

Title: TBA

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Abstract:

abelian GLSMs

Outline

- 1) GLSM w/ bdris
- 2) B-brane Topological & Grade Restriction Rule (GRR)
- 2) $Z_{\mathbb{P}^1}(B)$
- 3) $G = U(1)$
- 4) $G = U(2)$
- ...

1322 [hep-th]
PROGRESS)

1) 2d $N=(2,2)$ susy G.T.

$$(G = H \cdot U(1)^S, \rho_V: G \rightarrow SU(2), \text{ Cox. Vect space})$$

↑
SIMPLE

$$W: V \rightarrow E, R: \dots$$

↑
GAUGE INT.

abelian GLSMs

Outline

- 1) GLSM w/ bdris
- 2) B-brane Topological & Grade Restriction Rule (GRR)
- 2) $Z_{\mathbb{P}^1}(B)$
- 3) $G = U(1)$
- 4) $G = U(2)$
- ...

1332 [hep-th]
PROGRESS)

2) Z_d $N=(2,2)$ SUSY G.T.

$$(G = H \times U(1)^S, \rho_V: G \rightarrow SL(V), \text{ Cox. Vect. space})$$

SIMPLE

$$W: V \rightarrow \mathbb{C}, R: \mathbb{Z}(1) \rightarrow \mathfrak{gl}(V)$$

GAUGE INT.

$$t_i = \zeta_i - i\theta_i \in \mathbb{R} + i\mathbb{R} / 2\pi\mathbb{Z}$$

$i=1, \dots, S$

$$S_{\text{GISM}} = S_{\text{kin}} + S_W + S_{\text{FI}}$$

$$\mathcal{N} = \mathcal{Z}_B \subset \mathcal{N}^*(2,2)$$

$$S_{\text{FI}} = \int_{\Sigma} \left(\frac{i}{2\pi} \sum_{\alpha} D^{\alpha} - \lambda \right) \pm \int_{\text{Im}} \left(\frac{1}{2\pi} t_{\alpha} \sigma^{\alpha} \right)$$

$$t_i = \zeta_i - i\theta_i \in \mathbb{C}$$

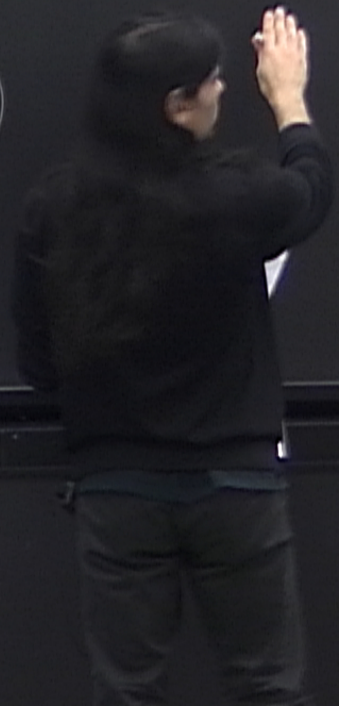
$$i=1, \dots, S$$

$$S_{\text{GWSM}} = S_{\text{kin}} + S_W + S_{\text{FI}}$$

$$\mathcal{N} = \mathbb{Z}_B \subset \mathcal{N} \cdot (2,2)$$

$$S_{\text{FI}} = \int_{\mathbb{Z}} \left(\frac{i}{2\pi} \zeta_i \bar{D}^2 - \frac{i\theta_i}{2\pi} F_i^2 \right) \pm \int_{\partial\mathbb{Z}} \gamma_m \left(\frac{1}{2\pi} t_i \sigma^m \right)$$

$$e^{-S_W}$$



$$t_i = \zeta_i - i\theta_i \in \mathbb{C}$$

$$i=1, \dots, N$$

$$S_{\text{GLSM}} = S_{\text{kin}} + S_W + S_{\text{FI}}$$

$$\mathcal{N} = \mathbb{Z}_B \subset \mathcal{N} \cdot (2, 2)$$

$$S_{\text{FI}} = \int_{\Sigma} \left(\frac{1}{2\pi} \zeta_i \hat{D} - i \frac{\theta_i}{2\pi} \hat{F}_i \right) \pm \int_{\partial \Sigma} \gamma_m \left(\frac{1}{2\pi} t_i \sigma^m \right)$$

