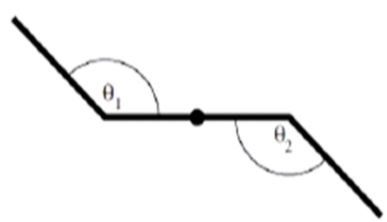
$$P \approx e(n - n_0)\Phi$$

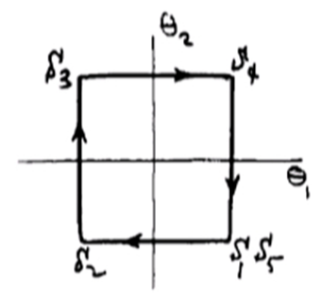


Life at low Reynolds numbers

$Re = vL/\nu << 1$ $\eta \Delta v = \nabla P$

- **Scallop theorem** (Purcell 1977) to achieve propulsion at low Reynolds number a swimmer must deform in a way that is not invariant under time-reversal.
- **Berry phase & non-abelian gauge theory:** Wilczek, Shapere (1989), Geometry of self-propulsion at low *Re*. Avron, Kenneth, Gat (2004).





nature physics

LETTERS

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Electron viscosity, current vortices and negative nonlocal resistance in graphene

Leonid Levitov[1*] and Gregory Falkovich[2,3*]

For ohmic transport the current-field relation $\mathbf{j} = \sigma\mathbf{E}$ is local, and as a result current is a potential vector field with zero vorticity. Indeed the relation $\mathbf{E} = -\nabla\phi$, where ϕ is electrostatic potential, yields $\nabla\times\mathbf{j} = 0$. Relating current density and flow velocity, $\mathbf{j} = ne\mathbf{v}$, and assuming constant particle number density n, we see that the velocity field itself is potential. Combining this with the continuity equation we write the incompressibility condition as Laplace's equation for the electric potential

$$\frac{ne}{\sigma}\nabla_i v_i = \nabla^2\phi = 0, \quad \nabla^2 = \partial_x^2 + \partial_y^2. \tag{1}$$

$$\mathbf{v} = \mathbf{z}\times\nabla\psi = (-\partial_y\psi, \partial_x\psi) \qquad \nabla^2\psi = 0$$

DC viscous flow

$$\eta\nabla^2 v_i = ne\nabla_i\phi, \quad \nabla_i v_i = 0.$$

$$\text{vorticity } \omega = \nabla\times\mathbf{v} = \mathbf{z}\nabla^2\psi \text{ is non-zero}$$

Purely viscous case

$$(\nabla^2)^2\psi = 0$$

Mixed Ohmic-viscous case

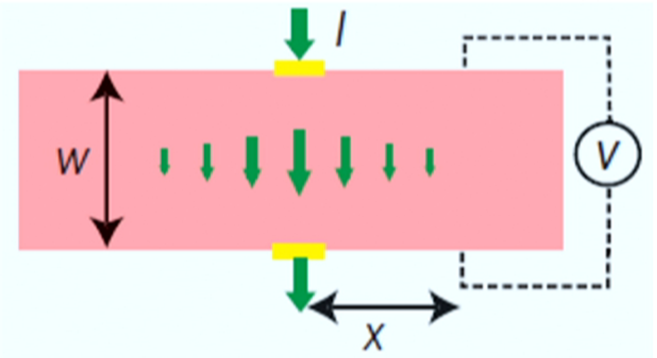
$$[\eta(\nabla^2)^2 - \rho(en)^2\nabla^2]\psi(x,y) = 0$$

$\rho = \gamma_p m/ne^2$ is resistivity

$$v_y(x,y)_{y=0,w} = \partial_x\psi(x,y)_{y=0,w} = \frac{I(x)}{en}$$

general boundary condition

$$v_\perp = 0, \quad v_\parallel = -\alpha\partial_\parallel P$$



Potential

$$\phi(x,y) = \frac{\beta I}{2} \int \mathrm{d}k e^{ikx} a_k [\sinh k(y-w) + \sinh ky]$$

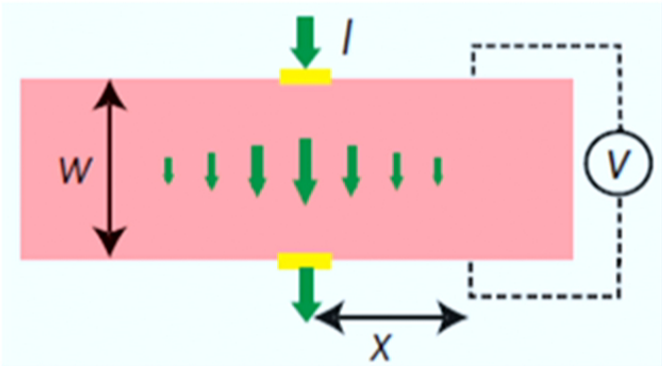
$$a_k = k \tanh(kw/2)/(kw + \sinh kw)$$

$|x|, |y| \ll w$

$$\phi(x,y) \approx \frac{\beta I}{2} \int \mathrm{d}k e^{ikx} |k| e^{-|k|y} = \frac{\beta I (y^2 - x^2)}{(y^2 + x^2)^2}$$

