Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 13

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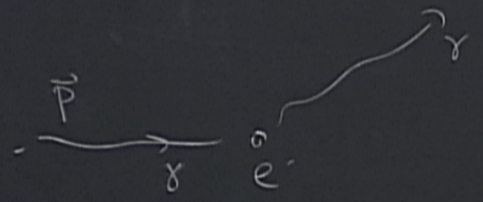
URL: http://pirsa.org/16040047

Abstract:

NORMAL COORDS IN e^- REST FRAME

$U_b^\mu = (1, 0)$ $\quad g_{\mu\nu} = (-1, \delta_{ij})$

$U_\gamma^\mu = (1, V_i^\gamma)$



$$\frac{d\sigma}{d\Omega} = \frac{3}{8\pi}\sigma_T\left(\frac{1+\cos^2\theta}{2}\right)$$

n_e = FREE ELECTRON DENSITY

ρ_γ = PHOTON ENERGY DENSITY

V_i^γ = PHOTON BULK VELOCITY

Π_i^γ = PHOTON MOMENTUM DENSITY $= \frac{4}{3}\rho_\gamma V_i^\gamma$

A SINGLE PHOTON w/ INCOMING MOMENTUM $\vec{p}_{\gamma i}$ TRANSFERS MOMENTUM TO BARYON FLUID AT THE RATE

$$\frac{d\vec{\pi}^b}{dt} = n_e \int d^2\hat{n}\, \frac{3}{8\pi}\, \sigma_T \left(\frac{1+\cos^2\theta}{2}\right)(1-\cos\theta)\, \vec{p}_{\gamma i}$$

$$= n_e \sigma_T \vec{p}_{\gamma i}$$

$\nabla_\mu T^{\mu i}_b$ = MOMENTUM / VOLUME / TIME

$$= n_e \sigma_T \pi^\gamma_i$$

$$= \frac{4}{3} \sigma_T n_e \rho_\gamma v^\gamma_i$$

$\nabla_\mu T^{\mu 0}_b$ = ENERGY / VOLUME / TIME

$= 0$

A SINGLE PHOTON w/ INCOMING MOMENTUM $\vec{p}_{in}$ TRANSFERS MOMENTUM TO BARYON FLUID AT THE RATE

$$\frac{d\vec{\pi}^b}{dt} = n_e \int d^2\hat{n}\, \frac{3}{8\pi}\, \sigma_T \left(\frac{1+\cos^2\theta}{2}\right)(1-\cos\theta)\, \vec{p}_{in}$$

