Title: Emergent Coulombic criticality and Kibble-Zurek scaling in a topological magnet

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Abstract: <p>When a classical system is driven through a continuous phase transition, its nonequilibrium response is universal and exhibits Kibble-Zurek scaling. We explore this dynamical scaling in the context of a three-dimensional topological magnet with fractionalized excitations, namely, the liquid-gas transition of the emergent mobile magnetic monopoles in dipolar spin ice. Using field-mixing and finite-size scaling techniques, we place the critical point of the liquid-gas line in the three-dimensional Ising universality class. We then demonstrate Kibble-Zurek scaling for sweeps of the magnetic field through the critical point. Unusually slow microscopic time scales in spin ice offer a unique opportunity to detect this universal nonequilibrium physics within current experimental capability.</p>

Emergent criticality and Kibble-Zurek scaling in a topological magnet

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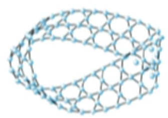
EPSRC

Engineering and Physical Sciences Research Council

HELMHOLTZ ASSOCIATION

Virtual Institute: New States of Matter and their Excitations



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12-04-2016

Perimeter Institute

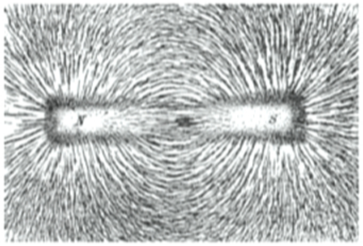
Waterloo (CA)

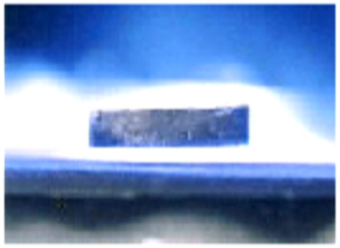
Outline

- context and motivation:
  - emergent phases of matter and spin ice
  - liquid-gas critical end point supported by fractionalised excitations
  - novel setting for critical and out-of-equilibrium behaviour in a 3D topological magnet
- (revised) finite-size scaling $\rightarrow$ 3D Ising universality
- non-equilibrium behaviour: Kibble-Zurek scaling in field-sweep magnetisation measurements
- relation to experiments and conclusions

Symmetries (particles) and phases of matter

many body systems: new ordered phases stem from symmetries present in the system at high energy / temperature





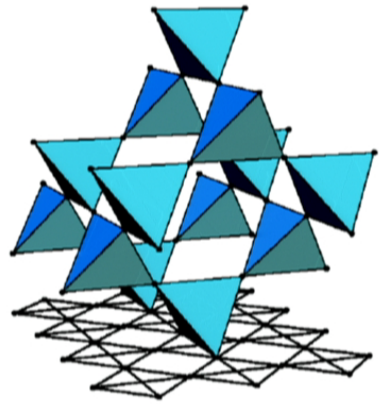
1937 Landau
1950 Ginzburg & Landau

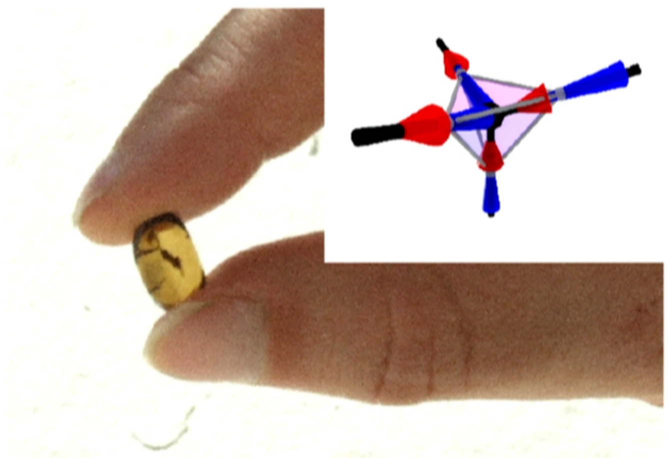
classification of phases and phase transitions based on *local* order parameters and symmetries of the system

- solid-liquid phases
- magnetism
- superconductivity and superfluidity

Spin Ice ($Dy_2Ti_2O_7$ and $Ho_2Ti_2O_7$)

- local [111] crystal field ~ 200 K
$\Rightarrow$ Ising spins
- large spins (15/2 and 8)
$\Rightarrow$ classical limit (small exchange ~ 1 K)
- large magnetic moment $\sim 10\,\mu_B$
$\Rightarrow$ long range dipolar interactions



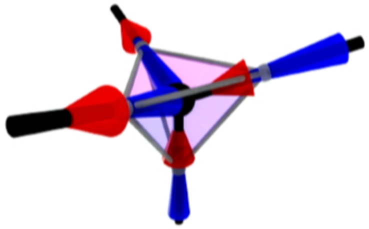


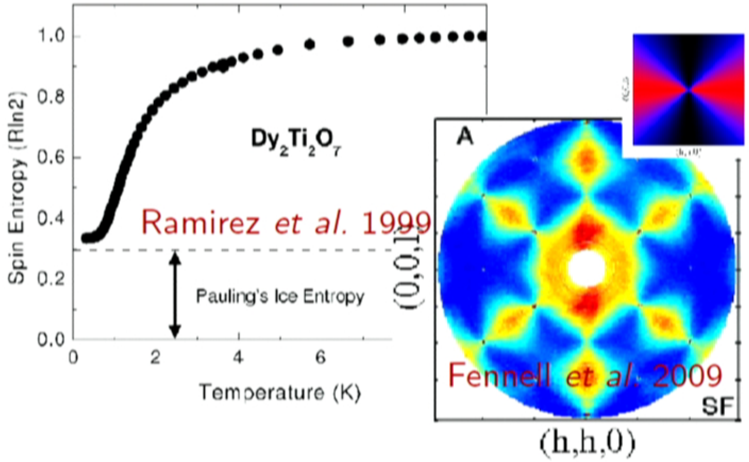
- dipolar interactions $\sim$ nn
- extensively degenerate GS
- emergent Gauge symmetry
- elementary excitations fractionalise dipoles into monopoles + long-range Coulomb interactions