$$\Sigma = \mathbb{D}^2$$

$$\delta \left(e^{-S_W} T_{\text{FM}} \left(P e^{-\int_{\partial \mathbb{D}^2} A_\tau d\tau} \right) \right) = 0$$



$$t_i = \zeta_i - i\theta_i \in \mathbb{R} + i\mathbb{R} / 2\pi\mathbb{Z}$$

$$i=1, -1, 5$$

$$+ \sum_W + \sum_{F \in J}$$

$$\Sigma = \mathbb{D}^2 \quad \delta \left(e^{-S_W} T_{\Sigma_M} \left(P e^{-\int_{\partial \mathbb{D}^2} A_\tau d\tau} \right) \right) = 0$$

(2.2)

$$\frac{i}{2\pi} \sum_i \left(D - i \frac{\theta_i}{2\pi} F_i \right) \pm \int_{\partial \Sigma} \gamma_m \left(\frac{1}{2\pi} t_i \sigma^m \right)$$

How to CONSTRUCT A_τ ?

- $M = M_0 \oplus M_1$ \mathbb{Z}_2 -graded (Chern-Pontr. V.S.)
- $Q(\phi) \in E_{\text{incl}}^{\text{chiral}}(M)$ $Q(\phi): M_{(2)} \rightarrow M_{(0)}$ $Q^2(\phi) = W(\phi) \mathbb{1}_M$

$$t_i = \zeta_i - i\theta_i \in \mathbb{R} + i\mathbb{R} / 2\pi\mathbb{Z}$$

$$i = 1, -1, 5$$

$$+ \sum_W + \sum_{F \in J}$$

$$\Sigma = \mathbb{D}^2 \quad \delta \left(e^{-S_W} \text{Tr}_M \left(P e^{-\int_{\partial \mathbb{D}^2} A_\mu dt} \right) \right) = 0$$

(2,2)

$$\frac{i}{2\pi} \sum_i \left(D_i^* - i \frac{\theta_i}{2\pi} F_i^* \right) \pm \int_{\partial \Sigma} \gamma_m \left(\frac{1}{2\pi} t_i \right)$$

How to construct A_μ ?

- $M = M_0 \oplus M_1$ \mathbb{Z}_2 -graded (Chern-Pontryagin V.S)
- $Q(\phi) \in E_{\text{incl}}^{\text{chiral}}(M)$ $Q(\phi): M_{(0)} \rightarrow M_{(1)}$ $Q^2(\phi) = W(\phi) \mathbb{1}_M$
- $\rho_Q: G \rightarrow \text{SL}(M)$, $\rho_Q: \mathfrak{g} \rightarrow \mathfrak{gl}(M)$ ← compatible w/ S_V, R

$$\mathcal{N} = \mathbb{Z}_2 \subset \mathcal{N}^*(2,2)$$

$$S_{FI} = \int_{\Sigma} \left(\frac{i}{2\pi} \tilde{\Sigma} \cdot D - \frac{i\theta_i}{2\pi} F_i \right) \pm \int_{\partial \Sigma} \gamma_m \left(\frac{1}{2\pi} t_m \sigma^m \right)$$

$$W(\lambda \phi) = \lambda^2 W(\phi)$$

How to construct $A_{\mathbb{Z}_2}$?

- $M = M_0 \oplus M_1$ \mathbb{Z}_2 -graded (Chern-Patton V.S.)
- $Q(\phi) \in E_{\text{ind}}^{\text{chiral}}(M)$ $Q(\phi): M_{(0)} \rightarrow M_{(1)} \in \mathcal{G}$
- $\mathcal{S}_Q: \mathcal{G} \rightarrow \text{SL}(M)$, $\mathcal{F}_+ \mathcal{U}(M) \rightarrow \mathcal{F}_- \mathcal{U}(M) \leftarrow$

$$\mathcal{B} = \mathcal{S}_Q(\mathcal{F}_+) \rightsquigarrow A_{\mathbb{Z}_2}$$

$$\mathcal{N} = \mathbb{Z}_2 \subset \mathcal{N} = (2, 2)$$

$$S_{FI} = \int_{\Sigma} \left(\frac{1}{2\pi} \tilde{\Sigma} \cdot D - i \frac{\theta_i}{2\pi} F_i \right) \pm \int_{\partial \Sigma} \gamma_{\text{int}} \left(\frac{1}{2\pi} t_a \sigma^a \right)$$

$$W(\lambda \phi) = \lambda^2 W(\phi)$$

$\delta(\epsilon) \quad \text{Is } M \text{ (I.E.)}$

How to construct $A_{\mathbb{Z}_2}$?

