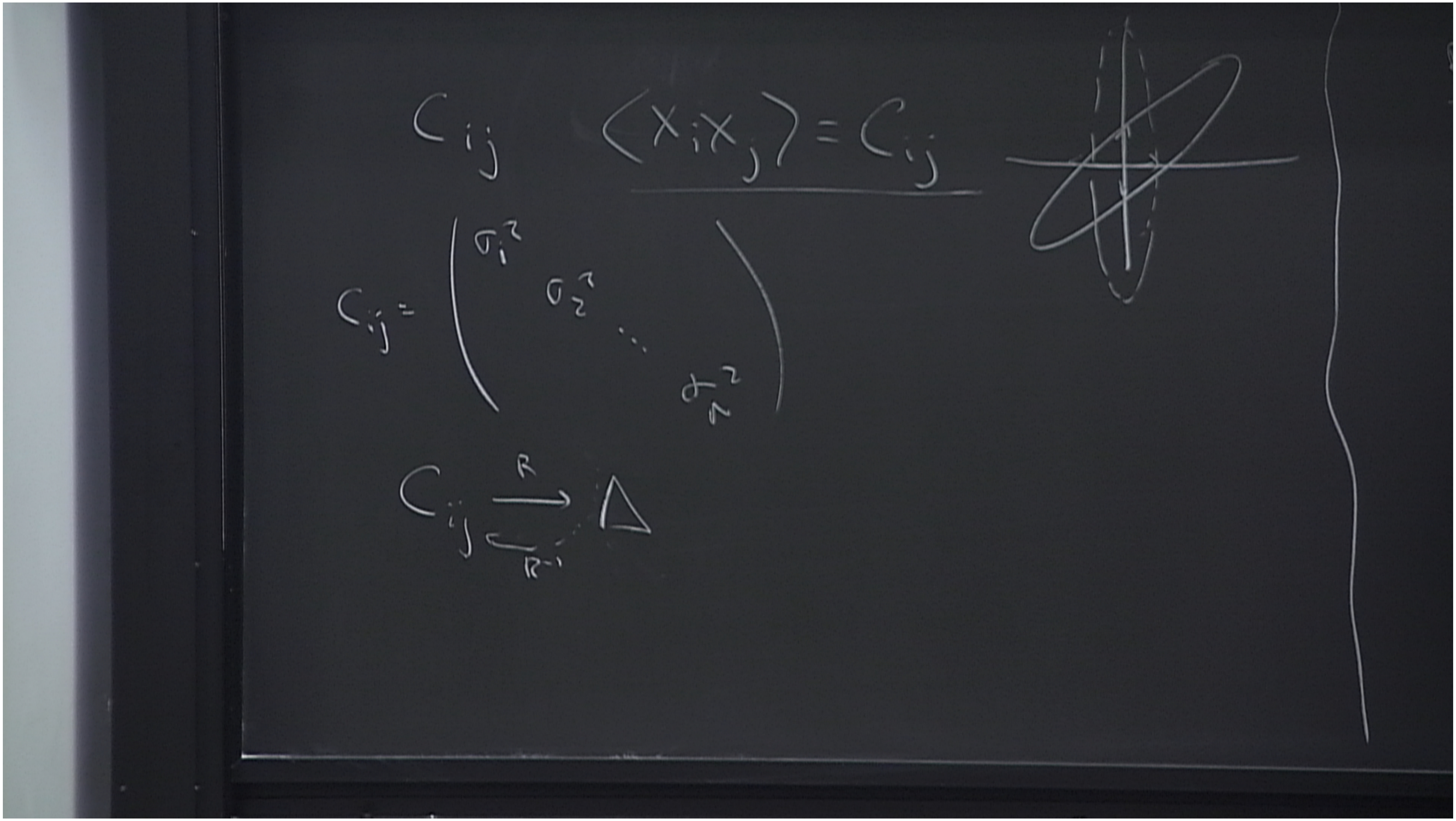
Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 3

Date: Apr 13, 2016 10:15 AM

URL: http://pirsa.org/16040037

Abstract:


C_{ij}
$\langle x_i x_j \rangle = C_{ij}$
$C_{ij} = \begin{pmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_n^2 \end{pmatrix}$
$C_{ij} \xrightarrow{R} \Delta$
R^{-1}

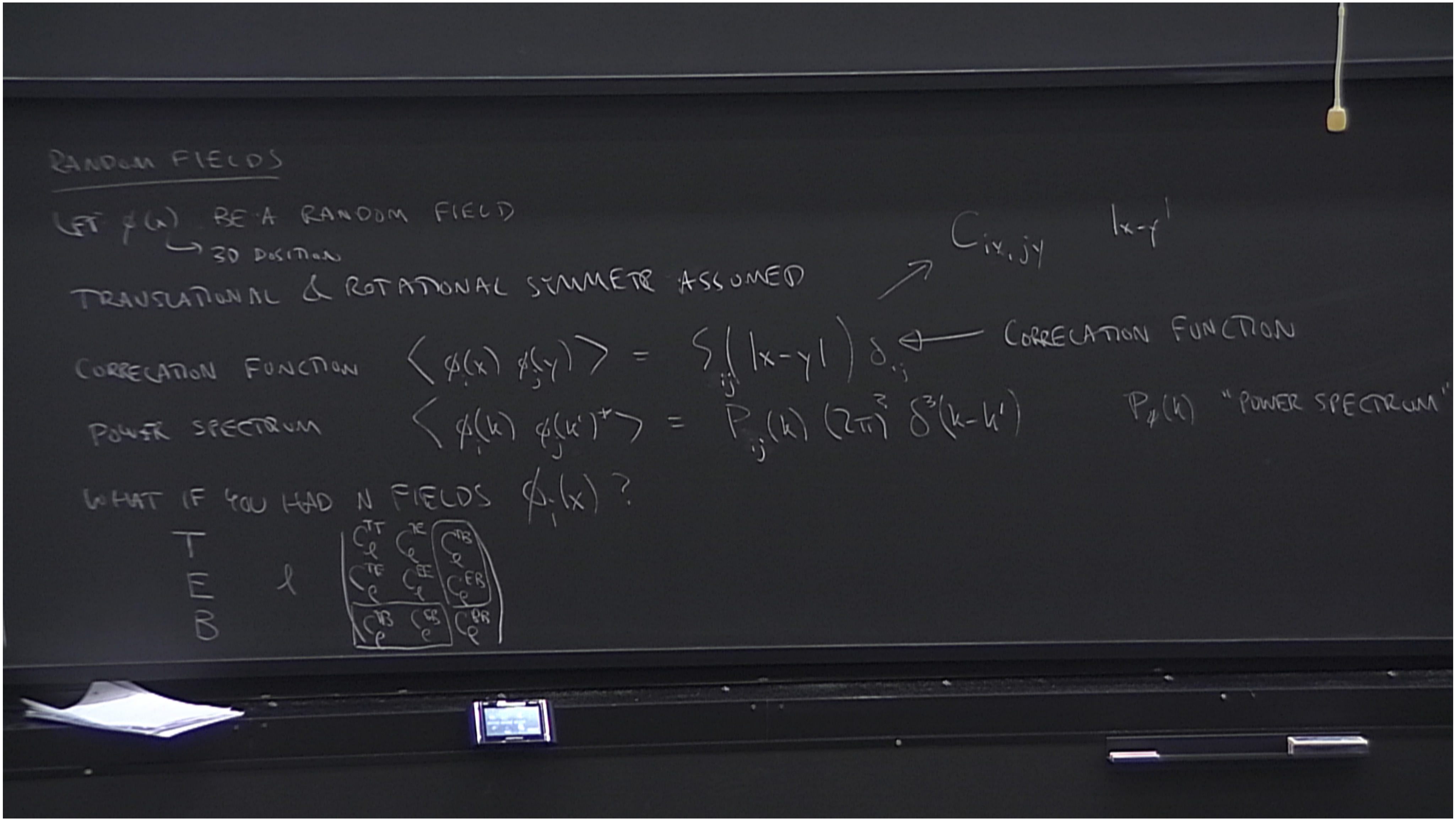
RANDOM FIELDS

LET $\phi(x)$ BE A RANDOM FIELD
$\hookrightarrow$ 3D POSITION

TRANSLATIONAL & ROTATIONAL SYMMETRY ASSUMED

CORRELATION FUNCTION $\langle \phi(x)\, \phi(y) \rangle = \xi(|x-y|)$ $\leftarrow$ CORRELATION FUNCTION

POWER SPECTRUM $\langle \phi(k)\, \phi(k')^{*} \rangle = P_\phi(k)\, (2\pi)^3\, \delta^3(k-k')$


