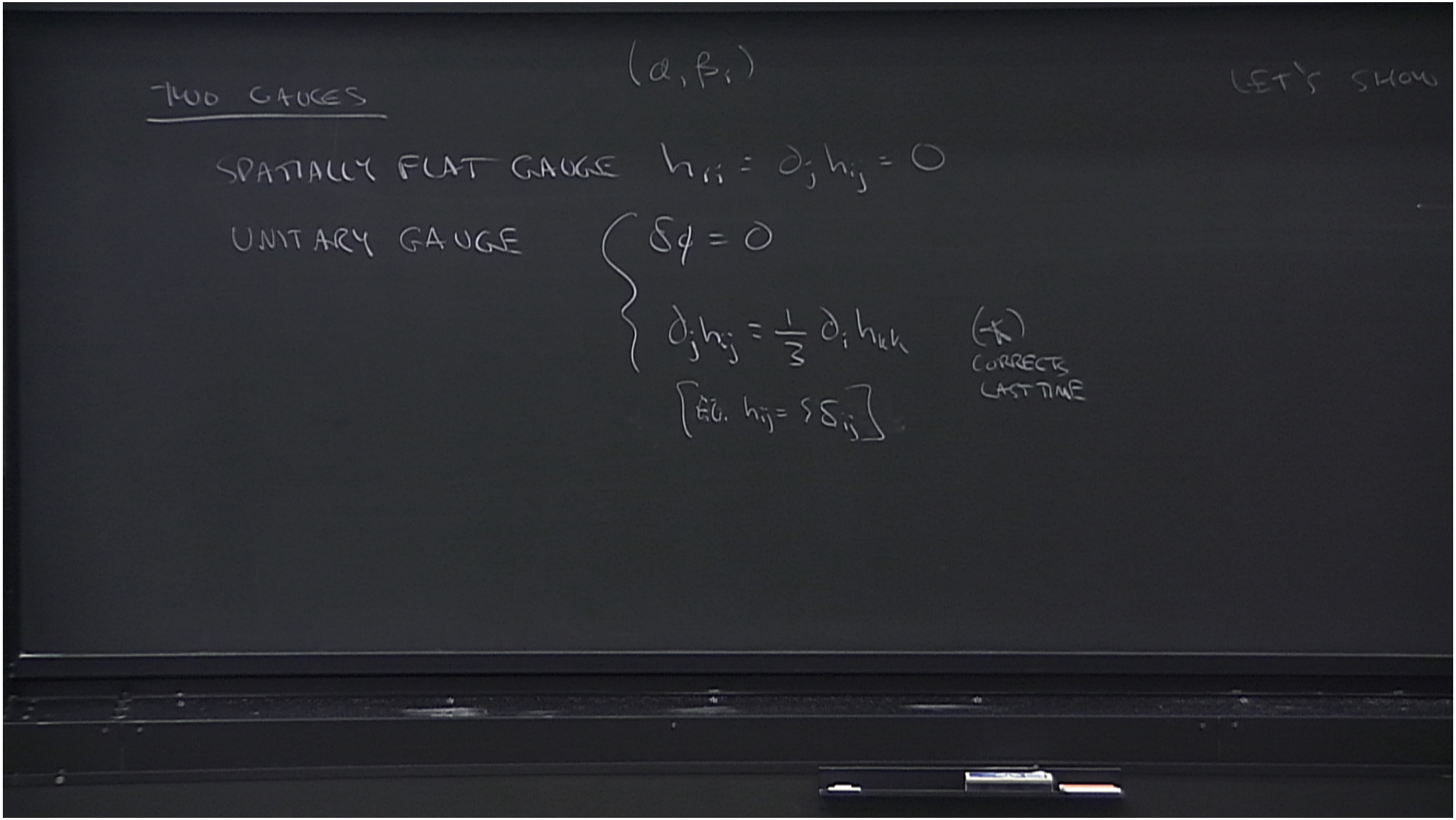
Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 9

