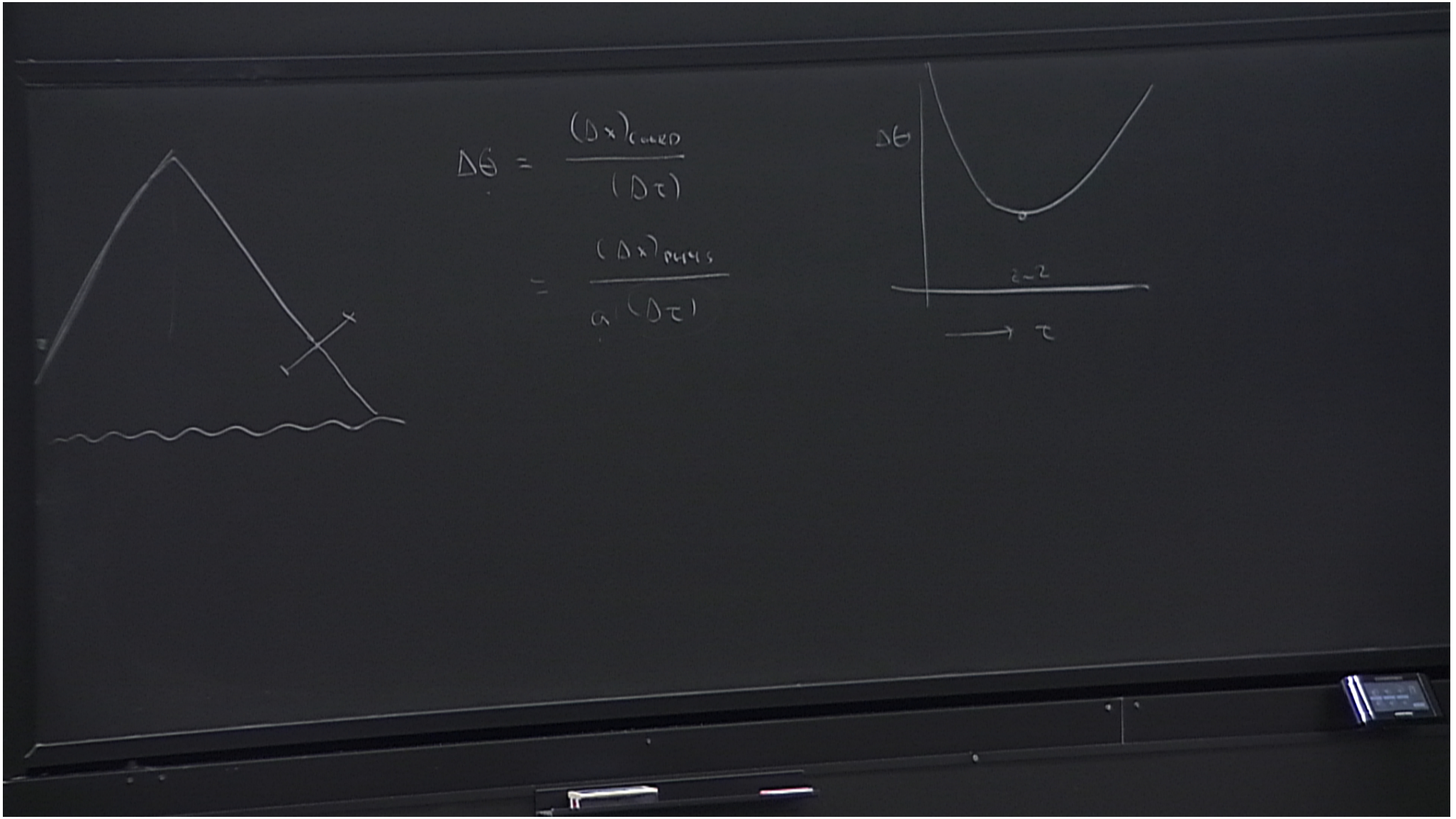
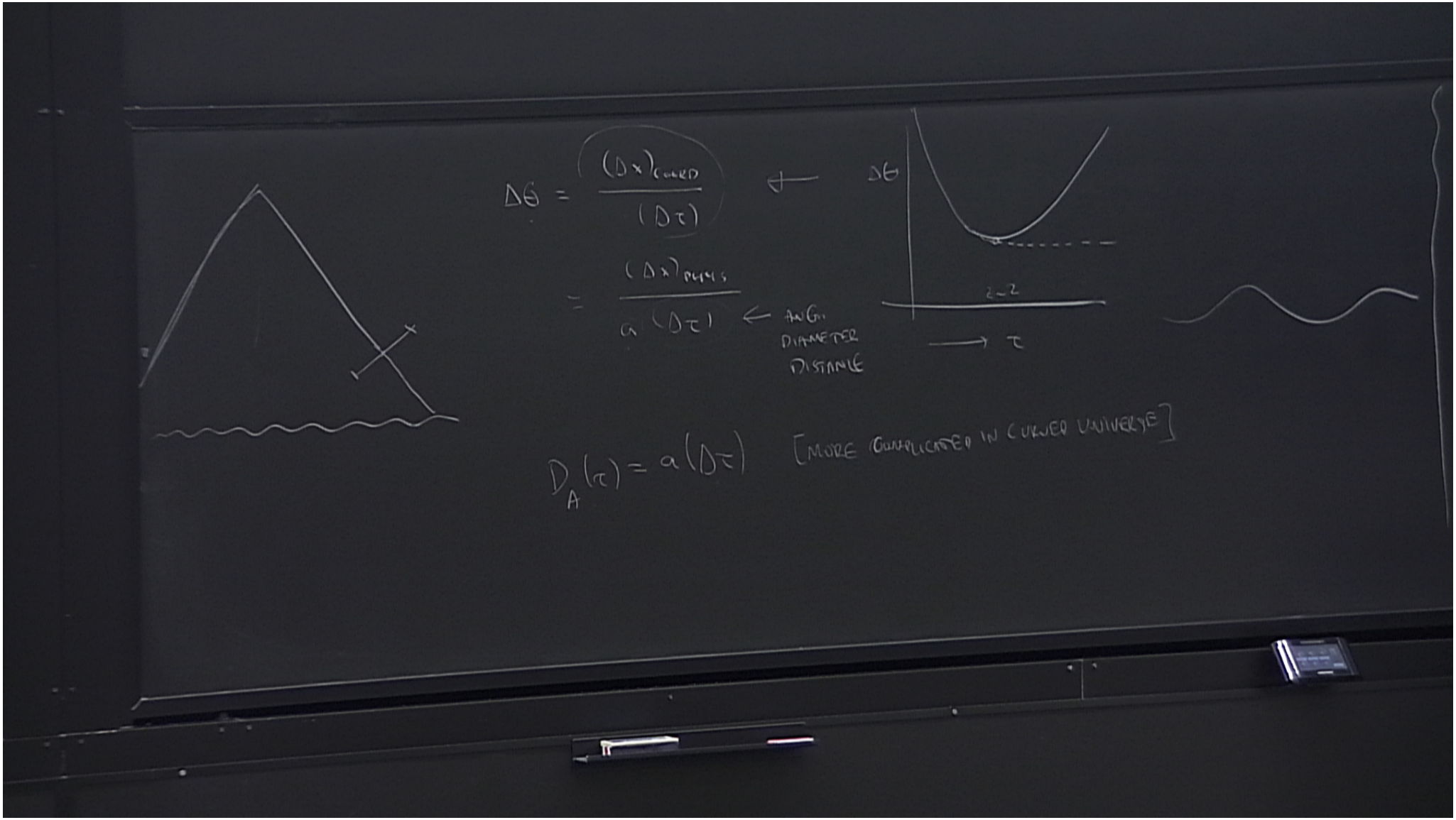
Title: PSI 2015/2016 Explorations in Cosmology - Kendrick Smith - 2