RANDOM FIELDS
LET φ(x) BE A RANDOM FIELD
3D POSITION
TRANSLATIONAL & ROTATIONAL SYMMETRY ASSUMED
C_{ix, jy} |x-y|
CORRELATION FUNCTION ⟨φ_i(x) φ_j(y)⟩ = ξ_{ij}(|x-y|) δ_{ij} ⟵ CORRELATION FUNCTION
POWER SPECTRUM ⟨φ_i(k) φ_j(k')*⟩ = P_{ij}(k) (2π)^3 δ^3(k-k')
P_φ(k) "POWER SPECTRUM"
WHAT IF YOU HAD N FIELDS φ_i(x) ?
T
E
B
ℓ
C_ℓ^TT C_ℓ^TE C_ℓ^TB
C_ℓ^TE C_ℓ^EE C_ℓ^EB
C_ℓ^TB C_ℓ^EB C_ℓ^BB

$$\langle \zeta(k)\, \zeta(k')^* \rangle = P_\zeta(k)\, (2\pi)^3 \delta^3(k-k')$$

$$\sim \frac{10^{-9}}{k^{2.96}}$$

$$\delta\rho(k, z=0)$$

$$\langle \delta\rho(k)\, \delta\rho(k')^* \rangle = P_m(k)\, (2\pi)^3 \delta^3(k-k')$$

$$\delta\rho(\vec{k}) = T(k)\, S(\vec{k}) \qquad T(k) \rightarrow \Omega_b, \Omega_m$$

$$\langle \delta\rho(\vec{k})\, \delta\rho(\vec{k})^{+} \rangle = T(k)^2 \langle S(\vec{k})\, S(\vec{k}')^{+} \rangle$$

$$= T(k)^2 \left(\frac{10^{-9}}{k^{2.96}} \right)$$

IF ϕ IS A R.F. W/ POWER SPECTRUM $P(k)$:

WHAT'S THE RMS SIZE OF A ϕ FLUCTUATION?

$$(\Delta\phi)_{RMS} = \left[\langle \phi(x)^2 \rangle\right]^{1/2}$$

$$= \left\langle \left(\int \frac{d^3k}{(2\pi)^3}\,\phi(k)\,e^{ik\cdot x}\right)\left(\int \frac{d^3k'}{(2\pi)^3}\,\phi(k')\,e^{ik'\cdot x}\right)\right\rangle^{1/2}$$

$$= \left[\int \frac{d^3k\,d^3k'}{(2\pi)^6}\,\langle \phi(k)\,\phi(k')\rangle\,e^{i(k+k')\cdot x}\right]^{1/2}$$

$$= \left[\int \frac{d^3k\,d^3k'}{(2\pi)^3}\,P(k)\,(2\pi)^3\,\delta^3(k+k')\,e^{i(k+k')\cdot x}\right]^{1/2}$$

IF ϕ IS A R.F. W/ POWER SPECTRUM $P(k)$:

WHAT'S THE RMS SIZE OF A ϕ FLUCTUATION?

$$(\Delta\phi)_{RMS} = \left[\langle \phi(x)^2 \rangle \right]^{1/2}$$

$$= \left\langle \left(\int \frac{d^3k}{(2\pi)^3} \phi(k)\, e^{ik\cdot x} \right) \left(\int \frac{d^3k'}{(2\pi)^3} \phi(k')\, e^{ik'\cdot x} \right) \right\rangle^{1/2}$$

$$= \left[\int \frac{d^3k\, d^3k'}{(2\pi)^6} \langle \phi(k)\, \phi(k') \rangle\, e^{i(k+k')\cdot x} \right]^{1/2}$$

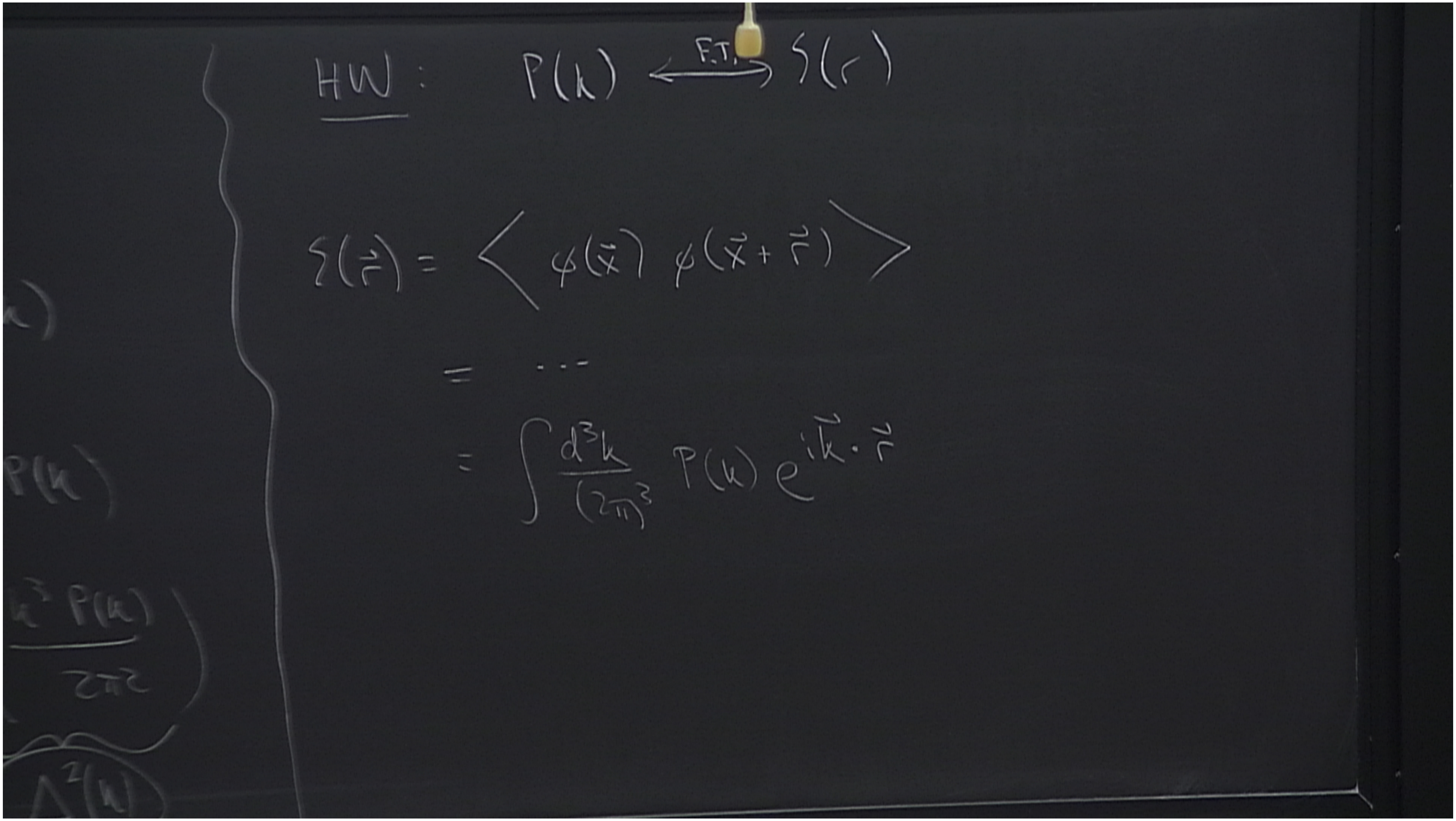
$$= \left[\int \frac{d^3k\, d^3k'}{(2\pi)^6} P(k)\, (2\pi)^3\, \delta^3(k+k')\, e^{i(k+k')\cdot x} \right]^{1/2}$$

