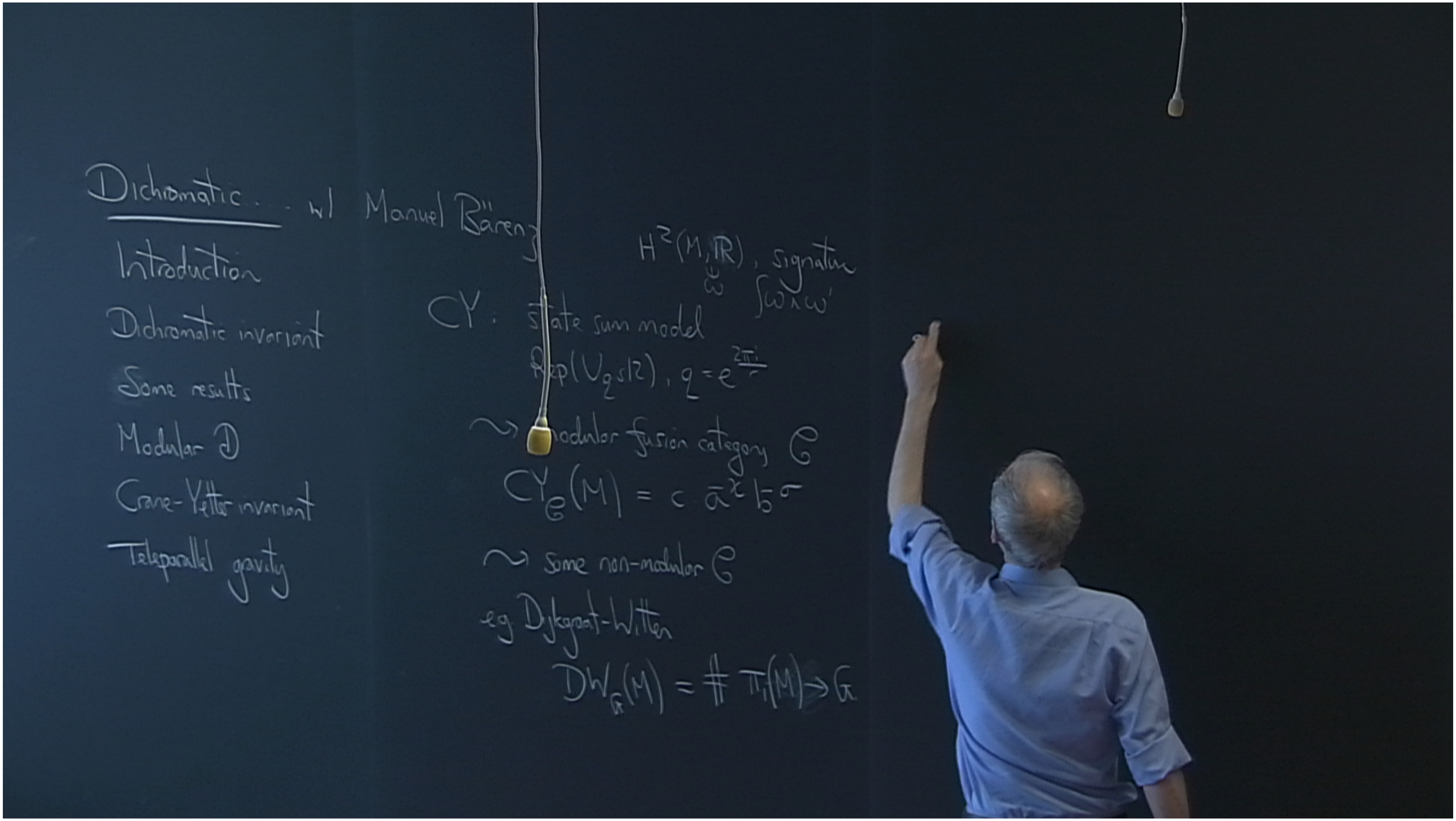
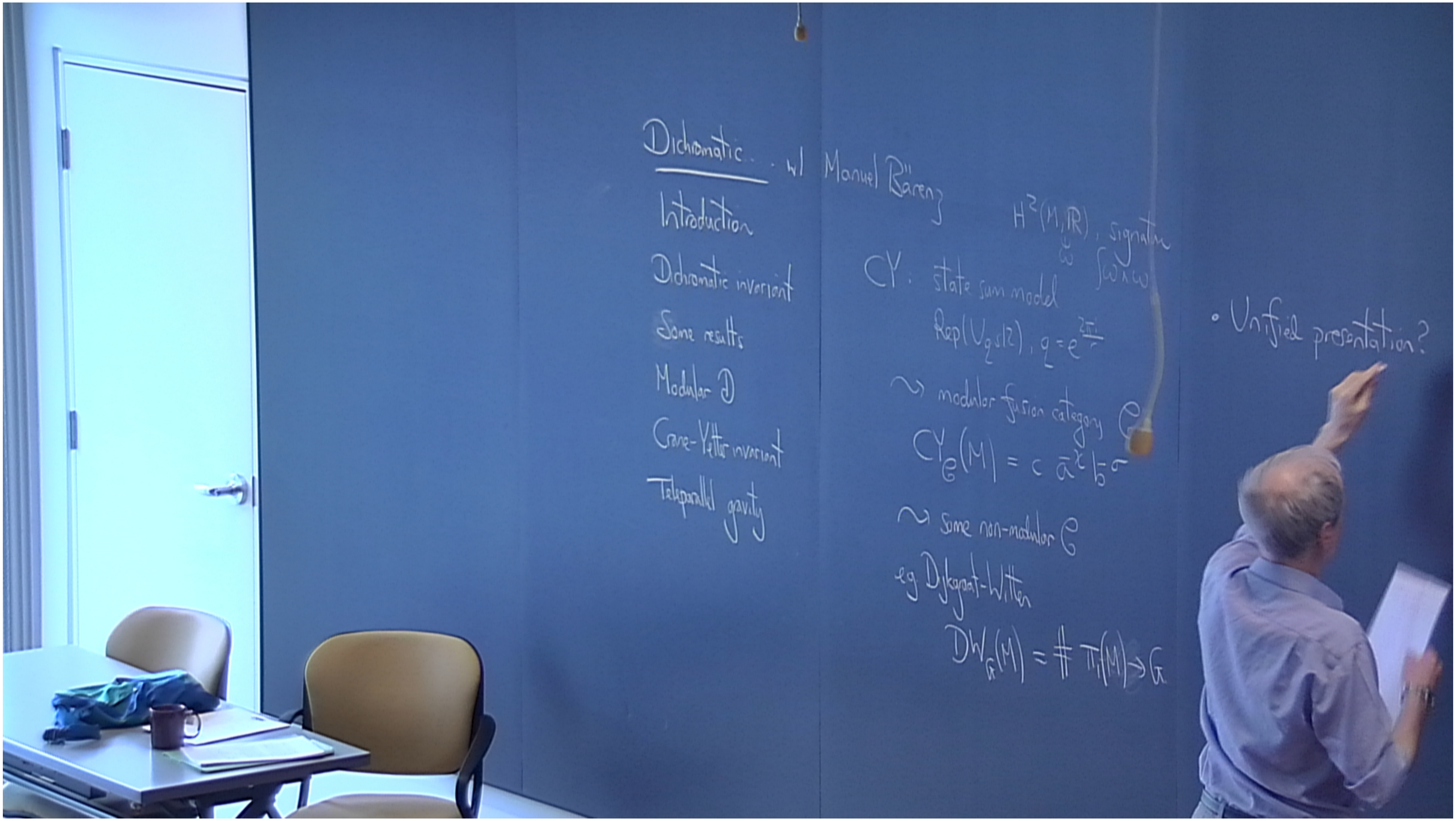
Title: TBA

Date: Apr 22, 2016 11:00 AM

URL: http://pirsa.org/16040064

Abstract:


Dichromatic ... w/ Manuel Bärenz
Introduction
Dichromatic invariant
Some results
Modular D
Crane-Yetter invariant
Teleparallel gravity
H^2(M, R), signature
ω ∫ω∧ω'
CY: state sum model
Rep(U_q sl2), q = e^(2πi/r)
modular fusion category C
CY_C(M) = c a^χ b^σ
some non-modular C
eg. Dijkgraaf-Witten
DW_G(M) = # π1(M) → G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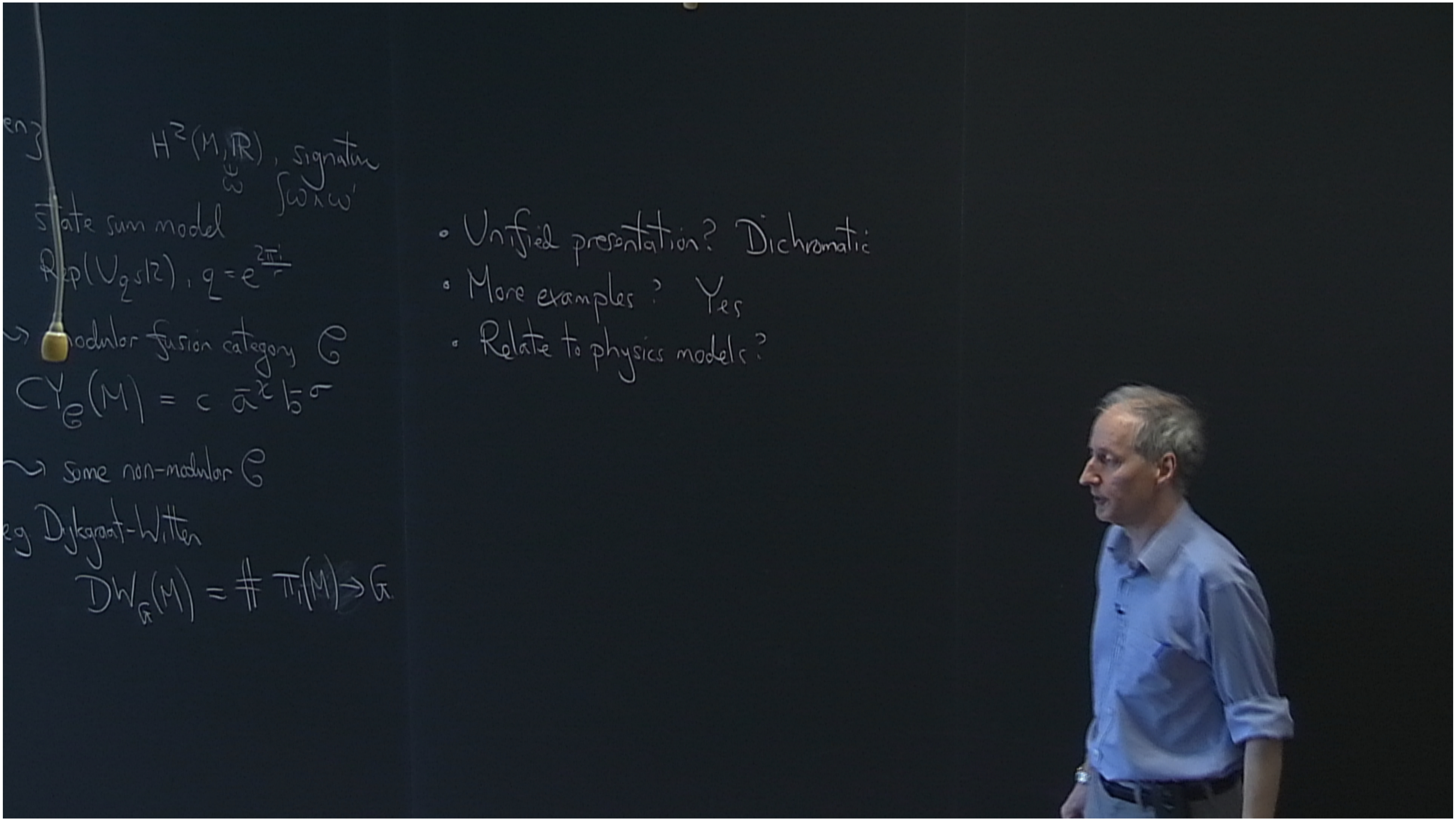
• Unified presentation?


$H^2(M, \mathbb{R})$, signature
$\int \omega \wedge \omega'$
state sum model
$\mathrm{Rep}(U_q sl2)$, $q = e^{\frac{2\pi i}{r}}$
modular fusion category $\mathcal{C}$
$CY_{\mathcal{C}}(M) = c\, \bar{a}^{\chi}\, b^{\sigma}$
some non-modular $\mathcal{C}$
eg Dijkgraaf-Witten
$DW_G(M) = \# \; \pi_1(M) \to G$
• Unified presentation? Dichromatic
• More examples? Yes
• Relate to physics models?

signature

$\omega \wedge \omega'$

$\mathcal{C}$

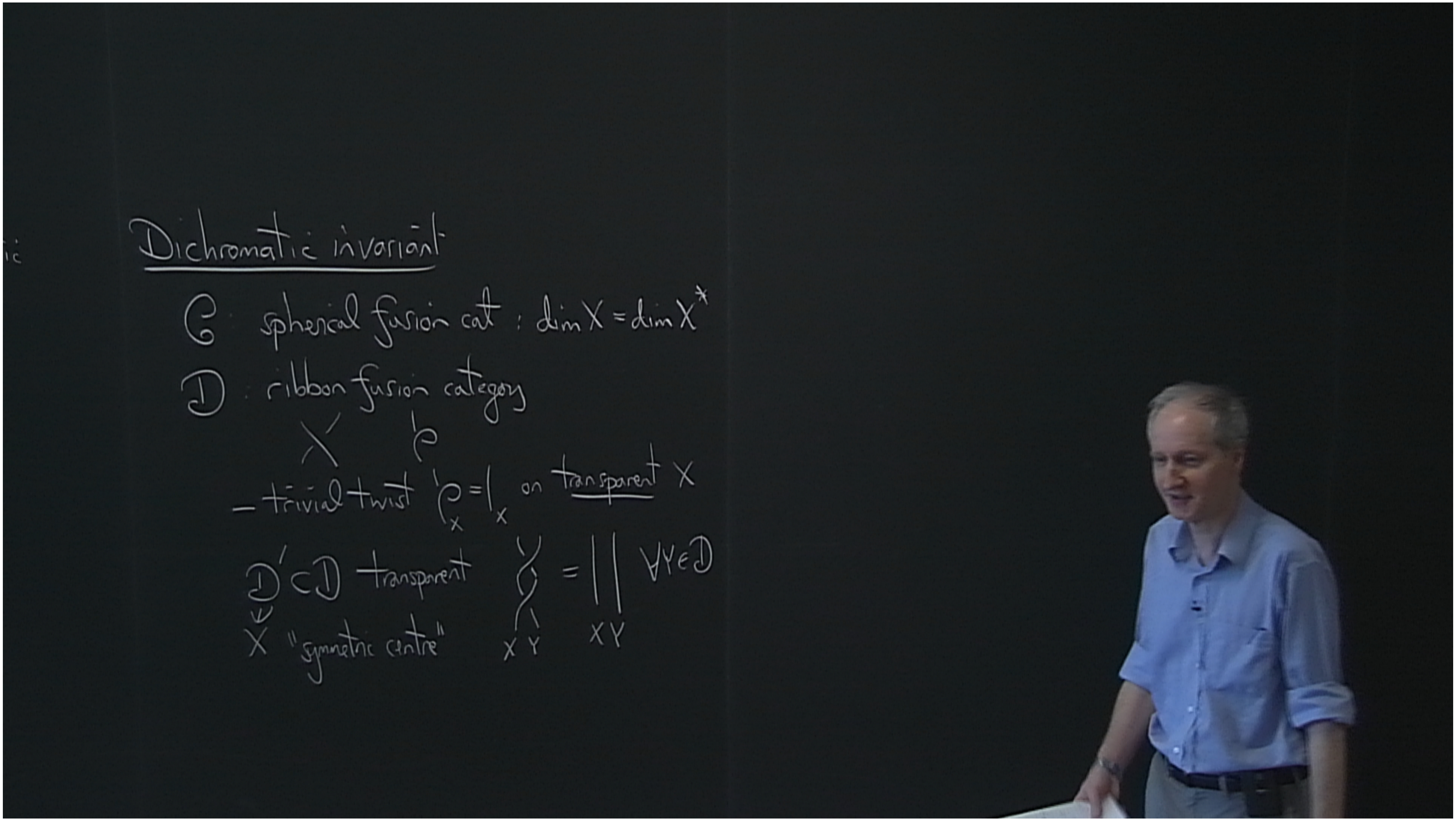
$\pi_1(M) \to G$

- Unified presentation? Dichromatic
- More examples? Yes
- Relate to physics models?
  - Walker-Wang
  - Teleparallel gravity

Dichromatic invariant

$\mathcal{C}$: spherical fusion cat : $\dim X = \dim X^*$

$\mathcal{D}$: ribbon fusion category


Dichromatic invariant
C : spherical fusion cat : dim X = dim X*
D : ribbon fusion category
– trivial twist = 1_X on transparent X
D′ ⊂ D transparent = ∀Y ∈ D
X Y X Y
"symmetric centre"

