Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 12

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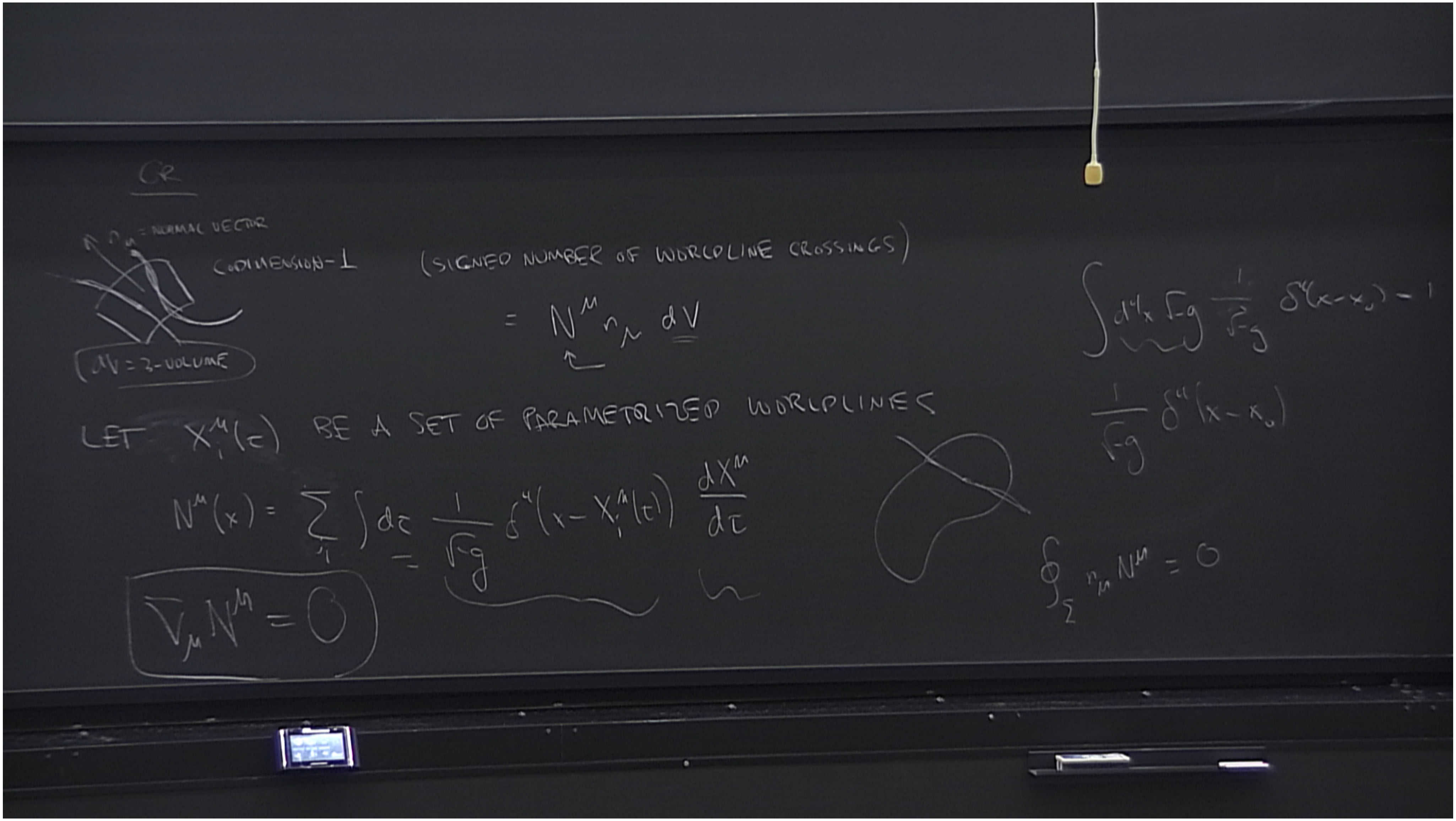
Abstract:

