Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 10

Date: Apr 22, 2016 10:15 AM

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Abstract:

$$g_{ij} = a^2(1+2h_{ij}) \qquad g^{ij} = a^{-2}(1-2h^{ij}+\dots)$$

$$h_{kk} = 0$$

$$\Gamma_{\mu\nu}^{\rho} = \frac{1}{2} g^{\rho\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right)$$

$$\Gamma_{ij}^{0} \supset \frac{1}{2} g^{00}\left(-\partial_0 g_{ij}\right)$$

$$\supset a^2 \dot{h}_{ij}$$

$$\Gamma_{0i}^{j} \supset \frac{1}{2} g^{jk}\left(\partial_0 g_{ik}\right)$$

$$\supset \frac{1}{2} a^{-2}\delta^{jk}\left(2a^2 \dot{h}_{ik}\right)$$

$$\supset \dot{h}_{ij}$$

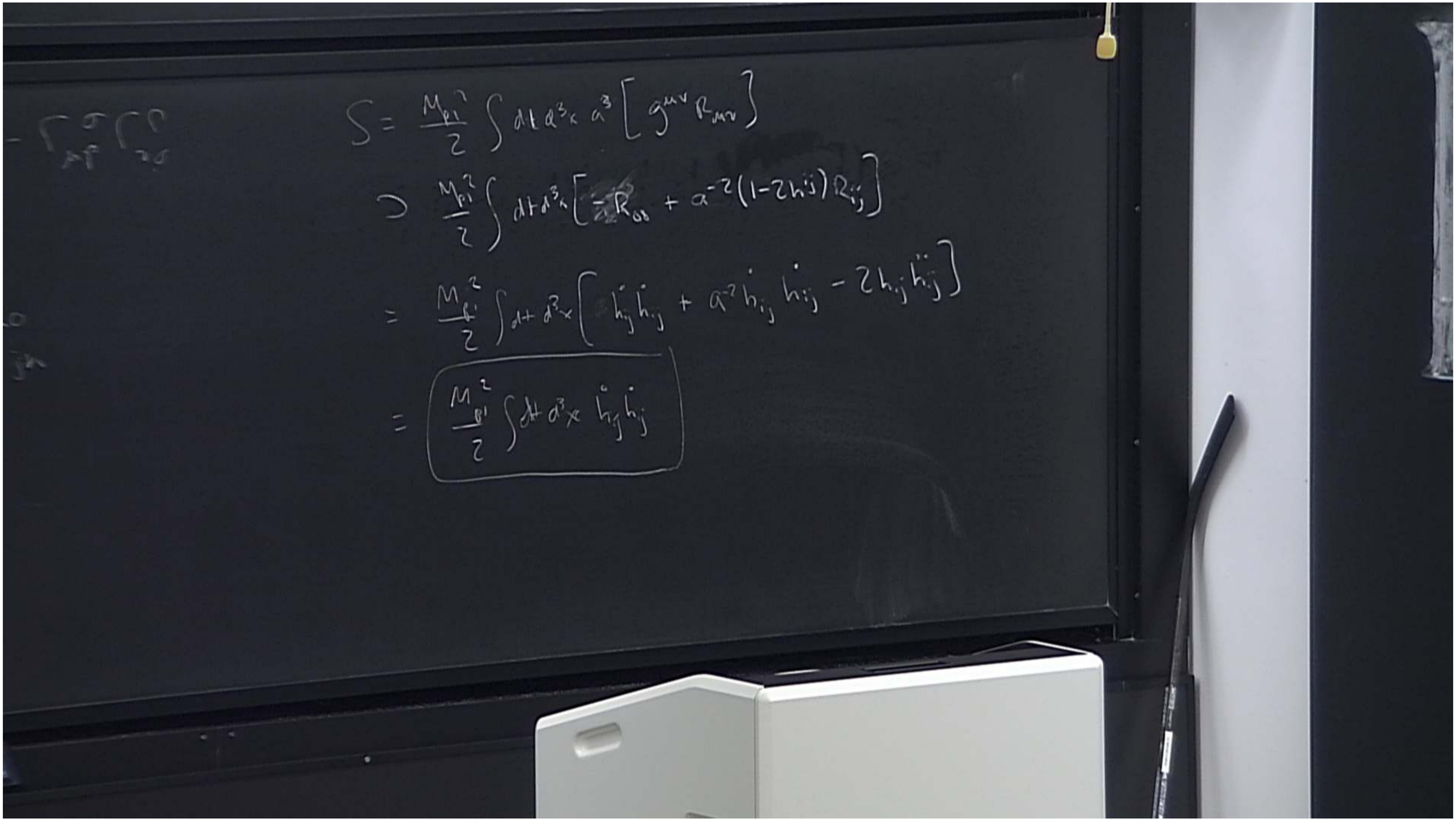
$$R_{\mu\nu} = \partial_\rho \Gamma_{\mu\nu}^{\rho} - \partial_\mu \Gamma_{\nu\rho}^{\rho} + \Gamma_{\mu\nu}^{\rho}\Gamma_{\rho\sigma}^{\sigma} - \Gamma_{\mu\rho}^{\sigma}\Gamma_{\nu\sigma}^{\rho}$$

