Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 8

Date: Apr 20, 2016 10:15 AM

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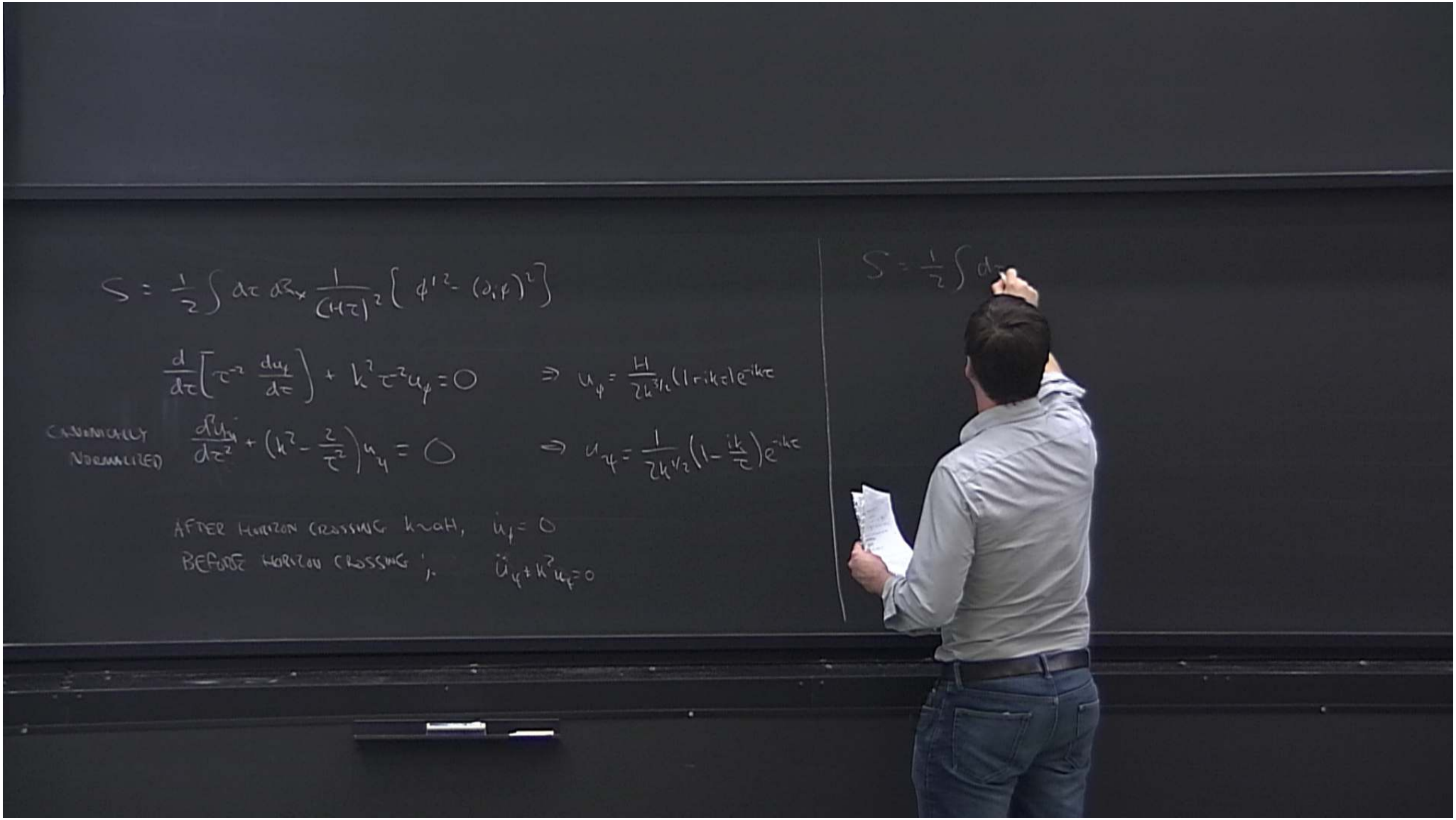
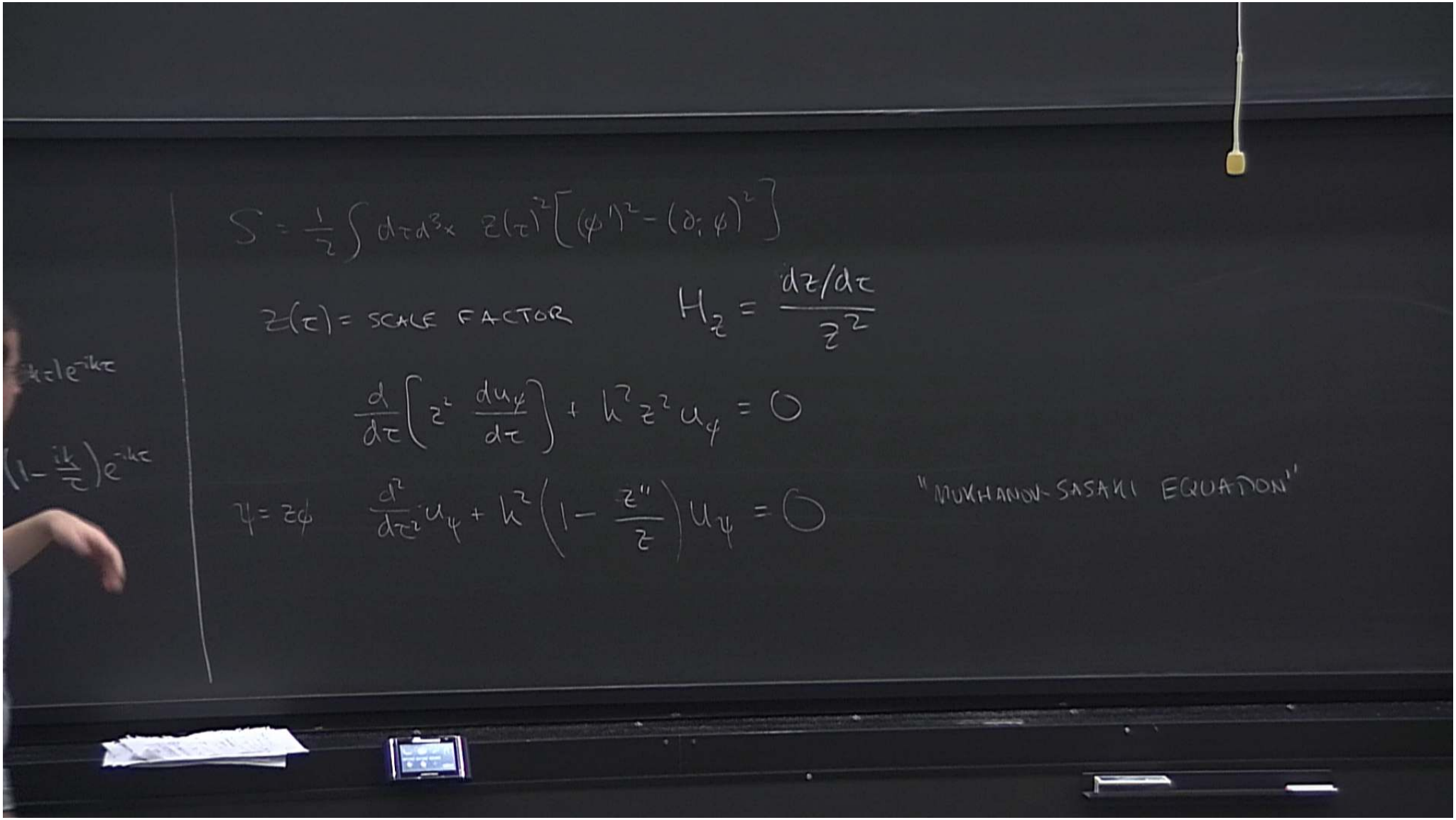
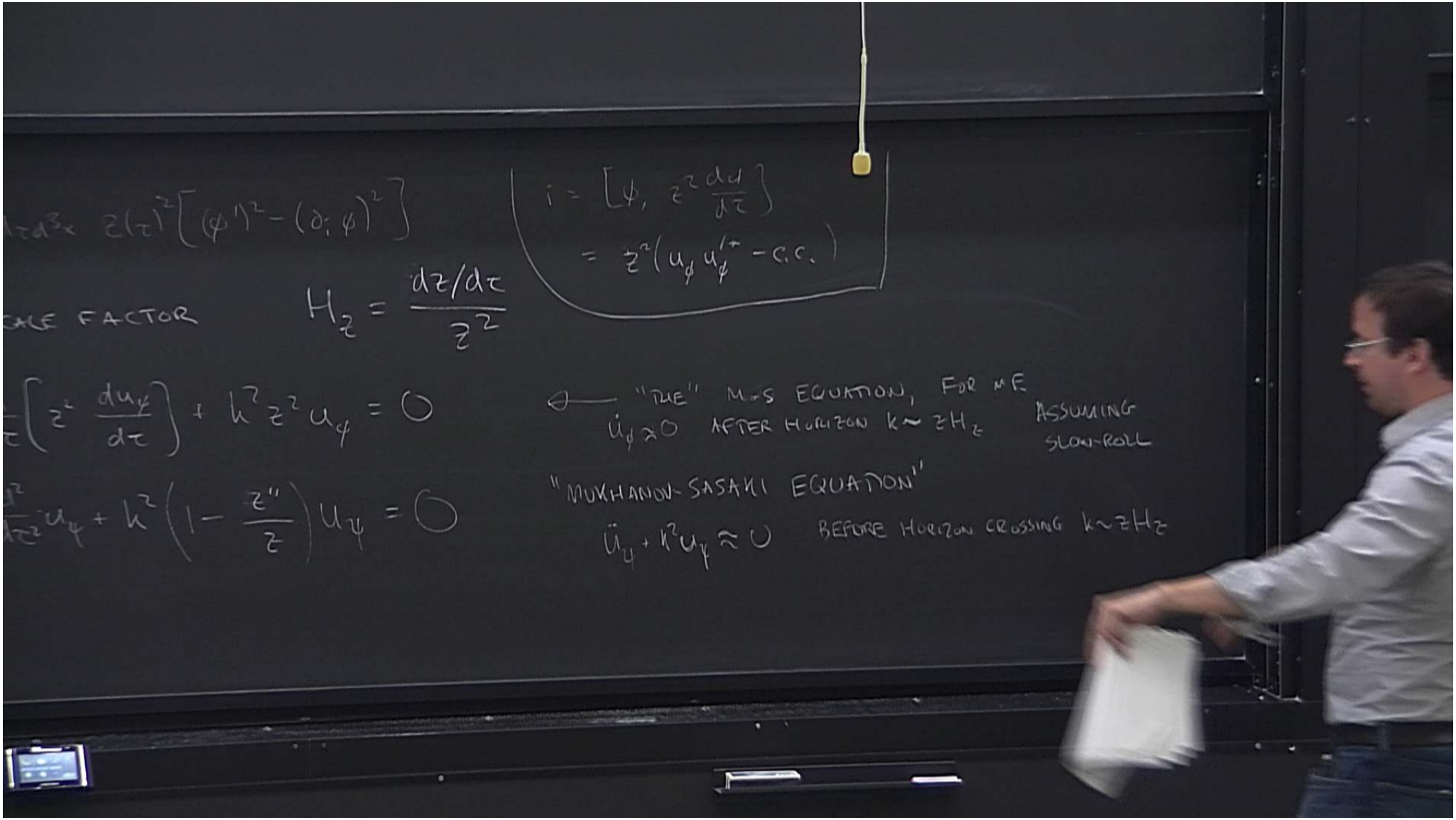
Abstract:

