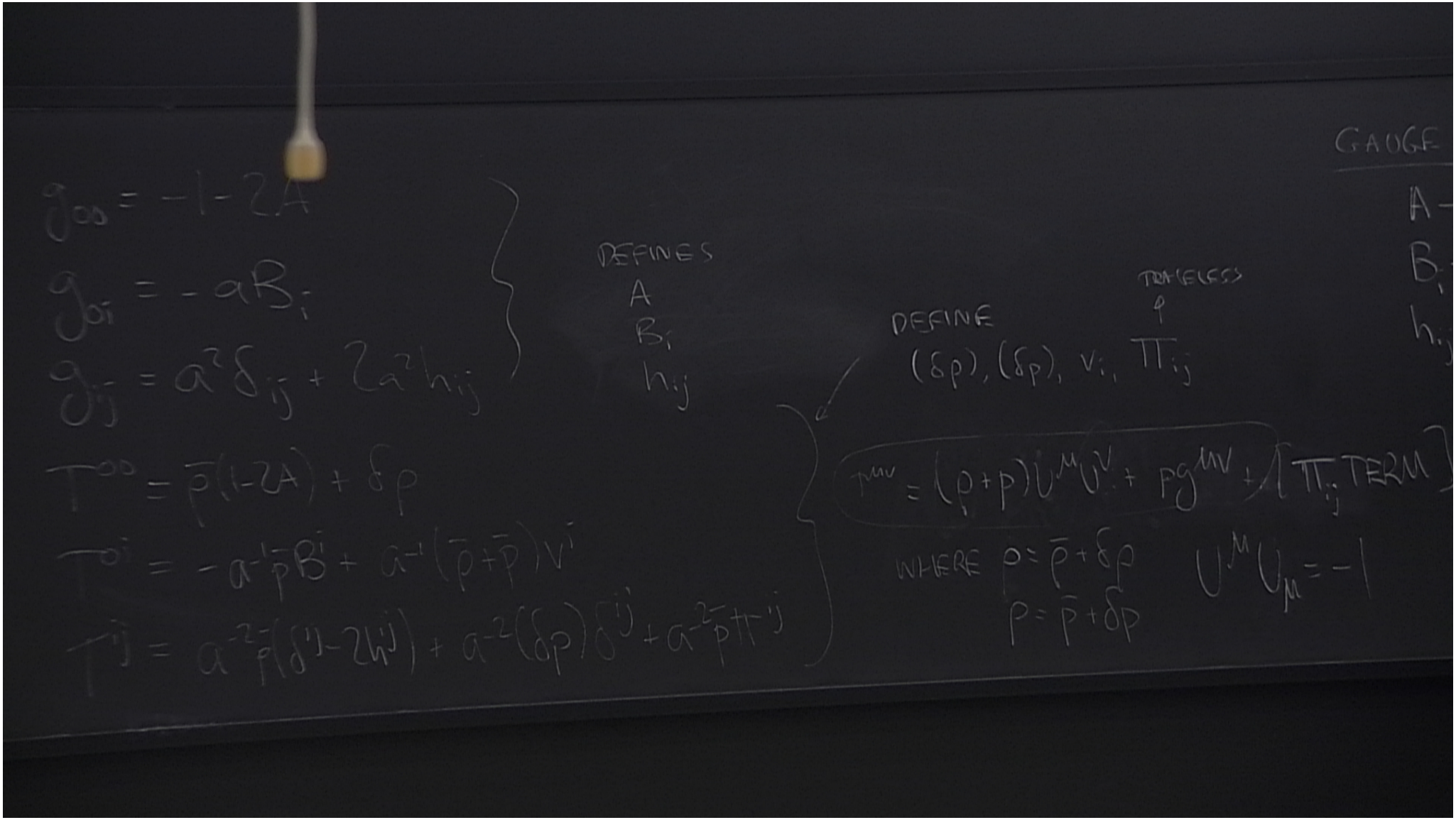
Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 5

Date: Apr 15, 2016 10:15 AM

URL: http://pirsa.org/16040039

Abstract:


$g_{00} = -1 - 2A$
$g_{0i} = -aB_i$
$g_{ij} = a^2\delta_{ij} + 2a^2 h_{ij}$
DEFINES
A
B_i
h_{ij}
DEFINE
$(\delta\rho), (\delta p), v_i, \Pi_{ij}$
TRACELESS
$T^{00} = \bar{\rho}(1-2A) + \delta\rho$
$T^{0i} = -a^{-1}\bar{p}B^i + a^{-1}(\bar{\rho}+\bar{p})v^i$
$T^{ij} = a^{-2}\bar{p}(\delta^{ij} - 2h^{ij}) + a^{-2}(\delta p)\delta^{ij} + a^{-2}\Pi^{ij}$
$T^{\mu\nu} = (\rho + p)U^\mu U^\nu + pg^{\mu\nu} + [\Pi_{ij}\text{ TERM}]$
WHERE $\rho = \bar{\rho} + \delta\rho$
$p = \bar{p} + \delta p$
$U^\mu U_\mu = -1$
GAUGE
A =
B_i
h_{ij}

GAUGE $x'^\mu = x^\mu + \varepsilon^\mu(x)$ $(\varepsilon^0, \varepsilon^i) = (\alpha, \beta^i)$

$A \to A - \dot{\alpha}$

$B_i \to B_i - a^{-1}\partial_i\alpha + a\dot{\beta}_i$

$h_{ij} \to -H\delta_{ij}\alpha - \frac{1}{2}(\partial_i\beta_j + \partial_j\beta_i)$