$$V(x) = \phi(x,w) - \phi(x,0)$$

$$V(x) \approx -\frac{2\beta}{x^2} I$$



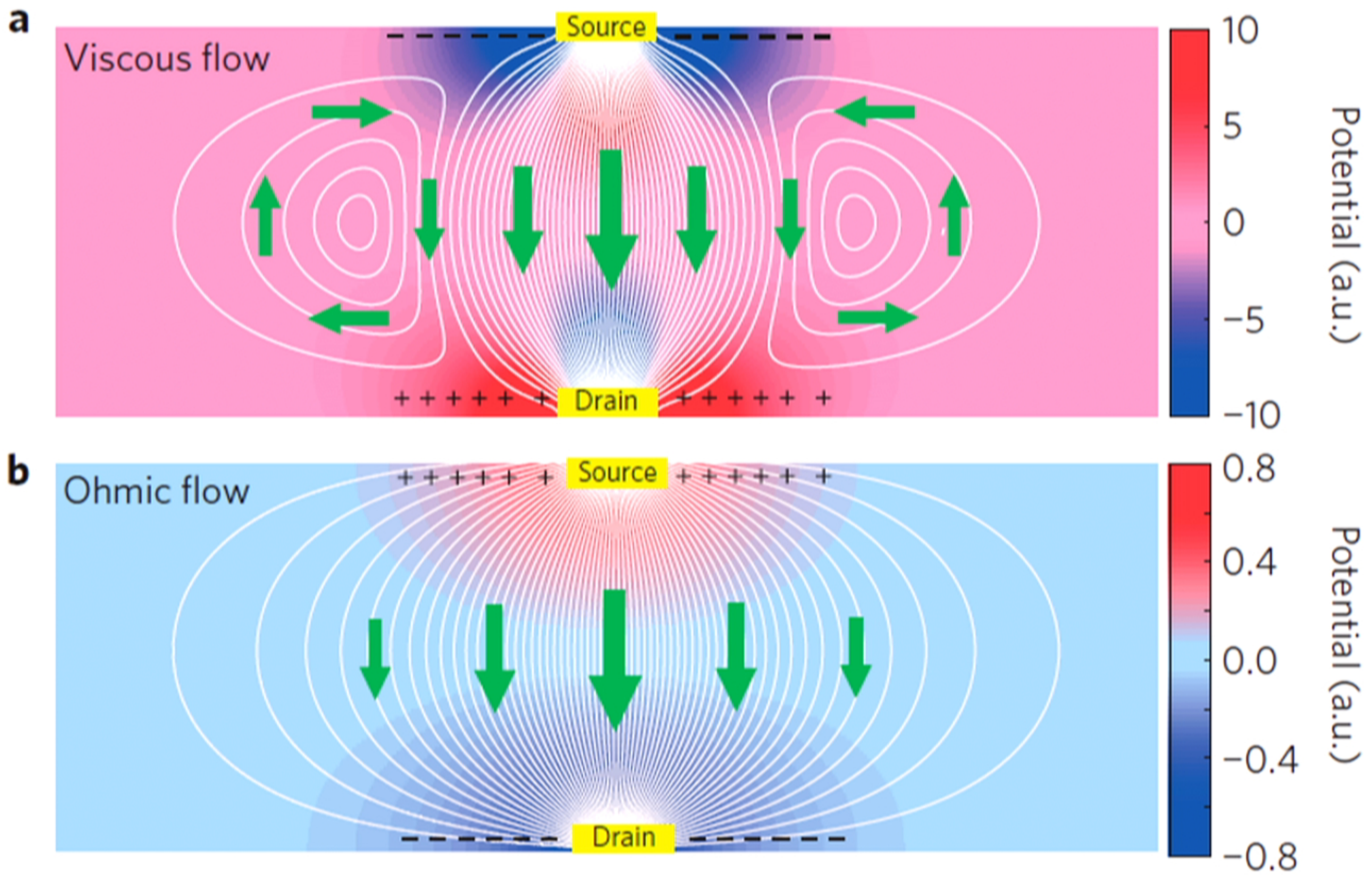


Figure 1 | Current streamlines and potential map for viscous and ohmic flows. White lines show current streamlines, colours show electrical potential

THE GENERAL OHMIC-VISCOUS CASE

$$[\eta(\nabla^2)^2 - \rho(en)^2\nabla^2]\psi(x,y) = 0$$

$$(\partial_y^2 - k^2)(\partial_y^2 - q^2)\psi_k(y) = 0, \quad q^2 = k^2 + \rho(en)^2/\eta$$

$$\psi_k(y) = -\frac{I}{enik}\sum_{\pm}\left[a_\pm \exp(\pm ky) + b_\pm \exp(\pm qy)\right]$$

$$a_+ = \frac{(e^{qw}-1)\,q}{(k-q)\left(1-e^{(k+q)w}\right) + (k+q)\left(e^{qw}-e^{kw}\right)},$$

$$a_- = \frac{e^{kw}\left(e^{qw}-1\right)q}{(k-q)\left(1-e^{(k+q)w}\right) + (k+q)\left(e^{qw}-e^{kw}\right)},$$

$$b_+ = \frac{\left(e^{kw}-1\right)k}{(q-k)\left(1-e^{(k+q)w}\right) + (k+q)\left(e^{kw}-e^{qw}\right)},$$

$$b_- = \frac{\left(e^{kw}-1\right)ke^{qw}}{(q-k)\left(1-e^{(k+q)w}\right) + (k+q)\left(e^{kw}-e^{qw}\right)}.$$

$$\epsilon = (enw)^2 \frac{\rho}{\eta}$$

Voltage and negative nonlocal resistance

$$V(x) = \frac{I\rho}{\pi} \int_{-\infty}^{\infty} \frac{dk}{k} e^{ikx} f(k), \quad f(k) = \frac{e^{kw} - 1}{e^{kw} + 1 - \frac{k}{q}\coth(qw)(e^{kw} - 1)}$$