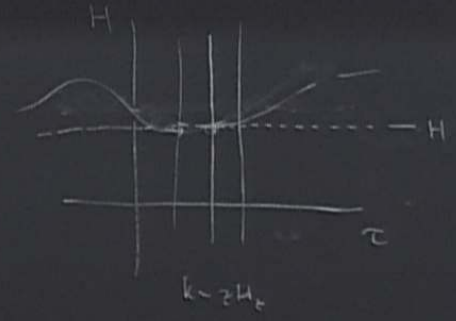

$S = \frac{1}{2}\int d\tau\, d^3x \frac{1}{(H\tau)^2}\left\{\phi'^2 - (\partial_i\phi)^2\right\}$
$\frac{d}{d\tau}\left[\tau^{-2}\frac{du_k}{d\tau}\right] + k^2\tau^2 u_k = 0 \quad \Rightarrow u_k = \frac{H}{2k^{3/2}}(1+ik\tau)e^{-ik\tau}$
CANONICALLY NORMALIZED
$\frac{d^2u_{kN}}{d\tau^2} + \left(k^2 - \frac{2}{\tau^2}\right)u_{kN} = 0 \quad \Rightarrow u_{kN} = \frac{1}{2k^{1/2}}\left(1 - \frac{ik}{\tau}\right)e^{-ik\tau}$
AFTER HORIZON CROSSING $k \sim aH$, $\dot{u}_k = 0$
BEFORE HORIZON CROSSING $\ddot{u}_k + k^2 u_k = 0$
$S = \frac{1}{2}\int d$


$S = \frac{1}{2}\int d\tau d^3x\ z(\tau)^2\left[(\varphi')^2 - (\partial_i\varphi)^2\right]$
$z(\tau)$ = SCALE FACTOR
$H_z = \frac{dz/d\tau}{z^2}$
$\frac{d}{d\tau}\left(z^2\frac{du_\varphi}{d\tau}\right) + k^2 z^2 u_\varphi = 0$
$\chi = z\varphi$
$\frac{d^2}{d\tau^2}u_\chi + k^2\left(1 - \frac{z''}{z}\right)u_\chi = 0$
"MUKHANOV-SASAKI EQUATION"