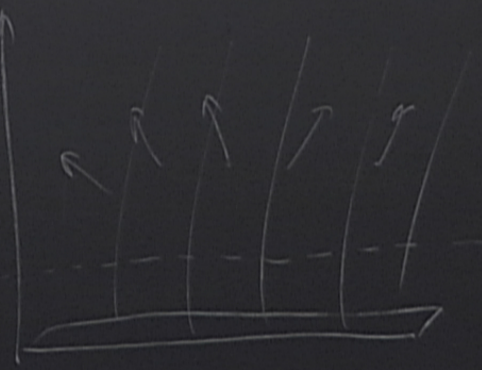
TRACELESS

Π_{ij}

$(\rho + p)U^\mu U^\nu + pg^{\mu\nu} + [\Pi_{ij}\ \text{TERM}]$

$\rho = \bar{\rho} + \delta\rho$

$U^\mu U_\mu = -1$



$$\Delta T^{\mu\nu} = -\mathcal{L}_\xi T^{\mu\nu} = -\varepsilon^\lambda \partial_\lambda \bar{T}^{\mu\nu} + (\partial_\lambda \varepsilon^\mu)\bar{T}^{\lambda\nu} + (\partial_\lambda \varepsilon^\nu)\bar{T}^{\mu\lambda}$$

E.G. $\Delta T^{00} = -\varepsilon^0 \partial_0 \bar{\rho} + 2(\partial_0 \varepsilon^0)\bar{T}^{00}$

$= -\alpha \dot{\bar{\rho}} + 2\dot{\alpha}\bar{\rho}$

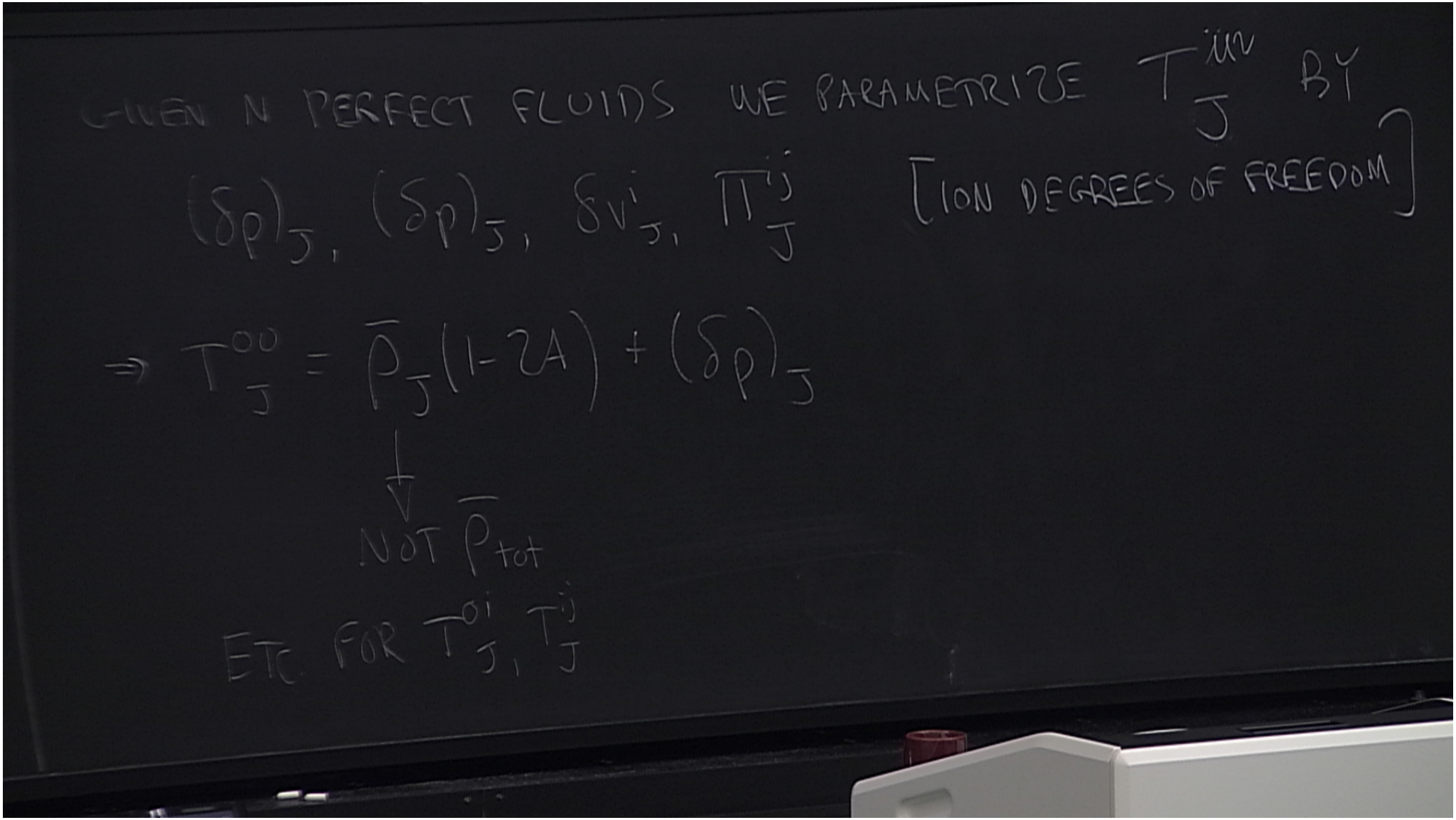
SINCE $T^{00} = \bar{\rho}(1-2A) + \delta\rho$ AND $\Delta A = -\alpha$

$\Rightarrow \Delta(\delta\rho) = -\dot{\bar{\rho}}\alpha$

SIMILARLY $\Delta(\delta p) = -\dot{\bar{p}}\alpha$

$\Delta v_i = \alpha\beta_i$

$\Delta \Pi_{ij} = 0$


GIVEN N PERFECT FLUIDS WE PARAMETRIZE $T^{\mu\nu}_J$ BY
$(\delta\rho)_J$, $(\delta p)_J$, δv^i_J, Π^{ij}_J [10N DEGREES OF FREEDOM]
$\Rightarrow T^{00}_J = \bar{\rho}_J(1-2A) + (\delta\rho)_J$
NOT $\bar{\rho}_{tot}$
ETC. FOR T^{0i}_J, T^{ij}_J