Frustration leads to (classical) degeneracy

dipolar interactions minimised by
2-in, 2-out ice rules ⇒ local constraint

Gingras *et al.* , Shastry *et al.* 1999-2001





six ground states per tetrahedron:

$$N_{\rm gs} = 2^N \left(\frac{6}{16}\right)^{N/2}$$

$$\mathcal{S} = \frac{N}{2} \ln \frac{3}{2}$$

extensive degeneracy

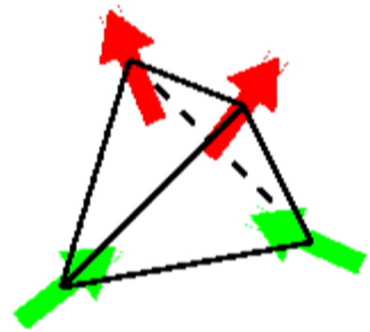
Is spin ice ordered or not?

No order as in ferromagnet

- extensive degeneracy

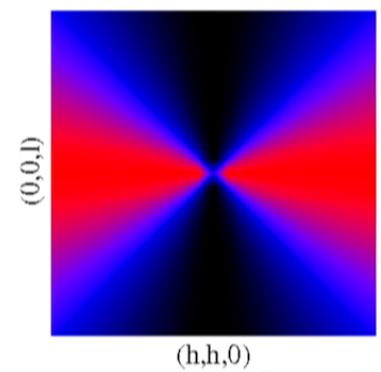
Not disordered like a paramagnet

- ice rules $\Rightarrow$ 'conservation law'



Consider magnetic moments $\vec{\mu}_i$ as a (lattice) 'flux' vector field

- Ice rules $\Leftrightarrow \nabla \cdot \vec{\mu} = 0 \;\Rightarrow\; \vec{\mu} = \nabla \times \vec{A}$
- Simplest assumption: free field
  $\mathcal{S} = (\kappa/2) \int |\nabla \times A|^2 \, dr^3$
- Local constr. $\Rightarrow$ emergent gauge symm.
  - $\rightarrow$ algebraic spin corr. $\sim \frac{3\cos^2\theta - 1}{r^3}$
  - $\rightarrow$ structure factor (saddle point)

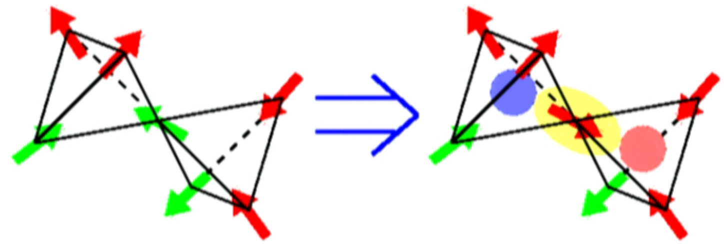


Deconfined elementary excitations

Ising spins:

→ excitation = spin reversal

→ two defective tetrahedra



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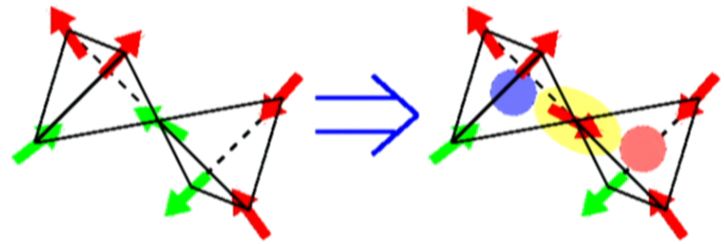