Dichromatic invariant

$\mathcal{C}$: spherical fusion cat : $\dim X = \dim X^*$

$\mathcal{D}$: ribbon fusion category

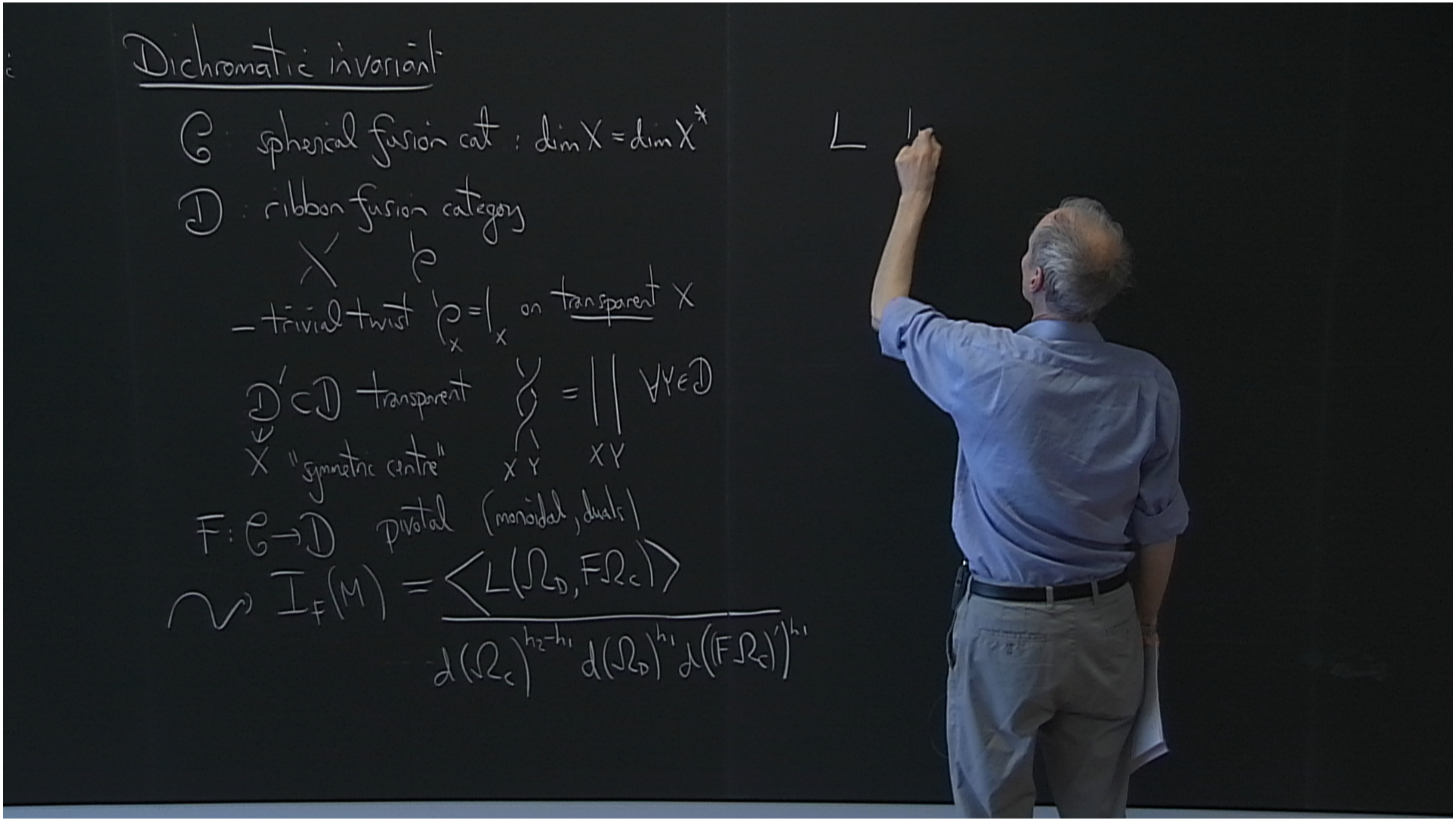
– trivial twist on transparent X

$\mathcal{D}' \subset \mathcal{D}$ transparent $= \;||\; \forall Y \in \mathcal{D}$

$X \in \mathcal{D}'$ "symmetric centre"

$F : \mathcal{C} \to \mathcal{D}$ pivotal (monoidal, duals)

$\rightsquigarrow I_F(M)$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– trivial twist = 1 on transparent X
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X "symmetric centre"
X Y
X Y
F : C → D pivotal (monoidal, duals)
I_F(M) = ⟨L(Ω_D, FΩ_C)⟩ / d(Ω_C)^{h_2−h_1} d(Ω_D)^{h_1} d((FΩ_C)')^{h_1}
L

riant

fusion cat : $\dim X = \dim X^*$

usion category

$\mathcal{C}$

wist $\theta_X = 1_X$ on transparent X

transparent $= \| \quad \forall Y \in \mathcal{D}$

$X \; Y \qquad X \; Y$

c centre"

pivotal (monoidal, duals)

$$) = \frac{\langle L(\Omega_{\mathcal{D}}, F\Omega_{\mathcal{C}}) \rangle}{d(\Omega_{\mathcal{C}})^{h_2-h_1} \, d(\Omega_{\mathcal{D}})^{h_1} \, d((F\Omega_{\mathcal{C}})')^{h_1}}$$

L link diagram
labelled with $\Omega_{\mathcal{D}}, F\Omega_{\mathcal{C}}$

$\langle \quad \rangle$ evaluation in $\mathcal{D}$

$$\Omega_{\mathcal{C}} = \sum_{X \in \Lambda_{\mathcal{C}}} d(X)\, X \, , \quad d(X) = \bigcirc_X$$

simple objects

Dichromatic invariant

$\mathcal{C}$: spherical fusion cat : $\dim X = \dim X^*$

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– trivial twist $\theta_X = 1_X$ on transparent X

$\mathcal{D}' \subset \mathcal{D}$ transparent $= \,||\, \forall Y \in \mathcal{D}$ $\quad X\,Y \qquad X\,Y$

$\mathcal{D}'$ "symmetric centre"

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L link diagram

labelled with Ω_D, $F\Omega_C$

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$\Lambda_{\mathcal{C}}$: simple objects

$$F\Omega_{\mathcal{C}} = \sum_{X \in \Lambda_{\mathcal{C}}} d(X) F(X) = \sum_{Y \in \Lambda_D} c_Y Y$$

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Dichromatic invariant

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L link diagram
labelled with Ω_D, $F\Omega_C$

$\langle \quad \rangle$ evaluation in $\mathcal{D}$

$$\Omega_{\mathcal{C}} = \sum_{X \in \Lambda_{\mathcal{C}}} d(X)\, X, \quad d(X) = \bigcirc_X$$

$$\rightsquigarrow d(\Omega_{\mathcal{C}}) = \sum d(X)^2$$

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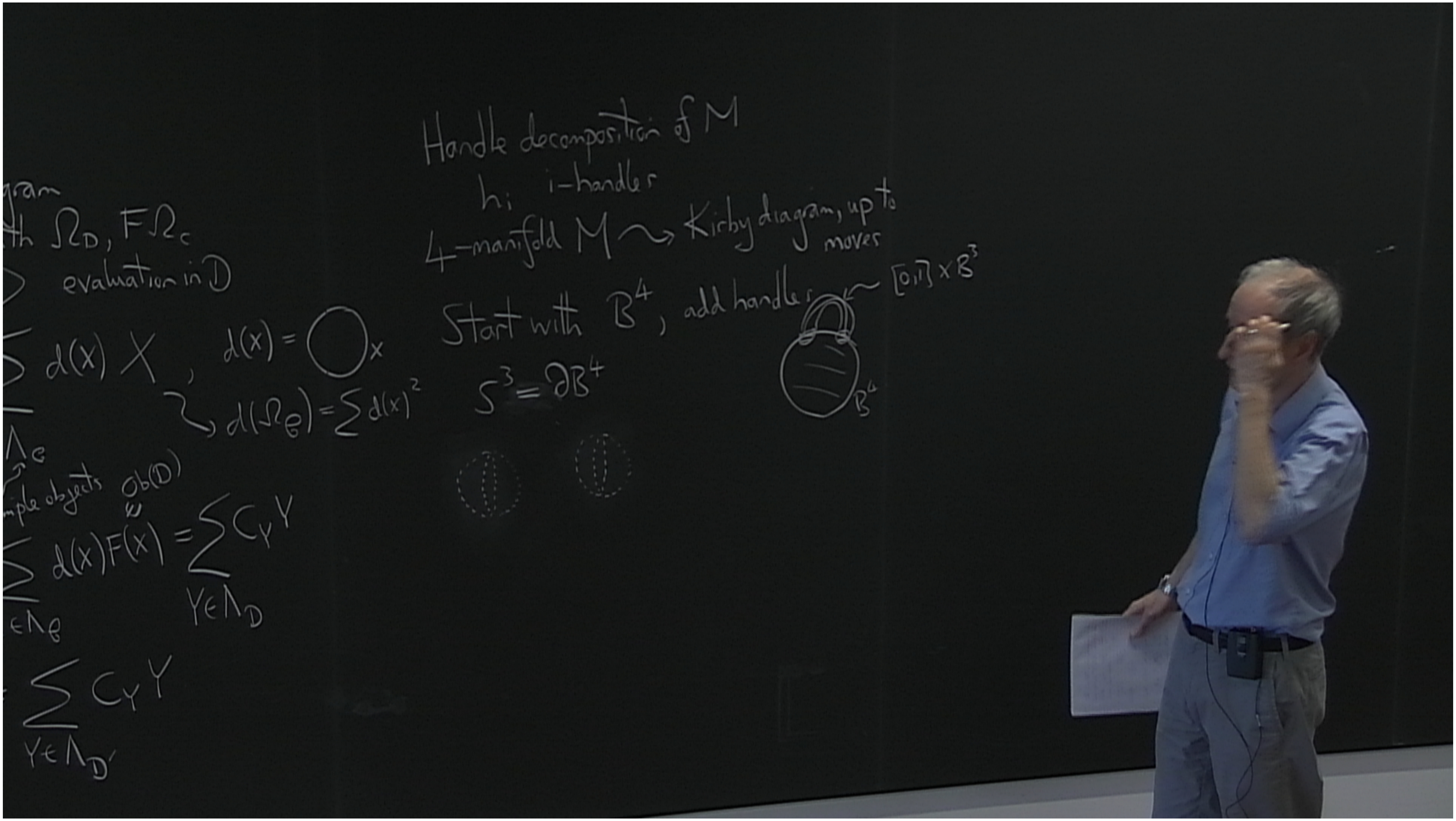
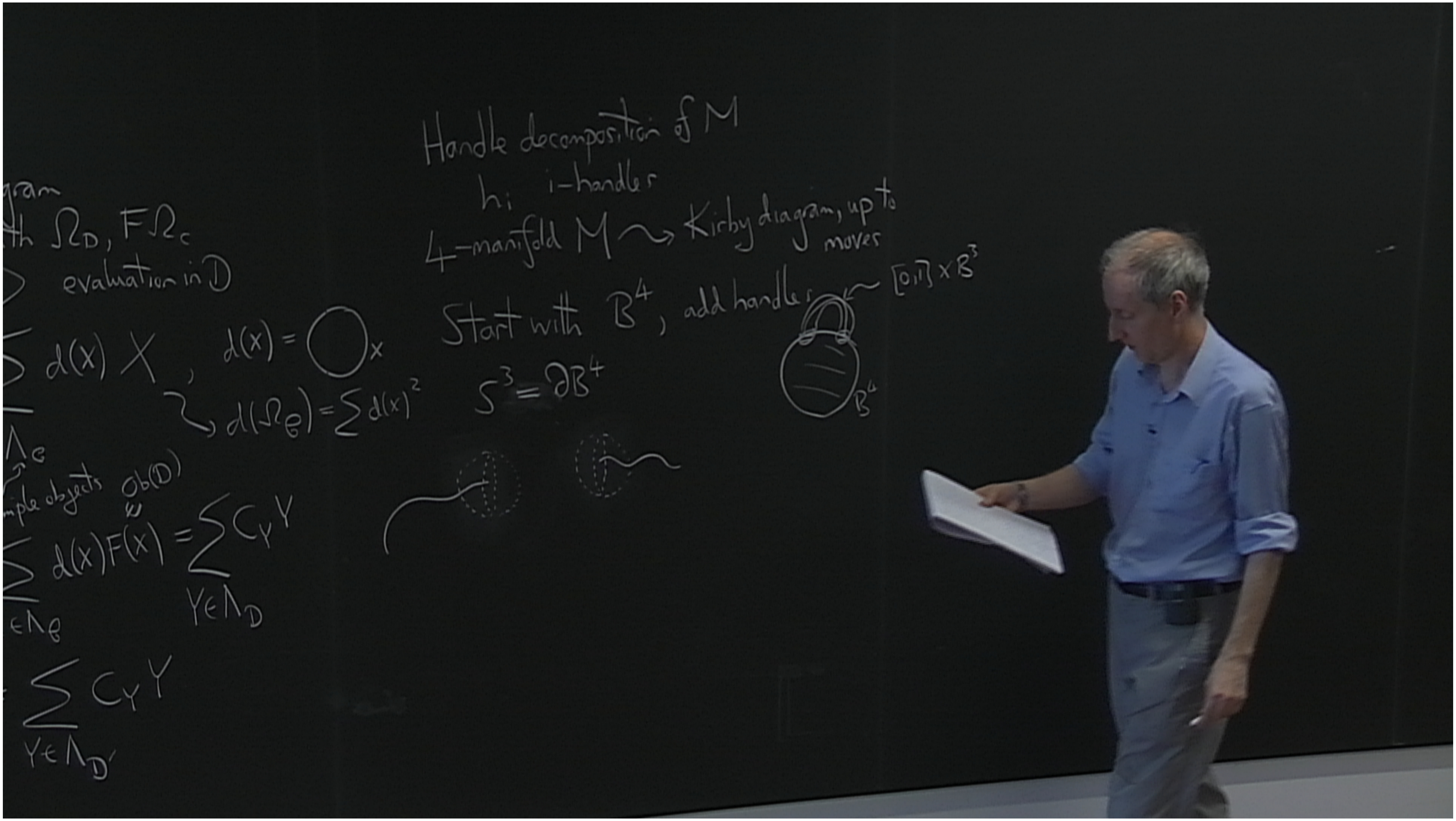

Handle decomposition of M
h_i i-handles
4-manifold M ⇝ Kirby diagram, up to moves
Start with B^4, add handles
$[0,1] \times B^3$
B^4
$S^3 = \partial B^4$
evaluation in D
$d(x) = \bigcirc_x$
$d(\Omega_\mathcal{B}) = \sum d(x)^2$
Ob(D)
$d(x)F(x) = \sum C_Y Y$
$Y \in \Lambda_D$
$\sum C_Y Y$
$Y \in \Lambda_{D'}$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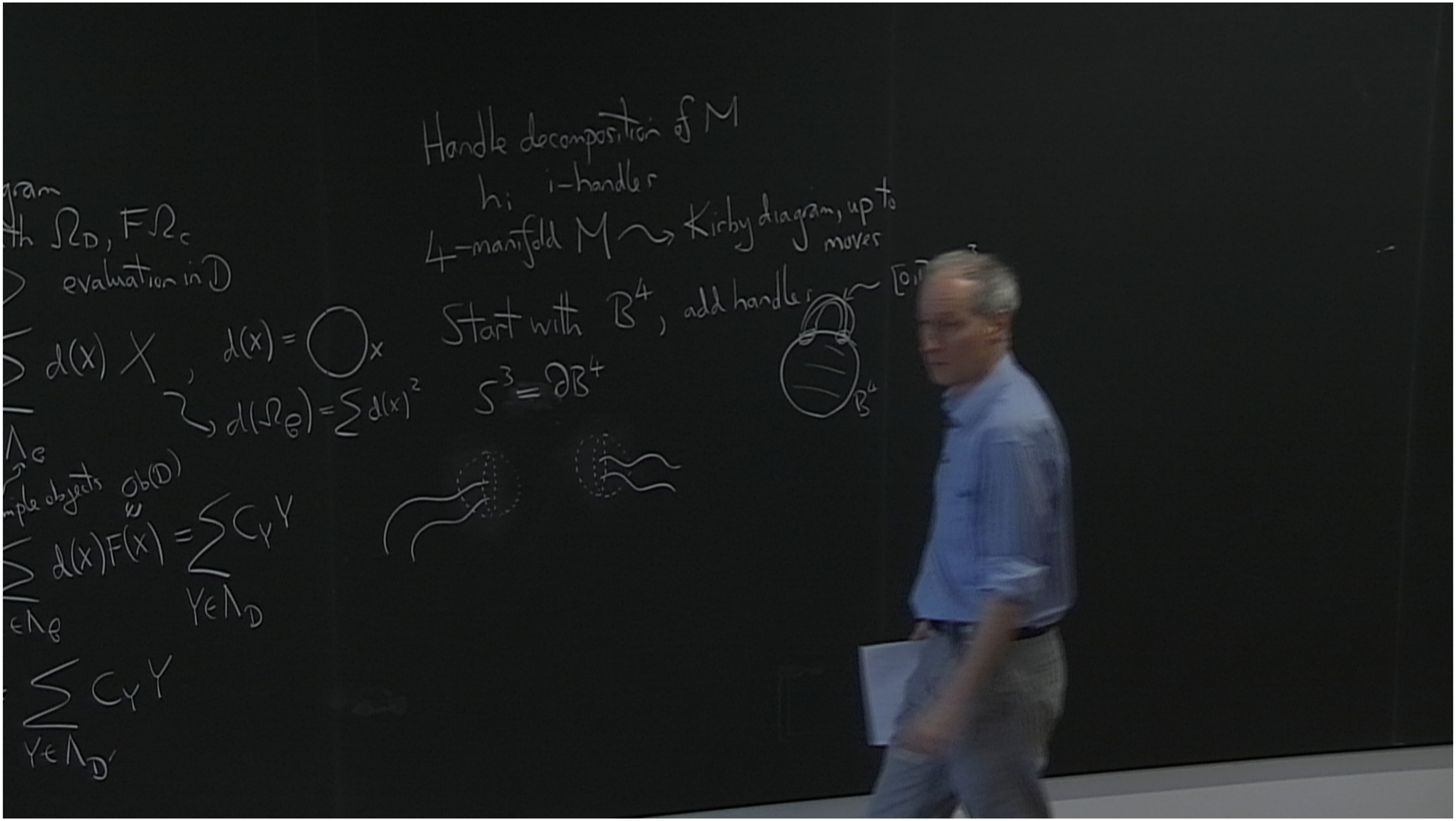