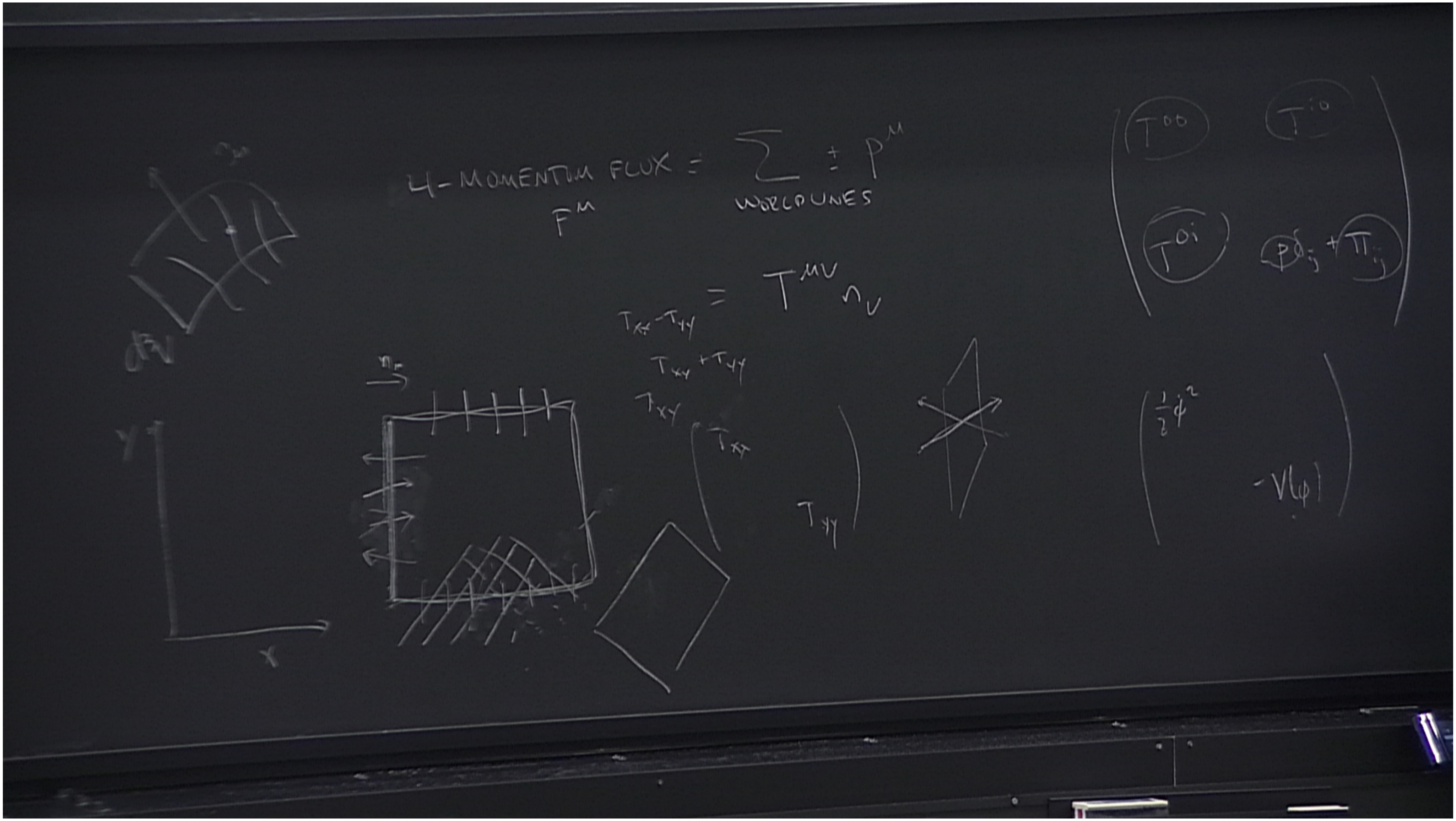

GR
n_μ = NORMAL VECTOR
CODIMENSION-1
dV = 3-VOLUME
(SIGNED NUMBER OF WORLDLINE CROSSINGS)
$= N^\mu n_\mu \, dV$
$\int d^4x \sqrt{-g} \frac{1}{\sqrt{-g}} \delta^4(x-x_0) = 1$
$\frac{1}{\sqrt{-g}} \delta^4(x-x_0)$
LET $X_i^\mu(\tau)$ BE A SET OF PARAMETRIZED WORLDLINES
$N^\mu(x) = \sum_i \int d\tau \frac{1}{\sqrt{-g}} \delta^4(x - X_i^\mu(\tau)) \frac{dX^\mu}{d\tau}$
$\oint_\Sigma n_\mu N^\mu = 0$
$\boxed{\nabla_\mu N^\mu = 0}$