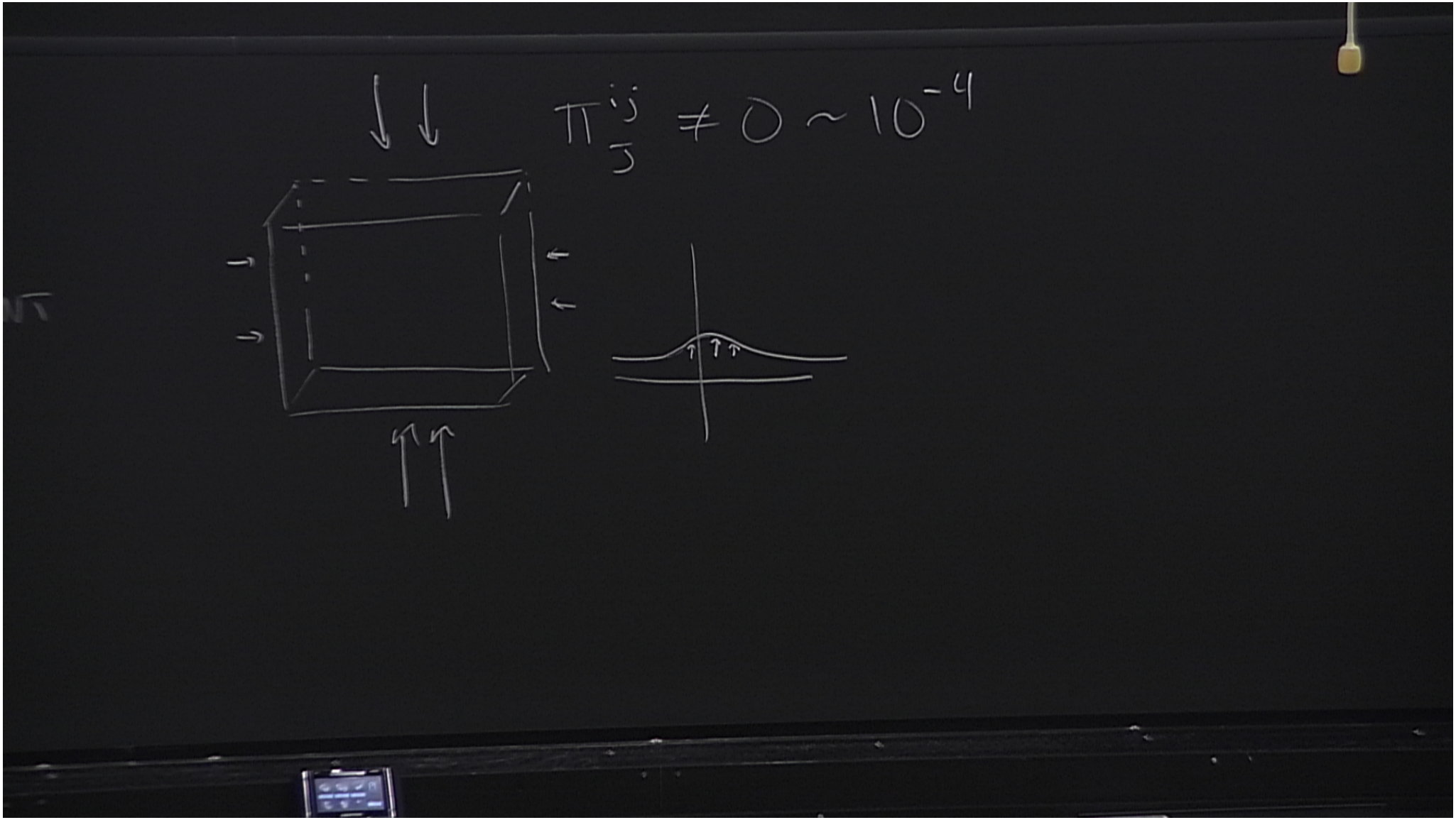
- VELOCITIES V^i_j ARE NOT GAUGE INVARIANT

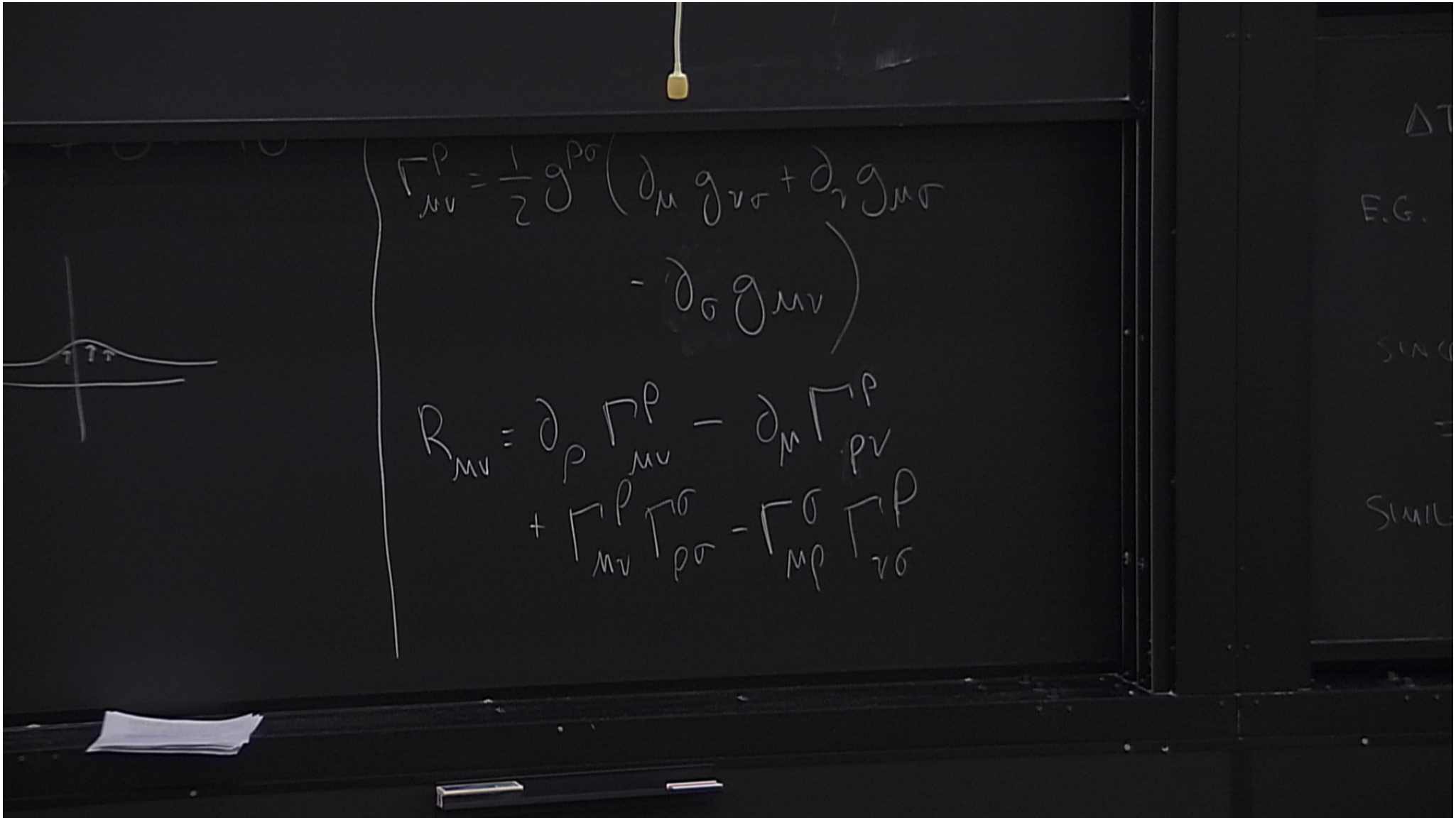
  BUT $(V^i_J - V^i_K)$ IS.