$$R_{00} \supset -\Gamma_{0i}^{j}\Gamma_{0j}^{i} - \cancel{\partial \Gamma_{0i}^{i}}$$

$$= -\dot{h}_{ij}\dot{h}_{ij}$$

$$R_{ij} \supset \partial_0 \Gamma_{ij}^{0} + \cancel{\Gamma_{ij}^{0}\Gamma_{0k}^{k}} - 2\Gamma_{i0}^{k}\Gamma_{jk}^{0}$$

$$= a^2 \ddot{h}_{ij} - 2a^2 \dot{h}_{ik}\dot{h}_{jk}$$



$S = \frac{M_{pl}^2}{2}\int dt\, d^3x\, a^3 \left[g^{\mu\nu} R_{\mu\nu} \right]$
$\supset \frac{M_{pl}^2}{2}\int dt\, d^3x \left[-R_{00} + a^{-2}(1-2h^{ij}) R_{ij} \right]$
$= \frac{M_{pl}^2}{2}\int dt\, d^3x \left[\dot{h}_{ij}\dot{h}_{ij} + a^{-2}\dot{h}_{ij}\dot{h}_{ij} - 2h_{ij}\ddot{h}_{ij} \right]$
$= \boxed{\frac{M_{pl}^2}{2}\int dt\, d^3x\, \dot{h}_{ij}\dot{h}_{ij}}$

$$\Gamma^{\sigma}_{\mu\rho}\,\Gamma^{\rho}_{\nu\sigma}$$

$$S = \frac{M_{pl}^2}{2}\int dt\, d^3x\, a^3 \left[g^{\mu\nu} R_{\mu\nu} \right]$$

$$\supset \frac{M_{pl}^2}{2}\int dt\, d^3x \left[-R_{00} + a^{-2}\left(1 - 2h^{ij}\right) R_{ij} \right]$$

$$= \frac{M_{pl}^2}{2}\int dt\, d^3x \left[\dot{h}_{ij}\dot{h}_{ij} - 2\dot{h}_{ij}\dot{h}_{ij} - 2h_{ij}\ddot{h}_{ij} \right]$$

$$= \boxed{\frac{M_{pl}^2}{2}\int dt\, d^3x\, \dot{h}_{ij}\dot{h}_{ij}}$$

TEST SCALAR IN FRW

$$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3} z^2 \left(\phi'^2 - k^2\phi^2 \right)$$

$$\Rightarrow \boxed{\frac{\partial}{\partial\tau}\left(z^2 \frac{\partial\phi}{\partial\tau}\right) + k^2\phi = 0}$$

$$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3} \left(\psi'^2 - \left(k^2 - \frac{z''}{z}\right)\psi^2 \right]$$