Date: Apr 21, 2016 10:15 AM

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Abstract:


(α, β_i)
LET'S SHOW
TWO GAUGES
SPATIALLY FLAT GAUGE $h_{ii} = \partial_j h_{ij} = 0$
UNITARY GAUGE
$\delta\varphi = 0$
$\partial_j h_{ij} = \frac{1}{3}\partial_i h_{kk}$
(*) CORRECTS LAST TIME
[EG. $h_{ij} = \zeta \delta_{ij}$]

LET'S SHOW EXPLICITLY THAT AN ARB METRIC CAN BE PUT INTO SPATIALLY FLAT GAUGE VIA A GAUGE TRANSFORMATION $x^\mu \to x^\mu + \xi^\mu(x)$ $\quad \xi^\mu = (\alpha, \beta_i)$

FORMULA SHEET

$$\begin{cases} S: & h_{ij} \to h_{ij} - 3H\alpha - \partial_i \beta^i \\ V_i: & \partial_j h_{ij} \to \partial_j h_{ij} - H\partial_i \alpha - \frac{1}{2}\left(\partial_i \partial_j B_j + \partial^2 B_i\right) \end{cases}$$

WANT TO SHOW THAT FOR ARB "SOURCES" (S, V_i) CAN SOLVE

$$3H\alpha + \partial_i B_i = S$$

$$H\partial_i \alpha + \frac{1}{2}\left(\partial_i \partial_j B_j + \partial^2 B_i\right) = V_i$$