- ANISOTROPIC STRESS $\underbrace{\Pi^i_j}_{J}$ IS GAUGE INVARIANT

- $\delta\rho_I$, δP_I ARE NOT GAUGE INVARIANT, BUT

$$\frac{\delta\rho_I}{\dot{\bar{\rho}}_I} - \frac{\delta\rho_J}{\dot{\bar{\rho}}_J}$$ IS GAUGE INVARIANT

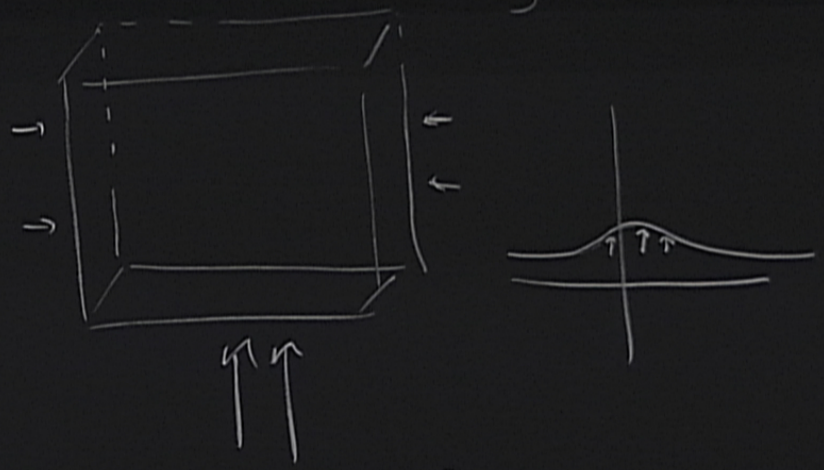

$\pi^{ij}_{J} \neq 0 \sim 10^{-4}$


$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right)$
$R_{\mu\nu} = \partial_\rho \Gamma^{\rho}_{\mu\nu} - \partial_\mu \Gamma^{\rho}_{\rho\nu} + \Gamma^{\rho}_{\mu\nu} \Gamma^{\sigma}_{\rho\sigma} - \Gamma^{\sigma}_{\mu\rho} \Gamma^{\rho}_{\nu\sigma}$
E.G.