$$\Rightarrow \boxed{\psi'' + \left(k^2 - \frac{z''}{z}\right)\psi = 0}$$

IF

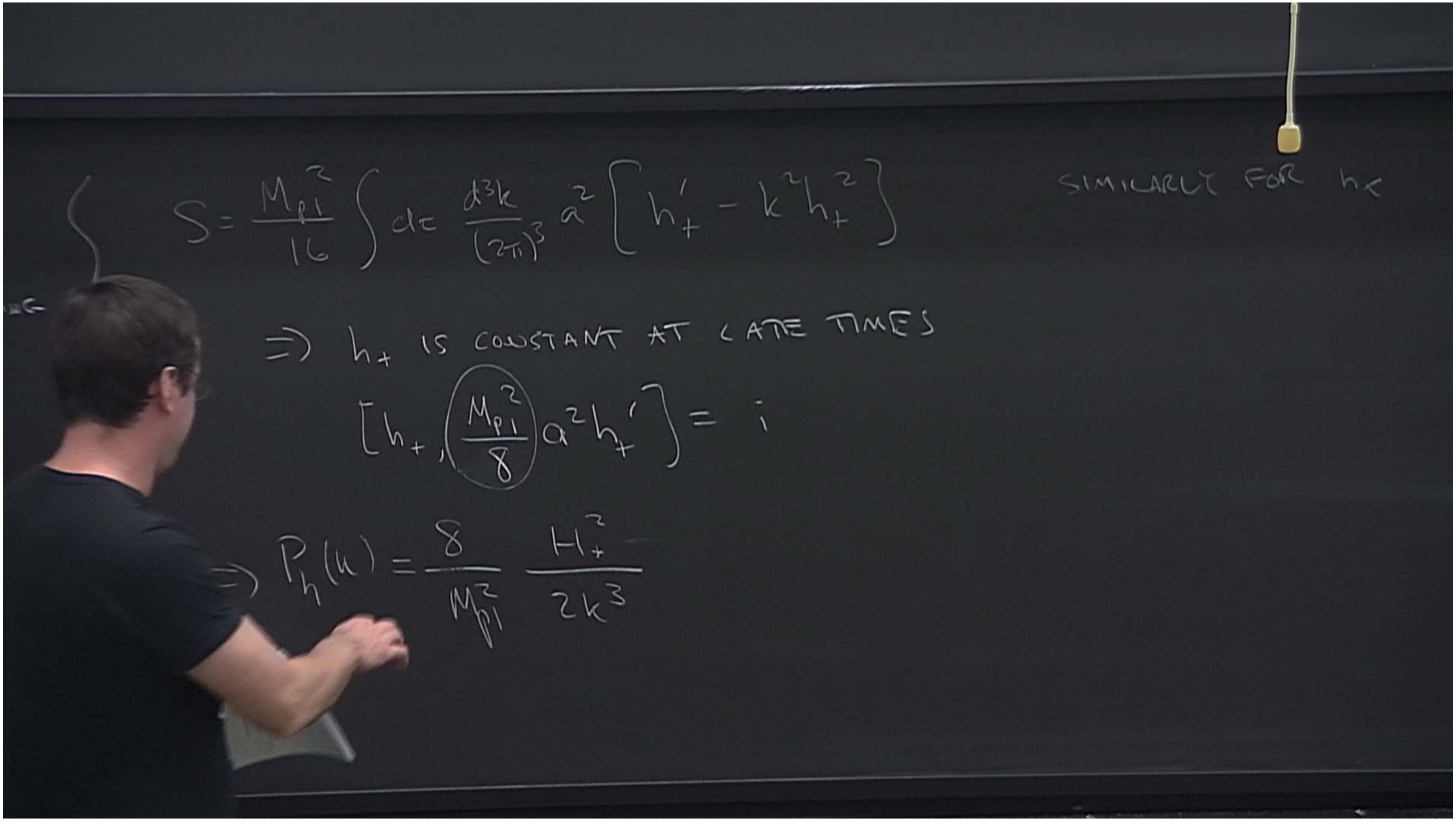
1) SLOW ROLL

2) NORMALIZATION $[\psi, \psi'] = [\phi, z^2\phi'] = i$

THEN $P_\phi(k)^2 = \frac{(H^2)_*}{2k^3}$ AT LATE TIMES

$\dot{\phi} = 0$ AFTER HORIZON CROSSING

$\psi = z\phi$


$S = \frac{M_{pl}^2}{16} \int d\tau \frac{d^3k}{(2\pi)^3} a^2 \left[h_+'^2 - k^2 h_+^2 \right]$
SIMILARLY FOR $h_\times$
$\Rightarrow h_+$ IS CONSTANT AT LATE TIMES
$\left[h_+ , \frac{M_{pl}^2}{8} a^2 h_+' \right] = i$
$\Rightarrow P_h(k) = \frac{8}{M_{pl}^2} \frac{H_+^2}{2k^3}$

$$S = \frac{M_{pl}^2}{16}\int d\tau \, \frac{d^3k}{(2\pi)^3}\, a^2 \left[h_+'^2 - k^2 h_+^2 \right]$$