- $M = M_0 \oplus M_1$ \mathbb{Z}_2 -graded (Chern-Paton V.S.)
- $Q(\phi) \in E_{\text{ind}}^{\text{chiral}}(M)$ $Q(\phi): M_{(0)} \rightarrow M_{(1)}$
- $\mathcal{S}_Q: \mathcal{G} \rightarrow \text{SL}(M)$, $\mathcal{F}_Q \rightarrow \mathcal{F}(M)$

$$B = (M, Q, \mathcal{S}_Q, \mathcal{F}_Q) \rightsquigarrow A_{\mathbb{Z}_2}$$

$$N = \mathbb{Z}_B \subset N^*(2,2)$$

$$S_{FI} = \int_{\Sigma} \left(\frac{i}{2\pi} \tilde{\Sigma} \cdot D - \frac{i\theta_i}{2\pi} F_i \right) \pm \int_{\partial \Sigma} \text{Tr} \left(\frac{1}{2\pi} t_a \sigma^a \right)$$

$$W(\lambda \phi) = \lambda^2 W(\phi)$$

How to construct A_{τ} ?

- $M = M_0 \oplus M_1$ \mathbb{Z}_2 -graded (Chern-Paton V.S.)
- $Q(\phi) \in \text{End}^{\text{odd}}(M)$ $Q(\phi): M_{(0)} \rightarrow M_{(1)}$
- $S_Q: G \rightarrow SL(M)$, $F_{\tau} \rightarrow \mathfrak{g}(M)$

$$B = (M, Q, S_Q, F_{\tau}) \rightsquigarrow A_{\tau}$$

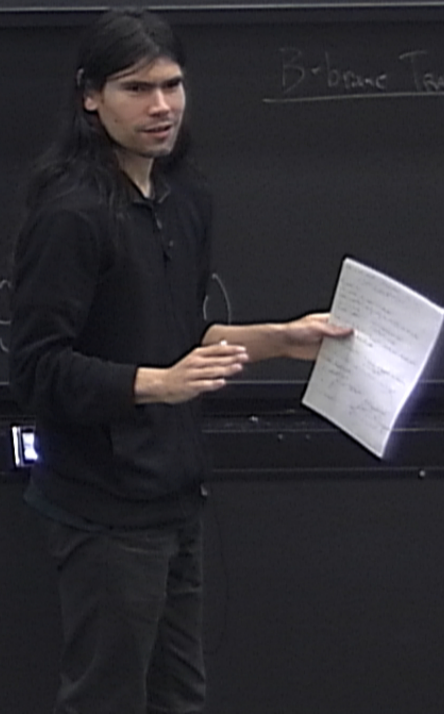
B.C.: \mathbb{F} -chirals ~ Neumann

$$\mathcal{L} - \int c t_a = \text{Lie}(T_a)c$$

↓
Vect

Asympt. of γ depend on some \sim

B-brane Transversal & GRR



$\delta(\epsilon) \in \text{Iso}_M(\pm \epsilon)$

$$\left(\frac{i \theta_i F_i}{2\pi} \right) \pm \int_{\partial \Sigma} \text{Yon} \left(\frac{1}{2\pi} t_i \sigma_i \right)$$

$$(\lambda \phi) = \lambda^2 W(\phi)$$

How to construct A_ϵ ?