E INVARIANT

NT, BUT

ARIANT



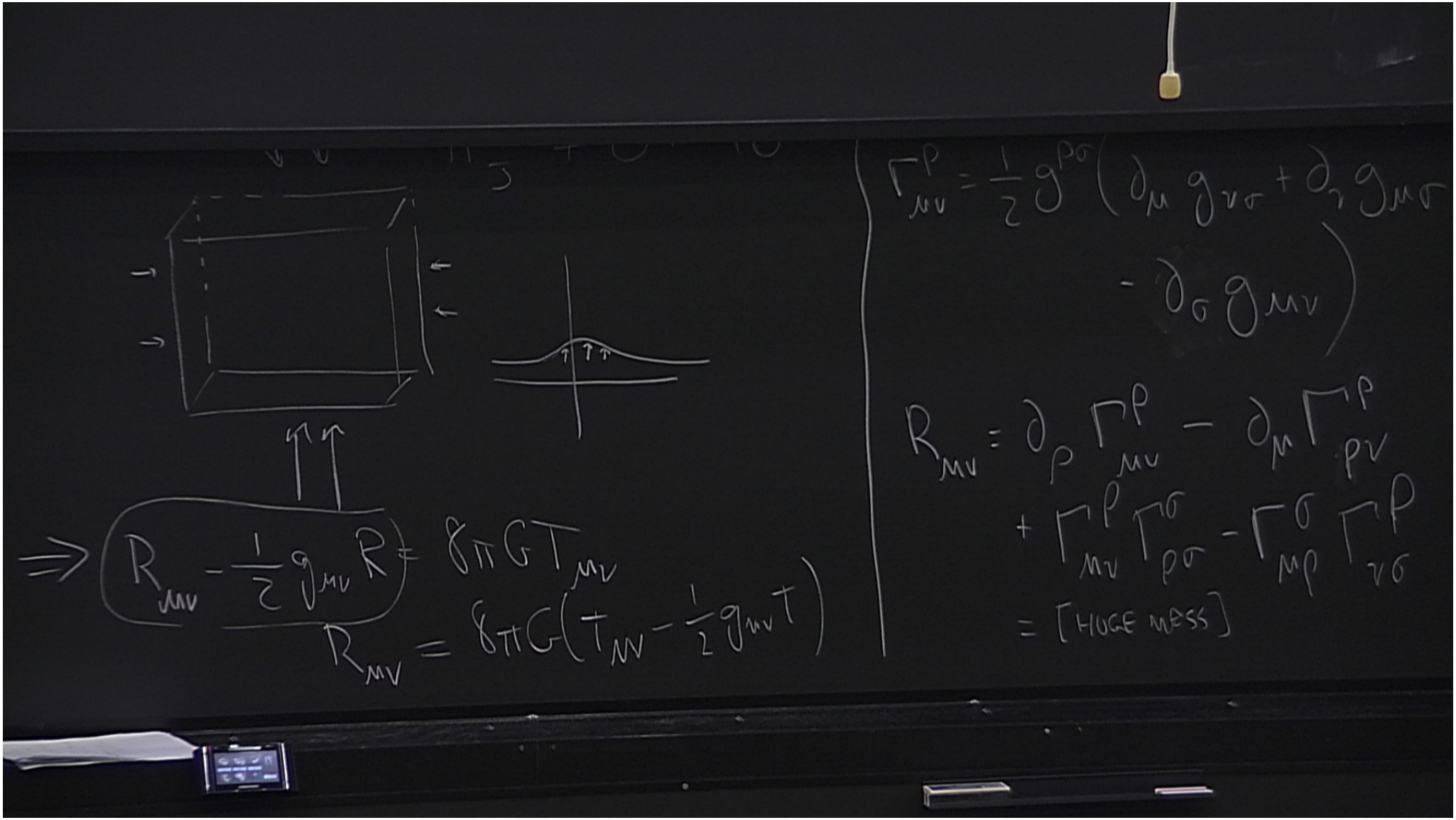
$$\Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$$

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\right.$$

$$R_{\mu\nu} = \partial_{\rho} \Gamma$$

$$+ \Gamma^{\rho}_{\mu\nu}$$


$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right)$
$R_{\mu\nu} = \partial_\rho \Gamma^{\rho}_{\mu\nu} - \partial_\mu \Gamma^{\rho}_{\rho\nu} + \Gamma^{\rho}_{\mu\nu} \Gamma^{\sigma}_{\rho\sigma} - \Gamma^{\sigma}_{\mu\rho} \Gamma^{\rho}_{\nu\sigma}$
$= [\text{HUGE MESS}]$
$\Rightarrow \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = 8\pi G T_{\mu\nu}$
$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$

$$\Gamma^0_{0i} = H\delta^j_i + \dot{h}^j_i - \frac{1}{2}a^{-1}\partial_i B^j + \frac{1}{2}a^{-1}\partial^j B_i$$

$$\Gamma^k_{ij} = \partial_i h^k_j + \partial_j h^k_i - \partial^k h_{ij} + aH\delta_{ij}B^k$$

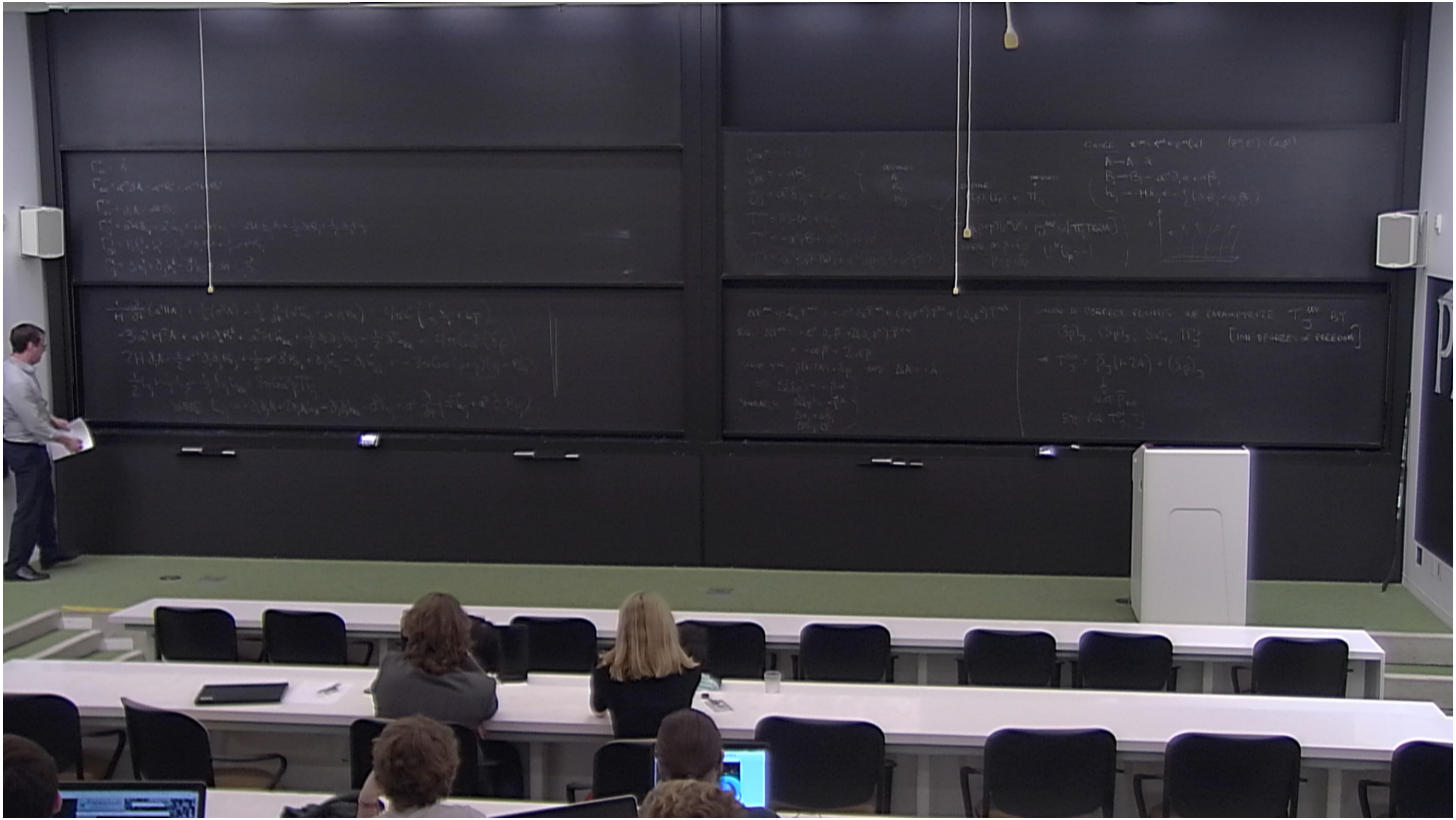
$$\frac{1}{H}\frac{\partial}{\partial t}\left(a^2 H A\right) + \frac{1}{3}\left(\partial^2 A\right) - \frac{1}{3}\frac{\partial}{\partial t}\left(a^2\dot{h}_{ii} + a\,\partial_i B_i\right) = 4\pi G\left(\frac{1}{3}\delta\rho + \delta P\right)$$

