Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 14

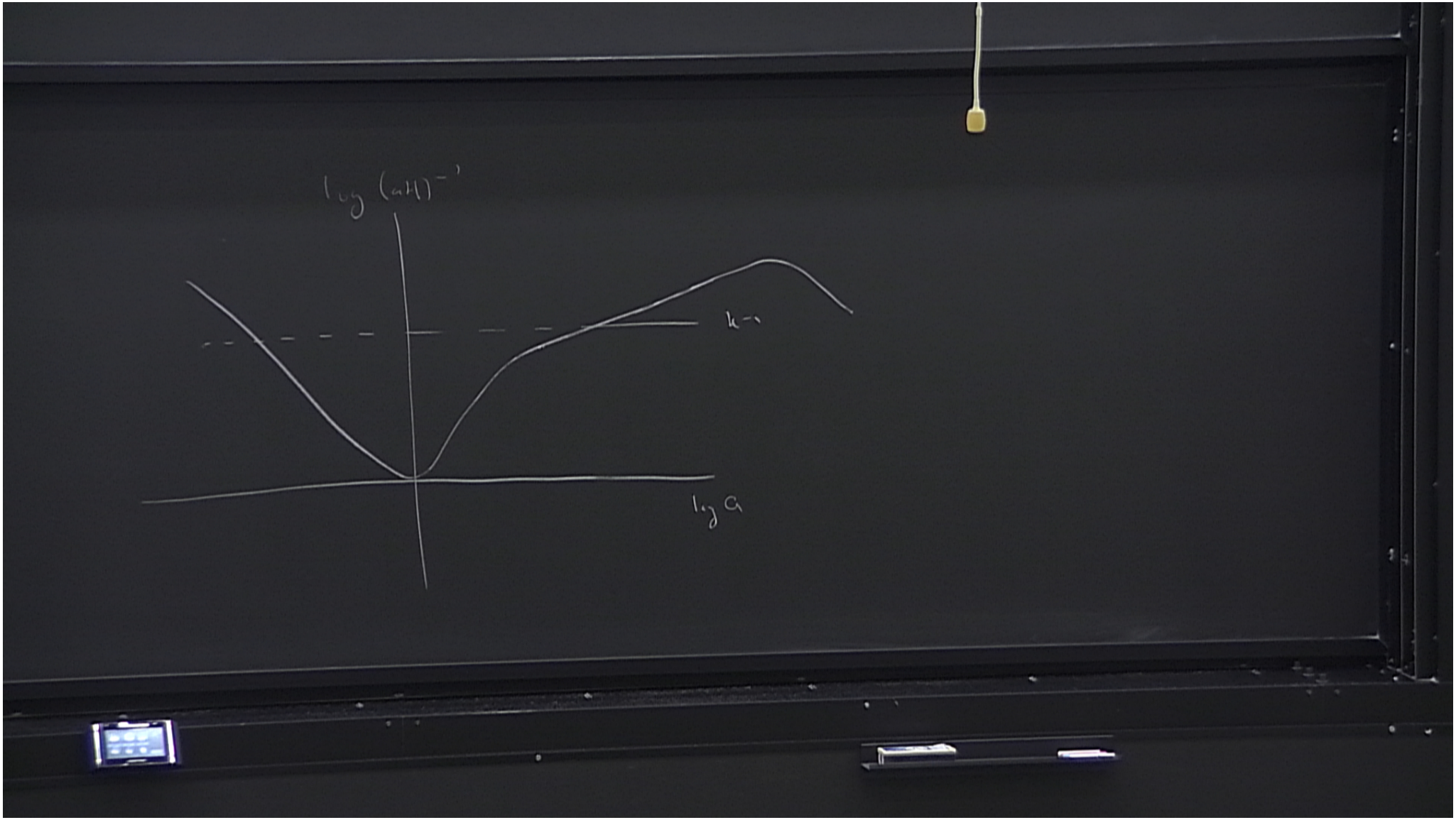
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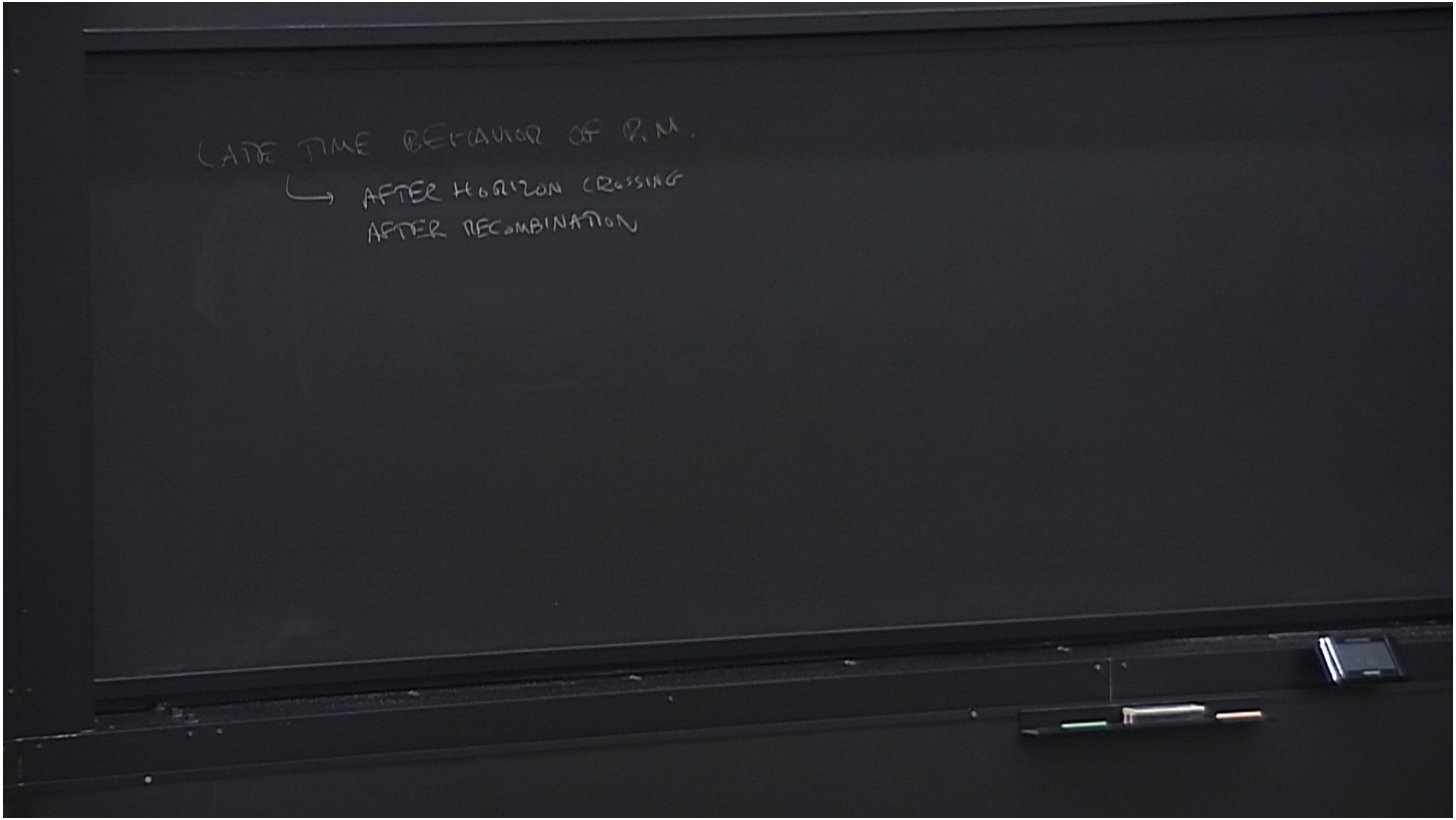
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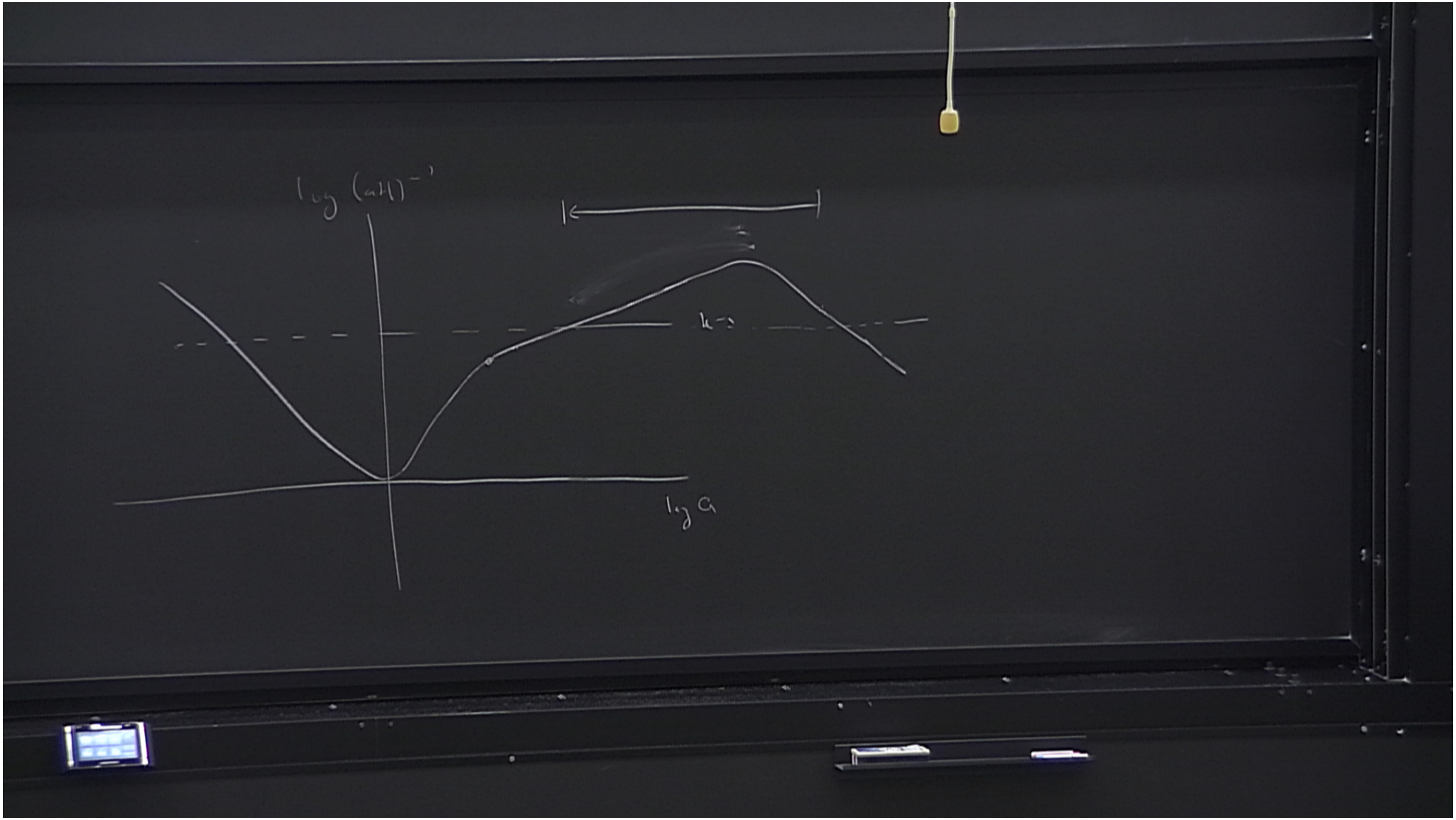
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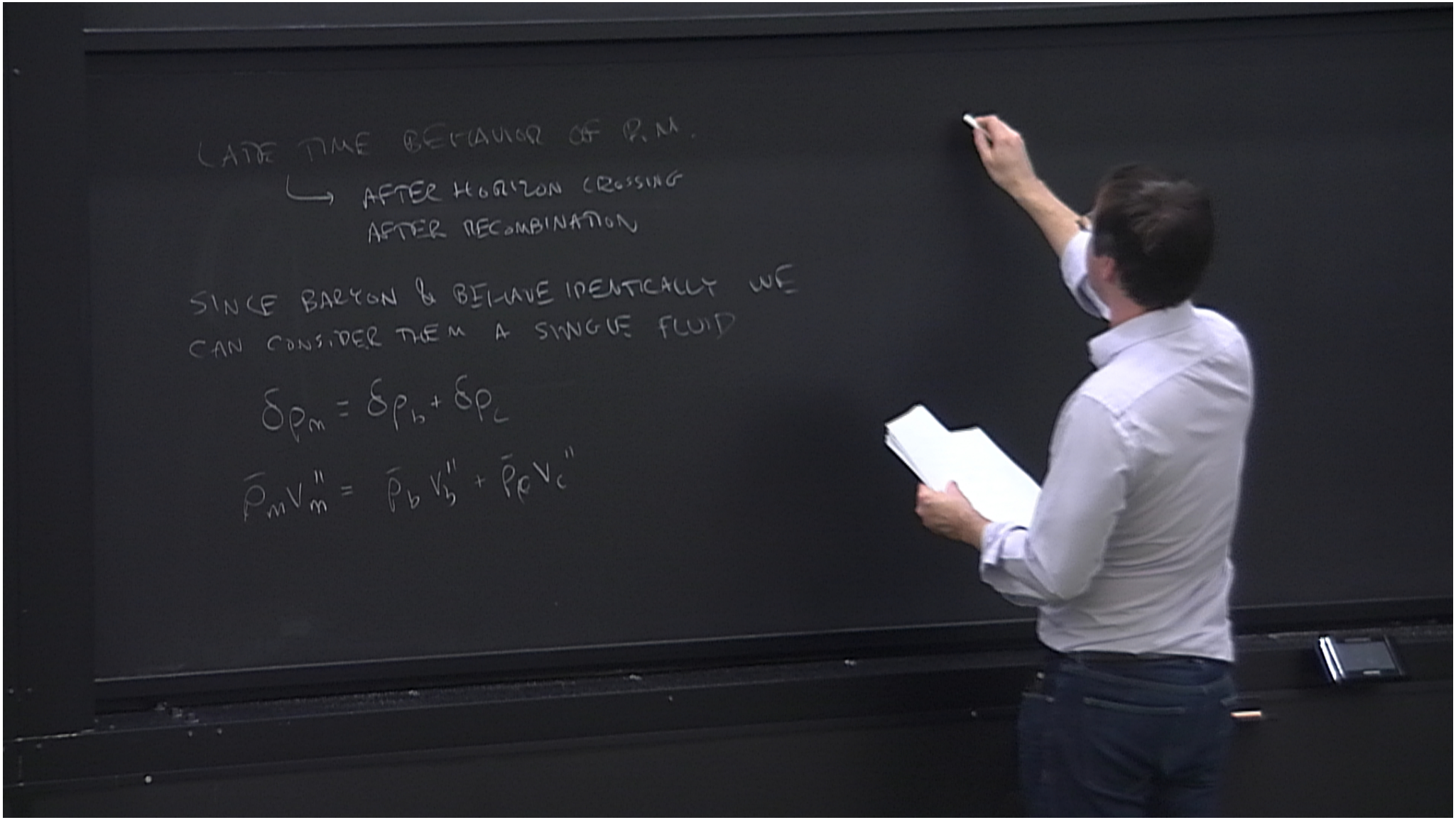