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Λ_C
Ob(D)
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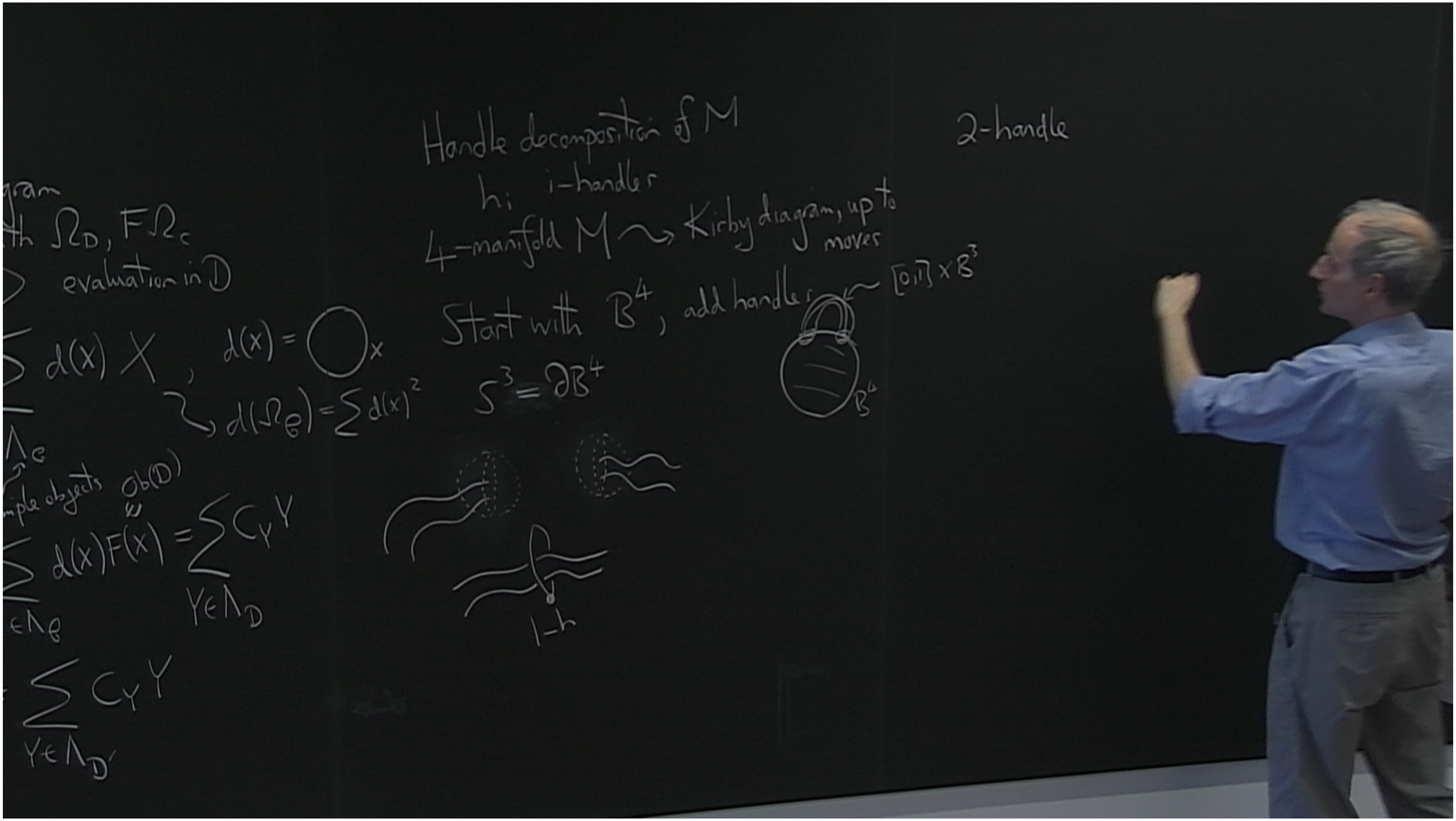
Y∈Λ_D
Σ C_Y Y
Y∈Λ_D'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B^4
1-h
2-handle
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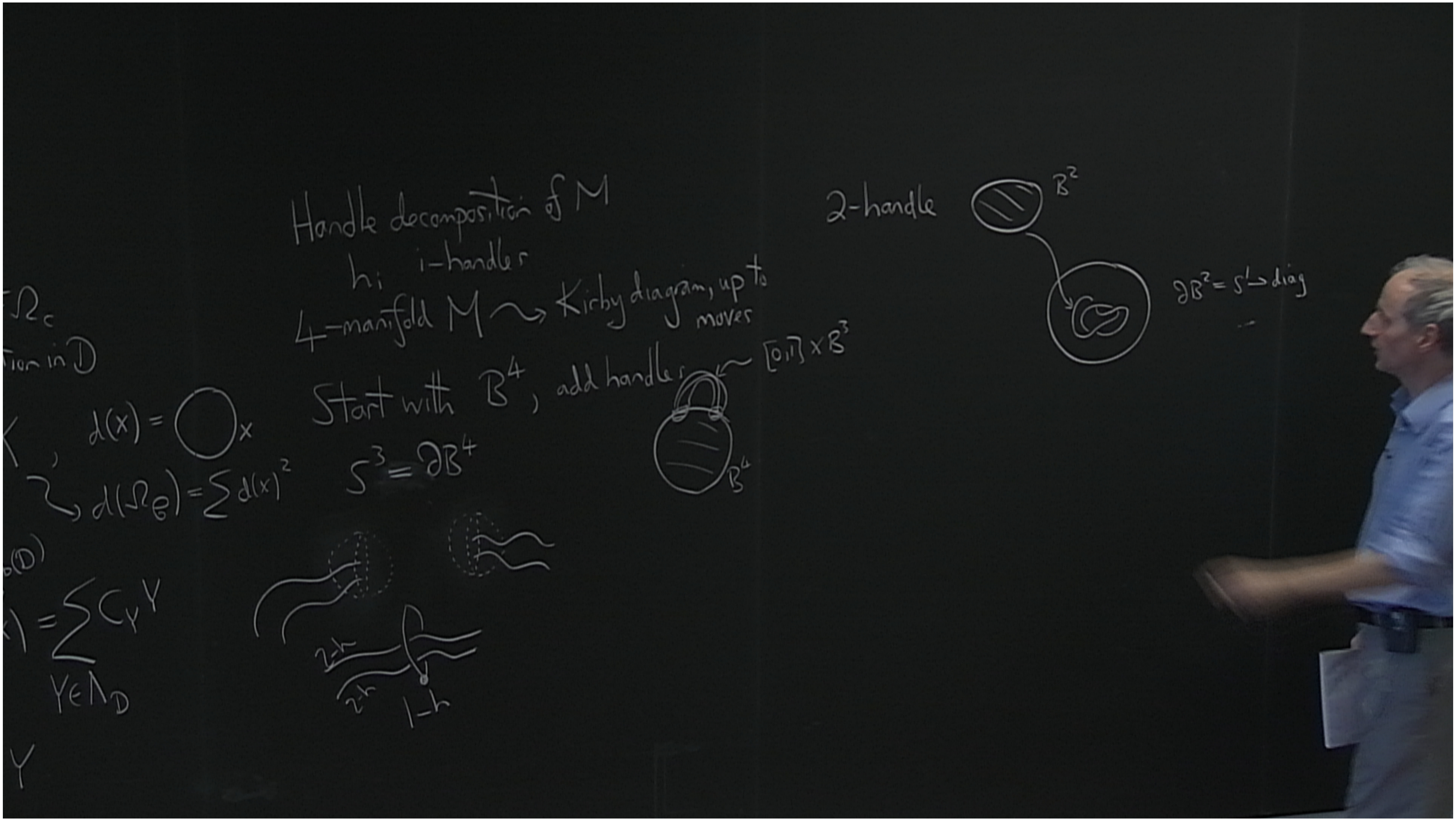
simple objects
Ob(D)
$\sum d(x)F(x) = \sum_{Y \in \Lambda_D} c_Y Y$
$\sum_{Y \in \Lambda_{D'}} c_Y Y$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Start with B^4, add handles
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B^4
2-handle
B^2
$\partial B^2 = S^1 \hookrightarrow$ diag
$d(x) =$
$d(\Omega_e) = \sum d(x)^2$
$Y \in \Lambda_D$
1-h
2-h

Ω_c

tion in D

$d(x) = \bigcirc_x$

$d(\Omega_\mathcal{C}) = \sum d(x)^2$

(D)

$) = \sum_{Y \in \Lambda_D} C_Y Y$

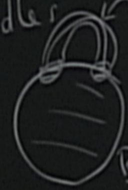
Y

Handle decomposition of M

h_i i-handles

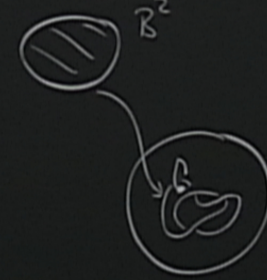
4-manifold M $\rightsquigarrow$ Kirby diagram, up to moves

Start with B^4, add handles $\leftarrow [0,1] \times B^3$

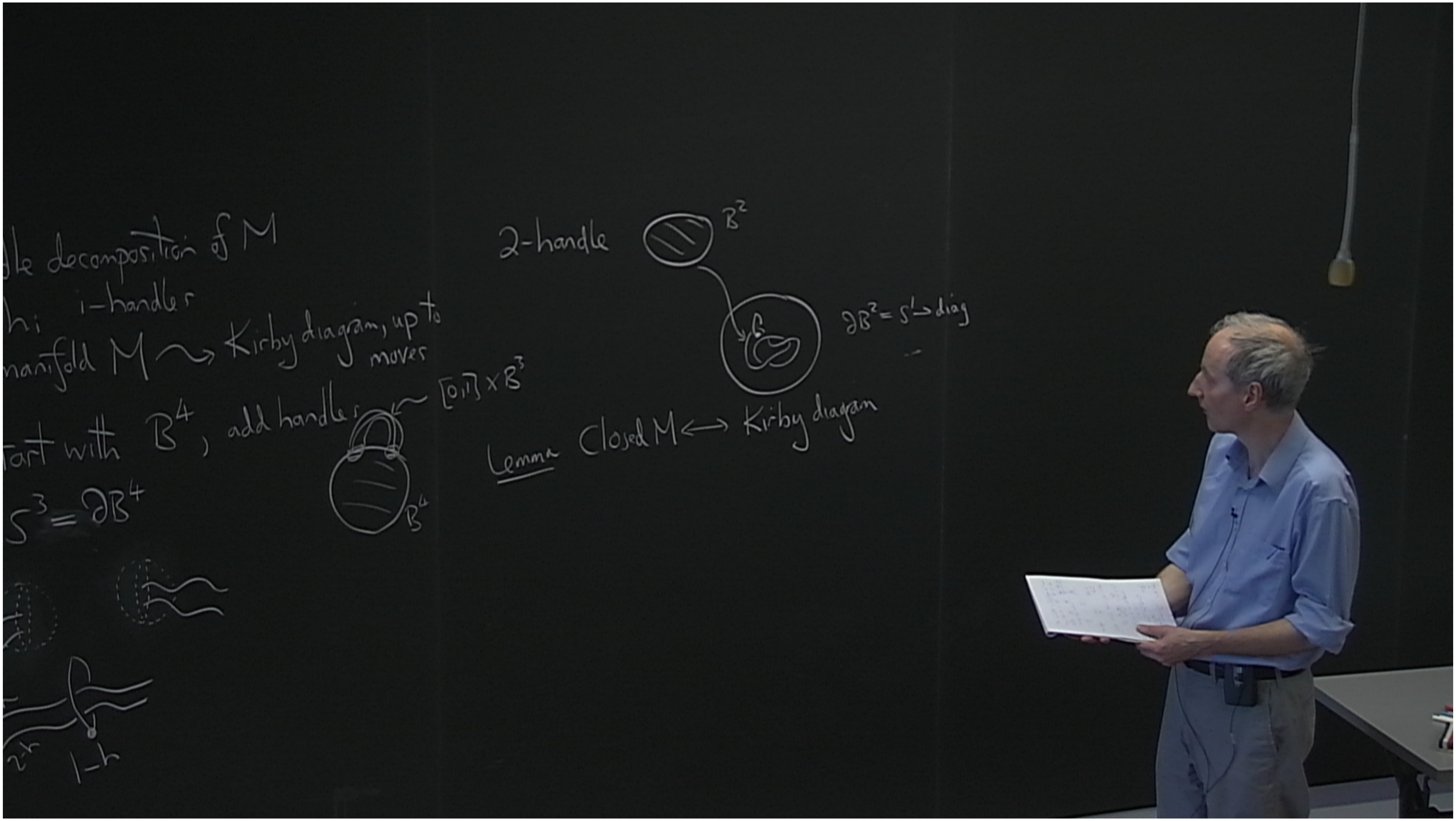


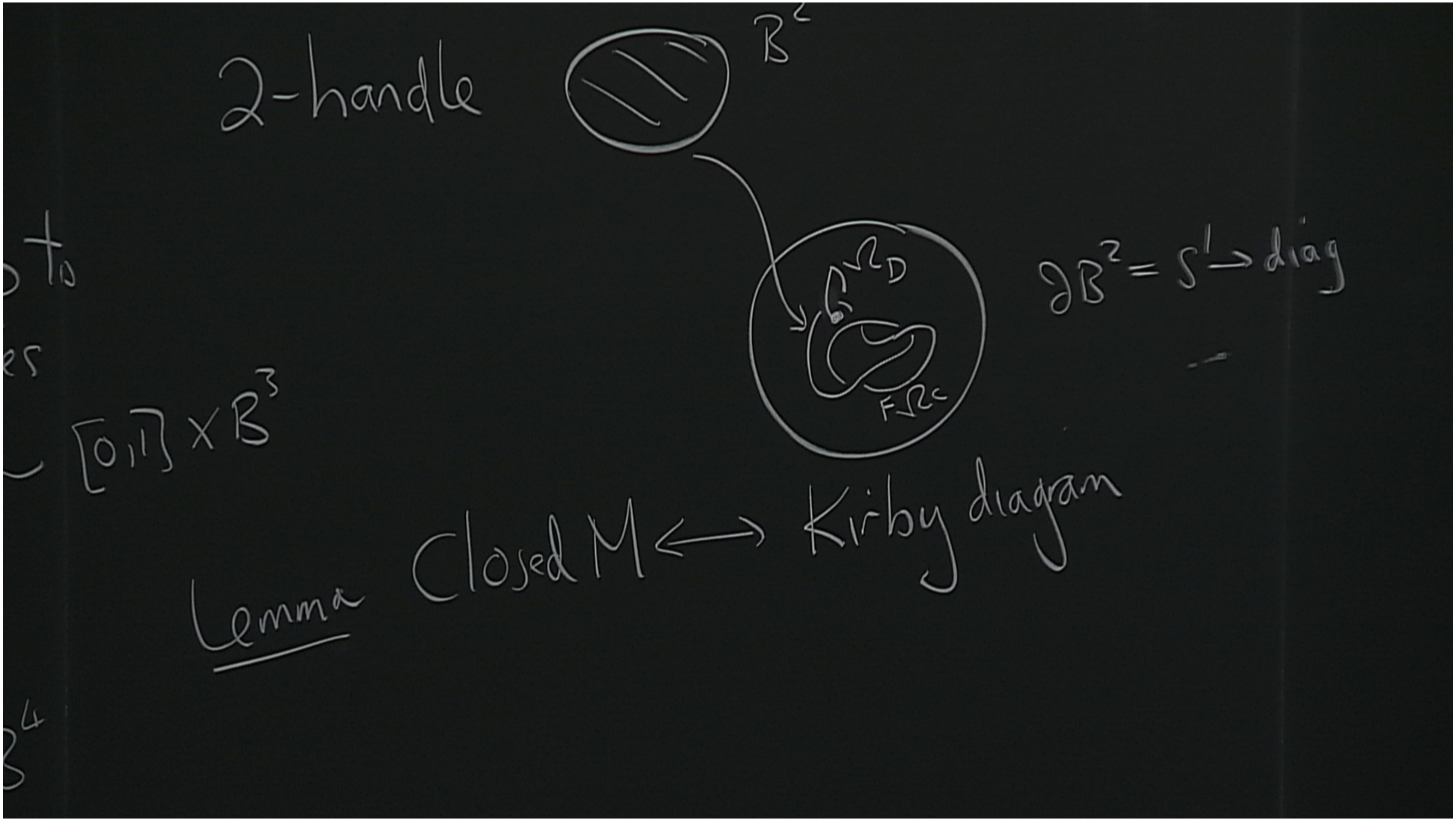
$S^3 = \partial B^4$

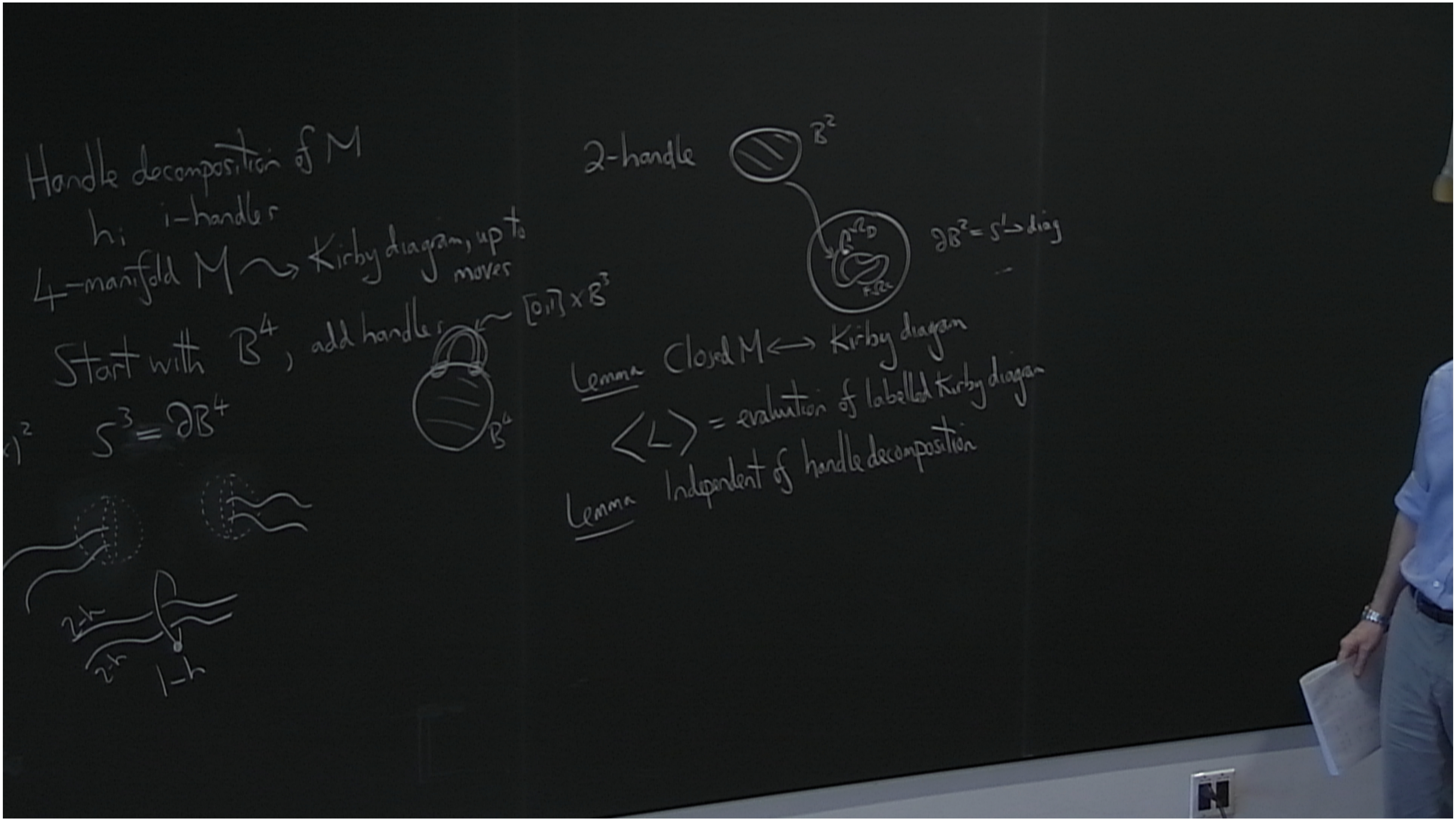
2-handle



$\partial B^2 = S^1 \hookrightarrow$ diag


decomposition of M
i-handles
manifold M ⟶ Kirby diagram, up to moves
with B^4, add handles
[0,1] × B^3
S^3 = ∂B^4
B^4
1-h
2-handle
B^2
∂B^2 = S^1 → diag
Lemma Closed M ⟷ Kirby diagram