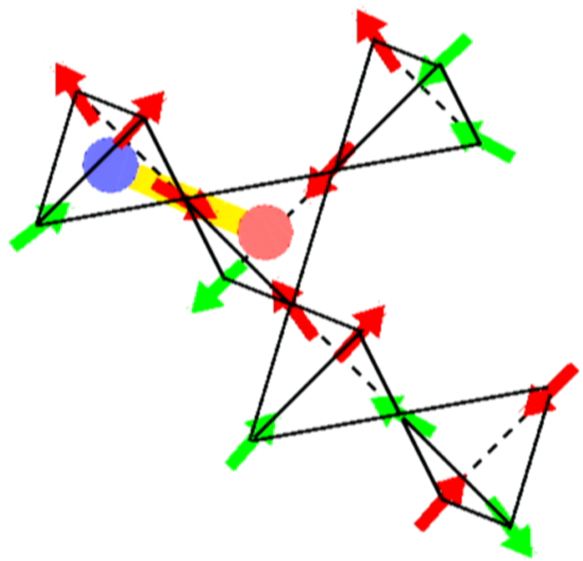

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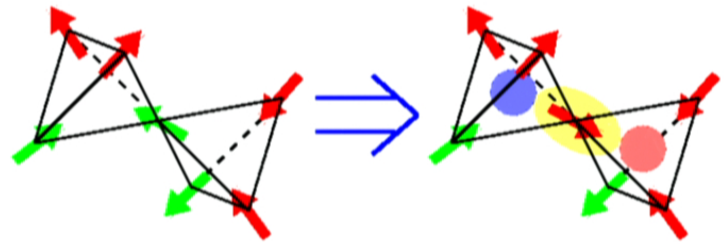
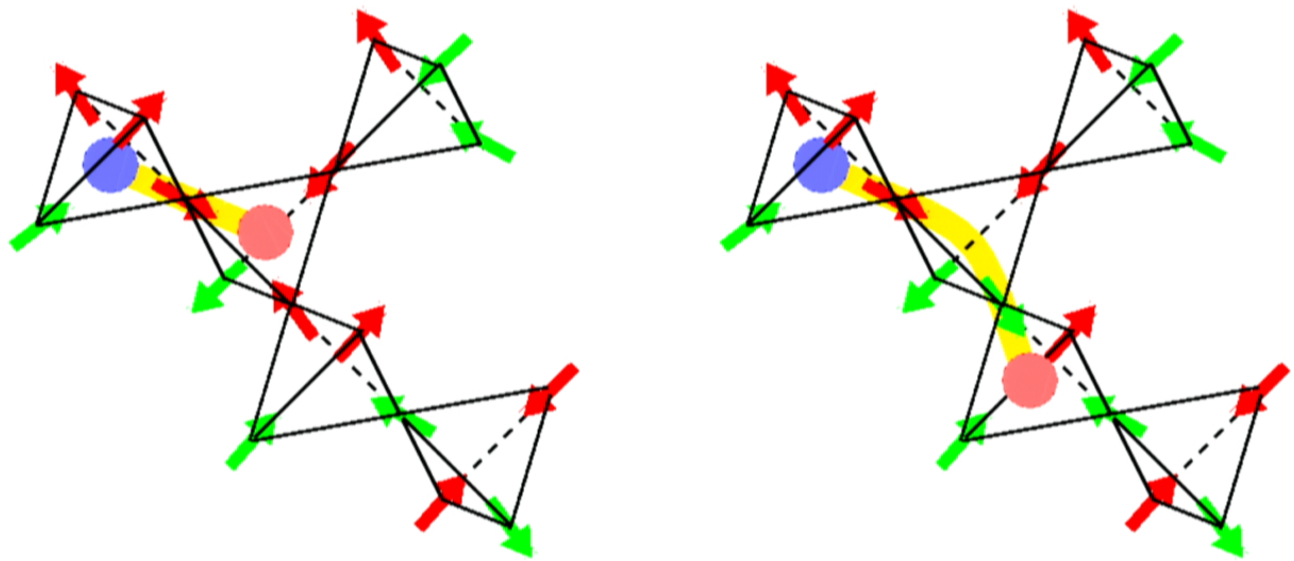
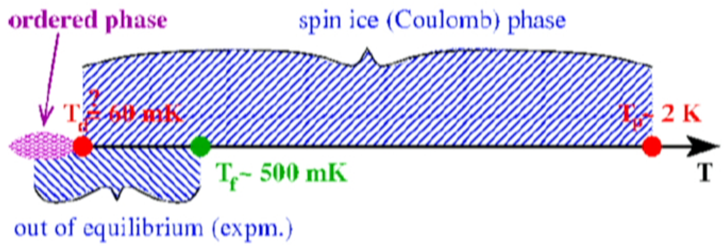
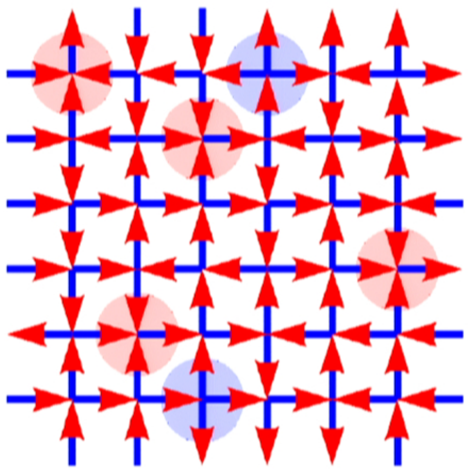


they can be separated at small (finite) energy cost!

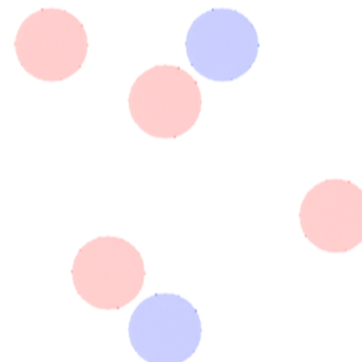


Spin ice as a Coulomb liquid



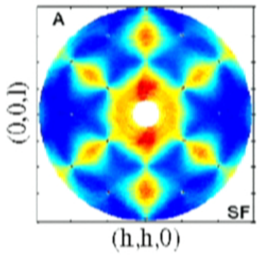


$\Leftrightarrow$

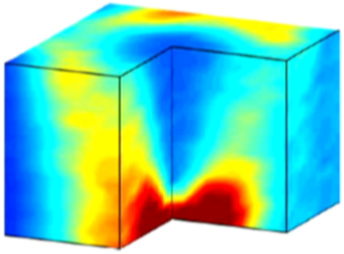


+ Coulomb interactions
+ entropic interactions
+ kinematic constraints

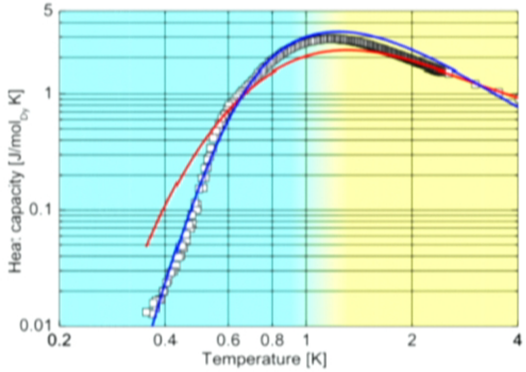
Key to understand thermodynamic properties