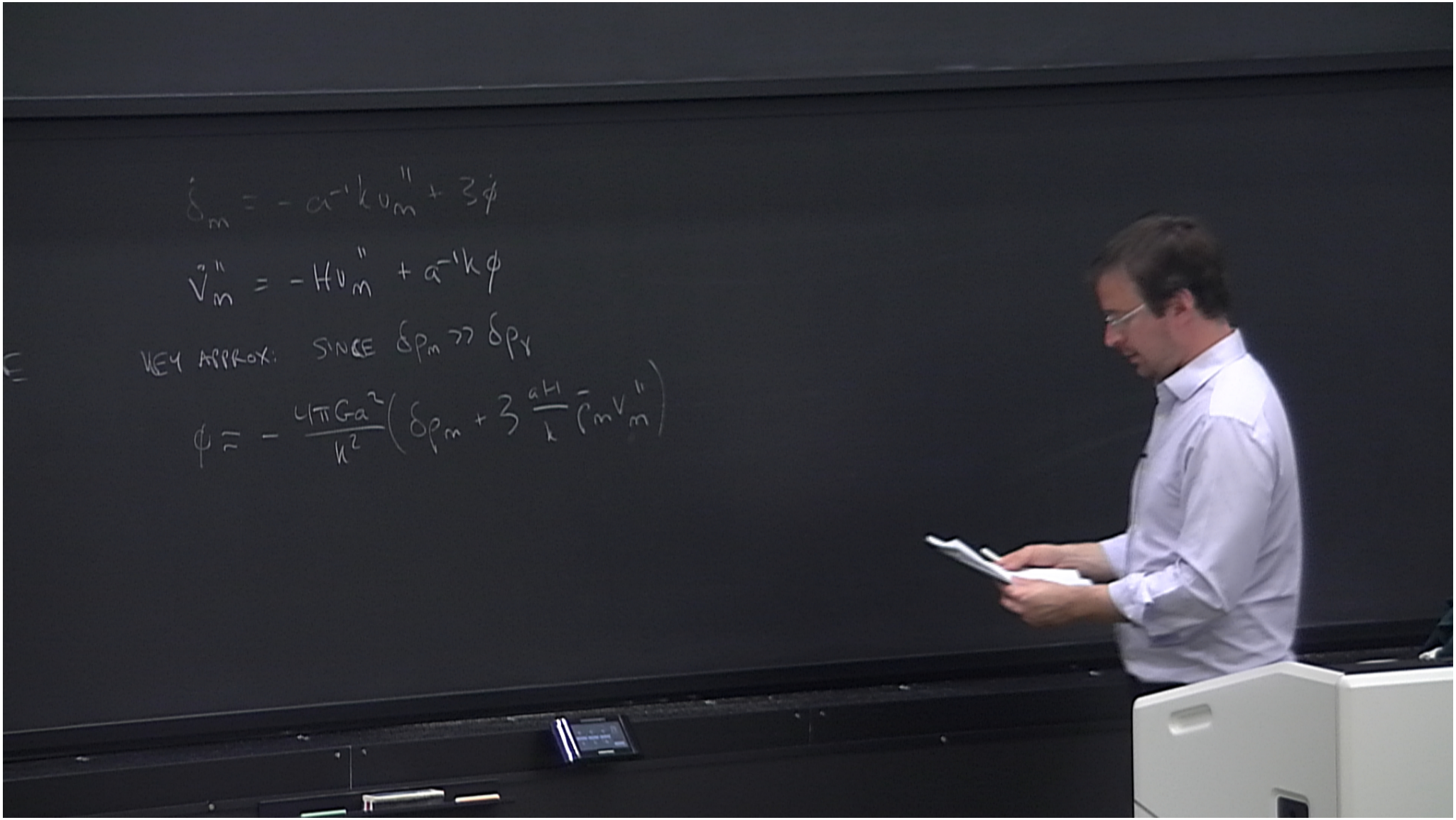


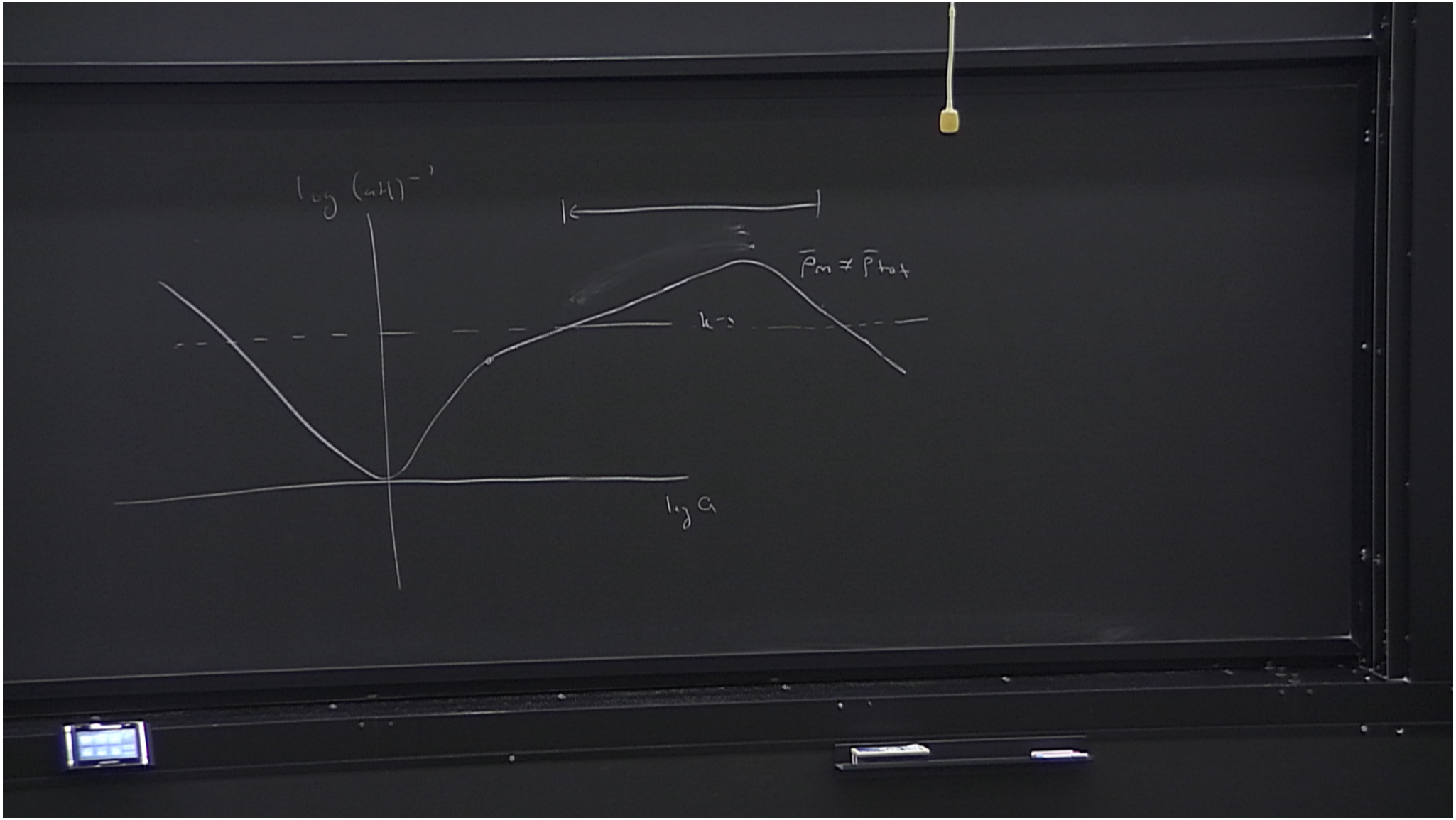

log (aH)⁻¹
k-1
log a

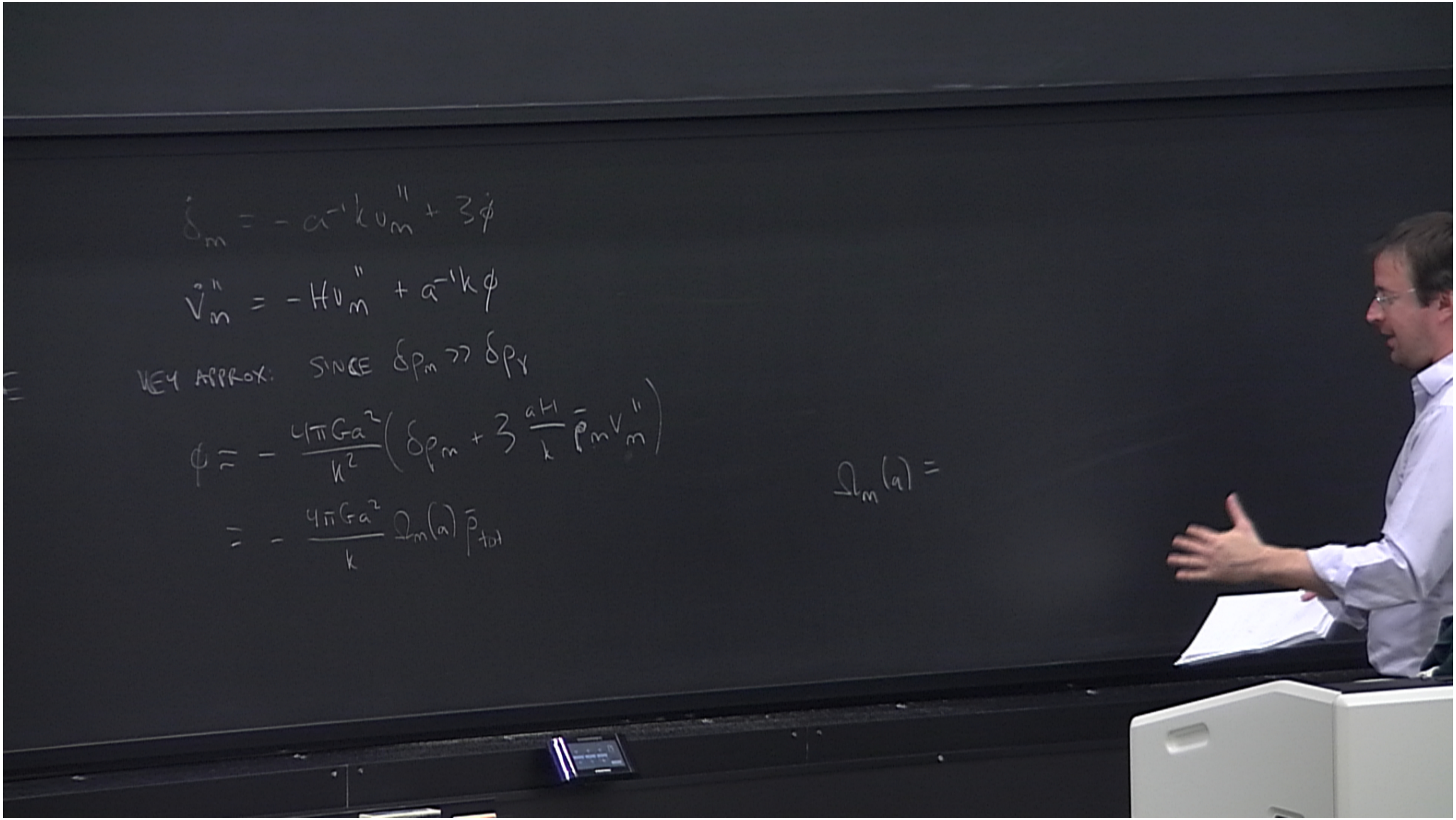

LATE TIME BEHAVIOR OF P.M.
↳ AFTER HORIZON CROSSING
AFTER RECOMBINATION

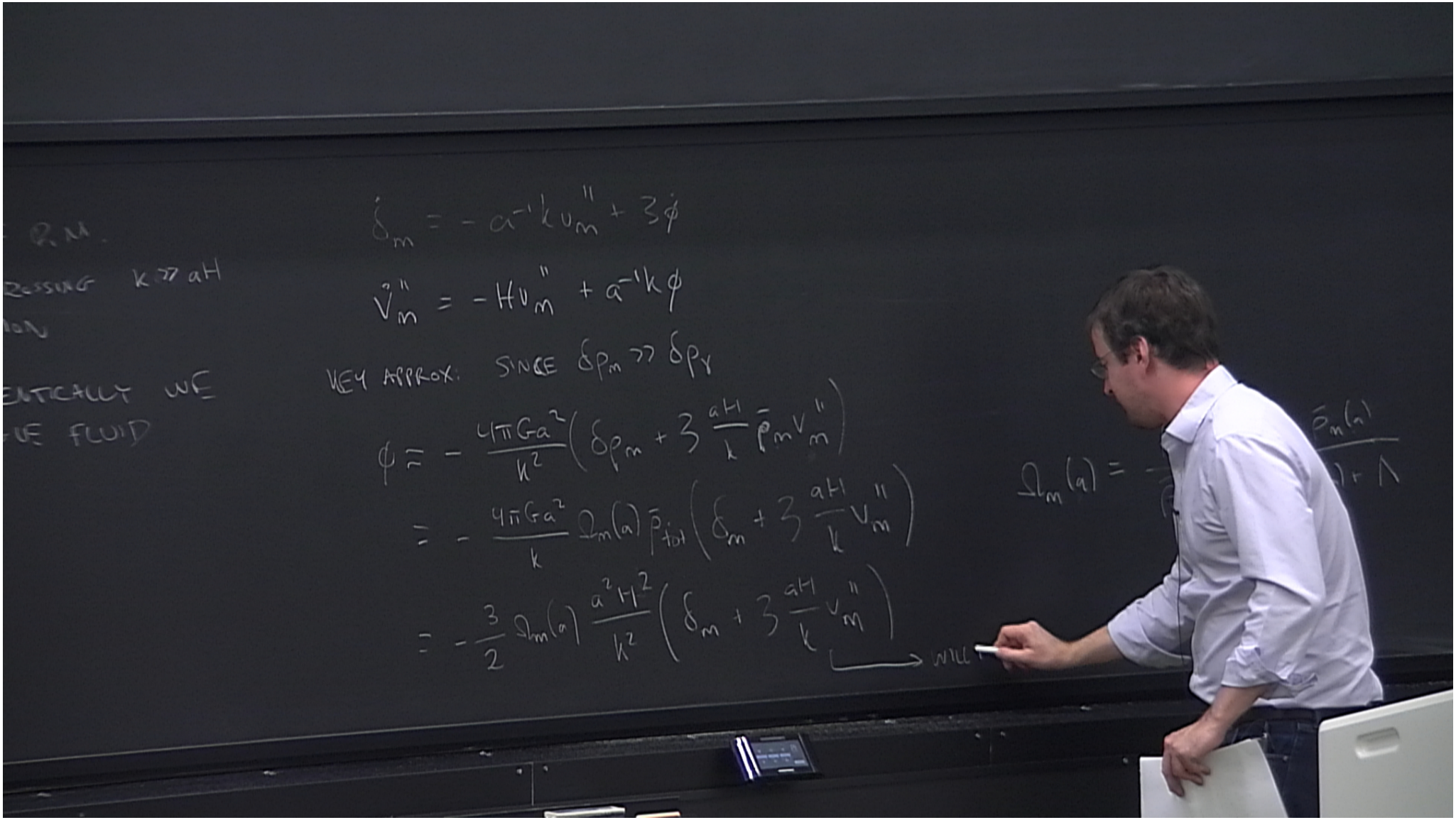

log (aH)^-1
k=3
log a

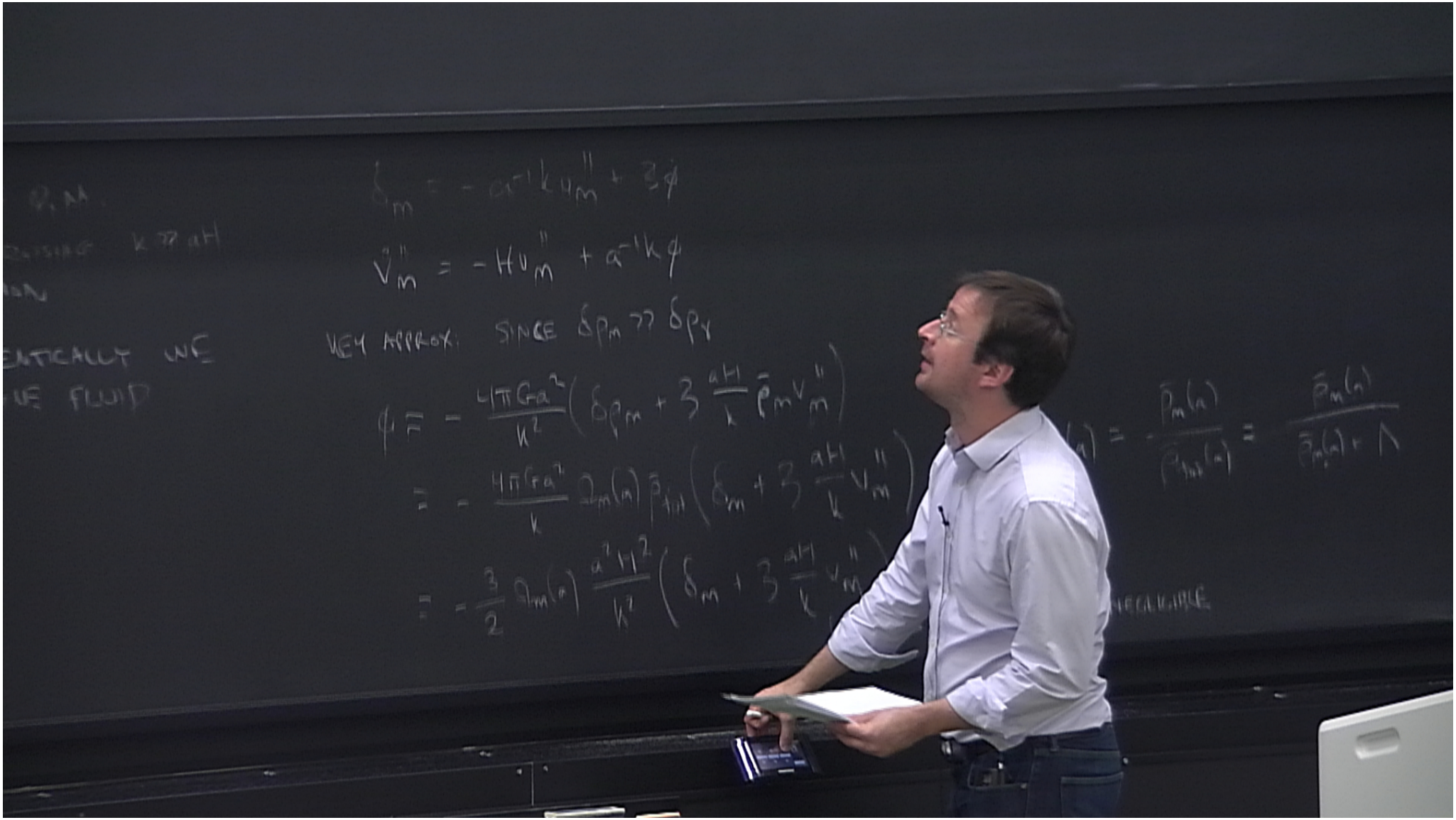

LATE TIME BEHAVIOR OF P.M.
↳ AFTER HORIZON CROSSING
AFTER RECOMBINATION
SINCE BARYON & DM BEHAVE IDENTICALLY WE
CAN CONSIDER THEM A SINGLE FLUID
$\delta\rho_m = \delta\rho_b + \delta\rho_c$
$\bar{\rho}_m V_m'' = \bar{\rho}_b V_b'' + \bar{\rho}_c V_c''$

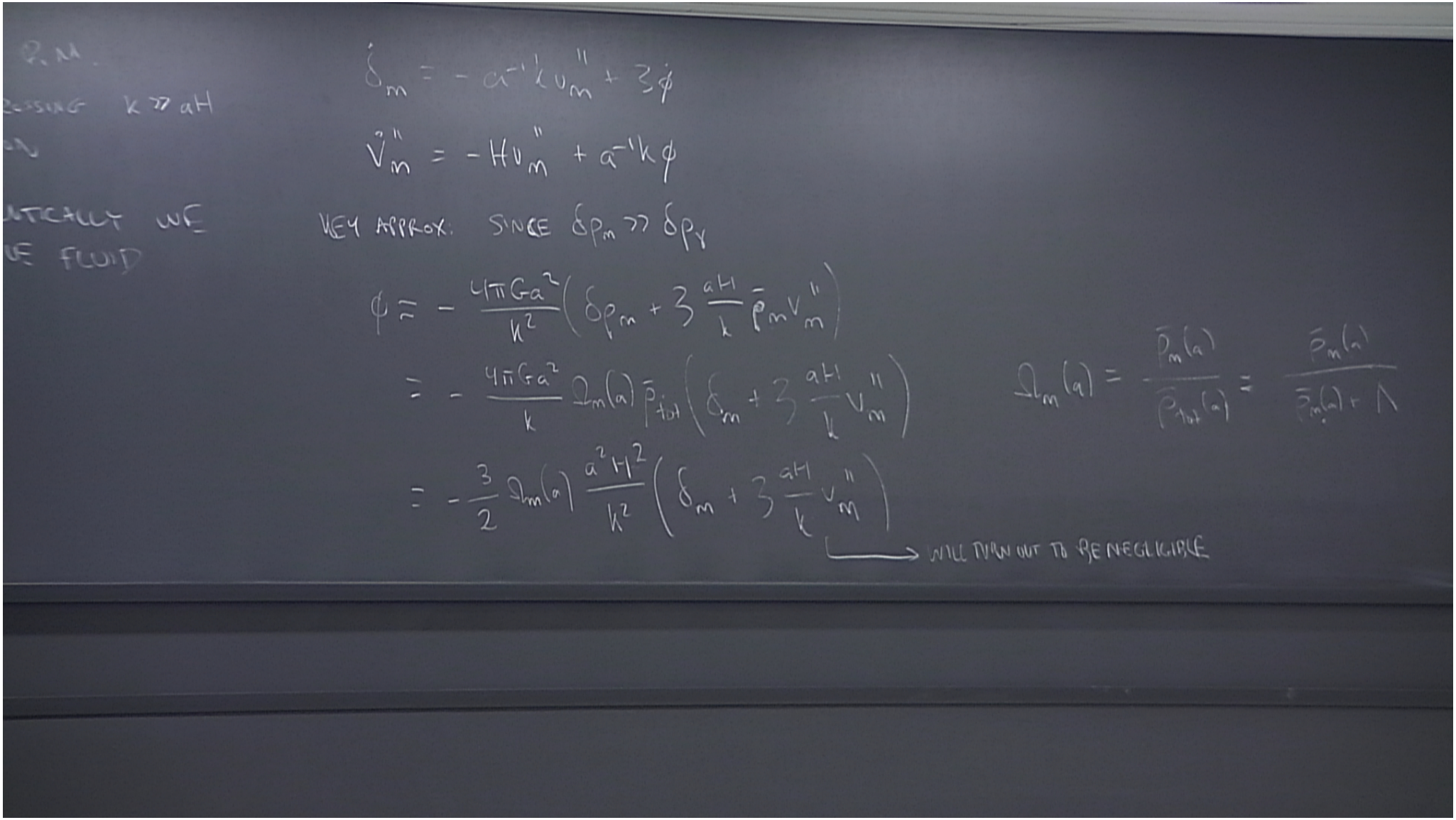

$\dot{\delta}_m = -a^{-1}k v_m'' + 3\dot{\phi}$
$\dot{V}_m'' = -H v_m'' + a^{-1}k\phi$
KEY APPROX: SINCE $\delta\rho_m \gg \delta\rho_r$
$\phi = -\frac{4\pi G a^2}{k^2}\left(\delta\rho_m + 3\frac{aH}{k}\bar{\rho}_m V_m''\right)$


log (a+1)^-1
$\bar{\rho}_m \neq \bar{\rho}_{tot}$
k=
log a


$\dot{\delta}_m = -a^{-1}kv_m'' + 3\dot{\phi}$
$\dot{v}_m'' = -Hv_m'' + a^{-1}k\phi$
KEY APPROX: SINCE $\delta\rho_m \gg \delta\rho_\gamma$
$\phi = -\frac{4\pi G a^2}{k^2}\left(\delta\rho_m + 3\frac{aH}{k}\bar{\rho}_m v_m''\right)$
$= -\frac{4\pi G a^2}{k}\Omega_m(a)\bar{\rho}_{tot}$
$\Omega_m(a) =$

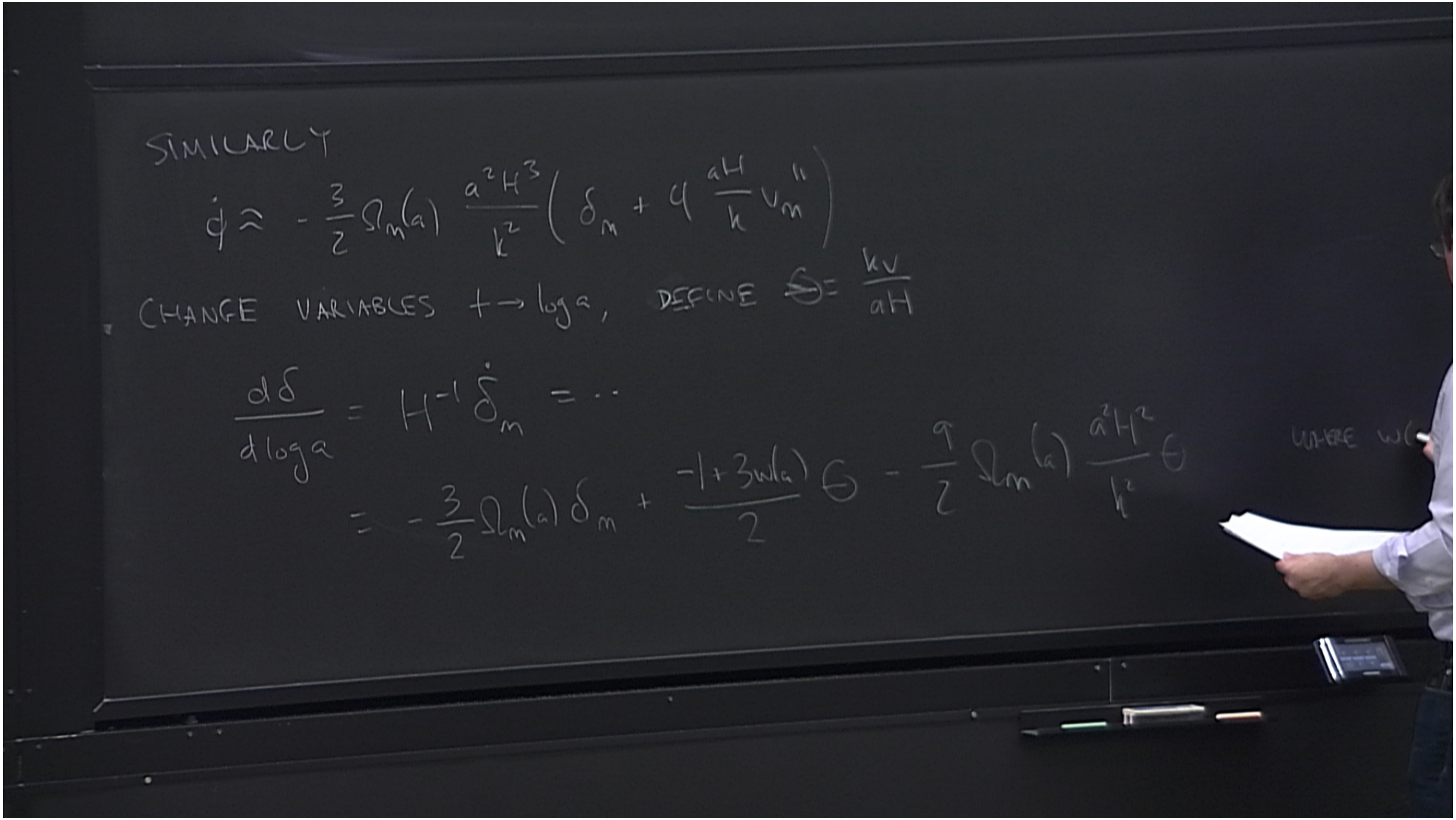

$\dot{\delta}_m = -a^{-1}kv_m'' + 3\dot{\phi}$
$\dot{v}_m'' = -Hv_m'' + a^{-1}k\phi$
KEY APPROX: SINCE $\delta\rho_m \gg \delta\rho_\gamma$
$\phi = -\frac{4\pi G a^2}{k^2}\left(\delta\rho_m + 3\frac{aH}{k}\bar{\rho}_m v_m''\right)$
$= -\frac{4\pi G a^2}{k}\Omega_m(a)\bar{\rho}_{tot}\left(\delta_m + 3\frac{aH}{k}v_m''\right)$
$= -\frac{3}{2}\Omega_m(a)\frac{a^2H^2}{k^2}\left(\delta_m + 3\frac{aH}{k}v_m''\right)$
$\Omega_m(a) =$


$\delta_m = -a^{-1}kv_m + 3\dot{\phi}$
$\dot{v}_m = -Hv_m + a^{-1}k\psi$
VERY APPROX: SINCE $\delta\rho_m \gg \delta\rho_\gamma$
$\psi = -\frac{4\pi G a^2}{k^2}\left(\delta\rho_m + 3\frac{aH}{k}\bar{\rho}_m v_m\right)$
$= -\frac{4\pi G a^2}{k}\Omega_m(a)\bar{\rho}_{tot}\left(\delta_m + 3\frac{aH}{k}v_m\right)$
$= -\frac{3}{2}\Omega_m(a)\frac{a^2H^2}{k^2}\left(\delta_m + 3\frac{aH}{k}v_m\right)$


$\dot{\delta}_m = -a^{-1} k v_m'' + 3\dot{\phi}$
$\dot{v}_m'' = -H v_m'' + a^{-1} k \phi$
KEY APPROX: SINCE $\delta\rho_m \gg \delta\rho_\gamma$
$\phi \simeq -\frac{4\pi G a^2}{k^2}\left(\delta\rho_m + 3\frac{aH}{k}\bar{\rho}_m v_m''\right)$
$= -\frac{4\pi G a^2}{k}\Omega_m(a)\bar{\rho}_{tot}\left(\delta_m + 3\frac{aH}{k}v_m''\right)$
$= -\frac{3}{2}\Omega_m(a)\frac{a^2H^2}{k^2}\left(\delta_m + 3\frac{aH}{k}v_m''\right)$
WILL TURN OUT TO BE NEGLIGIBLE
$\Omega_m(a) = \frac{\bar{\rho}_m(a)}{\bar{\rho}_{tot}(a)} = \frac{\bar{\rho}_m(a)}{\bar{\rho}_m(a) + \Lambda}$

SIMILARLY

$$\dot{\phi} \approx -\frac{3}{2}\Omega_m(a)\,\frac{a^2H^3}{k^2}\left(\delta_m + q\,\frac{aH}{k}v_m^{\parallel}\right)$$


SIMILARLY
$\dot{\delta} \approx -\frac{3}{2}\Omega_m(a)\frac{a^2H^3}{k^2}\left(\delta_m + q\frac{aH}{k}v_m^{\parallel}\right)$
CHANGE VARIABLES $t \to \log a$, DEFINE $\Theta = \frac{kv}{aH}$
$\frac{d\delta}{d\log a} = H^{-1}\dot{\delta}_m = \cdots$
$= -\frac{3}{2}\Omega_m(a)\delta_m + \frac{-1+3w(a)}{2}\Theta - \frac{q}{2}\Omega_m(a)\frac{a^2H^2}{k^2}\Theta$
WHERE $w($

SIMILARLY

$$\dot{\psi} \approx -\frac{3}{2}\Omega_m(a)\,\frac{a^2H^3}{k^2}\left(\delta_m + 4\,\frac{aH}{k}v_m^{\parallel}\right)$$