2-handle
B^2
∂B^2 = S^1 → diag
[0,1] × B^3
Lemma
Closed M ⟷ Kirby diagram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B^4
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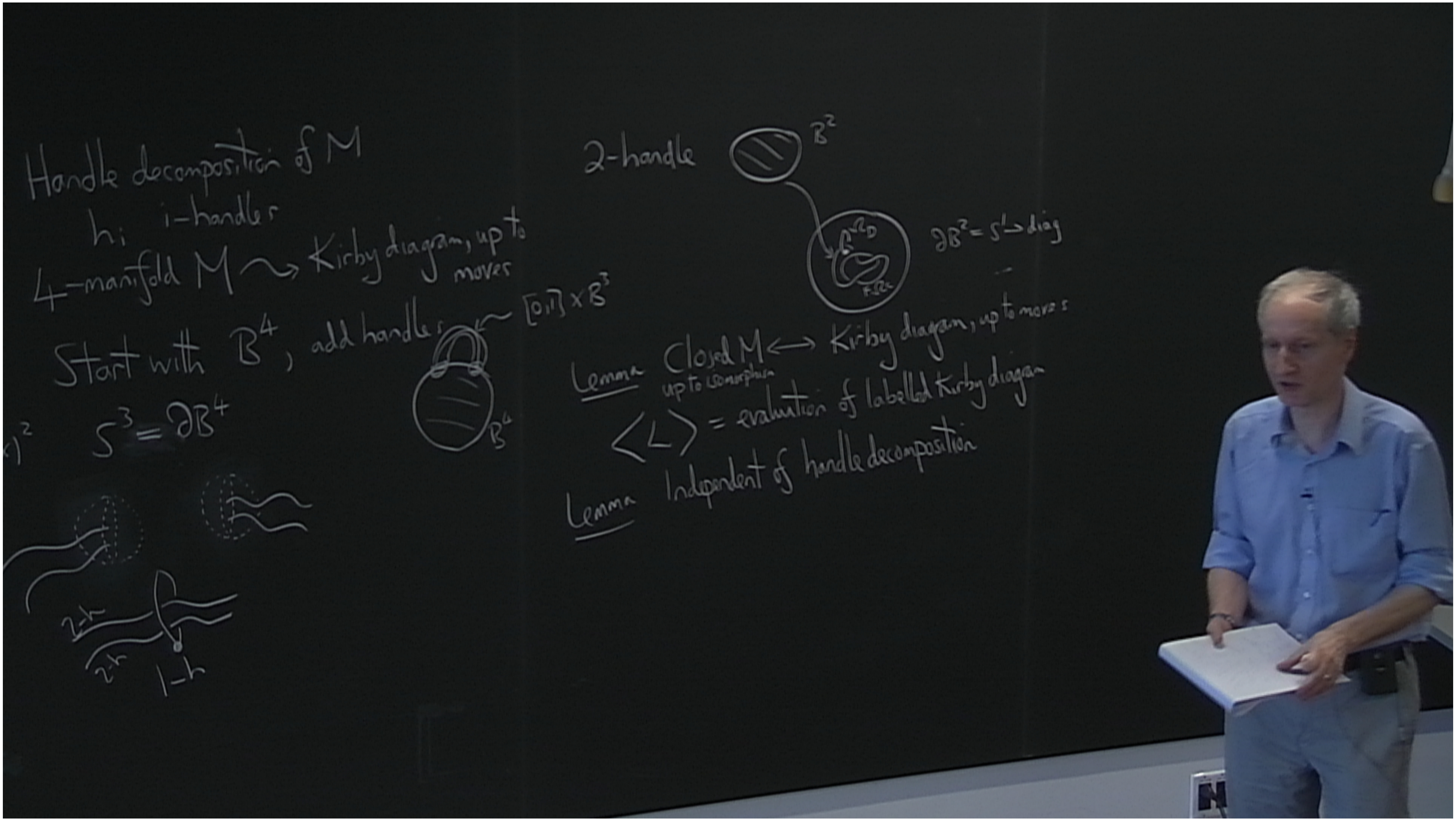
2-h
1-h
2-handle
B^2
∂B^2 = S^1 → diag
Lemma Closed M ⟷ Kirby diagram
⟨L⟩ = evalution of labelled Kirby diagram
Lemma Independent of handle decomposition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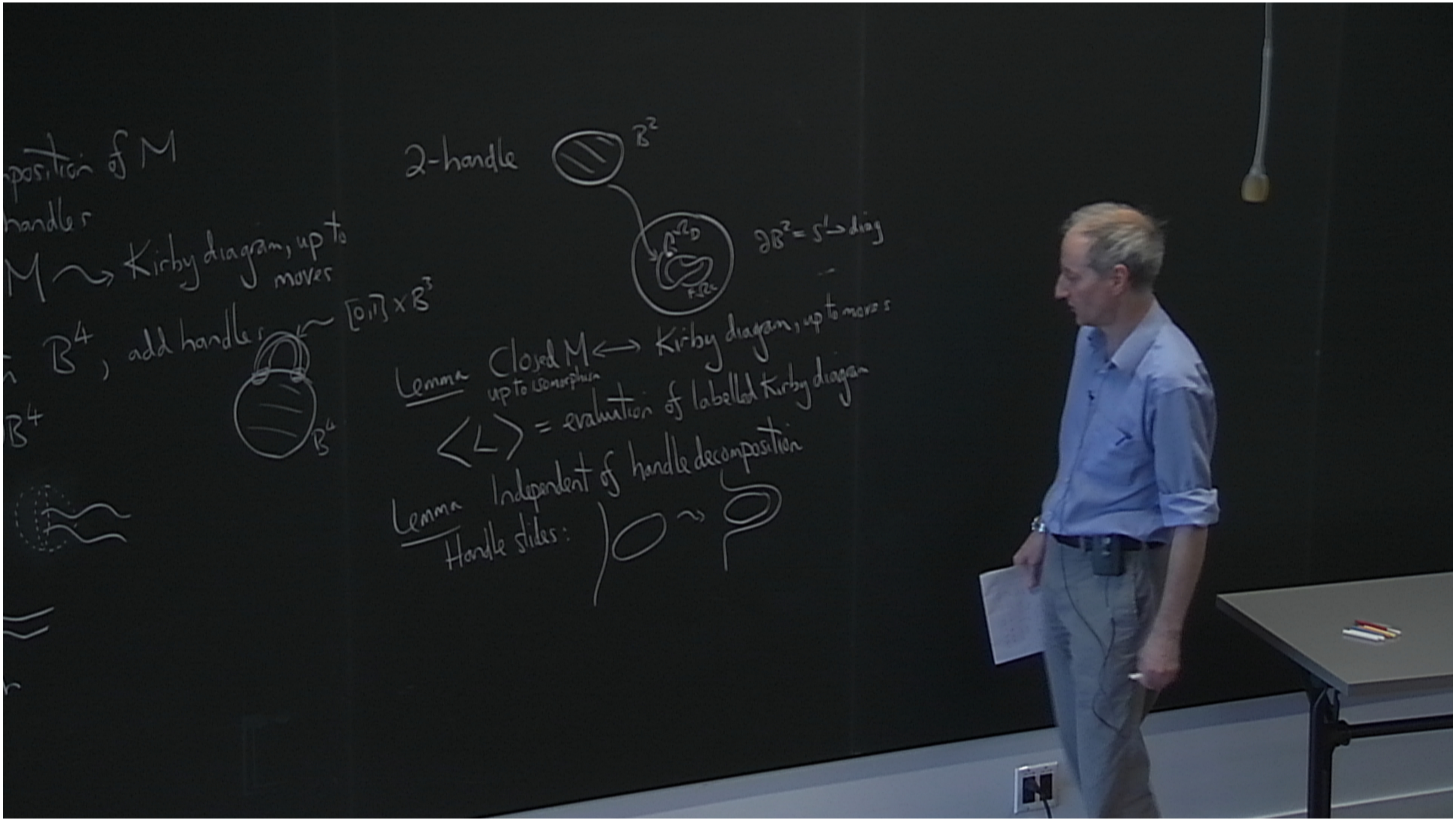
2-h
2-h
1-h
2-handle
B^2
∂B^2 = S^1 -> diag
Lemma Closed M <-> Kirby diagram, up to moves
up to isomorphism
<L> = evaluation of labelled Kirby diagram
Lemma Independent of handle decomposition