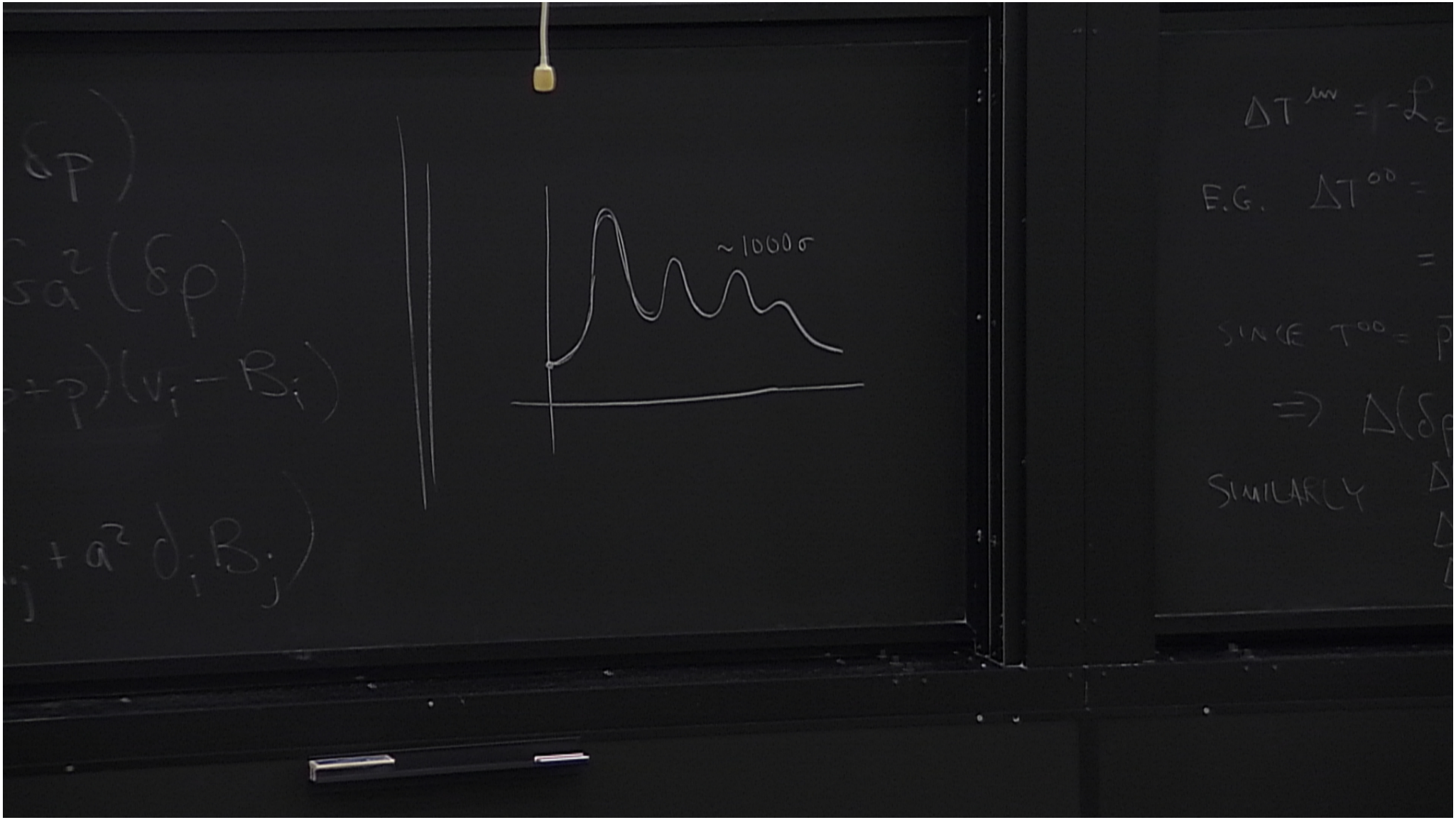
$$-3a^2H^2A + aH\partial_k B^k + a^2 H\dot{h}_{kk} + \frac{1}{2}\partial_i\partial_j h_{ij} - \frac{1}{2}\partial^2 h_{kk} = 4\pi G a^2(\delta\rho)$$

$$2H\partial_i A - \frac{1}{2}a^{-1}\partial_i\partial_j B_j + \frac{1}{2}a^{-1}\partial^2 B_i + \partial_j\dot{h}_{ij} - \partial_i\dot{h}_{jj} = -8\pi G a(\rho+P)(v_i - B_i)$$

$$\frac{1}{2}L_{ij} + \frac{1}{2}L_{ji} - \frac{1}{3}\delta_{ij}L_{kk} = 8\pi G a^2 \rho\,\Pi_{ij}$$