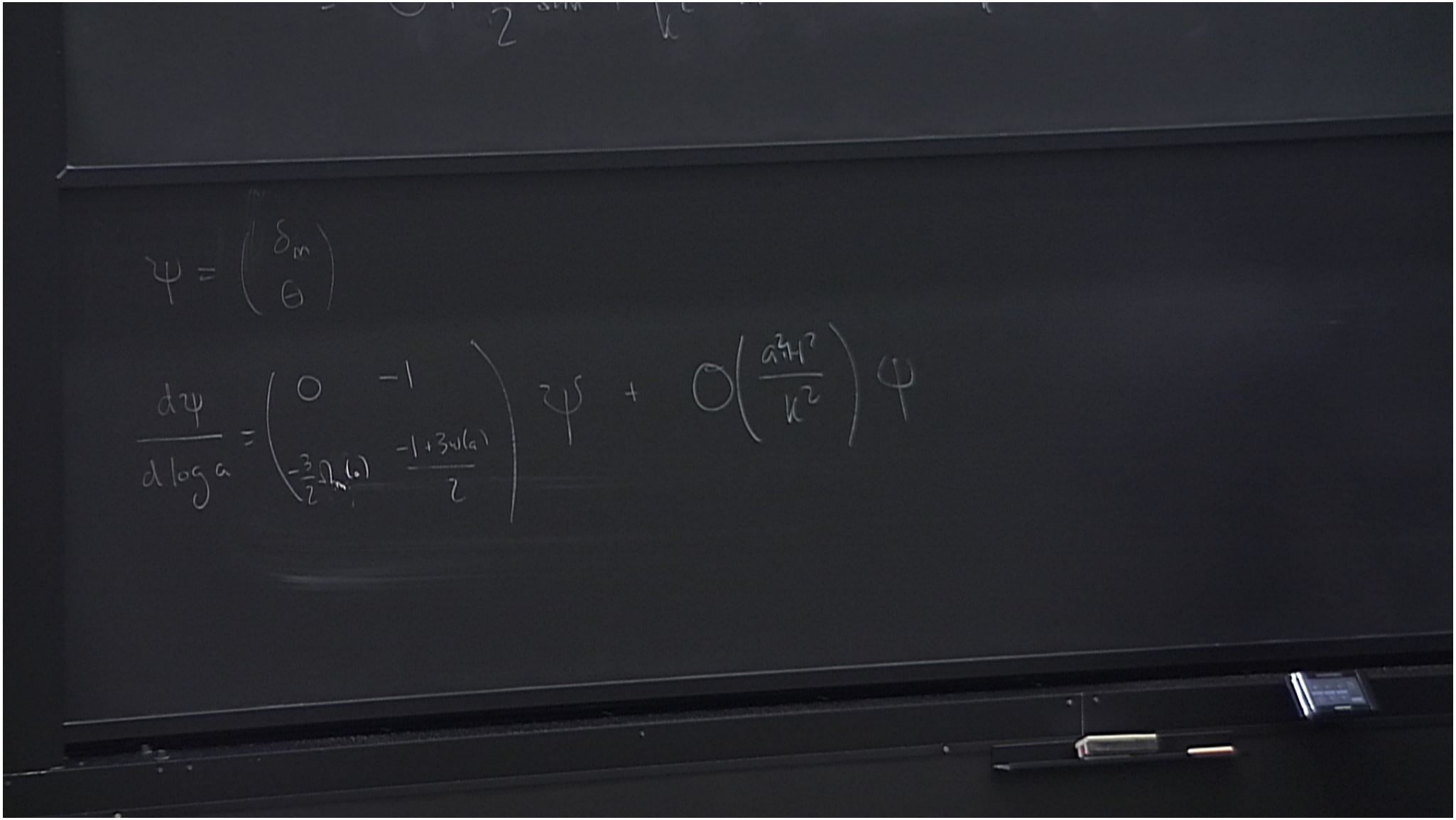
$$\frac{d\Theta}{d\log a} = -\frac{3}{2}\Omega$$

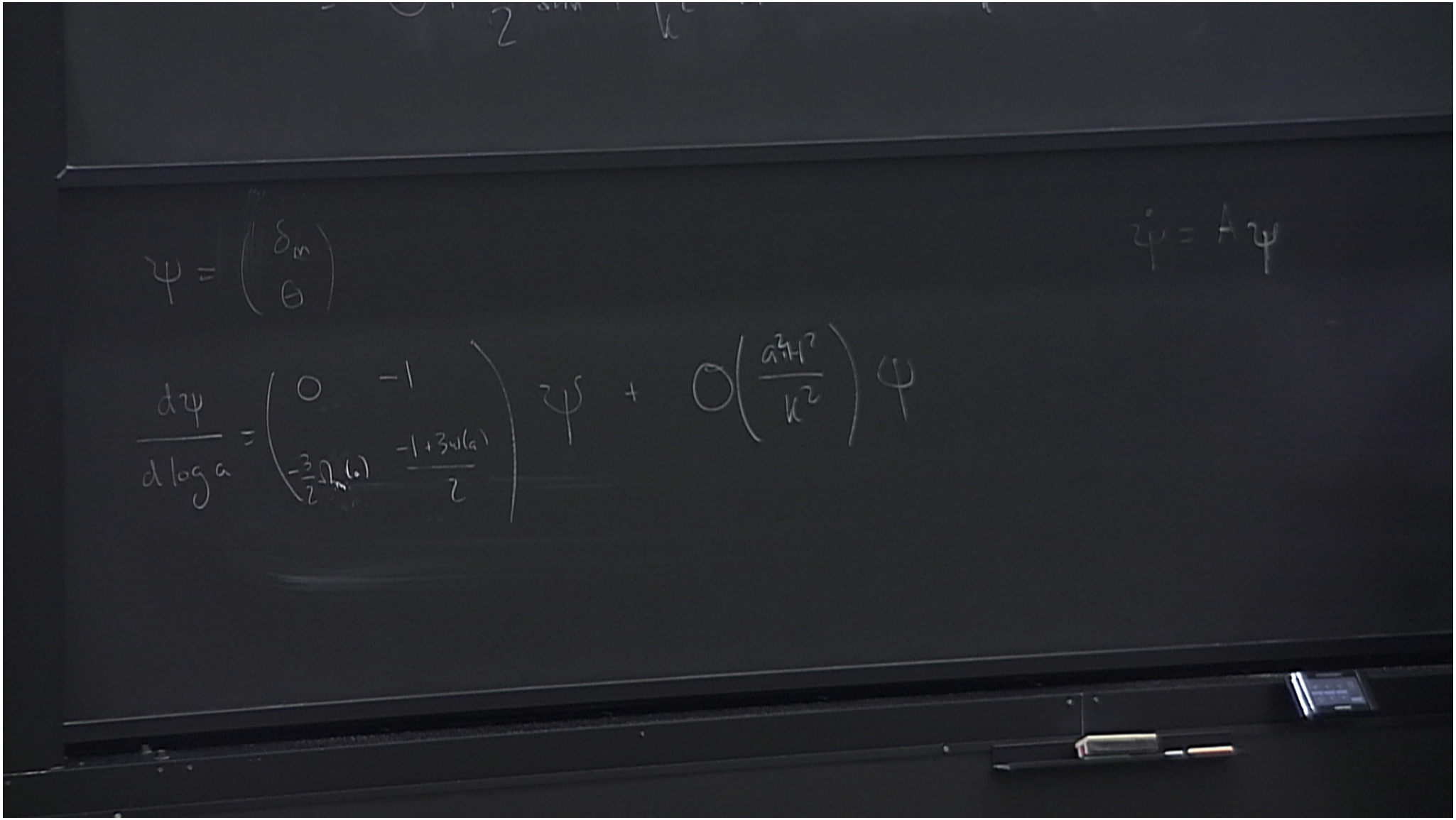
CHANGE VARIABLES $t \to \log a$, DEFINE $\Theta = \frac{kv}{aH}$

$$\frac{d\delta}{d\log a} = H^{-1}\dot{\delta}_m = \ldots$$

$$= -\Theta + \frac{9}{2}\Omega_m(a)\,\frac{a^2H^2}{k^2}\,\delta_m + 18\,\Omega_m(a)\,\frac{a^4H^4}{k^4}\,\Theta$$

WHERE $w(a) =$

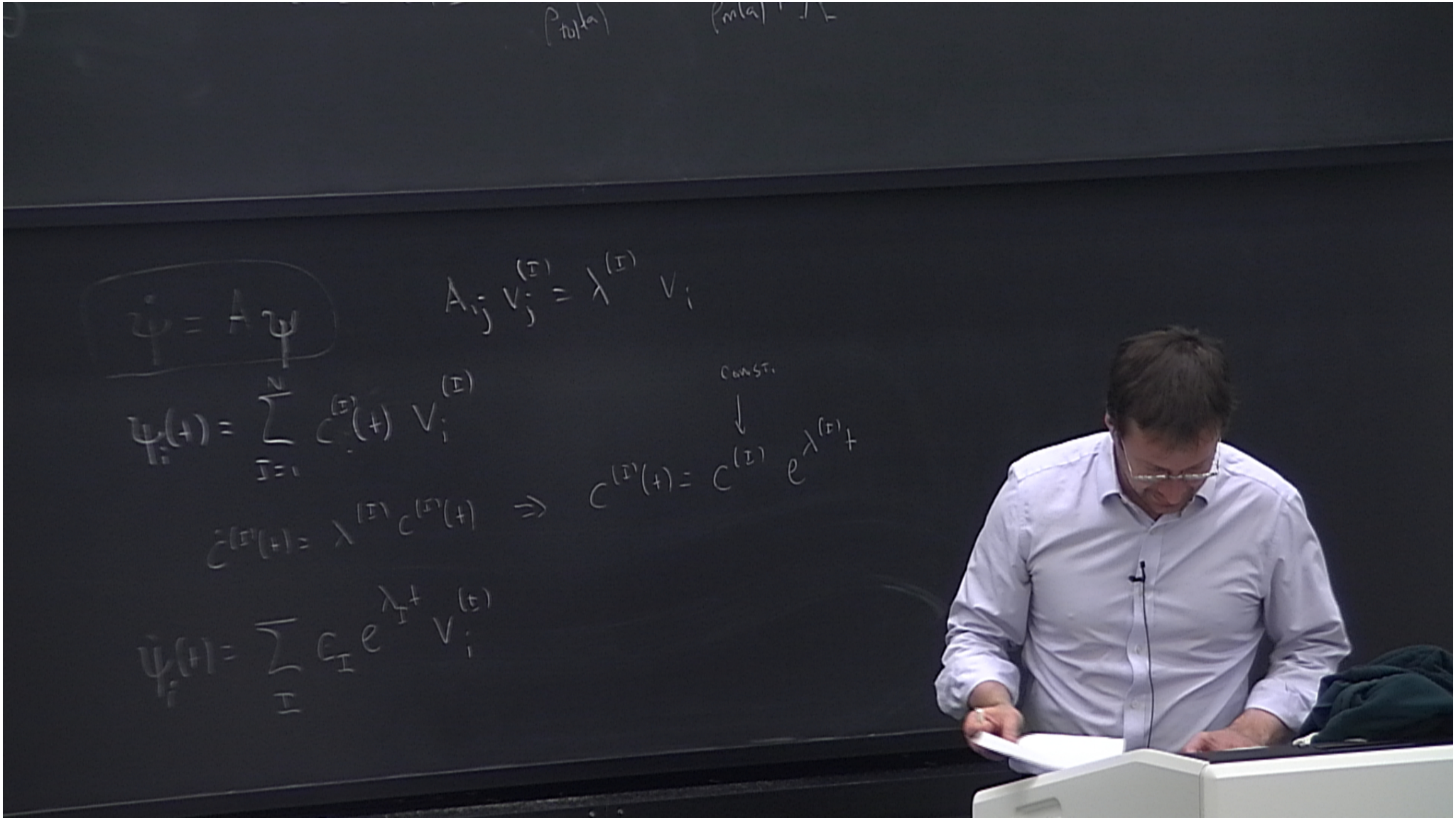

$\Psi = \begin{pmatrix} \delta_m \\ \theta \end{pmatrix}$
$\frac{d\Psi}{d\log a} = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}\Omega_m(a) & \frac{-1+3w(a)}{2} \end{pmatrix} \Psi + O\left(\frac{a^2H^2}{k^2}\right)\Psi$

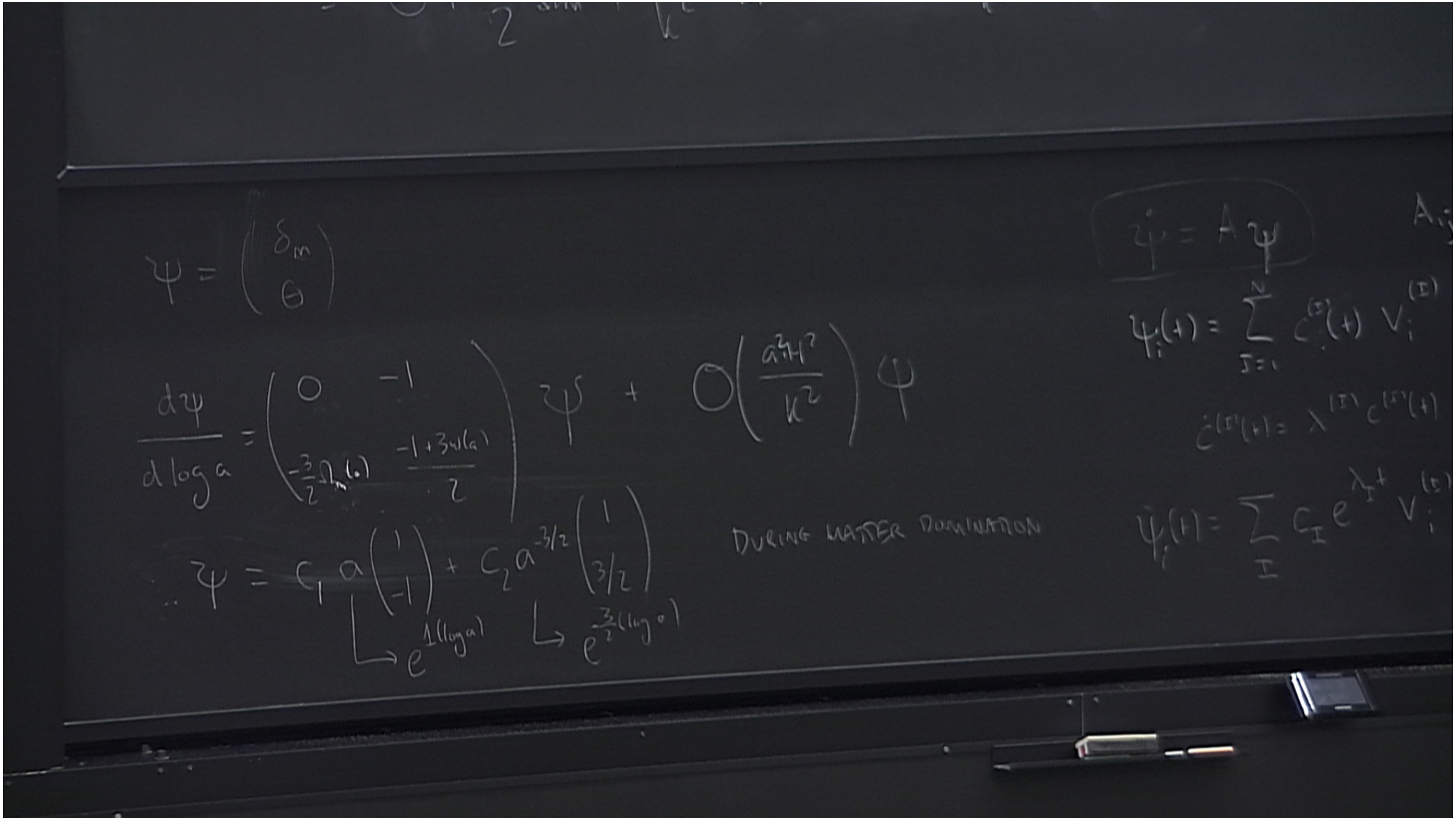

$\psi = \begin{pmatrix} \delta_m \\ \theta \end{pmatrix}$
$\dot{\psi} = A\psi$
$\frac{d\psi}{d\log a} = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}\Omega_m(a) & \frac{-1+3w(a)}{2} \end{pmatrix} \psi + O\left(\frac{a^2H^2}{k^2}\right)\psi$

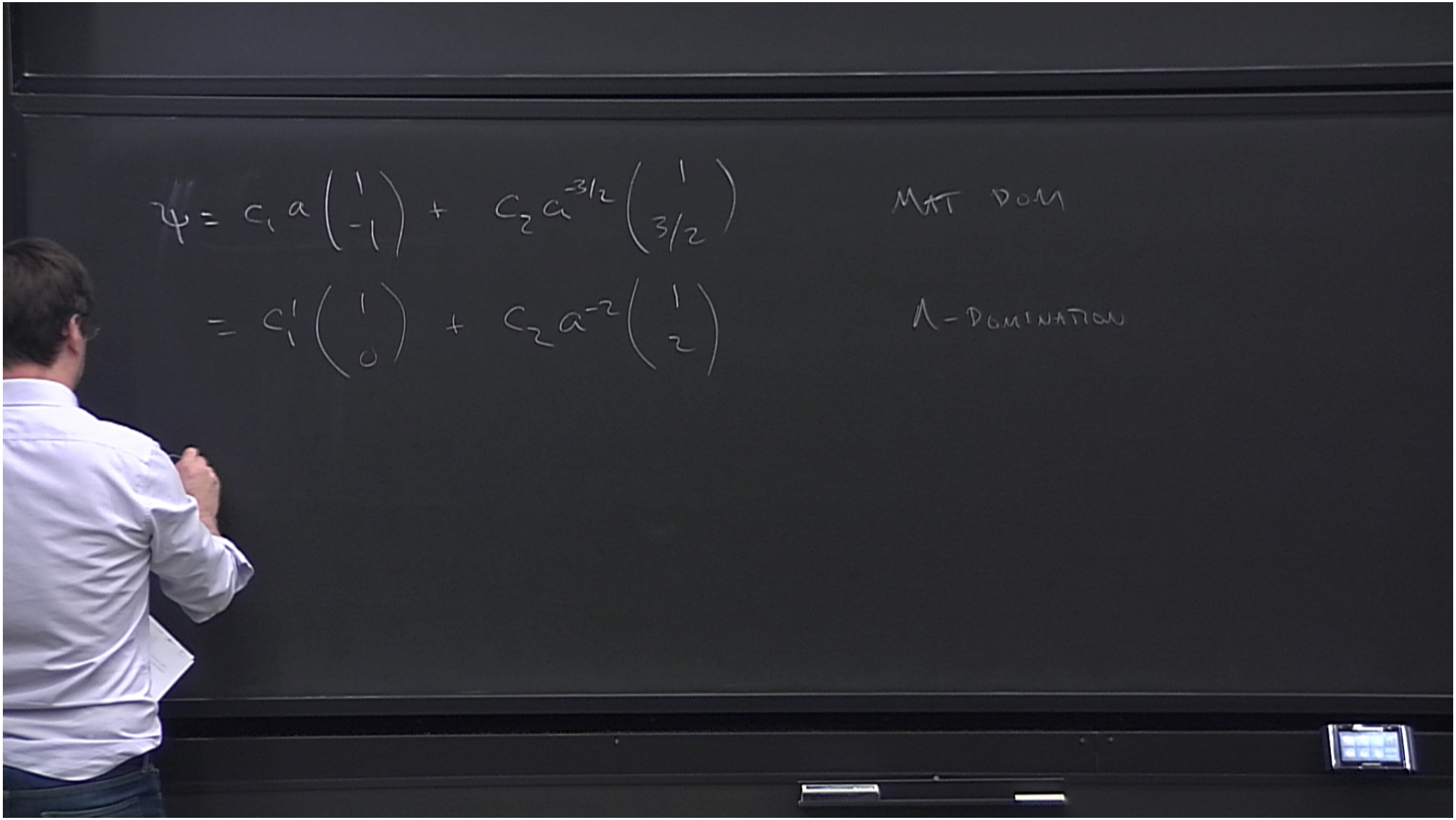
$$\frac{d\theta}{d\log a} = -\frac{3}{2}\Omega_m(a)\,\delta_m + \frac{-1 + 3w(a)}{2}\theta - \frac{9}{2}\Omega_m(a)\,\frac{a^2H^2}{k^2}\theta$$

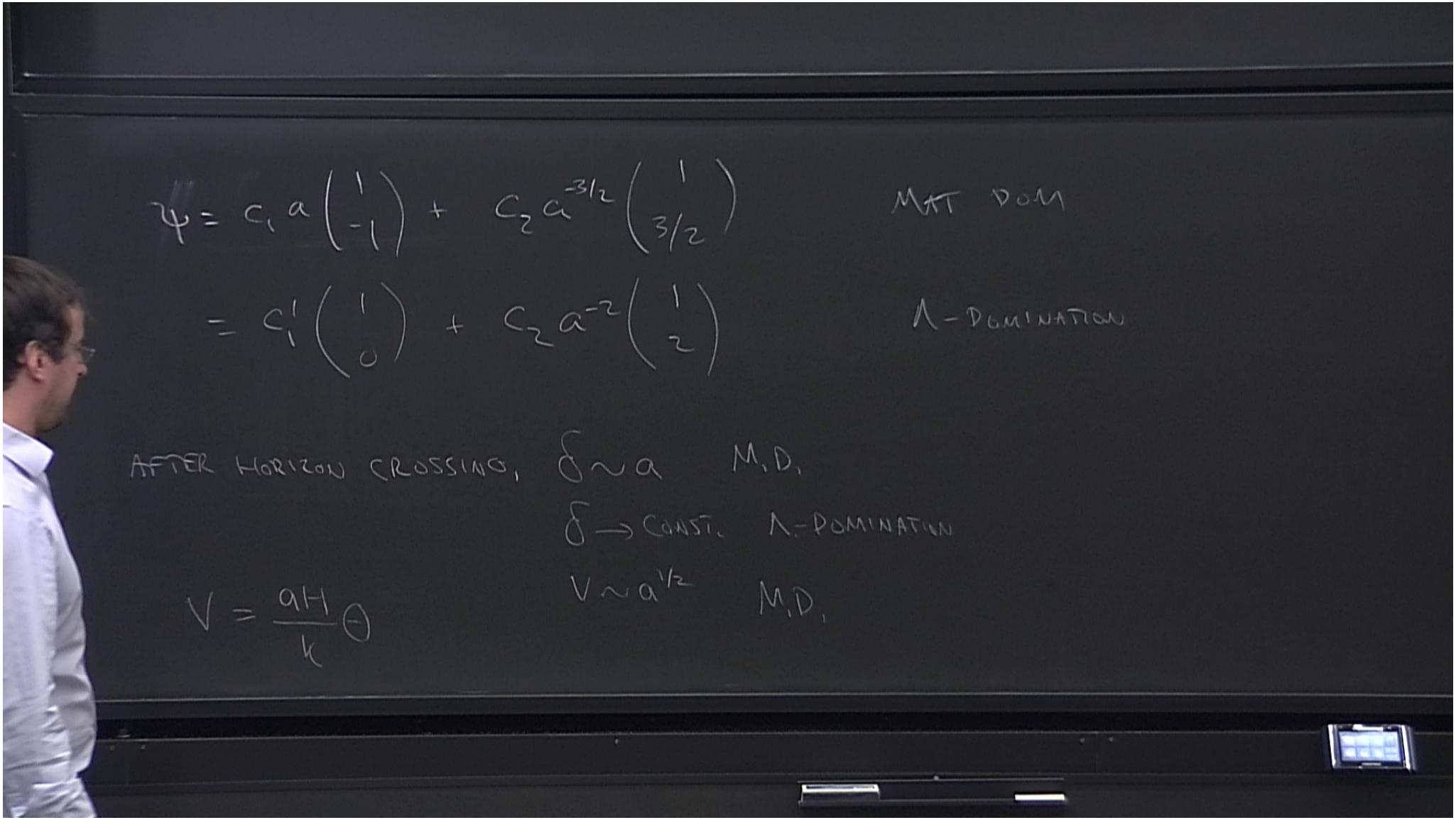
WHERE $w(a) = \dfrac{\bar{P}_{tot}(a)}{\bar{\rho}_{tot}(a)} = -\dfrac{\Lambda}{\bar{\rho}_m(a) + \Lambda}$