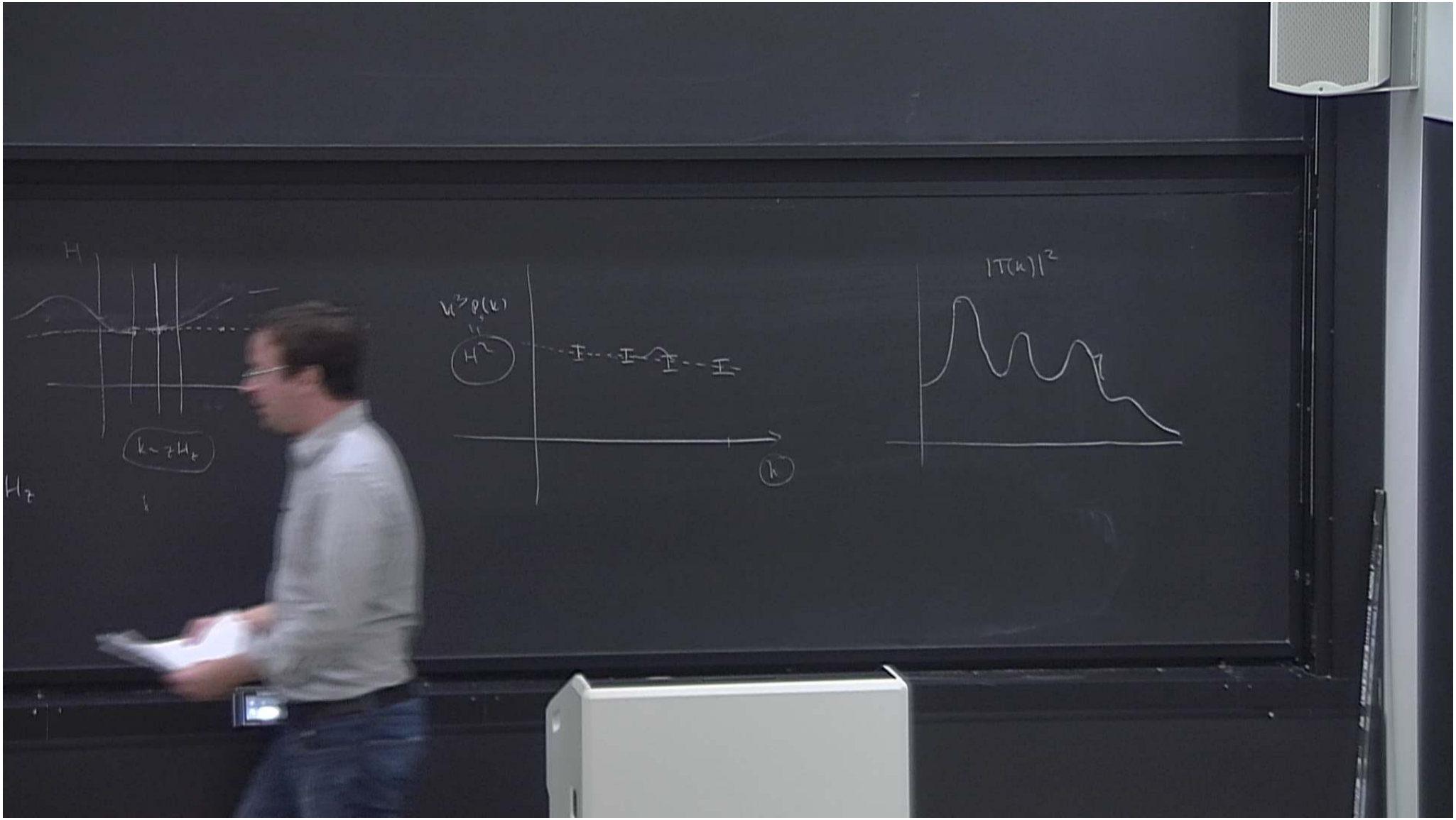

$d\tau d^3x \; z(\tau)^2\left[(\phi')^2-(\partial_i\phi)^2\right]$
$i = \left[\phi, z^2\frac{du}{d\tau}\right]$
$= z^2(u_\phi u_\phi'^* - c.c.)$
CALE FACTOR
$H_z = \frac{dz/d\tau}{z^2}$
$\frac{d}{d\tau}\left[z^2\frac{du_\phi}{d\tau}\right] + k^2 z^2 u_\phi = 0$
"THE" M-S EQUATION, FOR ME
$\dot{u}_\phi \approx 0$ AFTER HORIZON $k \sim zH_z$
ASSUMING SLOW-ROLL
$\frac{d^2}{d\tau^2}u_\psi + k^2\left(1-\frac{z''}{z}\right)u_\psi = 0$
"MUKHANOV-SASAKI EQUATION"
$\ddot{u}_\psi + k^2 u_\psi \approx 0$ BEFORE HORIZON CROSSING $k \sim zH_z$

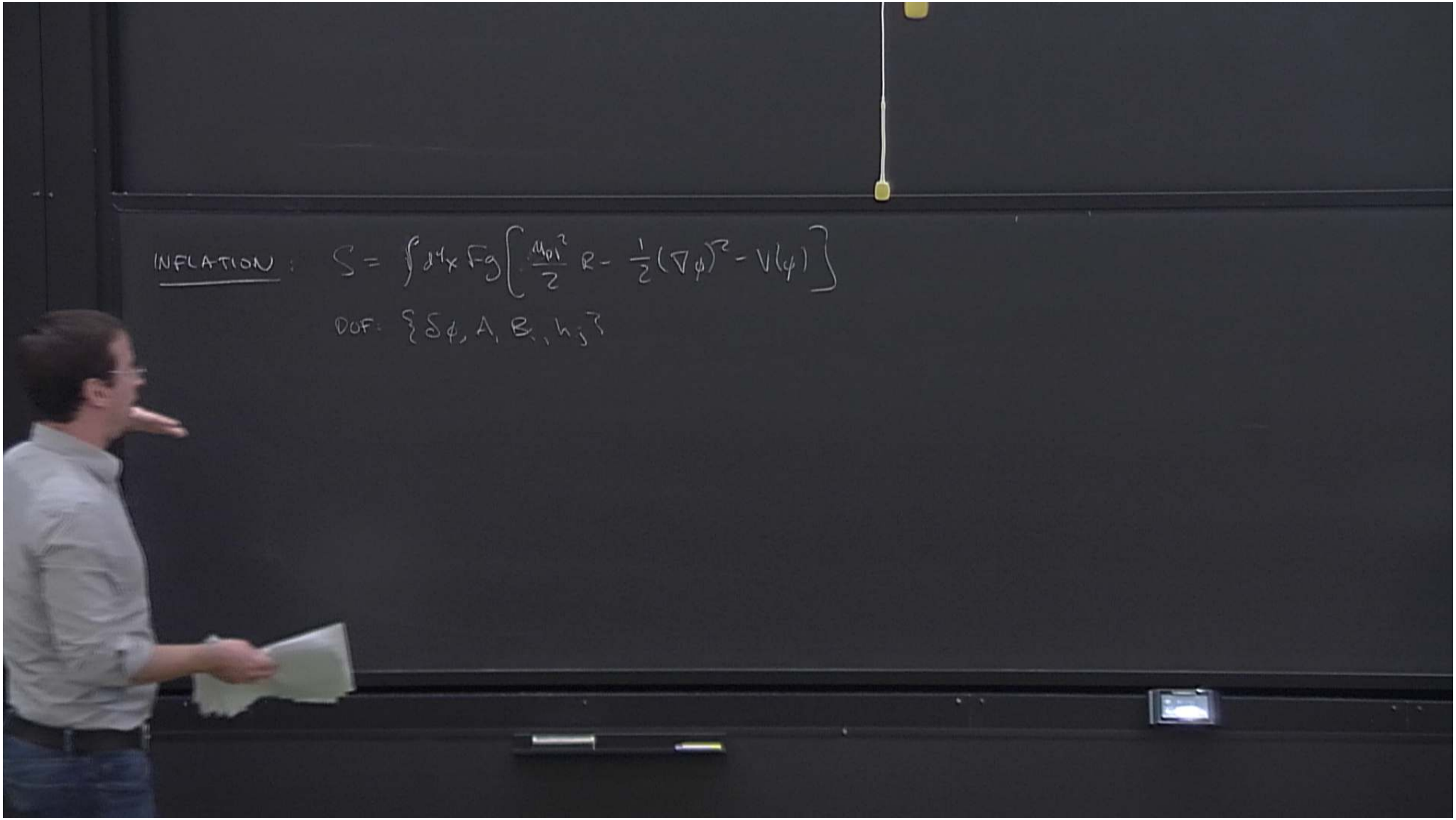
A USEFUL APPROXIMATION: IN SLOW ROLL, THE VALUE OF u_q AFTER HORIZON CROSSING IS