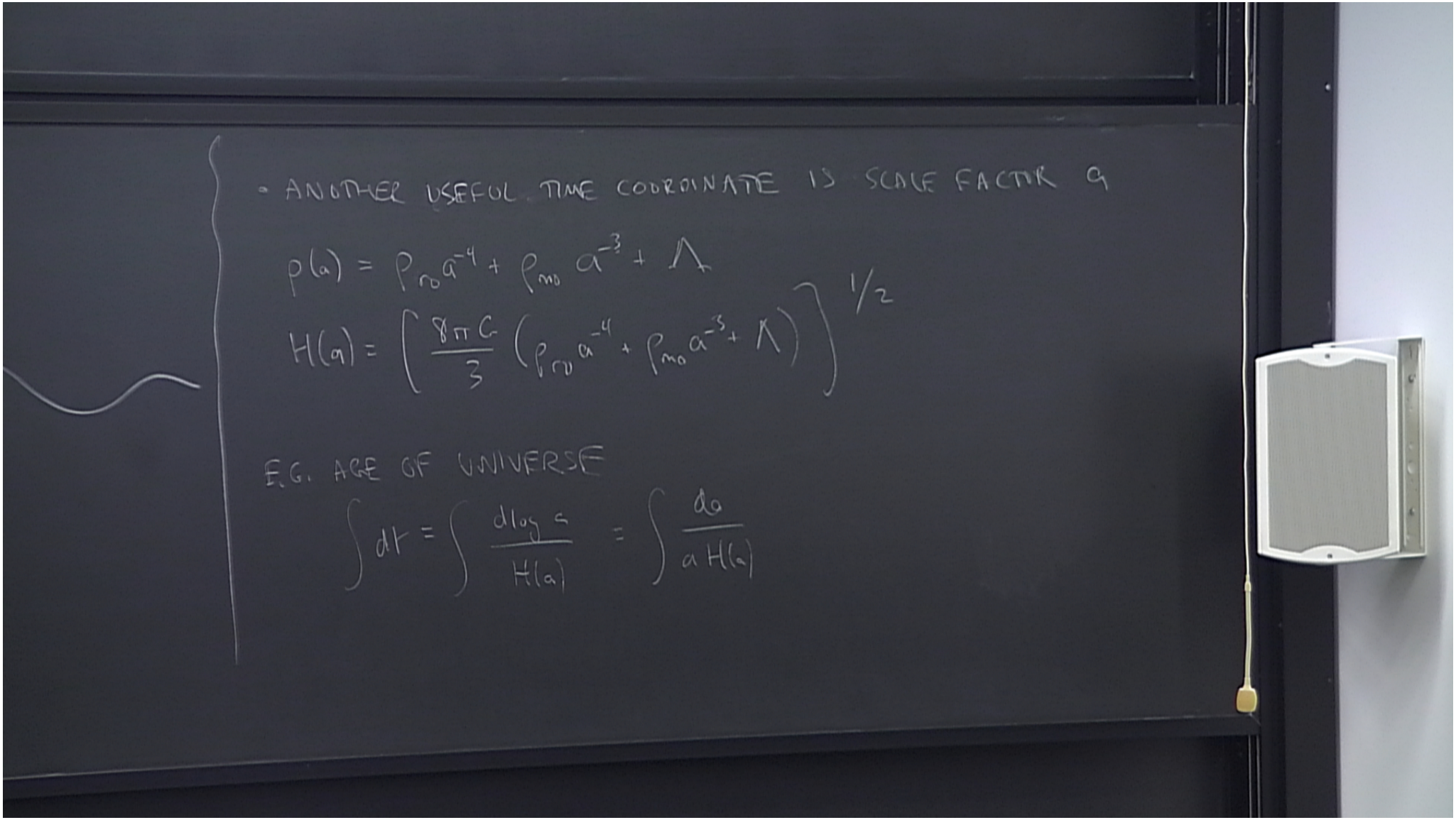
Date: Apr 12, 2016 10:15 AM

URL: http://pirsa.org/16040036

Abstract:


$\Delta\theta = \frac{(\Delta x)_{\text{CLOCK}}}{(\Delta\tau)}$
$= \frac{(\Delta x)_{\text{PHYS}}}{a\,(\Delta\tau)}$
$\Delta\theta$
τ


$\Delta\theta = \frac{(\Delta x)_{\text{CURVED}}}{(\Delta\tau)}$
$= \frac{(\Delta x)_{\text{PHYS}}}{a(\Delta\tau)}$
ANG. DIAMETER DISTANCE
$\Delta\theta$
$z \sim 2$
τ
$D_A(\tau) = a(\Delta\tau)$
[MORE COMPLICATED IN CURVED UNIVERSE]