$$= n_e \sigma_T \vec{p}_{in}$$

$\nabla_\mu T_b^{\mu i}$ = MOMENTUM / VOLUME / TIME

$= n_e \sigma_T \pi_i^\gamma$

$= \frac{4}{3} \sigma_T n_e \rho_\gamma v_i^\gamma$

$\nabla_\mu^{\mu 0}$ = ENERGY / VOLUME / TIME

$= 0$ SINCE $\beta \ll m_e$

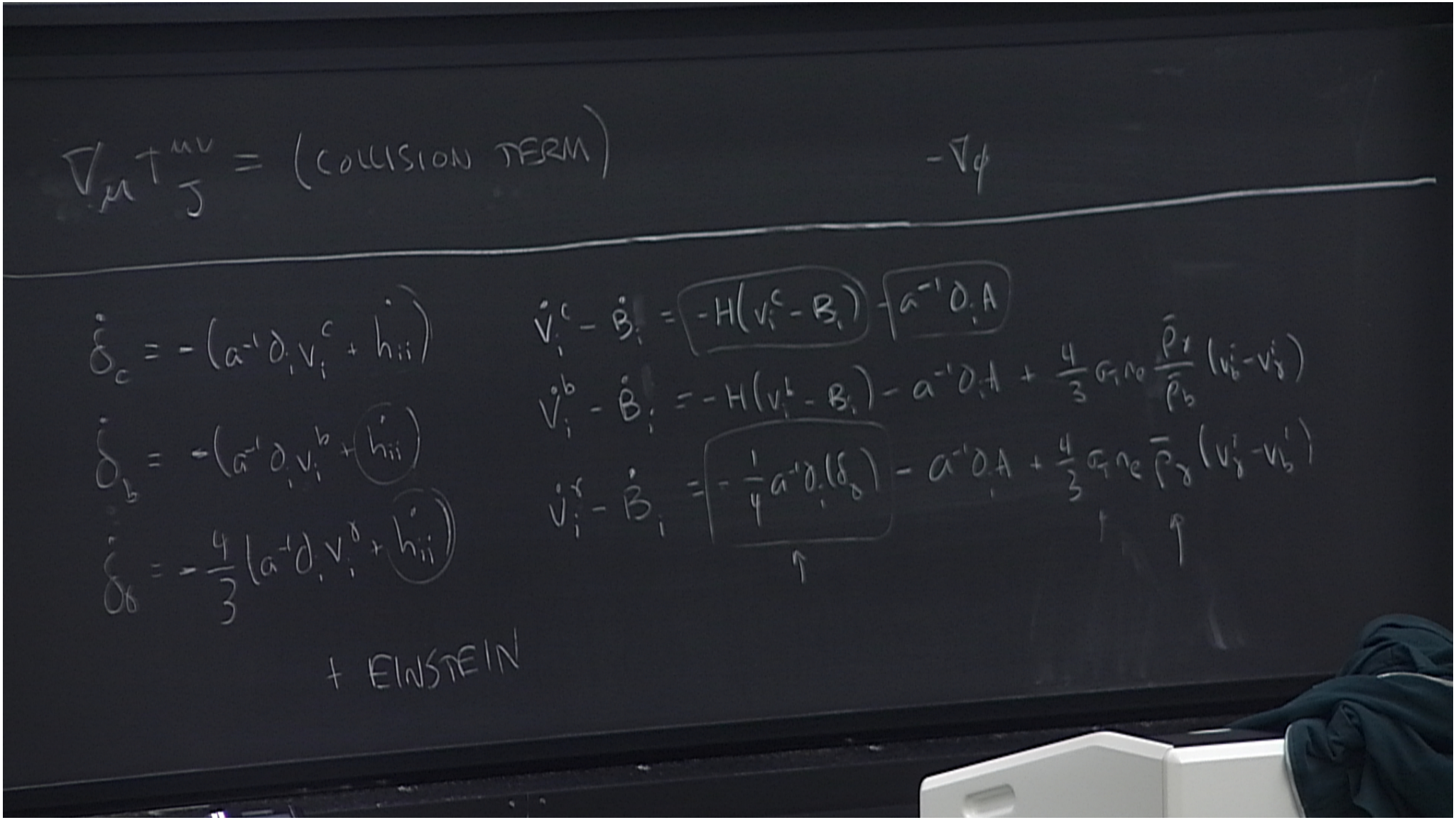
$$\nabla_\mu T^\mu_{\ b} = \left(0, \frac{4}{3}\sigma_T n_e \rho_\gamma v_\gamma^i\right)$$ [NORMAL COORDS, e^- REST FRAME]

$$= \frac{4}{3}\sigma_T n_e \rho_\gamma \left(U_\gamma^\nu - U_b^\nu\right)$$ [TRUE IN ALL FRAMES]

PERTURBED FRW FRAME: $U_\gamma^\nu = (1-A,\ a^{-1}v_\gamma^i)$

$$\nabla_\mu T^\mu_{\ b} = \left(0, \frac{4}{3}\sigma_T n_e \rho_\gamma a^{-1}(v_\gamma^i - v_b^i)\right)$$

$$\nabla_\mu T^\mu_{\ \gamma} = -\nabla_\mu T^\mu_{\ b}$$


$\nabla_\mu T^{\mu\nu}_J = (\text{COLLISION TERM})$
$-\nabla\phi$
$\dot{\delta}_c = -(a^{-1}\partial_i v_i^c + \dot{h}_{ii})$
$\dot{\delta}_b = -(a^{-1}\partial_i v_i^b + \dot{h}_{ii})$
$\dot{\delta}_\delta = -\frac{4}{3}(a^{-1}\partial_i v_i^\delta + \dot{h}_{ii})$
$\dot{v}_i^c - \dot{B}_i = -H(v_i^c - B_i) - a^{-1}\partial_i A$
$\dot{v}_i^b - \dot{B}_i = -H(v_i^b - B_i) - a^{-1}\partial_i A + \frac{4}{3}\sigma_T n_e \frac{\bar{\rho}_\gamma}{\bar{\rho}_b}(v_b^i - v_\gamma^i)$
$\dot{v}_i^\gamma - \dot{B}_i = -\frac{1}{4}a^{-1}\partial_i(\delta_\delta) - a^{-1}\partial_i A + \frac{4}{3}\sigma_T n_e \bar{\rho}_\gamma (v_\gamma^i - v_b^i)$
+ EINSTEIN