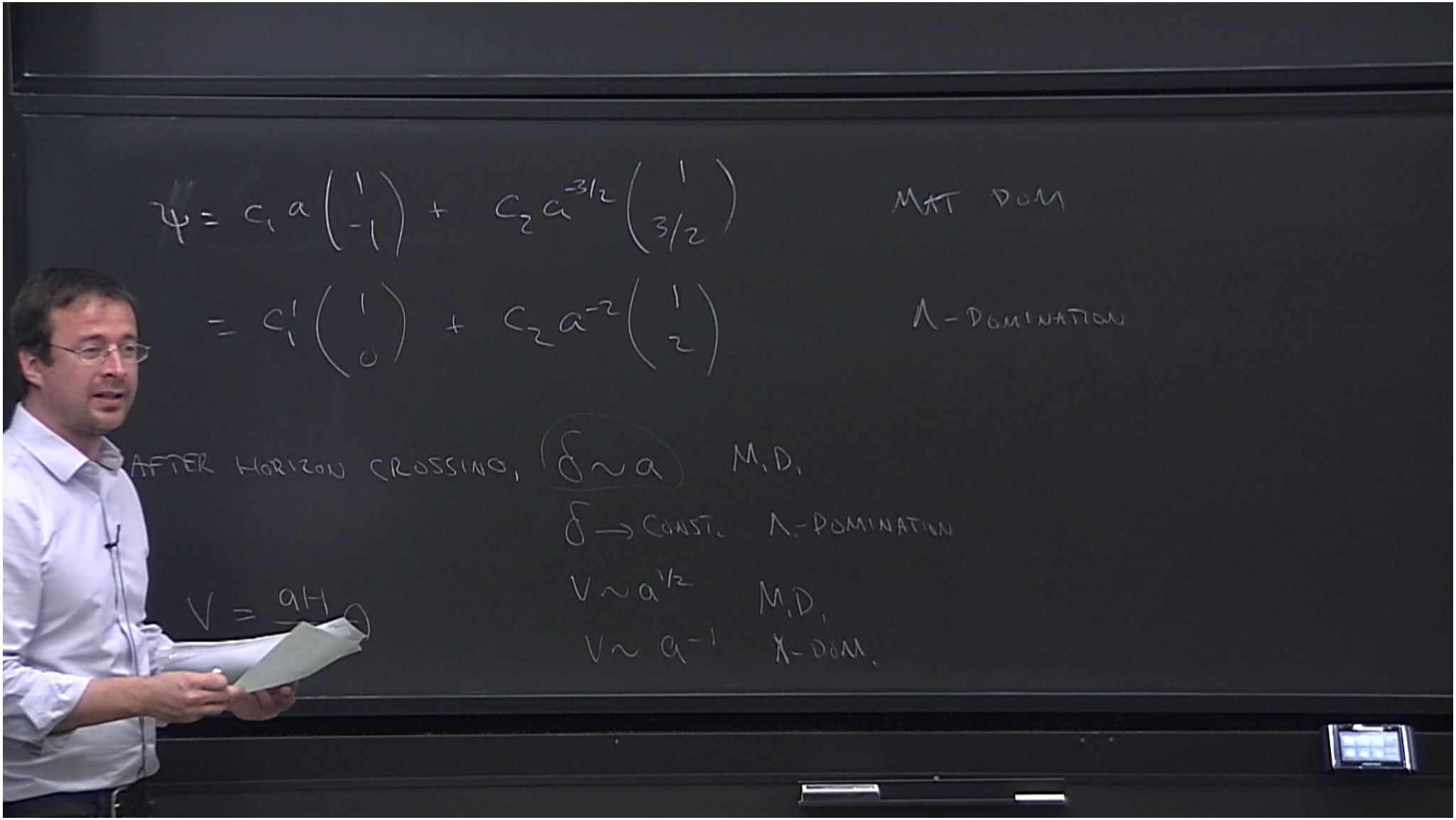
$\Lambda \cdot V_i^{(I)} = \lambda^{(I)}$


$\dot{\psi} = A\psi$
$A_{ij} V_j^{(I)} = \lambda^{(I)} V_i$
$\psi_i(t) = \sum_{I=1}^{N} c^{(I)}(t) V_i^{(I)}$
$\dot{c}^{(I)}(t) = \lambda^{(I)} c^{(I)}(t) \Rightarrow c^{(I)}(t) = c^{(I)} e^{\lambda^{(I)} t}$
Const.
$\psi_i(t) = \sum_I c_I e^{\lambda_I t} V_i^{(I)}$


$\psi = \begin{pmatrix} \delta_m \\ \theta \end{pmatrix}$
$\frac{d\psi}{d\log a} = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}\Omega_m(a) & \frac{-1+3w(a)}{2} \end{pmatrix} \psi + O\left(\frac{a^2H^2}{k^2}\right)\psi$
$\therefore \psi = c_1 a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 a^{-3/2} \begin{pmatrix} 1 \\ 3/2 \end{pmatrix}$
$\hookrightarrow e^{1(\log a)}$ $\hookrightarrow e^{-\frac{3}{2}(\log a)}$
DURING MATTER DOMINATION
$\dot{\psi} = A\psi$
$\psi_i(t) = \sum_{I=1}^{N} c^{(I)}(t)\, V_i^{(I)}$
$\dot{c}^{(I)}(t) = \lambda^{(I)} c^{(I)}(t)$
$\psi_i(t) = \sum_I c_I e^{\lambda_I t} V_i^{(I)}$


$\psi = c_1 a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 a^{-3/2} \begin{pmatrix} 1 \\ 3/2 \end{pmatrix}$
MAT DOM
$= c_1' \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 a^{-2} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
Λ-DOMINATION


$\psi = c_1 a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 a^{-3/2} \begin{pmatrix} 1 \\ 3/2 \end{pmatrix}$
MAT DOM
$= c_1' \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 a^{-2} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
Λ-DOMINATION
AFTER HORIZON CROSSING, $\delta \sim a$ M.D.
$\delta \to$ CONST. Λ-DOMINATION
$V \sim a^{1/2}$ M.D.
$V = \frac{aH}{k}\theta$


$\psi = c_1 a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 a^{-3/2} \begin{pmatrix} 1 \\ 3/2 \end{pmatrix}$
MAT DOM
$= c_1' \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 a^{-2} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
Λ-DOMINATION
AFTER HORIZON CROSSING, $\delta \sim a$ M.D.
$\delta \to$ CONST. Λ-DOMINATION
$V \sim a^{1/2}$ M.D.
$V \sim a^{-1}$ X-DOM.
$V = \frac{9H}{\ }$

$$\psi = c_1 a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 a^{-3/2} \begin{pmatrix} 1 \\ 3/2 \end{pmatrix}$$ MAT DOM

$$= c_1' \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 a^{-2} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ Λ-DOMINATION

AFTER HORIZON CROSSING, $\delta \sim a$ M.D.

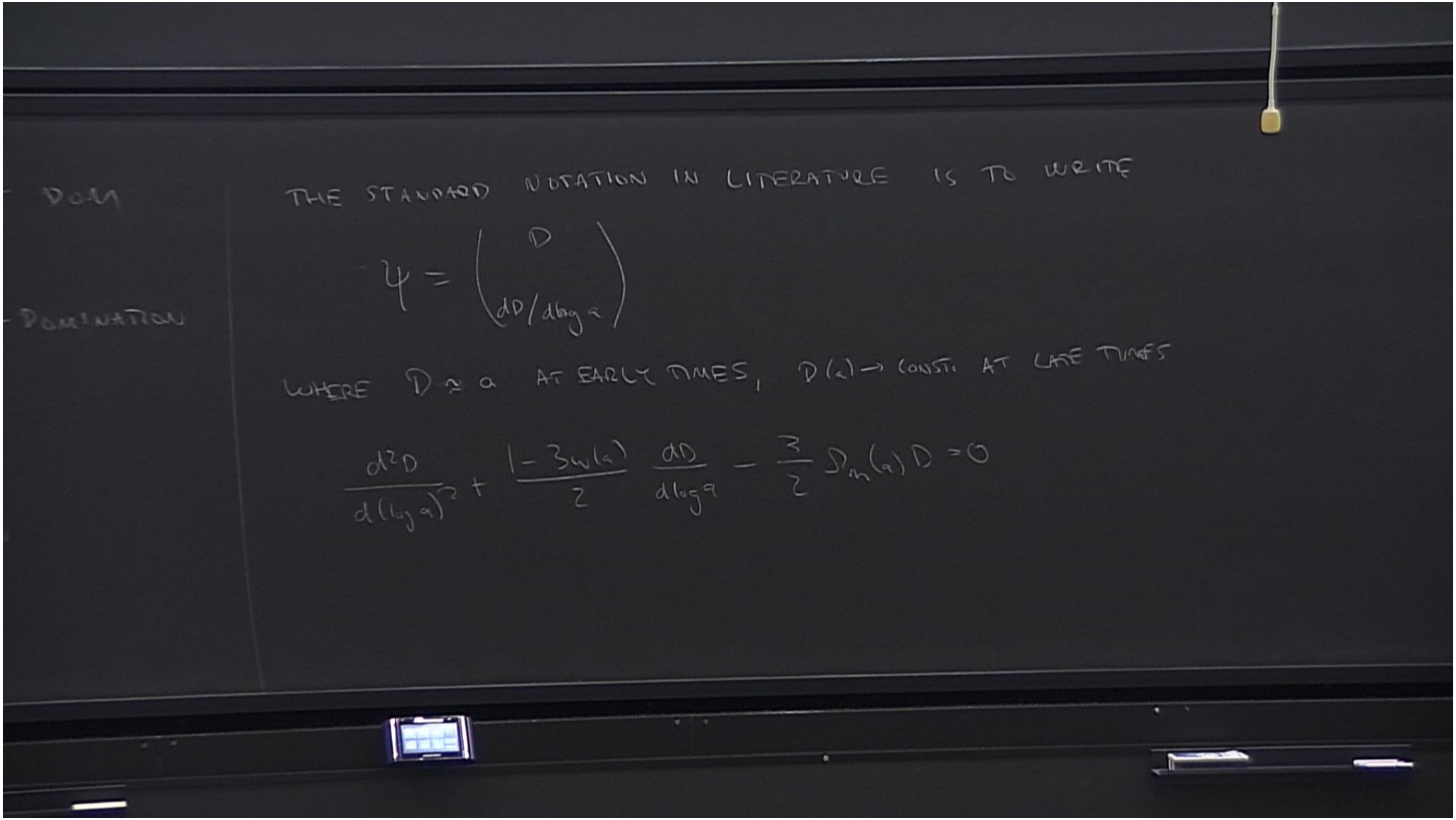
$\delta \to$ CONST. Λ-DOMINATION

$v = \frac{k}{aH} V$

$V = \frac{aH}{k} \theta$

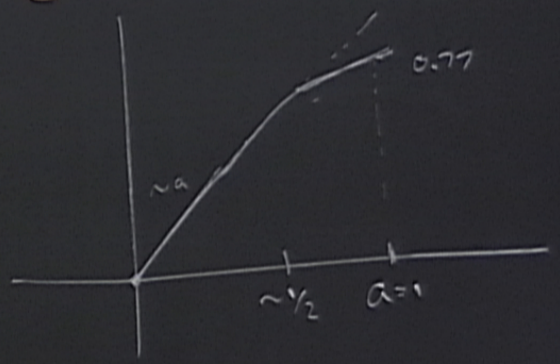
$V \sim a^{1/2}$ M.D.

$V \sim a^{-1}$ Λ-DOM.


DOM
DOMINATION
THE STANDARD NOTATION IN LITERATURE IS TO WRITE
$\psi = \begin{pmatrix} D \\ dD/d\log a \end{pmatrix}$
WHERE $D \simeq a$ AT EARLY TIMES, $D(a) \to$ CONST. AT LATE TIMES
$\frac{d^2D}{d(\log a)^2} + \frac{1-3w(a)}{2}\frac{dD}{d\log a} - \frac{3}{2}\Omega_m(a)D = 0$

THE STANDARD NOTATION IN LITERATURE IS TO WRITE

$$\psi = \begin{pmatrix} D \\ dD/d\log a \end{pmatrix}$$



WHERE $D \propto a$ AT EARLY TIMES, $D(a) \to$ CONST. AT LATE TIMES

$$\frac{d^2D}{d(\log a)^2} + \frac{1-3w(a)}{2}\frac{dD}{d\log a} - \frac{3}{2}\Omega_m(a)D = 0$$

D "GROWTH FUNCTION"

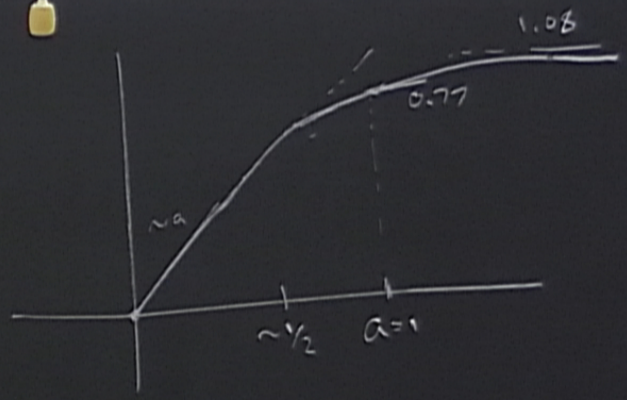
STANDARD NOTATION IN LITERATURE IS TO WRITE

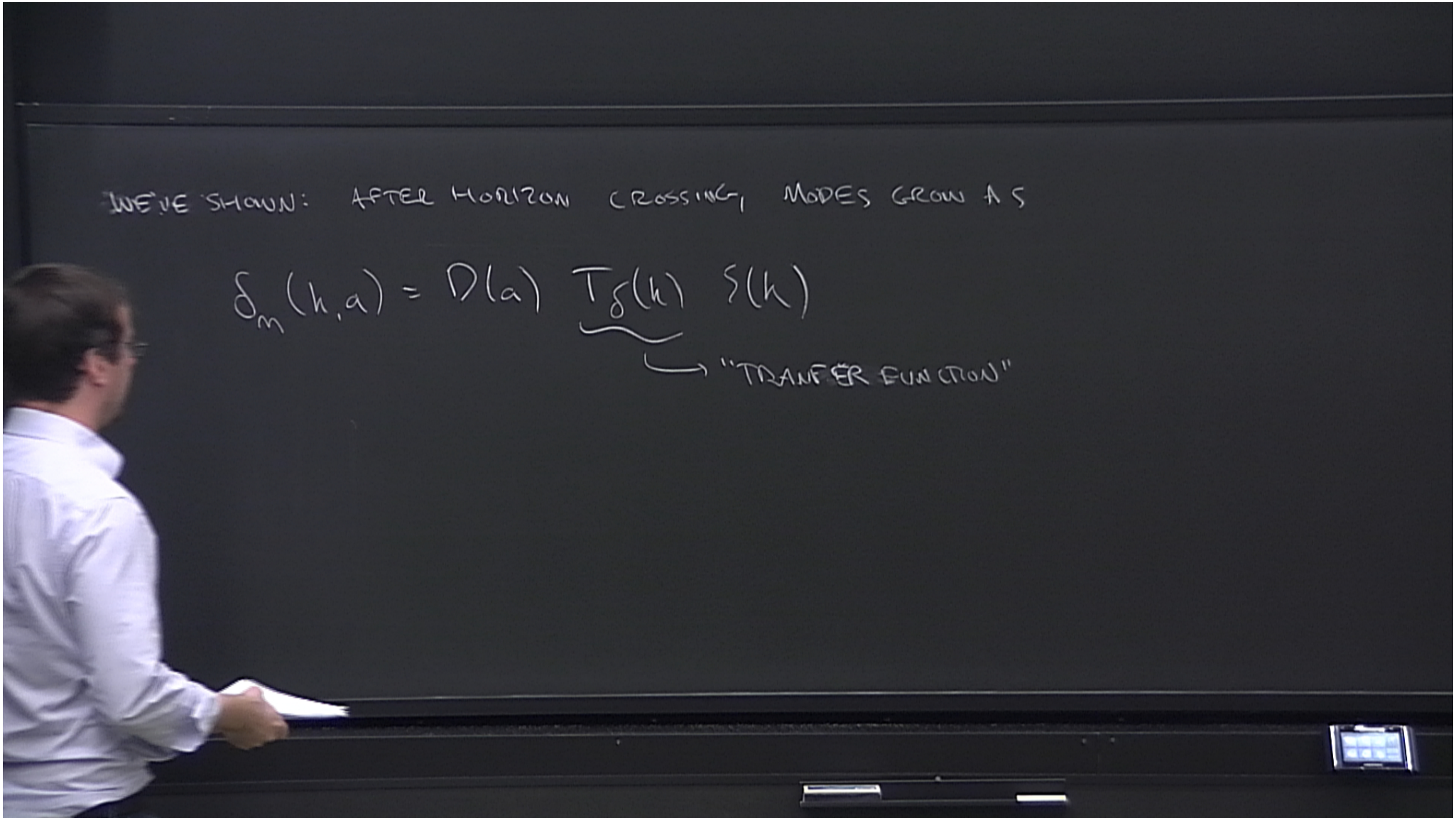
$$\psi = \begin{pmatrix} D \\ dD/d\log a \end{pmatrix}$$

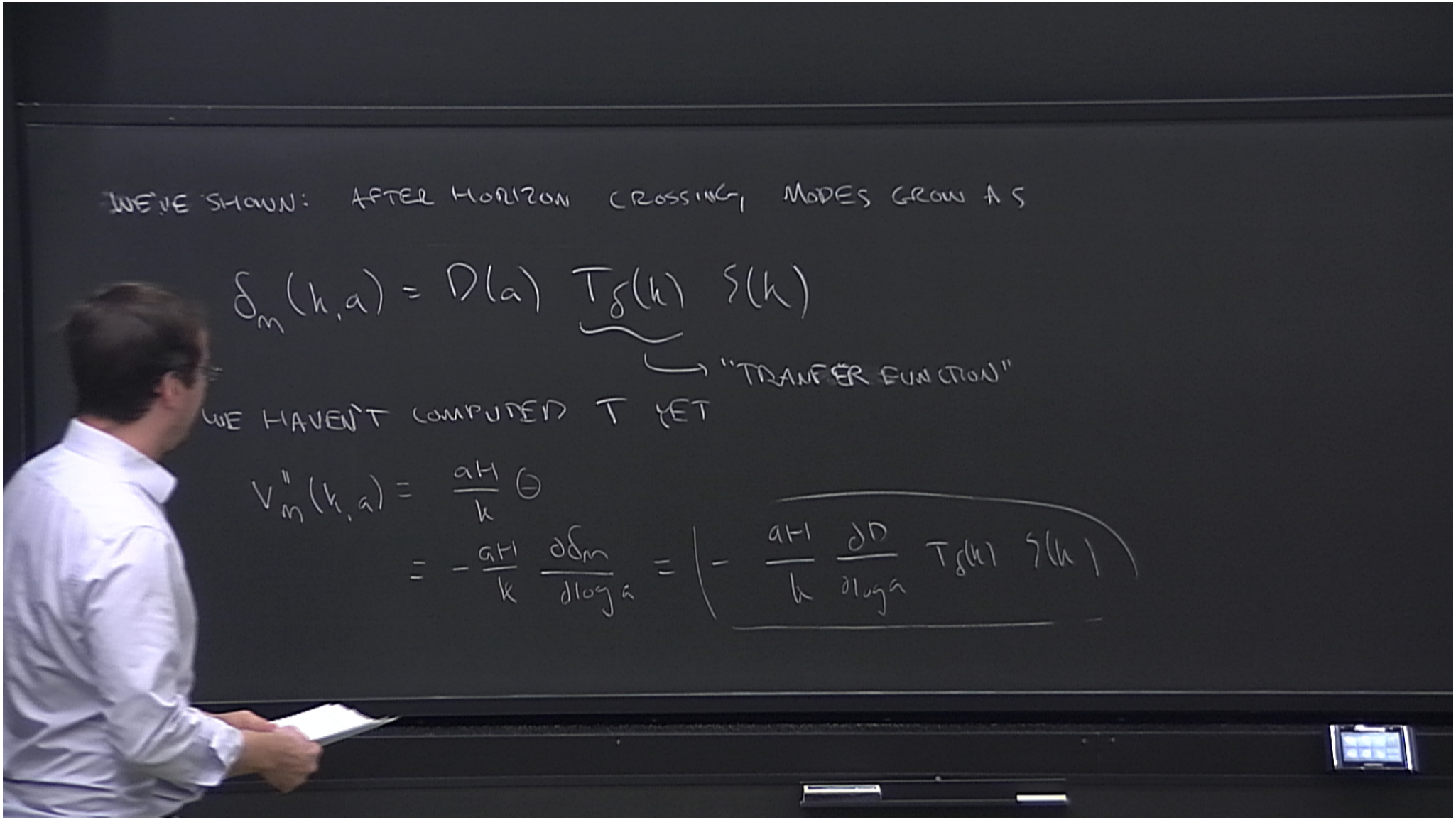
WHERE $D \propto a$ AT EARLY TIMES, $D(a) \to$ CONST. AT LATE TIMES

$$\frac{d^2D}{d(\log a)^2} + \frac{1 - 3w(a)}{2}\frac{dD}{d\log a} - \frac{3}{2}\Omega_m(a) D = 0$$

D "GROWTH FUNCTION"




WE'VE SHOWN: AFTER HORIZON CROSSING, MODES GROW AS
$\delta_m(k,a) = D(a)\, T_\delta(k)\, \zeta(k)$
"TRANSFER FUNCTION"


WE'VE SHOWN: AFTER HORIZON CROSSING, MODES GROW AS
$\delta_m(k,a) = D(a)\, T_\delta(k)\, \zeta(k)$
"TRANSFER FUNCTION"
WE HAVEN'T COMPUTED T YET
$V_m''(k,a) = \frac{aH}{k}\Theta$
$= -\frac{aH}{k}\frac{d\delta_m}{d\log a} = -\frac{aH}{k}\frac{dD}{d\log a}\, T_\delta(k)\, \zeta(k)$

WE'VE SHOWN: AFTER HORIZON CROSSING, MODES GROW AS

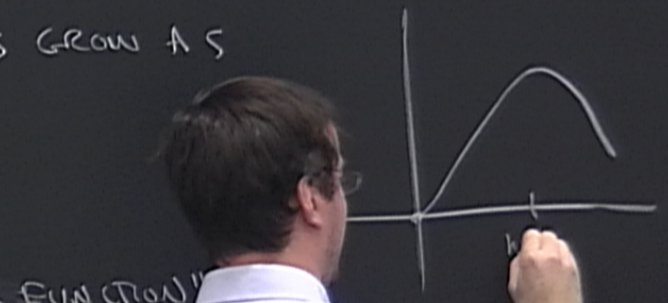
$$\delta_m(k,a) = D(a)\, T_\delta(k)\, \zeta(k)$$

"TRANSFER FUNCTION"

BUT WE HAVEN'T COMPUTED T YET

$$V_m''(k,a) = \frac{aH}{k}\Theta$$

$$= -\frac{aH}{k}\frac{d\delta_m}{d\log a} = \left(-\frac{aH}{k}\frac{dD}{d\log}\right.$$



WE'VE SHOWN: AFTER HORIZON CROSSING, MODES GROW AS

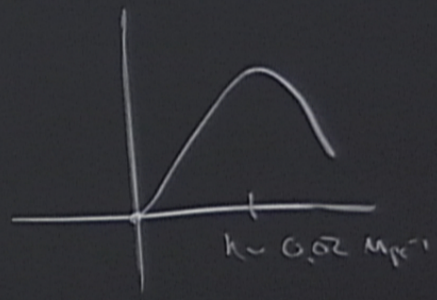
$$\delta_m(k,a) = D(a)\, T_\delta(k)\, \zeta(k)$$