SIMILARLY FOR $h_\times$

$\Rightarrow$ h_+ IS CONSTANT AT LATE TIMES

$$\left[h_+ , \frac{M_{pl}^2}{8} a^2 h_+' \right] = i$$

$$\Rightarrow P_h(k) = \frac{8}{M_{pl}^2} \, \frac{H_*^2}{2k^3}$$

INFLATON IN SPATIALLY FLAT GAUGE

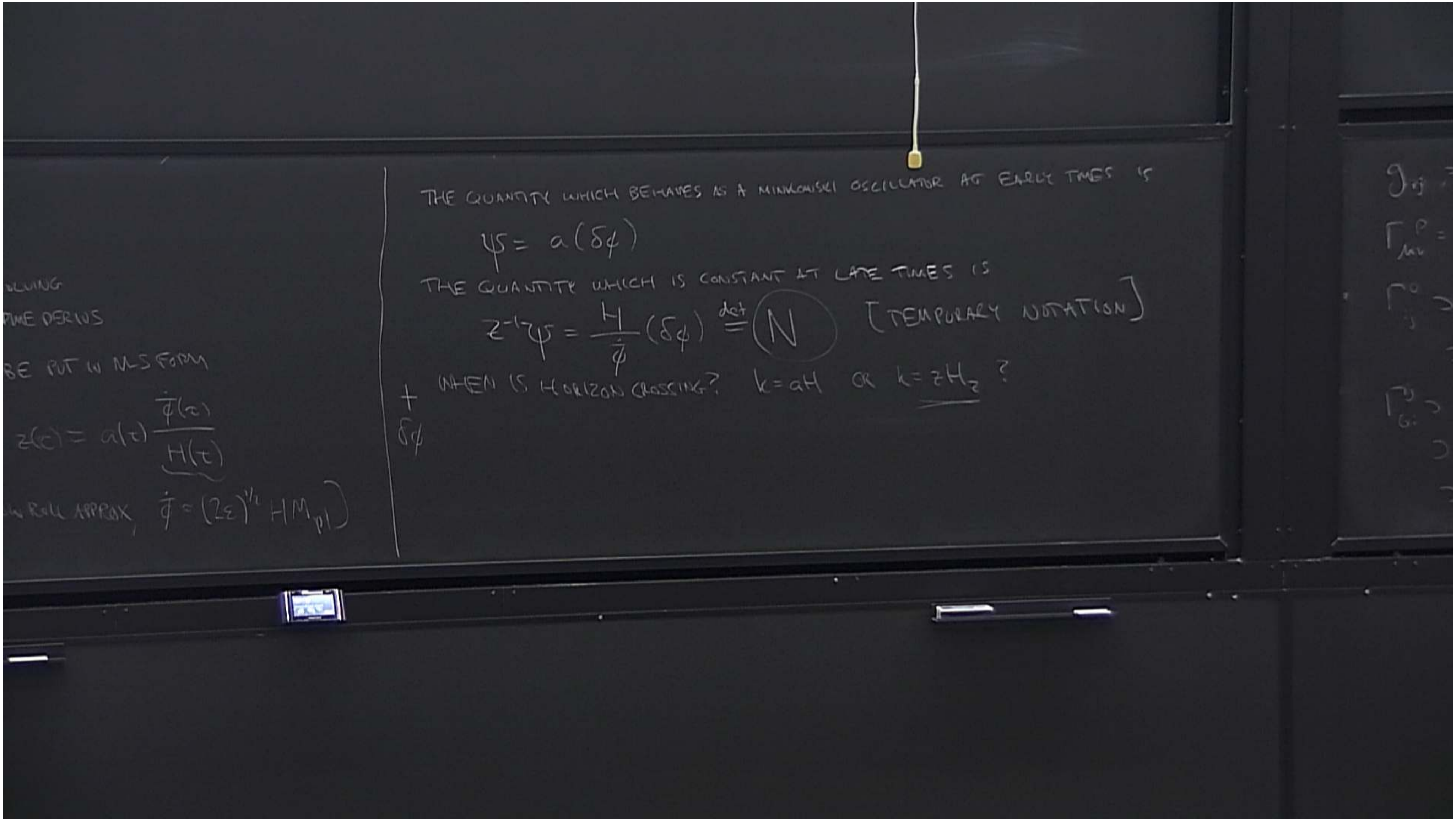
$\delta\phi$ SATISFIES A 2^{ND} ORDER EQ OF MOTION

$$\left(\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - a^2\partial^2 + V''(\bar\phi)\right)\delta\phi = \text{MESS INVOLVING 0 OR 1 TIME DERIVS}$$

TO BE SHOWN LATER: AFTER A LOT OF ALGEBRA, THIS CAN BE PUT IN M-S FORM

$$\frac{\partial^2(a\,\delta\phi)}{\partial\tau^2} + \left(k^2 - \frac{\partial_\tau^2 z}{z}\right)(a\,\delta\phi) \qquad \text{WHERE } z(\tau) = a(\tau)\frac{\dot{\bar\phi}(\tau)}{H(\tau)}$$

[IN SLOW ROLL APPROX, $\dot{\bar\phi} = (2\varepsilon)^{1/2} H M_{pl}$]


OLVING
TIME DERIVS
BE PUT IN M-S FORM
$z(\tau) = a(\tau)\frac{\dot{\phi}(\tau)}{H(\tau)}$
W ROLL APPROX, $\dot{\phi} = (2\varepsilon)^{1/2} H M_{pl}$
THE QUANTITY WHICH BEHAVES AS A MINKOWSKI OSCILLATOR AT EARLY TIMES IS
$\psi = a(\delta\phi)$
THE QUANTITY WHICH IS CONSTANT AT LATE TIMES IS
$z^{-1}\psi = \frac{H}{\dot{\phi}}(\delta\phi) \overset{def}{=} N$
[TEMPORARY NOTATION]
WHEN IS HORIZON CROSSING? $k = aH$ OR $k = zH_z$?
$+$
$\delta\phi$
g_{ij}
$\Gamma^{\rho}_{\mu\nu} =$

THE QUANTITY WHICH BEHAVES AS A MINKOWSKI OSCILLATOR AT EARLY TIMES IS

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[TEMPORARY NOTATION]

$+$ $\delta\phi$

WHEN IS HORIZON CROSSING? $k = aH$ OR $k = \underline{zH_z}$?