"SVT" DECOMPOSITION

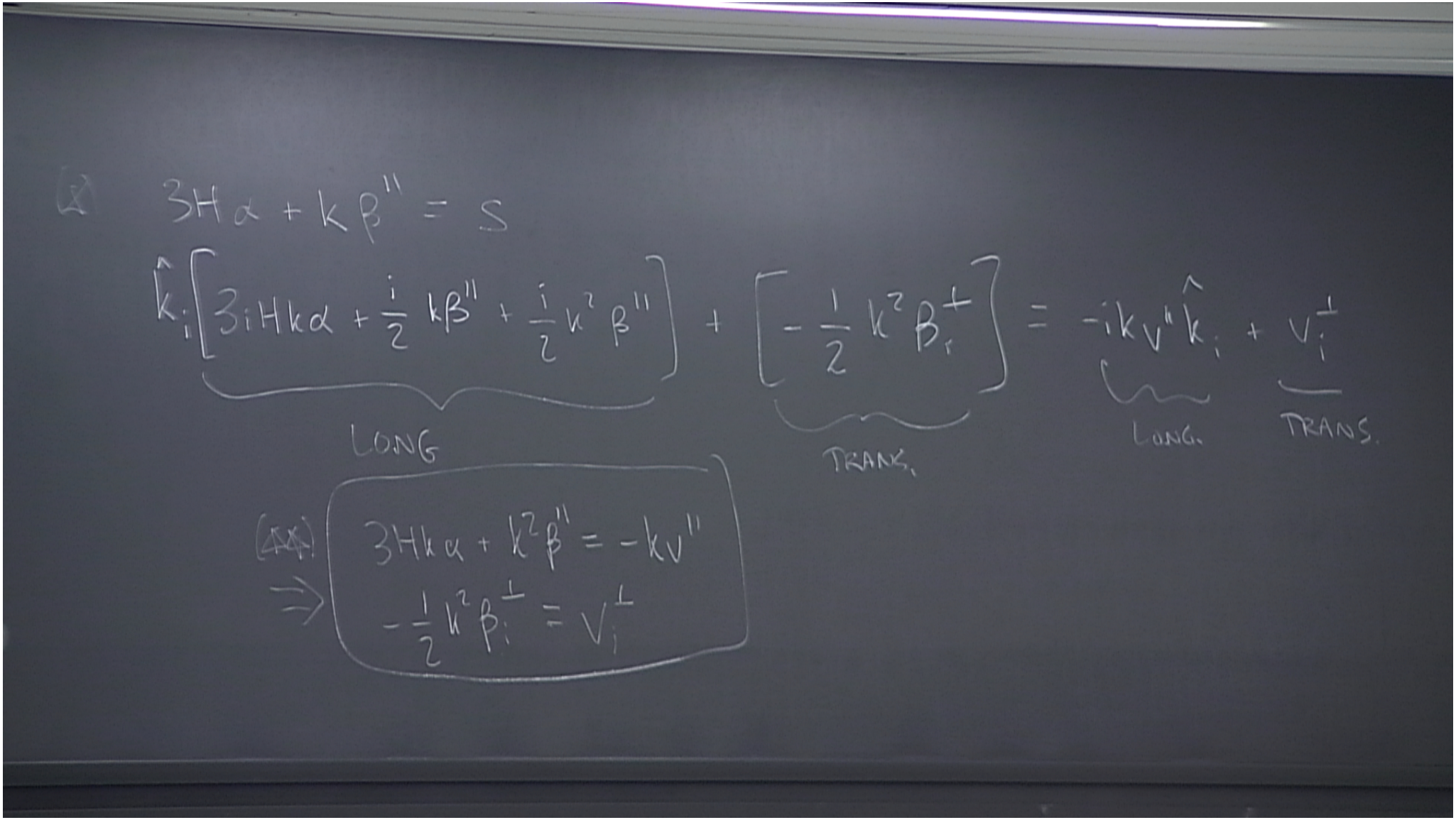
1) FOURIER SPACE

$$3Ha + ik_i\beta_i = S$$

$$iHk_i\alpha - \frac{1}{2}k_ik_j\beta_j - \frac{1}{2}k^2\beta_i = V_i$$

2) "SVT" DECOMPOSE MEANS: FOR EACH VECTOR V_i

$$V_i = \underbrace{-iV''\hat{k}_i}_{\text{LONGITUDINAL}} + \underbrace{V_i^{\perp}}_{\text{TRANSVERSE}} \qquad \text{WHERE } V_i^{\perp}k_i = 0$$


(i) $3H\alpha + k\beta'' = s$
$\hat{k}_i\left[3iHk\alpha + \frac{i}{2}k\beta'' + \frac{i}{2}k^2\beta''\right] + \left[-\frac{1}{2}k^2 B_i^{\perp}\right] = -ikv''\hat{k}_i + V_i^{\perp}$
LONG
TRANS.
LONG.
TRANS.
(ii) $3Hk\alpha + k^2\beta'' = -kv''$
$\Rightarrow -\frac{1}{2}k^2 B_i^{\perp} = V_i^{\perp}$

NOW SOLVE

$$\alpha = \frac{1}{2H}\left(s + \frac{v_{11}}{k}\right)$$

$$\beta'' = -\frac{1}{2k}\left(s + 3\frac{v_{11}}{k}\right)$$

$$\beta^{\perp} = -\frac{2}{k^2} v_i^{\perp}$$

FOR A SYMMETRIC TENSOR S_{ij}, THE SVT DECOMP. IS

$$S_{ij} = S^{(T)} \delta_{ij} + S''\left(\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij}\right) + \frac{1}{2}\left(\hat{k}_i S_j^{(m)} + S_i^{(m)} \hat{k}_j\right) + S_{ij}^{\perp}$$

"TRACE" "LONGITUDINAL" "MIXED" $S_i^{(m)} \hat{k}_i = 0$ "TRANSVERSE" $k_i S_{ij}^{\perp} = 0$

NOW SOLVE

$$\alpha = \frac{1}{2H}\left(s + \frac{V_{11}}{k}\right)$$

$$\beta^{11} = -\frac{1}{2k}\left(s + 3\frac{V_{11}}{k}\right)$$

$$\beta^{\perp} = -\frac{2}{k^2} V_i^{\perp}$$

$$S^{(T)} = \frac{1}{3}\delta^{ij} S_{ij}$$

T_{ij}

FOR A SYMMETRIC TENSOR S_{ij}, THE SVT DECOMP. IS

$$S_{ij} = S^{(T)}\delta_{ij} + S^{11}\left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right) + \frac{1}{2}\left(\hat{k}_i S_j^{(M)} + S_i^{(M)}\hat{k}_j\right) + S_{ij}^{\perp}$$