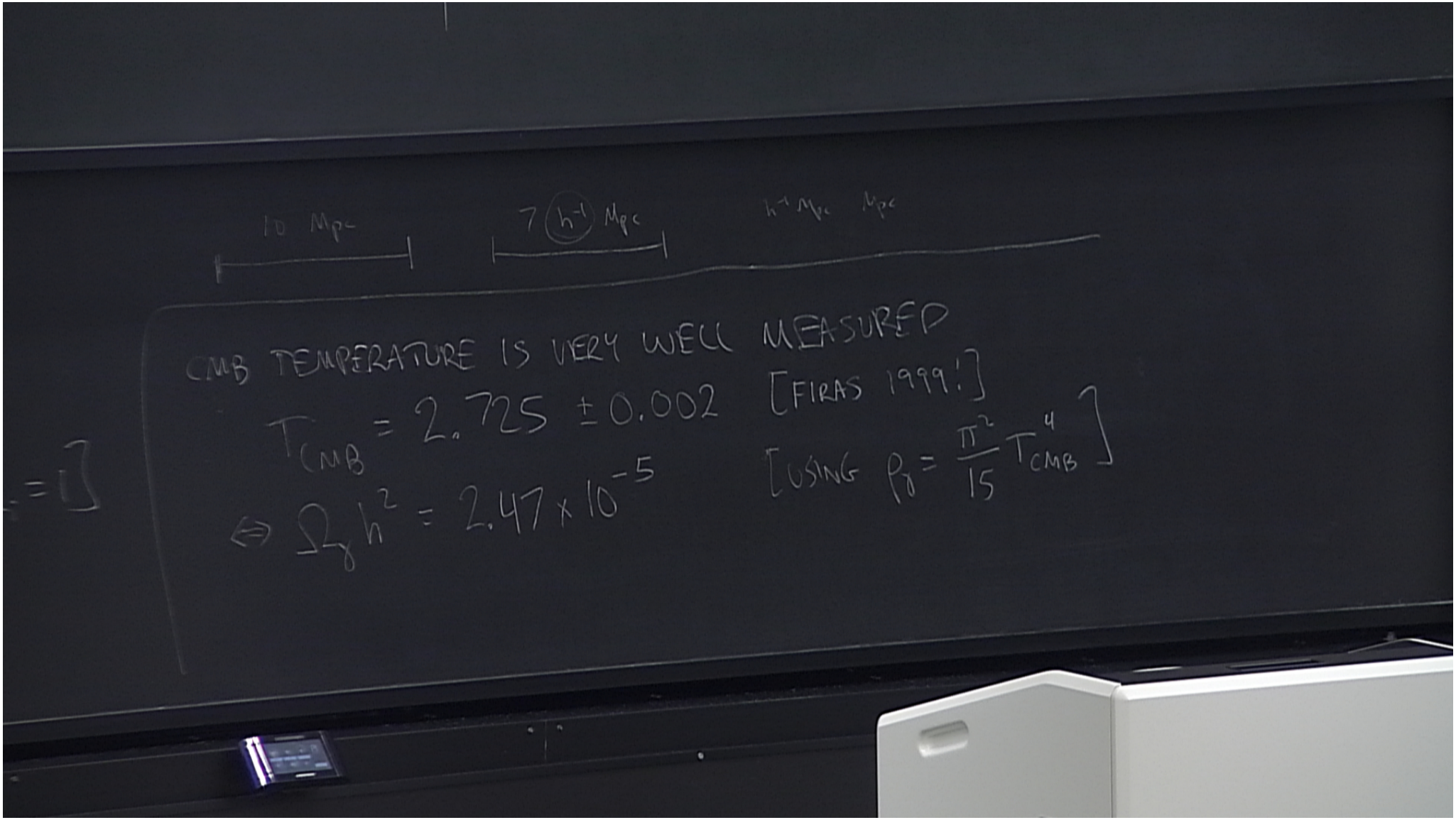

• ANOTHER USEFUL TIME COORDINATE IS SCALE FACTOR a
$\rho(a) = \rho_{r0} a^{-4} + \rho_{m0} a^{-3} + \Lambda$
$H(a) = \left(\frac{8\pi G}{3} \left(\rho_{r0} a^{-4} + \rho_{m0} a^{-3} + \Lambda \right) \right)^{1/2}$
E.G. AGE OF UNIVERSE
$\int dt = \int \frac{d\log a}{H(a)} = \int \frac{da}{a H(a)}$

- PARAMETERS OF EXP HISTORY $\{\rho_{r0}, \rho_{c0}, \rho_{b0}, \Lambda\}$

$$\rho_{m0} = \underbrace{\rho_{c0}}_{\text{CDM}} + \underbrace{\rho_{b0}}_{\text{BARYONIC}}$$

- WEIRD COSMOLOGISTS' PARAMETRIZATION
  - INSTEAD OF SPECIFYING ρ_{i0}, SPECIFY $\Omega_i \overset{\text{DEF}}{=} \dfrac{\rho_{i0}}{\sum_j \rho_{j0}}$ $\quad [\Rightarrow \sum \Omega_i = 1]$
  - INSTEAD OF SPECIFYING $\rho_0 = \sum_j \rho_{j0}$, SPECIFY

