Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 6

Date: Apr 18, 2016 10:15 AM

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Abstract:

$\nabla^\mu \phi \nabla_\mu \phi - V(\phi) \}$

$\left(\frac{1}{2} (\nabla^\rho \phi)(\nabla_\rho \phi) + V(\phi) \right)$

<u>BACKGROUND</u> $\phi = \phi(t)$ AND $ds^2 = -dt^2 + a(t)^2 dx^2$

$$\Rightarrow T^{\mu\nu} = \begin{pmatrix} \rho & \\ & a^{-2}\delta_{ij} p \end{pmatrix} \qquad \begin{aligned} \rho &= \tfrac{1}{2}\dot\phi^2 + V(\phi) \\ p &= \tfrac{1}{2}\dot\phi^2 - V(\phi) \end{aligned}$$

EOM OF INFLATON: $\nabla^2 \phi = V'(\phi)$

USEFUL GENERAL FORMULA: IN UNPERTURBED FRW, THE DIVERGENCE IS

$$\begin{aligned} \nabla_\mu X^\mu &= \partial_\mu X^\mu + \Gamma^\mu_{\mu\nu} X^\nu \\ &= \partial_\mu X^\mu + 3H X^0 \end{aligned} \qquad \begin{cases} \Gamma^\mu_{\mu 0} = 3H \\ \Gamma^\mu_{\mu i} = 0 \end{cases}$$

BACKGROUND $\phi = \phi(t)$ AND $ds^2 = -dt^2 + a(t)^2 dx^2$

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USEFUL GENERAL FORMULA: IN UNPERTURBED FRW, THE DIVERGENCE IS

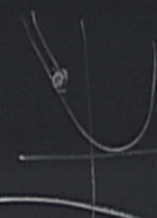
$$\nabla_\mu X^\mu = \partial_\mu X^\mu + \Gamma^\mu_{\mu\nu} X^\nu \qquad \begin{cases} \Gamma^\mu_{\mu 0} = 3H \\ \Gamma^\mu_{\mu i} = 0 \end{cases} \qquad \nabla_\mu X^\mu = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} X^\mu)$$

$$= \partial_\mu X^\mu + 3H X^0$$

$(\phi))$

$$\nabla^0 \phi = -\frac{\partial \phi}{\partial t}$$

$$\Rightarrow \nabla_\mu \nabla^\mu \phi = -\frac{\partial^2 \phi}{\partial t^2} - 3H \frac{\partial \phi}{\partial t}$$



"SLOW

EOM: $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

"HUBBLE FRICTION TERM"

$+$ FRIEDMANN $\left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{1}{3M_{pl}^2}\rho = \frac{1}{3M_{pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$

SLOW

E.C.

DETERMINE EVOLUTION OF BACKGROUND

"SLOW ROLL" APPROXIMATION TO EQS OF MOTION IS:

$$3H\dot{\phi} + V'(\phi) \approx 0$$

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 \approx \frac{1}{3M_{pl}^2} V(\phi)$$

SLOW ROLL IS NICE BECAUSE WE CAN CONVERT ANY QUANTITY TO LOCAL DERIVS OF ϕ

E.G. $$[H]_{SR} \approx M_{pl}^{-1}\left(\frac{V}{3}\right)^{1/3}$$

$$[\dot{\phi}]_{SR} \approx -\frac{V'}{3H} \approx -M_{pl}\frac{V'}{(3V)^{1/2}}$$

$$[\ddot{\phi}]_{SR} \approx -M_{pl}\frac{d}{d\phi}\left[\frac{V'}{(3V)^{1/2}}\right]\dot{\phi}_* = M_{pl}^2\left(\frac{V'V''}{3V} - \frac{V'^3}{6V^2}\right)$$

FOR WHICH POTENTIALS $V(\phi)$ IS SLOW-ROLL APPROX?

$$\ddot{\phi}_{SR} \ll 3H\dot{\phi}_{SR} \qquad \frac{1}{2}\dot{\phi}_{SR}^2 \ll V$$

$$\Rightarrow \quad M_{pl}^2\left[\frac{V'V''}{3V} - \frac{V'^3}{6V^2}\right] \ll V' \qquad M_{pl}^2 \frac{V'^2}{6V} \ll V$$

INTRODUCE THE SLOW ROLL PARAMETERS

$$\boxed{\epsilon \overset{def}{=} \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = M_{pl}^2\left(\frac{V''}{V}\right)} \leftarrow \text{"FLAT POTENTIAL"}$$