position of M
handles
M ⇝ Kirby diagram, up to moves
B^4, add handles
[0,1] × B^3
B^4
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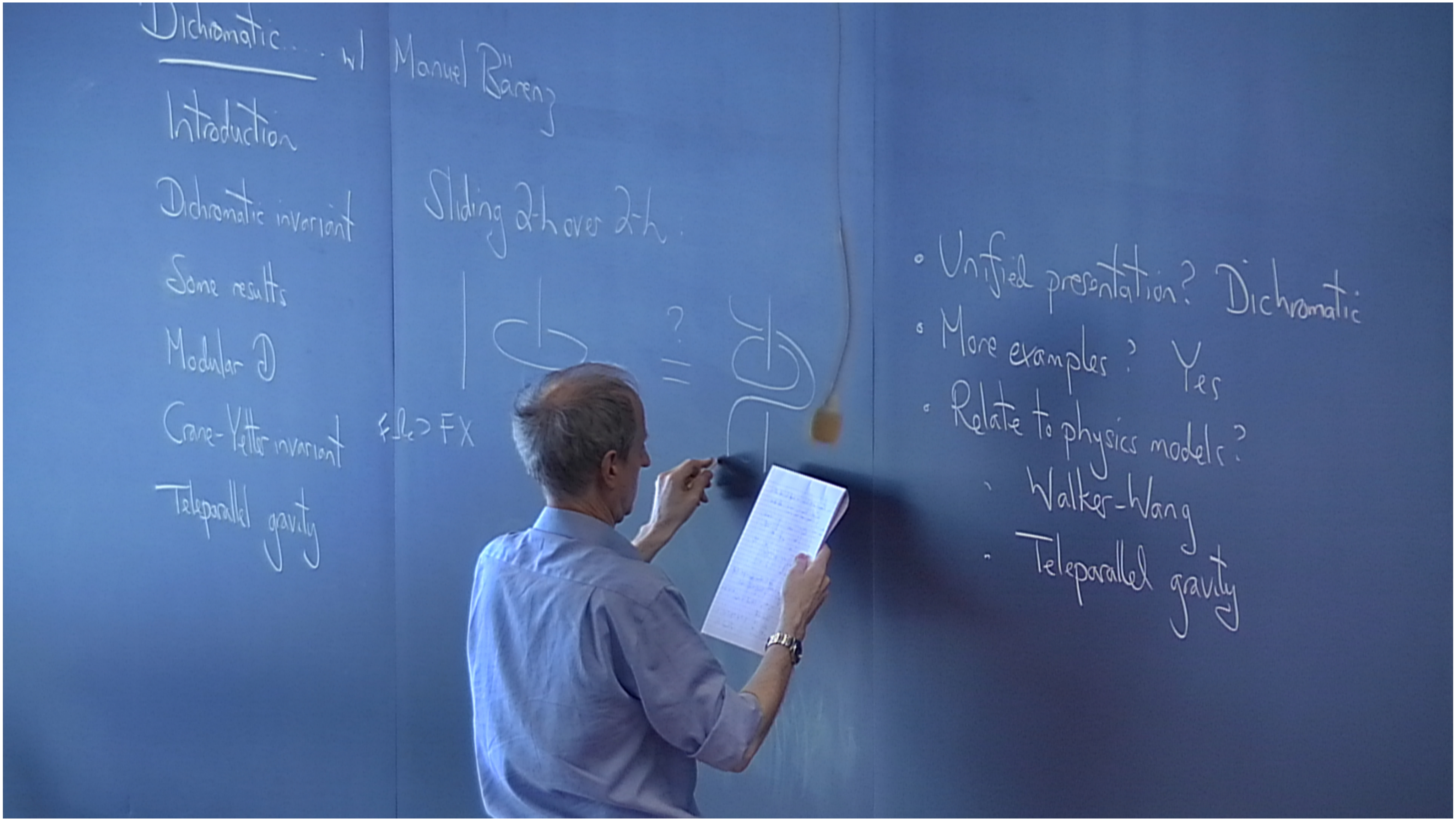
Handle slides: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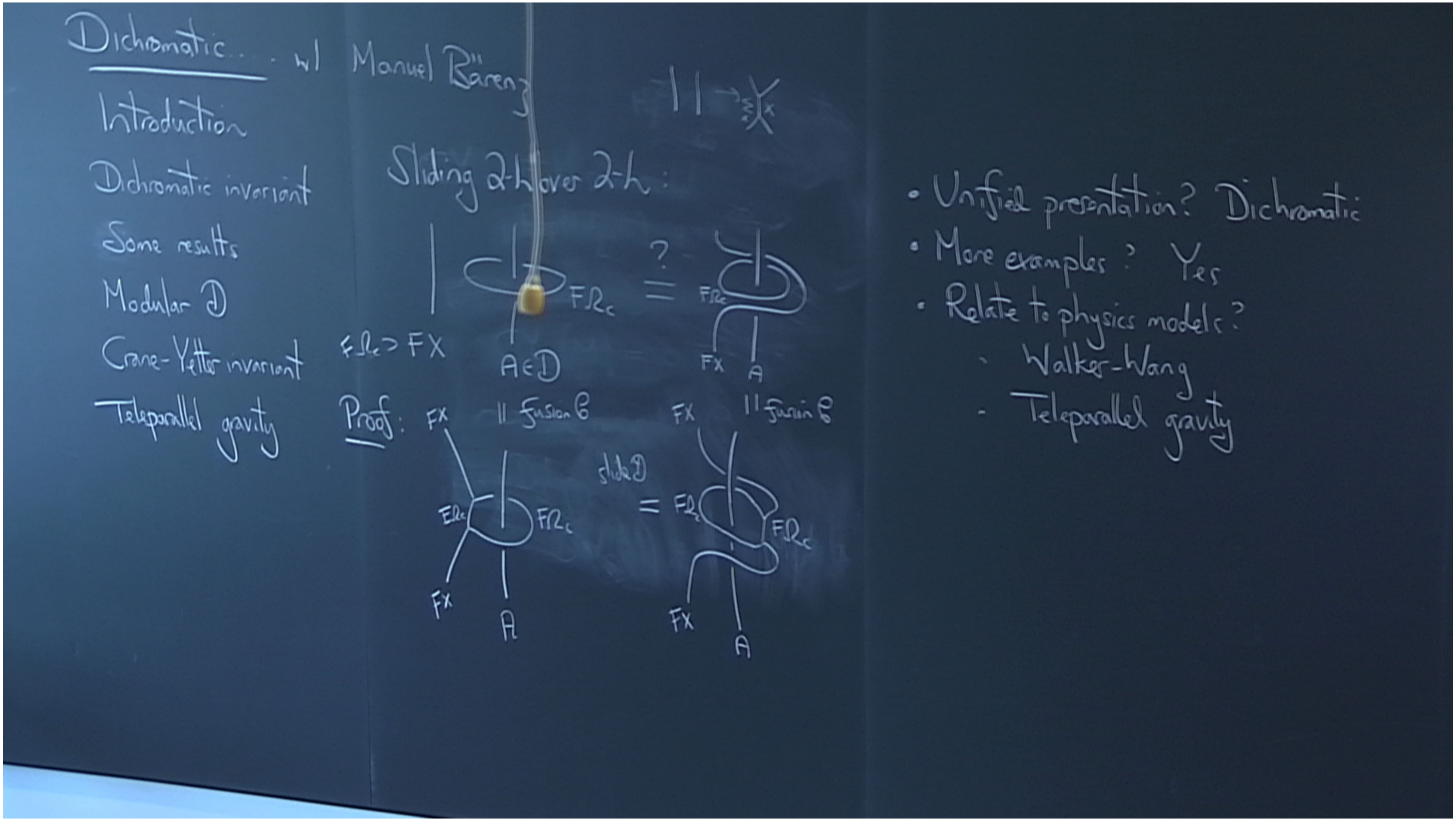
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?
=
FΩc
A∈D
FX
A
FΩc ⊃ FX
Proof:
|| fusion C
slide D
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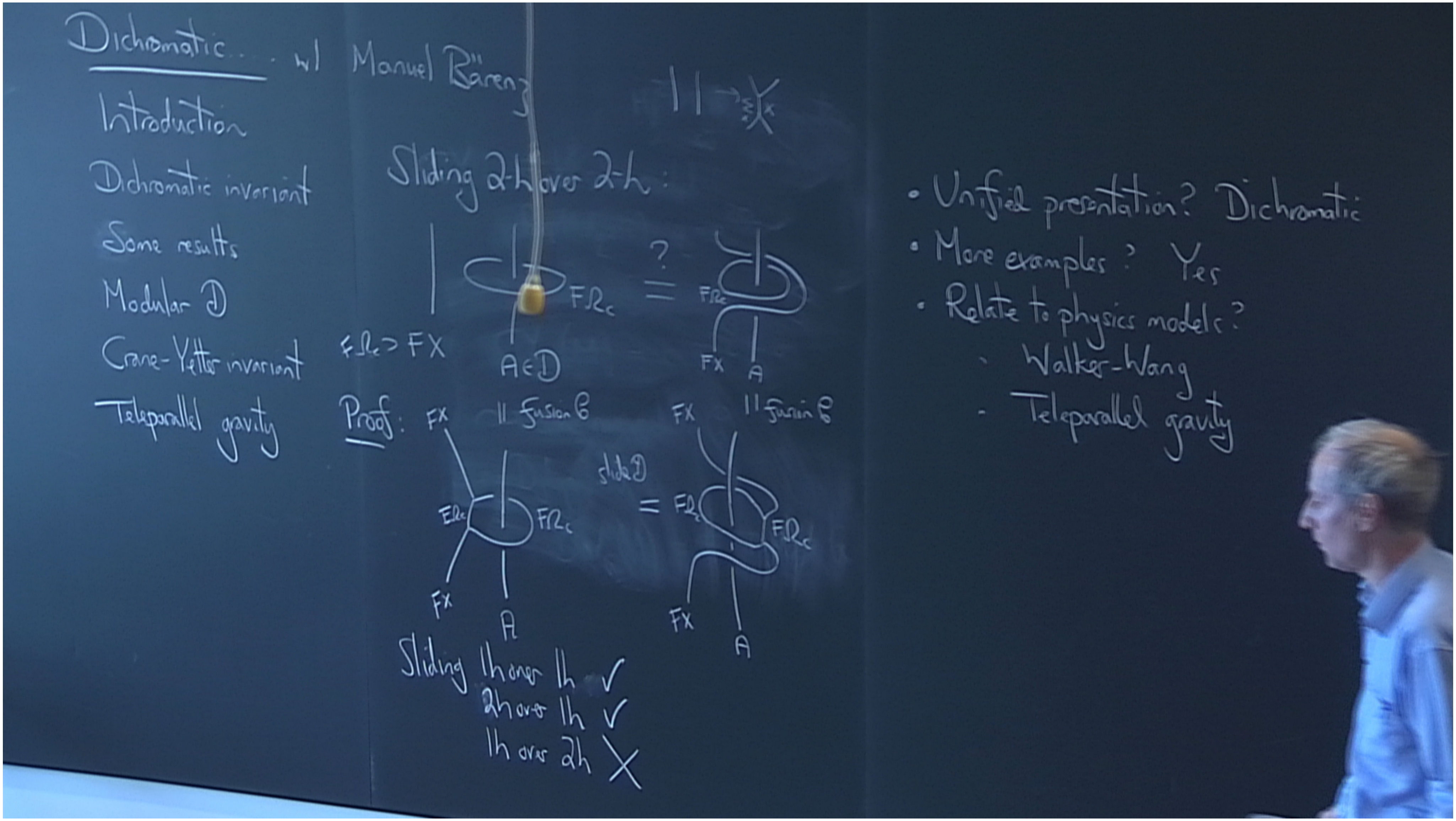
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FΩc
?
=
FΩc > FX
A ∈ D
FX
A
Proof: FX
‖ fusion C
slide D
FΩc
Fx
A
Sliding 1h over 1h ✓
2h over 1h ✓
1h over 2h ✗
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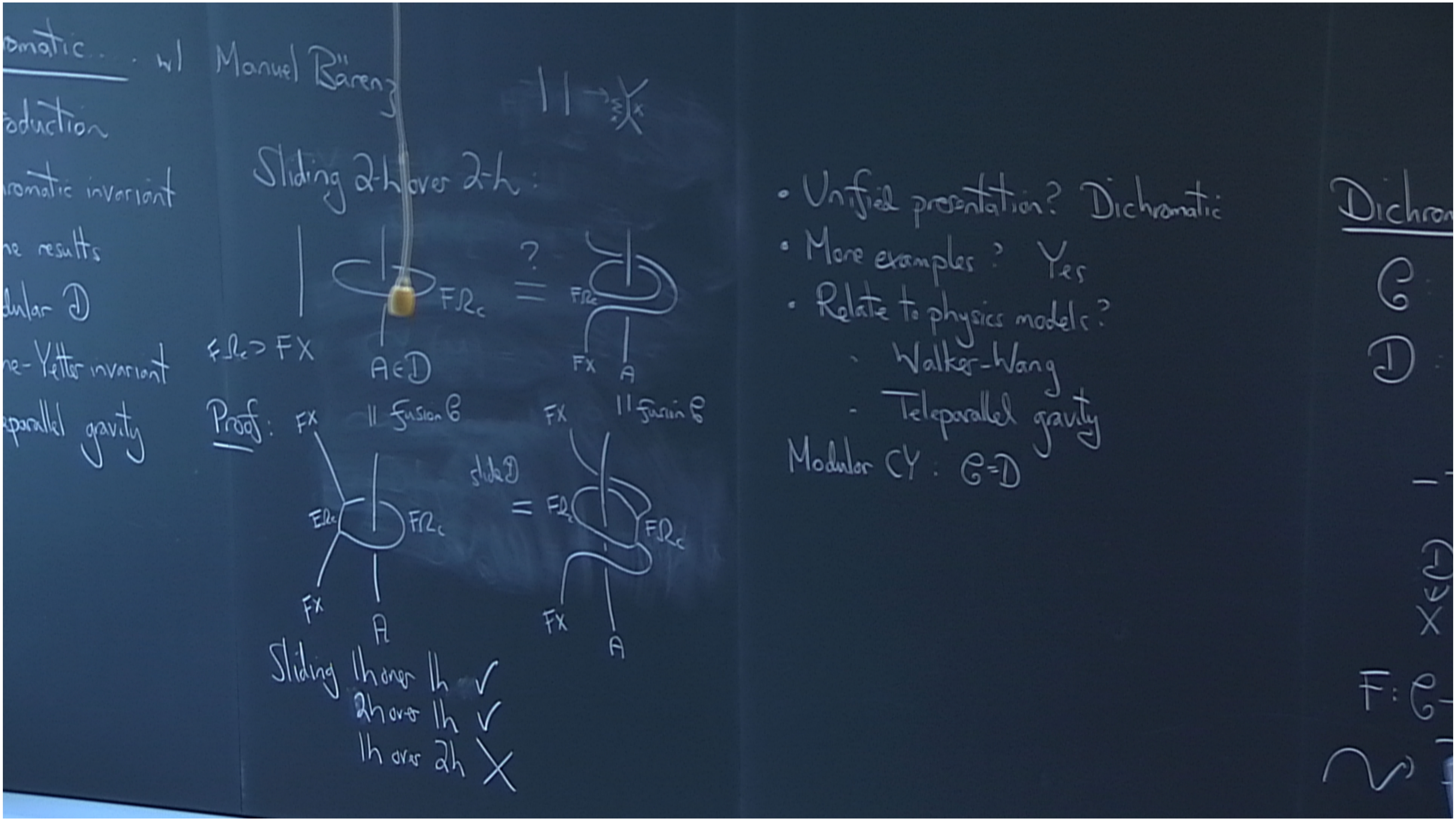

w/ Manuel Bärenz
Sliding 2-h over 2-h :
FΩc ⊃ FX
A ∈ D
Proof :
slide D
Sliding 1h over 1h ✓
2h over 1h ✓
1h over 2h X
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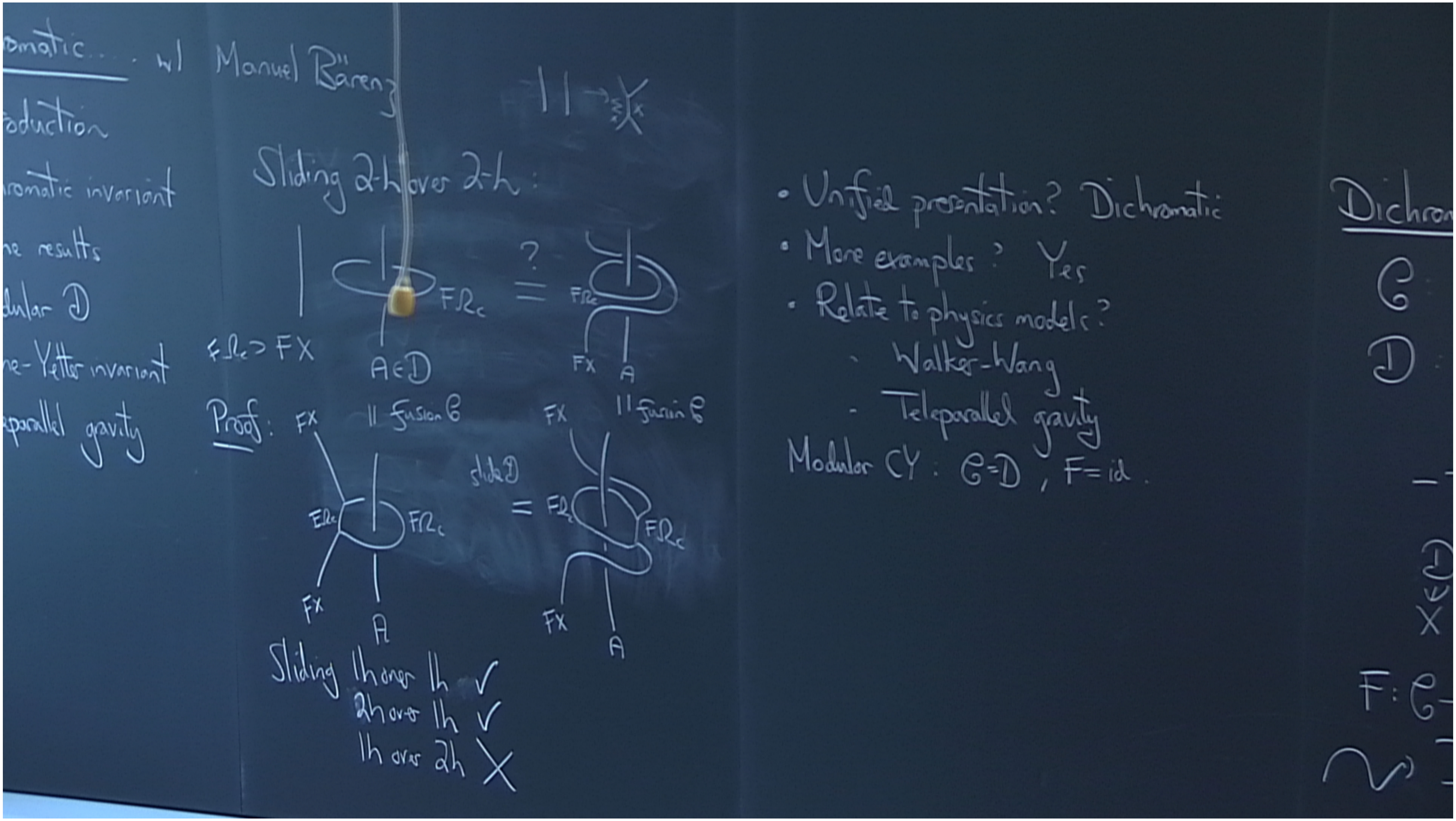
Modular CY : C = D
Dichromatic



w/ Manuel Bärenz
Sliding 2-h over 2-h :
?
=
FΩc
FΩc > FX
A∈D
FX
A
Proof:
FX
|| fusion C
slide D
Sliding 1h over 1h ✓
2h over 1h ✓
1h over 2h ✗
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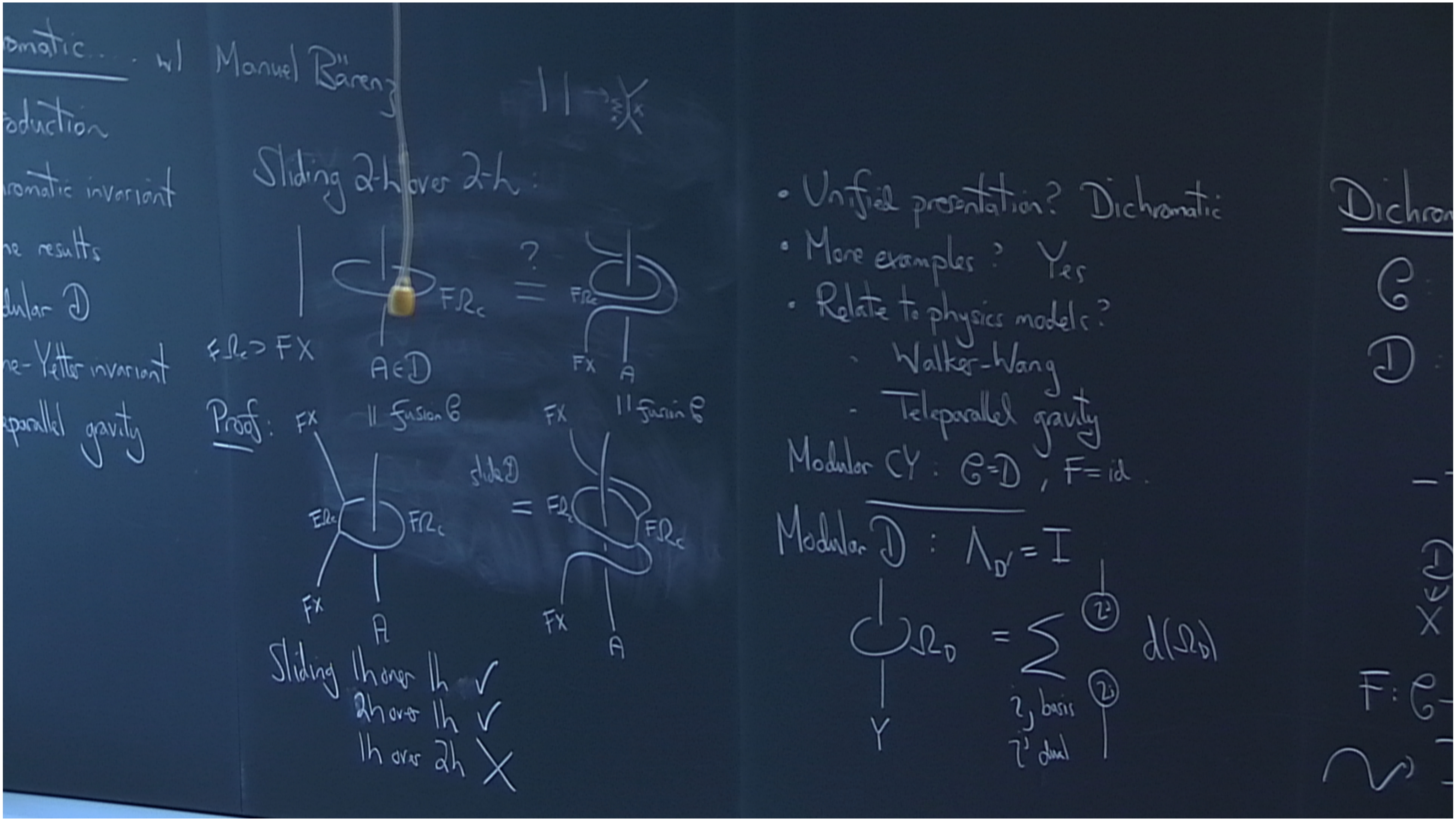
Modular CY : C=D , F=id .



w/ Manuel Bärenz
Sliding 2-h over 2-h :
FΩc
FΩc ⊃ FX
A ∈ D
FX
A
Proof:
fusion C
slide D
Sliding 1h over 1h ✓
2h over 1h ✓
1h over 2h X
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Modular CY : C=D , F=id .
Modular D : Λ_D' = I
d(Ω_D)
basis
dual

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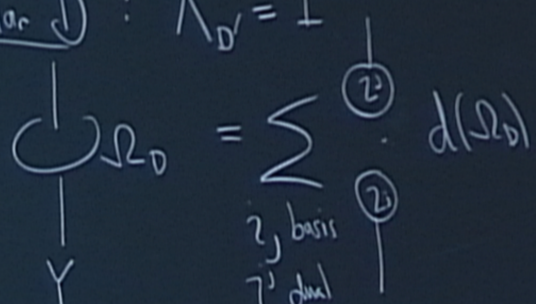
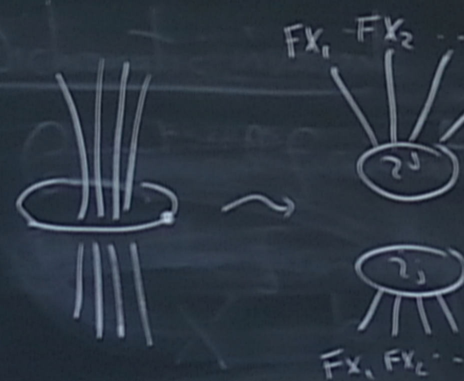
Modular CY: $\mathcal{C}=\mathcal{D}$, $F=\mathrm{id}$.