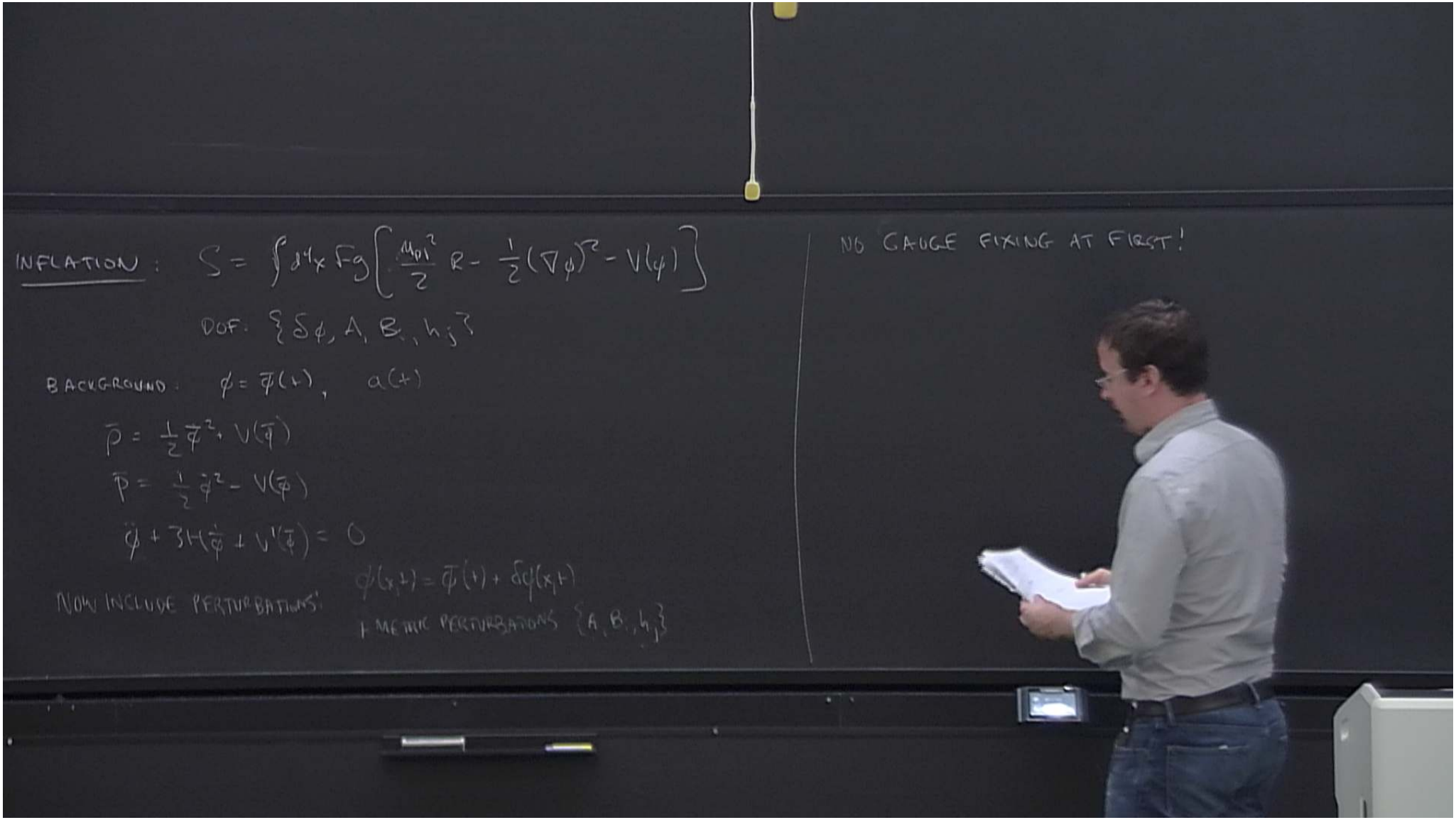
$$|u_q|^2 = \frac{(H_z)_*}{2k^3}\left\{1 + \text{TERMS OF } \mathcal{O}(\varepsilon,\eta)\right\}$$



WHERE $(\cdot)_*$ MEANS "EVALUATED AT HORIZON CROSSING" $k \sim zH_*$


H
k ~ zH_z
H_z
k^3 P(k)
H^2
k
|T(k)|^2


INFLATION: $S = \int d^4x \sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$
DOF: $\{\delta\phi, A, B_i, h_{ij}\}$


INFLATION : $S = \int d^4x \sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$
DOF: $\{\delta\phi, A, B_i, h_{ij}\}$
BACKGROUND: $\phi = \bar{\phi}(t)$, $a(t)$
$\bar{\rho} = \frac{1}{2}\dot{\bar{\phi}}^2 + V(\bar{\phi})$
$\bar{p} = \frac{1}{2}\dot{\bar{\phi}}^2 - V(\bar{\phi})$
$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V'(\bar{\phi}) = 0$
NOW INCLUDE PERTURBATIONS: $\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$
+ METRIC PERTURBATIONS $\{A, B_i, h_{ij}\}$
NO GAUGE FIXING AT FIRST!