E.G. $V(\phi) \sim \phi^p$

THIS BECOMES $\frac{\epsilon - \eta}{3} \ll 1 \qquad \frac{\epsilon}{3} \ll 1$

$$\Leftrightarrow \epsilon, \eta \ll 1$$

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← "FLAT POTENTIAL"

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$\eta, \varepsilon \sim M_{pl}^2\phi^{-2}$

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$$\Leftrightarrow \varepsilon, \eta \ll 1$$

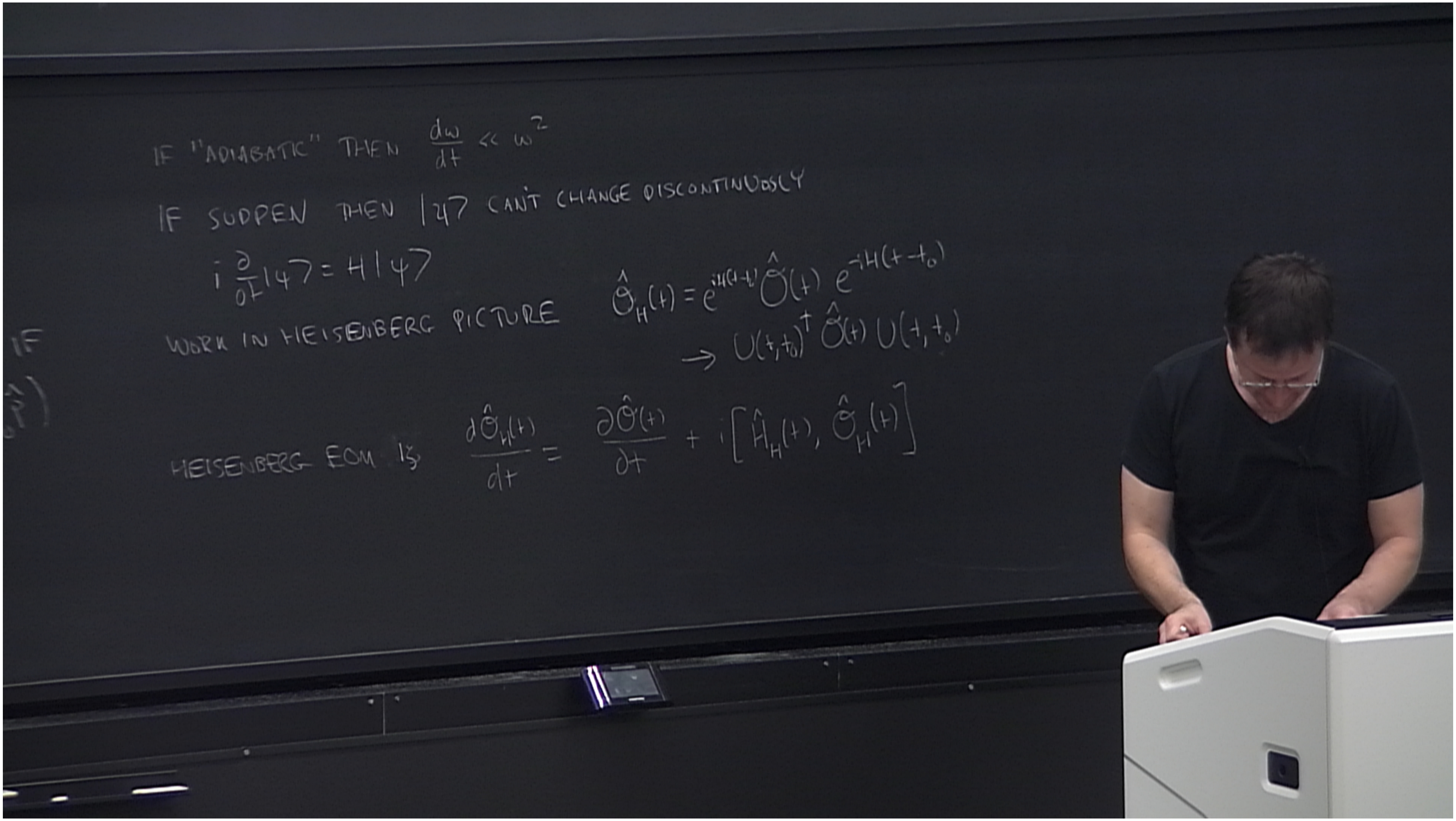
$$\left[\ddot\phi\right]_{SR} \approx -M_{pl}\frac{d}{d\phi}\left(\frac{V'}{(2V)^{1/2}}\right) \qquad \ddot\phi_{*} = M_{pl}^{2}\left(\frac{V''}{3V} - \frac{V'^{2}}{6V^{2}}\right)$$

