Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 7

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Abstract:

$$\hat{H}(t) = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\omega(t)^2\hat{x}^2$$

$\omega(t) \to \omega_0$ AS $t \to -\infty$

$|\psi\rangle = |0\rangle$ AS $t \to -\infty$

$$\hat{x}_H(t) = u(t)\,a + u^*(t)\,a^\dagger$$

$$\hat{p}_H(t) = \dot{u}(t)\,a + \dot{u}^*(t)\,a^\dagger$$

$$\ddot{u}(t) = -\omega(t)^2 u(t)$$

$$u(t) \to (2\omega_0)^{-1/2} e^{-i\omega_0 t}$$

(AN COMPUTE ANYTHING, E.G. $\langle\psi(t)|\hat{x}^2|\psi(t)\rangle = u(t)\,u(t)^*$

WAVEFUNCTIONS ARE GAUSSIAN

MASSLESS TEST SCALAR IN DE SITTER

$$S = \frac{1}{2}\int \sqrt{-g}\, g^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi)$$

$$S_\varphi = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\frac{1}{(H\tau)^2}\left[(\partial_\tau\varphi_k^*)(\partial_\tau\varphi_k) - (\partial_i\varphi_k)^*(\partial_i\varphi_k)\right]$$

"NATURAL"

$$S_\chi = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[(\partial_\tau\chi_k^*)(\partial_\tau\chi_k) - \left(k^2 - \frac{2}{\tau^2}\right)\chi_k^*\chi_k\right]$$

"CANONICALLY NORMALIZED"

$$\chi = \frac{a}{H\tau}$$

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"CANONICALLY NORMALIZED"

$$\chi = \frac{a}{H\tau}$$

"BUNCH-DAVIES VACUUM": EVERY χ_k IS IN ITS GROUND STATE

$$\dot{X} = \frac{dX}{dt} \qquad X' = \frac{dX}{d\tau} \quad \leftarrow \text{NOTATION}$$

$$\begin{cases} \hat{\psi}_k^H(\tau) = u_\psi(k,\tau)\, a_k + u_\psi^*(k,\tau)\, a_{-k}^\dagger \\ \hat{\psi}_k'^H(\tau) = u_\psi'(k,\tau)\, a_k + u_\psi'^*(k,\tau)\, a_{-k}^\dagger \end{cases} \qquad [a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^3(k-k')$$

WHERE $\quad \frac{\partial^2}{\partial\tau^2} u_\psi(k,\tau) + \left(k^2 - \frac{2}{\tau^2}\right) u_\psi(k,\tau) = 0$

$$u_\psi(k,\tau) \to (2k)^{-1/2} e^{-ik\tau} \quad \text{AS } \tau \to -\infty$$

$$\begin{cases} \hat{\psi}_k^H(\tau) = u_\psi(k,\tau)\, a_k + u_\psi^*(k,\tau)\, a_{-k}^\dagger \\ \hat{\pi}_k^H(\tau) = u_\psi'(k,\tau)\, a_k + u_\psi'^*(k,\tau)\, a_{-k}^\dagger \end{cases} \qquad [a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^3(k-k')$$

WHERE $\dfrac{\partial^2}{\partial\tau^2} u_\psi(k,\tau) + \left(k^2 - \dfrac{2}{\tau^2}\right) u_\psi(k,\tau) = 0 \qquad (\star)$

$$u_\psi(k,\tau) \to (2k)^{-1/2} e^{-ik\tau} \quad \text{AS } \tau \to -\infty$$

EQ $(\star)$ HAS AN ANALYTIC SOLUTION:

$$u_\psi(k,\tau) = (2k)^{-1/2}\left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}$$

KIND OF HARD TO FIND. E.G. $u'' + \left(k^2 - \frac{2-\varepsilon}{\tau^2}\right) u \Rightarrow u \propto \tau^{1/2} H_\nu(k\tau)$ $\varepsilon = m^2/H^2$ $\nu = \sqrt{\frac{9}{4} - \varepsilon}$

