Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 15

Date: Apr 29, 2016 10:15 AM

URL: http://pirsa.org/16040049

Abstract:

$\boxed{k \ll aH}$

NEWTONIAN GAUGE: $B'' = h'' = 0$, $A = \Phi$ $\quad h^{(1)} = -\Psi$

$$\Phi = -\dot{\alpha} + \cdots \qquad \dot{\alpha} + H\alpha = \lambda + \frac{8\pi G a^2 \bar{\rho}\, \Pi}{k^2}$$

$$\Psi = H\alpha - \lambda + \cdots$$

WHICH GAUGE SYMMETRIES THE METRIC IN NEWTONIAN GAUGE?

AT $k \neq 0$, NONE $\quad \varepsilon^{\mu} = \left(\alpha e^{ik\cdot x},\ i\beta_i e^{ik\cdot x} \right)$

$$B'' \to B'' + a^{-1} k \alpha + a\dot{\beta}$$

$$h'' \to h'' + k\beta$$

AT $k=0$, LOTS!

$$B_i \to B_i - a^{-1}\partial_i\alpha + a\dot{\beta}_i$$

$$h_{ij} \to h_{ij} - H\delta_{ij}\alpha - \frac{1}{2}(\partial_i\beta_j + \partial_j\beta_i)$$

$$\left.\begin{aligned} \alpha &= \alpha(t) \\ \beta_i &= -\lambda x_i \end{aligned}\right\}$$

$$\Phi \to \Phi - \dot{\alpha}$$

$$\Psi \to \Psi + H\alpha - \lambda$$

$\dot{\beta}$:

$\frac{1}{2}(\partial_i \beta_j + \partial_j \beta_i)$

$$\delta\phi \to \delta\phi - \dot{\bar{\phi}}\,\alpha$$

$$\delta\rho \to \delta\rho - \dot{\bar{\rho}}\,\alpha$$

GET SOLUTION TO $k=0$ EQS OF MOTION

$\Rightarrow$ GAUGE MODE

E.G. PERFECT FL

CAN THIS SOLUTION BE EXTENDED TO $k \neq 0$?

ONE MIGHT THINK THE ANSWER IS ALWAYS "YES"

E.G. MATTER SECTOR W/ METRIC FLUCTUATIONS NEGLECTED

$\delta\phi = \mathcal{O}(k^0)$

SINGLE FIELD INFLATION

$$\left(\frac{\partial^2}{\partial t^2} + 3H \frac{\partial}{\partial t} + V''(\bar{\phi}) \right) \delta\phi = -\frac{k^2}{a^2} (\delta\phi)$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\ddot{\Phi} - 3a^2 H \dot{\psi} - k^2 \psi = 4\pi G a^2 (\delta\rho)$$

$$\cancel{(a^2 H^2 \Phi) - \frac{1}{3} k^2 \Phi + \frac{d}{dt}(a^2 \dot{\psi}) = 4\pi G a^2 \left(\frac{1}{3}\delta p + \delta p \right)}$$