$\eta \to 0$ $\quad x \gg w/\sqrt{\epsilon}$ $\quad |k| \ll \sqrt{\epsilon}/w$ $\quad q^2 = k^2 + \rho(en)^2/\eta$

$$R_{nl}(x) = \frac{V(x)}{I} = \frac{\rho}{\pi} \int \frac{e^{ikx} dk}{k} \tanh(kw/2) = \frac{\rho}{\pi} \ln|\coth(\pi x/2w)| > 0$$

$\rho \to 0$ $\quad R_{\rm nl}(x) = V(x)/I$ $\quad$ *everywhere negative*

$\epsilon \gg 1$ (corresponding to high resistivity or low viscosity)

$|x| \lesssim \frac{w}{\sqrt{\epsilon}}$ $\quad R_{\rm nl}(x) = -2\beta/x^2$

$\frac{w}{\sqrt{\epsilon}} \lesssim |x| \lesssim w$ $\quad R_{nl}(x) = \frac{\rho}{\pi} \ln 2w/\pi x$

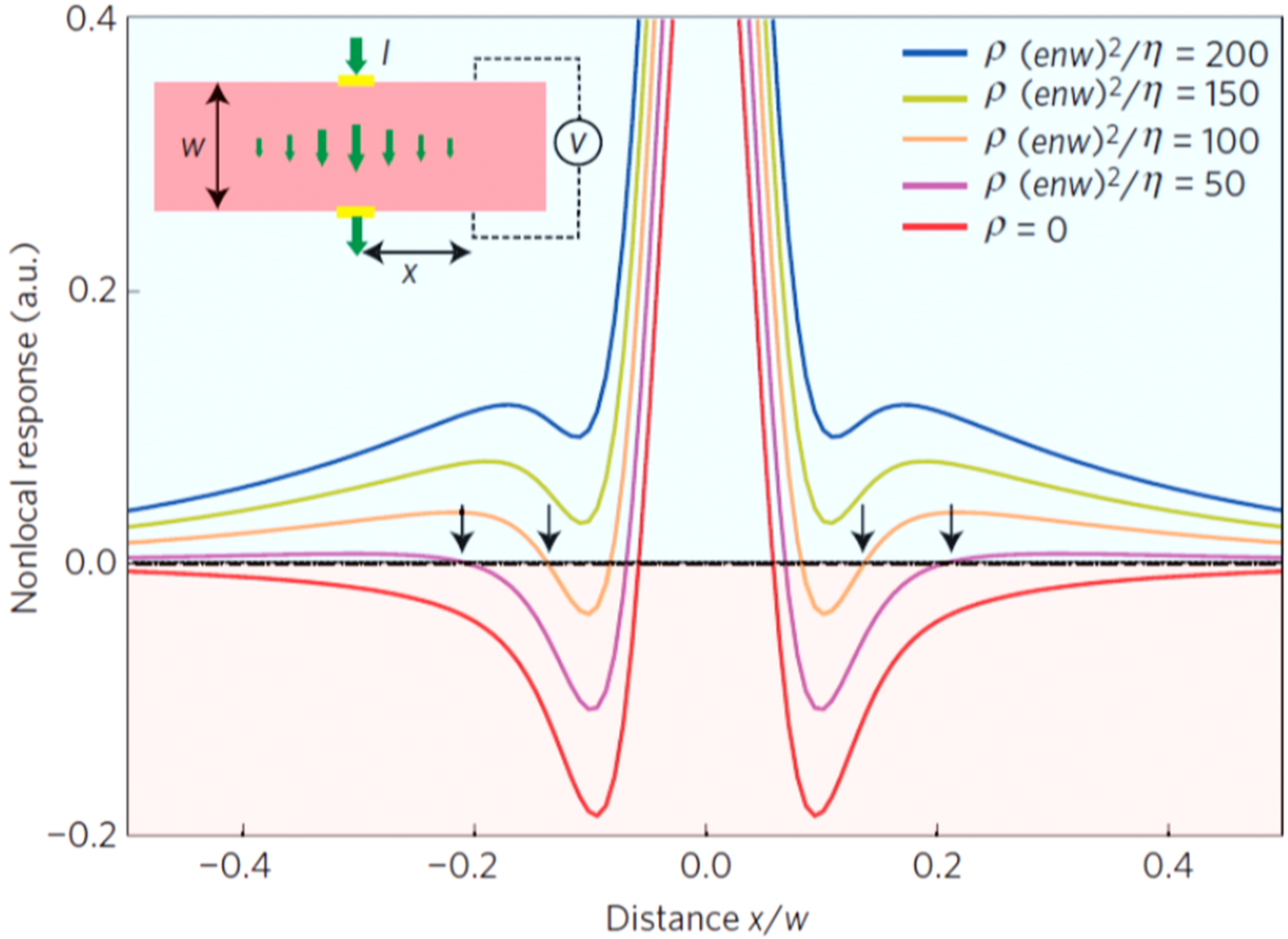


Figure 2 | Nonlocal response for different resistivity-to-viscosity ratios ρ/η

$$\epsilon = \rho(enw)^2/\eta \approx 2\gamma_{ee}\gamma_p(w/v_F)^2$$

THE ROBUSTNESS OF THE NEGATIVE NONLOCAL RESISTANCE

replacing the delta function for the current source by a Lorentzian

$$nev_y = Ia/\pi(x^2 + a^2) \qquad V(x) = -\frac{2\beta I(x^2 - a^2)}{(x^2 + a^2)^2}$$

Two types of small contacts (*l*<<*w*):

1) narrow graphene channels shaped through etching, 2) metal leads.

For $x \ll w$

$$V(x) = \beta \int_{-\infty}^{\infty} dk\, I(k) e^{ikx} |k| = -\beta \int_{-\infty}^{\infty} dx' \left(\frac{I(x')}{(x - x' + i0)^2} + \text{c.c.} \right)$$