$$= \left[\int \frac{d^3k}{(2\pi)^3} P(k) \right]^{1/2}$$

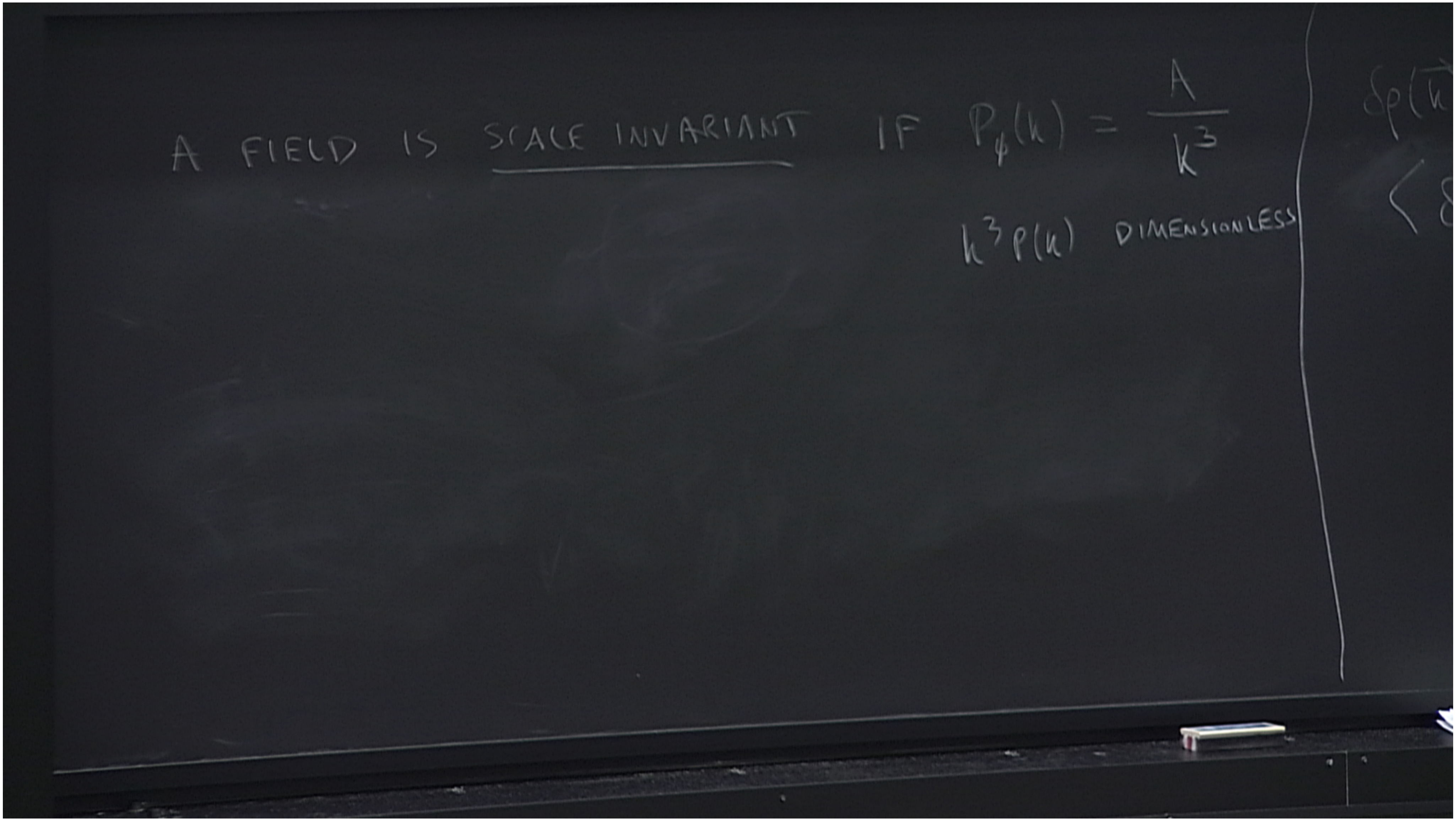
$$\langle (\Delta\phi)^2 \rangle = \int \frac{d^3k}{(2\pi)^3} P(k)$$

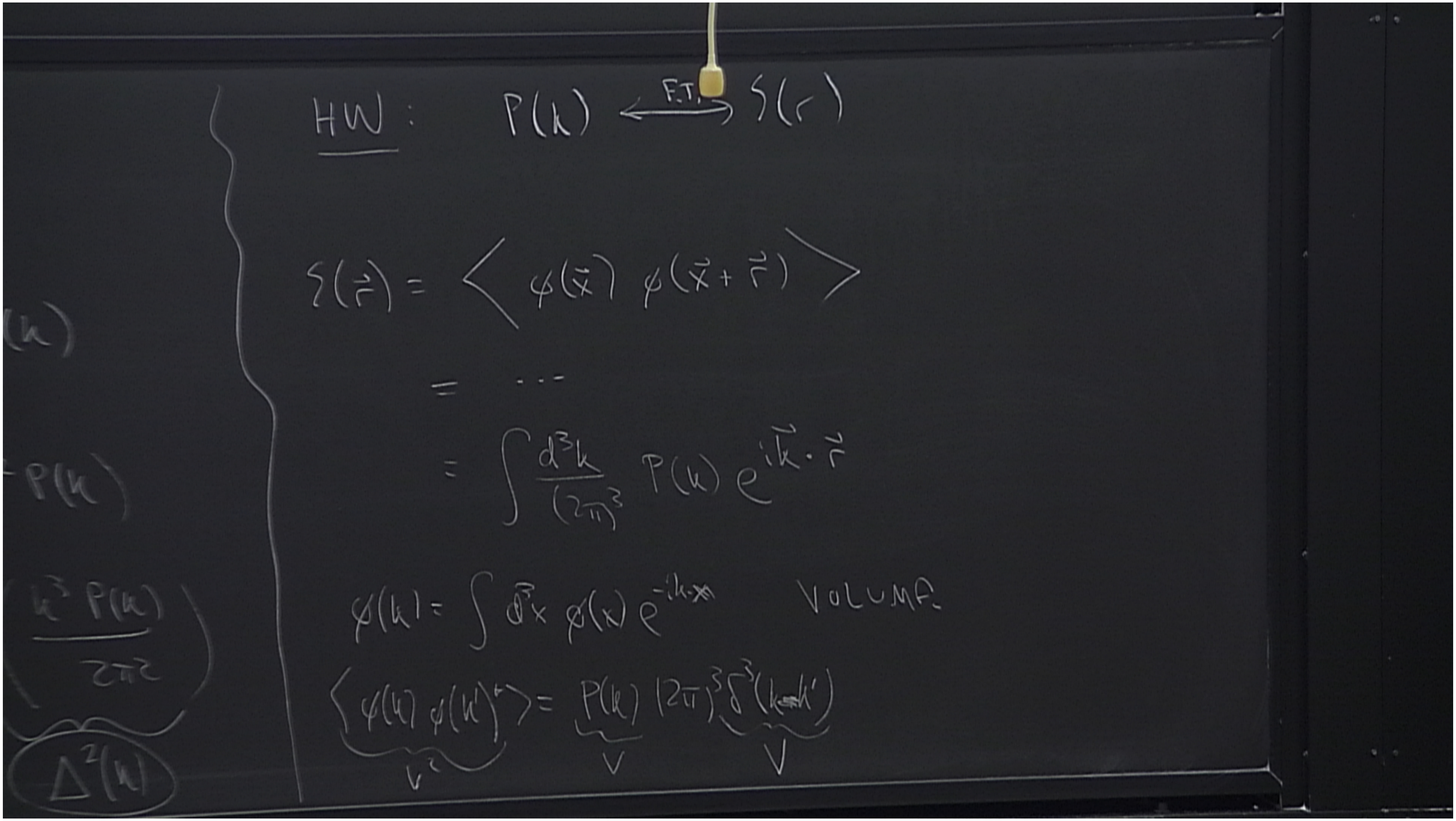
$$= \int_0^\infty \frac{dk}{2\pi^2} k^2 P(k)$$

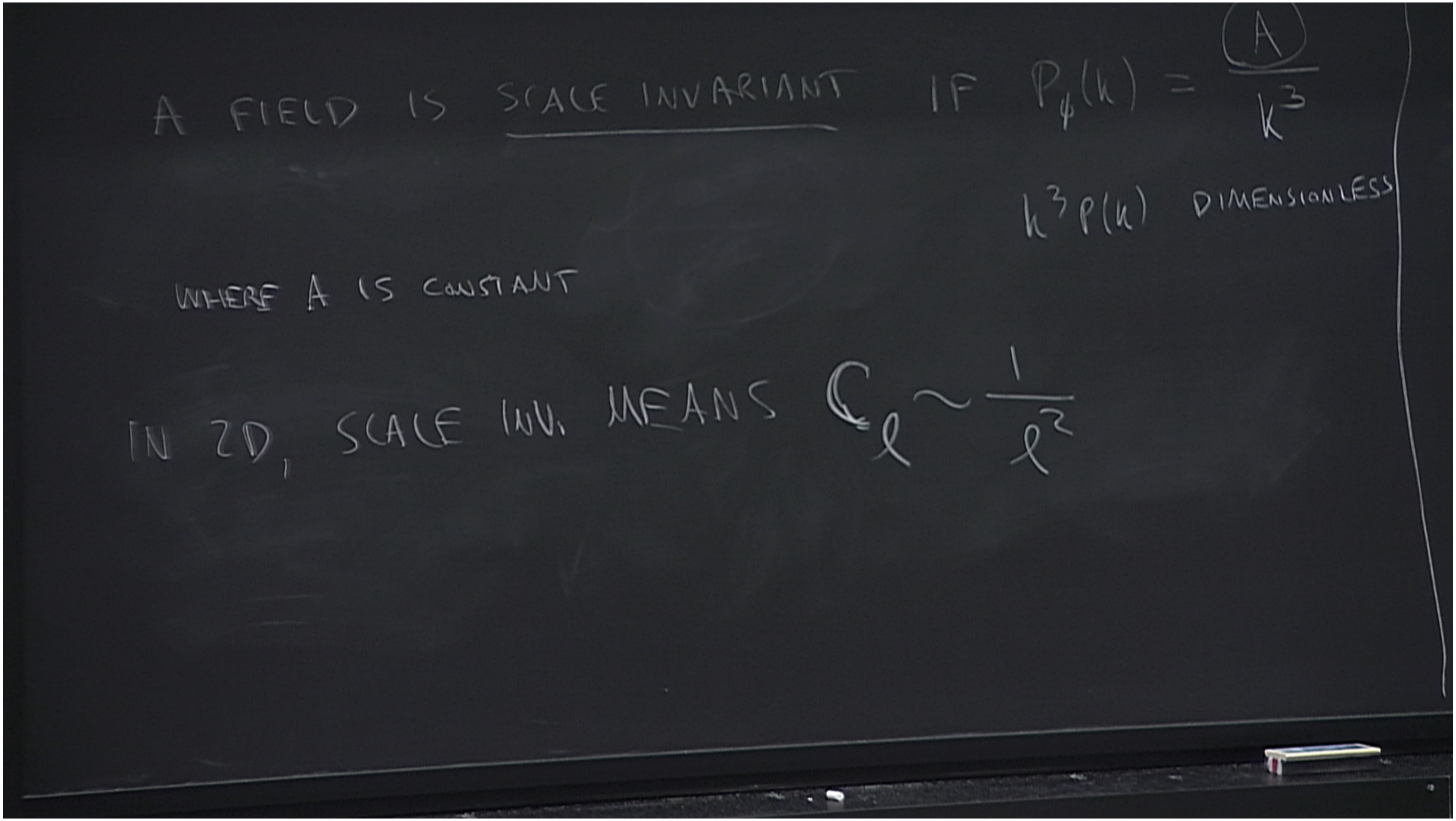
$$= \int d\log k \underbrace{\left(\frac{k^3 P(k)}{2\pi^2} \right)}_{\Delta^2(k)}$$