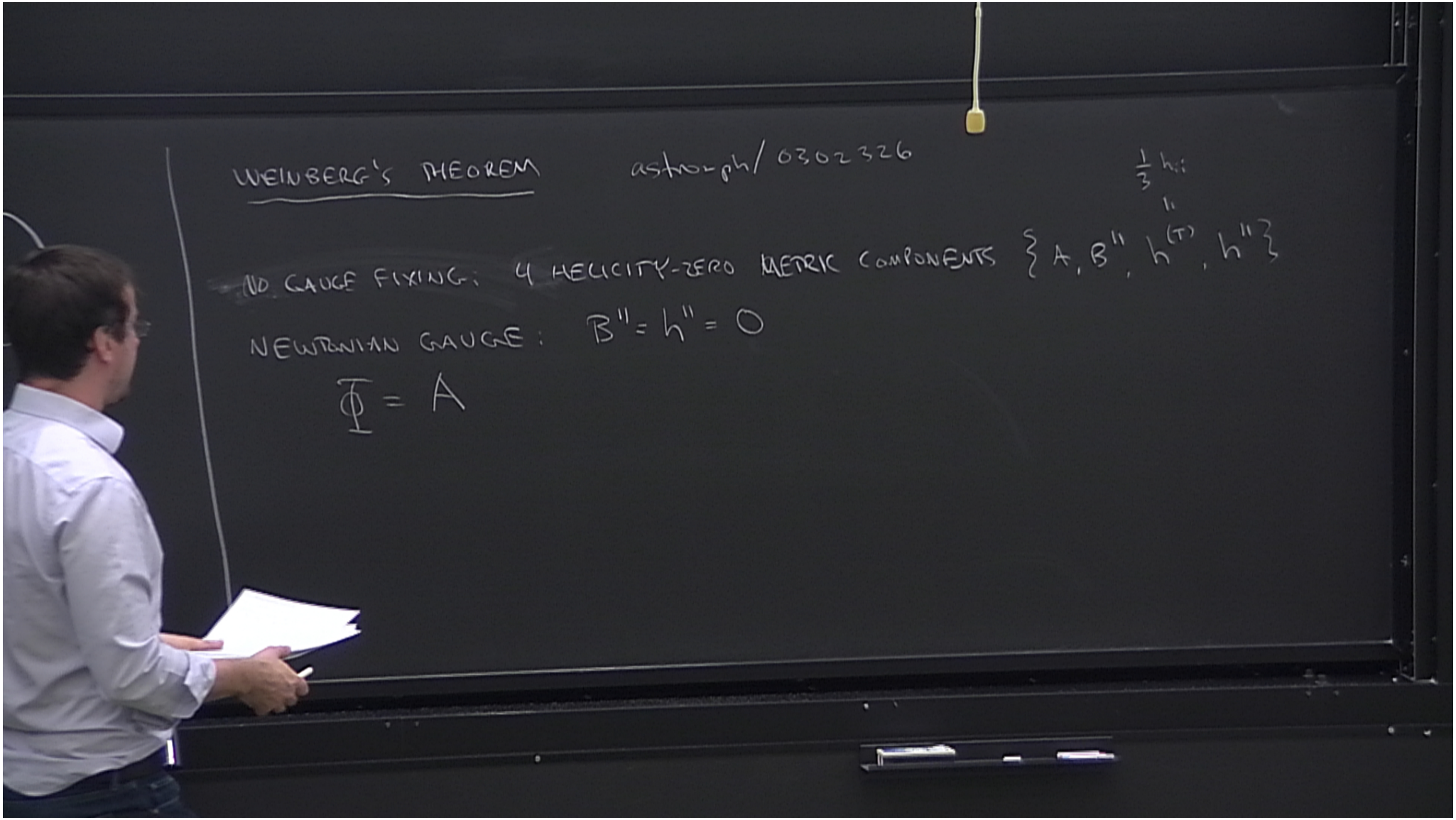
"TRANSFER FUNCTION"

BUT WE HAVEN'T COMPUTED T YET

$$V_m''(k,a) = \frac{aH}{k}\Theta$$

$$= -\frac{aH}{k}\frac{d\delta_m}{d\log a}$$




WEINBERG'S THEOREM
astro-ph/0302326
$\frac{1}{3}h_{ii}$
$\parallel$
NO GAUGE FIXING: 4 HELICITY-ZERO METRIC COMPONENTS $\{A, B'', h^{(T)}, h''\}$
NEWTONIAN GAUGE: $B'' = h'' = 0$
$\Phi = A$

WEINBERG'S THEOREM astro-ph/0302326

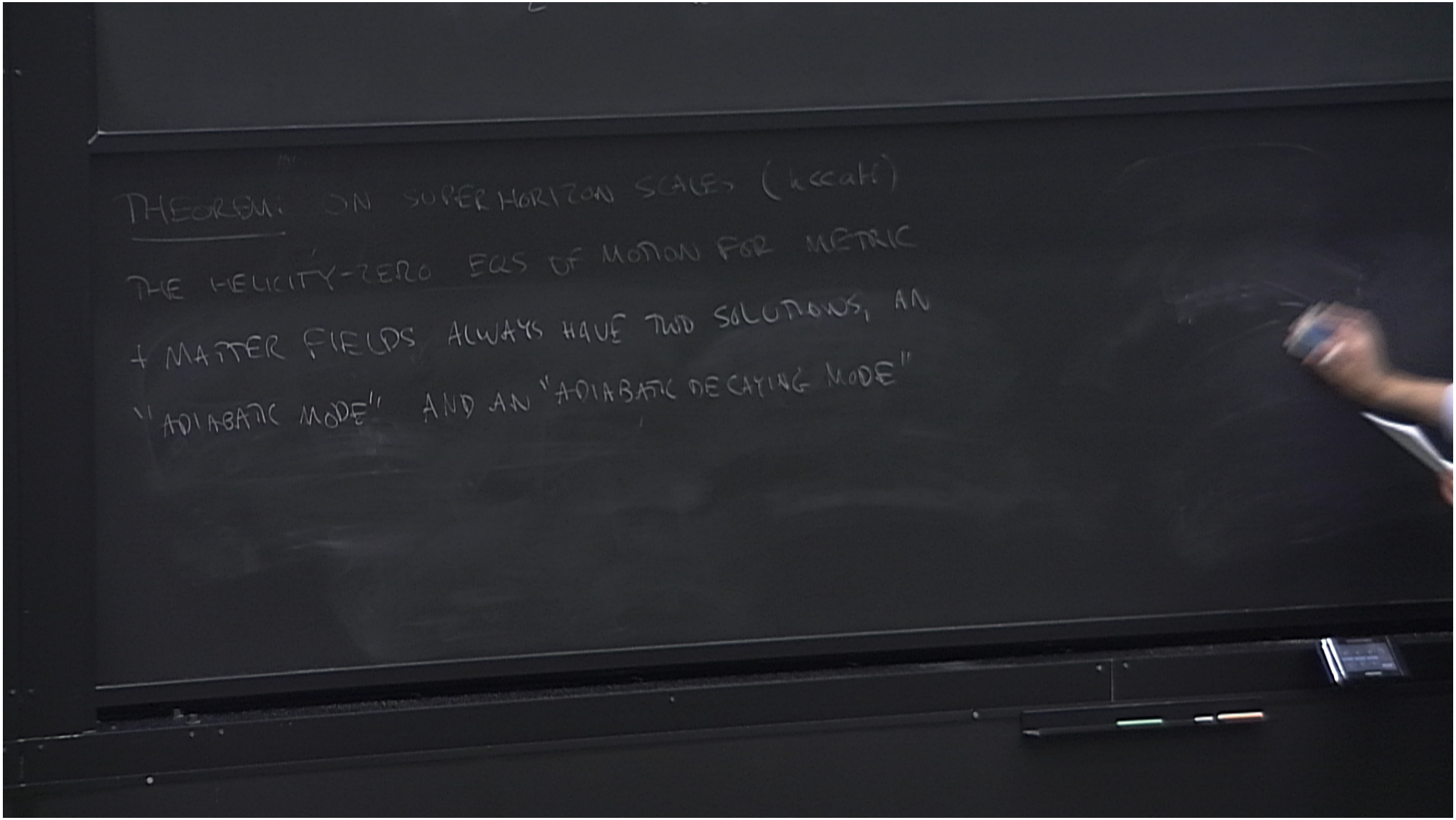
$\frac{1}{3}h_{ii}$

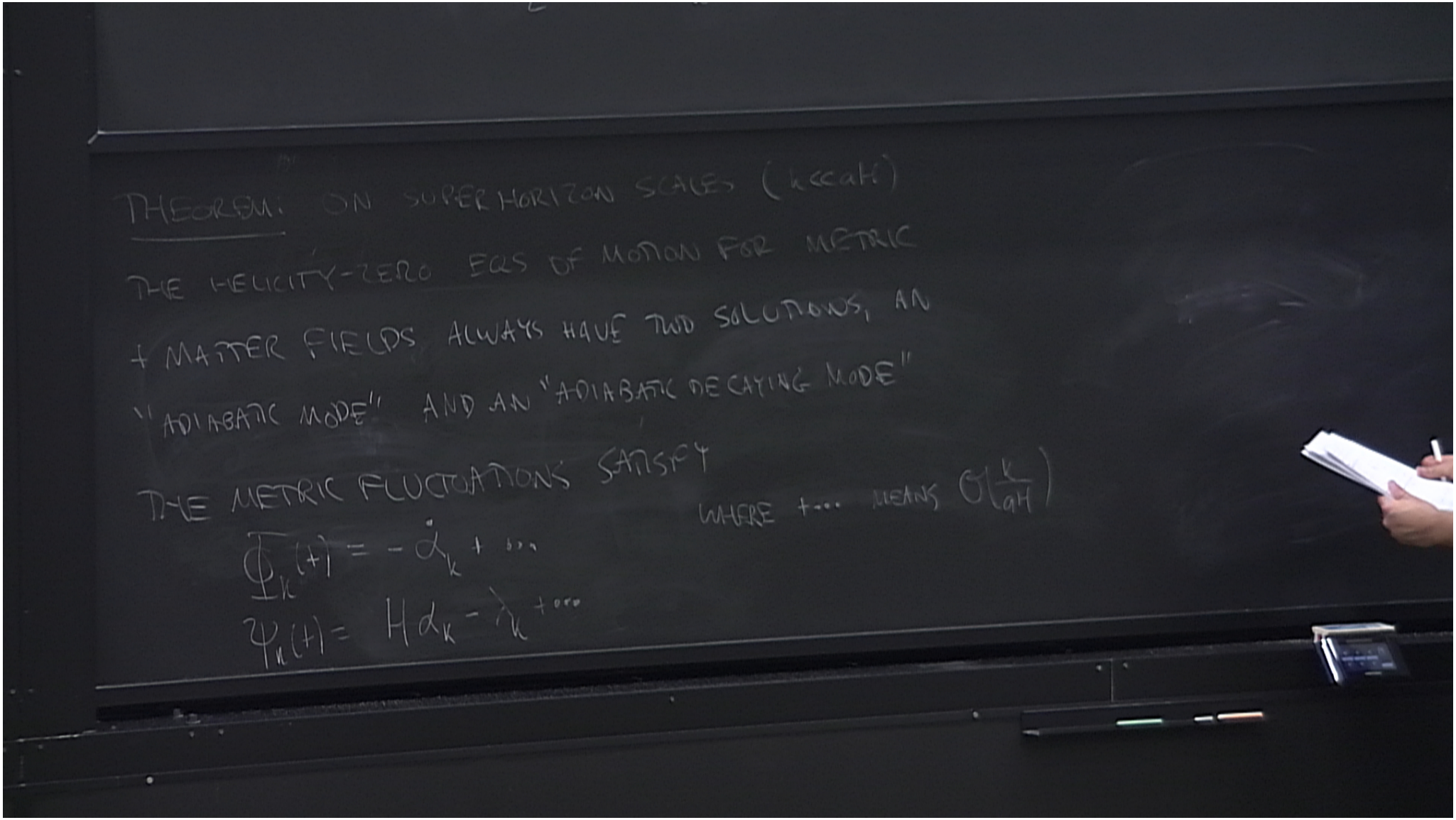
NO GAUGE FIXING: 4 HELICITY-ZERO METRIC COMPONENTS $\{A, B'', h^{(T)}, h''\}$

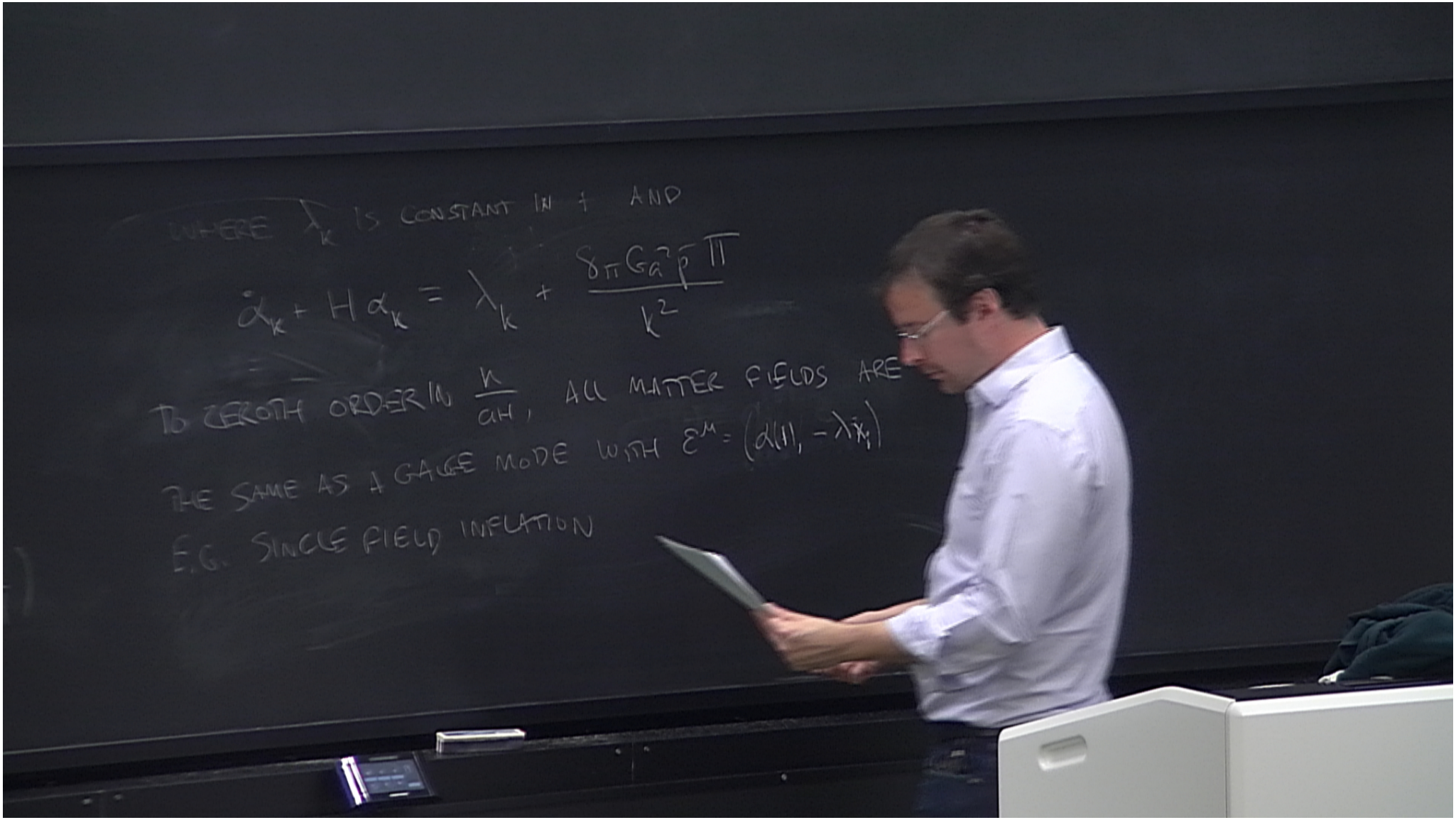
NEWTONIAN GAUGE: $B'' = h'' = 0$

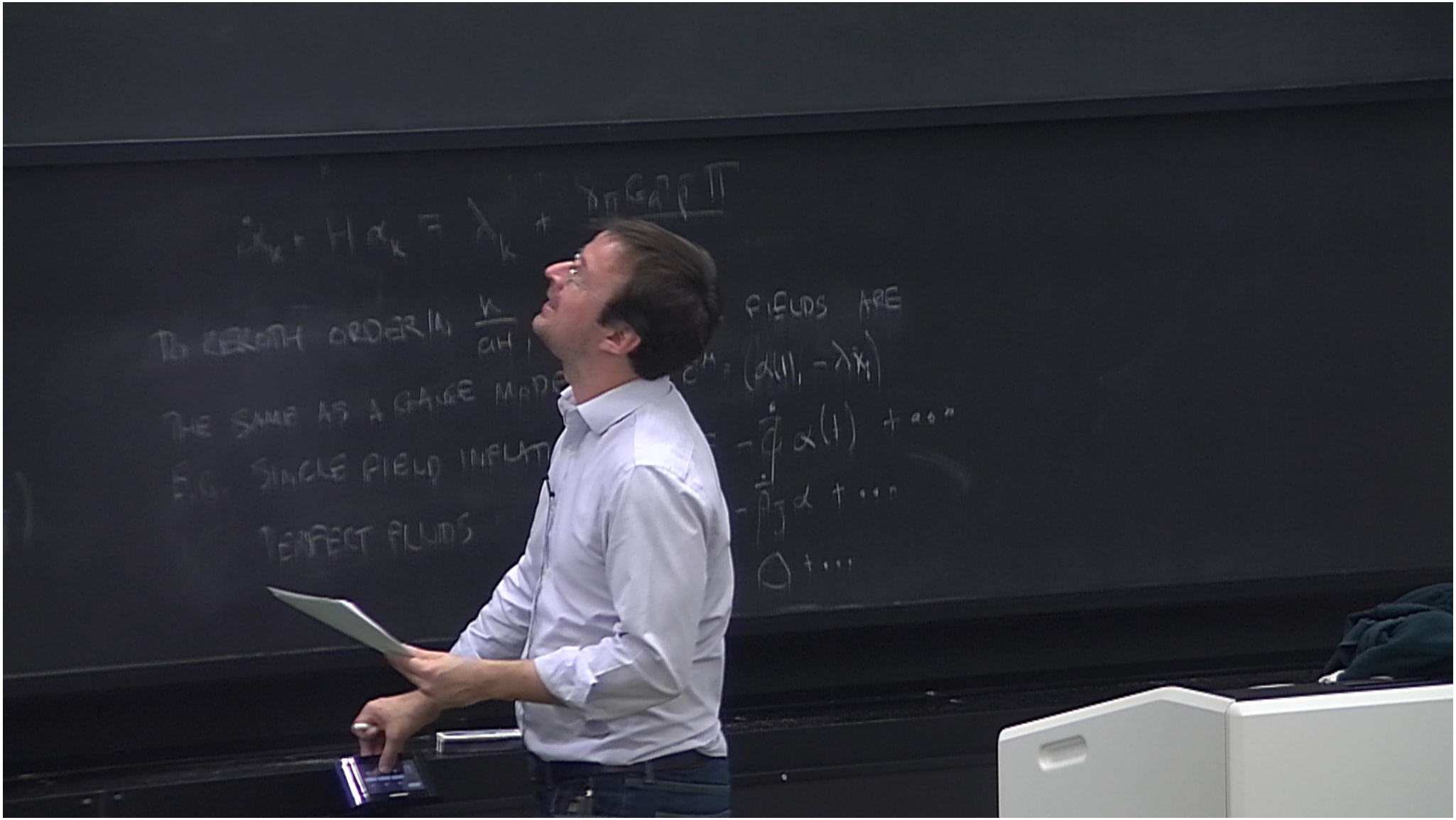
$\Phi = A$

$\Psi = -h^{(T)}$ $[= \Phi$ IF $\Pi'' = 0]$


THEOREM: ON SUPERHORIZON SCALES (k<<aH)
THE HELICITY-ZERO EQS OF MOTION FOR METRIC
+ MATTER FIELDS ALWAYS HAVE TWO SOLUTIONS, AN
"ADIABATIC MODE" AND AN "ADIABATIC DECAYING MODE"


THEOREM: ON SUPERHORIZON SCALES ($k \ll aH$)
THE HELICITY-ZERO EQS OF MOTION FOR METRIC
+ MATTER FIELDS ALWAYS HAVE TWO SOLUTIONS, AN
"ADIABATIC MODE" AND AN "ADIABATIC DECAYING MODE"
THE METRIC FLUCTUATIONS SATISFY
WHERE +... MEANS $O(\frac{k}{aH})$
$\Phi_k(t) = -\dot{\alpha}_k + \ldots$
$\Psi_k(t) = H\alpha_k - \lambda_k + \ldots$


WHERE λ_k IS CONSTANT IN τ AND
$\dot{\alpha}_k + H\alpha_k = \lambda_k + \frac{8\pi G a^2 \bar{p}\,\Pi}{k^2}$
TO ZEROTH ORDER IN $\frac{k}{aH}$, ALL MATTER FIELDS ARE
THE SAME AS A GAUGE MODE WITH $\xi^\mu = (\alpha(\tau), -\lambda \tilde{x}_i)$
E.G. SINGLE FIELD INFLATION


FIELDS ARE
THE SAME AS A GAUGE
E.G. SINGLE FIELD
PERFECT FLUIDS

WHERE λ_k IS CONSTANT IN τ AND

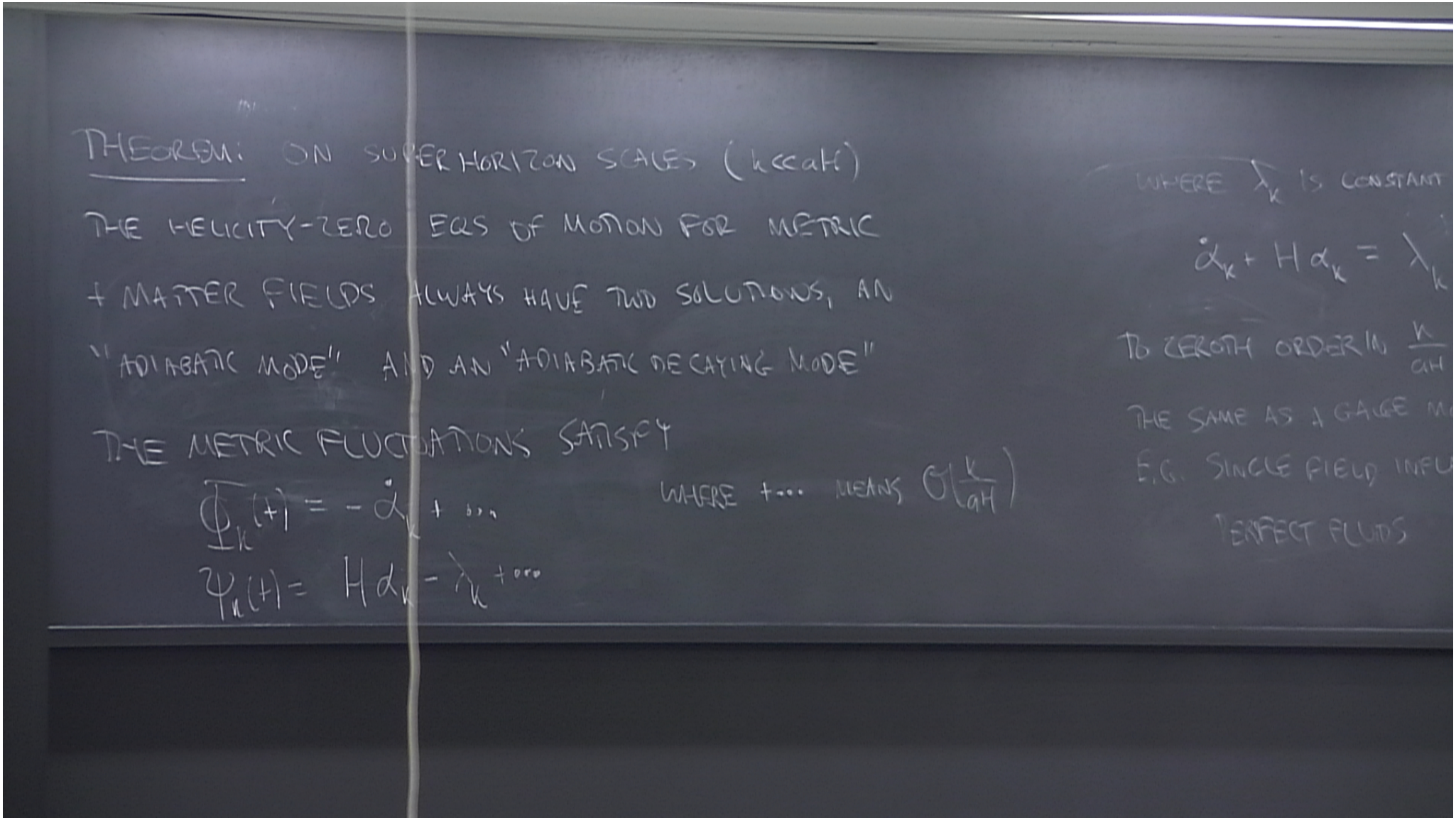
$$\dot{\alpha}_k + H\alpha_k = \lambda_k + \frac{8\pi G a^2 \bar{p}\,\Pi}{k^2}$$