For current constant at -*l*<*x*<*l*

$$V(x) = -\frac{2\beta}{x^2 - \ell^2} I$$

$$I(x)_{y=0,w} = \begin{cases} 3I(\ell^2 - x^2)/4\ell^3, & |x| \le \ell \\ 0, & |x| \ge \ell \end{cases} \qquad V(x) = \frac{3\beta I}{\ell^3} \left(2\ell + x \ln \frac{|x - \ell|}{|x + \ell|} \right)$$

$$V(x) = \begin{cases} V, & |x| < l \\ V\left(1 - \dfrac{|x|}{\sqrt{x^2 - l^2}}\right), & |x| > l \end{cases} \qquad I(x) = \frac{\lambda}{2\pi\beta} (\ell^2 - x^2)^{1/2}$$

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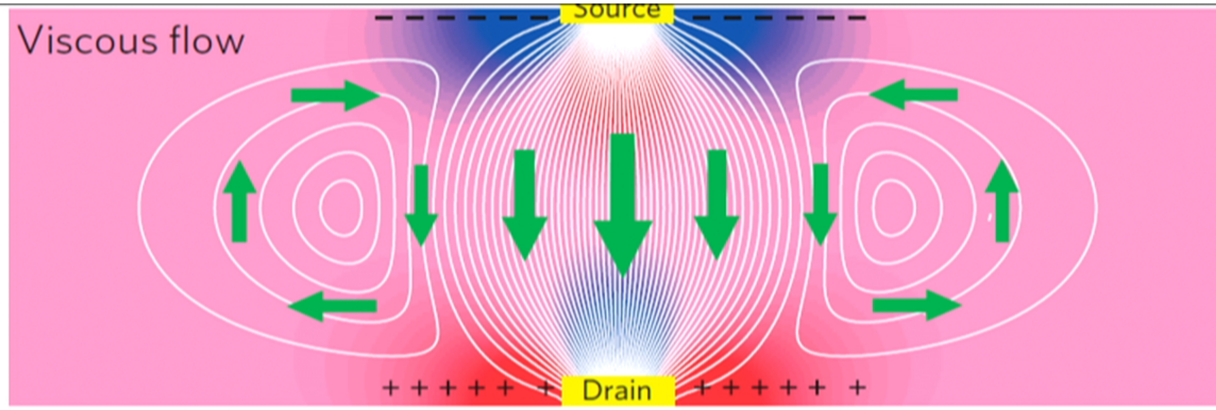


This singular behavior changes upon introducing partial-slip boundary conditions (finite edge resistivity): $v_x(x,y)_{y=0} = -\partial_y \psi(x,y)_{y=0} = \alpha(x)\partial_x \phi(x,y)_{y=0}$

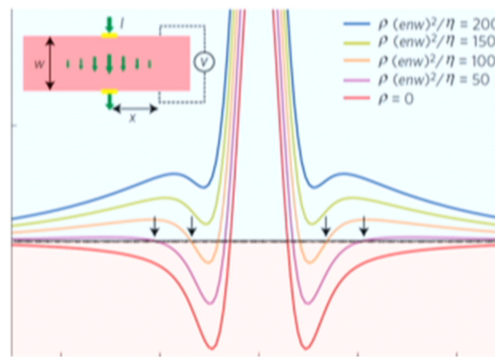
$$\phi_k(y) = b_k[\sinh k(y-w) + \sinh ky]$$

$$b_k = \frac{I\eta}{\pi(en)^2}\frac{k\tanh(kw/2)}{kw + \sinh kw + 2(\eta\alpha/ne)k^2\sinh kw}$$

The potential near the contact

$$\phi(x,y) = \beta\int_{-\infty}^{\infty} dk\, I(k)e^{ikx-y}|k|^{-1}$$

For point contact, negative singularity $-x^{-2}$ is replaced by weaker positive singularity $\ln(1/x)$.
For a finite contact, the potential is regular.