"TRACE" "LONGITUDINAL" "MIXED" $S_i^{(M)}\hat{k}_i = 0$ "TRANSVERSE" $k_i S_{ij}^{\perp} = 0$

EINSTEIN'S EQS IN SPATIALLY GAUGE

TWO OF E'S EQS:

$$-3a^2H^2(A) + aH\partial_j(B_j) = 4\pi G a^2(\delta\rho)$$

$$2H\partial_i A - \frac{1}{2}a^{-1}\partial_i\partial_j B_j + \frac{1}{2}a^{-1}\partial^2 B_i = -8\pi G a(\bar{\rho}+\bar{p})(v_i - B_i)$$

SOLVE FOR (A, B_i) GIVEN STRESS-ENERGY USING SVT PROCEDURE

SVT $\quad -3a^2H^2A + kaHB'' = 4\pi G a^2(\delta\rho)$

$\Rightarrow \quad -2HkA = -8\pi G a(\bar{\rho}+\bar{p})(v'' - B'')$

$\quad -\frac{1}{2}a^{-1}k^2B^{\perp} = -8\pi G a(\bar{\rho}+\bar{p})(v'' - B'')$

"TRACE" "LONGITUDINAL"

$$A = \frac{4\pi G a}{Hk}(\bar{\rho}+\bar{p})(v'' - B'')$$

$$\Rightarrow \quad B'' = \frac{4\pi G a}{Hk}(\delta\rho) + \frac{12\pi G a^2}{k^2}(\bar{\rho}+\bar{p})(v'' - B'')$$

$$B^{\perp} = \frac{16\pi G a^2}{k^2}(\bar{\rho}+\bar{p})(v^{\perp} - B^{\perp})$$

ONLY PART OF THE METRIC REMAINING IS $h_{ij}^{\perp}$ [TRANSVERSE TRACELESS]

$$\frac{1}{2}\ddot{h}_{ij} + \frac{1}{2}\dot{h}_{ij} - \frac{1}{3}\dot{h}_{kk} = 8\pi G a^2 \bar{p}\,\Pi_{ij}$$

"TRACE" "LONGITUDINAL"