$$\cancel{k(H\Phi + \dot{\psi}) = 4\pi G a (\bar{\rho} + \bar{p}) v''}$$

$$-k^4(\Phi - \psi) = 8\pi a^2 \bar{p}\, \Pi'' \Leftarrow 0 = 0$$

LEADING ORDER IN K

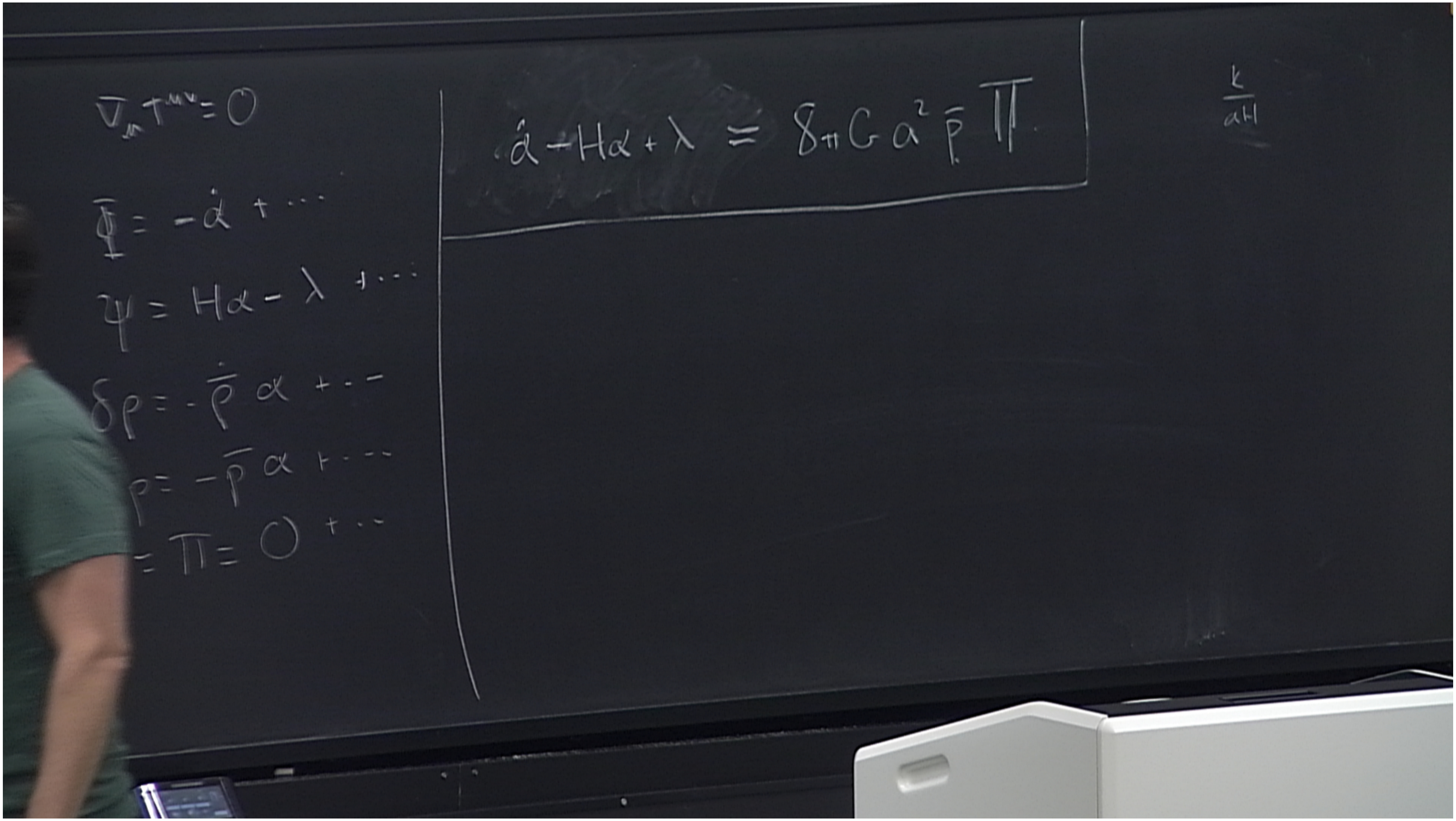
$$\Phi = -\dot{\alpha} + \cdots$$

$$\psi = H\alpha - \lambda$$

$$\delta\rho = -\dot{\bar{\rho}}\,\alpha +$$

$$\delta p = -\dot{\bar{p}}\,\alpha +$$

$$V = \Pi = 0 +$$



$\nabla_\mu T^{\mu\nu} = 0$
$\Phi = -\dot{\alpha} + \cdots$
$\Psi = H\alpha - \lambda + \cdots$
$\delta\rho = -\dot{\bar{\rho}}\,\alpha + \cdots$
$p = -\bar{p}\,\alpha + \cdots$
$= \Pi = 0 + \cdots$
$\dot{\alpha} - H\alpha + \lambda = 8\pi G a^2 \bar{p}\,\Pi$
$\frac{k}{aH}$

$k \ll aH$

NEWTONIAN GAUGE: $B^{\parallel} = h^{\parallel} = 0$, $A = \Phi$ $h^{(+)} = -\Psi$

$\Phi = -\dot{\alpha} + \cdots$

$\Psi = H\alpha - \lambda + \cdots$

$\delta\rho = -\dot{\bar{\rho}}\alpha + \cdots$

$\dot{\alpha} + H\alpha = \lambda + \frac{8\pi G a^2 \bar{\rho}\Pi}{k^2}$

$\alpha(t) = \frac{1}{a(t)} \int_{-\infty}^{t} dt' \left[\lambda a(t') + \frac{8\pi G a^3 \bar{\rho}\Pi}{k^2} \right]$

ADIABATIC DECAYING MODE. $\lambda = 0$ $\alpha \sim 1/a$

WHICH GAUGE SYMMETRIES THE METRIC IN NEWTONIAN GAUGE?