- $M = M_0 \oplus M_1$ \mathbb{Z}_2 -graded (Chern-Pontr. V.S.)
- $Q(\phi) \in \text{End}^{\text{odd}}(M)$ $Q(\phi): M_{(a)} \rightarrow M_{(b)}$ $Q^2(\phi) = W(\phi) \mathbb{1}_M$
- $S_Q: G \rightarrow \text{SL}(M)$, $\Gamma \times \mathbb{R} \rightarrow \mathfrak{g}(M) \leftarrow \text{Compatible w/ } S_V, R$

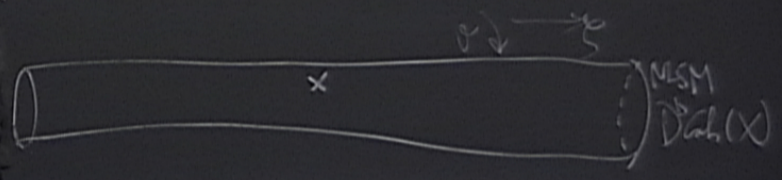
$$\sim \rightarrow A_\epsilon$$

NEUMANN

$$t_\infty \mathbb{C} = \text{Lie}(T_G) \mathbb{C}$$

Asympt. of ψ depending on some $\text{Yon}(V)$

THE TRANSPORT & GRR
HYPERSURFACE ($G = U(1)$)



$$\delta(\epsilon) \text{ is } \mathbb{Z}_2 \text{-graded}$$

How to construct A_ϵ ?

$$\left(\frac{i\theta_i F_i}{2\pi} \right) \pm \int_{\partial \Sigma} \left(\frac{1}{2\pi} t_{\alpha\beta} \sigma^{\alpha\beta} \right)$$

$$(\lambda \phi) = \lambda^2 W(\phi)$$

- $M = M_0 \oplus M_1$ \mathbb{Z}_2 -graded (Chern-Pontr. V.S.)
- $Q(\phi) \in \text{End}^{\text{odd}}(M)$ $Q(\phi): M_{(0)} \rightarrow M_{(1)}$ $Q^2(\phi) = W(\phi) \mathbb{1}_M$
- $S_Q: G \rightarrow SL(M)$, $F_x \underline{u}(x) \rightarrow \mathfrak{g}(M) \leftarrow \text{Compatible w/ } S_V, R$

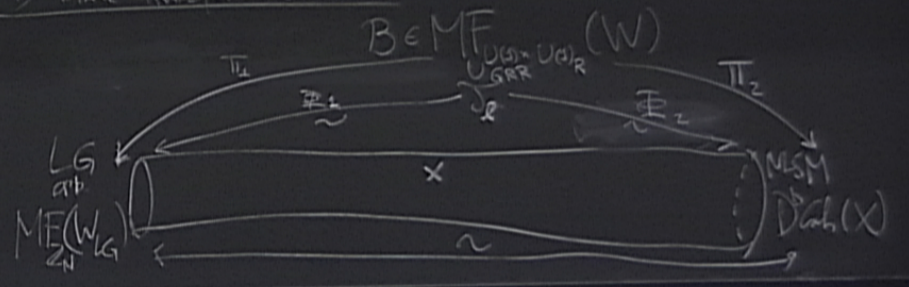
B-Field Transport & GRR

$\rightsquigarrow A_\epsilon$

NEUMANN

$$t_{\alpha\beta} c = \text{Lie}(T_\alpha)c$$

Asympt. of γ depending on some $\int_{\text{loop}} W_{\text{eff}}[B]$



$Z_{D^2}(\mathcal{B})$ | Rigid SUSY on D^2 w/ $ds^2 = \frac{4r^2 |dz|^2}{(1+|z|^2)^2}$
 can be exactly computed

$$Z_{D^2}(\mathcal{B}) = (\Lambda r)^{2c} \sigma \left(\prod_{\alpha > 0} \alpha(\sigma) \operatorname{sinh}(\pi \alpha(\sigma)) \right) \prod_{j=1}^{\dim V} \Gamma(iQ_j(\sigma) + R_{j/2}) e^{it(\sigma)} \int_{\mathcal{B}}$$

$Z_{\mathbb{D}^2}(B)$ | Regular susy on \mathbb{D}^2 w/ $ds^2 = \frac{4r^2 |dz|^2}{(1+|z|^2)^2}$
 can be exactly computed

$$Z_{\mathbb{D}^2} = \int_{\gamma \subset \mathbb{R}^2} d^l \sigma \left(\prod_{\alpha \in \mathfrak{g}} \alpha(\sigma) \sinh(\tau \alpha(\sigma)) \right) \prod_{j=1}^{l-V} \Gamma(iQ_j(\sigma) + R_{j/2}) e^{i\tau(\sigma)} f_B(\sigma)$$