$$A = \frac{4\pi G a}{Hk}(\bar{\rho}+\bar{p})(v''-B'')$$

4/10 $\nabla_\mu T^{\mu\nu}$

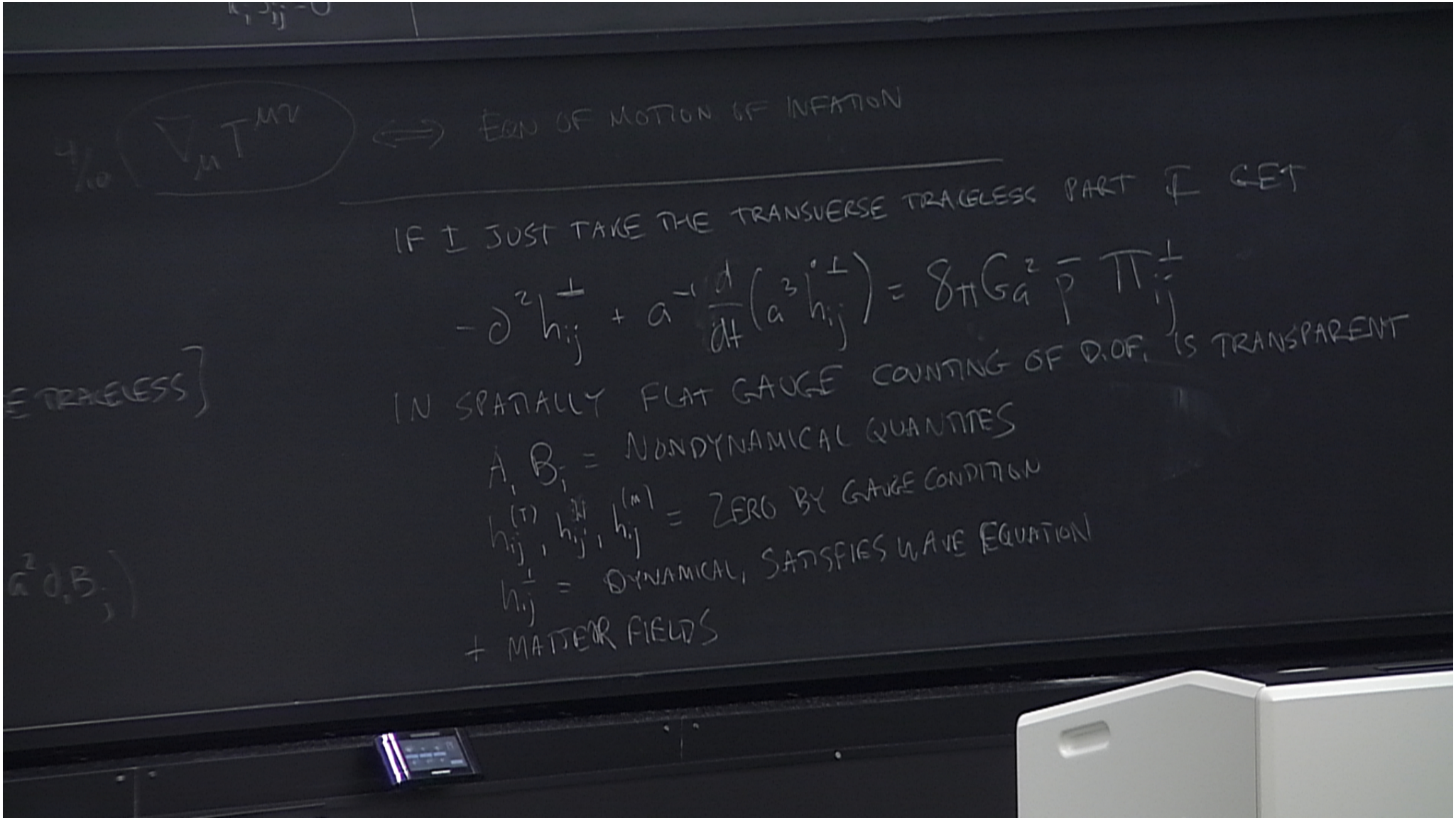
$$\Rightarrow \quad B' = \frac{4\pi G a}{Hk}(\delta\rho) + \frac{12\pi G a^2}{k^2}(\bar{\rho}+\bar{p})(v''-B'')$$

$$B^{\perp} = \frac{16\pi G a^2}{k^2}(\bar{\rho}+\bar{p})(v^{\perp}-B^{\perp})$$

ONLY PART OF THE METRIC REMAINING IS $h_{ij}^{\perp}$ [TRANSVERSE TRACELESS]

$$\frac{1}{2}L_{ij} + \frac{1}{2}L_{ji} - \frac{1}{3}L_{kk} = 8\pi G a^2 \bar{p}\,\Pi_{ij}$$

WHERE $L_{ij} = -\partial_i\partial_j A - \partial^2 h_{ij} + a^{-1}\frac{d}{dt}\left(a^3 \dot{h}_{ij} + a^2 \partial_{(i}B_{j)}\right)$