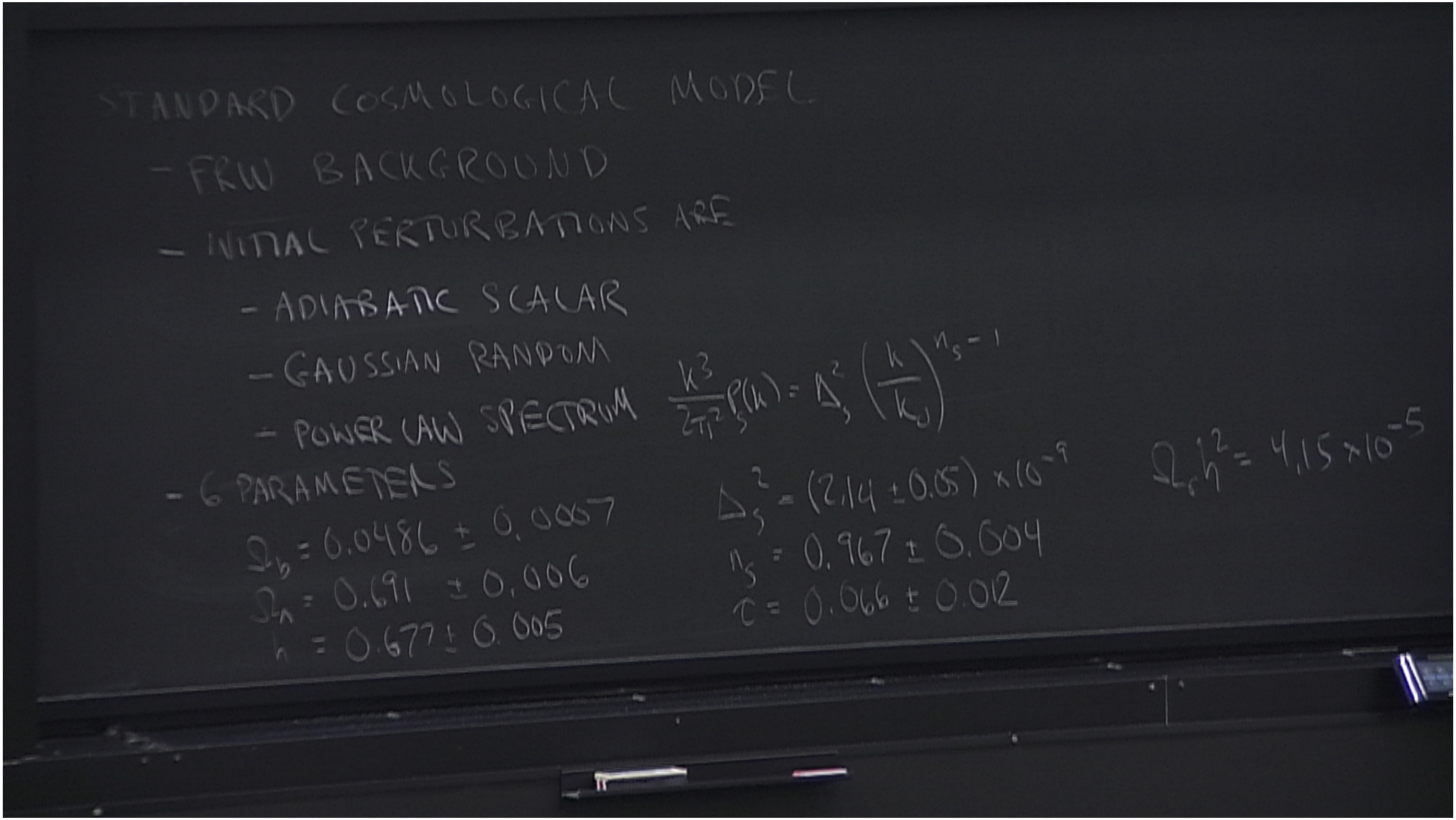
$$H_0 = \left(\frac{8\pi G}{3}\rho_0\right)^{1/2} = 100h \text{ km sec}^{-1} \text{Mpc}^{-1}$$