WHERE $L_{ij} = -\partial_i\partial_j A + 2\partial_i\partial_k h_{jk} - \partial_i\partial_j h_{kk} - \partial^2 h_{ij} + a^{-1}\frac{\partial}{\partial t}\left(a^2\dot{h}_{ij} + a^2\partial_i B_j\right)$




~1000σ
E.G.
SINCE
SIMILARLY

X^μ = VECTOR FIELD

$$\int d^4x \, (\partial_\mu X^\mu) = 0$$

$$\int d^4x \sqrt{-g} \, (\partial_\mu X^\mu) \neq 0$$

$$\int d^4x \, (\nabla_\mu X^\mu) \neq 0$$

$$\int d^4x \sqrt{-g} \, (\nabla_\mu X^\mu) = 0$$

$$\boxed{\nabla_\mu X^\mu = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} X^\mu)}$$

$$\int d^4x \sqrt{-g} \, (\nabla_\mu X^{\mu\nu}) \, Y_\nu = \int d^4x \, [illegible]$$

$$= - \int d^4x \sqrt{-g} \, X^{\mu\nu} (\nabla_\mu Y_\nu)$$

$$\int d^4x \sqrt{-g} \left(\nabla_\mu X^{\mu\nu}\right) Y_\nu = \int d^4x \sqrt{-g} \left[\nabla_\mu \left(X^{\mu\nu} Y_\nu\right) - X^{\mu\nu} \nabla_\mu Y_\nu\right]$$

$$= -\int d^4x \sqrt{-g}\, X^{\mu\nu} \left(\nabla_\mu Y_\nu\right)$$

$$\int d^4x \sqrt{-g} \left(\nabla_\mu \nabla_\nu X^\rho\right) Y_\rho^{\mu\nu} = -\int d^4x \sqrt{-g} \left(\nabla_\mu X^\rho\right) \left(\nabla_\nu Y_\rho^{\mu\nu}\right)$$

$$\int d^4x \sqrt{-g} \left(\nabla_\mu X^{\mu\nu}\right) Y_\nu = \int d^4x \sqrt{-g} \left[\nabla_\mu \left(X^{\mu\nu} Y_\nu\right) - X^{\mu\nu} \nabla_\mu Y_\nu\right]$$

$$= -\int d^4x \sqrt{-g}\, X^{\mu\nu} \left(\nabla_\mu Y_\nu\right)$$

$$\int d^4x \sqrt{-g} \left(\nabla_\mu \nabla_\nu X^\rho\right) Y_\rho^{\mu\nu} = -\int d^4x \sqrt{-g} \left(\nabla_\mu X^\rho\right) \left(\nabla_\nu Y_\rho^{\mu\nu}\right)$$

$$- \int d^4x \sqrt{-g}\, R_{\mu\nu\sigma}{}^{\rho} X^\sigma Y_\rho^{\mu\nu}$$

$$\int d^4x \sqrt{-g} \left(\nabla_\mu X^{\mu\nu}\right) Y_\nu = \int d^4x \sqrt{-g} \left[\nabla_\mu (X^{\mu\nu} Y_\nu) - X^{\mu\nu} \nabla_\mu Y_\nu\right]$$

$$= -\int d^4x \sqrt{-g}\, X^{\mu\nu} (\nabla_\mu Y_\nu)$$

$$\int d^4x \sqrt{-g} \left(\nabla_\mu \nabla_\nu X^\rho\right) Y_\rho^{\mu\nu} = -\int d^4x \sqrt{-g} \left(\nabla_\mu X^\rho\right) \left(\nabla_\nu Y_\rho^{\mu\nu}\right)$$

$$- \int d^4x \sqrt{-g}\, R_{\mu\nu\sigma}{}^{\rho} X^\sigma Y_\rho^{\mu\nu}$$

$$\int d^4x \sqrt{-g} \left(\nabla_\mu \nabla_\nu f\right) Y^{\mu\nu} \stackrel{?}{=} -\int d^4x \sqrt{-g} \left(\nabla_\mu f\right) \left(\nabla_\nu Y^{\mu\nu}\right)$$

SINGLE FIELD SLOW ROLL

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{Pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right) \qquad \left[M_{Pl}^2 = \frac{1}{8\pi G} \right]$$

FIELD CONTENT: $g_{\mu\nu}, \phi$

$V(\phi)$ FLAT POTENTIAL

TECHNICAL FLATNESS CONDITION:

$$\left.\begin{aligned} \varepsilon &= \frac{M_{Pl}^2}{2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 \ll 1 \\ \eta &= M_{Pl}^2 \left(\frac{V''(\phi)}{V(\phi)} \right) \ll 1 \end{aligned}\right\} \text{"SLOW ROLL PARAMETERS"}$$

ϕ-EOM: $0 = \frac{\delta S}{\delta \phi} = \nabla_\mu \nabla^\mu \phi - V'(\phi)$

STRESS ENERGY TENSOR: $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$

$=$

ϕ-EOM: $0 = \frac{\delta S}{\delta \phi} = \nabla_\mu \nabla^\mu \phi - V'(\phi)$

STRESS ENERGY TENSOR: $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S^{\text{non-EH}}}{\delta g^{\mu\nu}}$

=

ϕ-EOM: $0 = \frac{\delta S}{\delta \phi} = \nabla_\mu \nabla^\mu \phi - V'(\phi)$

$\frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} = -$

STRESS ENERGY TENSOR: $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S^{\text{non-EH}}}{\delta g^{\mu\nu}}$

$= g_{\mu\nu}\left(-\frac{1}{2}\nabla^\rho \phi \nabla_\rho \phi - V(\phi)\right)$

$$\Gamma^{ij}_{0i} = H\delta^j_i + \dot{h}^j_i - \frac{1}{2}a^{-1}\partial_i B^j + \frac{1}{2}a^{-1}\partial^j B_i$$