$\nabla_\mu T^{\mu\nu}$
EQN OF MOTION OF INFATION
IF I JUST TAKE THE TRANSVERSE TRACELESS PART I GET
$-\partial^2 h_{ij}^{\perp} + a^{-1}\frac{d}{dt}\left(a^3 \dot{h}_{ij}^{\perp}\right) = 8\pi G a^2 \bar{p}\,\Pi_{ij}^{\perp}$
IN SPATIALLY FLAT GAUGE COUNTING OF D.OF. IS TRANSPARENT
A, B_i = NONDYNAMICAL QUANTITES
$h_{ij}^{(T)}, h_{ij}^{(V)}, h_{ij}^{(m)}$ = ZERO BY GAUGE CONDITION
$h_{ij}^{\perp}$ = DYNAMICAL, SATISFIES WAVE EQUATION
+ MATTER FIELDS

ϕ - EOM IN SPATIALLY FLAT GAUGE

$$\left(\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - a^{-2}\partial^2 + V''(\bar{\phi})\right)\delta\phi = \dot{\phi}^2(\dot{A} - a^{-1}\partial_i B - \cancel{h^i_j}) - 2V'(\bar{\phi})A$$

$$-kB'$$

$$A = \frac{4\pi G a}{Hk}(\bar{\rho}+\bar{P})(v'-B'')$$

$$B'' = \frac{4\pi G a}{Hk}(\delta\rho) + \frac{12\pi G a^2}{k^2}(\bar{\rho}+\bar{P})(v-B'')$$

$$\delta\rho = \dot{\bar{\phi}}\frac{\partial(\delta\phi)}{\partial t} - \dot{\bar{\phi}}A + V'(\bar{\phi})\delta\phi$$

$$(\bar{\rho}+\bar{P})(v_i - B_i) = -a^{-1}\dot{\bar{\phi}}(\delta\phi)$$

$$-\partial_i\alpha + \frac{1}{2}\left(\partial_i\partial_j B_j + \partial^2 B_i\right) = V_i$$

GE

$$\delta_{0i} = \dot{\phi}^2\left(\dot{A} - a^{-1}\partial_i B - \cancel{h_{jj}^{\cdot}}\right) - 2V'(\bar{\phi})\,A$$

$-kB''$

"SCALAR SECTOR MESS"

$\bar{F})(V^{\cdot} - B'')$

$$a^{-1}\frac{d}{dt}\left(a^3 \dot{h}^{\perp}_{ij}\right) + k^2 h^{\perp}_{ij} = 0$$

LET'S ANALYZE THIS EQ. & FIND QUANTUM FLUCTUATIONS

SWITCH TO CONFORMAL TIME

$$\frac{d}{d\eta}\left(a^2 \frac{dh^{\perp}_{ij}}{d\eta}\right) + a^2 k^2 h^{\perp}_{ij} = 0$$

THIS IS THE MUKHANOV-SASAKI EQUATION! $(z = a)$

$h^{\perp}_{ij}$ FREEZES AFTER HORIZON CROSSING

$a h^{\perp}_{ij}$ BEHAVES AS HARMONIC OSCILLATOR BEFORE H.C.

NEED NORMALIZATION OF MODE FUNCTION, GO BACK TO ACTION

APPEARS TO REQUIRE $S_{EH} = \frac{M_{Pl}^2}{2}\int \sqrt{-g}R$ TO SECOND ORDER!

WE JUST NEED THE TERMS W/ TWO TIME DERIVATIVES ON h_{ij}

FROM FORMULA SHEET:

$$\Gamma^0_{ij} \supset a^2 \dot{h}_{ij} \qquad \Gamma^j_{0i} \supset \dot{h}^j_i$$

NEED NORMALIZATION OF MODE FUNCTION, GO BACK TO ACTION

APPEARS TO REQUIRE $S_{EH} = \frac{M_{Pl}^2}{2}\int \sqrt{-g}R$ TO SECOND ORDER!

