Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 7

Date: Apr 19, 2016 10:15 AM

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Abstract:

$$\hat{H}(t) = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\omega(t)^2\hat{x}^2$$

$\omega(t) \to \omega_0$ AS $t \to -\infty$

$|\psi\rangle = |0\rangle$ AS $t \to -\infty$

$$\hat{x}_H(t) = u(t)\,a + u^*(t)\,a^\dagger$$

$$\hat{p}_H(t) = \dot{u}(t)\,a + \dot{u}^*(t)\,a^\dagger$$

$$\ddot{u}(t) = -\omega(t)^2 u(t)$$

$$u(t) \to (2\omega_0)^{-1/2} e^{-i\omega_0 t}$$

CAN COMPUTE ANYTHING, E.G. $\langle\psi(t)|\hat{x}^2|\psi(t)\rangle = u(t)u(t)^*$

WAVEFUNCTIONS ARE GAUSSIAN

MA

MASSLESS TEST SCALAR IN DE SITTER

$$S = -\frac{1}{2}\int \sqrt{-g}\, g^{\mu\nu}(\partial_\mu \phi)(\partial_\nu \phi)$$

$$S_\phi = -\frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\frac{1}{(H\tau)^2}\left[(\partial_\tau \phi_k^*)(\partial_\tau \phi_k) - (\partial_i \phi_k)^*(\partial_i \phi_k)\right]$$ "NATURAL"

$$S_\psi = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[(\partial_\tau \psi_k^*)(\partial_\tau \psi_k) - \left(k^2 - \frac{2}{\tau^2}\right)\psi_k^* \psi_k\right]$$ "CANONICALLY NORMALIZED"

$$\psi = \frac{\phi}{H\tau}$$

MASSLESS TEST SCALAR IN DE SITTER

$$S = -\frac{1}{2}\int \sqrt{-g}\, g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$S_\phi = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3} \frac{1}{(H\tau)^2}\left[(\partial_\tau \phi_k^*)(\partial_\tau \phi_k) - (\partial_i \phi_k)^*(\partial_i \phi_k)\right]$$ "NATURAL"

$$S_\psi = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[(\partial_\tau \psi_k^*)(\partial_\tau \psi_k) - \left(k^2 - \frac{2}{\tau^2}\right)\psi_k^* \psi_k\right]$$ "CANONICALLY NORMALIZED" $\psi = \frac{\phi}{H\tau}$

"BUNCH-DAVIES VACUUM": EVERY ψ_k IS IN ITS GROUND STATE

$\dot{X} = \frac{dX}{dt}$ $\quad X' = \frac{dX}{d\tau}$ $\leftarrow$ NOTATION

$$\begin{cases} \hat{\psi}^H_k(\tau) = u_\psi(k,\tau)\, a_k + u^*_\psi(k,\tau)\, a^\dagger_{-k} \\ \hat{\psi}'^H_k(\tau) = u'_\psi(k,\tau)\, a_k + u'^*_\psi(k,\tau)\, a^\dagger_{-k} \end{cases}$$

$$[a_k, a^\dagger_{k'}] = (2\pi)^3 \delta^3(k-k')$$

WHERE $\dfrac{\partial^2}{\partial\tau^2} u_\psi(k,\tau) + \left(k^2 - \dfrac{2}{\tau^2}\right) u_\psi(k,\tau) = 0$

$$u_\psi(k,\tau) \to (2k)^{-1/2} e^{-ik\tau} \quad \text{AS } \tau \to -\infty$$

$$\begin{cases} \hat{\psi}^{H}_{k}(\tau) = u_\psi(k,\tau)\, a_k + u^*_\psi(k,\tau)\, a^\dagger_{-k} \\ \hat{\psi}'^{H}_{k}(\tau) = u'_\psi(k,\tau)\, a_k + u'^*_\psi(k,\tau)\, a^\dagger_{-k} \end{cases}$$