HW: $P(k) \overset{F.T.}{\longleftrightarrow} \xi(r)$
$\xi(\vec{r}) = \langle \phi(\vec{x})\, \phi(\vec{x}+\vec{r}) \rangle$
$= \ldots$
$= \int \frac{d^3k}{(2\pi)^3} P(k)\, e^{i\vec{k}\cdot\vec{r}}$

A FIELD IS **SCALE INVARIANT** IF $P_\phi(k) = \frac{A}{k^3}$


A FIELD IS SCALE INVARIANT IF $P_\phi(k) = \frac{A}{k^3}$
$k^3 P(k)$ DIMENSIONLESS

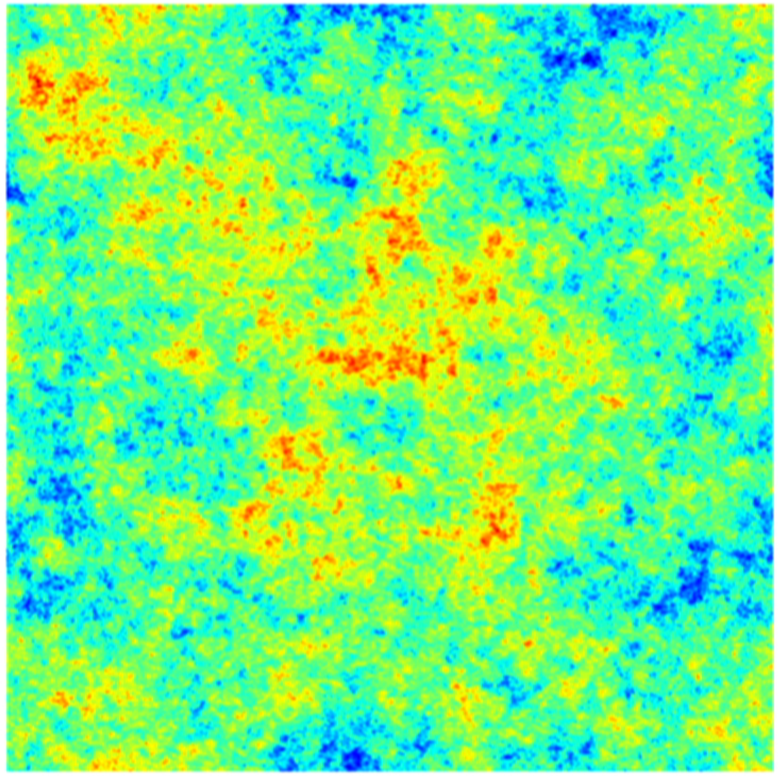

HW: P(k) ⟵F.T.⟶ ξ(r)
ξ(r⃗) = ⟨ φ(x⃗) φ(x⃗ + r⃗) ⟩
= ...
= ∫ d³k/(2π)³ P(k) e^{ik⃗·r⃗}
φ(k) = ∫ d³x φ(x) e^{-ik·x}
VOLUME
⟨φ(k) φ(k')*⟩ = P(k) (2π)³ δ³(k−k')
Δ²(k)
k³ P(k) / 2π²


A FIELD IS SCALE INVARIANT IF $P_\psi(k) = \frac{A}{k^3}$
$k^3 P(k)$ DIMENSIONLESS
WHERE A IS CONSTANT
IN 2D, SCALE INV. MEANS $C_\ell \sim \frac{1}{\ell^2}$

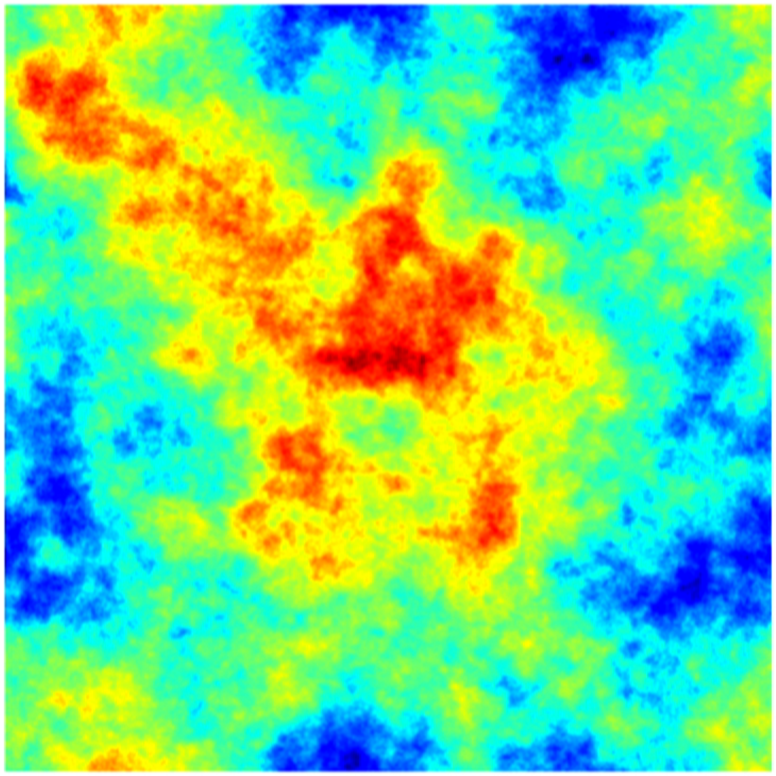
Random field slide show

Kendrick Smith
PSI cosmology, 2016

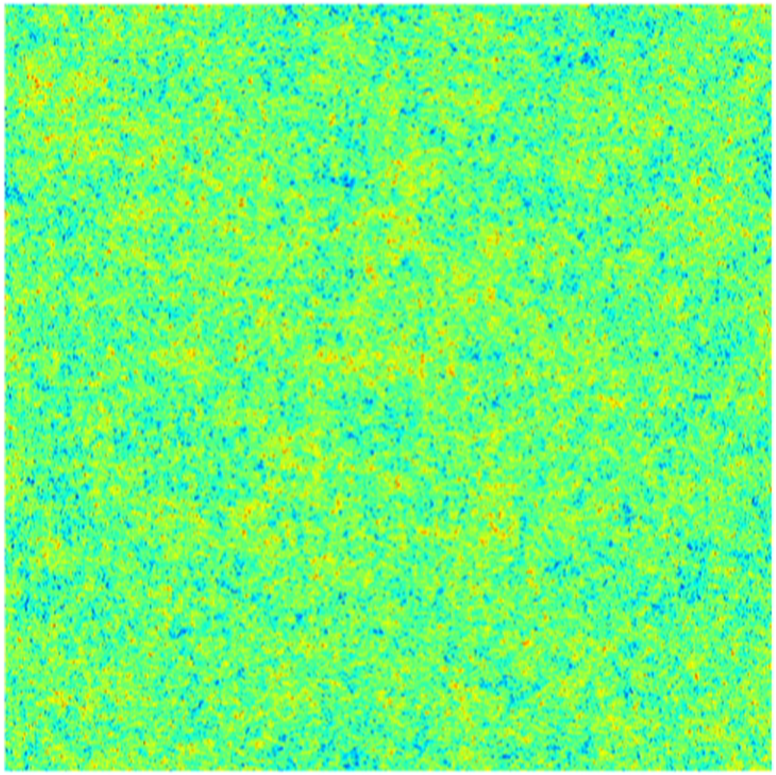
Scale invariant 2D Gaussian random field: $C(l) \propto l^{-2}$