LET'S COMPARE

SCATTERING RATE = $\sigma_T n_e$

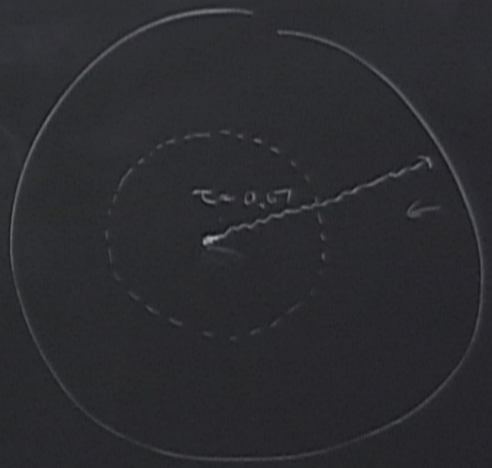
HUBBLE RATE = H

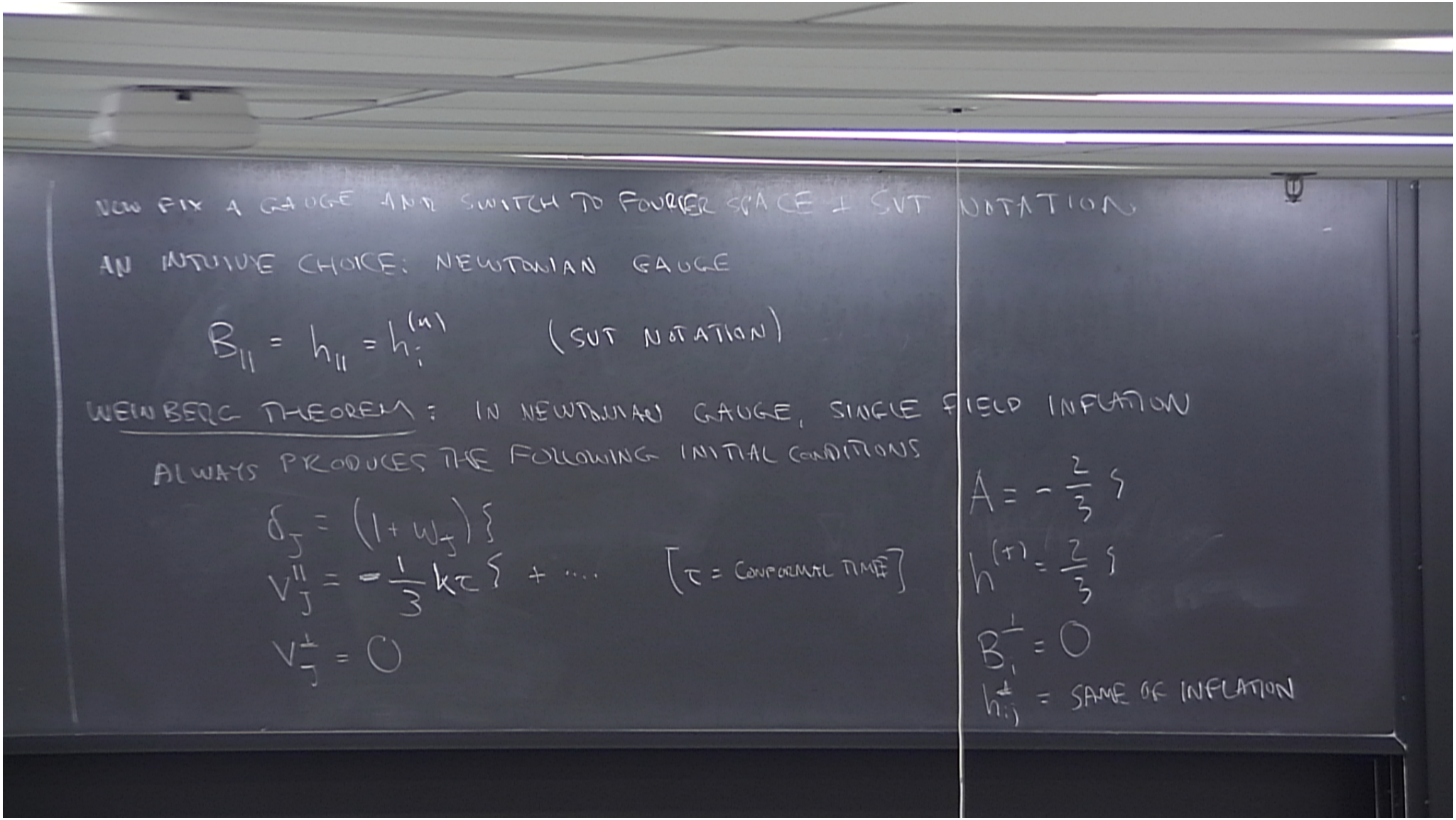
DURING MATTER DOMINATION

$\sigma_T n_e = (1.4 \times 10^{-4}\ \mathrm{Gyr}^{-1})\ a^{-3}$

$H \approx (1.4 \times 10^{-2}\ \mathrm{Gyr}^{-1})\ a^{-3/2}$

AT RECOMBINATION, $\sigma_T n_e \gg H$




NOW FIX A GAUGE AND SWITCH TO FOURIER SPACE + SVT NOTATION
AN INTUIVE CHOICE: NEWTONIAN GAUGE
$B_{11} = h_{11} = h_{:}^{(n)}$ (SVT NOTATION)
WEINBERG THEOREM: IN NEWTONIAN GAUGE, SINGLE FIELD INFLATION
ALWAYS PRODUCES THE FOLLOWING INITIAL CONDITIONS
$\delta_J = (1+w_J)\,\}$
$V_J^{\parallel} = -\frac{1}{3}k\tau\,\}$ + ... [τ = CONFORMAL TIME]
$V_J^{\perp} = 0$
$A = -\frac{2}{3}\,\}$
$h^{(T)} = \frac{2}{3}\,\}$
$B_i^{\perp} = 0$
$h_{ij}^{\perp}$ = SAME OF INFLATION

IN NEWTONIAN GAUGE, EINSTEIN'S EQS (SVT)