SLOW ROLL ALSO MEANS "IN APPROPRIATE UNITS, NOTHING CHANGES MUCH IN A HUBBLE TIME"

E.G. $\dfrac{\dot\phi}{H}$ = CHANGE IN ϕ PER HUBBLE

$$= (2\varepsilon)^{1/2} M_{pl}$$

$$\frac{\dot H}{H} = -\varepsilon H \iff w_{SR} = -1 + \frac{2}{3}\varepsilon \iff a \sim e^{Ht}$$


IF "ADIABATIC" THEN $\frac{d\omega}{dt} \ll \omega^2$
IF SUDDEN THEN $|\psi\rangle$ CAN'T CHANGE DISCONTINUOUSLY
$i \frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$
WORK IN HEISENBERG PICTURE $\hat{O}_H(t) = e^{iH(t-t_0)}\hat{O}(t)\, e^{-iH(t-t_0)}$
$\rightarrow U(t,t_0)^\dagger \hat{O}(t)\, U(t,t_0)$
HEISENBERG EOM IS $\frac{d\hat{O}_H(t)}{dt} = \frac{\partial \hat{O}(t)}{\partial t} + i\left[\hat{H}_H(t), \hat{O}_H(t)\right]$

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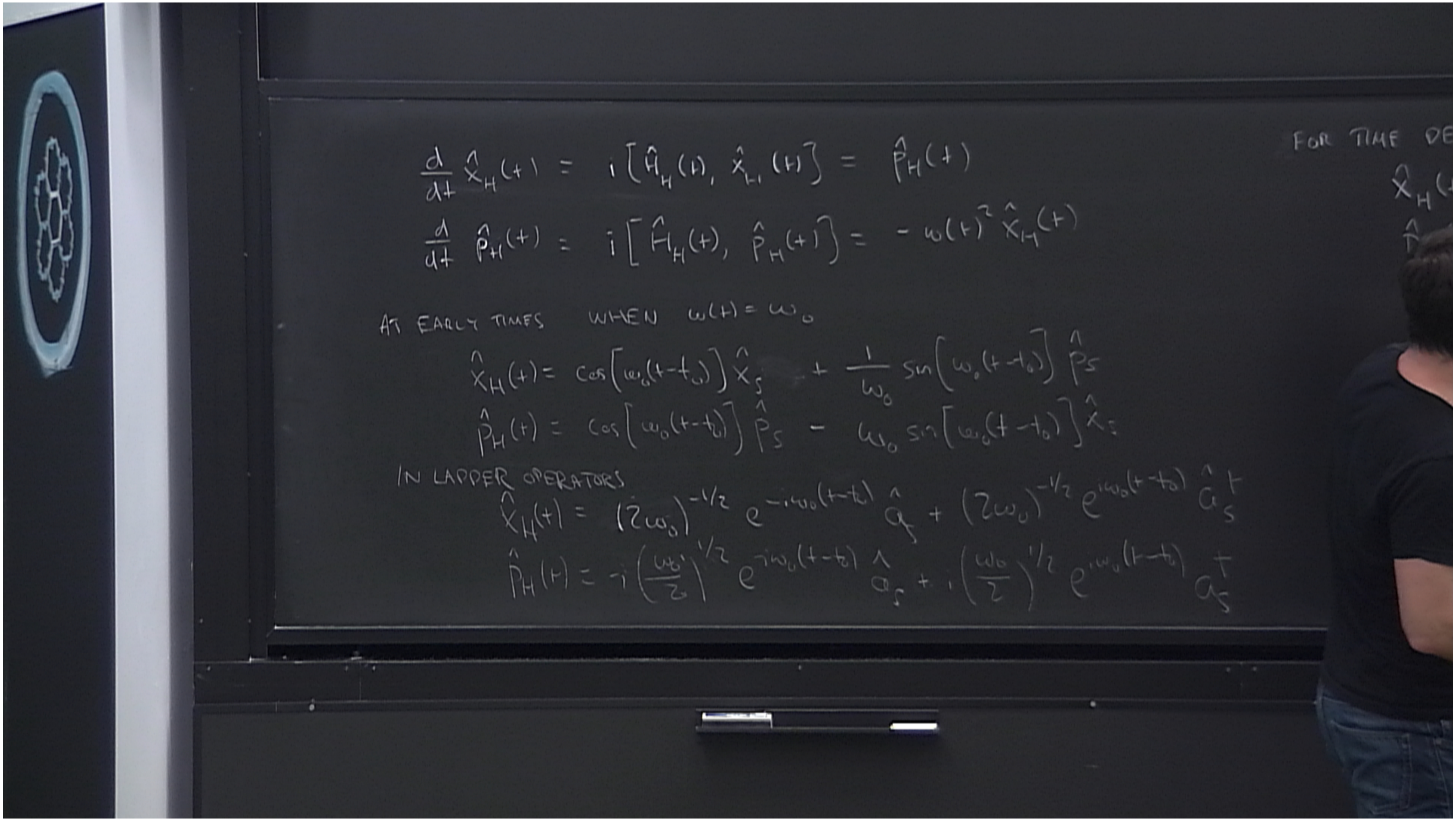
HEISENBERG EOM IS $\frac{d\hat{O}_H(t)}{dt} = \frac{\partial \hat{O}(t)}{\partial t} + i\left[\hat{H}_H(t), \hat{O}_H(t)\right]$