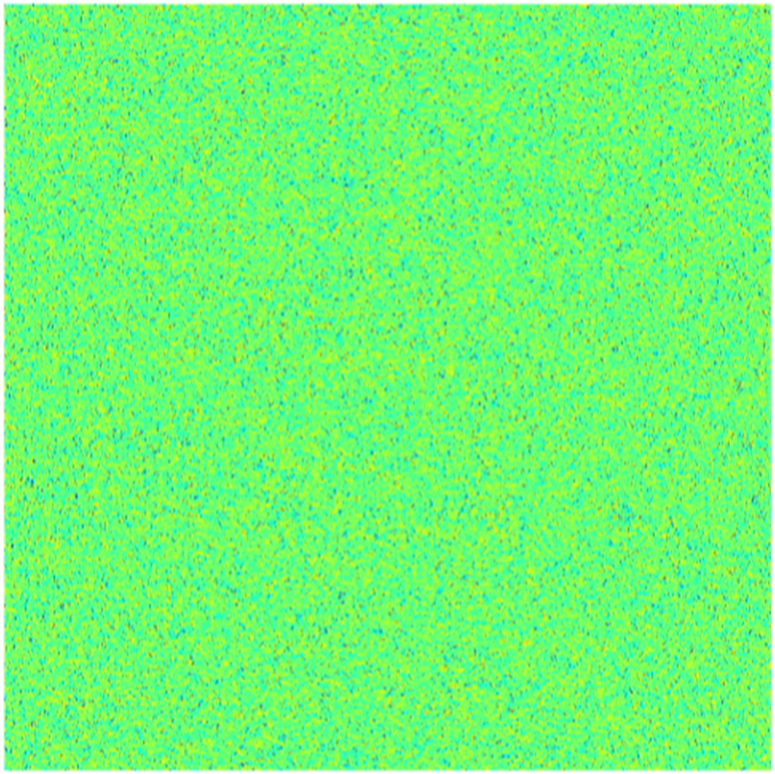
"Red" power law spectrum: $C(l) \propto l^{-3}$



"Blue" power law spectrum: $C(l) \propto l^{-1}$

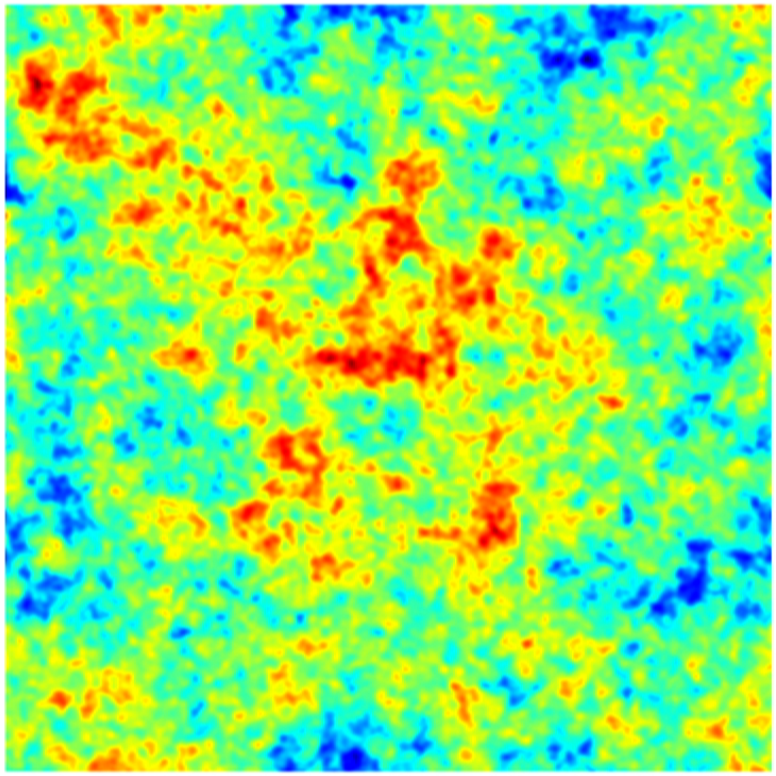


White noise: C(l) = constant

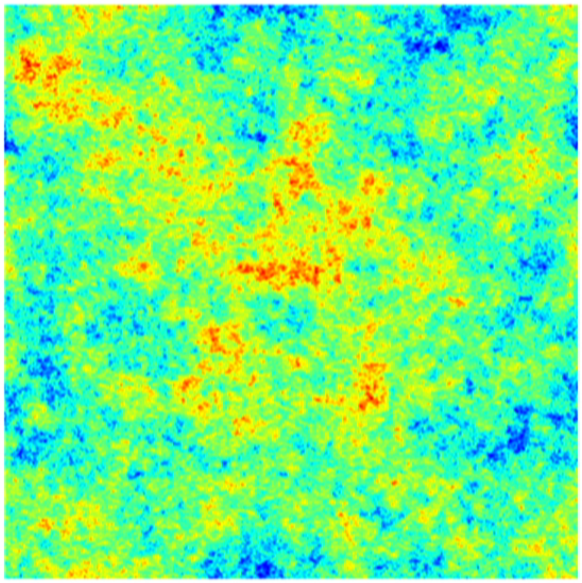


No correlation between pixels in this case!

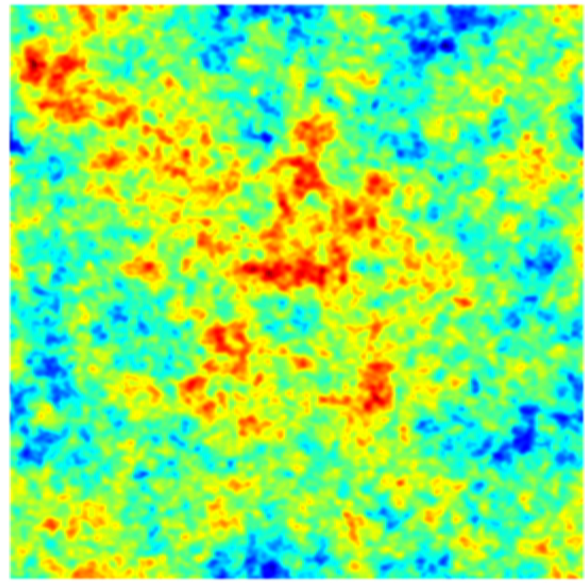
Scale invariant power spectrum with cutoff: $C(l) \propto l^{-2} \exp(-(l/l_0)^2)$



Not self-similar: Scale invariant on scales larger than characteristic scale l_0^{-1}, fluctuations are smoothed out below this scale