$l_{\mathfrak{g}} = \dim(\mathfrak{T}_{\mathfrak{g}})$

$$Z_B(B) = (\Lambda r)^{k_2} \int_{\gamma_{ct,c}} d^{k_2} \sigma \left(\prod_{\alpha \in \mathcal{A}} \alpha(\sigma) \text{Im}(\alpha(\sigma)) \right) \prod_{j=1}^l L'(iQ_j(\sigma) + k_{j,2}) e^{-\int_B \sigma}$$

$$l_2 = \dim(T_{\mathcal{G}})$$

$$Z_B(B) = \int_{\gamma_{ct,c}} d^{k_2} \sigma e^{-\int_B \sigma} F_B(\sigma)$$

Def: γ ADMISSIBLE (For F_B $B \in \mathbb{R}^+$)

i) Lagrangian

ii) Cont. def of $\gamma_R := \{ \gamma_m(\sigma) = 0 \}$ s.t. doesn't go through poles of $F_B(\sigma)$

iii) $|e^{-\int_B \sigma} F_B(\sigma)| \rightarrow 0$ in ALL ASYMPT. DIR. of γ

P

$$Z_B(B) = (\Lambda r)^{k_2} \int_{\gamma_{ct,c}} d^k \sigma \left(\prod_{\alpha \in \mathcal{P}} \alpha(\sigma) \operatorname{Im} h(\alpha(\sigma)) \right) \prod_{j=1}^l L'(i\Omega_j(\sigma) + k_j/2) e^{-\int_B \sigma}$$

$$l_0 = \dim(T_{\mathcal{G}})$$

$$Z_D(B) = \int_{\gamma_{ct,c}} d^k \sigma e^{-\int_B \sigma} F_B(\sigma)$$

Def: γ admissible (for F_B , $B \in t$)

Proposal

i) Lagrangian

ii) Cont. def of $\gamma_R := \{ \gamma_{\text{Im}}(\sigma) \}$ s.t. doesn't go through poles of $F_B(\sigma)$

iii) $|e^{-\int_B \sigma} F_B(\sigma)| \rightarrow 0$ in t. dir. of γ

$$(\Delta r)^{-2} \int_{\gamma_C} d^k \sigma \left(\prod_{i=1}^k \alpha_i(\sigma) \right) \text{Im} h(\tau \alpha(\sigma)) \prod_{j=1}^l \left[(i Q_j(\sigma) + R_{j,1/2}) e^{-\dots} f_B(\sigma) \right]$$

$$l_G = \dim(T_G)$$

$$Z_D(B) = \int_{\gamma_C} d^k \sigma e^{-i \tau(\sigma)} F_B(\sigma)$$

QUESTION (For $B \in \mathbb{R}^d$)

deg of $\gamma_R := \{ \text{Im}(\sigma) = 0 \}$ s.t. doesn't go through poles of $F_B(\sigma)$
 $\left| \frac{1}{F_B(\sigma)} \right| \rightarrow 0$ IN ALL ASYMPT. DIR. of γ

PROPOSAL:

B is GRR IF \exists ADMISSIBLE γ_t
 for all $\xi = \text{Re}(t)$ and all $\sigma \in T_t$.

$\delta(\epsilon) \in \text{Iso}_M(\mathbb{C})$

How to construct A_ϵ ?

$$\left(\frac{i\partial\bar{\partial}}{2\pi} F_\epsilon^i \right) \pm \int_{\partial\mathbb{D}} \gamma_{\text{int}} \left(\frac{1}{2\pi} t_i \sigma^i \right)$$

$$(\lambda\phi) = \lambda^2 W(\phi)$$

- $M = M_0 \oplus M_1$ \mathbb{Z}_2 -graded (Chern-Pontryagin V.S.)
- $Q(\phi) \in \text{End}^{\text{odd}}(M)$ $Q(\phi): M_0 \rightarrow M_1$ $Q^2(\phi) = W(\phi) \mathbb{1}_M$
- $\mathcal{S}_Q: \mathcal{G} \rightarrow \text{SL}(M)$, $\Gamma_x: \mathcal{U}(M) \rightarrow \mathfrak{g}(M) \leftarrow \text{Compatible w/ } \mathcal{S}_V, R$