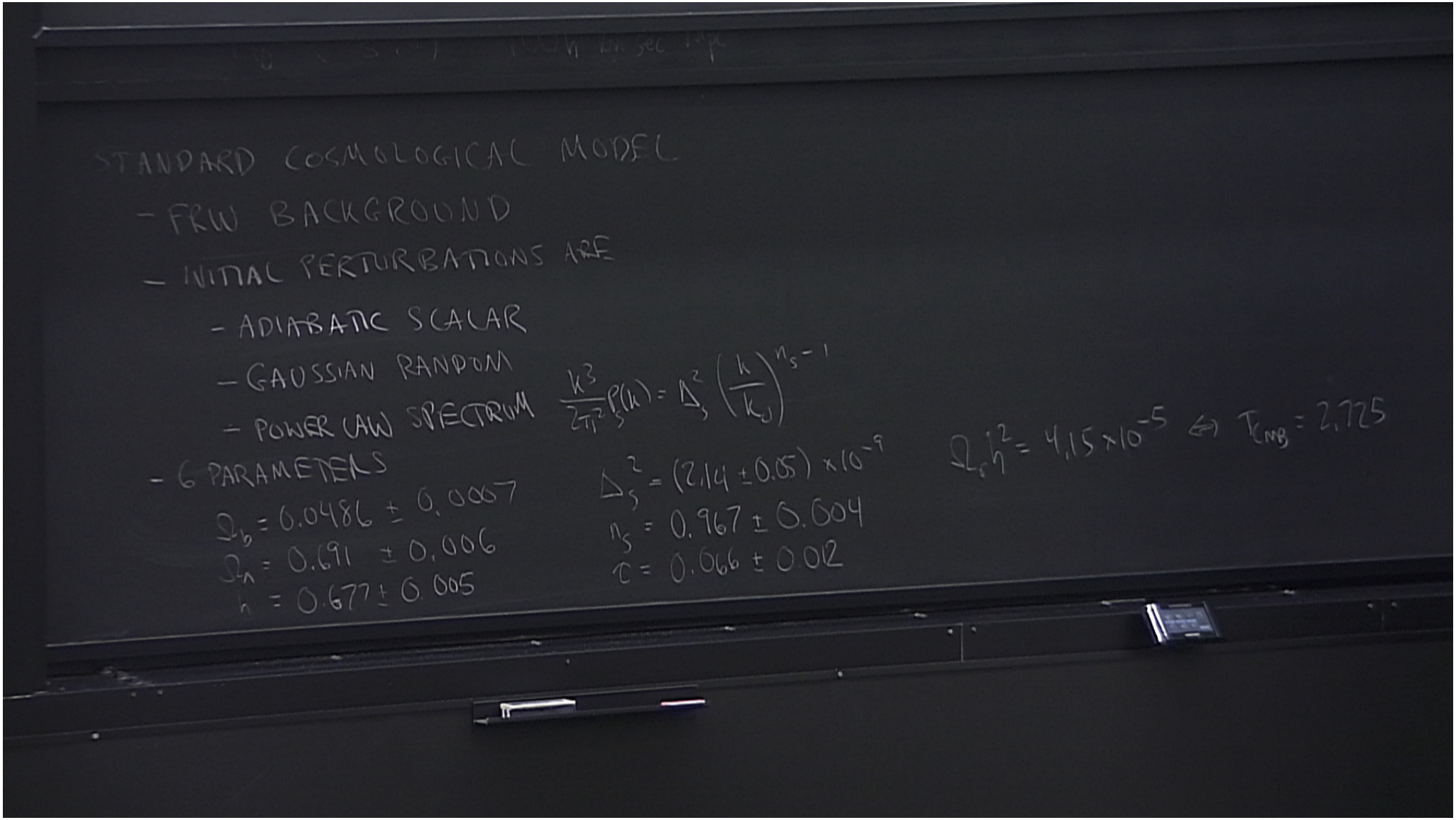

10 Mpc
7 (h⁻¹) Mpc
h⁻¹ Mpc
Mpc
CMB TEMPERATURE IS VERY WELL MEASURED
$T_{CMB} = 2.725 \pm 0.002$ [FIRAS 1999!]
$\Leftrightarrow \Omega_\gamma h^2 = 2.47 \times 10^{-5}$
[USING $\rho_\gamma = \frac{\pi^2}{15} T_{CMB}^4$]

ADDING NEUTRINO (ASSUMING MASSLESS):

$\Omega_\nu h^2 = 1.68 \times 10^{-5}$ [USING $\rho_\nu = \frac{7\pi^4}{40} T_\nu^4$, $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_{CMB}$]

$\Rightarrow \Omega_r h^2 = 4.15 \times 10^{-5}$


STANDARD COSMOLOGICAL MODEL
- FRW BACKGROUND
- INITIAL PERTURBATIONS ARE
- ADIABATIC SCALAR
- GAUSSIAN RANDOM
- POWER LAW SPECTRUM
$\frac{k^3}{2\pi^2} P(k) = \Delta_s^2 \left(\frac{k}{k_0}\right)^{n_s - 1}$
- 6 PARAMETERS
$\Omega_b = 0.0486 \pm 0.0007$
$\Omega_\Lambda = 0.691 \pm 0.006$
$h = 0.677 \pm 0.005$
$\Delta_s^2 = (2.14 \pm 0.05) \times 10^{-9}$
$n_s = 0.967 \pm 0.004$
$\tau = 0.066 \pm 0.012$
$\Omega_c h^2 = 4.15 \times 10^{-5}$