AT $k \neq 0$, NONE $\varepsilon^{\mu} = (\alpha e^{ik\cdot x}, i\beta_i e^{ik\cdot x})$

$B^{\parallel} \to B^{\parallel} + a^{-1} k\alpha + a\dot{\beta}$

$h^{\parallel} \to h^{\parallel} + k\beta$

AT $k = 0$,

B_i

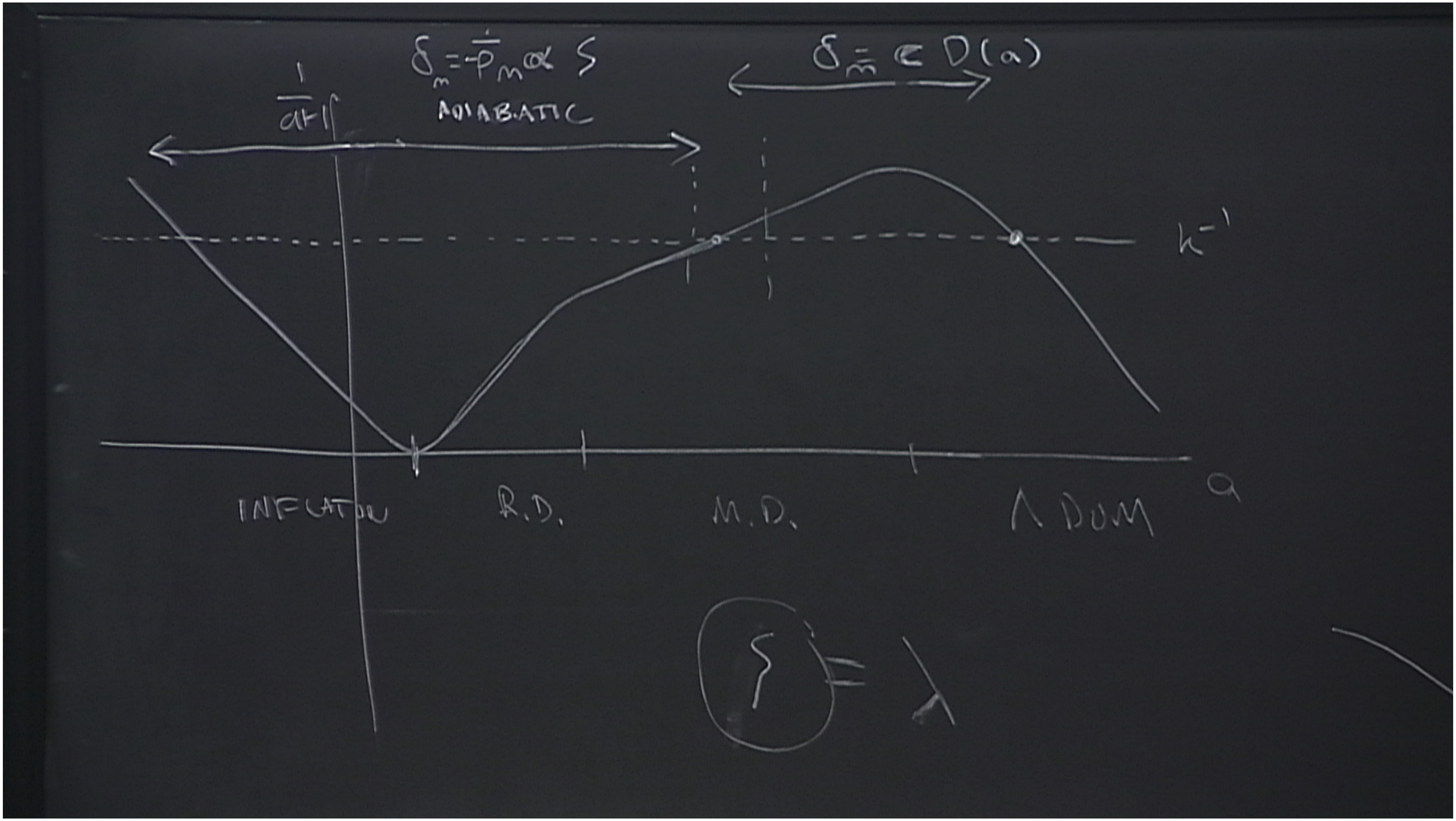
h_{ij}

$\alpha = \alpha($

$\beta_i = -$

Φ

Ψ


$\delta_m = \frac{\dot{\rho}_m}{\dots} \propto \zeta$
ADIABATIC
$\delta_m = \epsilon D(a)$
$\frac{1}{aH}$
k^{-1}
INFLATION
R.D.
M.D.
Λ DOM
a
$\zeta = \lambda$