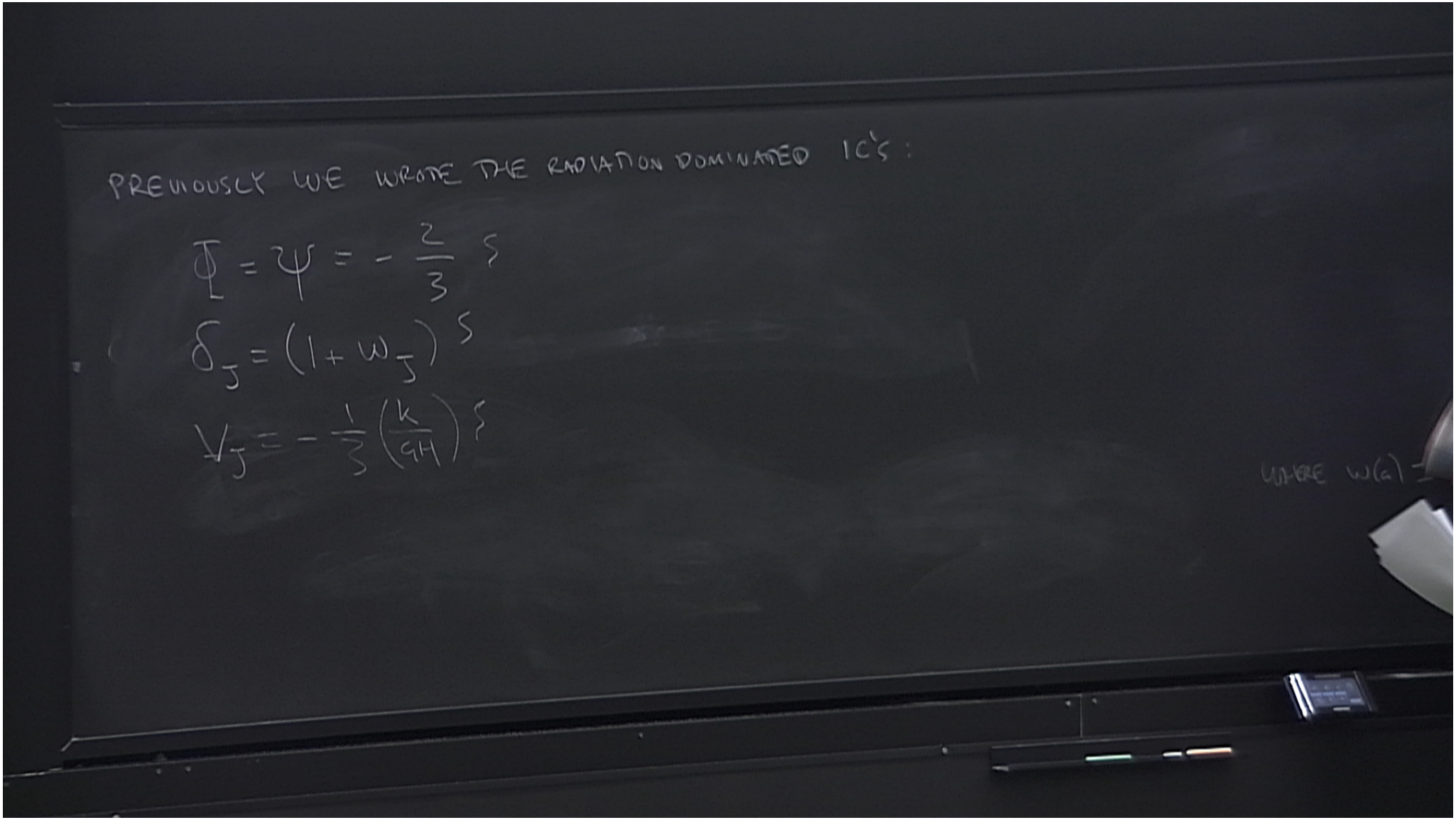
TO ZEROTH ORDER IN $\frac{k}{aH}$, ALL MATTER FIELDS ARE THE SAME AS A GAUGE MODE WITH $\xi^\mu = (\alpha(\tau), -\lambda \hat{x}_i)$

E.G. SINGLE FIELD INFLATION $\delta\phi = -\dot{\bar{\phi}}\,\alpha(\tau) + \ldots$

PERFECT FLUIDS $\delta\rho_J = -\dot{\bar{\rho}}_J \alpha + \ldots$

$\delta v_J = 0 + \ldots$


THEOREM: ON SUPERHORIZON SCALES ($k \ll aH$)
THE HELICITY-ZERO EQS OF MOTION FOR METRIC
+ MATTER FIELDS ALWAYS HAVE TWO SOLUTIONS, AN
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$\Phi_k(t) = -\dot{\alpha}_k + \dots$
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WHERE $+\dots$ MEANS $O(\frac{k}{aH})$
WHERE λ_k IS CONSTANT
$\dot{\alpha}_k + H\alpha_k = \lambda_k$
TO ZEROTH ORDER IN $\frac{k}{aH}$
THE SAME AS
E.G. SINGLE FIELD INFL
PERFECT FLUIDS


PREVIOUSLY WE WROTE THE RADIATION DOMINATED IC'S:
$\Phi = \Psi = -\frac{2}{3}\zeta$
$\delta_J = (1+w_J)\zeta$
$V_J = -\frac{1}{3}\left(\frac{k}{aH}\right)\zeta$
WHERE $w(a) =$

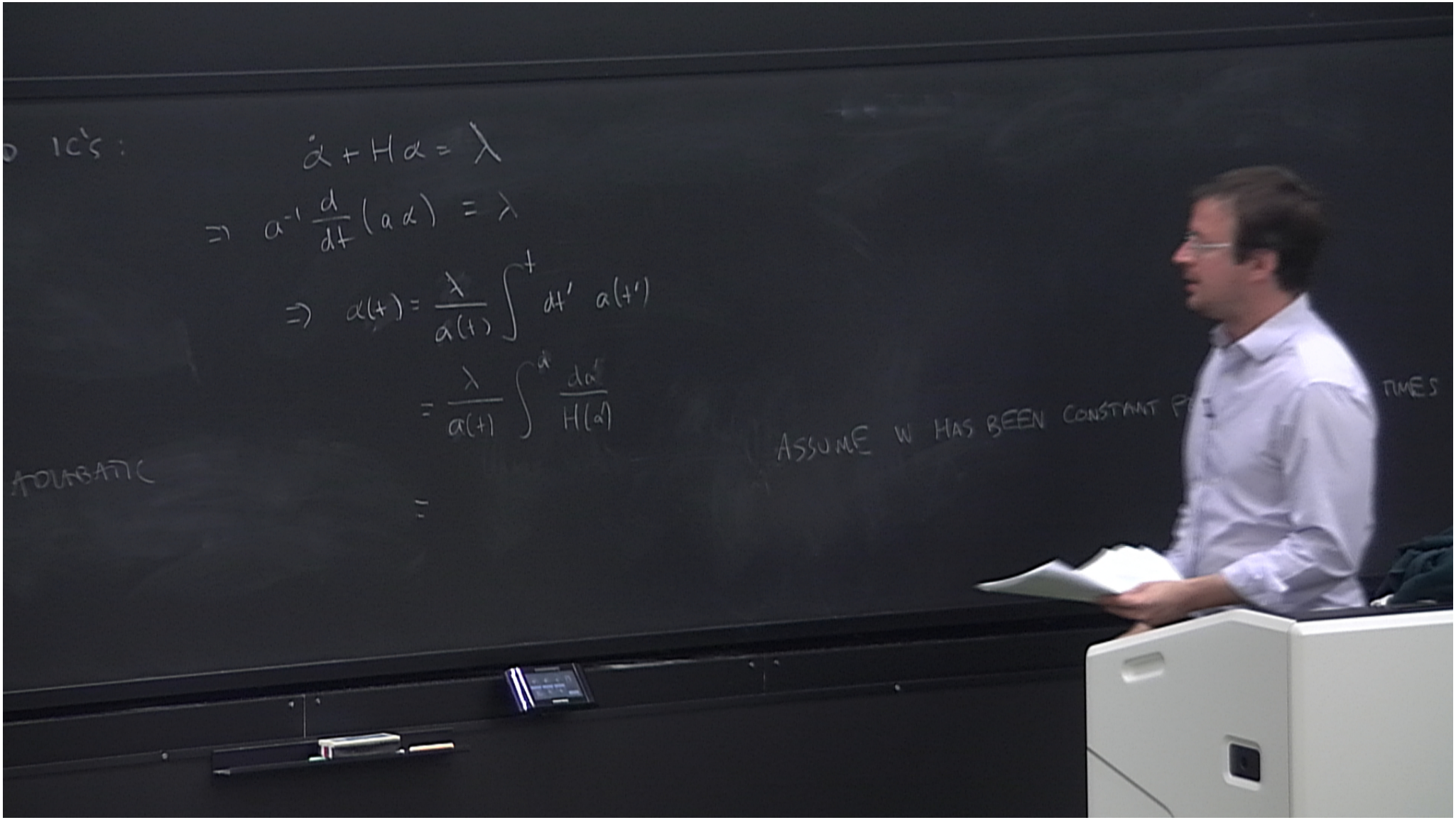
PREVIOUSLY WE WROTE THE RADIATION DOMINATED IC'S:

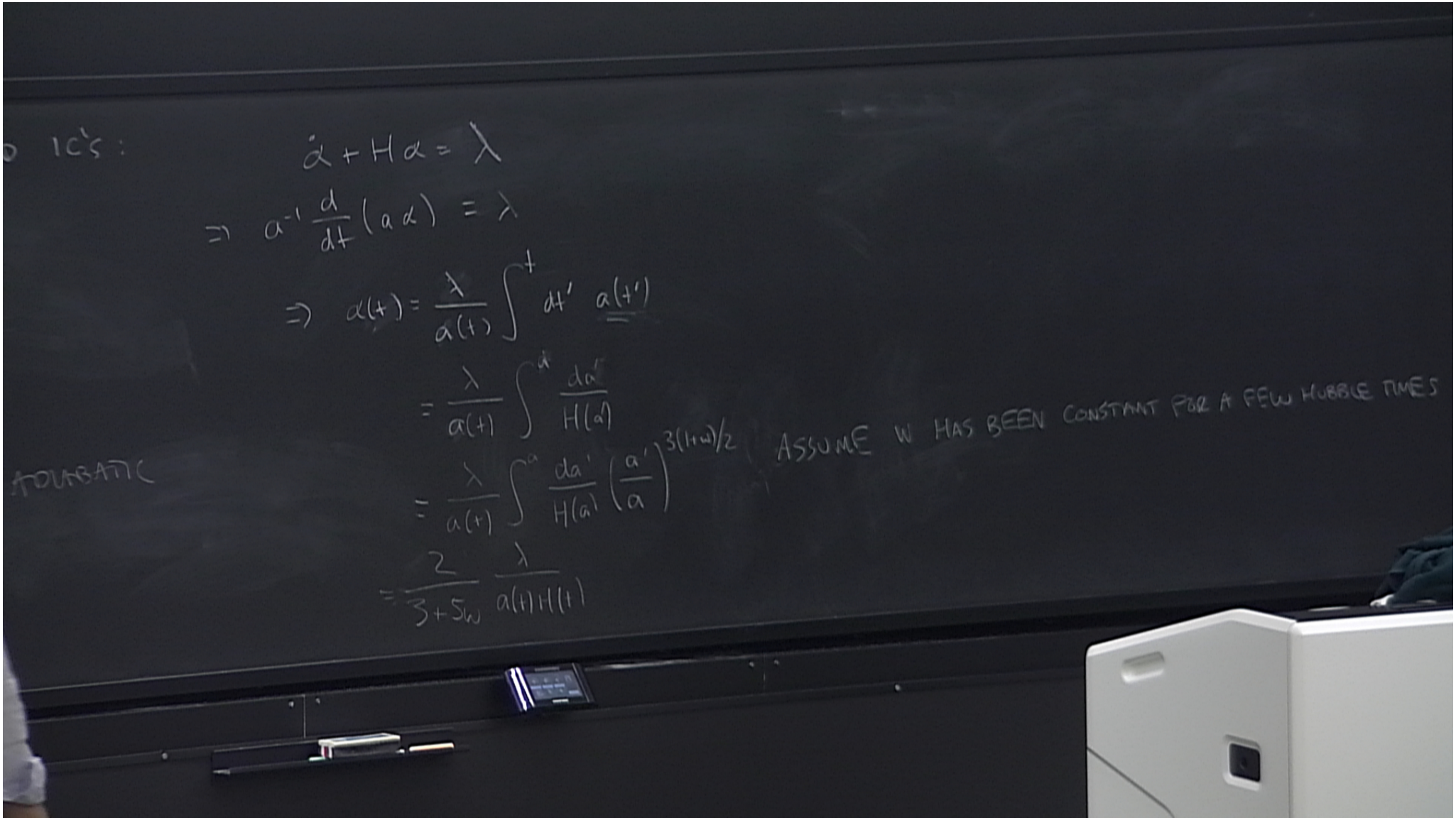
$$\Phi = \Psi = -\frac{2}{3}\zeta$$

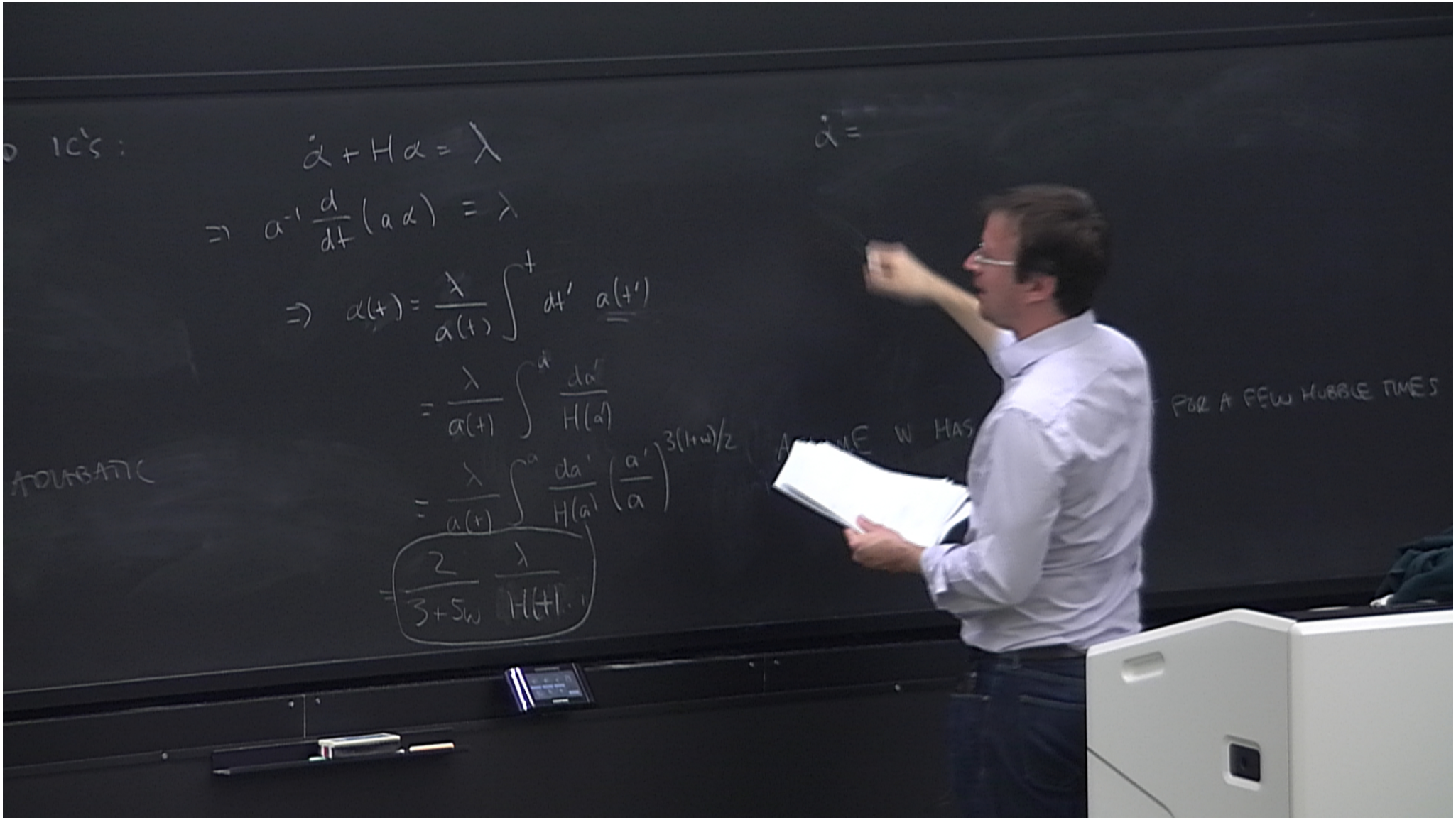
$$\delta_J = (1+w_J)\zeta$$

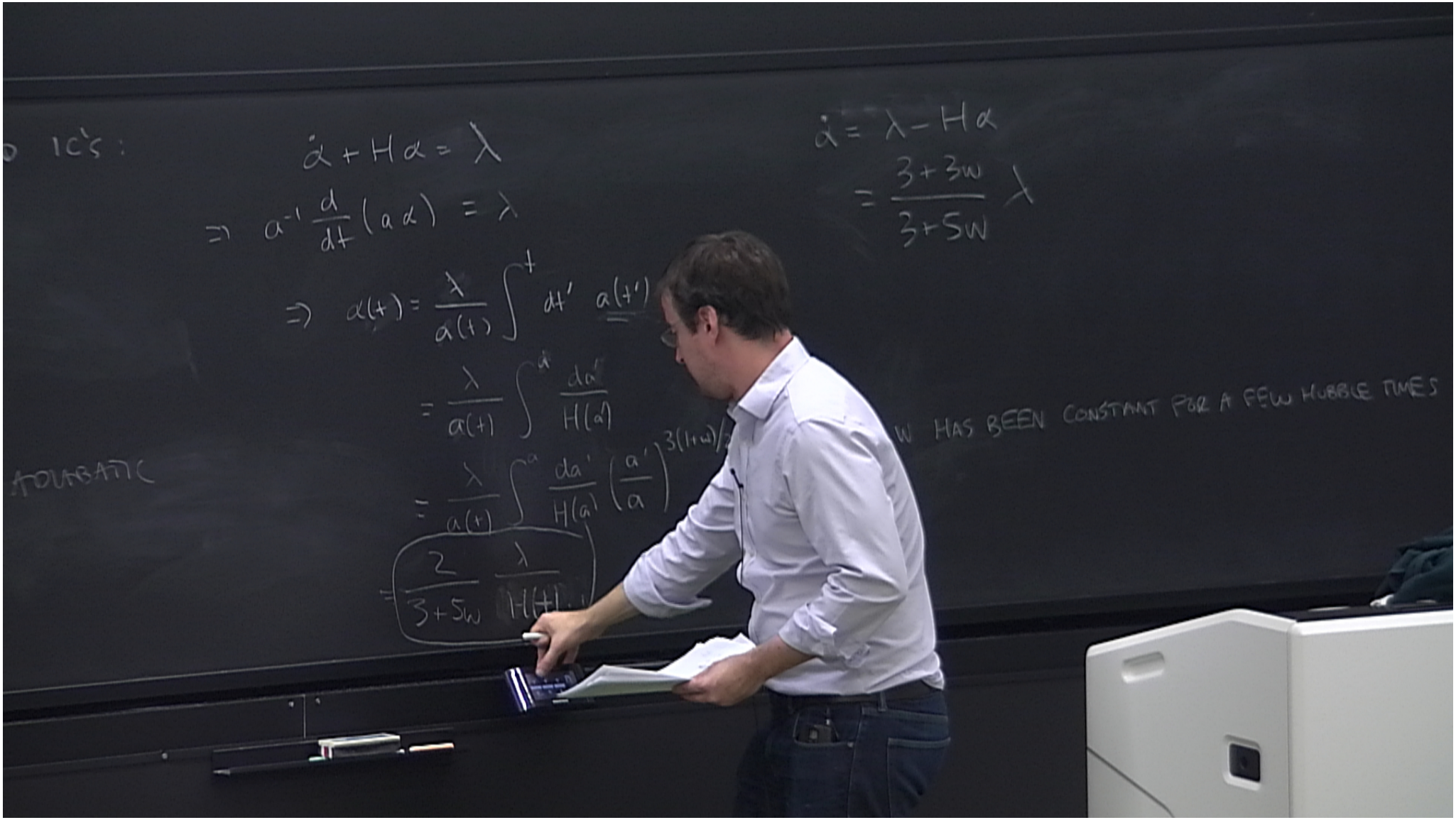
$$V_J = -\frac{1}{3}\left(\frac{k}{aH}\right)\zeta$$

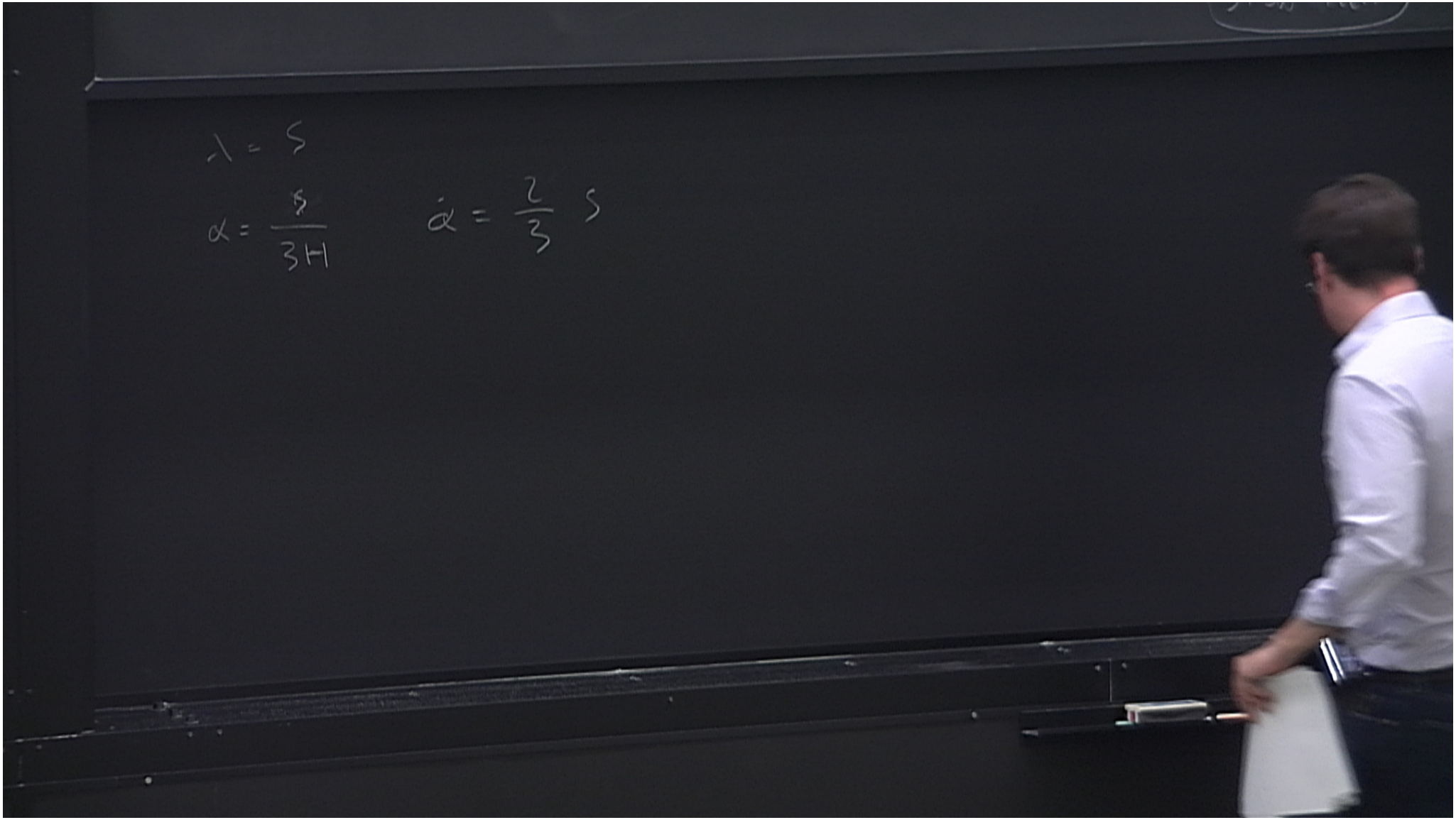
I'LL CHECK THAT THIS IS THE SAME AS THE ADIABATIC MODE WITH $\lambda = \zeta$