REMINDER $h^{(T)} = \frac{1}{3} h_{..}$

HELICITY ZERO

$$\begin{cases} -3a^2H^2A + 3a^2H\dot{h}^{(T)} + k^2 h^{(T)} = 4\pi G a^2 (\delta\rho) \\ HA - \dot{h}^{(T)} = 4\pi G a (\bar{\rho} + \bar{p}) \frac{v'_{\parallel}}{k} \\ -k^2(A + h^{(T)}) = 8\pi G a^2 \Pi^{\parallel} \end{cases}$$

"VECTOR MODE"

HELICITY 1

$$\frac{1}{2} k^2 B^i_{\perp} = 8\pi G a^2 (\bar{\rho} + \bar{p})(v^i_{\perp} - B^i_{\perp})$$

HELICITY 2

$$\left(a^2 \frac{d^2}{dt^2} + 3a^2 H \frac{d}{dt} + k^2\right) h^{\perp}_{ij} = 8\pi G a^2 \Pi^{\perp}_{ij} = 0$$

"TENSOR MODE"

$h^{(T)} = \frac{1}{3} h_{ii}$

HELICITY ZERO

LAST EINSTEIN EQ $\Rightarrow$ $A = -h^{(T)} = \Phi$

↑ NEWTONIAN GAUGE NOTATIONAL CONVENTION

FIRST + SECOND EINSTEIN EQS

$\dot{\Phi} = -\frac{4\pi G a^2}{k^2}\left(\delta\rho + \frac{3aH}{k}(\bar{\rho}+\bar{p})V''\right)$