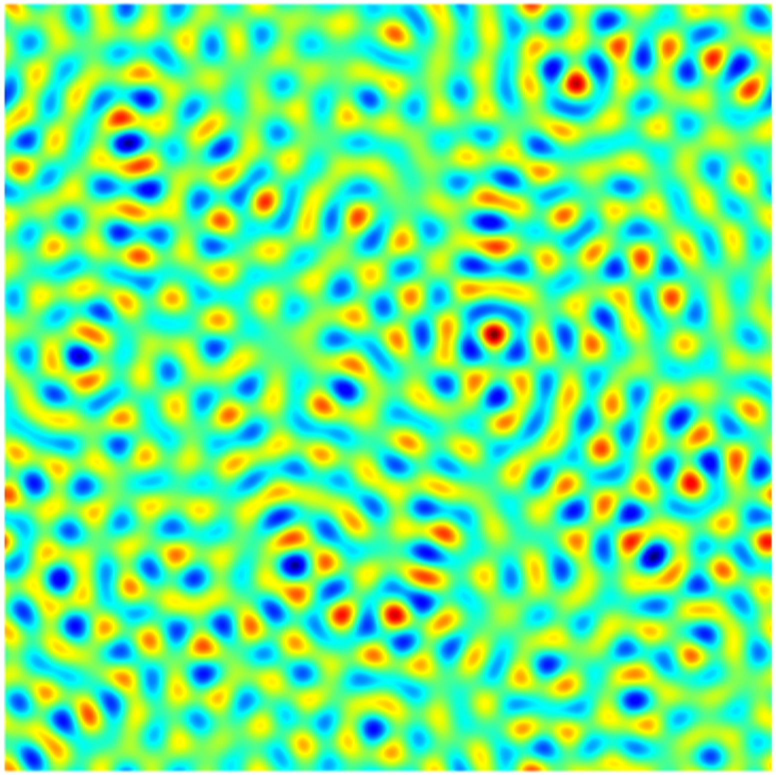


Scale invariant power spectrum: $C(l) \propto l^{-2}$



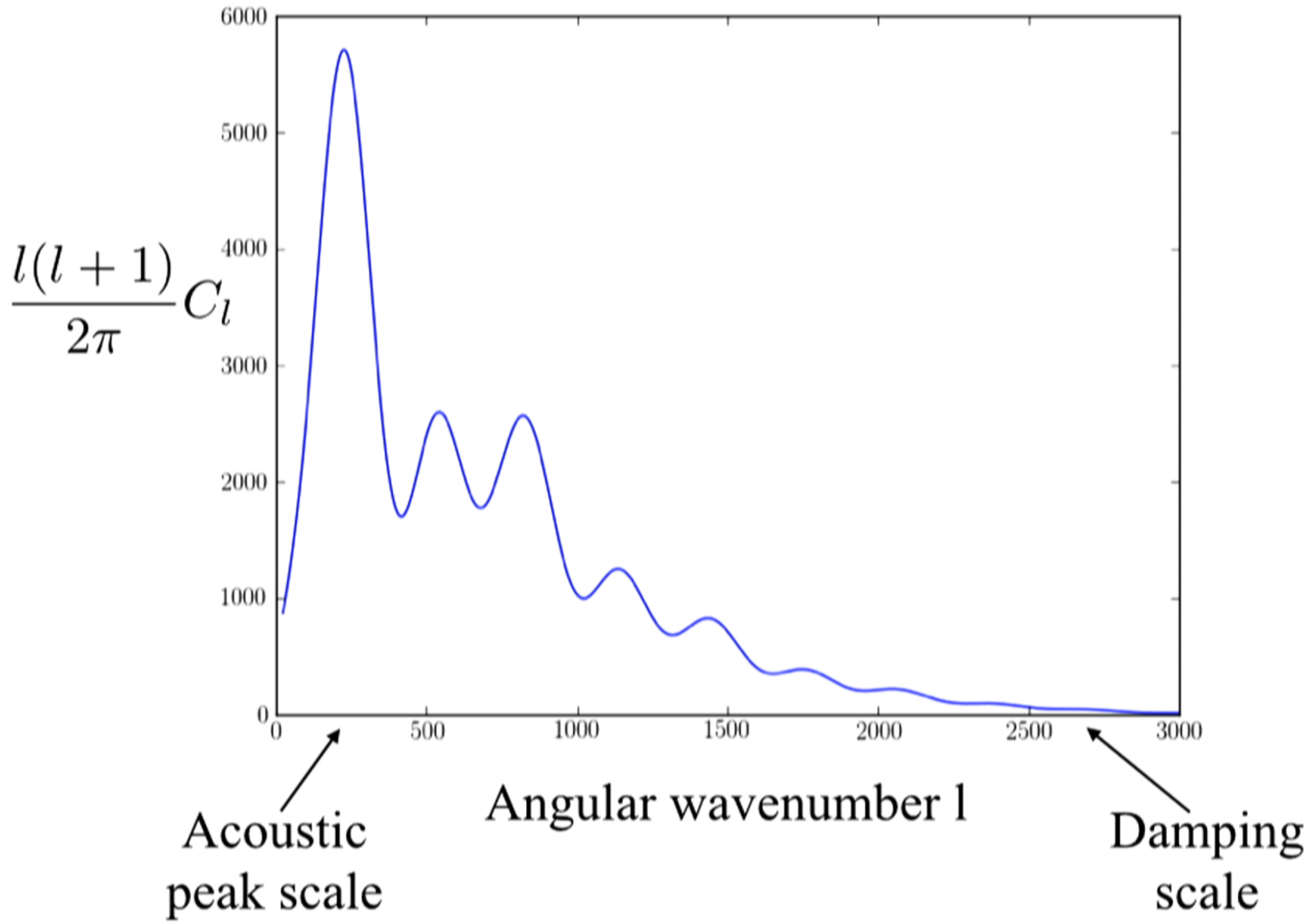
Scale invariant power spectrum with cutoff: $C(l) \propto l^{-2} \exp(-(l/l_0)^2)$

Delta function power spectrum: $P(l) \propto \delta(l-l_0)$

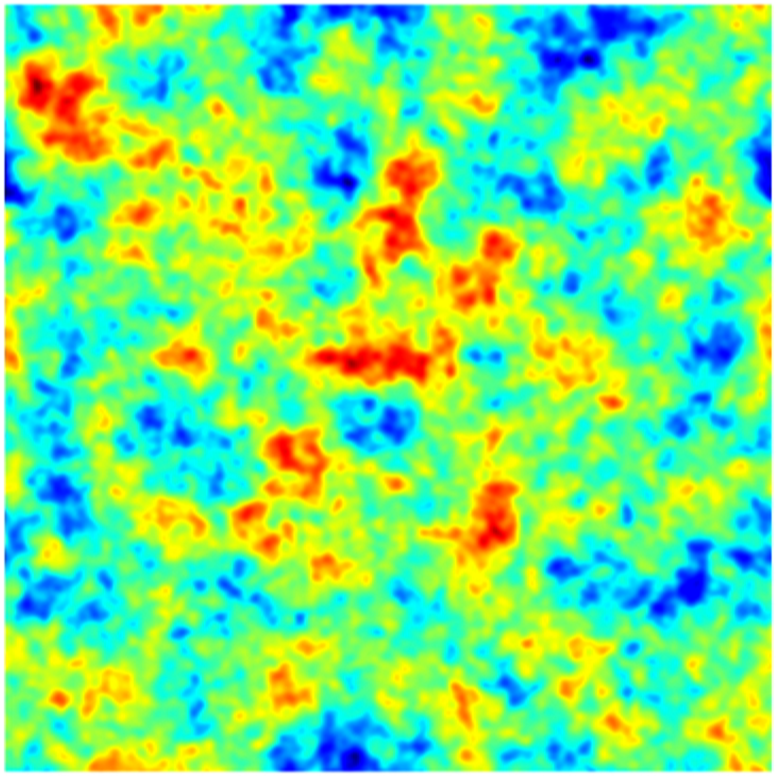


Very strongly defined characteristic scale

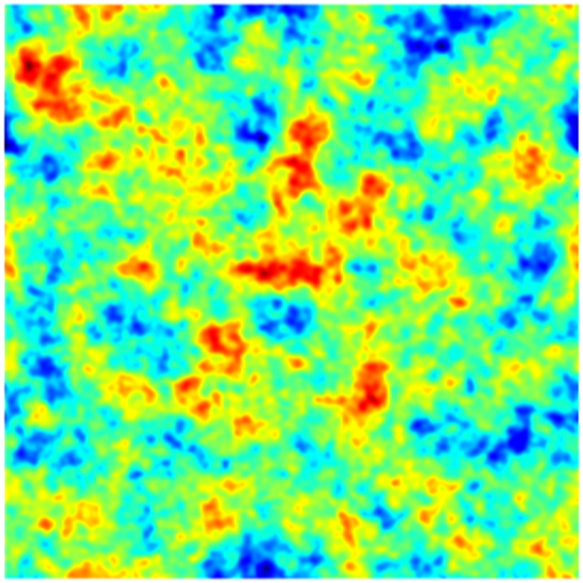
CMB power spectrum



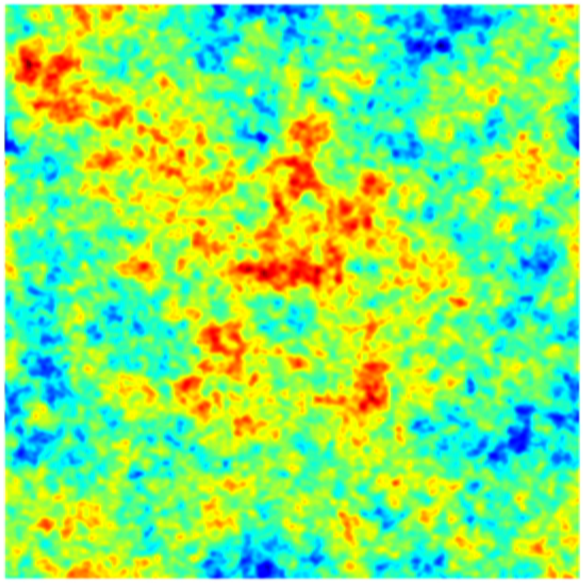
CMB map (simulated)



A Gaussian map with the CMB power spectrum



CMB power spectrum



Scale invariant power spectrum with cutoff: $P(l) \propto l^{-2} \exp(-(l/l_0)^2)$