$$k = zH_z = z\frac{\partial_\tau z}{z^2} = \partial_\tau(\log z)$$

$$= \partial_\tau(\log a) + \partial_\tau(\cancel{\log}\,\dot{\phi}) - \partial_\tau(\cancel{\log}\,H)$$

$$= aH$$

WHERE $z(\tau) = a(\tau) \frac{\quad}{H(\tau)}$

[IN SLOW ROLL APPROX, $\dot{\phi} = (2\varepsilon)^{1/2} H M_{pl}$]

$\delta\phi$

$$k = zH_z = z\frac{\partial_\tau z}{z^2} = \partial_\tau(\log z)$$
$$= \partial_\tau(\log a) + \partial_\tau(\cancel{\log}\,\dot{\phi}) - \partial_\tau(\cancel{\log}\,H)$$
$$= aH$$

[$\psi = a\,\delta\phi$]
TEST SCALAR

$$[\delta\phi,\ a^2(\delta\phi')] = i \iff [\psi,\ \psi'] = i$$

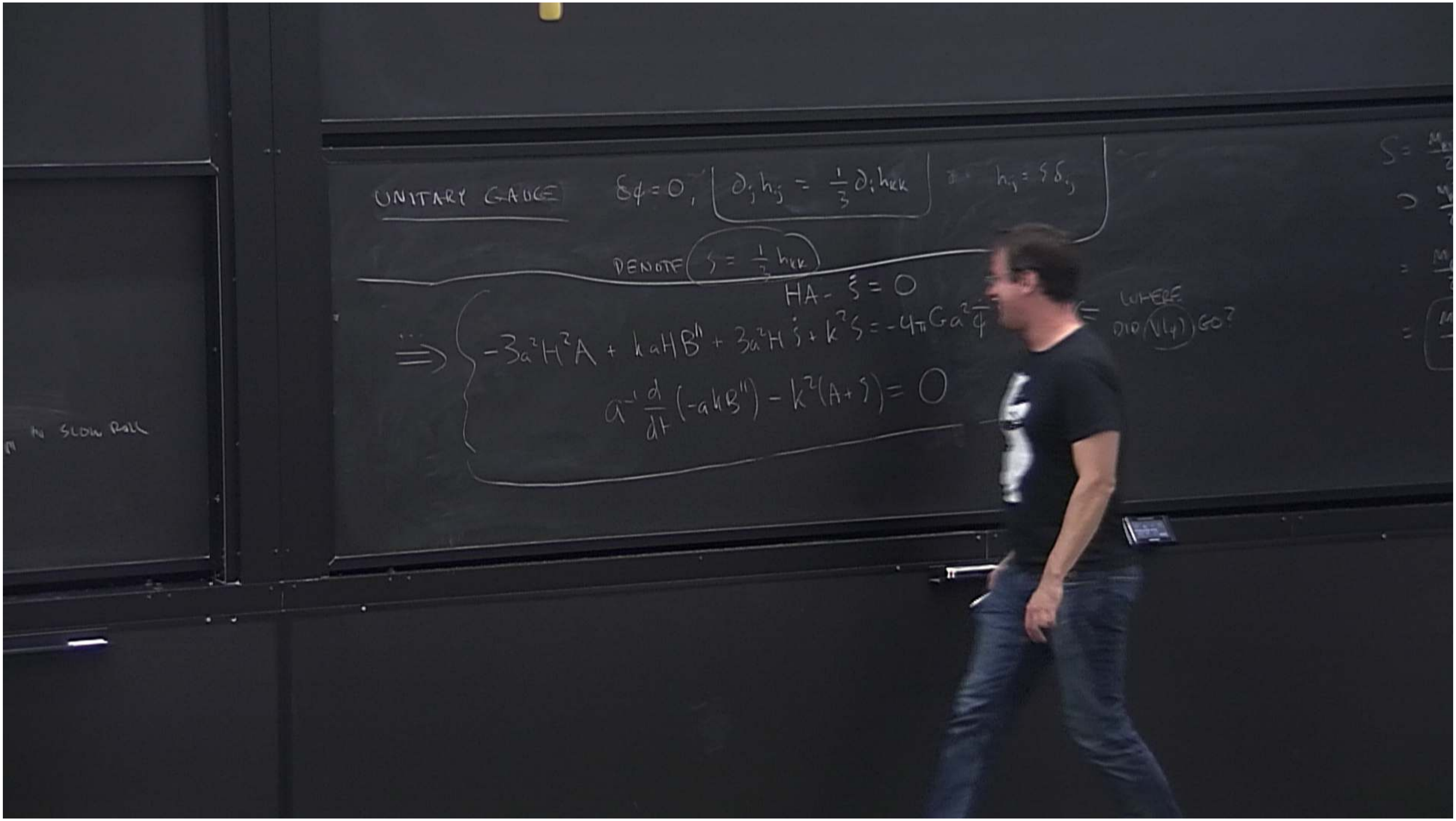
$$N = z\psi \iff [N,\ z^2 N'] = i$$