$\dot{\Phi} = -H\bar{q} + 4\pi G a(\bar{\rho}+\bar{p})\frac{V'}{k}$

$\rightarrow$ ZERO FOR ALL TIMES

FLUID EQS IN NEWTONIAN GAUGE

$$\dot{\delta}_c = a^{-1} k v_c'' + 3\dot{\Phi}$$

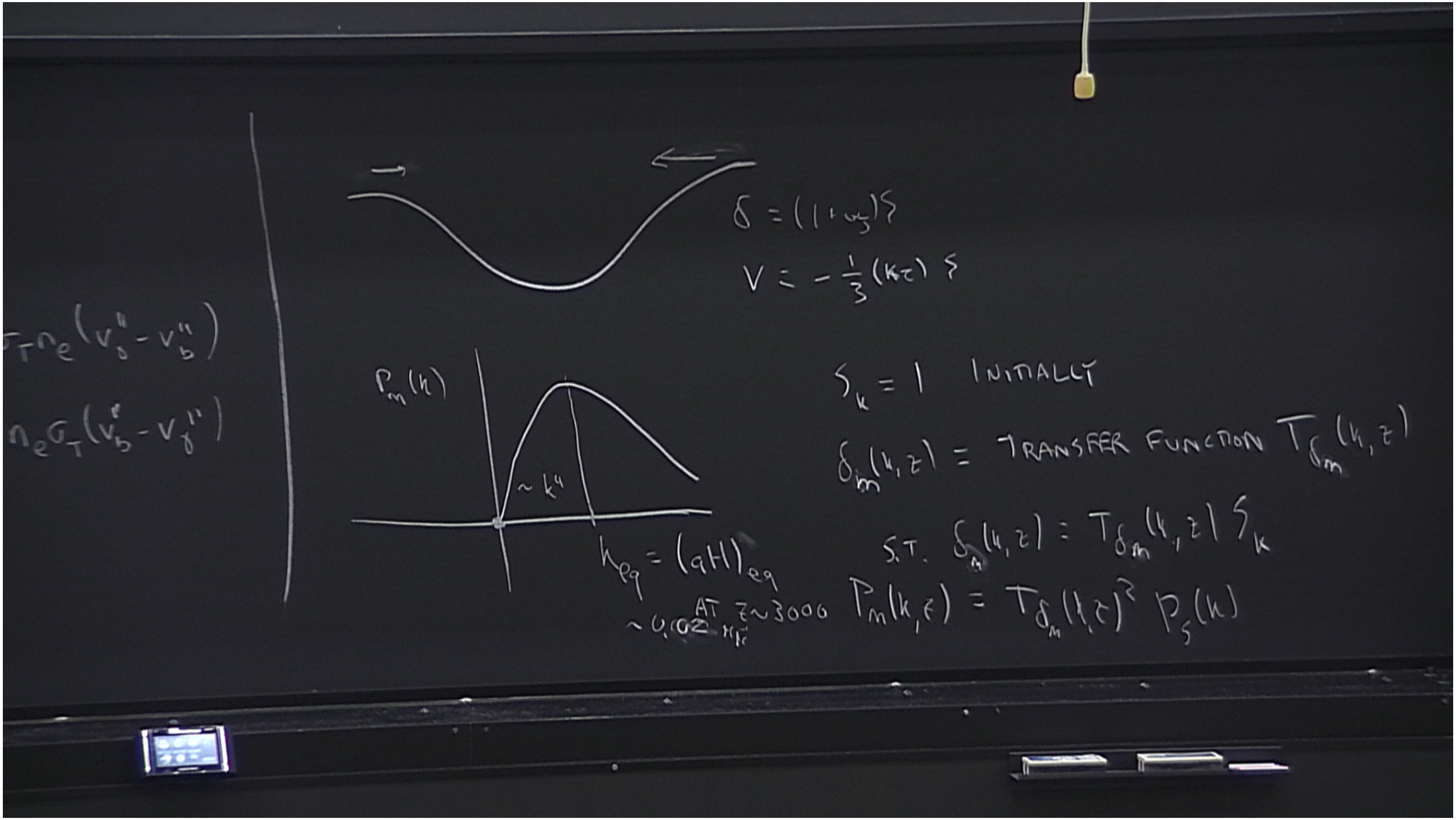
$$\dot{\delta}_b = a^{-1} k v_b'' + 3\dot{\Phi}$$