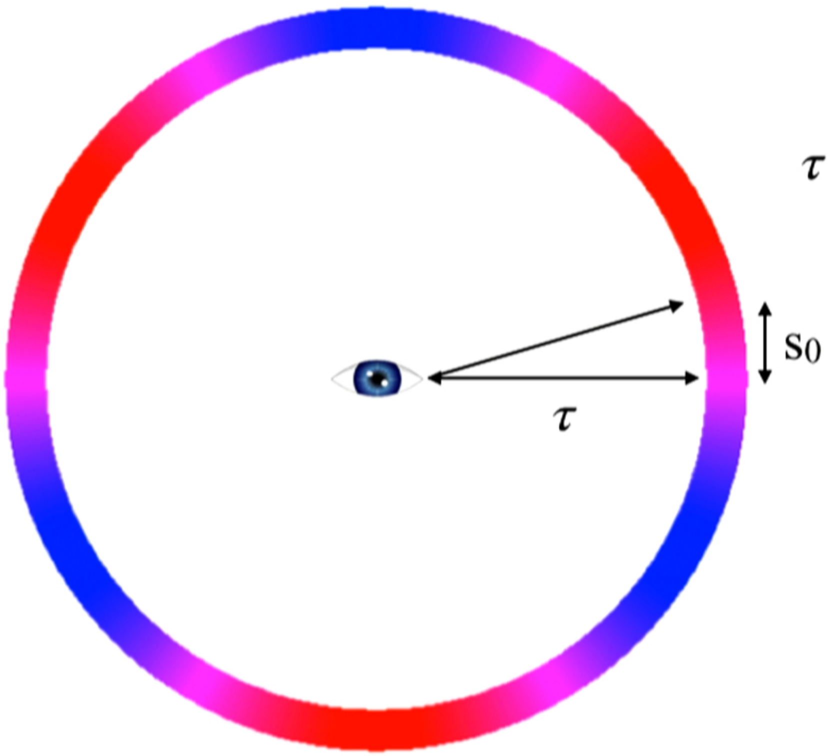
Just for fun: characteristic scales in the CMB (derivation will come later in the class!)

Before the CMB was formed, the universe is a plasma, inhomogeneities propagate as sound waves.

Define the "sound horizon" s_0 to be the maximum distance that a sound wave can travel between the big bang and the formation of the CMB.

We see the sound horizon as a characteristic angle on the sky

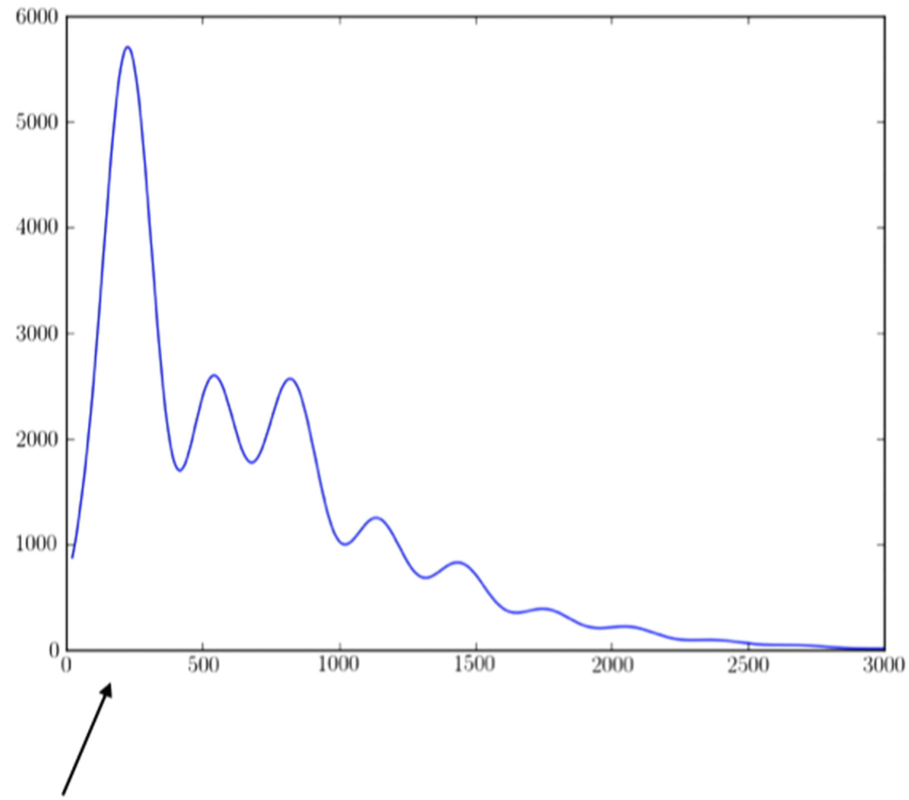
$$\Delta\theta = s_0 / \tau$$



s_0 = sound horizon

τ = conformal time to CMB

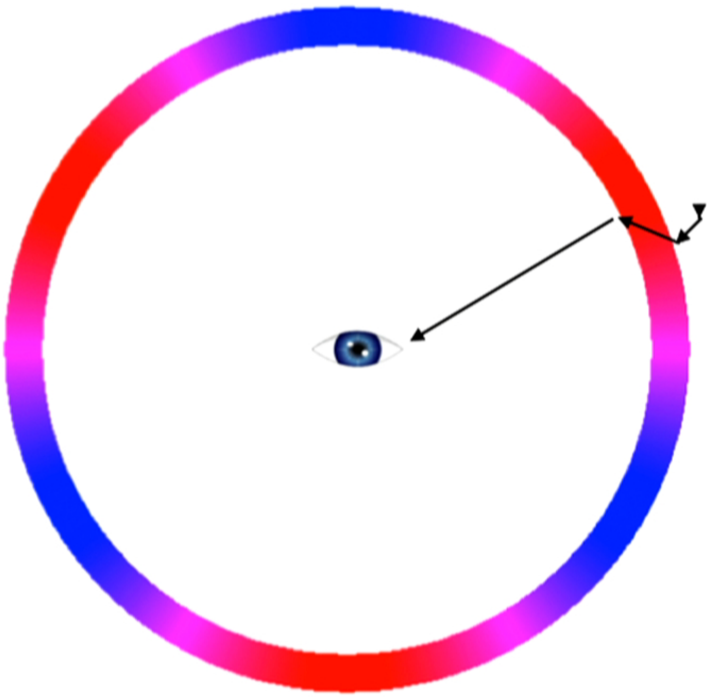
The acoustic peak scale is determined by $\Delta\theta$



Acoustic peak scale $l_a \sim \pi / \Delta\theta = \pi\ (s_0/\tau)$

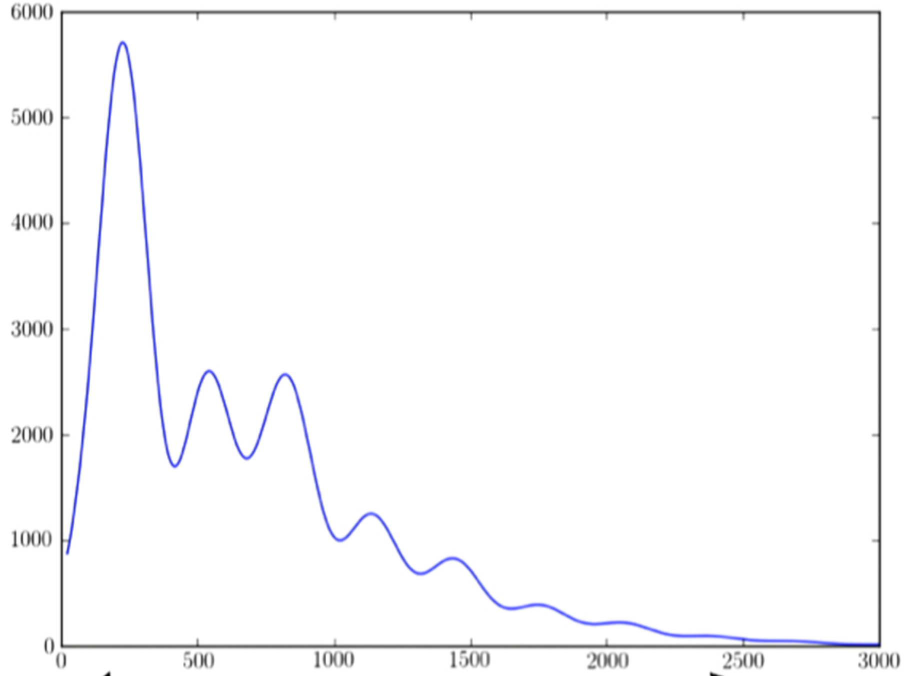
Imagine tracing a CMB photon back in time.