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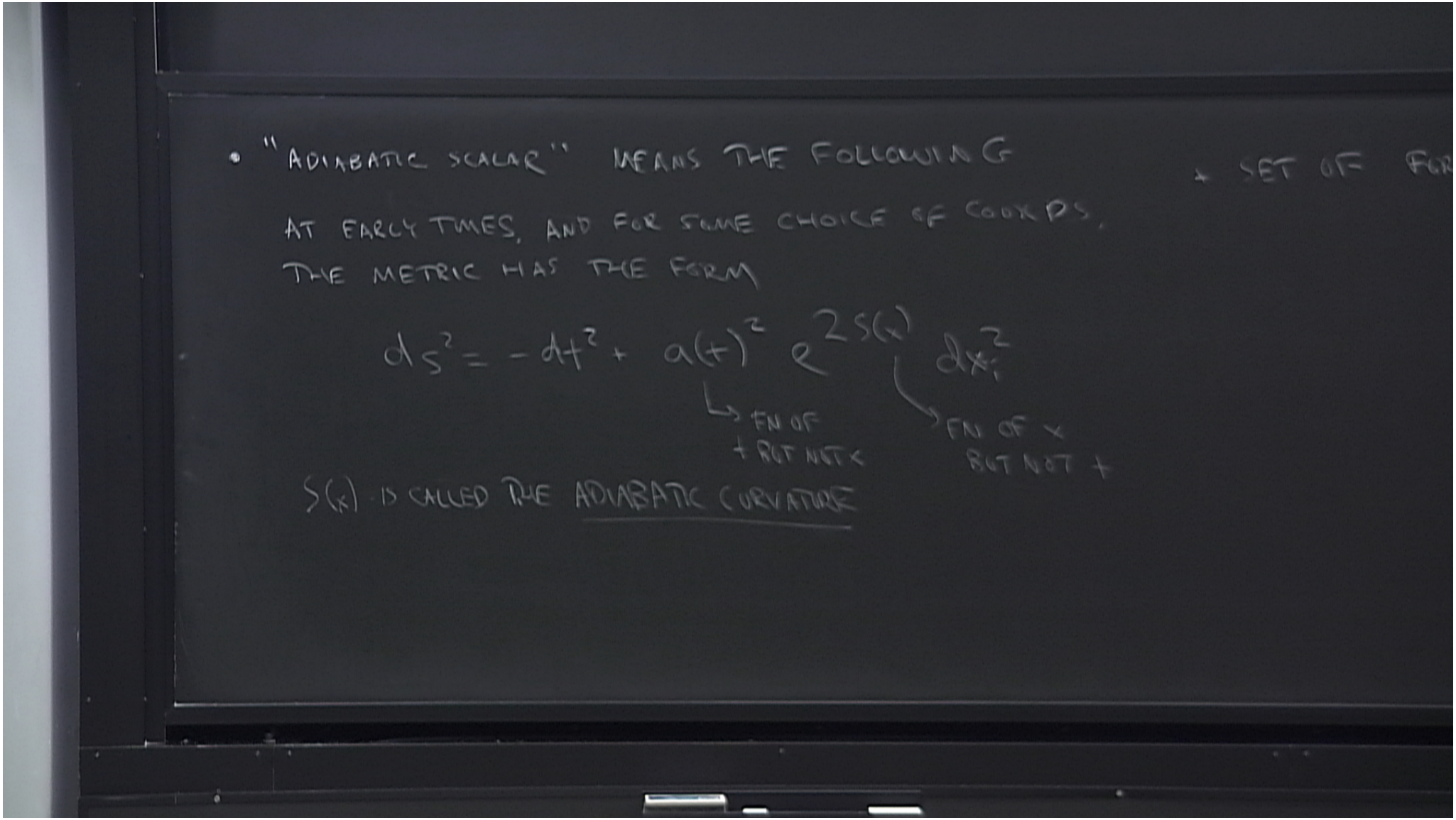
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$\Rightarrow \Omega_r h^2 = 4.15 \times 10^{-5}$

OBSERVER'S TIME COORDINATE: $a = (1+z)^{-1}$ "REDSHIFT z"

IN AN EXPANDING UNIVERSE, PHOTONS REDSHIFT AS $\nu \propto a^{-1} = (1+z)$

FOR A GENERAL FRW GEODESIC, $p_{phys} \sim a^{-1}$


• "ADIABATIC SCALAR" MEANS THE FOLLOWING
+ SET OF FOR
AT EARLY TIMES, AND FOR SOME CHOICE OF COORDS,
THE METRIC HAS THE FORM
$ds^2 = -dt^2 + a(t)^2 e^{2\zeta(x)} dx_i^2$
FN OF + BUT NOT x
FN OF x BUT NOT +
$\zeta(x)$ IS CALLED THE ADIABATIC CURVATURE

A SET OF FORMULAS FOR LEADING BEHAVIOR AS $\tau \to 0$ IN ALL OTHER FIELDS IN TERMS OF δ. E.G. MATTER DENSITY

$$\rho_m(x,\tau) = \rho_{m0}\, a^{-3}\left[1 - \frac{1}{3}\tau^2 \partial^2 \delta(x) + \cdots O(\tau^3)\right]$$

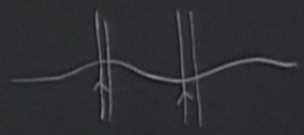
THE RADIATION DENSITY

$$\rho_r(x,\tau) = \rho_{r0}\, a^{-4}\left[1 - \frac{4}{9}\tau^2 \partial^2 \delta(x) + \cdots\right]$$

TO BE ELABORATED & EXPLAINED LATER!

+ SET OF FORMULAS FOR LEADING BEHAVIOR AS $\tau \to 0$ IN ALL OTHER FIELDS IN TERMS OF S. E.G. MATTER DENSITY

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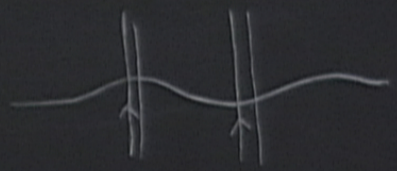
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ION DENSITY

$$x,\tau) = \rho_{r0}\, a^{-4}\left[1 - \frac{4}{9}\tau^2 \partial^2 \delta(x) + \cdots\right]$$

DIAGRAM: SINGLE FIELD INFLATION ALWAYS GENERATES THE ADIABATIC MODE

ABORATED & EXPLAINED LATER!