mag. corr.: pinch-points
Fennel et al 2009, Kadowaki et al 2009

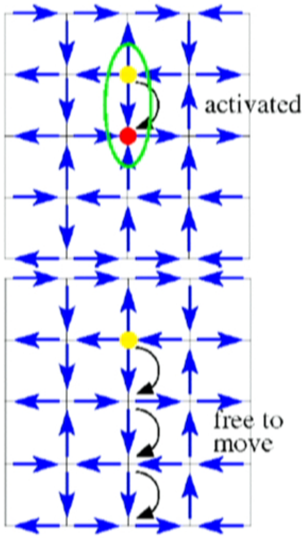


mag. corr.: "Dirac" strings
Morris et al. 2009



Debye-Hückel heat cap. Morris et al. '09

Monopoles act as facilitators of spin dynamics



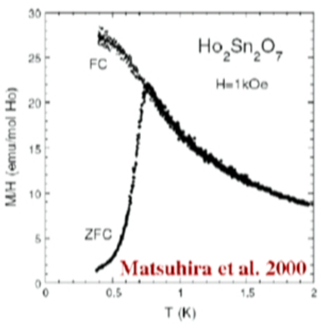
magnetic response ⇔ monopole motion

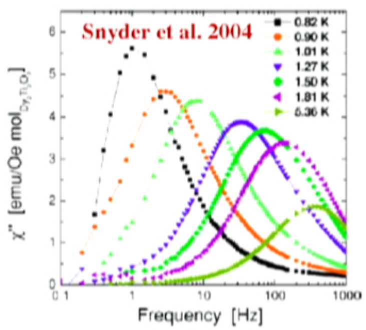
e.g., Ryzhkin 2005, Jaubert *et al.* 2009

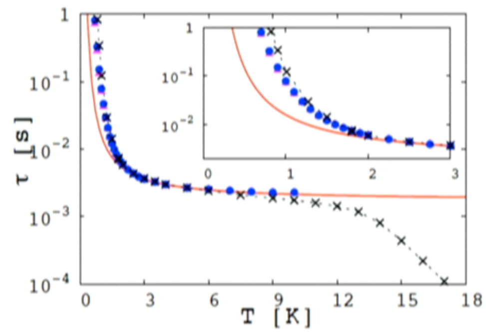
$$\Rightarrow \tau \sim \tau_0/\rho(T)$$

$T \lesssim 1$ K: paucity of monopoles

$(\rho \sim e^{-4.35/T})$ $\Rightarrow \tau \sim \tau_0\, e^{4.35/T}$

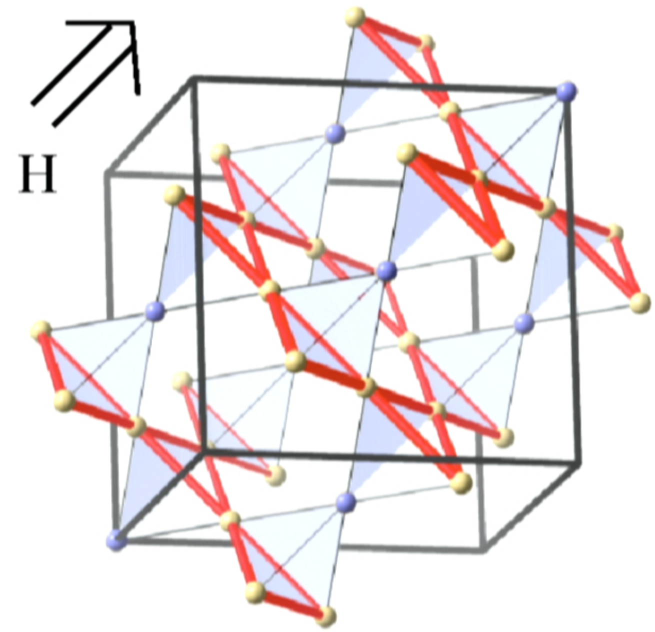






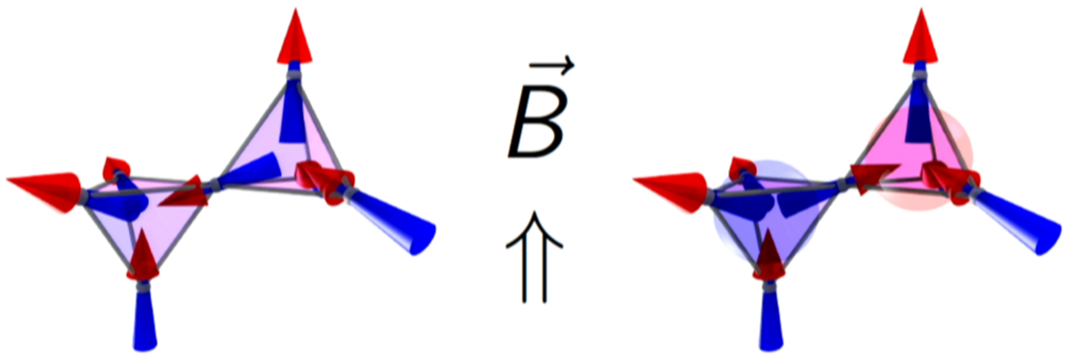
External [111] field ⇔ chemical potential