Modular $\mathcal{D}$: $\Lambda_{\mathcal{D}'}=I$





$\sqrt{\Omega_{\mathcal{C}}}$

$F\Omega_{\mathcal{C}}$

$(F\Omega_{\mathcal{C}}$

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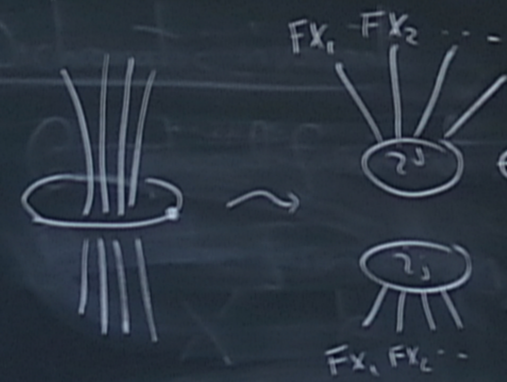
Modular CY : $\mathcal{C} = \mathcal{D}$, $F = \mathrm{id}$.

<u>Modular $\mathcal{D}$</u> : $\Lambda_{\mathcal{D}} = I$

$$Y \;\; \Omega_{\mathcal{D}} = \sum_{\substack{\tau_j \text{ basis} \\ \tau^j \text{ dual}}} \tau^j \cdot \tau_j \; d(\Omega_{\mathcal{D}})$$



$\in \mathcal{D}(I, Fx_1 \otimes Fx_2 \otimes \ldots)$ L

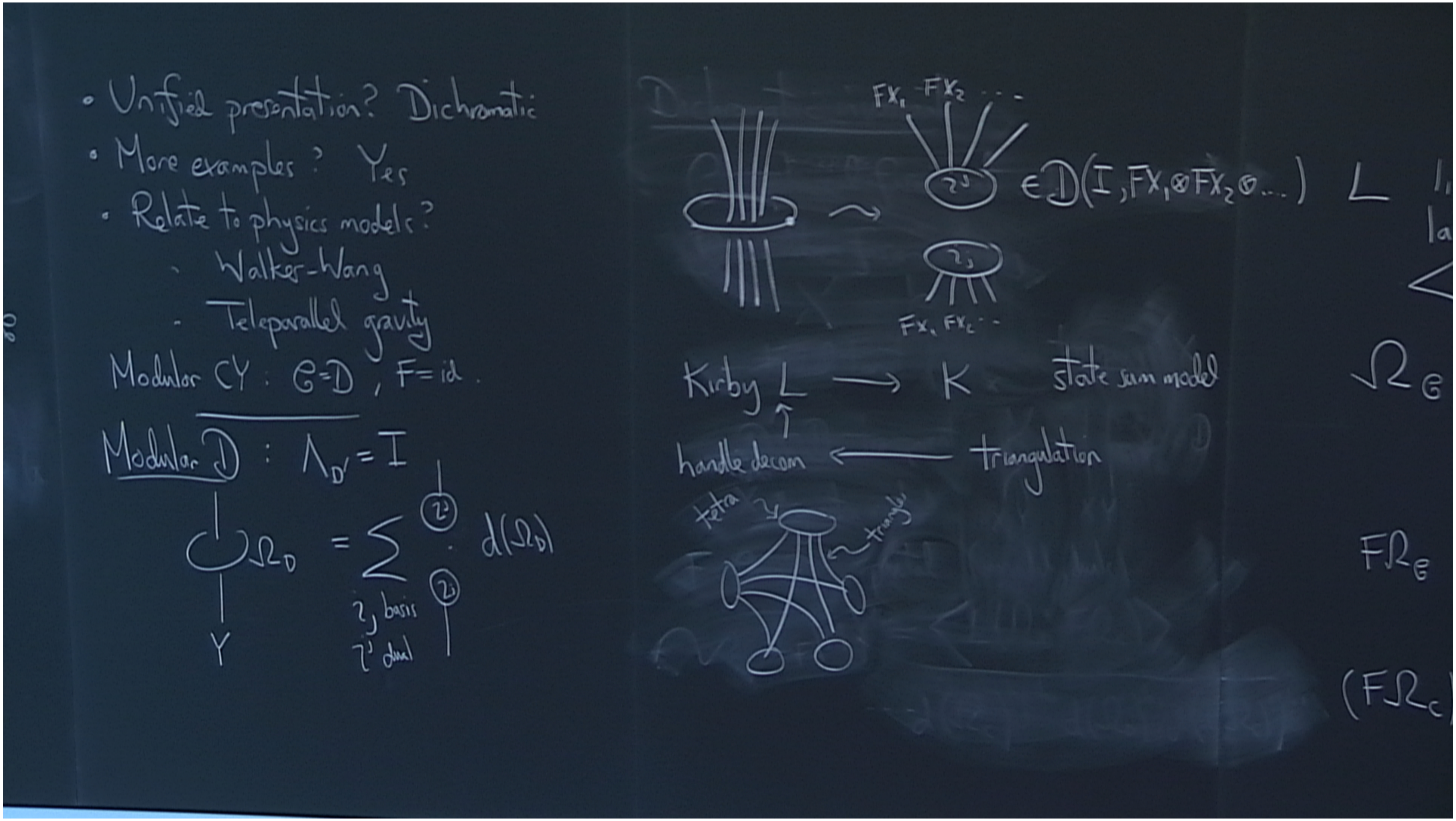
Kirby $L \longrightarrow K$ state sum model

handle decom $\longleftarrow$ triangulation

$\Omega_{\mathcal{C}}$

$F\Omega_{\mathcal{C}}$

$(F\Omega_{\mathcal{C}})$



• Unified presentation? Dichromatic
• More examples? Yes
• Relate to physics models?
- Walker-Wang
- Teleparallel gravity
Modular CY : C=D, F=id.
Modular D : Λ_D' = I
Ω_D = Σ d(Ω_D)
i, basis
i^j dual
Y
FX_1 FX_2 . . .
∈ D(I, FX_1 ⊗ FX_2 ⊗ ...)
FX_1 FX_2 . .
Kirby L → K state sum model
handle decom ← triangulation
tetra
triangle
Ω_C
FΩ_C
(FΩ_C)

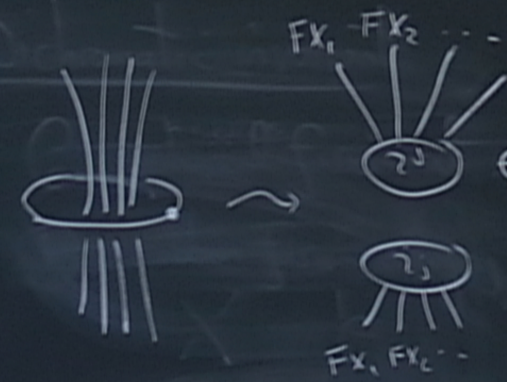
- Unified presentation? Dichromatic
- More examples? Yes
- Relate to physics models?
  - Walker-Wang
  - Teleparallel gravity

Modular CY : $\mathcal{C} = \mathcal{D}$, $F = id$.

Modular $\mathcal{D}$: $\Lambda_{\mathcal{D}'} = I$

$$\Omega_{\mathcal{D}} = \sum_{\substack{i_j \text{ basis} \\ i^j \text{ dual}}} i^j \cdot i_j \quad d(\Omega_{\mathcal{D}})$$

Y



$\in \mathcal{D}(I, FX_1 \otimes FX_2 \otimes \ldots)$ L

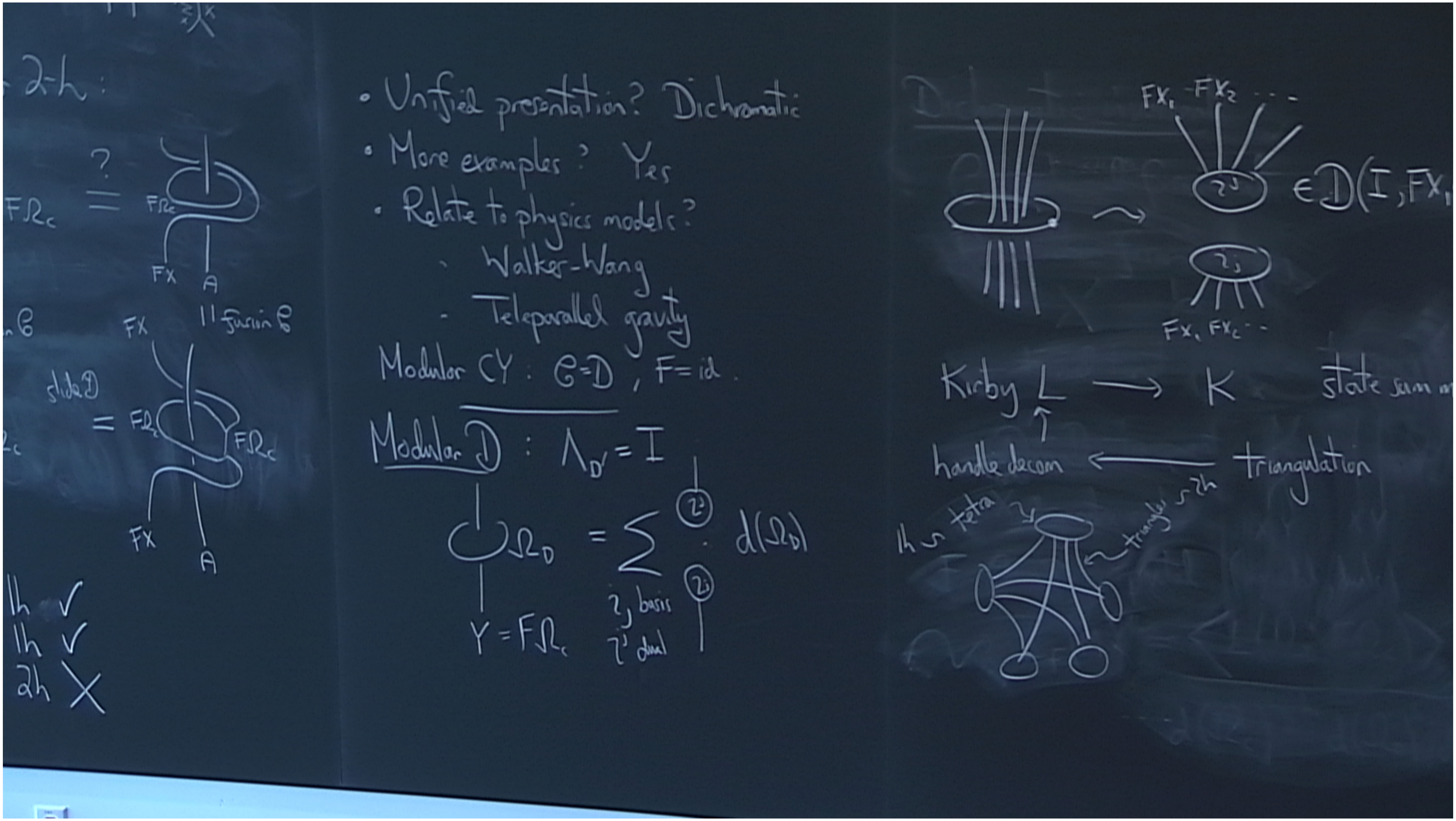
Kirby $L \longrightarrow K$ state sum model