TO BE ELABORATED & EXPLA

$\delta(x)^* = \delta(x)$ $\quad$ $\delta(k)^* = \delta(-k)$

RECALL DEFINITION OF POWER SPECTRUM:

$$\langle \delta(\vec{k})\, \delta(\vec{k}')^* \rangle = P_\delta(k)(2\pi)^3 \delta^3(\vec{k}-\vec{k}')$$

$e^{-i(\vec{k}-\vec{k}')\cdot\Delta\vec{x}} = e^{-i\vec{k}\cdot\Delta\vec{x}}\, e^{i\vec{k}'\cdot\Delta\vec{x}}$

$\Delta\vec{x}$ →

$$\delta(\vec{k}) \to e^{i\vec{k}\cdot\Delta\vec{x}}\, \delta(\vec{k})$$

$$\langle \delta(\vec{k}_1)\, \delta(\vec{k}_2) \rangle = P_\delta(k_1)(2\pi)^3 \delta^3(\vec{k}_1+\vec{k}_2)$$

E ELABORATED & EXPLAINED LATER!

$S(\vec{k}) \to e^{i\vec{k}\cdot\vec{\Delta x}} S(\vec{k})$ "GAUSSIAN RANDOM"

S IS A GAUSSIAN FIELD IF EACH FOURIER MODE $S(\vec{k})$ IS AN INDEPENDENT GAUSSIAN RANDOM NUMBER, WITH k-DEPENDENT VARIANCE GIVEN BY $P_S(k)$

A REAL RANDOM VARIABLE x IS GAUSSIAN $\iff$ $p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$

WHERE $\sigma^2 = \langle x^2 \rangle$

A COMPLEX R.V. z IS GAUSSIAN IF Re(z), Im(z) ARE INDEPENDENT GAUSSIANS W/ VARIANCE $\frac{1}{2}\langle z^* z \rangle$.

ELABORATED & EXPLAINED LATER!

$S(\vec{k}) \to e^{i\vec{k}\cdot\vec{\Delta x}} S(\vec{k})$ "GAUSSIAN RANDOM"

$S(x)$ $\mathrm{Re}(S_k)$ $\mathrm{Im}(S_k)$ $\vec{k}$



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LOGICAL MODEL

ROUND

RBATIONS ARE

SCALAR

RANDOM

SPECTRUM $\frac{k^3}{2\pi^2} P_S(k) = \Delta_S^2 \left(\frac{k}{k_0}\right)^{n_S - 1}$

$= 0.0007$

0.006

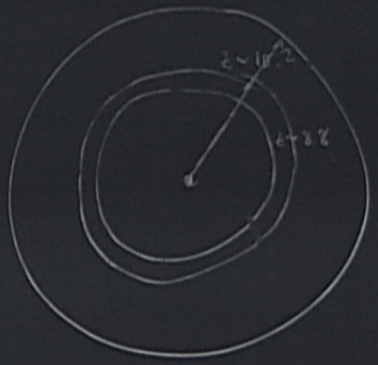
0.005

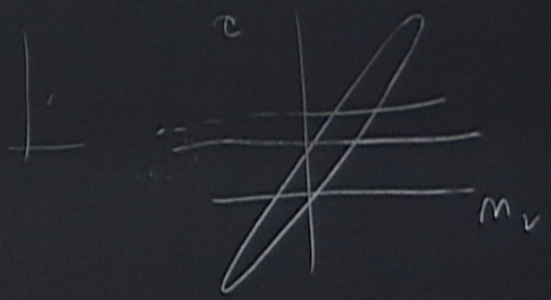
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- INSTEAD OF SPECIFYING ρ_{i0}, SPECIFY $\lambda_i = \Sigma_i \rho_{i0}$

MORE OF RANDOM FIELDS

- GAUSSIAN RANDOM NUMBER IN 1 VARIABLE

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$\langle x^2 \rangle = \sigma^2 \qquad \langle x^{2N+1} \rangle = 0$$

$$\langle x^4 \rangle = 3\sigma^4$$

$$\langle x^6 \rangle = 15\sigma^6$$

$$\vdots$$

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$\langle$ xxxxxx $\rangle = 15\sigma^6$

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$X = X_1 + \cdots + X_N$

INDEPENDENT
IDENTICALLY DISTRIBUTED
RANDOM VARIABLES

$N \gg 1$

$\Rightarrow$ GAUSSIAN

$\langle x^6 \rangle = 15\sigma^6$ $\Rightarrow$ GAUSSIAN

N-DIMENSIONAL GAUSSIAN x_i

VARIANCE $\sigma^2 \Rightarrow \langle x_i x_j \rangle = C_{ij}$ "COVARIANCE MATRIX"

PDF IS $p(x_i) = \frac{1}{\sqrt{2\pi \, \mathrm{Det}(C)}} \exp\left(-\frac{1}{2} x^T C^{-1} x\right)$

BASIC CALCULATIONAL TOOL IS WICK'S THEOREM, E.G.

$\langle x_i x_j x_k x_l \rangle = C_{ij} C_{kl} + C_{ik} C_{jl} + C_{il} C_{jk}$

CMB TEM

$T_{(M}$

$\Leftrightarrow \Omega$