Viscous heat production $W = \eta \left(\partial_i v_j + \partial_j v_i\right)^2 = 2\eta |(\partial_x + i\partial_y)^2 \psi(x,y)|^2$

versus Joule heat production

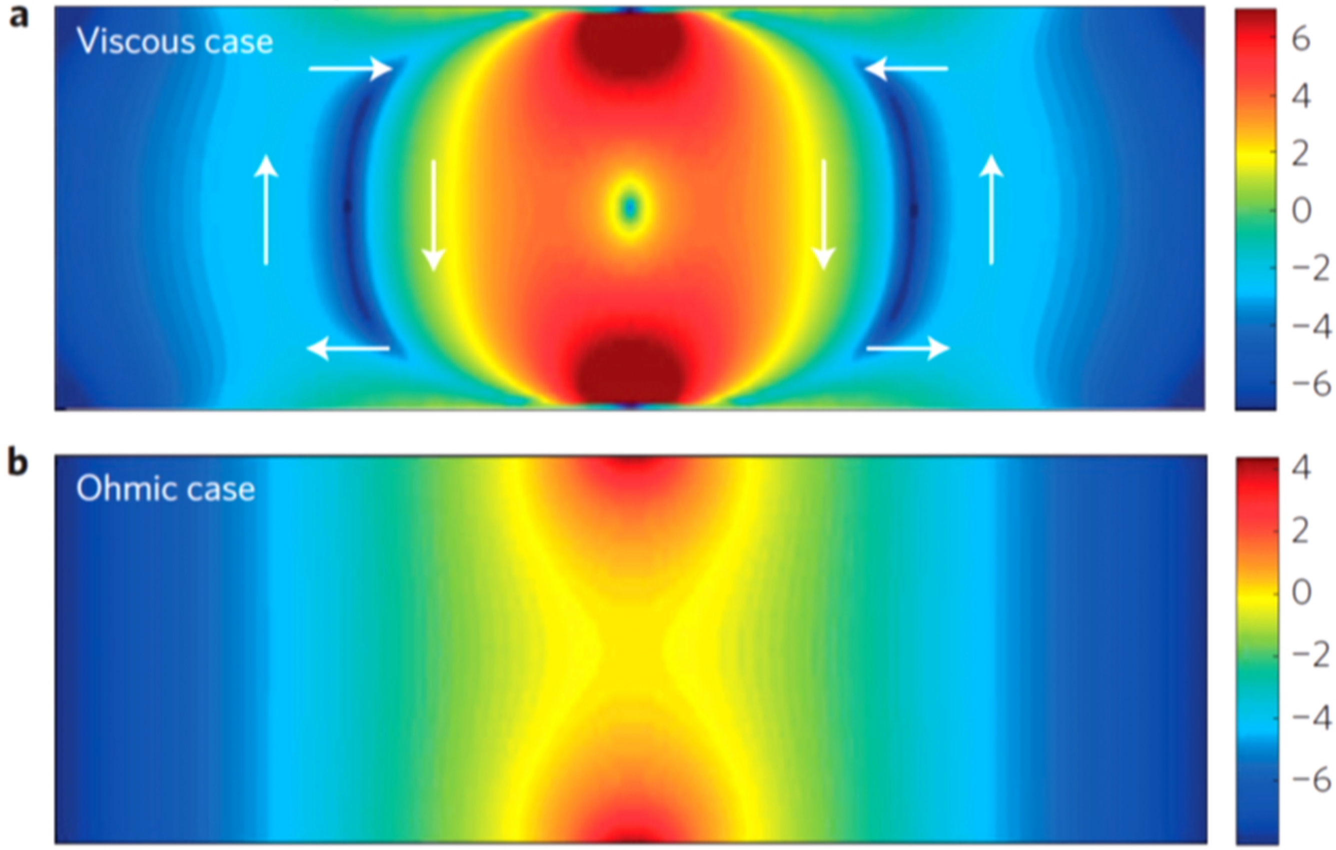


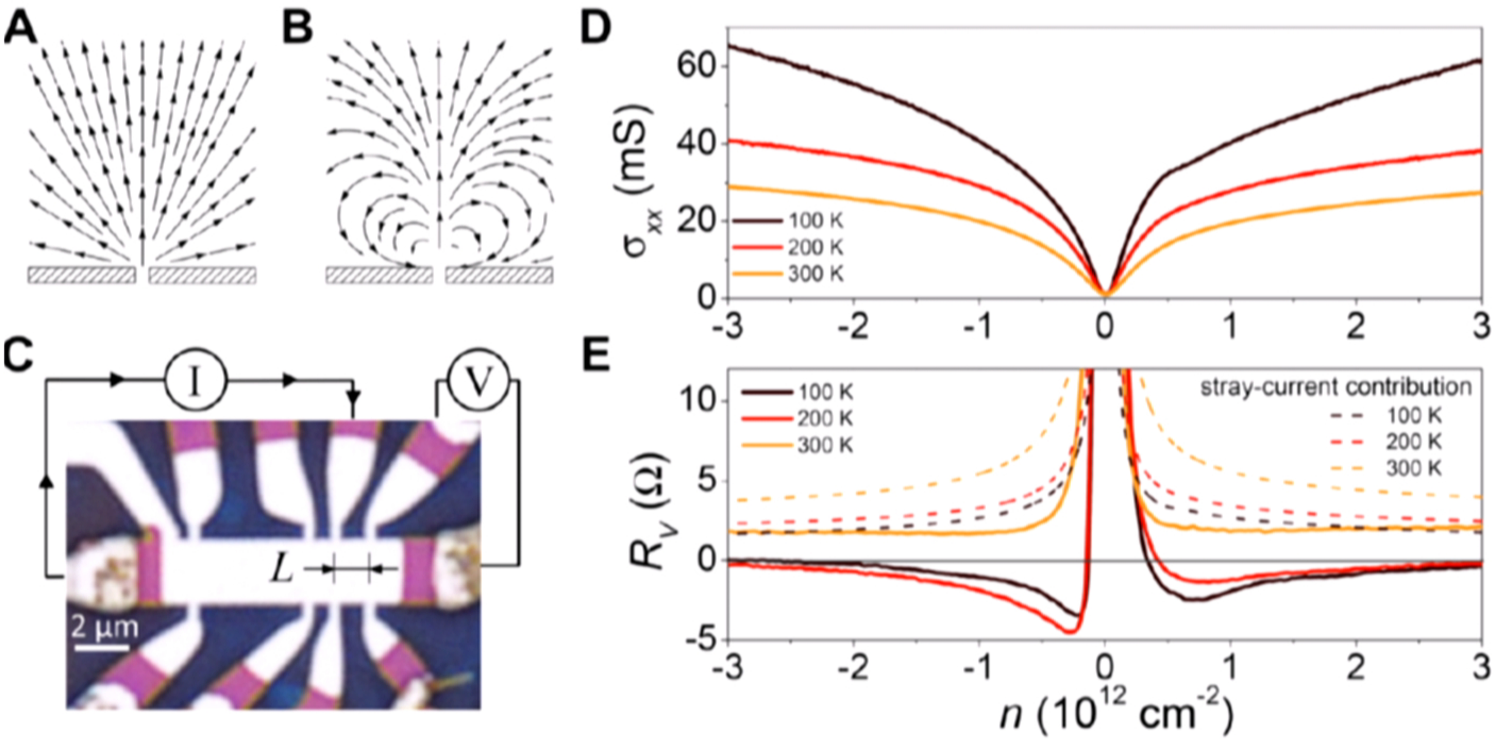
Figure 3 | Heating patterns for viscous and ohmic flows

Negative local resistance caused by viscous electron backflow in graphene

Science

11 February 2016

D. A. Bandurin,[1] I. Torre,[2] R. Krishna Kumar,[1,3] M. Ben Shalom,[1,4] A. Tomadin,[5] A. Principi,[6] G. H. Auton,[4] E. Khestanova,[1,4] K. S. Novoselov,[4] I. V. Grigorieva,[1] L. A. Ponomarenko,[1,3] A. K. Geim,[1*] M. Polini[7*]