- pyrochlore lattice = alternating kagome and triangular layers



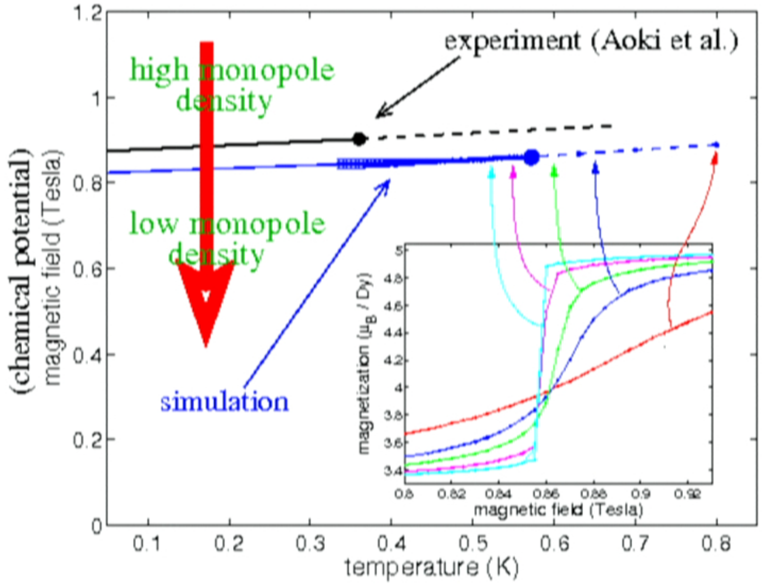
External [111] field ⇔ chemical potential

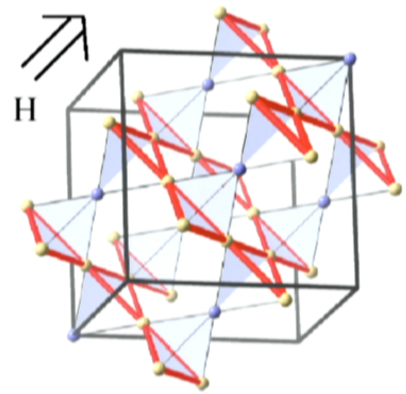
- pyrochlore lattice = alternating kagome and triangular layers
- (field couples more strongly to triangular than kagome spins)
- [111] saturated phase ⇔ fully packed monopoles *(ionic crystal)*



Liquid gas phase diagram

CC, Moessner, Sondhi 2008





mag. $\propto$ monopole dens.

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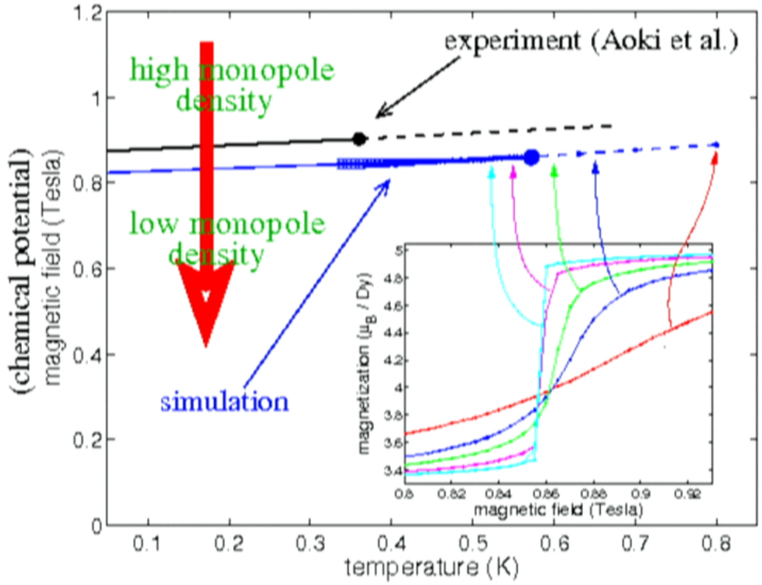
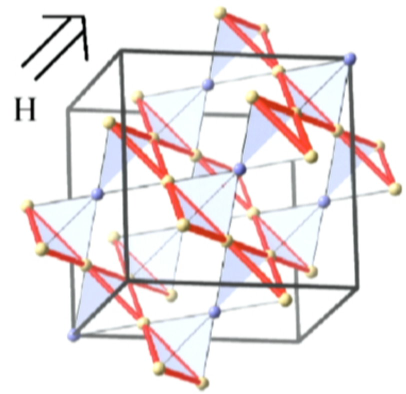




mag. $\propto$ monopole dens.

Novel setting for critical and nonequil. behaviour

- liquid-gas driven by emergent fractionalized excitations
  (not trivially related to original $\mathbb{Z}_2$ spin degrees of freedom)

- underlying spin 2in-2out 'vacuum' described by topological free gauge field theory

- spin dynamics occurs via monopole motion
  $\rightarrow$ local and global kinematic constraints
  CMS PRL 2010; Mostame et al. PNAS 2014

slow time scales ($\sim$ ms) $\rightarrow$ real time dynamics expm. accessible

(Revised) finite-size scaling at the critical point

liquid-gas behaviour: generally expected Ising criticality

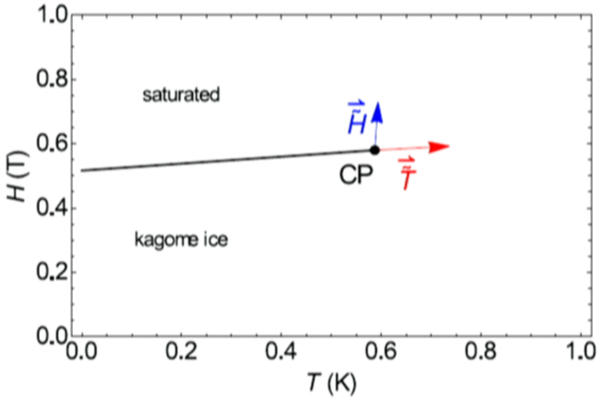
Fisher 1994; Luijten et al. 2002

energy and density (equivalently, E and M) are not the critical scaling operators: Rehr et al. 1973; Bruce et al. 1992; Wilding 1997