IC's:
$\dot{\alpha} + H\alpha = \lambda$
$\Rightarrow a^{-1}\frac{d}{dt}(a\alpha) = \lambda$
$\Rightarrow \alpha(t) = \frac{\lambda}{a(t)}\int^{t} dt'\, a(t')$
$= \frac{\lambda}{a(t)}\int^{a} \frac{da'}{H(a')}$
ADIABATIC
ASSUME W HAS BEEN CONSTANT
TIMES


IC's:
$\dot{\alpha} + H\alpha = \lambda$
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$= \frac{\lambda}{a(t)}\int^{a} \frac{da'}{H(a')}$
$= \frac{\lambda}{a(t)}\int^{a} \frac{da'}{H(a')}\left(\frac{a'}{a}\right)^{3(1+w)/2}$
$= \frac{2}{3+3w}\frac{\lambda}{a(t)H(t)}$
ADIABATIC
ASSUME W HAS BEEN CONSTANT FOR A FEW HUBBLE TIMES


IC's:
$\dot{\alpha} + H\alpha = \lambda$
$\Rightarrow a^{-1}\frac{d}{dt}(a\alpha) = \lambda$
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$= \frac{2}{3+3w}\frac{\lambda}{H(t)}$
ADIABATIC
FOR A FEW HUBBLE TIMES


IC's:
$\dot{\alpha} + H\alpha = \lambda$
$\Rightarrow a^{-1}\frac{d}{dt}(a\alpha) = \lambda$
$\Rightarrow \alpha(t) = \frac{\lambda}{a(t)}\int_{t}^{t} dt'\, a(t')$
$= \frac{\lambda}{a(t)}\int_{a}^{a} \frac{da'}{H(a')}$
$= \frac{\lambda}{a(t)}\int_{a}^{a} \frac{da'}{H(a')}\left(\frac{a'}{a}\right)^{3(1+w)/2}$
$= \frac{2}{3+5w}\frac{\lambda}{H(t)}$
$\dot{\alpha} = \lambda - H\alpha$
$= \frac{3+3w}{3+5w}\lambda$
W HAS BEEN CONSTANT FOR A FEW HUBBLE TIMES
ADIABATIC


$\lambda = s$
$\alpha = \frac{s}{3H}$
$\dot{\alpha} = \frac{2}{3} s$

THEOREM: ON SUPERHORIZON SCALES ($k \ll aH$)

THE HELICITY-ZERO EQS OF MOTION FOR METRIC + MATTER FIELDS ALWAYS HAVE TWO SOLUTIONS, AN "ADIABATIC MODE" AND AN "ADIABATIC DECAYING MODE"

THE METRIC FLUCTUATIONS SATISFY

$$\Phi_k(t) = -\dot{\alpha}_k + \dots$$

$$\Psi_k(t) = H\alpha_k - \lambda_k + \dots$$

WHERE $+\dots$ MEANS $\mathcal{O}\left(\frac{k}{aH}\right)$

WHERE λ_k IS CONSTA

$$\dot{\alpha}_k + H\alpha_k = \lambda$$

TO ZEROTH ORDER IN $\frac{k}{}$

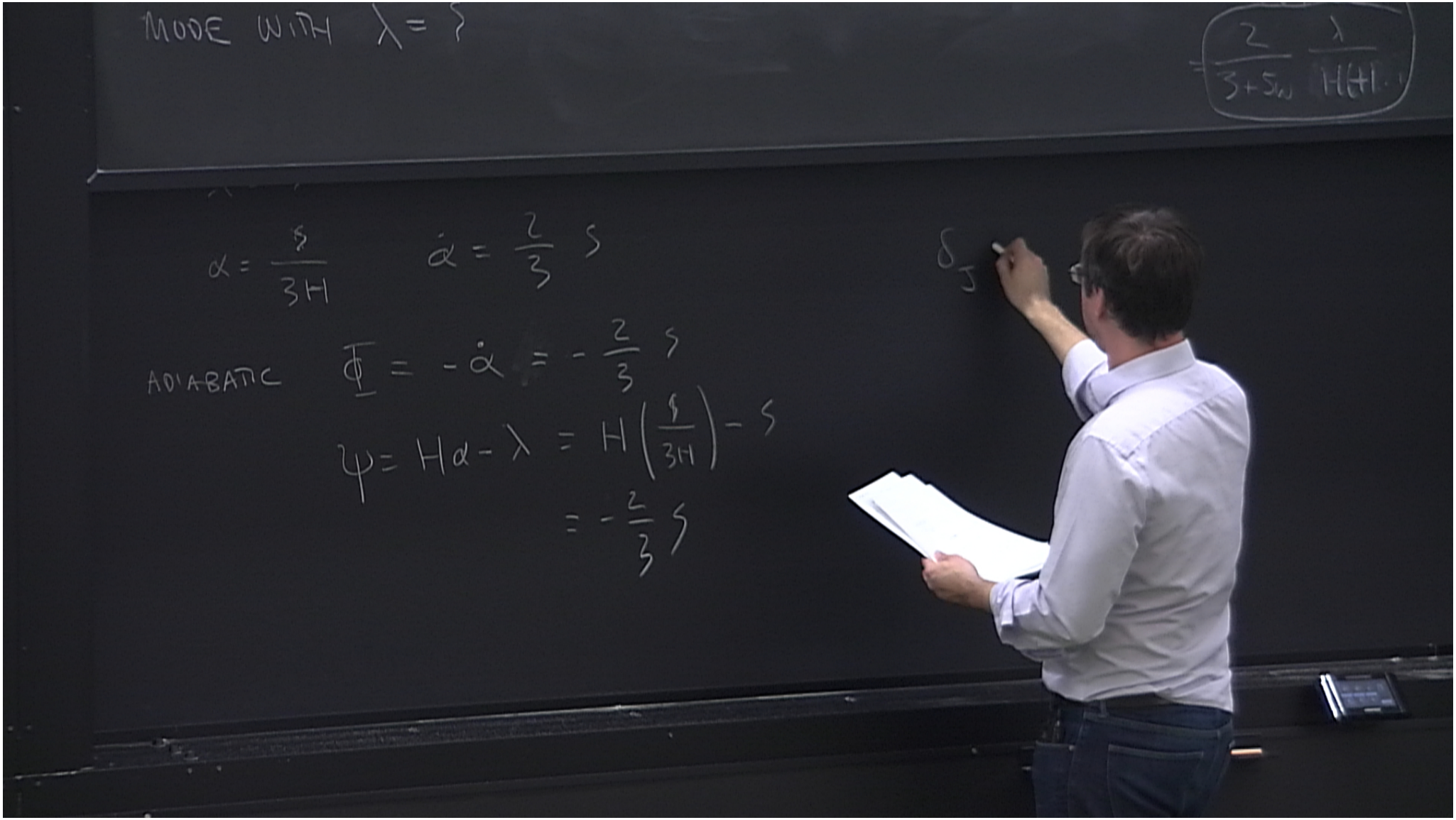
THE SAME AS A GAUGE

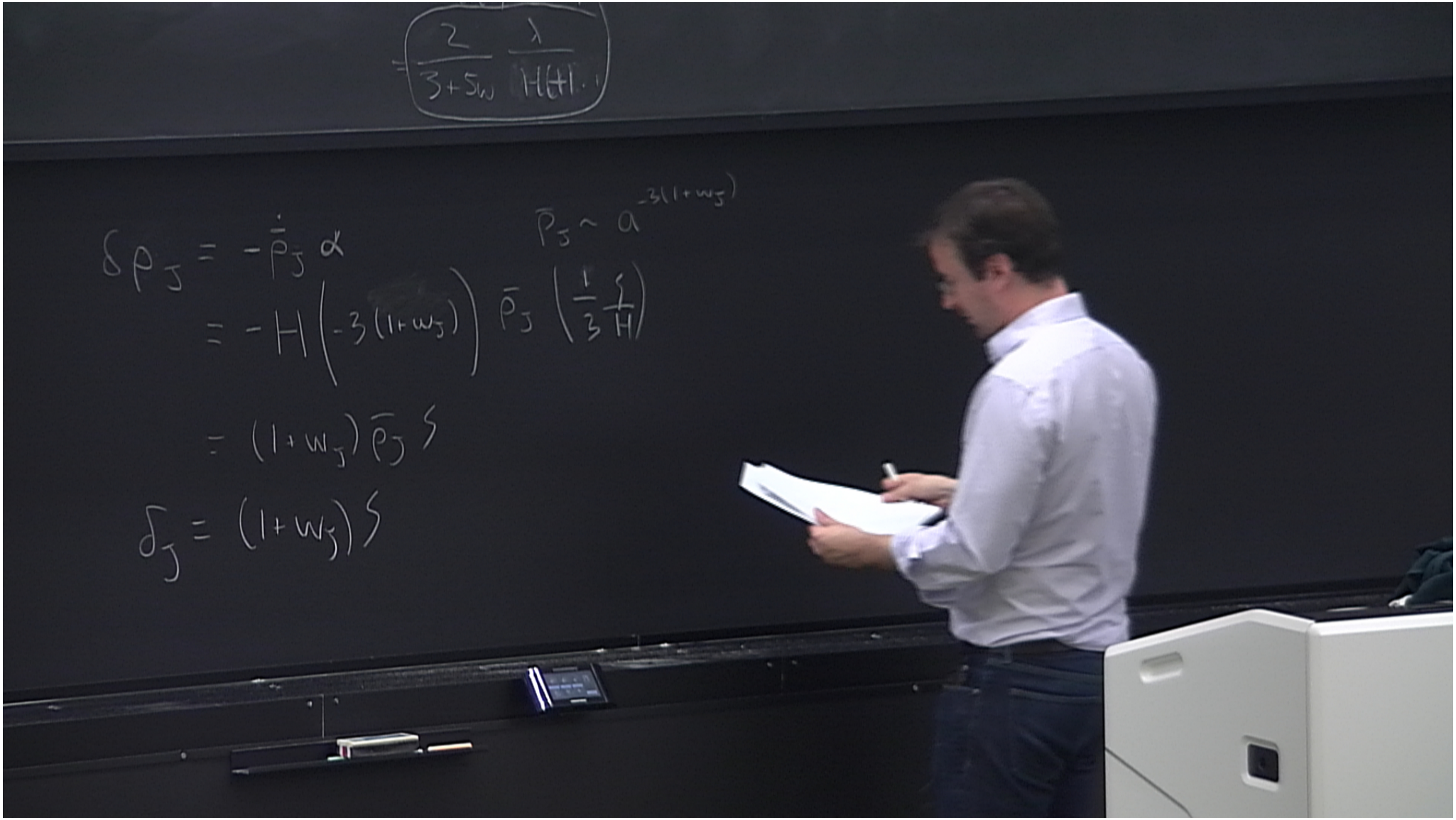
E.G. SINGLE FIELD IN

PERFECT FLUIDS

PREVIOUSLY WE WROTE THE RADIATION DOMINATED IC'S:

$$\dot{\alpha} + H\alpha = \lambda$$


MODE WITH $\lambda = s$
$\alpha = \frac{s}{3H}$
$\dot{\alpha} = \frac{2}{3} s$
ADIABATIC
$\Phi = -\dot{\alpha} = -\frac{2}{3} s$
$\psi = H\alpha - \lambda = H\left(\frac{s}{3H}\right) - s$
$= -\frac{2}{3} s$
$\frac{2}{3+5w} \frac{\lambda}{H(1+w)}$
δ_J


$= \frac{2}{3+5w} \frac{\lambda}{H(t)}$
$\bar{\rho}_J \sim a^{-3(1+w_J)}$
$\delta\rho_J = -\dot{\bar{\rho}}_J \alpha$
$= -H\left(-3(1+w_J)\right)\bar{\rho}_J \left(\frac{1}{3}\frac{\delta}{H}\right)$
$= (1+w_J)\bar{\rho}_J \delta$
$\delta_J = (1+w_J)\delta$


$\bar{\rho}_J \sim a^{-3(1+w_J)}$
$(V_J) = 0$
$\bar{\rho}_J \left(\frac{1}{3}\frac{\zeta}{H}\right)$

I'll CHECK THAT THIS IS THE SAME AS THE ABOVE

MOVE WITH $\lambda = s$

$$= \frac{\lambda}{a(t)} \left(\frac{da}{H(a)} \left(\frac{1}{a} \right) \right)$$

$$= \frac{2}{3+5w} \frac{\lambda}{H(t)}$$

$$\alpha = \frac{s}{3H} \qquad \dot{\alpha} = \frac{2}{3} s$$

ADIABATIC $\quad \Psi = -\dot{\alpha} = -\frac{2}{3} s + \cdots$

$$\psi = H\alpha - \lambda = H\left(\frac{s}{3H}\right) - s$$

$$= -\frac{2}{3} s$$

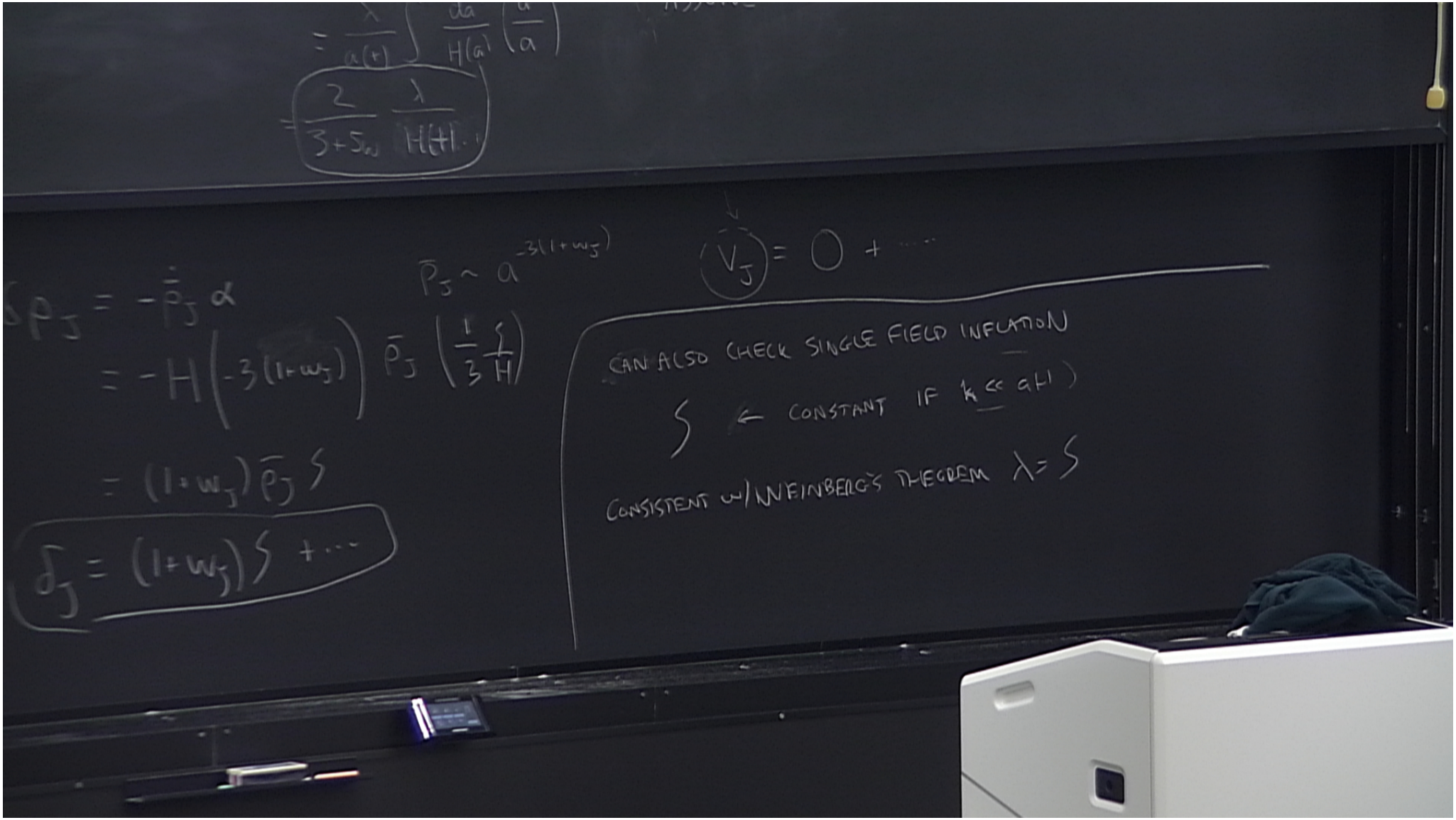
$$\delta\rho_J = -\dot{\bar{\rho}}_J \alpha$$

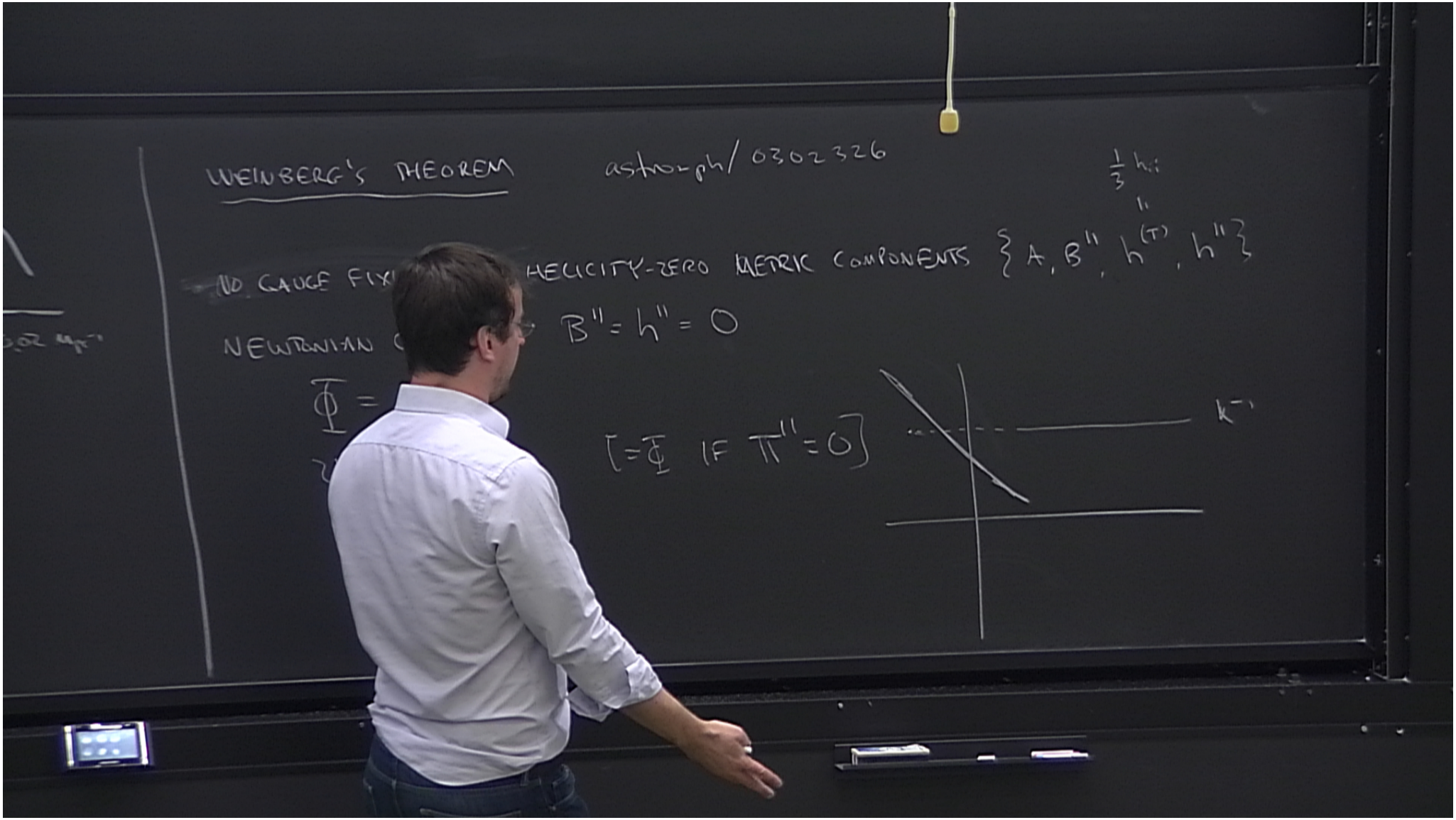
$$= -H\left(-3(1+w_J)\right) \bar{\rho}_J \left(\frac{1}{3}\frac{s}{H}\right)$$