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WHERE $$\frac{\partial^2}{\partial\tau^2} u_\psi(k,\tau) + \left(k^2 - \frac{2}{\tau^2}\right) u_\psi(k,\tau) = 0 \qquad (*)$$

$$u_\psi(k,\tau) \to (2k)^{-1/2} e^{-ik\tau} \quad \text{AS } \tau \to -\infty$$

EQ (*) HAS AN ANALYTIC SOLUTIONS:

$$u_\psi(k,\tau) = (2k)^{-1/2}\left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}$$

KIND OF HARD TO FIND. E.G. $u'' + \left(k^2 - \frac{2-\varepsilon}{\tau^2}\right) u \Rightarrow u \propto \tau^{1/2} H_\nu(k\tau)$ $\quad \nu = \sqrt{\frac{9}{4} - \varepsilon}$ $\quad \varepsilon = m^2/H^2$

OF COURSE WE ALSO HAVE

$$\hat{\phi}^H_k(\tau) = u_\phi(k,\tau)\hat{a}_k + u^*_\phi(k,\tau) a^\dagger_{-k}$$

$$\hat{\phi}'^H_k(\tau) = u'_\phi(k,\tau)\hat{a}_k + u'^*_\phi(k,\tau) a^\dagger_{-k}$$

WHERE $u_\phi = H\tau u_\psi = \frac{H}{2k^{3/2}}(1 + ik\tau)e^{-ik\tau}$

THE FORM OF THE MODE FUNCTION YOU'LL FIND IN BOOKS

A FUNDAMENTAL RESULT! THE POWER SPECTRUM $P_\phi(k)$

AT LATE TIMES

$$\langle \psi(t) | \hat{\phi}_k^{\dagger} \hat{\phi}_{k'} | \psi(t) \rangle = \langle 0 | \hat{\phi}_k^{H\dagger} \hat{\phi}^H_{k'} | 0 \rangle$$

$$= \langle 0 | \left(u_\phi(k,\tau) a_{-k} \right) \left(u_\phi^*(k,\tau) \, a^\dagger_{-k} \right) | 0 \rangle$$

$$= u_\phi(k,\tau)^* u_\phi(k,\tau) \, (2\pi)^3 \delta^3(k-k')$$

$$= \frac{H^2}{2k^3} \left(1 + k^2 \tau^2 \right) (2\pi)^3 \delta^3(k-k')$$

LATE TIMES $\longrightarrow$ $\underbrace{\frac{H^2}{2k^3}}(2\pi)^3\delta^3(k-k')$

$$\boxed{P(k) = \frac{H^2}{2k^3}}$$

DID WE NEED TO CANONICALLY NORMALIZE?
WHY NOT JUST COMPUTE u_ϕ BY SOLVING THIS EQ?

$$\frac{d}{d\tau}\left(\tau^{-2}\frac{du_\phi}{d\tau}\right) + k^2\tau^{-2}u_\phi = 0$$

PROBLEM: HOW DO YOU GET THE NORMALIZATION?

$$u_\phi(k,\tau) = A(k)\,(1+ik\tau)\,e^{-ik\tau}$$

$$\uparrow$$

$$?$$

IT CAN BE INFERRED FROM THE CANONICAL COMMUTATOR

$$\left[\hat{\phi}^H_k(\tau),\ \hat{P}^H_{\phi k'}(\tau)\right] = i(2\pi)^3\delta^3(k+k') \qquad \text{WHERE } P = \frac{\partial S}{\partial \phi'}$$

$$i = \left\langle 0 \left| \left[\hat{\phi}_k(\tau), \frac{1}{(H\tau)^2} \hat{\phi}'_{k'}(\tau) \right] \right| 0 \right\rangle$$

$$= \frac{1}{(H\tau)^2} \left\langle 0 \left| \left(u_{-k} \, a_{-k} \right) \left(u'^{*}_{-k'} \, a^{\dagger}_{-k'} \right) \right| 0 \right\rangle - c.c.$$