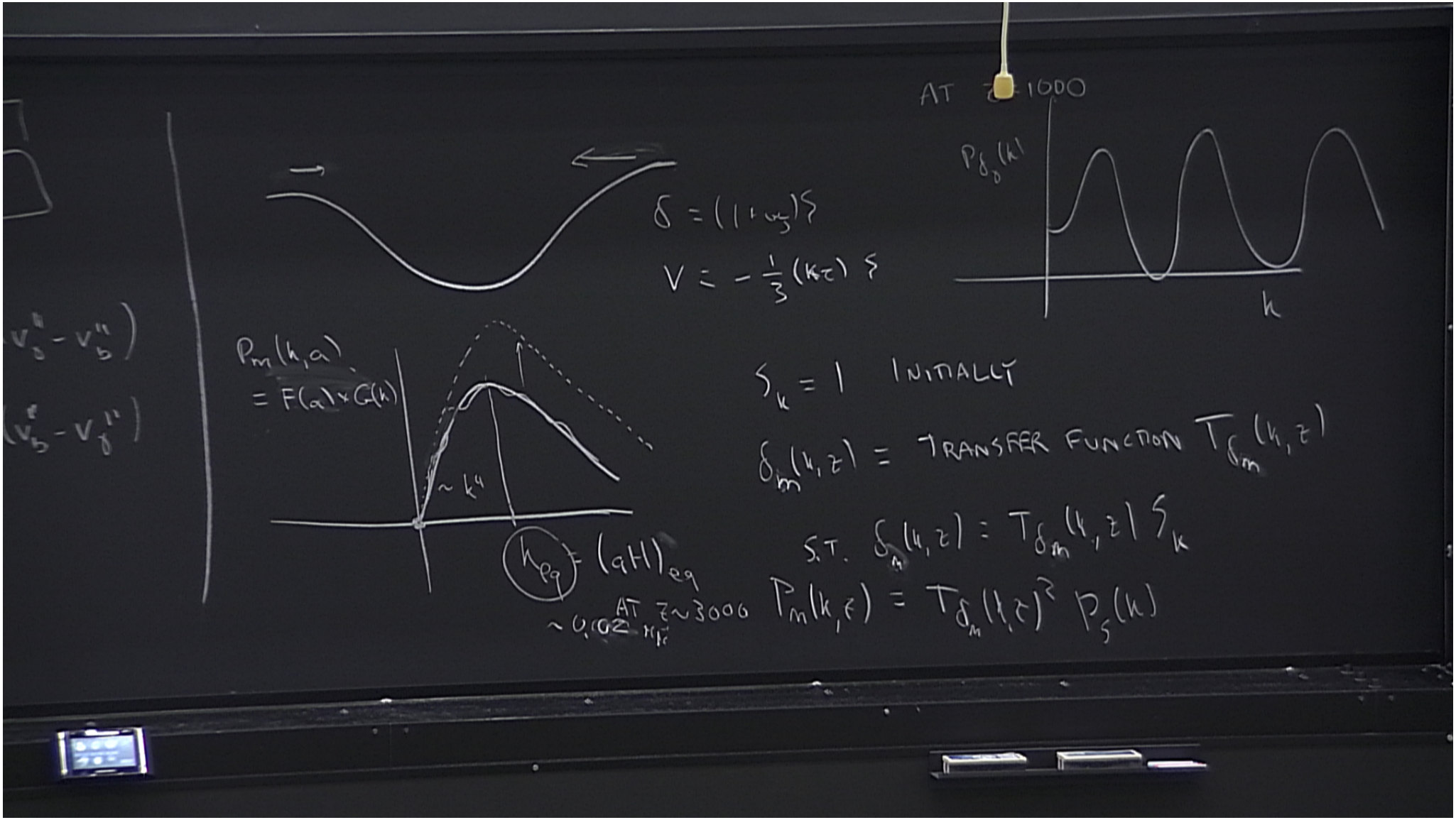
$$\dot{\delta}_\gamma = \frac{4}{3} a^{-1} k V_\gamma'' + 4\dot{\Phi}$$