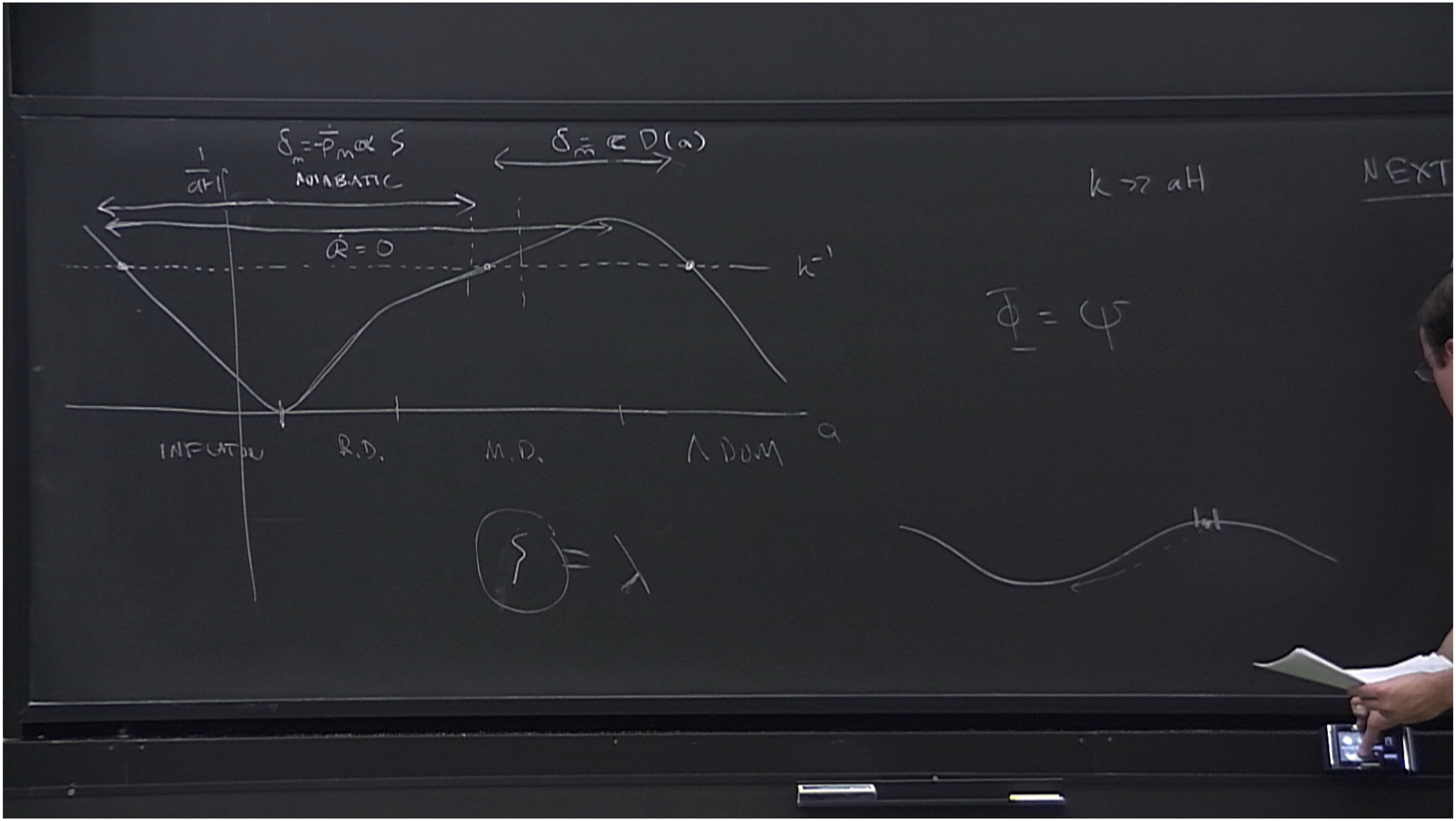
NEXT: CONSERVATION LAW

$$\mathcal{R} = -\Psi - \frac{aH}{k} v''$$ IN NEWTONIAN GAUGE

$$\dot{\mathcal{R}} = \frac{H}{\bar{\rho}+\bar{p}}\left[-\delta p + a\dot{\bar{p}}\frac{v}{k} + \frac{2}{3}\frac{\bar{p}}{\bar{\rho}+\bar{p}}\Pi\right]$$

$= 0$ DURING MATTER DOMINATION;

$= 0$ ADIABATIC MODE $(\mathcal{R} = \lambda = \zeta)$


ADIABATIC
Ṙ = 0
k >> aH
NEXT
k⁻¹
Φ = Ψ
R.D.
M.D.
Λ DOM
a

DURING MATTER DOMINATION: $D(a) \approx a$

$$H^2 = H_0^2 \Omega_m a^{-3} \qquad \begin{pmatrix}\delta \\ \Theta\end{pmatrix} = \begin{pmatrix}1 \\ -1\end{pmatrix} a$$