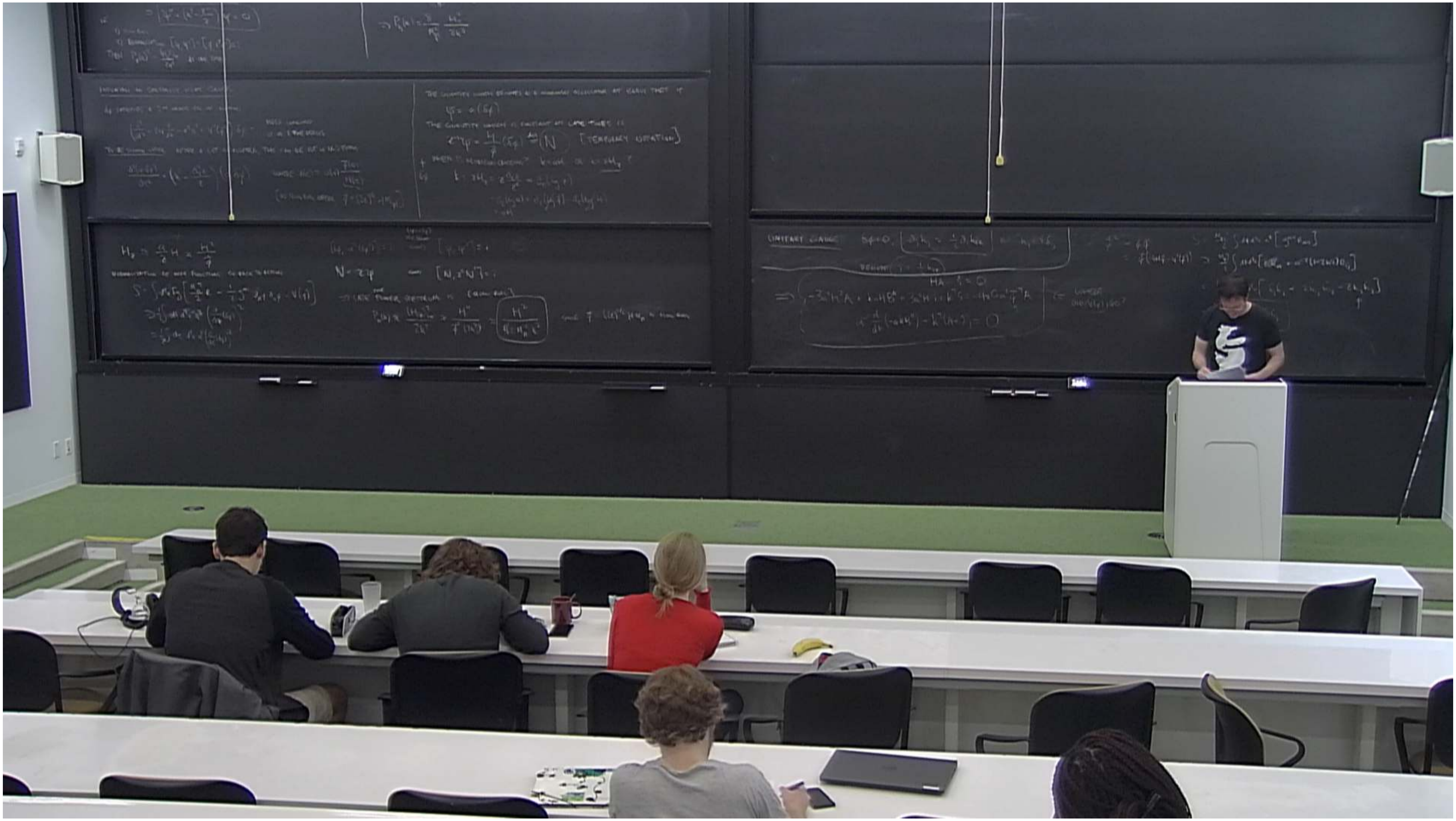
ACTION

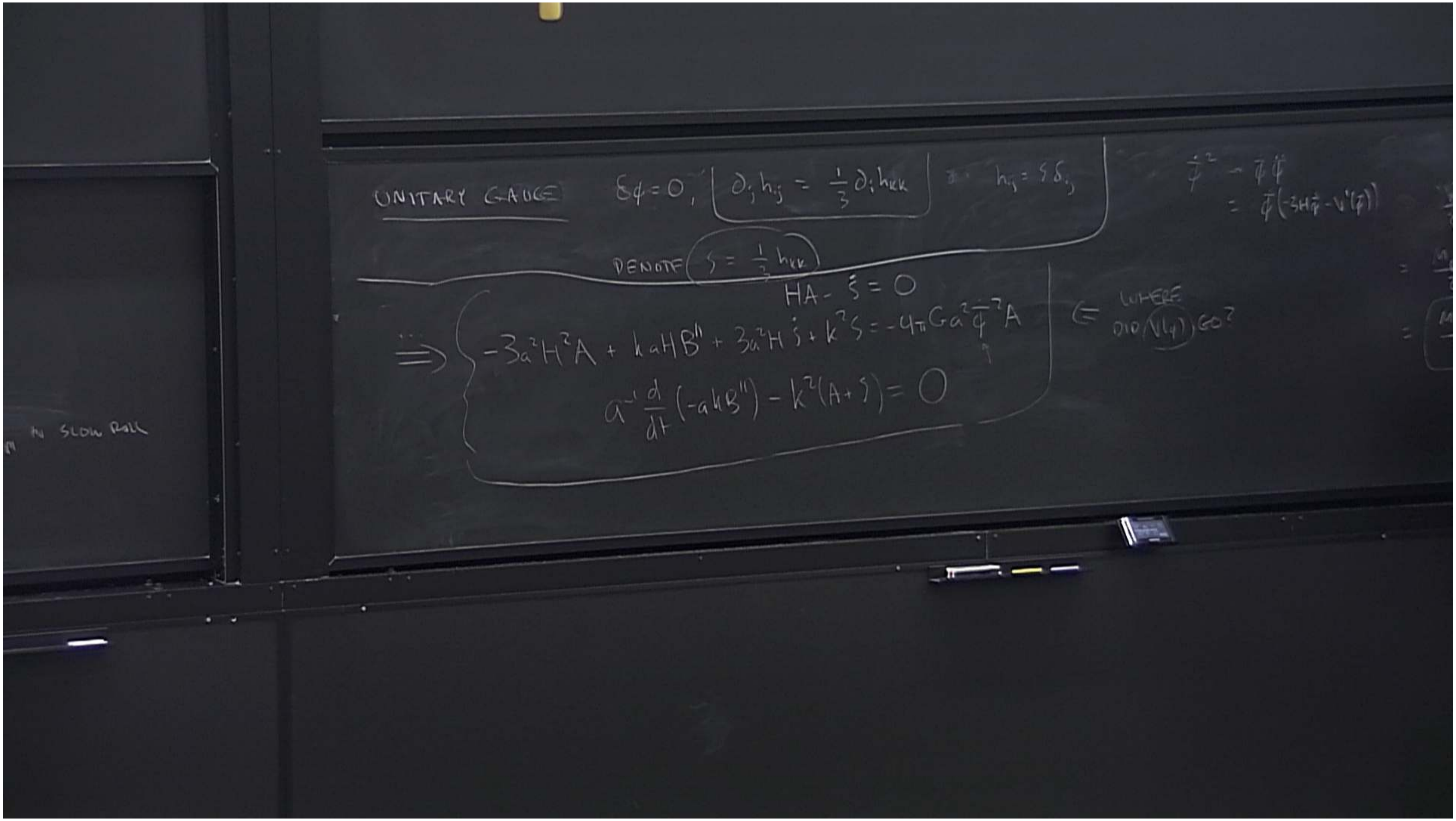
$\phi\,\partial_\tau\phi - V(\phi)]$

$\Rightarrow$ LATE-TIME POWER SPECTRUM IS

$$P_N(k) = \frac{(H_z)^2_*}{2k^3}$$


UNITARY GAUGE
$\delta\phi = 0$, $\partial_j h_{ij} = \frac{1}{3}\partial_i h_{kk}$
$h_{ij} = \zeta\delta_{ij}$
DENOTE $\zeta = \frac{1}{3} h_{kk}$
$HA - \dot{\zeta} = 0$
$-3a^2H^2A + kaHB'' + 3a^2H\dot{\zeta} + k^2\zeta = -4\pi G a^2\dot{\phi}$
WHERE DID $V(\phi)$ GO?
$a^{-1}\frac{d}{dt}(-akB'') - k^2(A+\zeta) = 0$
IN SLOW ROLL




UNITARY GAUGE
$\delta\phi = 0$, $\partial_j h_{ij} = \frac{1}{3}\partial_i h_{kk}$ $\Rightarrow$ $h_{ij} = \zeta\delta_{ij}$
DENOTE $\zeta = \frac{1}{3}h_{kk}$
$HA - \dot{\zeta} = 0$
$-3a^2H^2A + kaHB'' + 3a^2H\dot{\zeta} + k^2\zeta = -4\pi G a^2\dot{\bar{\phi}}^2 A$
$a^{-1}\frac{d}{dt}(-akB'') - k^2(A+\zeta) = 0$
WHERE DID $V(\phi)$ GO?
$\dot{\bar{\phi}}^2 - \bar{\phi}\ddot{\phi}$
$= \bar{\phi}(-3H\dot{\bar{\phi}} - V'(\bar{\phi}))$
IN SLOW ROLL

$h_{ij} = \zeta \delta_{ij}$

$\dot{\bar\varphi}^2 \to \dot{\bar\varphi}\,\ddot{\bar\varphi}$
$= \dot{\bar\varphi}\left(-3H\dot{\bar\varphi} - V'(\bar\varphi)\right)$

$\Rightarrow \frac{\partial}{\partial\tau}\left(a^2 H^{-2} \dot{H} \frac{\partial\zeta}{\partial\tau}\right) + a^2 H^{-2} \dot{H} k^2 \zeta = 0$

M-S FORM $\quad$ ζ CONST. AFTER h.c.