$$\langle\psi(t)|\hat{O}|\psi(t)\rangle = \langle\psi_0|\hat{O}_H(t)|\psi_0\rangle$$


$\frac{d}{dt}\hat{x}_H(t) = i[\hat{H}_H(t), \hat{x}_H(t)] = \hat{p}_H(t)$
$\frac{d}{dt}\hat{p}_H(t) = i[\hat{H}_H(t), \hat{p}_H(t)] = -\omega(t)^2\hat{x}_H(t)$
AT EARLY TIMES WHEN $\omega(t) = \omega_0$
$\hat{x}_H(t) = \cos[\omega_0(t-t_0)]\hat{x}_S + \frac{1}{\omega_0}\sin[\omega_0(t-t_0)]\hat{p}_S$
$\hat{p}_H(t) = \cos[\omega_0(t-t_0)]\hat{p}_S - \omega_0\sin[\omega_0(t-t_0)]\hat{x}_S$
IN LADDER OPERATORS
$\hat{x}_H(t) = (2\omega_0)^{-1/2}e^{-i\omega_0(t-t_0)}\hat{a}_S + (2\omega_0)^{-1/2}e^{i\omega_0(t-t_0)}\hat{a}_S^\dagger$
$\hat{p}_H(t) = -i\left(\frac{\omega_0}{2}\right)^{1/2}e^{-i\omega_0(t-t_0)}\hat{a}_S + i\left(\frac{\omega_0}{2}\right)^{1/2}e^{i\omega_0(t-t_0)}a_S^\dagger$
FOR TIME DE