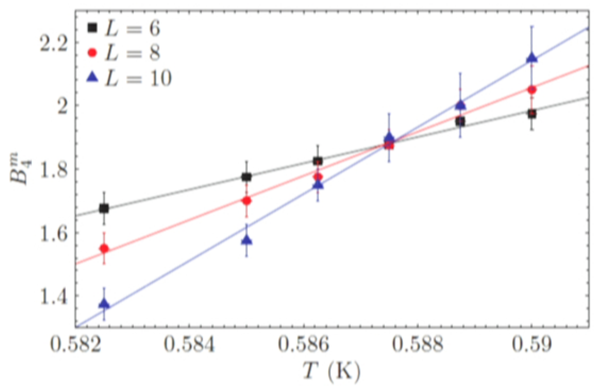
$$\tilde{E} = E + r\,M$$
$$\tilde{M} = M + s\,E$$

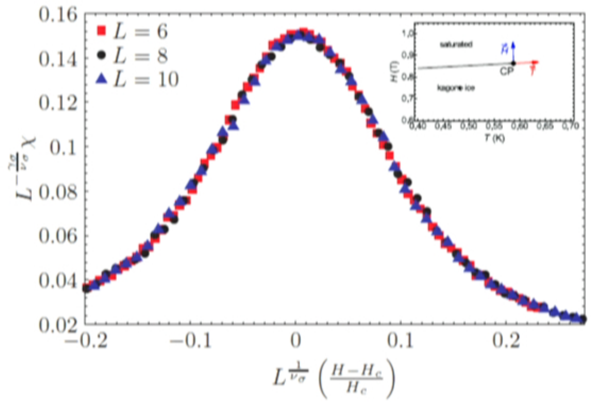
(T and H are not the temperature-like and field-like directions)

$$F(E, M; T, H) \simeq F_{\text{Ising}}(\tilde{E}, \tilde{M}; \tilde{T}, \tilde{H})$$



(Revised) finite-size scaling at the critical point





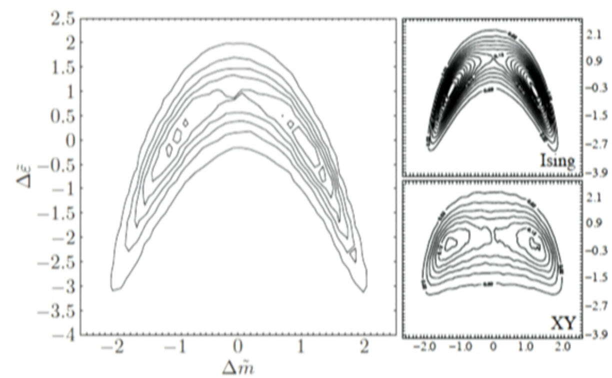
- Binder cumulants $\rightarrow (T_c, H_c)$
- finite-size scaling of mag. susc.:

$$\nu_\sigma = 0.41 \pm 0.01$$
$$\gamma_\sigma = 0.76 \pm 0.02$$

(cf. 3D Ising: $\nu_\sigma = 0.40$, $\gamma_\sigma = 0.79$)

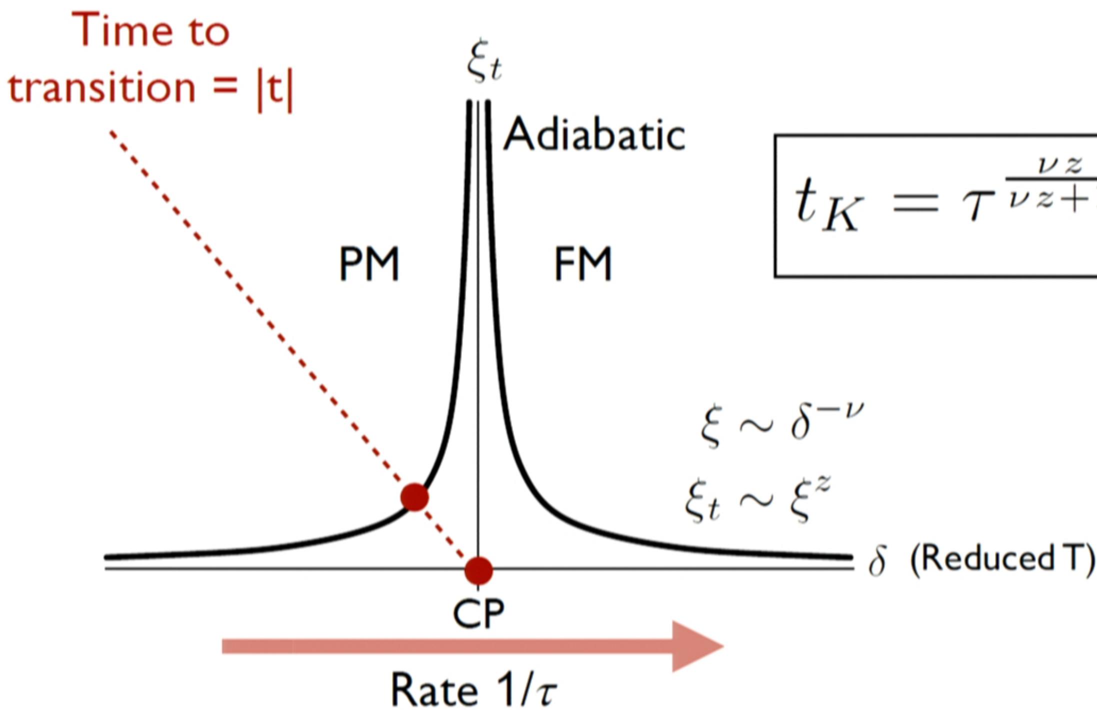
- joint prob. dist. $\Delta\tilde{M}$ vs. $\Delta\tilde{E}$

$\Delta X = (X - \langle X \rangle)/\sqrt{\langle \Delta X^2 \rangle}$



Finite time (Kibble-Zurek) scaling

courtesy of A.Chandran



$$t_K = \tau^{\frac{\nu z}{\nu z + 1}}$$

Finite time (Kibble-Zurek) scaling

courtesy of A.Chandran

$$t_K = \tau^{\frac{\nu z}{\nu z + 1}}$$

$$l_K = t_K^{1/z}$$

Universality, scaling theory?