SR

ENERGY E $\longleftrightarrow$ 4-MOMENTUM $p^{\mu} = (E, \vec{p})$

NUMBER DENSITY n $\longleftrightarrow$ NUMBER CURRENT $N^{\mu} = (n, n\vec{v})$

ENERGY DENSITY ρ $\longleftrightarrow$ STRESS ENERGY $T^{\mu\nu}$


4-MOMENTUM FLUX = ∑ ± p^μ
WORLDLINES
F^μ
= T^μν n_ν
T_xx - T_yy
T_xy + T_yy
T_xy
T_xx
T_yy
T^00
T^i0
T^0i
pδ_ij + π_ij
1/2 φ̇^2
- V(φ)

COVARIANT FORMULA:

$X_i^\mu(\tau)$ = N PARAMETRIZED WORLDLINES $i = 1 \cdots N$

PROPER TIME $\frac{dx^\mu}{dt} = p^\mu$

$$\nabla_\mu T_J^{\mu\nu} = 0$$

$$\Rightarrow T^{\mu\nu} = \sum_i \int d\tau \, \frac{1}{\sqrt{g}} \, \delta^4(x - X_i^\mu(\tau)) \, \frac{dX_i^\mu}{d\tau} \frac{dX_i^\nu}{d\tau}$$

$\nabla_\mu T^{\mu\nu} = 0$ IF TRAJECTORIES ARE GEODESICS

$$-\frac{2}{\sqrt{-g}} \frac{\delta S^n}{\delta g^{\mu\nu}} = T_{\mu\nu}$$

COVARIANT FORMULA:

$X^{\mu}_{i}(\tau) = \overset{N}{\text{PARAMETRIZED}}$ WORLDLINES $\quad i = 1 \cdots N$

PROPER TIME $\quad \dfrac{dx^{\mu}}{d\tau} = p^{\mu}$

$$\nabla_{\mu} T^{\mu\nu}_{J} = 0 \qquad \Rightarrow \quad T^{\mu\nu} = \sum_{i} \int d\tau \, \frac{1}{\sqrt{-g}} \, \delta^{4}(x - X^{\mu}_{i}(\tau)) \, \frac{dX^{\mu}_{i}}{d\tau} \frac{dX^{\nu}_{i}}{d\tau}$$

$$-\frac{2}{\sqrt{-g}} \frac{\delta S^{n}}{\delta g^{\mu\nu}} = T_{\mu\nu}$$

$\nabla_{\mu} T^{\mu\nu} = 0$ IF TRAJECTORIES ARE GEODESICS

$$= \sum_{i} \int d\tau \, \frac{1}{\sqrt{-g}} \, \delta^{4}(x - X^{\mu}_{i}(\tau)) \, \underbrace{\frac{D^{2} X^{\nu}_{i}}{D\tau^{2}}}_{= 0 \text{ FOR GEODESIC}}$$

NEXT FEW LECTURES: MODEL UNIVERSE AS SUM OF 3 PERFECT FLUIDS (BARYONS, CDM, PHOTONS) IN FIRST ORDER PERTURBATION THEORY

- AN APPROXIMATION WHICH NEGLECTS HIGHER MULTIPOLES + ANISOTROPIC STRESS
- NEUTRINOS OMITTED

- EACH SPECIES HAS A BACKGROUND DENSITY $\bar{\rho}_J$ AND PRESSURE $\bar{p}_J$ $\left(J \in \{b, c, \gamma\}\right)$

  \+ FLUCTUATIONS $\{\delta\rho^J, v_i^J, \delta p^J\}$

- FOR $J \in \{b, c\}$ WE HAVE $\delta p^J = 0$ $\left[\text{ORDER } \frac{v^2}{c^2}\right]$