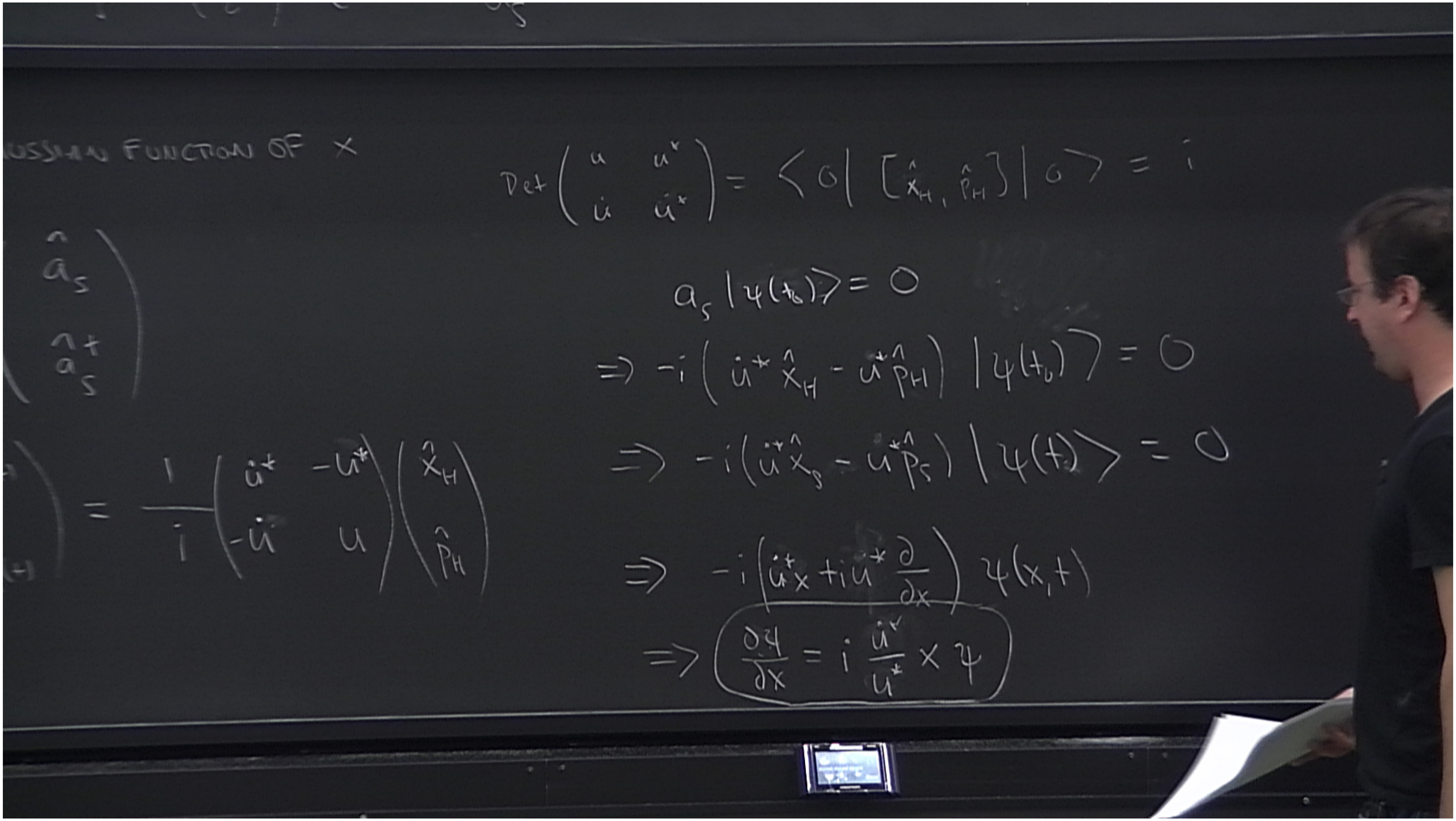
THE WAVEFUNCTION $\psi(x,t)$ IS A GAUSSIAN FUNCTION OF x

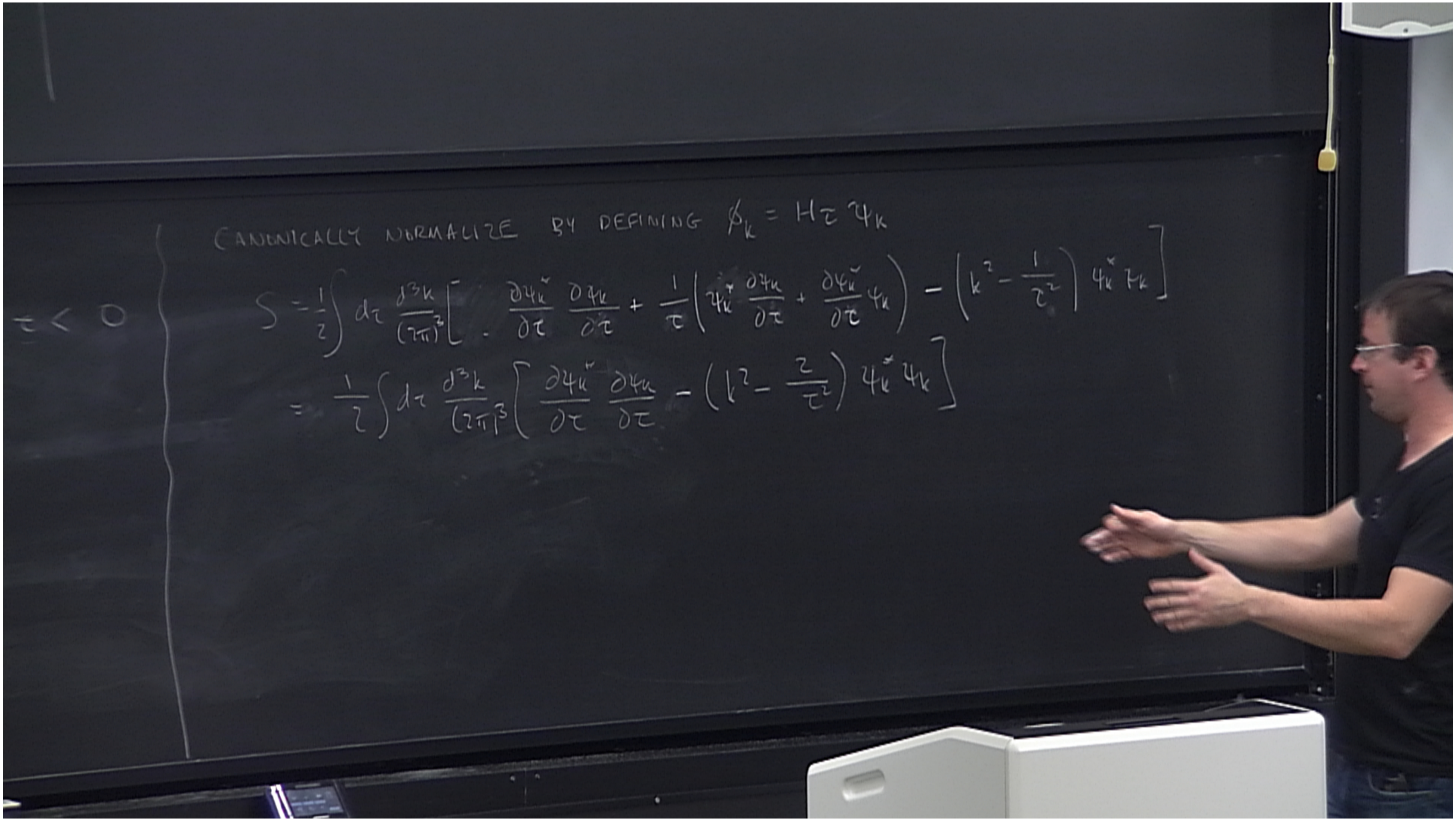
LET'S PROVE THIS!