handle decom $\longleftarrow$ triangulation

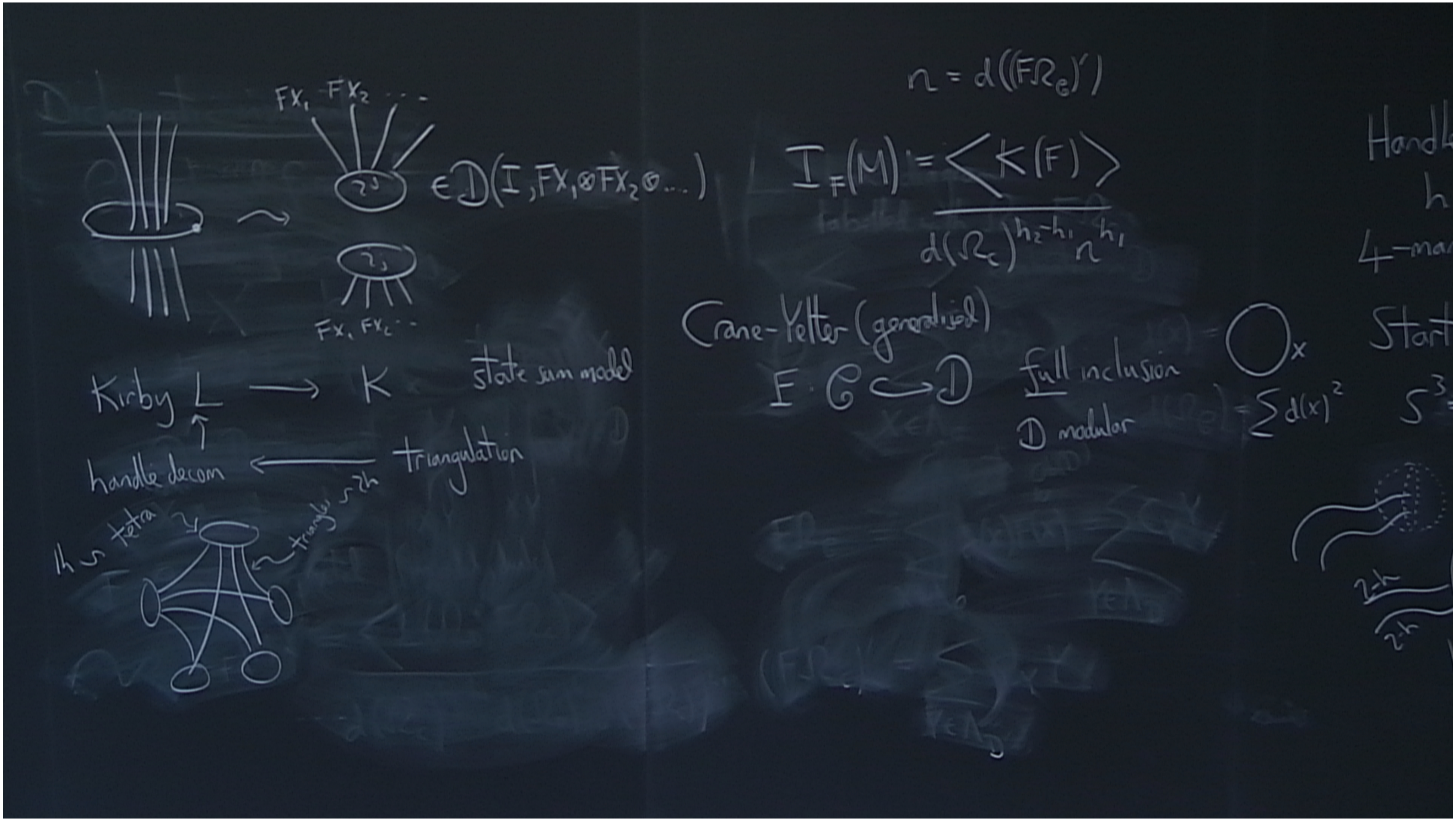
$\Omega_{\mathcal{C}}$

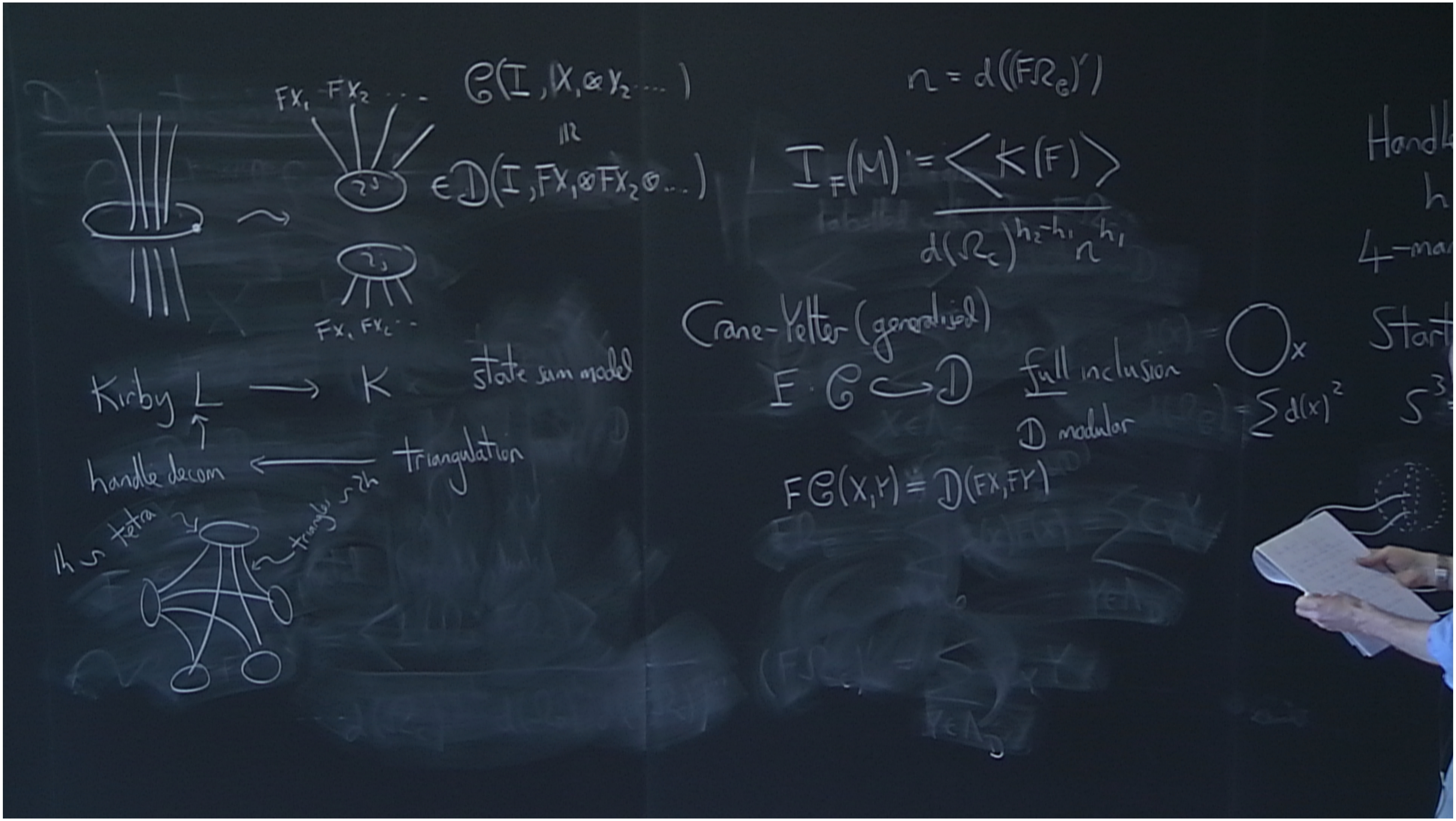
$F\Omega_{\mathcal{C}}$

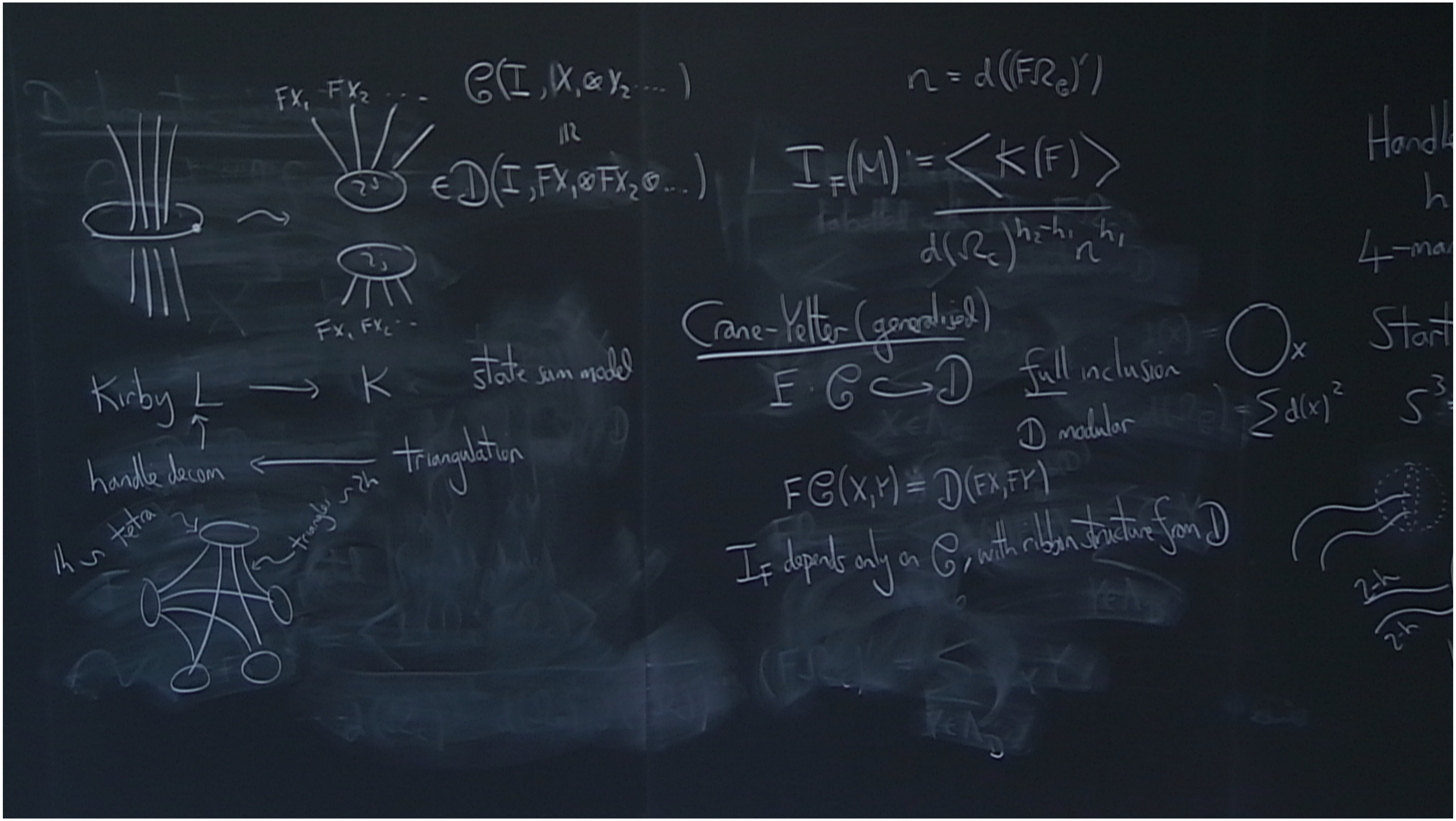
$(F\Omega_{\mathcal{C}})$



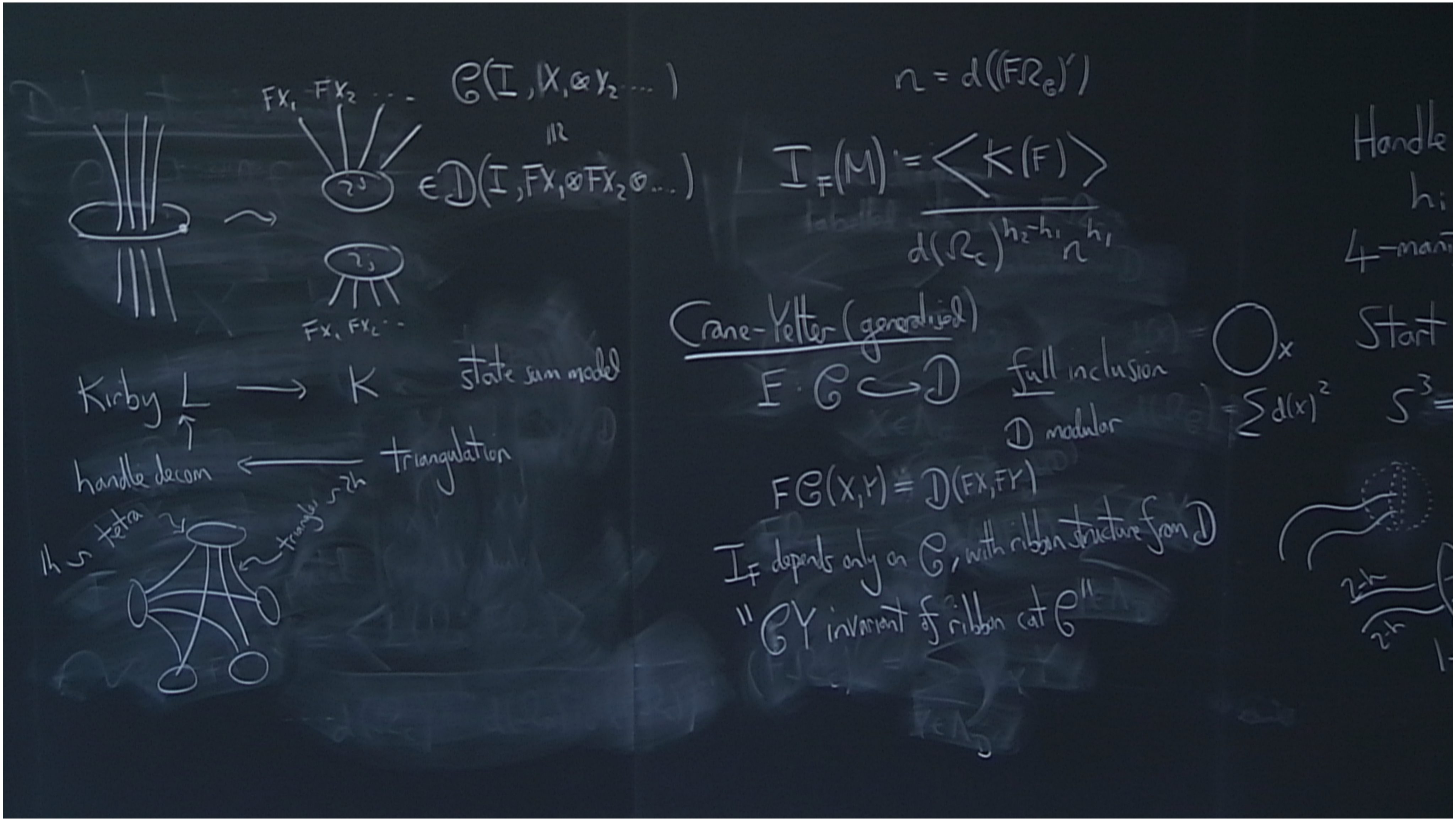
2-h :
?
=
FΩc
FX
A
FX
|| fusion 𝒞
slide 𝒟
= FΩc
FΩc
FX
A
1h ✓
1h ✓
2h X
• Unified presentation? Dichromatic
• More examples? Yes
• Relate to physics models?
- Walker-Wang
- Teleparallel gravity
Modular CY : 𝒞=𝒟 , F=id .
Modular 𝒟 : Λ_D' = I
Ω_D = Σ d(Ω_D)
Y = FΩc
basis
dual
FX₁ FX₂ . . .
∈ 𝒟(I, FX
FX₁ FX₂ . . .
Kirby L → K state sum
handle decom ← triangulation
1h ~ tetra
triangle ~ 2h


Fx1 Fx2 . . .
∈ D(I, Fx1 ⊗ Fx2 ⊗ ...)
Fx1 Fx2 . .
Kirby L → K
state sum model
handle decom ← triangulation
1h ~ tetra
triangle ~ 2h
n = d((FΩ_C)′)
I_F(M) = <K(F)> / d(Ω_C)^(h2−h1) n^(h1)
Crane-Yetter (generalised)
F : C ↪ D
full inclusion
D modular
= Σ d(x)^2
Handl
h
4-man
Start
S^3


Fx₁ Fx₂ . . .
C(I, X₁⊗X₂ . . .)
≅
∈ D(I, FX₁⊗FX₂⊗ . . .)
Fx₁ Fx₂ . . .
Kirby L → K state sum model
handle decom ← triangulation
h ↔ tetra
triangles ↔ 2h
n = d((FΩ_C)′)
I_F(M) = ⟨K(F)⟩ / d(Ω_C)^(h₂−h₁) n^(h₁)
Crane-Yetter (generalised)
F: C ↪ D full inclusion
D modular
FC(X,Y) = D(FX,FY)
= Σ d(x)²
Handl
4-man
Start
S³



FX1 FX2 ...
C(I, X1 ⊗ Y2 ...)
≅
∈ D(I, FX1 ⊗ FX2 ⊗ ...)
FX1 FX2 ...
Kirby L → K state sum model
handle decom ← triangulation
1h ~ tetra
triangle ~ 2h
n = d((FΩC)′)
IF(M) = ⟨K(F)⟩ / d(ΩC)^(h2−h1) n^h1
Crane-Yetter (generalised)
F : C ↪ D full inclusion
D modular
FC(X,Y) = D(FX,FY)
IF depends only on C, with ribbon structure from D
Σ d(x)²
Handl
h
4-man
Start
S³
2-h
2-h


FX₁ FX₂ . . .
𝒞(I, X₁⊗Y₂ . . .)
≅
∈ 𝒟(I, FX₁⊗FX₂⊗ . . .)
FX₁ FX₂ . . .
Kirby L ⟶ K
state sum model
handle decom ⟵ triangulation
1h ~ tetra
triangle ~ 2h
n = d((FΩ_𝒞)′)
I_F(M) = ⟨K(F)⟩ / d(Ω_𝒞)^{h₂−h₁} n^{h₁}
Crane-Yetter (generalised)
F: 𝒞 ↪ 𝒟
full inclusion
𝒟 modular
F𝒞(X,Y) ≅ 𝒟(FX,FY)
I_F depends only on 𝒞, with ribbon structure from 𝒟
"CY invariant of ribbon cat 𝒞"
Σ d(x)²
Handle
4-mani
Start
S³ =
2-h
2-h

$y_2 \dots)$

$\Omega = d((F\Omega_{\mathcal{C}})')$

$FX_2 \otimes \dots)$

$$I_F(M) := \frac{\langle K(F) \rangle}{d(\Omega_{\mathcal{C}})^{h_2 - h_1} \, \Omega^{h_1}}$$

model

<u>Crane-Yetter (generalised)</u>

$F: \mathcal{C} \hookrightarrow \mathcal{D}$ full inclusion

$\mathcal{D}$ modular

$F\mathcal{C}(X,Y) = \mathcal{D}(FX, FY)$

I_F depends only on $\mathcal{C}$, with ribbon structure from $\mathcal{D}$

"$\mathcal{C}Y$ invariant of ribbon cat $\mathcal{C}$"

e.g. ribbon $\mathcal{C} \rightsquigarrow Z(\mathcal{C})$ Drinfeld

$$CY_{\mathcal{C}}(S^1 \times \underline{S^1 \times S^2}) = |\Lambda_{\mathcal{C}'}| \, d(\Omega_{\mathcal{C}})$$

$$CY_{\mathcal{C}}(S^1 \times S^3) = d(\Omega_{\mathcal{C}})$$

$\Rightarrow$ state space dim TQFT $\quad |\Lambda_{\mathcal{C}'}| \quad S^1 \times S^2$

$\quad 1 \quad S^3$

(Walker-Wang)

$\mathcal{C} = Rep(G)$, trivial ribbon

$$CY_{\mathcal{C}} = DW_G$$

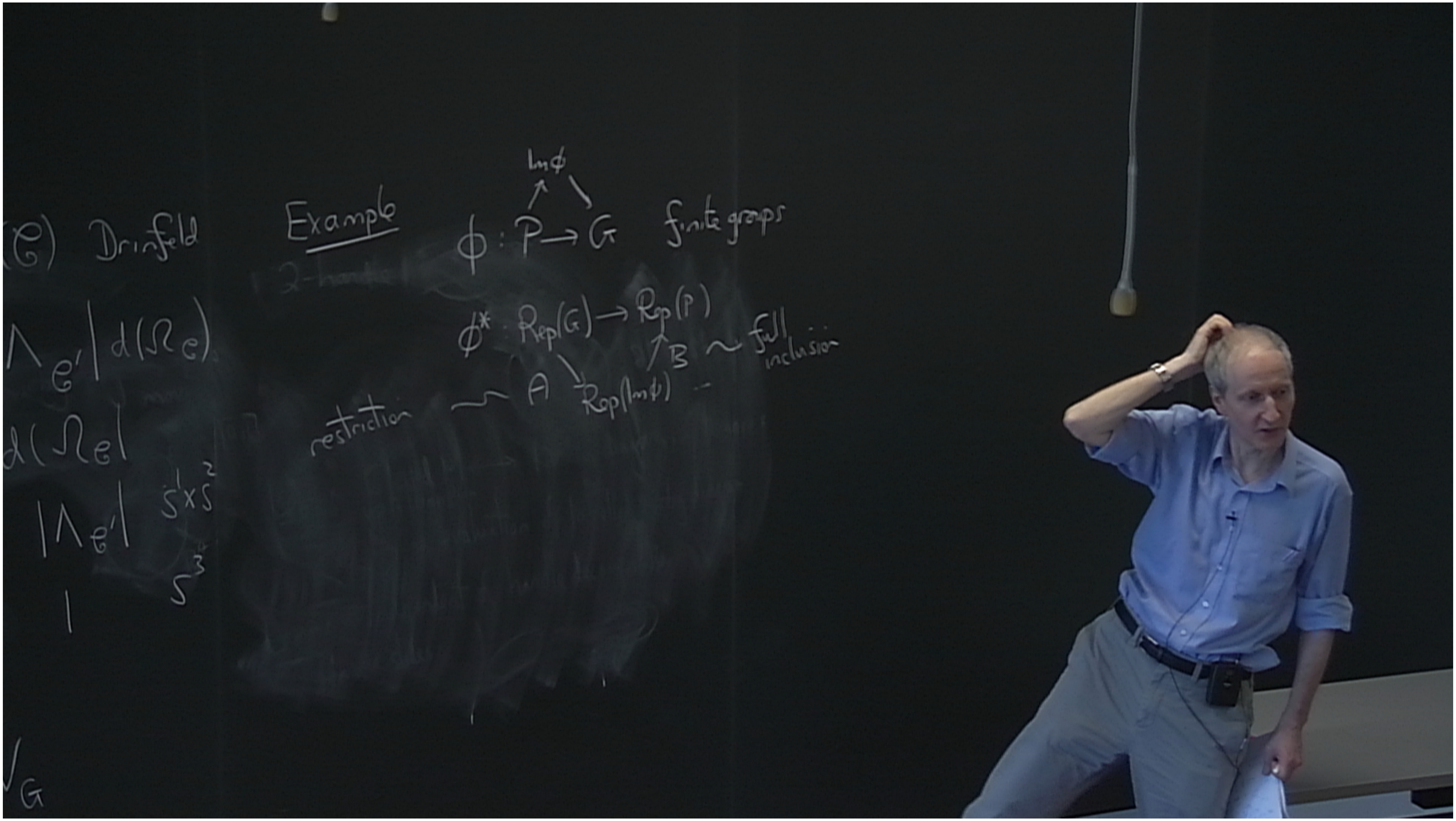
2-handle

$\times B^3$

Lemma Clo… up to …

$\langle\langle L \rangle\rangle$

Lemma Indep… Handle slide…


Drinfeld
Example
Im ϕ
ϕ : P → G finite groups
ϕ* : Rep(G) → Rep(P)
B ~ full inclusion
restriction ~ A
Rep(Im ϕ)