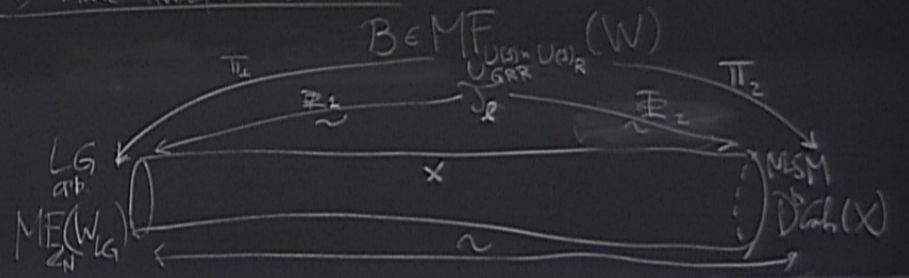
$\gamma \rightsquigarrow A_\epsilon$

NEUMANN


$$t_\infty \mathbb{C} = \text{Lie}(T_G) \mathbb{C}$$

Asympt. of γ depending on some $\gamma_{\text{int}}(W_{\text{eff}}[B])$

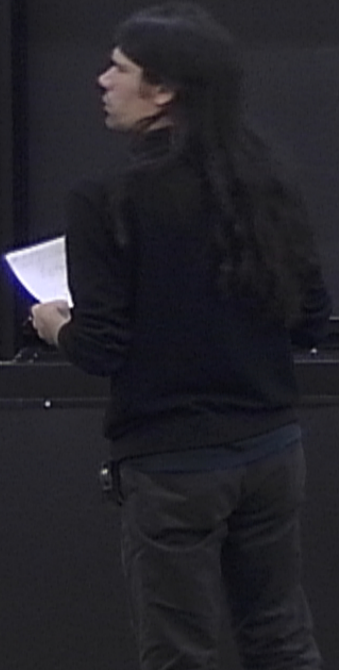
B-brane Transport & GRR




2) Logarithm
 iii) Cont. deg of $\gamma_R := \{ \gamma_{\text{Im}(\sigma)=0} \}$ st. doesn't go through poles of $F_B(\sigma)$
 iii) $|e^{s\tau(\sigma)} F_B(\sigma)| \rightarrow 0$ in ALL ASYMPT. DIR. of γ
 Conj: $Z_{\mathbb{P}^2}(B) = \underbrace{\int_{\mathbb{R}\mathbb{R}}}_{\text{central charge}} \langle 0 | B_{LR} \rangle_{\mathbb{R}\mathbb{R}} =$



$B = \text{GRR}$
 for all $\zeta =$
 $\theta \in T_e.$



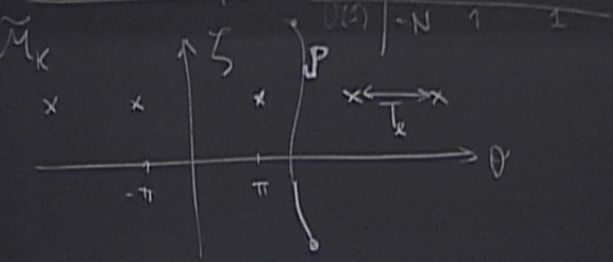
2) Lagrangian
 iii) Cont. disc of $\gamma_R := \{z \in \mathbb{C} \mid \text{Im}(z) = 0\}$ s.t. doesn't go through poles of $F_B(\sigma)$
 iii) $|e^{z\tau(\sigma)} F_B(\sigma)| \rightarrow 0$ in ALL ASYMPT. DIR. of γ
 Conj: $Z_B(B) = Z(B_{IR}) = \langle 0 \mid B_{IR} \rangle_{RR}$



$B = GR$
 for all $\zeta =$
 $\theta \in T_\zeta$.