$$\begin{pmatrix} \hat{x}_H(t) \\ \hat{p}_H(t) \end{pmatrix} = \begin{pmatrix} u(t) & u^*(t) \\ \dot{u}(t) & \dot{u}^*(t) \end{pmatrix} \begin{pmatrix} \hat{a}_s \\ \hat{a}_s^+ \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \hat{a}_s \\ \hat{a}_s^+ \end{pmatrix} = \begin{pmatrix} u & u^* \\ \dot{u} & \dot{u}^* \end{pmatrix}^{-1} \begin{pmatrix} \hat{x}_H(t) \\ \hat{p}_H(t) \end{pmatrix} = \frac{1}{i} \begin{pmatrix} \dot{u}^* & -\dot{u} \\ -u^* & u \end{pmatrix} \begin{pmatrix} \hat{x}_H \\ \hat{p}_H \end{pmatrix}$$

Det (


USSIAN FUNCTION OF X
$\mathrm{Det}\begin{pmatrix} u & u^* \\ \dot u & \dot u^* \end{pmatrix} = \langle 0 | [\hat x_H, \hat p_H] | 0 \rangle = i$
$\begin{pmatrix} \hat a_s \\ \hat a_s^+ \end{pmatrix}$
$= \frac{1}{i}\begin{pmatrix} \dot u^* & -u^* \\ -\dot u & u \end{pmatrix}\begin{pmatrix} \hat x_H \\ \hat p_H \end{pmatrix}$
$a_s |\psi(t_0)\rangle = 0$
$\Rightarrow -i(\dot u^* \hat x_H - u^* \hat p_H)|\psi(t_0)\rangle = 0$
$\Rightarrow -i(\dot u^* \hat x_s - u^* \hat p_s)|\psi(t)\rangle = 0$
$\Rightarrow -i\left(\dot u^* x + i u^* \frac{\partial}{\partial x}\right)\psi(x,t)$
$\Rightarrow \boxed{\frac{\partial \psi}{\partial x} = i\,\frac{\dot u^*}{u^*}\, x\, \psi}$


$\tau < 0$
CANONICALLY NORMALIZE BY DEFINING $\phi_k = H\tau\psi_k$
$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[\frac{\partial\psi_k^*}{\partial\tau}\frac{\partial\psi_k}{\partial\tau} + \frac{1}{\tau}\left(\psi_k^*\frac{\partial\psi_k}{\partial\tau} + \frac{\partial\psi_k^*}{\partial\tau}\psi_k\right) - \left(k^2 - \frac{1}{\tau^2}\right)\psi_k^*\psi_k\right]$
$= \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[\frac{\partial\psi_k^*}{\partial\tau}\frac{\partial\psi_k}{\partial\tau} - \left(k^2 - \frac{2}{\tau^2}\right)\psi_k^*\psi_k\right]$