OF COURSE WE ALSO HAVE

$$\hat{\phi}_k^H(\tau) = u_\phi(k,\tau)\hat{a}_k + u_\phi^*(k,\tau)\, a_{-k}^\dagger$$

$$\hat{\phi}_k'^H(\tau) = u_\phi'(k,\tau)\hat{a}_k + u_\phi'^*(k,\tau)\, a_{-k}^\dagger$$

WHERE $u_\phi = H\varepsilon u_\psi = \frac{H}{2k^{3/2}}(1 + ik\tau)\, e^{-ik\tau}$

↳ THE FORM OF THE MODE FUNCTION YOU'LL FIND IN BOOKS

A FUNDAMENTAL RESULT! THE POWER SPECTRUM $P_\phi(k)$

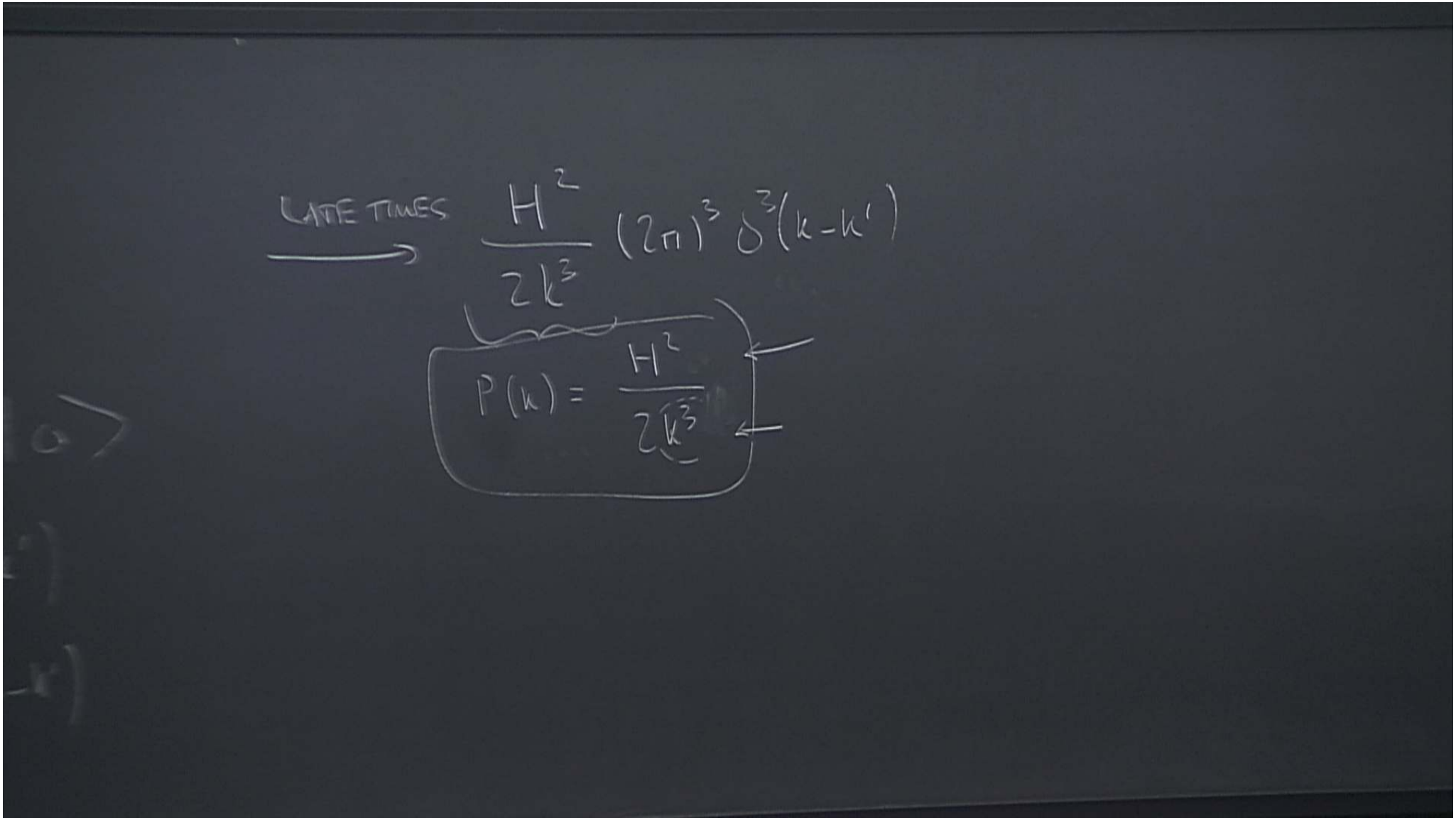
AT LATE TIMES

$$\langle \psi(t) | \hat{\phi}_k^\dagger \hat{\phi}_{k'} | \psi(t) \rangle = \langle 0 | \hat{\phi}_k^{H\dagger} \hat{\phi}_{k'}^{H} | 0 \rangle$$

$$= \langle 0 | \left(u_\phi(k,\tau) a_{-k} \right) \left(u_\phi^*(k,\tau) a_{-k}^\dagger \right) | 0 \rangle$$

$$= u_\phi(k,\tau)^* u_\phi(k,\tau) (2\pi)^3 \delta^3(k-k')$$

$$= \frac{H^2}{2k^3} \left(1 + k^2\tau^2\right) (2\pi)^3 \delta^3(k-k')$$


LATE TIMES
$\frac{H^2}{2k^3}(2\pi)^3\delta^3(k-k')$
$P(k) = \frac{H^2}{2k^3}$

DID WE NEED TO CANONICALLY NORMALIZE?
WHY NOT JUST COMPUTE u_ϕ BY SOLVING THIS EQ?

$$\frac{d}{d\tau}\left(\tau^{-2}\frac{du_\phi}{d\tau}\right) + k^2\tau^{-2}u_\phi = 0$$

PROBLEM: HOW DO YOU GET THE NORMALIZATION?