$\hookrightarrow e^{\lambda \log a}$ $\lambda = 1$

$$\delta_m = c\, D(a)$$

$$\Theta = \frac{k v''}{aH} = -c\, D(a)$$

$$\Psi = \Phi = -\frac{3}{2}\frac{a^2 H^2}{k^2}\delta_m$$

$$\Rightarrow \mathcal{R} = -\Psi - \frac{aH}{k} v''$$

$$= \frac{5}{2} c\, D(a) \frac{a^2 H^2}{k^2} = \frac{5}{2} c \frac{\Omega_m H_0^2}{k^2}$$

$$\mathcal{R}_{IC} = \mathcal{R}_{MD}$$

$$\zeta = \frac{5}{2} c \frac{\Omega_m H_0^2}{k^2}$$

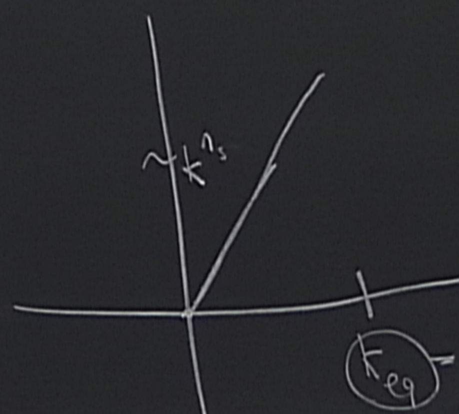
$$c = \frac{2}{5}\frac{k^2}{\Omega_m H_0^2}\zeta$$