Finite time (Kibble-Zurek) scaling

universal behaviour: (see e.g., Chandran et al. PRB '12; Liu et al. PRB '14)

$$\lim_{\substack{\tau\to\infty \\ x/l_{\mathrm{KZ}},\, t/t_{\mathrm{KZ}}\ \mathrm{fixed}}} \mathcal{O}(x, t; \tau) = \frac{1}{l_{\mathrm{KZ}}^{\Delta}} \mathcal{G}\left(\frac{x}{l_{\mathrm{KZ}}}, \frac{t}{t_{\mathrm{KZ}}}\right)$$

for example: magnetisation $(\Delta = d - \nu_\sigma^{-1})$

$$\langle m(t)\rangle \sim \frac{1}{l_{\mathrm{KZ}}^{\Delta}} \mathcal{G}\left(\frac{t}{t_{\mathrm{KZ}}}\right) \qquad \begin{aligned} l_{\mathrm{KZ}} &= \tau^{\frac{\nu}{\nu z+1}} \\ t_{\mathrm{KZ}} &= \tau^{\frac{\nu z}{\nu z+1}} \end{aligned}$$

in linear field sweeps across the critical point at rate $1/\tau$

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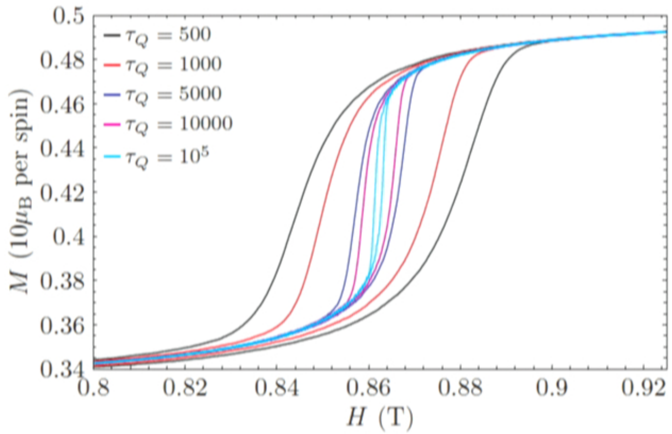
Finite time (Kibble-Zurek) scaling

sweeps at const. rate $1/\tau_Q$: high→low→high field H, across CP



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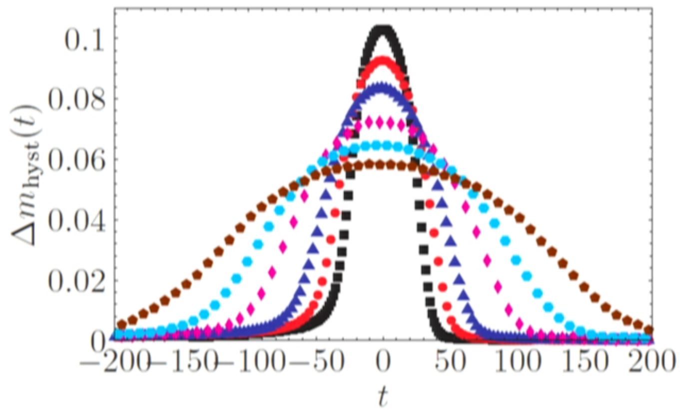

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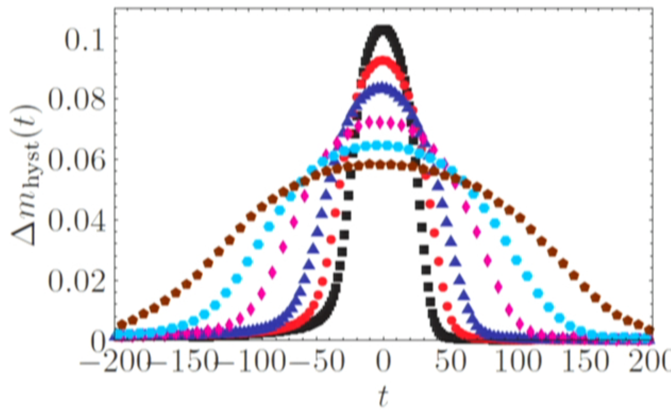