The CMB is formed when the universe transitions from opaque to transparent. Because this process isn't instantaneous, a CMB photon undergoes a little random walk (length of walk is exaggerated in the picture)



Let λ be the total distance traversed during the random walk

Characteristic scales in the CMB



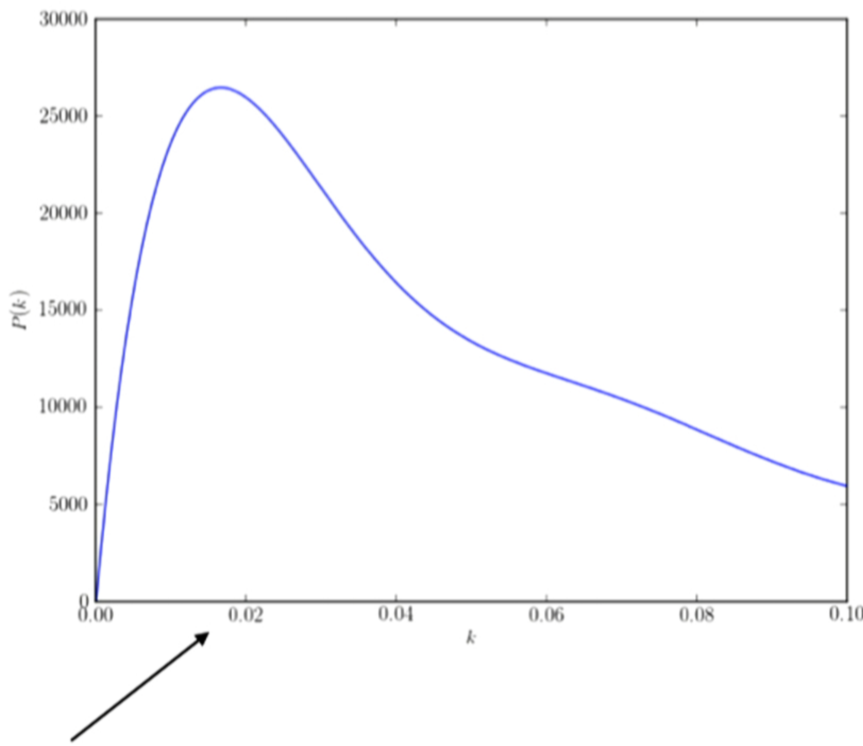
Acoustic peak scale

$l_a \sim \pi / \Delta\theta = \pi\, (s_0 / \tau)$

Damping scale

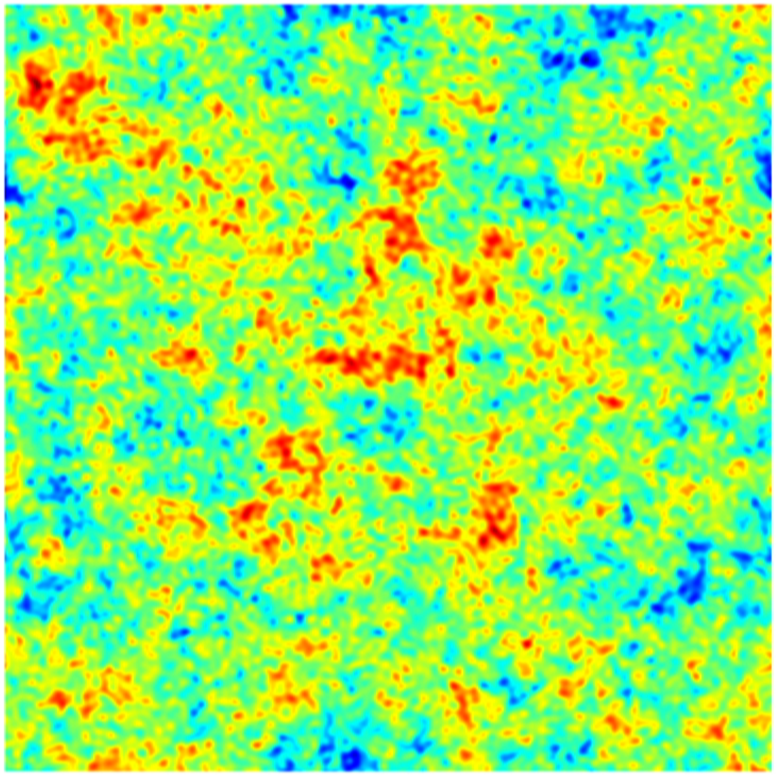
$l_d \sim (\lambda/\tau)$

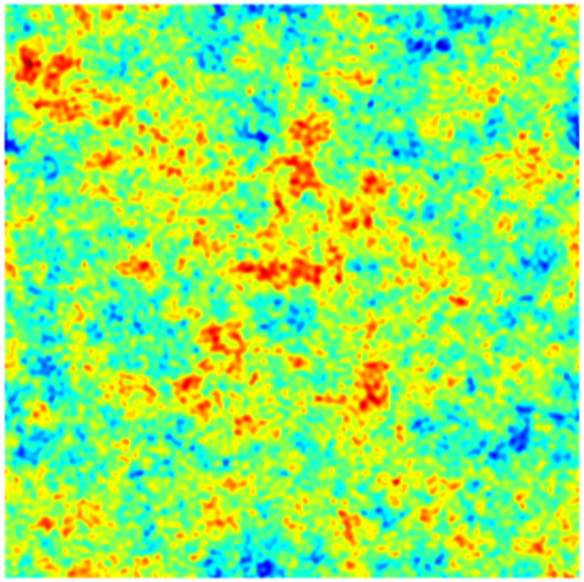
Matter power spectrum in the late universe (a 3D field)



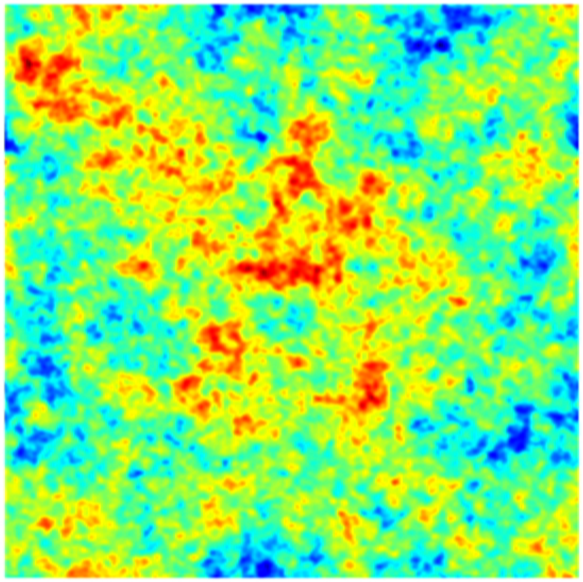
Characteristic scale! (Turns out to be the Hubble length evaluated at matter radiation equality)

Simulated Gaussian field with same power spectrum as the density field in the late universe





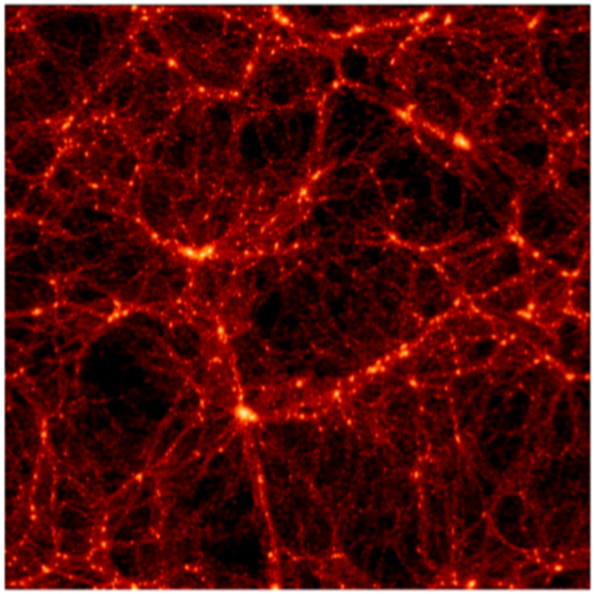
Matter power spectrum

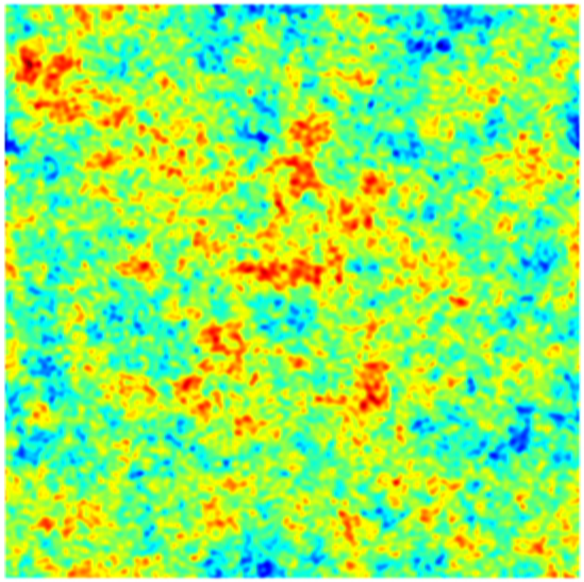


Scale invariant power spectrum with cutoff: $P(l) \propto l^{-2} \exp(-(l/l_0)^2)$

The density field in the late universe is nonlinear, and modelling it as a Gaussian field is generally not a good approximation

These two maps have roughly the same power spectrum!





Perhaps surprisingly, Gaussian statistics are a good approximation on large scales (low k)