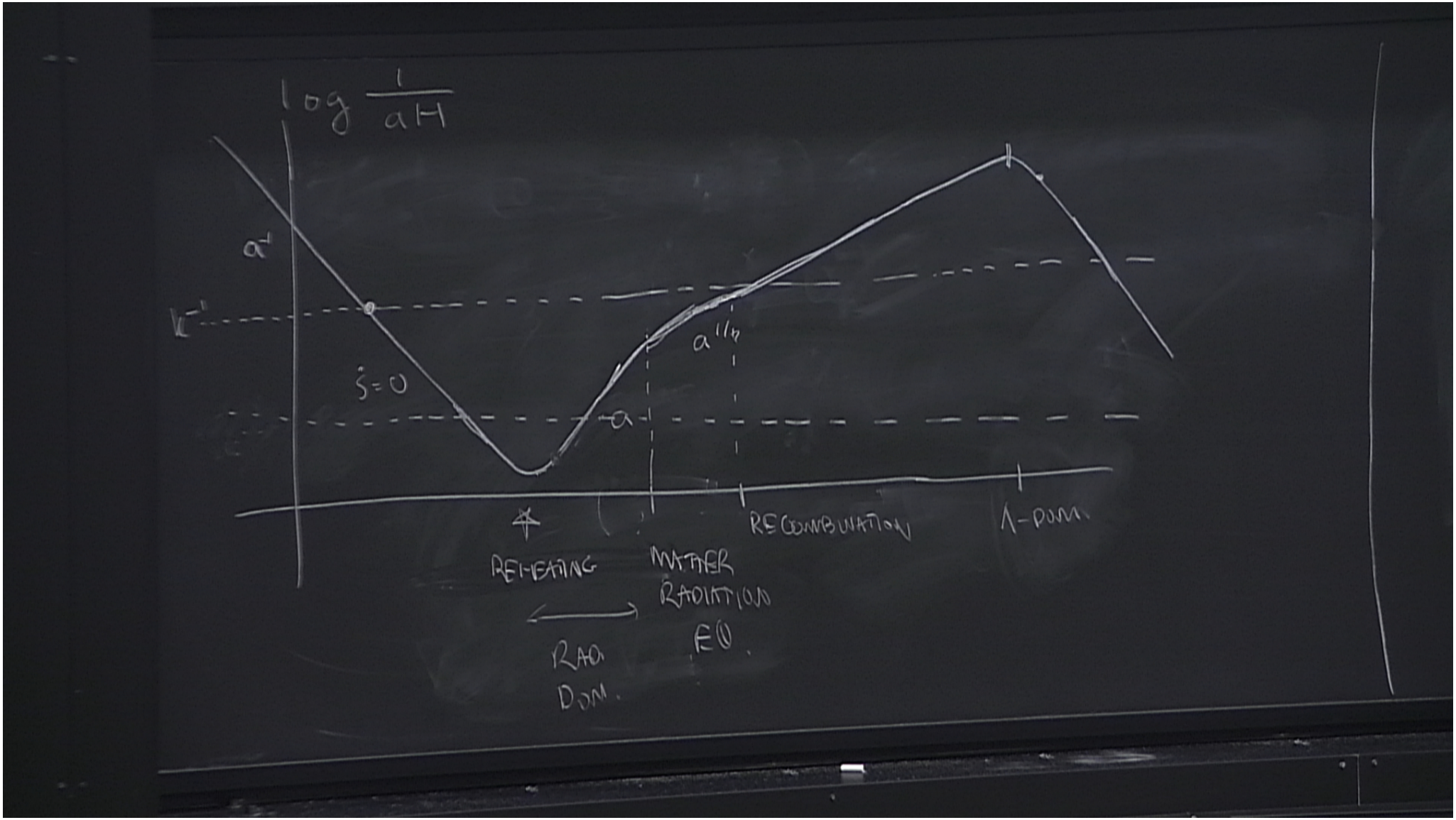
$$\dot{v}_c'' = -H v_c'' + a^{-1} k \Phi$$