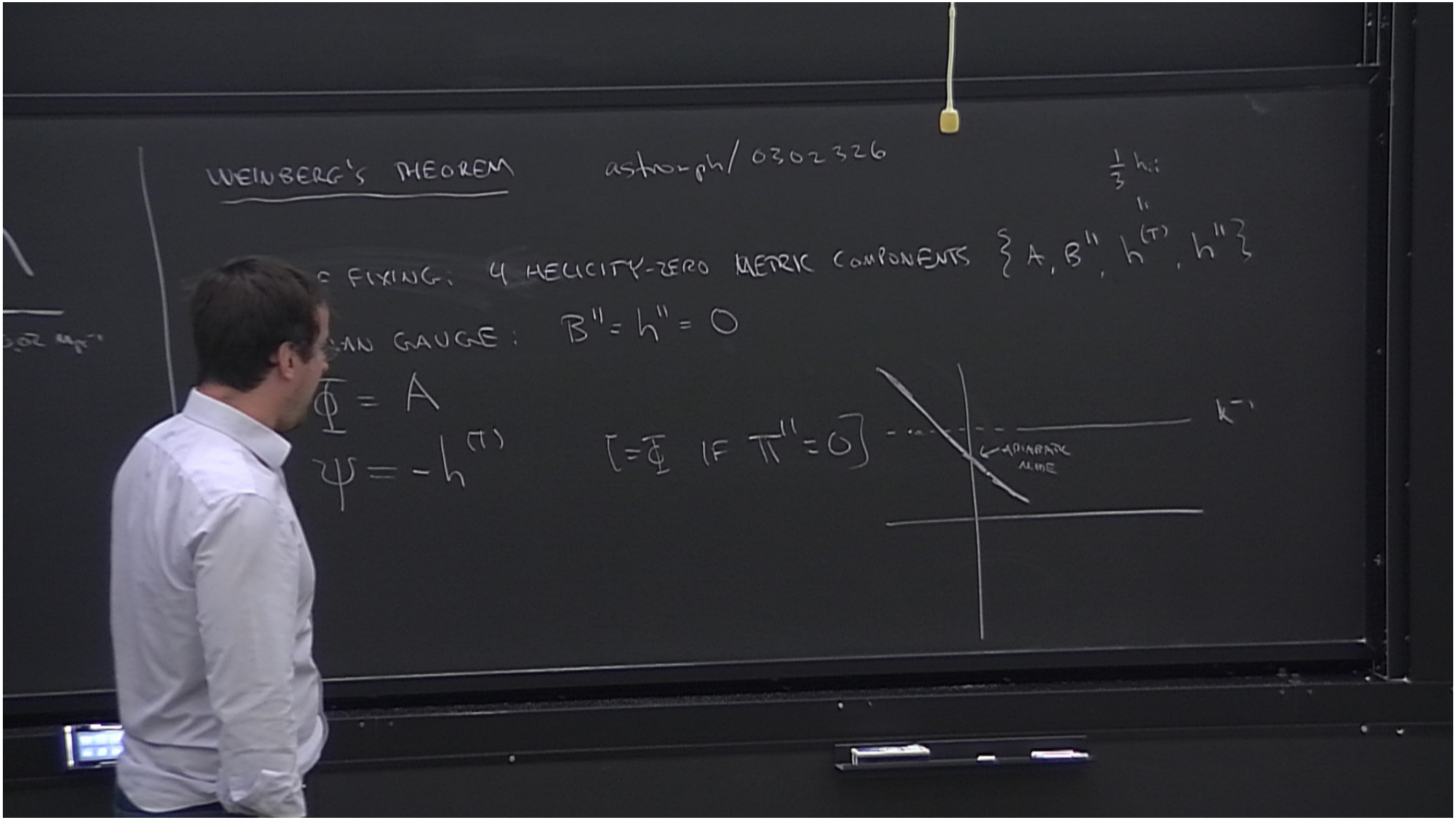
$$= (1+w_J)\bar{\rho}_J s$$

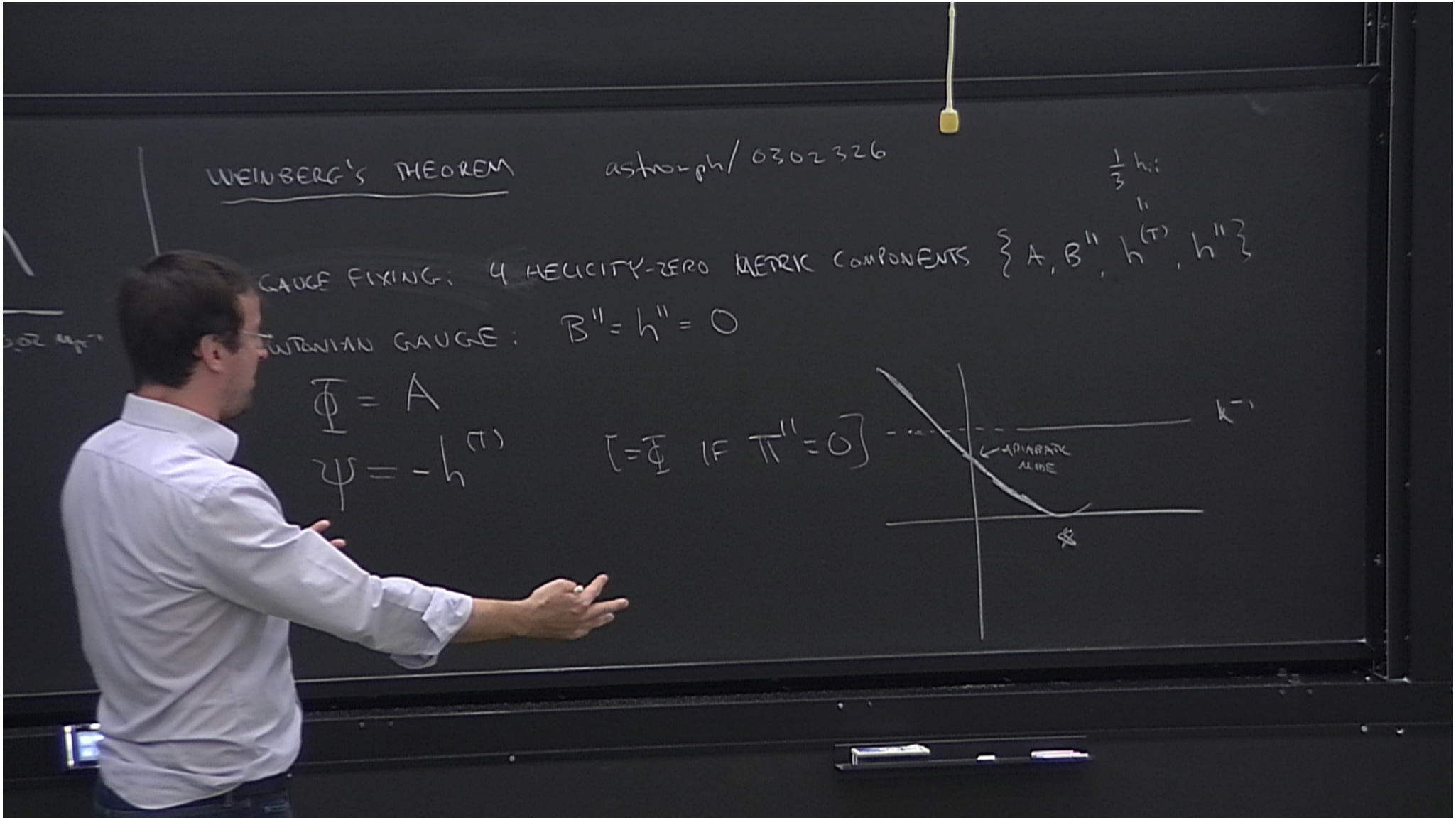
$$\bar{\rho}_J \sim a^{-3(1+w_J)}$$

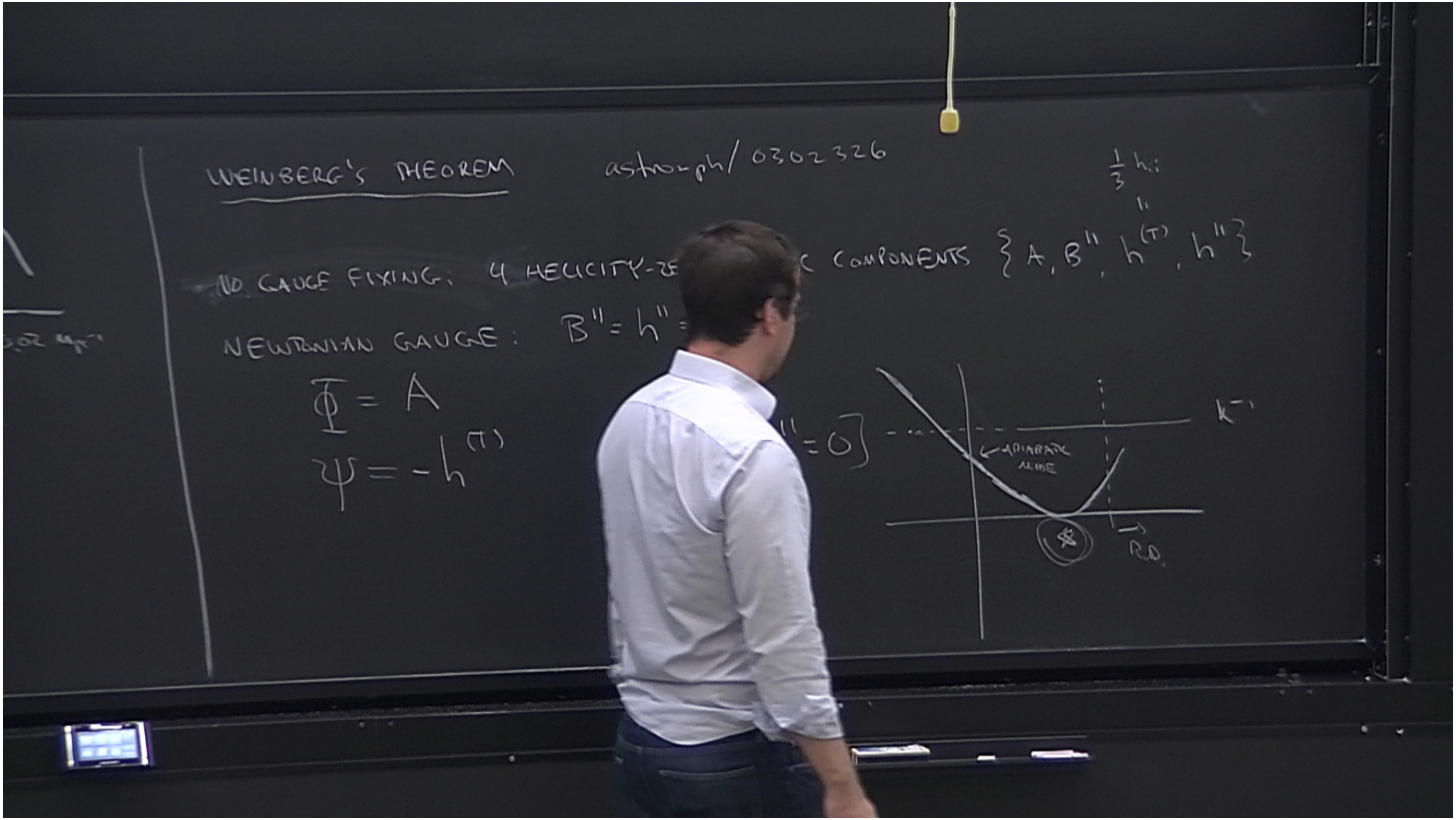
$$\boxed{\delta_J = (1+w_J) s + \cdots}$$

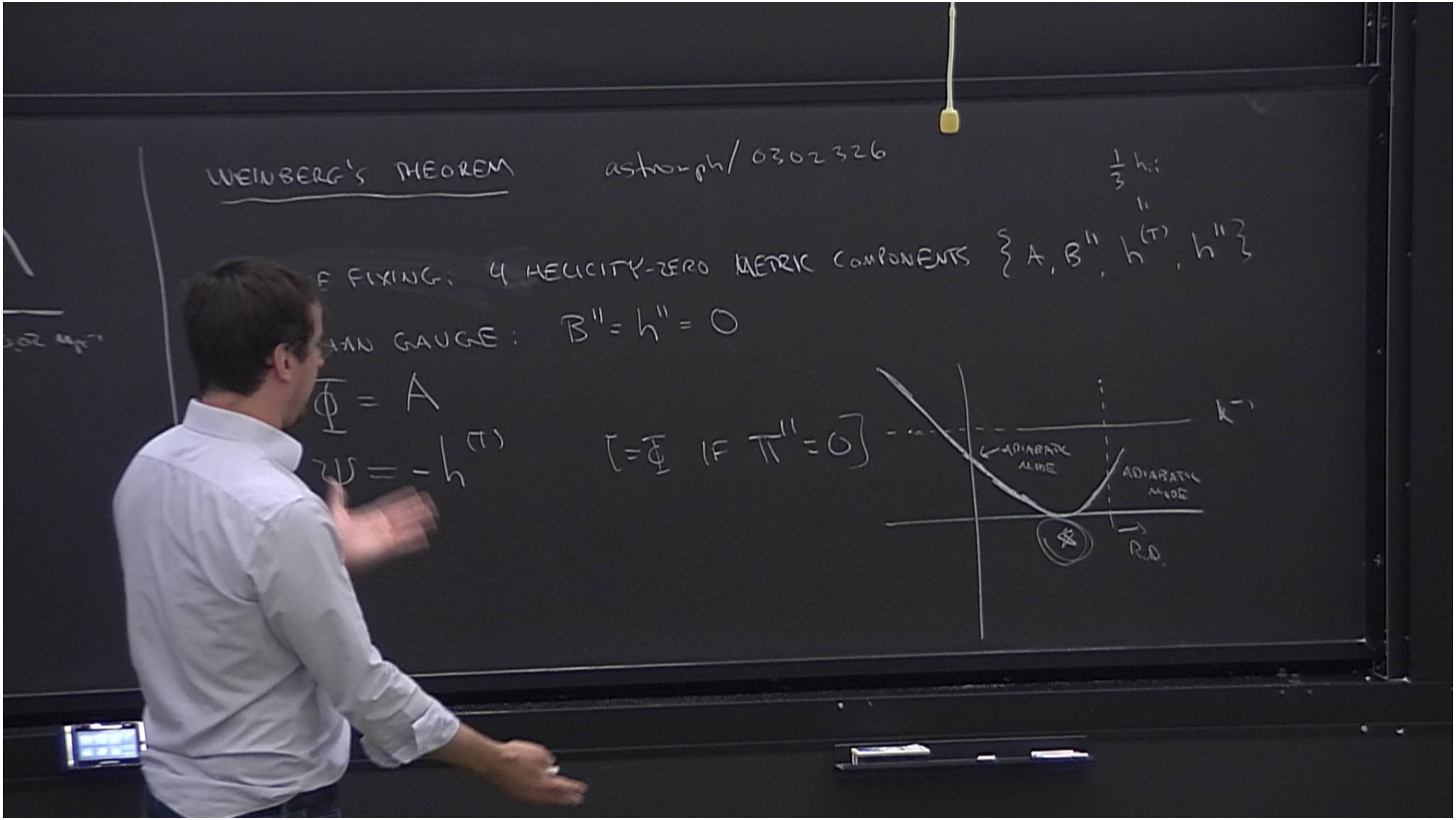

$\frac{\lambda}{a(t)} \int \frac{da}{H(a)} \left(\frac{a}{a}\right)$
$= \frac{2}{3+5_w} \frac{\lambda}{H(t)}$
$\bar{\rho}_J \sim a^{-3(1+w_J)}$
$\delta\rho_J = -\dot{\bar{\rho}}_J \alpha$
$= -H(-3(1+w_J))\bar{\rho}_J \left(\frac{1}{3}\frac{\zeta}{H}\right)$
$= (1+w_J)\bar{\rho}_J \zeta$
$\delta_J = (1+w_J)\zeta + \cdots$
$V_J = 0 + \cdots$
CAN ALSO CHECK SINGLE FIELD INFLATION
ζ ← CONSTANT IF $k \ll aH$)
CONSISTENT W/ WEINBERG'S THEOREM $\lambda = \zeta$

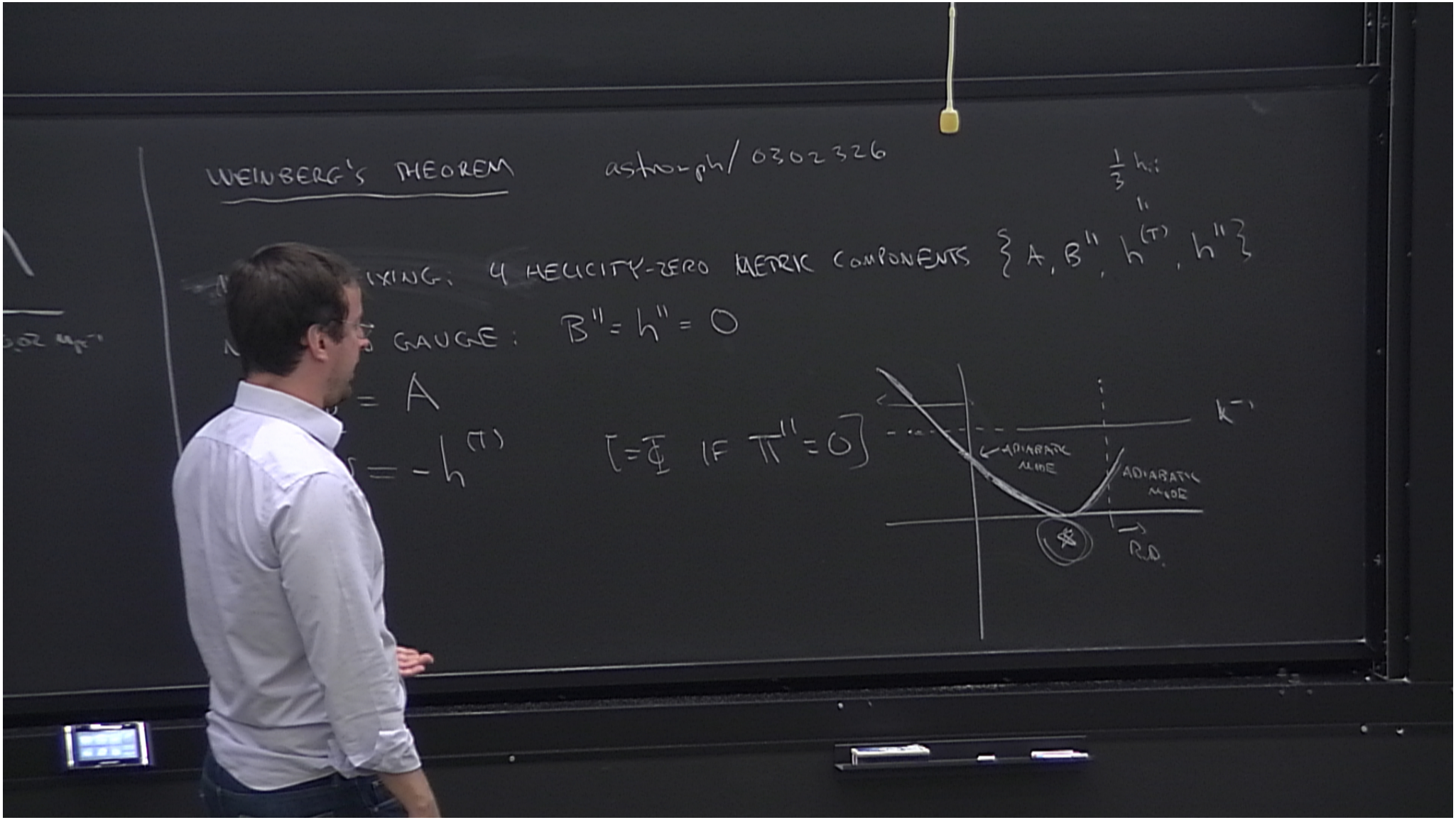

WEINBERG'S THEOREM
astro-ph/0302326
NO GAUGE FIX
HELICITY-ZERO METRIC COMPONENTS
NEWTONIAN
B'' = h'' = 0
[= Φ IF π'' = 0]
k

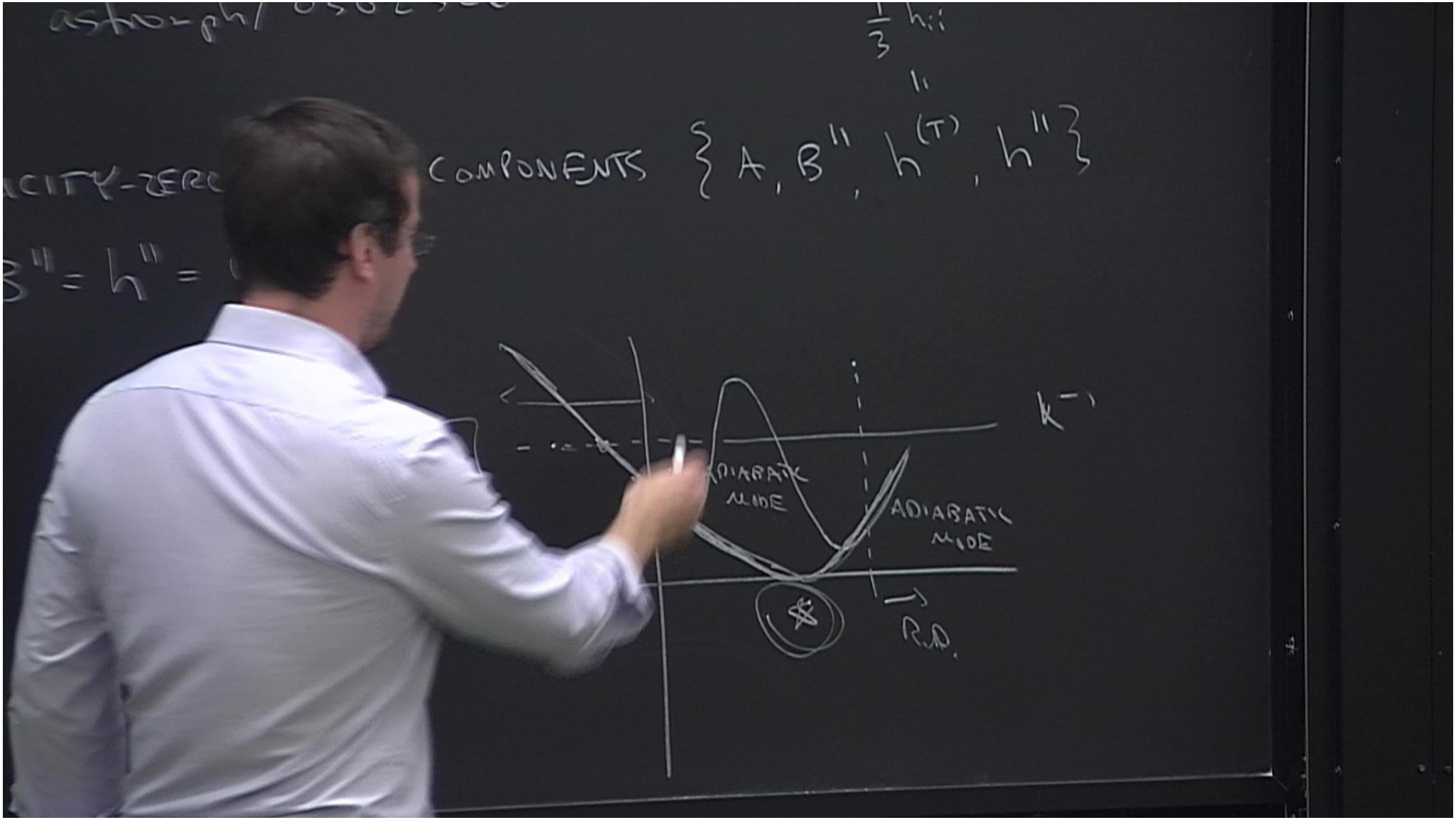

WEINBERG'S THEOREM
astro-ph/0302326
1/3 h::
1..
FIXING: 4 HELICITY-ZERO METRIC COMPONENTS {A, B'', h^(T), h''}
GAUGE: B'' = h'' = 0
Φ = A
Ψ = -h^(T)
[= Φ IF Π'' = 0]
k^-1
ADIABATIC


WEINBERG'S THEOREM
astro-ph/0302326
GAUGE FIXING: 4 HELICITY-ZERO METRIC COMPONENTS
NEWTONIAN GAUGE: B'' = h'' = 0
Φ = A
ψ = -h^(T)
[= Φ IF π'' = 0]
ADIABATIC MODE


WEINBERG'S THEOREM
astro-ph/0302326
NO GAUGE FIXING:
NEWTONIAN GAUGE:
$\Phi = A$
$\Psi = -h^{(T)}$
ADIABATIC MODE
RD


WEINBERG'S THEOREM
astro-ph/0302326
$\frac{1}{3}$ h::
1.
F FIXING: 4 HELICITY-ZERO METRIC COMPONENTS $\{A, B'', h^{(T)}, h''\}$
IN GAUGE: $B'' = h'' = 0$
$\Phi = A$
$\Psi = -h^{(T)}$ $[= \Phi$ IF $\Pi'' = 0]$
ADIABATIC MODE
ADIABATIC MODE
R.D.


WEINBERG'S THEOREM
astro-ph/0302326
$\frac{1}{3}h_{ii}$
$\{A, B'', h^{(T)}, h''\}$
GAUGE: $B'' = h'' = 0$
$= A$
$= -h^{(T)}$
$[= \Phi$ IF $\Pi'' = 0]$
ADIABATIC MODE
ADIABATIC MODE
R.D.
k^{-1}


COMPONENTS
ADIABATIC MODE
ADIABATIC MODE
R.D.

astro-ph/

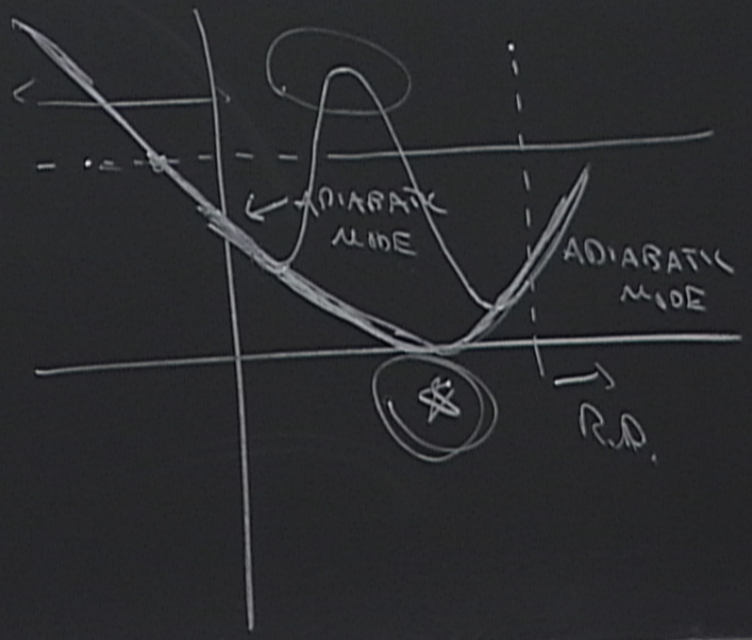
$\frac{1}{3} h_{ii}$

$\equiv$

CITY-ZERO METRIC COMPONENTS $\{A, B^{\perp}, h^{(T)}, h^{\perp}\}$

$B^{\perp} = h^{\perp} = 0$

$[= \underline{T} \quad \text{IF} \quad \Pi^{\perp} = 0]$



astro-ph/

$\frac{1}{3} h_{ii}$

=

CITY-ZERO METRIC COMPONENTS $\{A, B^{\perp}, h^{(T)}, h^{\perp}\}$

$B^{\perp} = h^{\perp} = 0$

$[= \Phi$ IF $\Pi^{\perp} = 0]$