$$= \frac{u(k,\tau)\, u'^{*}(k,\tau) - u^{*}(k,\tau)\, u'(k,\tau)}{(H\tau)^2}$$

$$= \frac{A^{*}A\,(1+ik\tau)\, k^2 \tau - c.c.}{(H\tau)^2} = \underbrace{A^{*}A \, \frac{2ik^3}{H^2}} \qquad \Rightarrow |A|^2 = \frac{H}{2k^3}$$

$$(\partial_\tau^2 \psi) + \left(k^2 - \frac{2}{\tau^2}\right)\psi = 0$$

$$(\partial_\tau^2 \phi) + \underbrace{\frac{2}{(-\tau)}}_{\text{HUBBLE FRICTION}}(\partial_\tau \phi) + k^2 \phi = 0$$

$$\frac{1}{aH} = \text{COMOVING HUBBLE LENGTH}$$

$$= \text{"CAUSAL HORIZON"}$$

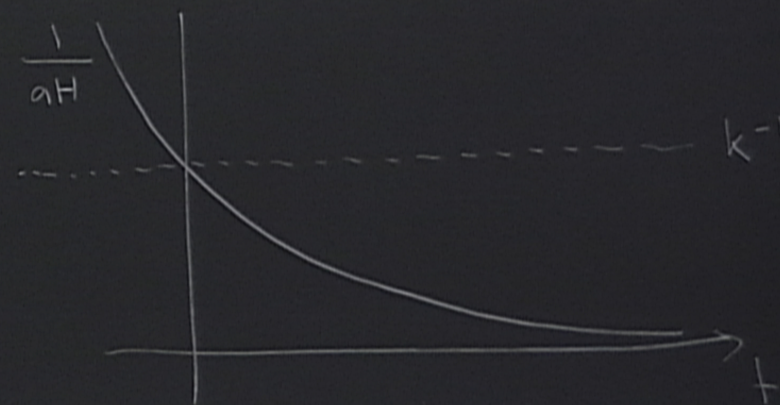
$$= -\tau = \frac{1}{H} e^{-Ht}, \text{ A DECREASING FUNCTION}$$