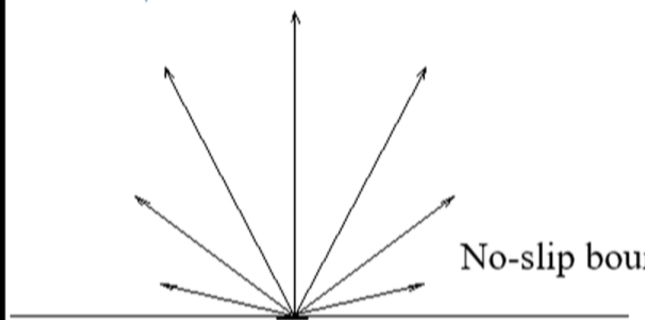
I = 0.3 µA; L = 1 µm; W = 2.5 µm

Measured viscosity ≈ 0.1 m² s⁻¹

Negative voltage needs neither vortex nor backflow

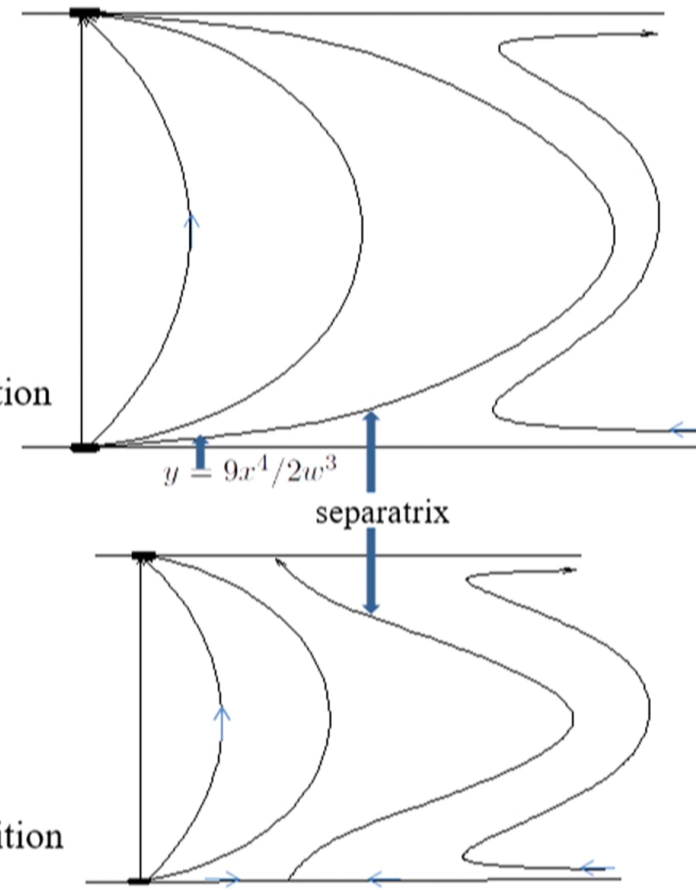
Flow in a half-plane

$$\psi = -2\theta + \sin 2\theta$$



$$\phi = \mathrm{Re}\, z^{-2} = -\cos 2\theta r^{-2}$$

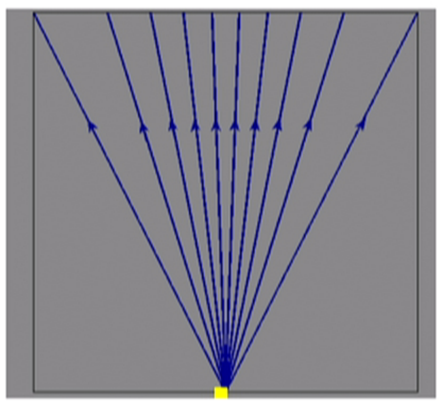
No-slip boundary condition

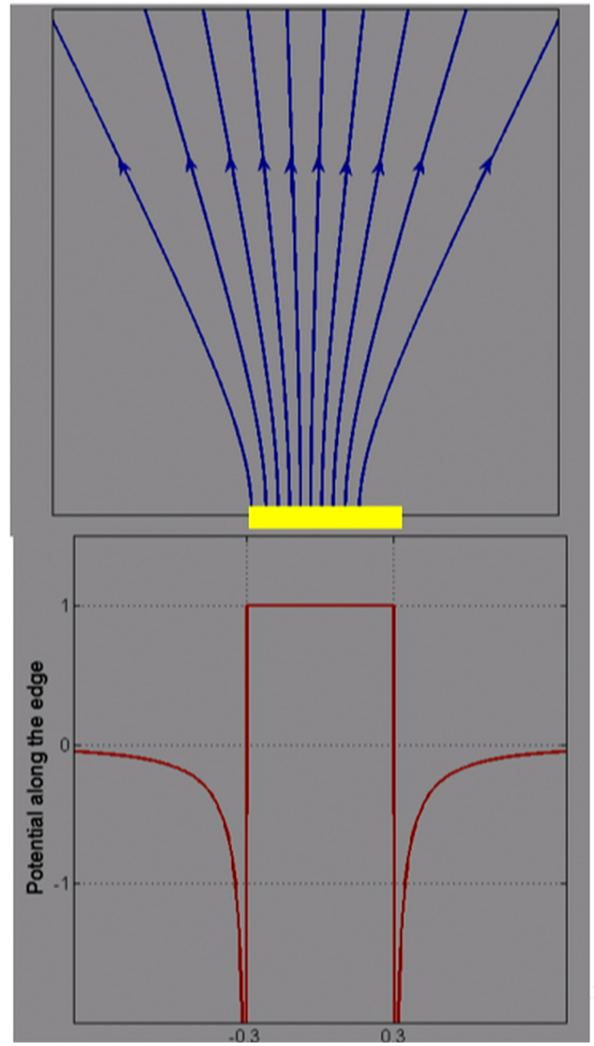


No-stress boundary condition

Current and voltage distribution in a VPC

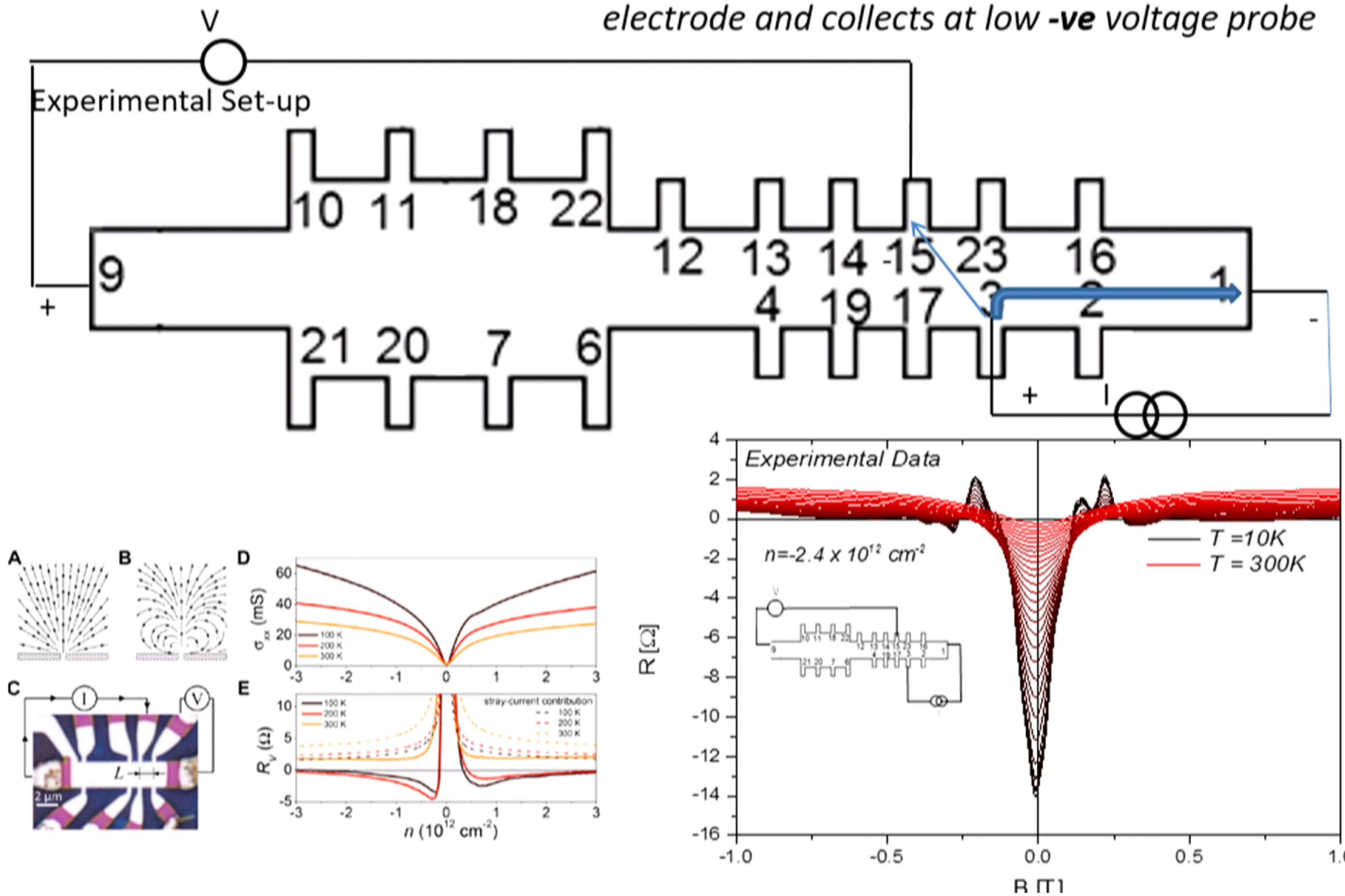
- Directional effect for current injected through VPC
- Low-current, high-vorticity regions
- Negative voltage outside VPC
- Divergence near VPC edges
- Up-converting DC-current transformer





Focussing in Ballistic Geometry

*Negative voltage drop measured as charge propagates ballistic ally from **+ve** current electrode and collects at low **-ve** voltage probe*

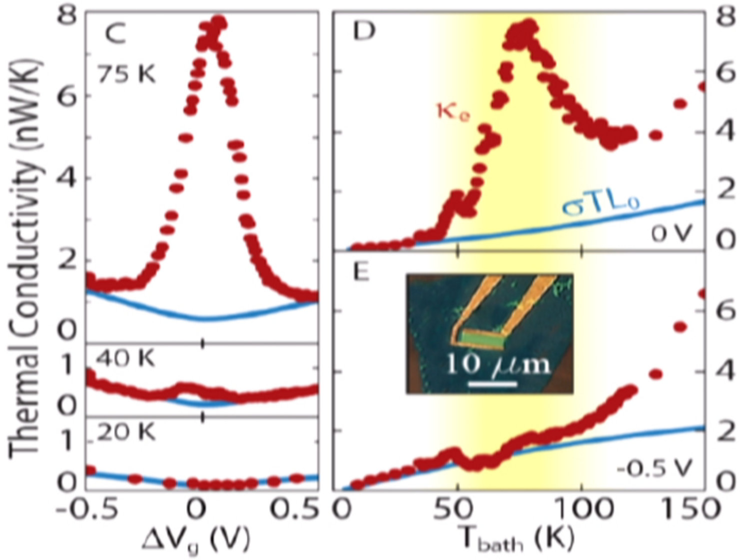


Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Science
10.1126/science.aad0343 (2016)

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] Kin Chung Fong[5*]

Interactions between particles in quantum many-body systems can lead to collective behavior described by hydrodynamics. One such system is the electron-hole plasma in graphene near the charge neutrality point, which can form a strongly coupled Dirac fluid. This charge neutral plasma of quasi-relativistic fermions is expected to exhibit a substantial enhancement of the thermal conductivity, thanks to decoupling of charge and heat currents within hydrodynamics. Employing high sensitivity Johnson noise thermometry, we report an order of magnitude increase in the thermal conductivity and the breakdown of the Wiedemann-Franz law in the thermally populated charge neutral plasma in graphene. This result is a signature of the Dirac fluid, and constitutes direct evidence of collective motion in a quantum electronic fluid.



Moral:

viscosity makes current-voltage relation nonlocal opening new possibilities

future "viscous electronics" may need *fluid mechanics*