$$u_\phi(k,\tau) = A(k)(1+ik\tau)e^{-ik\tau}$$

↑
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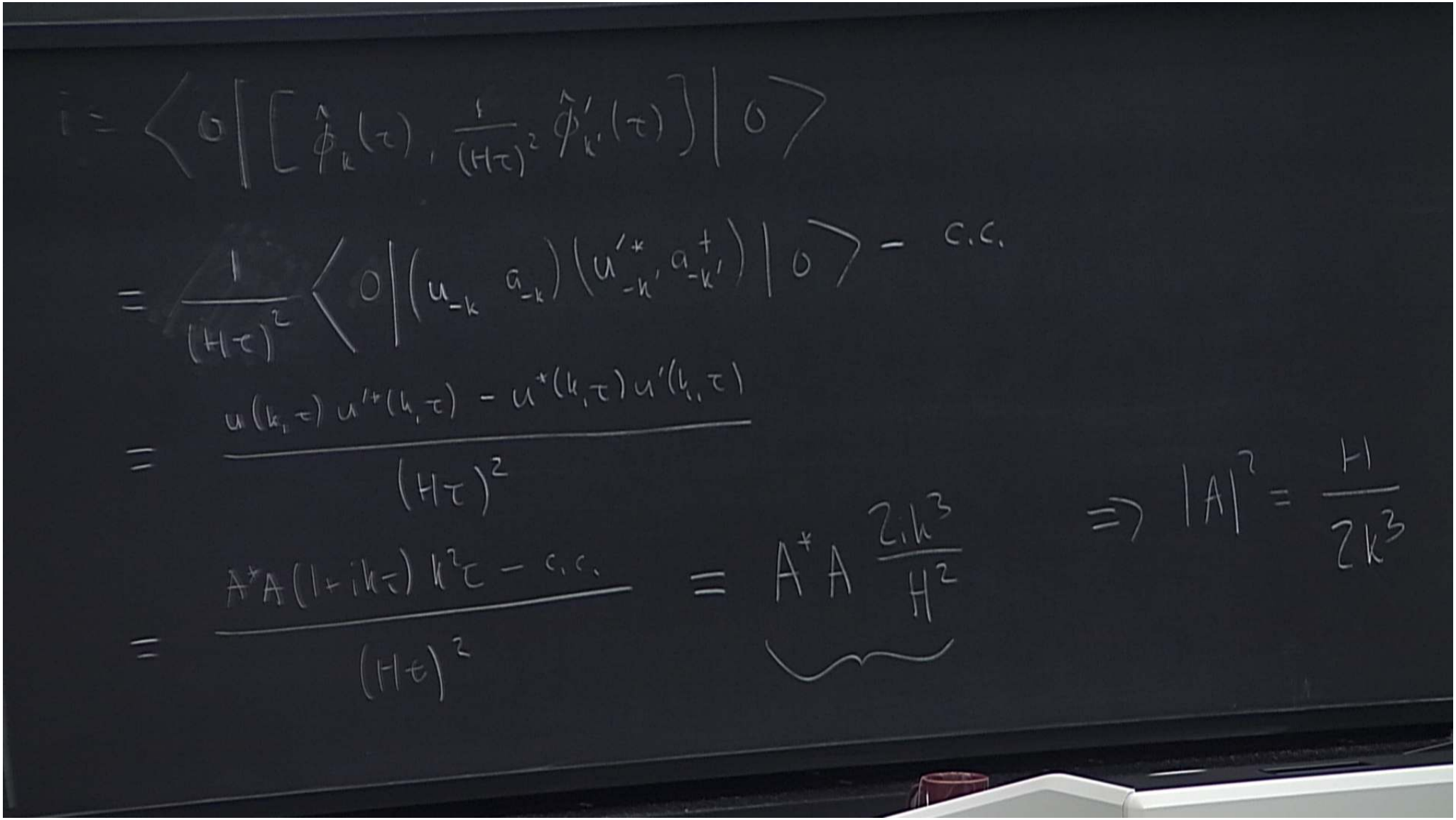
IT CAN BE INFERRED FROM THE CANONICAL COMMUTATOR

$$\left[\hat{\phi}^H_k(\tau),\ \hat{P}^H_{k'}(\tau)\right] = i(2\pi)^3\delta^3(k+k') \quad \text{WHERE } P = \frac{\partial S}{\partial \phi'}$$


$i = \langle 0 | \left[\hat{\phi}_k(\tau), \frac{1}{(H\tau)^2} \hat{\phi}'_{k'}(\tau) \right] | 0 \rangle$
$= \frac{1}{(H\tau)^2} \langle 0 | \left(u_{-k} \; a_{-k} \right) \left(u'^*_{-k'} \; a^\dagger_{-k'} \right) | 0 \rangle - c.c.$
$= \frac{u(k,\tau) u'^*(k,\tau) - u^*(k,\tau) u'(k,\tau)}{(H\tau)^2}$
$= \frac{A^* A (1 + ik\tau) k^2 \tau - c.c.}{(H\tau)^2} = A^* A \frac{2ik^3}{H^2}$
$\Rightarrow |A|^2 = \frac{H}{2k^3}$

$$(\partial_\tau^2 \psi) + \left(k^2 - \frac{2}{\tau^2}\right)\psi = 0$$

$$(\partial_\tau^2 \psi) + \frac{2}{(-\tau)}(\partial_\tau \psi) + k^2 \psi = 0$$