$^2 A \quad \Leftarrow$ WHERE DID $V(\varphi)$ GO?

$z^2 = a^2 H^{-2} \dot{H}$
$= \underbrace{4\pi G}_{1}\, a^2 H^{-2} \dot{\bar\varphi}^2$

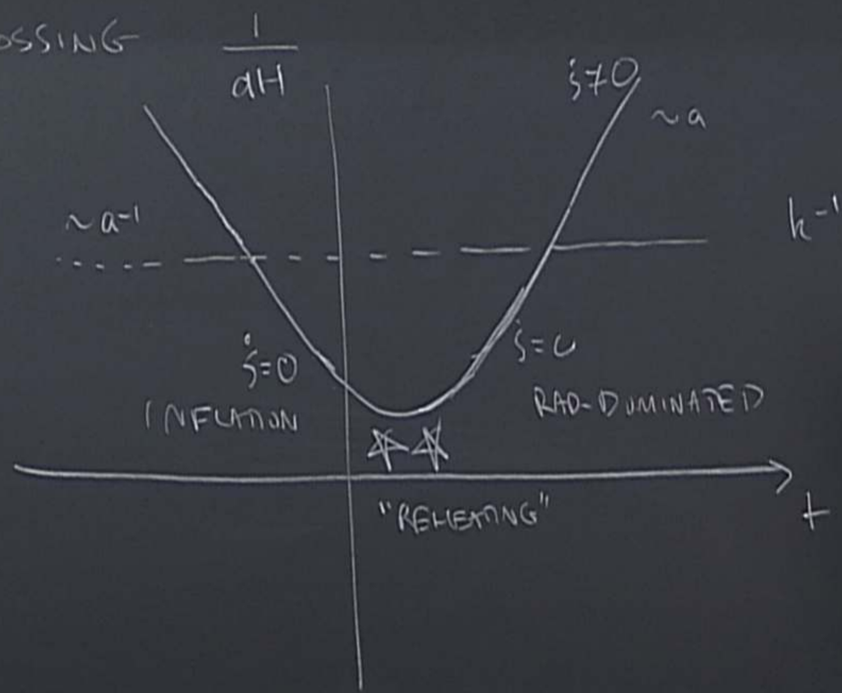
$\dot{H} = 4\pi G(\bar\rho + \bar p)$
$= -4\pi G \dot{\bar\varphi}^2$

$\dot{\zeta} \to 0$ AFTER HORIZON CROSSING

$$\frac{\partial}{\partial\tau}\left[z^2 \frac{\partial\zeta}{\partial\tau}\right] + k^2 \zeta = 0$$

WHERE $z = \frac{\dot{\phi}}{H}\zeta$

$$\boxed{P_\zeta(k) = \frac{H_*^2}{4 M_{Pl}^2 \varepsilon k^3}}$$



THEOREM (WEINBERG): SINGLE-FIELD INFLATION $\Rightarrow$ ADIABATIC INITIAL CONDITION

PHENOMENOLOGY

$$P_h(k) = \frac{8}{M_{pl}^2} \frac{H_*^2}{2k^3}$$

OBSERVABLES

"AMPLITUDE" $\Delta_h^2(k_*) = \frac{k^3}{2\pi^2} P_h(k) = \frac{2}{\pi^2} \frac{H_*^2}{M_{pl}^2}$ $\quad [k_* = 0.05\ M_{pc}^{-1}]$

"TILT" $n_t = \frac{d \log \Delta_h^2(k)}{d \log k}$

$= 0.05\ M_{pl}^{-1}]$

$(X)_*$ MEANS "EVALUATE X AT HORIZON CROSSING" $\quad k = aH$

$$d\log k = d\log a + \underline{d\log H} \rightarrow \text{SLOW ROLL SUPPRESSED}$$

$$= H\,dt$$

$$= \frac{H}{\dot{\phi}}\,d\phi$$

$$= (2\varepsilon)^{-1/2} M_{pl}^{-1}\,d\phi \qquad \text{SINCE } \dot{\bar{\phi}} = (2\varepsilon)^{1/2} H M_{pl}$$

$$\frac{dX_*}{d\log k} = (2\varepsilon)^{1/2} M_{pl} \frac{dX}{d\phi}$$

$= 0.05\ M_{pl}^{-1}]$

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