ϕ-EOM: $$0 = \frac{\delta S}{\delta \phi} = \nabla_\mu \nabla^\mu \phi - V'(\phi)$$

$$\frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}$$

STRESS ENERGY TENSOR: $$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S^{\text{non-EH}}}{\delta g^{\mu\nu}}$$

$$= g_{\mu\nu}\left(-\frac{1}{2}\nabla^\rho \phi \nabla_\rho \phi - V(\phi)\right) - \frac{2}{\sqrt{-g}}\left(\sqrt{-g}\right)\left(-\frac{1}{2}\nabla_\mu \phi \nabla_\nu \phi\right)$$

$$= \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu}(\nabla^\rho \phi)(\nabla_\rho \phi) - V(\phi) g_{\mu\nu}$$

BACKGROUND SOLUTION, I.E. ZEROTH ORDER IN PERTURBATION THEORY

$$\phi = \bar{\phi}(t)$$

$$T_{\mu\nu} = \begin{pmatrix} \dot{\phi}^2 & \\ & 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -1 & \\ & \delta_{ij} \end{pmatrix}(-\dot{\phi}^2) - V(\phi)\begin{pmatrix} -1 & \\ & \delta_{ij} \end{pmatrix}$$

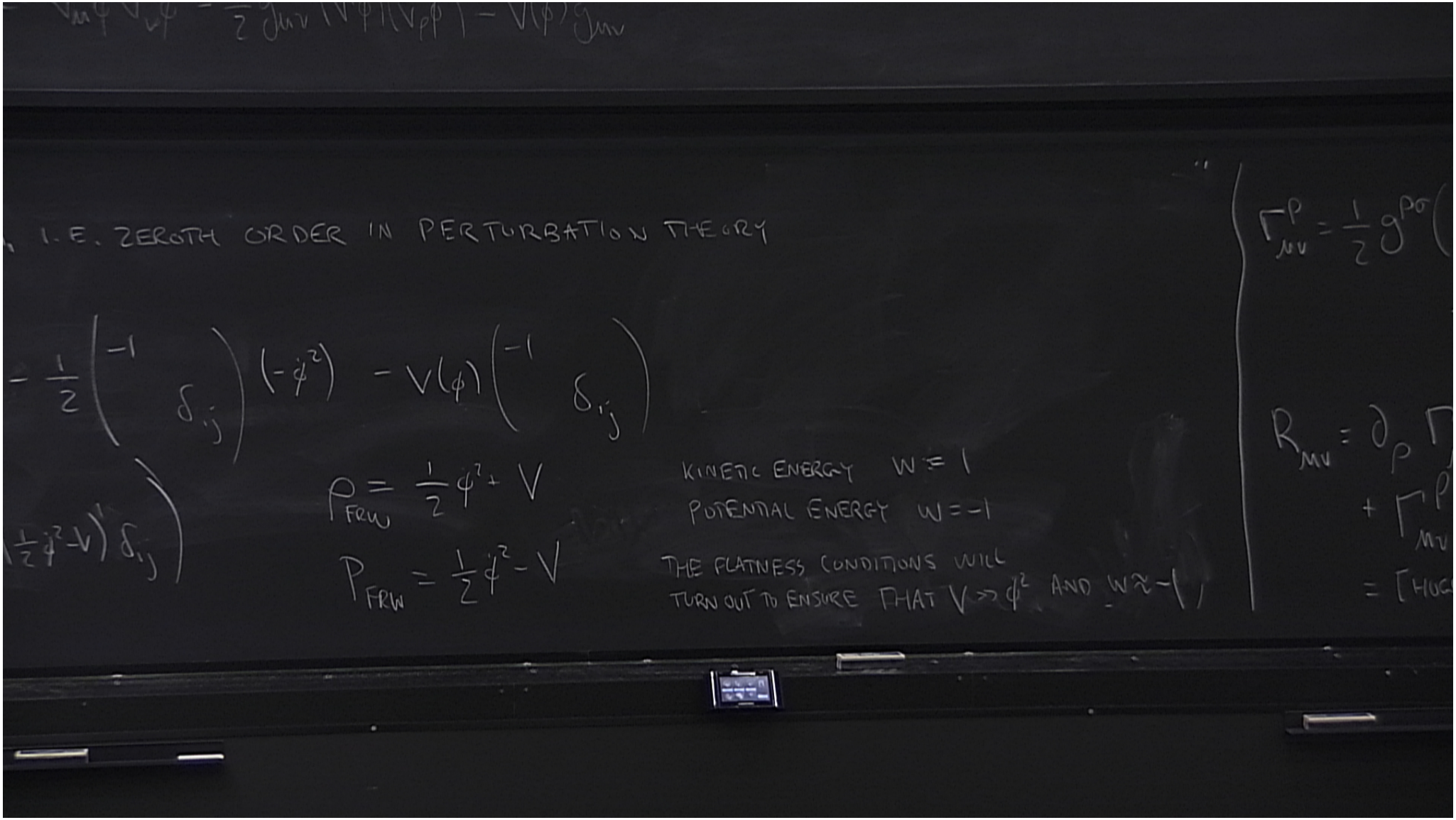
$$= \begin{pmatrix} \frac{1}{2}\dot{\phi}^2 + V & \\ & (\frac{1}{2}\dot{\phi}^2 - V)\,\delta_{ij} \end{pmatrix}$$

$$\rho_{FRW} = \frac{1}{2}\dot{\phi}^2 + V$$

$$P_{FRW} = \frac{1}{2}\dot{\phi}^2 - V$$

KINETIC ENERGY $w = 1$

POTENTIAL ENERGY $w = -1$


I.E. ZEROTH ORDER IN PERTURBATION THEORY
$\rho_{FRW} = \frac{1}{2}\dot{\phi}^2 + V$
$P_{FRW} = \frac{1}{2}\dot{\phi}^2 - V$
KINETIC ENERGY $W = 1$
POTENTIAL ENERGY $W = -1$
THE FLATNESS CONDITIONS WILL TURN OUT TO ENSURE THAT $V \gg \dot{\phi}^2$ AND $W \approx -1$