Abelian Example

$U(\zeta)$	$\begin{matrix} P & X_1 & \dots & X_N \\ -N & 1 & & 1 \end{matrix}$
------------	---------------------------------------------------------------------




2) Lagrangian
 ii) Cont. deg of $\gamma_R := \{ \gamma_m(\sigma) = 0 \}$ s.t. doesn't go through poles of $F_B(\sigma)$
 iii) $|e^{i\zeta(\sigma)} F_B(\sigma)| \rightarrow 0$ in ALL ASYMPT. DIR. of γ

Conj: $Z_B(B) = \mathcal{Z}(B_{IR}) = \langle 0 | B_{LR} \rangle_{RR}$

Central charge

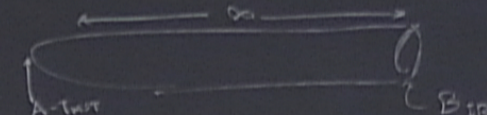
Abelian Example

$\exists \gamma$ ADM. FOR ALL $\rho \in \mathcal{D}$
 IF $-\frac{N}{2\pi} < \frac{\rho}{2\pi} + \eta < \frac{N}{2\pi}$
 weights of S_R

2) Logarithm

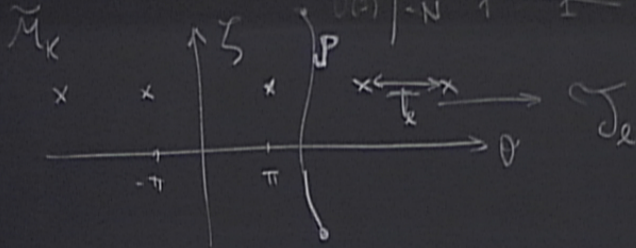
ii) Cont. log of $\gamma_R := \{ \operatorname{Im}(\sigma) = 0 \}$ s.t. doesn't go through poles of $F_B(\sigma)$

iii) $|e^{i\tau(\sigma)} F_B(\sigma)| \rightarrow 0$ in ALL ASYMPT. DIR. of γ

Conj: $Z_B^2(B) = \int_{\mathbb{R}^2} \langle \cdot | B_{LR} \rangle_{\mathbb{R}^2} =$ 

B_{LR}
for all $\zeta =$
 $\theta \in T_2$.

Abelian Example



$\exists \gamma$ s.t. FOR ALL $\rho \in \mathcal{D}$

IF $-\frac{N}{2} < \frac{\rho}{2\pi} + \tau < \frac{N}{2}$
weights of S_R

$$\mathcal{N} = \mathbb{Z}_B \subset \mathcal{N}^*(2,2)$$

$$S_{FI} = \int_{\mathbb{Z}} \left(\frac{1}{2\pi} \sum_i D_i^2 - i \frac{\theta_i}{2\pi} F_i \right) \pm \int_{\mathbb{Z}} \gamma_m \left(\frac{1}{2\pi} t_m \sigma_m^2 \right)$$

$$W(\lambda \phi) = \lambda^2 W(\phi)$$

How to construct $A_{\mathbb{Z}}$?

- $M = M_0 \oplus M_1 \mathbb{Z}_2$ -graded (Chern-Pontryagin V.S.)
- $Q(\phi) \in \text{End}^{\text{odd}}(M)$ $Q(\phi): M_0 \rightarrow M_1$
- $\mathcal{S}_Q: G \rightarrow \text{SL}(M)$, $\mathbb{Z}_2 \rightarrow \mathbb{Z}_2$

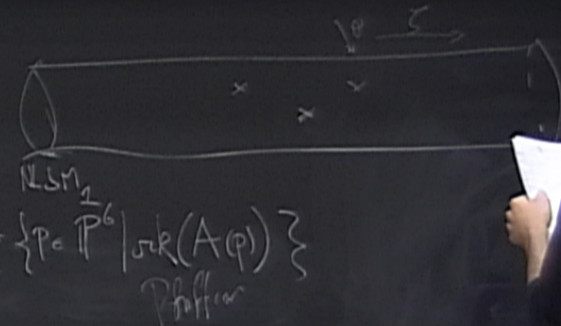
$$G = \text{U}(2)$$

	P^a	X_a
$G = \text{U}(2)$	\det^{-1}	2

$a=1, \dots, 7$ $\beta, \alpha=1, 2$

$$W = A_{\alpha}^{ij} P^{\alpha} X_i^{\alpha} X_j^{\beta} \epsilon_{\alpha\beta}$$