$\ddot{\bar{\varphi}} + 3H\dot{\bar{\varphi}} + V'(\bar{\varphi}) = 0$

INCLUDE PERTURBATIONS: $\varphi(x,t) = \bar{\varphi}(t) + \delta\varphi(x,t)$

+ METRIC PERTURBATIONS $\{A, B_i, h_{ij}\}$

$\nabla_\mu \nabla^\mu$ $\varphi = \bar{\varphi} + \delta\varphi$

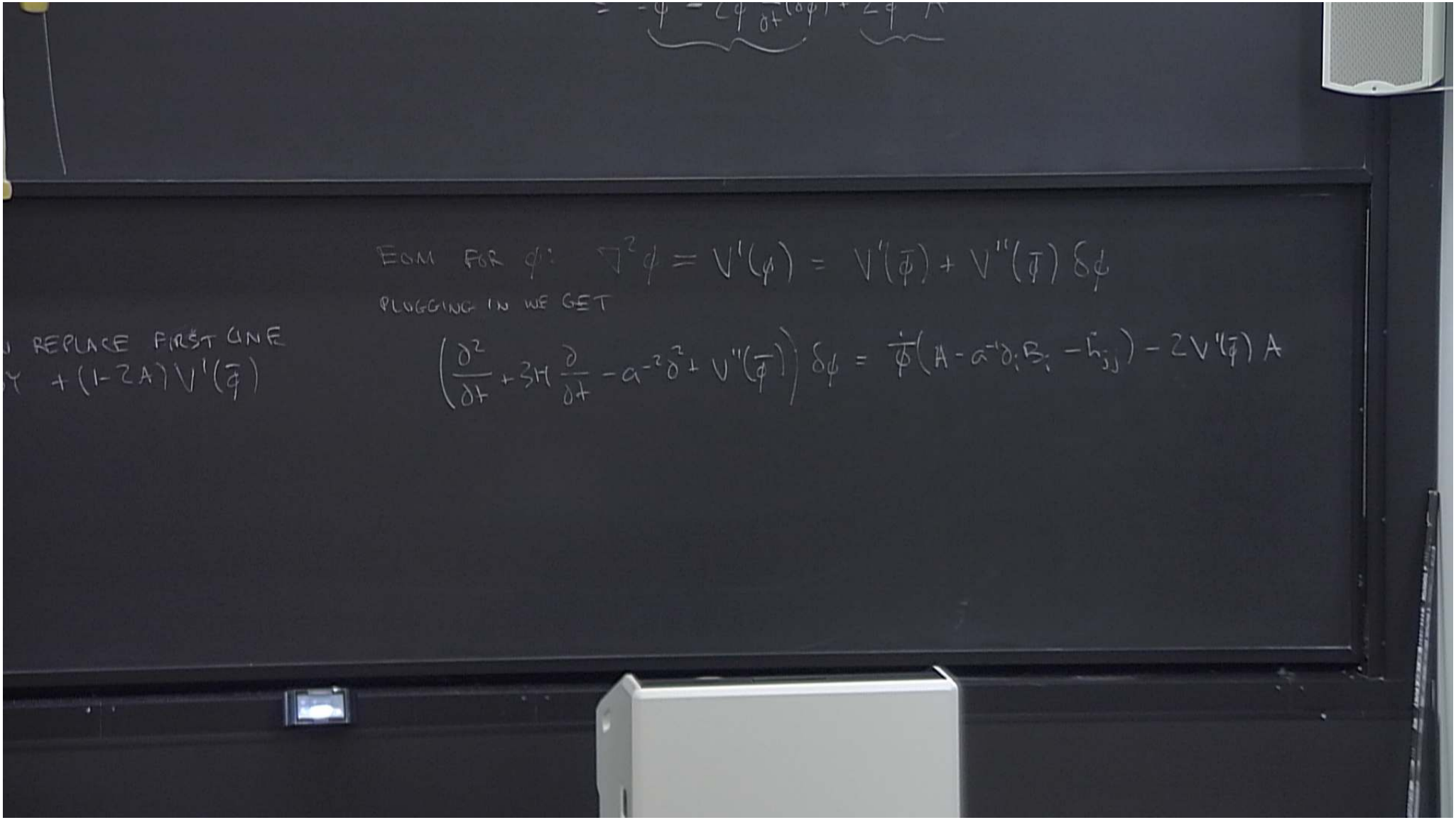
HARDER CALCULATION: $(\nabla_\mu \nabla^\mu \varphi)$ [HW]