↑ HUBBLE FRICTION

$\frac{1}{aH}$ = COMOVING HUBBLE LENGTH

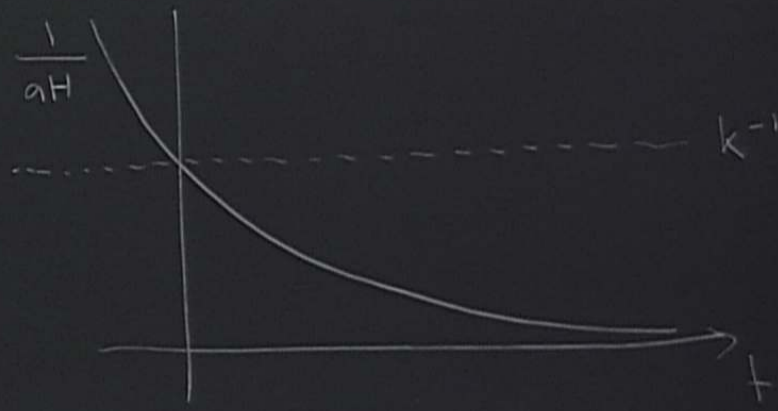
= "CAUSAL HORIZON"

$= -\tau = \frac{1}{H} e^{-Ht}$, A DECREASING FUNCTION

SIMPLE AT EARLY TIMES $\tau \ll -k^{-1}$
(FREE FIELD IN MINKOWSKI)

SIMPLE AT LATE TIMES $\tau \gg -k^{-1}$

ψ = CONSTANT, PINNED BY HUBBLE FRICTION



AT EARLY TIMES $\tau \ll -k^{-1}$
E FIELD IN MINKOWSKI)

AT LATE TIMES $\tau \gg -k^{-1}$
CONSTANT, PINNED BY HUBBLE FRICTION

$\rho = \psi^* \psi$

WAVELENGTH ↓ HORIZON ↓

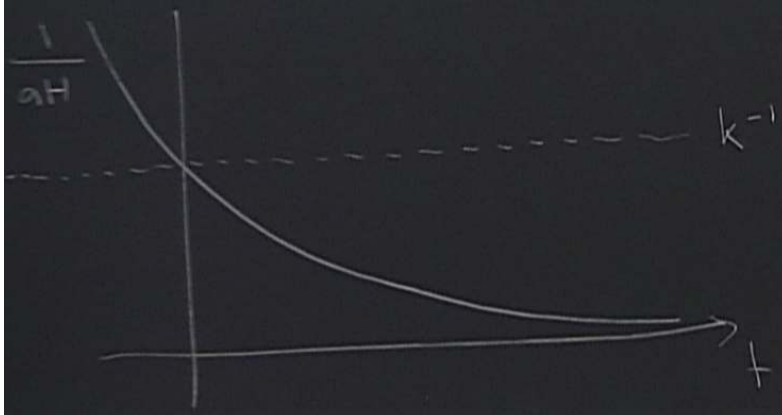
EARLY TIMES $(k^{-1} \ll (aH)^{-1})$

MODE "THINKS" IT'S A FREE FIELD IN MINKOWSKI

LATE TIMES $(k^{-1} \gg (aH)^{-1})$

ϕ = CONST. BY CAUSALITY

- FLUCTUATIONS "BECOME CLASSICAL" AT LATE TIMES



$\rho = \psi^* \psi$

$\phi_k^{(H)} = u_k a_k + u_k^* a_{-k}^\dagger$

$u_k = (1 + ik\tau) e^{-ik\tau}$

$\frac{du}{d\tau} = k^2 \tau e^{-ik\tau}$

$\rightarrow 0$ AT LATE TIMES

$\star$ $[\phi, \phi'] \rightarrow 0$ AT LATE TIMES

$\Rightarrow [\phi, \tau^{-1}\phi'] \rightarrow$ CONSTANT

WAVELENGTH ↓ k^{-1} $\ll$ $(aH)^{-1}$ ↓ HORIZON

EARLY TIMES

MODE "THINKS" IT'S A FREE FIELD IN MINKOWSKI

LATE TIMES $(k^{-1} \gg (aH)^{-1})$

ϕ = CONST. BY CAUSALITY

- FLUCTUATIONS "BECOME CLASSICAL" AT LATE TIMES
- PURE STATE