WE JUST NEED THE TERMS W/ TWO TIME DERIVATIVES ON h_{ij}

FROM FORMULA SHEET:

$$\Gamma^0_{ij} \supset a^2 \dot{h}_{ij} \qquad \Gamma^j_{0i} \supset \dot{h}^j_i - 2h^{jk}\dot{h}_{ik}$$

$$R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\mu\nu} - \partial_\mu \Gamma^\rho_{\nu\rho} + \Gamma^\rho_{\mu\nu}\Gamma^\lambda_{\rho\lambda} - \Gamma^\lambda_{\mu\rho}\Gamma^\rho_{\nu\lambda}$$

$$R_{00} \supset -\left(-2\dot{h}^{jk}\dot{h}_{jk}\right)$$

$$S_{EH} = \frac{M_{pl}^2}{2} \int d\tau\, d^3x\, a^2 \left(\frac{\partial h_{ij}}{\partial \tau}\right)\left(\frac{\partial h_{ij}}{\partial \tau}\right)$$

COSMOLOGISTS' CONVENTION FOR GW AMPLITUDE

$$h_{ij}(\vec{k}) = h_+(\vec{k}) \begin{pmatrix} \frac{1}{4} & & \\ & -\frac{1}{4} & \\ & & 0 \end{pmatrix} + h_\times(\vec{k}) \begin{pmatrix} 0 & \frac{1}{4} & \\ \frac{1}{4} & 0 & \\ & & 0 \end{pmatrix}$$

$$S_{EH} = \frac{M_{pl}^2}{2} \int d\tau\, d^3x\, a^2 \left[\frac{1}{8}\left(\frac{\partial h_+}{\partial \tau}\right)^2 + \frac{1}{8}\left(\frac{\partial h_\times}{\partial \tau}\right)^2\right]$$

SO THE CONJUGATE MOMENTA ARE

$$\Pi_+ = \frac{\delta S_{EH}}{\delta(\partial_\tau h_+)} = \frac{M_{Pl}^2}{8} a^2 \left(\frac{\partial h_+}{\partial t}\right)$$

& LIKEWISE FOR "x"

TO CALCULATE MODE FUNCTIONS WE WOULD INTEGRATE

$$\frac{d}{d\tau}\left[a^2 \frac{du}{d\tau}\right] = k^2 a^2 u$$

W/ NORMALIZATION

$$i = \left[\hat{h}_+, \hat{\Pi}'_+\right] = \frac{M_{Pl}^2}{8}\left(u u'^* - u' u^*\right)$$

"TRACE" "LONGITUDINAL" (MIXED) $k_i S_{ij} = 0$

SO THE CONJUGATE MOMENTA ARE

$$\Pi_+ = \frac{\delta \mathcal{L}_{EH}}{\delta(\partial_\tau h_+)} = \frac{M_{Pl}^2}{8} a^2 \left(\frac{\partial h_+}{\partial \tau}\right)$$ & LIKEWISE FOR "x"

TO CALCULATE MODE FUNCTIONS WE WOULD INTEGRATE

$$\frac{d}{d\tau}\left[a^2 \frac{du}{d\tau}\right] = k^2 a^2 u$$

NORMALIZATION

$$i = \left[\hat{h}_+, \hat{\Pi}'_+\right] = \frac{M_{Pl}^2}{8}\left(u u'^* - u' u^*\right)$$ → =1 BEFORE

$\partial_i S_{ij} = 0$

$$P_h(k) \approx \frac{8}{M_{Pl}^2}\left(\frac{H_*^2}{2k^3}\right)$$

$M_{Pl}^2 \hookrightarrow 1$ PREVIOUSLY

WHERE $(\cdot)_*$ = EVALUATED WHEN $k = aH$

$\dot{H} > 0$ →



- ALMOST SCALE INVARIANT

  INFLATION PREDICTS A "RED" POWER SPECTRUM: $P(k) = k^{-3-\varepsilon}$ $(\varepsilon > 0)$
- SIZE OF GW (STRAIN) IS $\sim \frac{H}{M_{Pl}}$
- CURRENT UPPER LIMIT (CMB): $\frac{H^2}{M_{Pl}^2} \lesssim 10^{-9}$

WILL IMPROVE SOON!