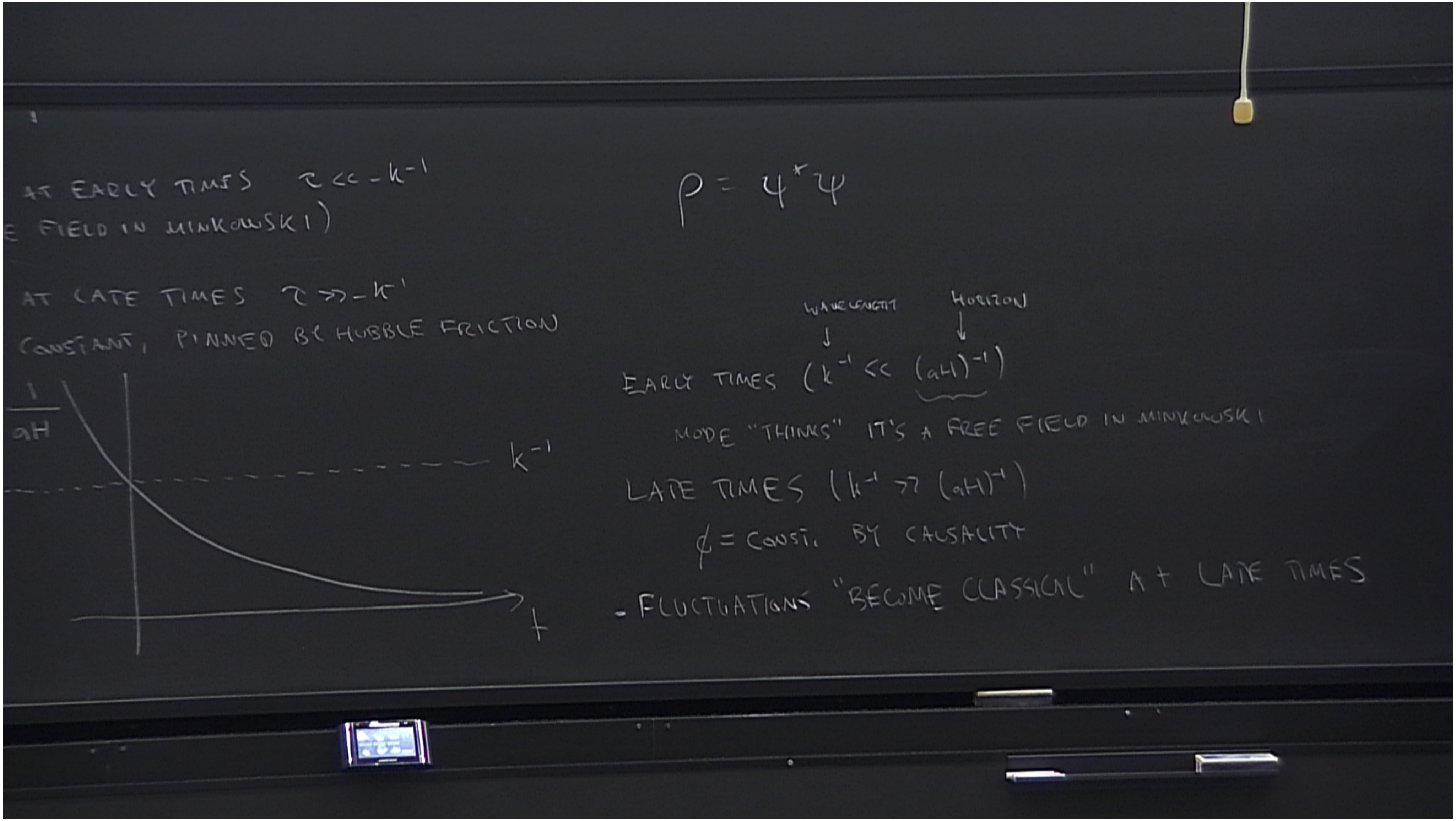
SIMPLE AT EARLY TIMES $\tau \ll -k^{-1}$

(FREE FIELD IN MINKOWSKI)

SIMPLE AT LATE TIMES $\tau \gg -k^{-1}$

ϕ = CONSTANT, PINNED BY HUBBLE FRICTION




AT EARLY TIMES τ << -k⁻¹
E FIELD IN MINKOWSKI)
AT LATE TIMES τ >> -k⁻¹
CONSTANT, PINNED BY HUBBLE FRICTION
1/aH
k⁻¹
t
ρ = ψ*ψ
WAVELENGTH
HORIZON
EARLY TIMES (k⁻¹ << (aH)⁻¹)
MODE "THINKS" IT'S A FREE FIELD IN MINKOWSKI
LATE TIMES (k⁻¹ >> (aH)⁻¹)
ϕ = CONST. BY CAUSALITY
- FLUCTUATIONS "BECOME CLASSICAL" AT LATE TIMES

$\rho = \psi^* \psi$

$\phi'^{H}_{k} = u'_k a_k + u'^{*}_k a^{\dagger}_{-k}$

$\rightarrow 0$ AT LATE TIMES

$u_k = (1 + ik\tau) e^{-ik\tau}$

$\frac{du}{d\tau} = k^2 \tau e^{-ik\tau}$

$\star$ $[\phi, \phi'] \rightarrow 0$ AT LATE TIMES

$\Rightarrow [\phi, \tau^{-1}\phi'] \rightarrow$ CONSTANT

EARLY TIMES $\left(\overset{\text{WAVELENGTH}}{\overset{\downarrow}{k^{-1}}} \ll \overset{\text{HORIZON}}{\overset{\downarrow}{\underbrace{(aH)^{-1}}}}\right)$

MODE "THINKS" IT'S A FREE FIELD IN MINKOWSKI

LATE TIMES $(k^{-1} \gg (aH)^{-1})$

ϕ = CONST. BY CAUSALITY

- FLUCTUATIONS "BECOME CLASSICAL" AT LATE TIMES
- PURE STATE