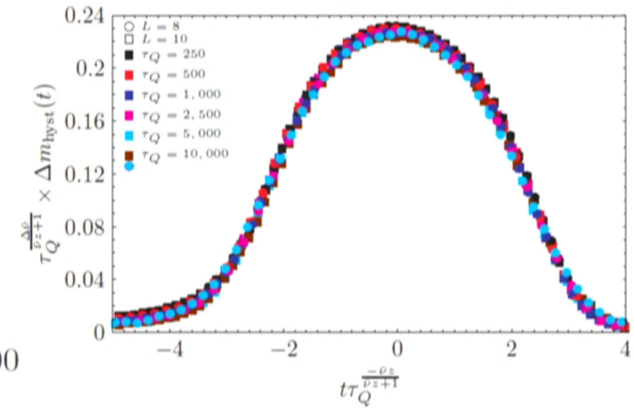

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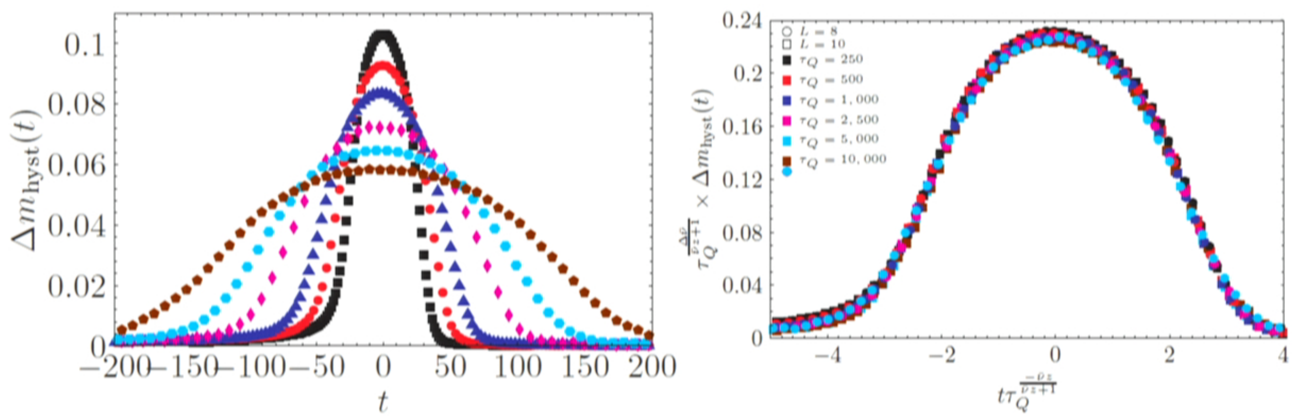
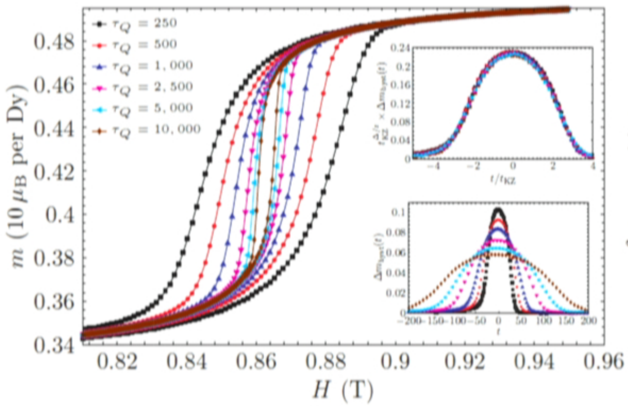
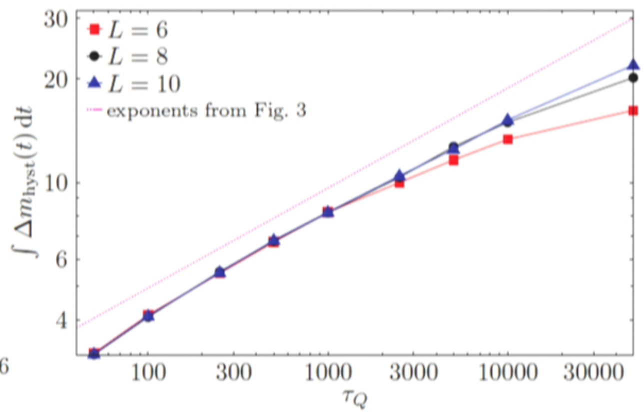


- clear evidence of scaling behaviour over 2 decades in τ_Q
- fitting parameters: $\nu_\sigma = 0.42 \pm 0.01$ and $z = 1.85 \pm 0.05$ (consistent with 3D Ising criticality)
- [Metropolis single-spin-flip 3D Ising: $z \simeq 2$ in Wansleben and Landau PRB 1991]

Finite time (Kibble-Zurek) scaling





- scaling of hysteresis area

$$\text{Area} \sim \tau_Q^{\left(1-\frac{\Delta}{z}\right)\frac{\nu_\sigma z}{\nu_\sigma z+1}}$$

- high-speed cutoff $\tau_Q^{(\min)} \sim 10$ MC step
- low-speed cutoff set by system size
  (MC with $L = 10$ cubic unit cells, $\tau_Q^{(\max)} \sim 10^5$ MC step)

Experiments

experimental evidence for KZ scaling of defect density exists but decisive tests of scaling of dynamical response functions are lacking

- dynamical rate in spin ice HTO and DTO ~ 1 kHz **and** described well by single-spin-flip Monte Carlo simulations
  Jaubert et al. Nat. Phys. 2009
- our simulations translate into field sweeps in the range of 0.03 Tesla/s to 0.7 Tesla/s
- fields swept $0.5 - 1$ Tesla at 0.59 K (expm. $0.3 - 0.4$ K)

$\Rightarrow$ Kibble-Zurek scaling **accessible in existing state of the art spin ice experiments** (e.g., field sweeps by Slobinski et al. PRL 2010)

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Conclusions

Hamp et al. PRB 2015

- study of critical scaling in and out of equilibrium in a 3D topological magnet
- liquid-gas behaviour driven by emergent fractionalised excitations
- (revised) finite size scaling places the critical end point in the 3D Ising universality class

  out of equilibrium: evidence of Kibble-Zurek scaling of dynamical response functions
- experimental parameters within existing field-sweep capability