- FOR PHOTONS, $g_{\mu\nu} T_\gamma^{\mu\nu} = 0$

  ↑ MASSLESS

FOR PERFECT FLUID $T_\gamma^{\mu\nu} = (\rho + p) U^\mu U^\nu + p g^{\mu\nu}$

WHERE $U^\mu U_\mu = -1$ AND $\rho = \bar{\rho} + \delta\rho$, $p = \bar{p} + \delta p$

$$0 = g_{\mu\nu} T_\gamma^{\mu\nu} = -(\rho + p) + 4p = -\rho + 3p$$

$$= \underbrace{(-\bar{\rho} + 3\bar{p})}_{\to 0} + (-\delta\rho + 3\delta p) \Rightarrow \boxed{\delta p_J = \frac{1}{3}\delta\rho_J}$$

- EACH SPECIES HAS A BACKGROUND DENSITY $\bar{\rho}_J$ AND PRESSURE $\bar{p}_J$ $\left(J \in \{b, c, \gamma\}\right)$

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- FOR PHOTONS, $g_{\mu\nu} T^{\mu\nu}_{\gamma} = 0$

  ↑
  MASSLESS

FOR PERFECT FLUID $T^{\mu\nu}_{\gamma} = (\rho + p) U^\mu U^\nu + p g^{\mu\nu}$ WHERE $U^\mu U_\mu = -1$ AND $\rho = \bar{\rho} + \delta\rho$, $p = \bar{p} + \delta p$

$$0 = g_{\mu\nu} T^{\mu\nu}_{\gamma} = -(\rho + p) + 4p = -\rho + 3p$$

$$= \underbrace{(-\bar{\rho} + 3\bar{p})}_{0} + (-\delta\rho + 3\delta p) \Rightarrow \boxed{\delta p_J = \frac{1}{3}\delta\rho_J}$$

$$\bar{p}_J = \frac{1}{3}\bar{\rho}_J$$

NOT GENERALLY TRUE THAT $\bar{p}_J = w_J \bar{\rho}_J \Rightarrow \delta p_J = w_J \delta \rho_J$

[COUNTEREXAMPLE: SINGLE FIELD INFLATION] $\frac{\bar{p}}{\bar{\rho}} \neq \frac{\delta p}{\delta \rho}$

TOTAL STRESS ENERGY APPEARING IN EINSTEIN EQS

$\bar{\rho} = \sum_J \bar{\rho}_J \qquad \bar{p} = \sum_J \bar{p}_J$

$\delta\rho = \sum_J \delta\rho_J \qquad \delta p = \sum_J \delta p_J$

$$\boxed{(\bar{\rho} + \bar{p}) v = \sum_J (\bar{\rho}_J + \bar{p}_J) v_J}$$

CONSISTENT WITH

$T_{\mu\nu} = \sum_J T_J^{\mu\nu}$

AND $T_{0i} = (\bar{\rho} + \bar{p}) v_i$

FINALLY WE DEFINE

$$\delta_J = \frac{\delta\rho_J}{\bar{\rho}_J} \quad \text{"FRACTIONAL OVERDENSITY"}$$

DYNAMICAL QUANTITIES: $\{\delta\rho_J,\ v^i_J,\ \text{METRIC}\}$

EQS MOTION FOR THE FLUIDS $\qquad \nabla_\mu T^{\mu\nu}_J = (\text{FORCE})$ SCHEMATICALLY