(G) Drinfeld

$\Lambda_{e'} \,|\, d(\Omega_e)$

$d(\Omega_e|$

$|\Lambda_{e'}|$ $\quad S^1 \times S^2$

1 $\quad S^3$

V_G

<u>Example</u>

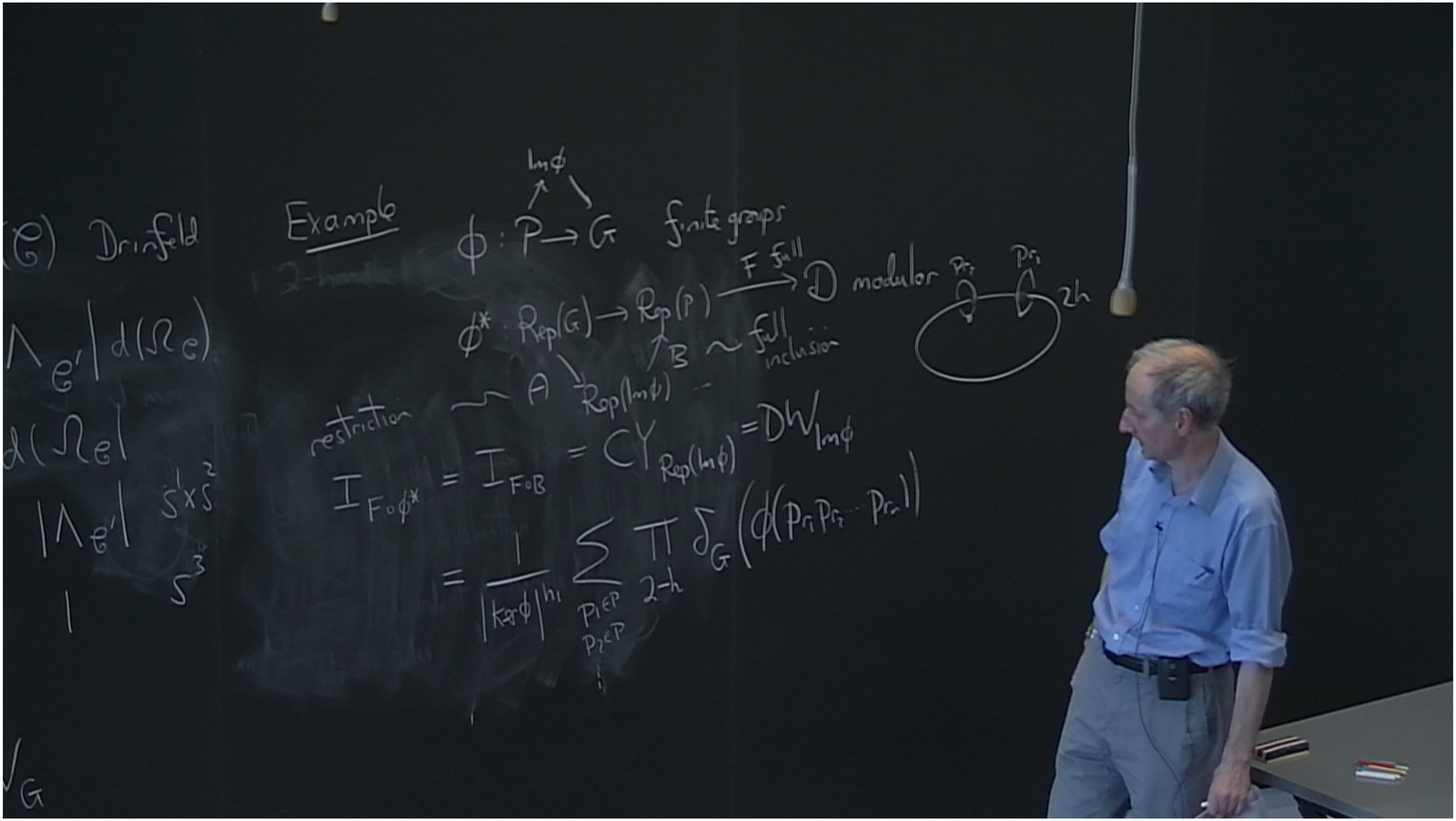
$$\begin{tikzcd} & \mathrm{Im}\,\phi \arrow[dr] & \\ \phi : P \arrow[ur] \arrow[rr] & & G \end{tikzcd}$$ finite groups

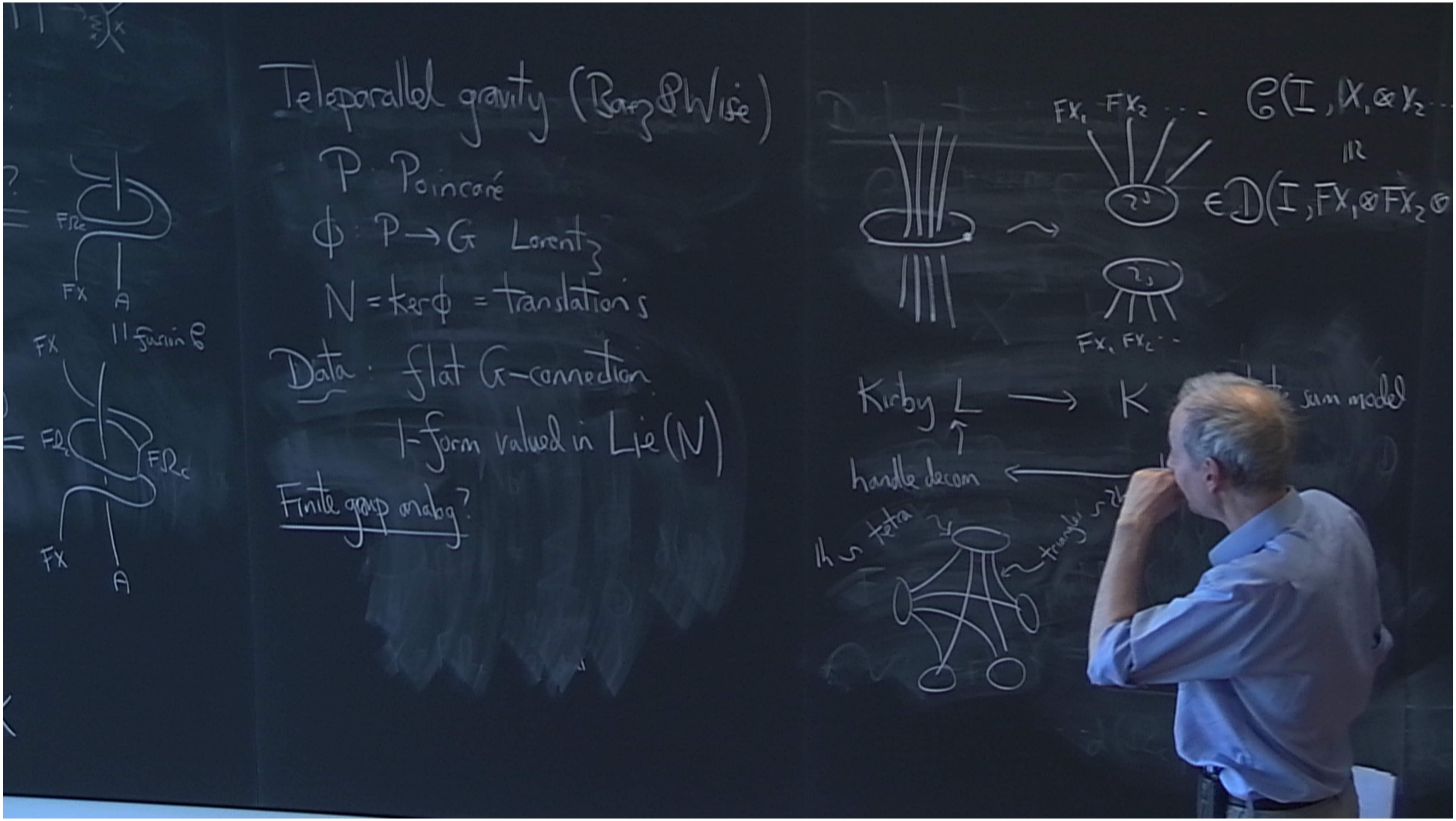
$$\begin{tikzcd} \phi^* : \mathrm{Rep}(G) \arrow[r] \arrow[dr, "A"'] & \mathrm{Rep}(P) \arrow[r, "F"] & \mathcal{D} \text{ modular} \\ & \mathrm{Rep}(\mathrm{Im}\,\phi) \arrow[u, "B"'] & \end{tikzcd}$$

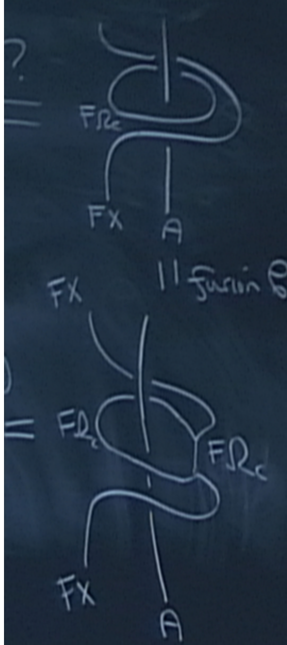
B ~ full inclusion

restriction ~ A

$$I_{F\circ\phi^*} = I_{F\circ B}$$


Drinfeld
Example
Im ϕ
ϕ : P → G finite groups
ϕ* : Rep(G) → Rep(P) F full → D modular
B ~ full inclusion
restriction ~ A Rep(Im ϕ)
I_{F∘ϕ*} = I_{F∘B} = CY_{Rep(Im ϕ)} = DW_{Im ϕ}
= 1/|Ker ϕ|^{h_1} Σ_{p_1∈P, p_2∈P, …} Π_{2-h} δ_G(ϕ(p_{r_1} p_{r_2} ⋯ p_{r_n}))
2h


Teleparallel gravity (Baez & Wise)
P : Poincaré
ϕ : P → G Lorentz
N = ker ϕ = translations
Data: flat G-connection
1-form valued in Lie(N)
Finite group analog?
Kirby L → K
handle decom
tetra
sum model
FX
A
fusion



Teleparallel gravity (Baez & Wise)

P : Poincaré

$\phi : P \to G$ Lorentz

$N = \ker\phi$ = translations

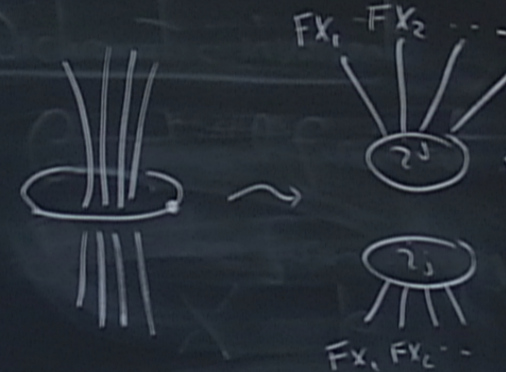
Data: flat G-connection

1-form valued in $\mathrm{Lie}(N)$

Finite group analog? $g \in G$ for each 1-h

relation $\prod g = e$ each 2-h

$n \in N = \ker\phi$ for each 1-h.



$\mathcal{C}(I, X_1 \otimes X_2 \cdots)$

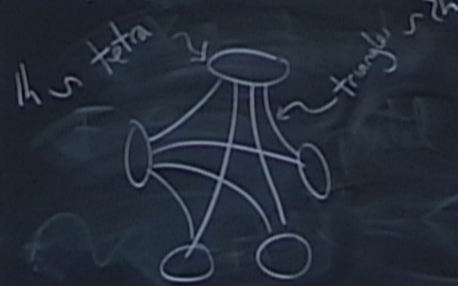
$\cong$

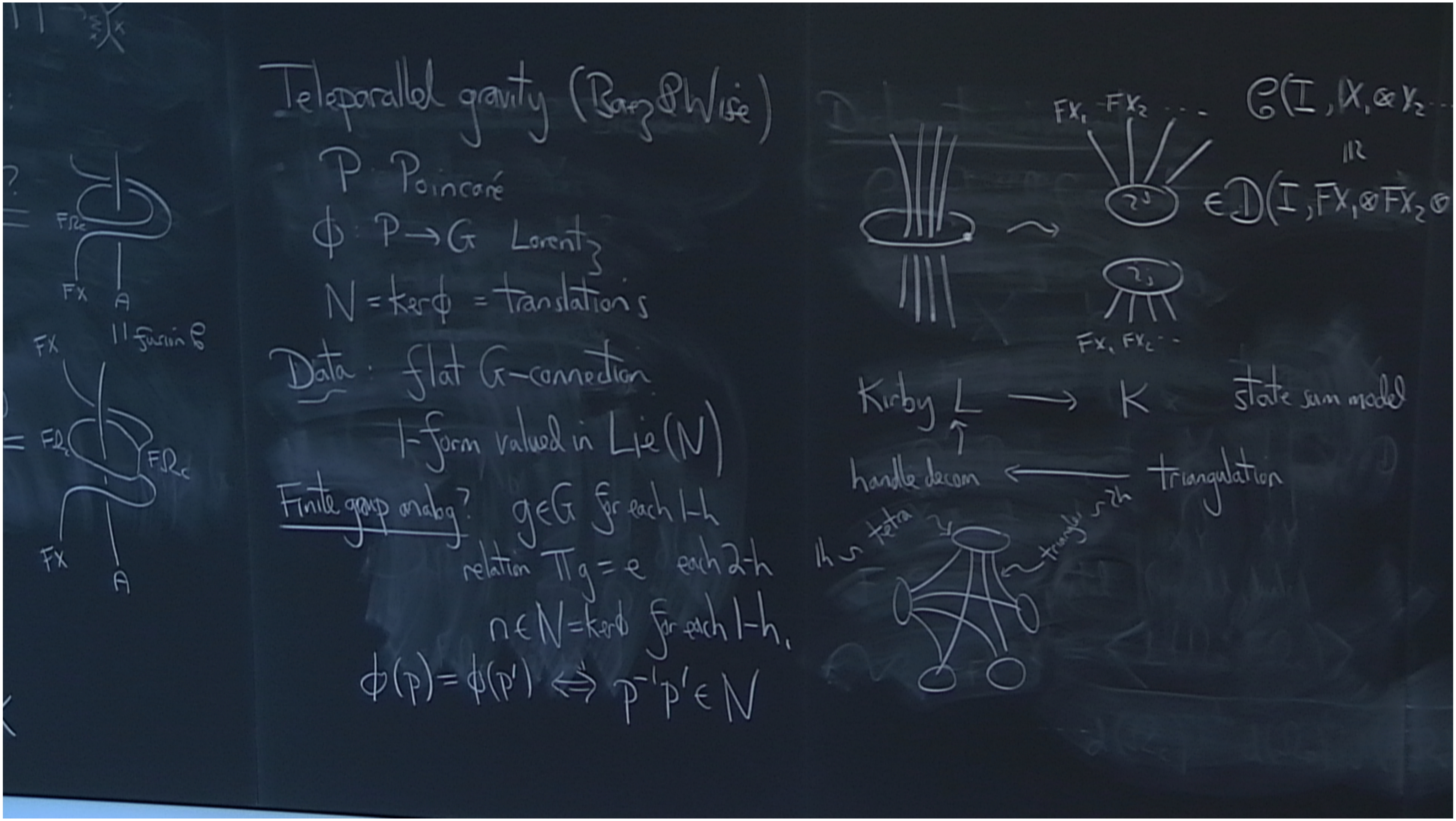
$\in \mathcal{D}(I, FX_1 \otimes FX_2 \otimes \cdots$

Kirby $L \longrightarrow K$ state sum model

$\uparrow$

handle decom $\longleftarrow$ triangulation





Teleparallel gravity (Baez & Wise)
P: Poincaré
φ: P → G Lorentz
N = ker φ = translations
Data: flat G-connection
1-form valued in Lie(N)
Finite group analog? g ∈ G for each 1-h
relation ∏g = e each 2-h
n ∈ N = ker φ for each 1-h.
φ(p) = φ(p′) ⇔ p⁻¹p′ ∈ N
FX₁ FX₂ ...
C(I, X₁ ⊗ X₂ ...
≅
∈ D(I, FX₁ ⊗ FX₂ ⊗
FX₁ FX₂ ...
Kirby L → K state sum model
handle decom ← triangulation
1h ↔ tetra
triangle ↔ 2h
FX
A
FX
A
FΩ
FΩc
|| fusion

(G) Drinfeld

$\backslash_{e'} | d(\Omega_e)$

$(\Omega_e|$

$|\Lambda_{e'}|$ $\quad S^1 \times S^2$

$|$ $\quad S^3$

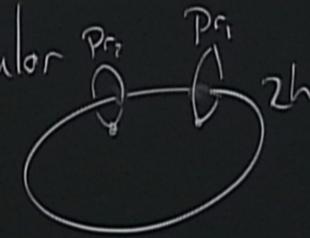
G

Example

$$\begin{tikzcd} & \mathrm{Im}\,\phi \arrow[dr] & \\ P \arrow[ur] \arrow[rr] & & G \end{tikzcd}$$

$\phi : P \to G$ finite groups

$$\phi^* : \mathrm{Rep}(G) \to \mathrm{Rep}(P) \xrightarrow{F \text{ full}} \mathcal{D} \text{ modular}$$



$\mathrm{Rep}(G) \xrightarrow{A} \mathrm{Rep}(\mathrm{Im}\,\phi) \xrightarrow{B} \mathrm{Rep}(P)$

restriction $\sim A$ $\qquad$ $B \sim$ full inclusion

$$I_{F \circ \phi^*} = I_{F \circ B} = CY_{\mathrm{Rep}(\mathrm{Im}\,\phi)} = DW_{\mathrm{Im}\,\phi}$$

$$= \frac{1}{|\mathrm{Ker}\,\phi|^{h_1}} \sum_{\substack{p_1 \in P \\ p_2 \in P \\ \vdots}} \prod_{2\text{-}h} \delta_G \left(\phi(p_{r_1} p_{r_2} \cdots p_{r_n}) \, \phi(p_{r_1}) \phi(p_{r_2}) \cdots \right)$$