$\{A_{\alpha}\}_{\alpha=1}^2$ 7x7 ANTI-SYM. generic



$$G = U(2) \quad \left| \begin{array}{c} 1 \\ \det^{-1} \end{array} \right. \quad 2$$

$$W = A_{\alpha}^{ij} P^{\alpha} X_{\alpha}^{\alpha} X_{\beta}^{\beta} \epsilon_{\alpha\beta}$$

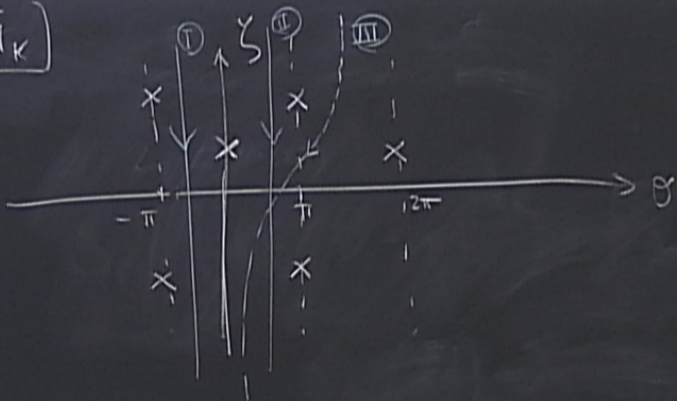
$i A_{\alpha}^{ij}$ 7x7 ANTI-SYM. symmetric

$$X_i = \left\{ P \in \mathbb{P}^6 \mid \text{rk}(A(P)) \right\}$$

Plücker

CI. $Gr(2, 7)$

\tilde{M}_K



Rep. of $U(2)$
 \mathbb{C} -Trivial Rep $S \cong \mathbb{C}^2$ -fund.
 $\Gamma(m) = \mathbb{C} \otimes (\det S)^m$
 $S^l(m) = \text{Sym}^l S \otimes (\det S)^m$

$$G = U(2) \mid \det^{-1} \quad 2$$

$$W = A_{\alpha}^{ij} P_{\alpha} X_i^{\alpha} X_j^{\beta} \epsilon_{\alpha\beta}$$

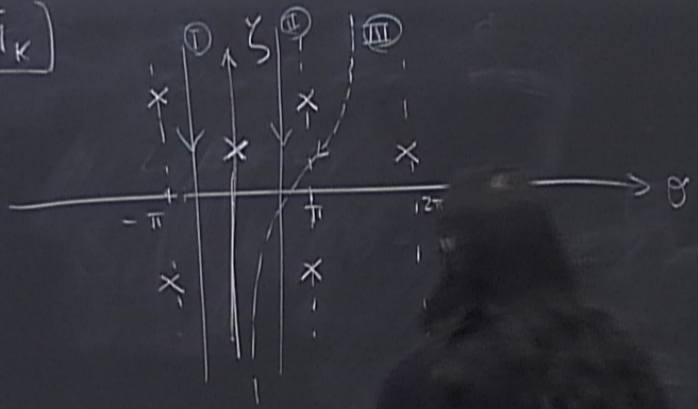
$\{A_{\alpha}^{ij}\}_{\alpha=1}^7$ 7x7 ANTI-SYM. generic

$$X_i = \{P_{\alpha} P^{\alpha} | \text{rk}(A(p))\}$$

Phaffian

$$CI. Gr(2,7)[7] = X$$

\tilde{M}_K



Rep. of $U(2)$
 \mathbb{C} -Ternary Rep $S \cong \mathbb{C}^2$ -fund.
 $C(m) = C \otimes (\det S)^m$
 $S^l(m) = \text{Sym}^l S \otimes (\det S)^m$

C	$C(1)$...	$C(6)$	$C(7)$
S	$S(1)$...	$S(6)$	$S(7)$
$S^2 S$	$S^2 S(1)$		$S^2 S(6)$	

$$\gamma \subset \mathbb{C}^2$$