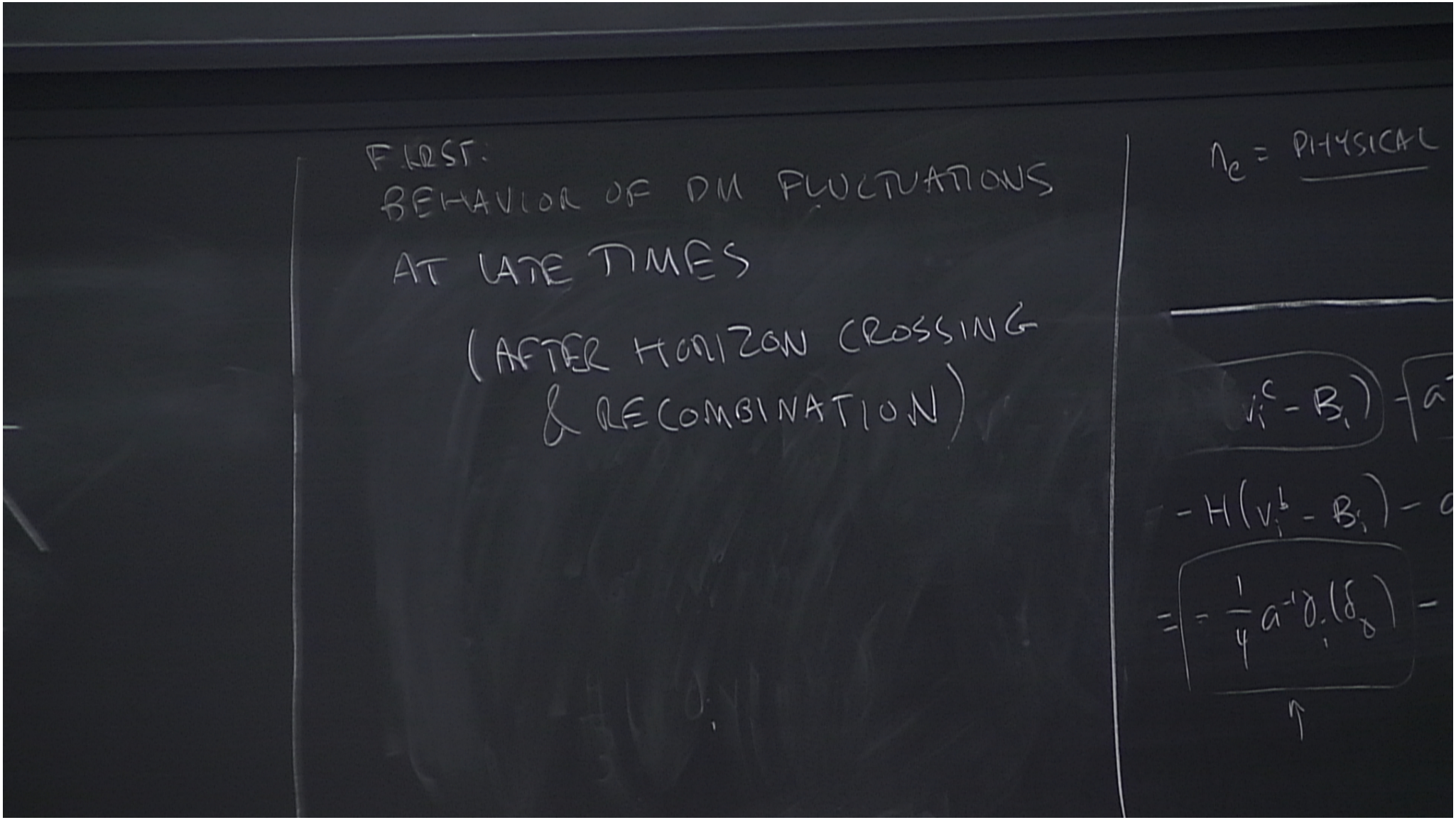
$$\dot{v}_b'' = -H v_b'' + a^{-1} k \Phi + \frac{4}{3} \frac{\bar{\rho}_\gamma}{\bar{\rho}_b} \sigma_T n_e (v_\gamma'' - v_b'')$$

$$\dot{V}_\gamma'' = \frac{1}{4} a^{-1} k \delta_\gamma + a^{-1} k \Phi + n_e \sigma_T (v_b'' - V_\gamma'')$$


$\delta = (1+w)\zeta$
$V = -\frac{1}{3}(kz)\zeta$
$P_m(k)$
$\sim k^4$
$k_{eq} = (aH)_{eq}$
~ 0.02
AT $z \sim 3000$
$\zeta_k = 1$ INITALLY
$\delta_m(k,z)$ = TRANSFER FUNCTION $T_{\delta_m}(k,z)$
S.T. $\delta_m(k,z) = T_{\delta_m}(k,z)\,\zeta_k$
$P_m(k,z) = T_{\delta_m}(k,z)^2\, P_\zeta(k)$


AT z = 1000
$P_{\delta_0}(k)$
k
$\delta = (1+w_3)\zeta$
$V = -\frac{1}{3}(kz)\zeta$
$P_m(k,a)$
$= F(a) \times G(k)$
$\sim k^4$
$k_{eq} = (aH)_{eq}$
~ 0.02 AT $z \sim 3000$
$\zeta_k = 1$ INITALLY
$\delta_m(k,z)$ = TRANSFER FUNCTION $T_{\delta_m}(k,z)$
S.T. $\delta_m(k,z) = T_{\delta_m}(k,z)\,\zeta_k$
$P_m(k,z) = T_{\delta_m}(k,z)^2\, P_\zeta(k)$


log 1/aH
a'
k⁻¹
ä=0
a
a''/a'
RECOMBINATION
Λ-DOM.
RE-HEATING
MATTER
RADIATION
EQ.
RAD
DOM.


FIRST:
BEHAVIOR OF DM FLUCTUATIONS
AT LATE TIMES
(AFTER HORIZON CROSSING
& RECOMBINATION)
Λ_c = PHYSICAL
$v_i^c - B_i$
$-H(v_i^b - B_i)$
$= -\frac{1}{4} a^{-1} \partial_i(\delta_\gamma)$