$$\delta_m = \frac{2}{5}\frac{k^2}{\Omega_m H_0^2} D(a)\, \zeta$$

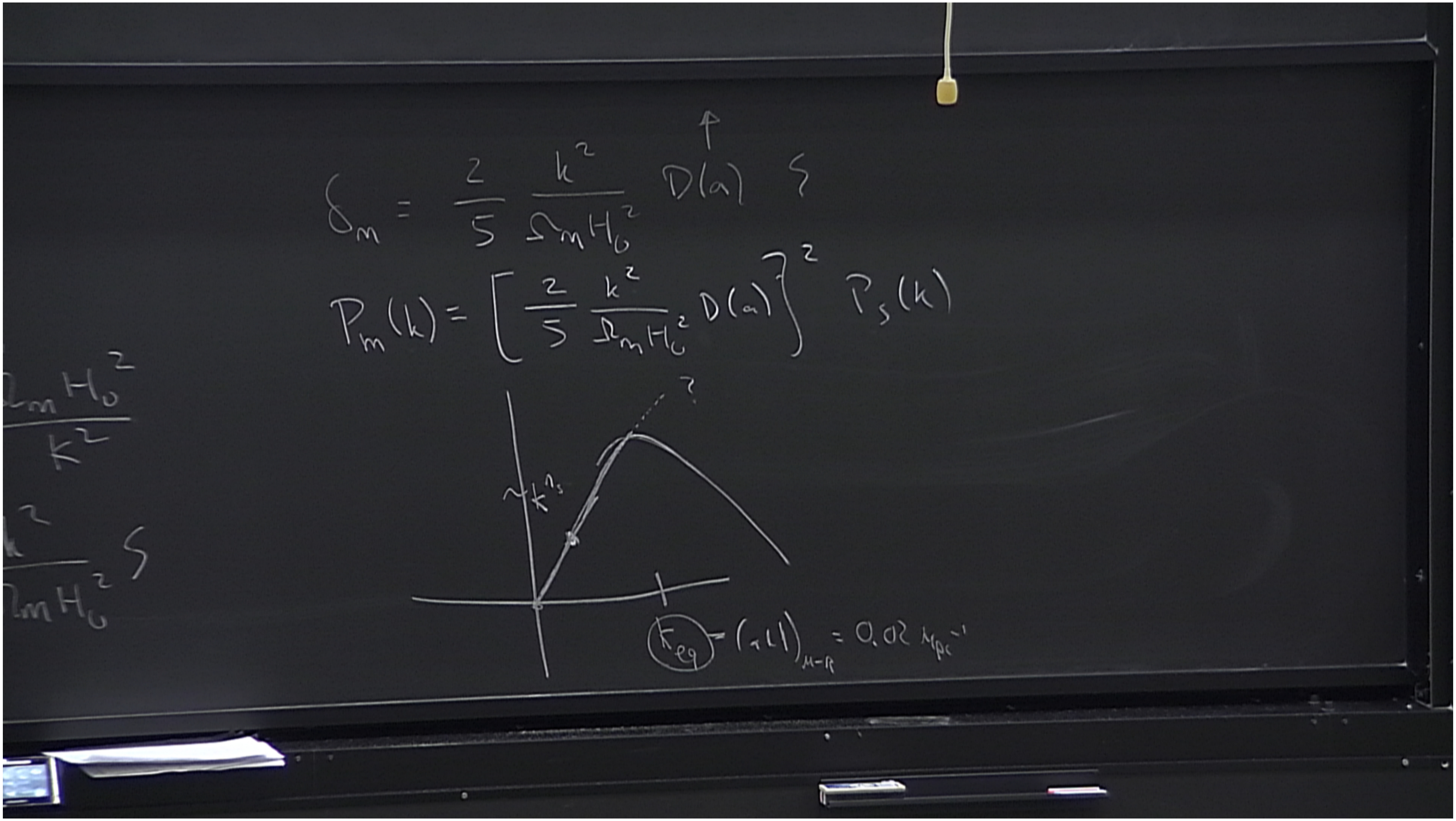
$$P_m(k) = \left[\frac{2}{5}\frac{k^2}{\Omega_m H_0^2} D(a)\right]^2 P_\zeta(k)$$

$$\frac{\Omega_m H_0^2}{k^2}$$

$$\frac{k^2}{\Omega_m H_0^2}\zeta$$



$k_{eq} = (aH)_{M-R} = 0.02\ \mathrm{Mpc}^{-1}$


$\delta_m = \frac{2}{5} \frac{k^2}{\Omega_m H_0^2} D(a)\, \zeta$
$P_m(k) = \left[\frac{2}{5} \frac{k^2}{\Omega_m H_0^2} D(a) \right]^2 P_\zeta(k)$
$\sim k^{n_s}$
?
$k_{eq} = (aH)_{M-R} = 0.02\ \mathrm{Mpc}^{-1}$