START WITH LHS:

$$\nabla_\mu T^{\mu 0}_J = \frac{\partial}{\partial t}(\delta\rho_J) + 3H\Big(\delta\rho_J + \underbrace{\delta p_J}_{= \, w_J\delta\rho_J}\Big) + (\bar\rho_J + \bar p_J)\left(a^{-1}\partial_i v^J_i + \dot h_{ii}\right)$$

$$= \frac{\partial}{\partial t}(\bar\rho_J\delta_J) + 3H(1+w_J)\bar\rho_J\delta_J + (1+w_J)\bar\rho_J\left(a^{-1}\partial_i v^J_i + \dot h_{ii}\right)$$

$$=$$

MATICALLY

$$\bar{\rho}_J \propto a^{-3(1+w_J)}$$

$$v_i^J + \dot{h}_{ii})$$

$$\Rightarrow \frac{\partial}{\partial t}\bar{\rho}_J = -3(1+w_J) H \bar{\rho}_J$$

$$(a^{-1}\partial_i v_i^J + \dot{h}_{ii})$$

$$\boxed{\nabla_\mu N^\mu = 0}$$

EQS MOTION FOR THE FLUIDS

$$\boxed{\nabla_\mu T^{\mu\nu}_J = (\text{FORCE}) \quad \text{SCHEMATICALLY}}$$

START WITH LHS:

$$\nabla_\mu T^{\mu 0}_J = \frac{\partial}{\partial t}(\delta\rho_J) + 3H\Big(\delta\rho_J + \overbrace{\delta p_J}^{w_J\delta\rho_J \,=}\Big) + (\bar\rho_J + \bar p_J)(a^{-1}\partial_i v_i^J + \dot h_{ii})$$

$$= \frac{\partial}{\partial t}(\bar\rho_J\delta_J) + 3H(1+w_J)\bar\rho_J\delta_J + (1+w_J)\bar\rho_J(a^{-1}\partial_i v_i^J + \dot h_{ii})$$

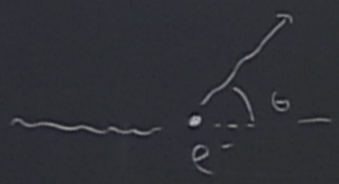
$$= \cancel{-3(1+w_J)H\bar\rho_J\delta_J} + \bar\rho_J\dot\delta_J + \cancel{3H(1+w_J)\bar\rho_J\delta_J} + (1+w_J)\bar\rho_J(a^{-1}\partial_i v_i^J + \dot h_{ii})$$

$$= \bar\rho_J\Big[\dot\delta_J + (1+w_J)(a^{-1}\partial_i v_i^J + \dot h_{ii})\Big]$$

$$= \bar{\rho}_J \left(\delta_J + (1+w_J)(a^{-1}\partial_i v^i_J + h_{ii}) \right)$$

SIMILARLY $\nabla^\mu T^J_{\mu i} = a(1+w_J)\bar{\rho}_J\left[\left(\frac{\partial}{\partial t} + H(1-3w_J)\right)(v_i^J - B_i) + a^{-1}\partial_i A\right] + w_J\, \partial_i(\delta_J)$

BARYONS & PHOTONS INTERACT BY THOMSON SCATTERING



IN REST FRAME OF BARYONS WE HAVE

$$\frac{d\sigma}{d\Omega} = \frac{3}{8\pi}\sigma_T\left(\frac{1+\cos^2\theta}{2}\right)$$

CAN WE TRANSLATE THIS TO EXPRESSIONS FOR $\nabla_\mu T^{\mu\nu}_b$, $\nabla_\mu T^{\mu\nu}_c$?

$w_j \, \partial_i(\delta_j)$

ANALYZE IN NORMAL COORDINATES, IN REST FRAME OF e^-

$U_b^\mu = (1, 0, 0, 0)$ $\quad g_{\mu\nu} = (-1, 1, 1, 1)$

$U_\gamma^\mu = (1, v_i^\gamma)$

n_e = FREE ELECTRON DENSITY

ρ_γ = PHOTON ENERGY DENSITY

WHY $\frac{4}{3}$?

V_γ^i = PHOTON BULK VELOCITY

Π_γ^i = PHOTON MOMENTUM DENSITY $= \frac{4}{3}\rho_\gamma V_i^\gamma = (\rho_\gamma + P_\gamma) V_i^\gamma$

$P_\gamma = \frac{1}{3}\rho_\gamma$

$= T^{0i}$

$$T^{\mu\nu} = (\rho + p) U^\mu U^\nu + p\, g^{\mu\nu}$$

$$= \begin{pmatrix} \rho & (\rho+p)v \\ (\rho+p)v & p\delta_{ij} \end{pmatrix}$$