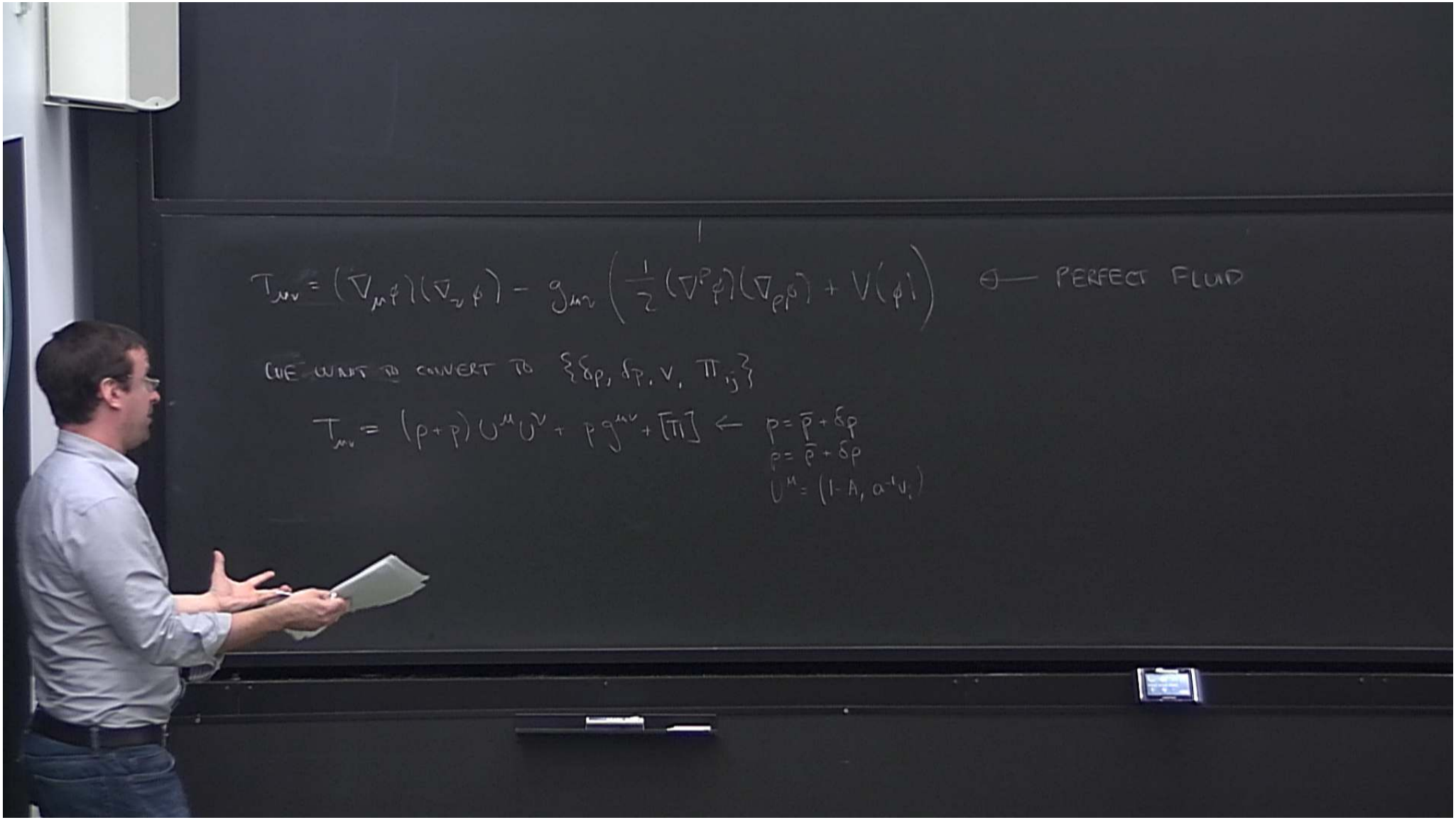
EOM FOR φ: $\nabla^2\varphi =$

$$\nabla^2\varphi = -(1-2A)\left(\ddot{\bar{\varphi}} + 3H\dot{\bar{\varphi}}\right) \quad \Leftarrow \text{CAN REPLACE FIRST LINE BY } +(1-2A)V'(\bar{\varphi})$$

$$+\left(-\frac{\partial^2}{\partial t^2} - 3H\frac{\partial}{\partial t} + a^{-2}\partial_i^2\right)(\delta\varphi)$$

$$+\dot{\bar{\varphi}}\left(A - a^{-1}\partial_i B_i - \dot{h}_{ii}\right)$$


EOM FOR ϕ: $\nabla^2\phi = V'(\phi) = V'(\bar{\phi}) + V''(\bar{\phi})\,\delta\phi$
PLUGGING IN WE GET
$\left(\frac{\partial^2}{\partial t} + 3H\frac{\partial}{\partial t} - a^{-2}\partial^2 + V''(\bar{\phi})\right)\delta\phi = \dot{\bar{\phi}}(A - a^{-1}\partial_i B_i - \dot{h}_{jj}) - 2V'(\bar{\phi})\,A$
REPLACE FIRST ONE
$+ (1-2A)V'(\bar{\phi})$


$T_{\mu\nu} = (\nabla_\mu \phi)(\nabla_\nu \phi) - g_{\mu\nu}\left(\frac{1}{2}(\nabla^\rho \phi)(\nabla_\rho \phi) + V(\phi)\right)$
← PERFECT FLUID
WE WANT TO CONVERT TO $\{\delta\rho, \delta p, v, \Pi_{ij}\}$
$T_{\mu\nu} = (\rho + p) U^\mu U^\nu + p g^{\mu\nu} + [\Pi]$ ←
$p = \bar{p} + \delta p$
$\rho = \bar{\rho} + \delta\rho$
$U^\mu = (1 - A, a^{-1} v_i)$

$$T_{\mu\nu} = \underbrace{(\nabla_\mu \phi)(\nabla_\nu \phi)}_{(\rho+p)U_\mu U_\nu} - g_{\mu\nu}\left(\underbrace{\frac{1}{2}(\nabla^\rho \phi)(\nabla_\rho \phi) + V(\phi)}_{p}\right) \leftarrow \text{PERFECT FLUID}$$

$$\bar{\rho} + \delta\rho = \frac{1}{2}(\nabla_\rho \phi)(\nabla^\rho \phi) + \ldots$$

$$=$$

WE WANT TO CONVERT TO $\{\delta\rho, \delta p, v, \Pi_{ij}\}$

$$T_{\mu\nu} = (\rho + p)U^\mu U^\nu + p g^{\mu\nu} + [\Pi] \leftarrow \begin{array}{l} \rho = \bar{\rho} + \delta\rho \\ p = \bar{p} + \delta p \\ U^\mu = (1 - A, a^{-1} v_i) \end{array} \leftarrow U^\mu U_\mu = -1$$

$\rho = \bar{\rho} + \delta\rho$

$U^\mu = (1 - A,\ a^{-1} v_i) \quad \Leftarrow \quad U^\mu U_\mu = -1$

$\delta\rho = \frac{\dot{\bar{\rho}}}{\dot{\bar{\phi}}} \frac{\partial(\delta\phi)}{\partial t} + \dot{\bar{\phi}}^2 A$

$\delta\rho = \frac{\dot{\bar{\rho}}}{\dot{\bar{\phi}}} \frac{\partial(\delta\phi)}{\partial t} + \dot{\bar{\phi}}^2 A$

$$V_i = \cdots = B_i - a^{-1} \frac{\partial_i(\delta\phi)}{\dot{\bar{\phi}}}$$

NOTE: IN EINSTEIN'S EQS THE COMBINATION WHICH APPEARS IS

$$(\bar{\rho} + \bar{p})(v_i - B_i) = -a^{-1} \dot{\bar{\phi}}\, \partial_i(\delta\phi)$$

$\rho = \bar{\rho} + \delta\rho$

$U^\mu = (1 - A,\ a^{-1} v_i) \quad \longleftarrow \quad U^\mu U_\mu = -1$

$\delta\rho = \dot{\bar{\phi}} \frac{\partial(\delta\phi)}{\partial t} + \dot{\bar{\phi}}^2 A - V'(\bar{\phi})\delta\phi$

$\delta\rho = \dot{\bar{\phi}} \frac{\partial(\delta\phi)}{\partial t} + \dot{\bar{\phi}}^2 A + V'(\bar{\phi})\delta\phi$

THE COMBINATION WHICH ... IS

$a^{-1} \dot{\bar{\phi}}\, \partial_i(\delta\phi)$

GAUGE FIXING

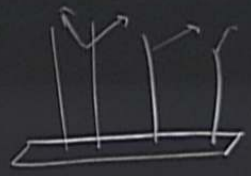
- TWO GAUGES

"SPATIALLY FLAT GAUGE"

$h_{ii} = 0 \qquad \partial_j h_{ij} = 0$

"UNITARY GAUGE"

$\delta\phi = 0 \qquad \partial_j h_{ij} = 0$



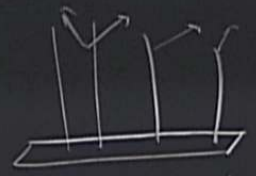
$U^\mu U_\mu = -1$

$$\delta\rho = \dot{\bar{\phi}}\frac{\partial(\delta\phi)}{\partial t} + \dot{\bar{\phi}}^2 A - V'(\bar{\phi})\delta\phi$$

$$\delta\rho = \dot{\bar{\phi}}\frac{\partial(\xi\dot{\phi})}{\partial t} + \dot{\bar{\phi}}^2 A + V'(\bar{\phi})\delta\phi$$

PEARS IS

GAUGE FIXING



- TWO GAUGES

"SPATIALLY FLAT GAUGE"

$h_{ii} = 0 \qquad \partial_j h_{ij} = 0$

$(\alpha, \beta^i) = \varepsilon^\mu$

"UNITARY GAUGE"

$\delta\phi = 0 \qquad \partial_j h_{ij} = 0$

$h_{ij}(\vec{k})$ $\quad h_{ij}(\vec{k}) = k_j h_{i}$

$\vec{k} = k\hat{z}$ $\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$U^\mu U_\mu = -1$

$$\delta\rho = \dot{\bar{\phi}}\frac{\partial(\delta\phi)}{\partial t} + \dot{\bar{\phi}}^2 A - V'(\bar{\phi})\delta\phi$$

$$\delta\rho = \dot{\bar{\phi}}\frac{\partial(\delta\phi)}{\partial t} + \dot{\bar{\phi}}^2 A + V'(\bar{\phi})\delta\phi$$

PEARS IS

GAUGE FIXING

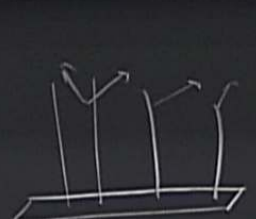
- TWO GAUGES

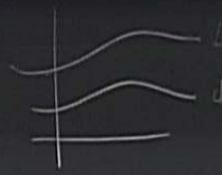
"SPATIALLY FLAT GAUGE"

$h_{ii} = 0 \qquad \partial_j h_{ij} = 0$ ← SPATIAL SLICE HAS GRAVITY WAVES BUT NO POTENTIAL WELLS

"UNITARY GAUGE"

$\delta\phi = 0 \qquad \partial_j h_{ij} = 0$ ← CONVERTED $\delta\phi$ FLUCTUATIONS TO POTENTIAL WELLS





$h_{ij}(\vec{k})$

$h_{ij}(\vec{k}) = k_j h_{ij}(\vec{k}) = 0$

$(\alpha, \beta^i) = \varepsilon^\mu$

$\vec{k} = k\hat{z}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = h_{ij}^{+}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} h_{ij}^{\times}$$