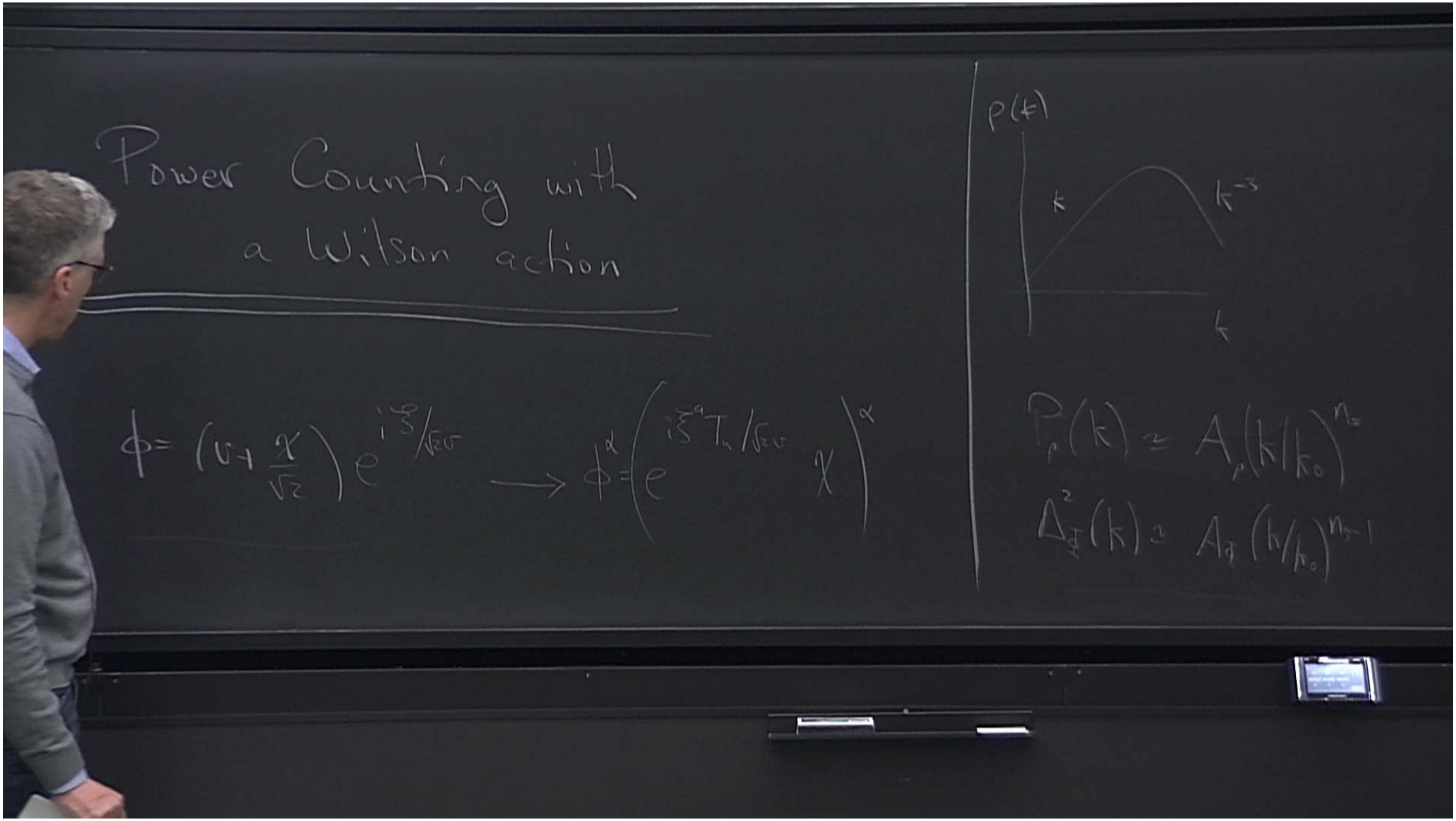
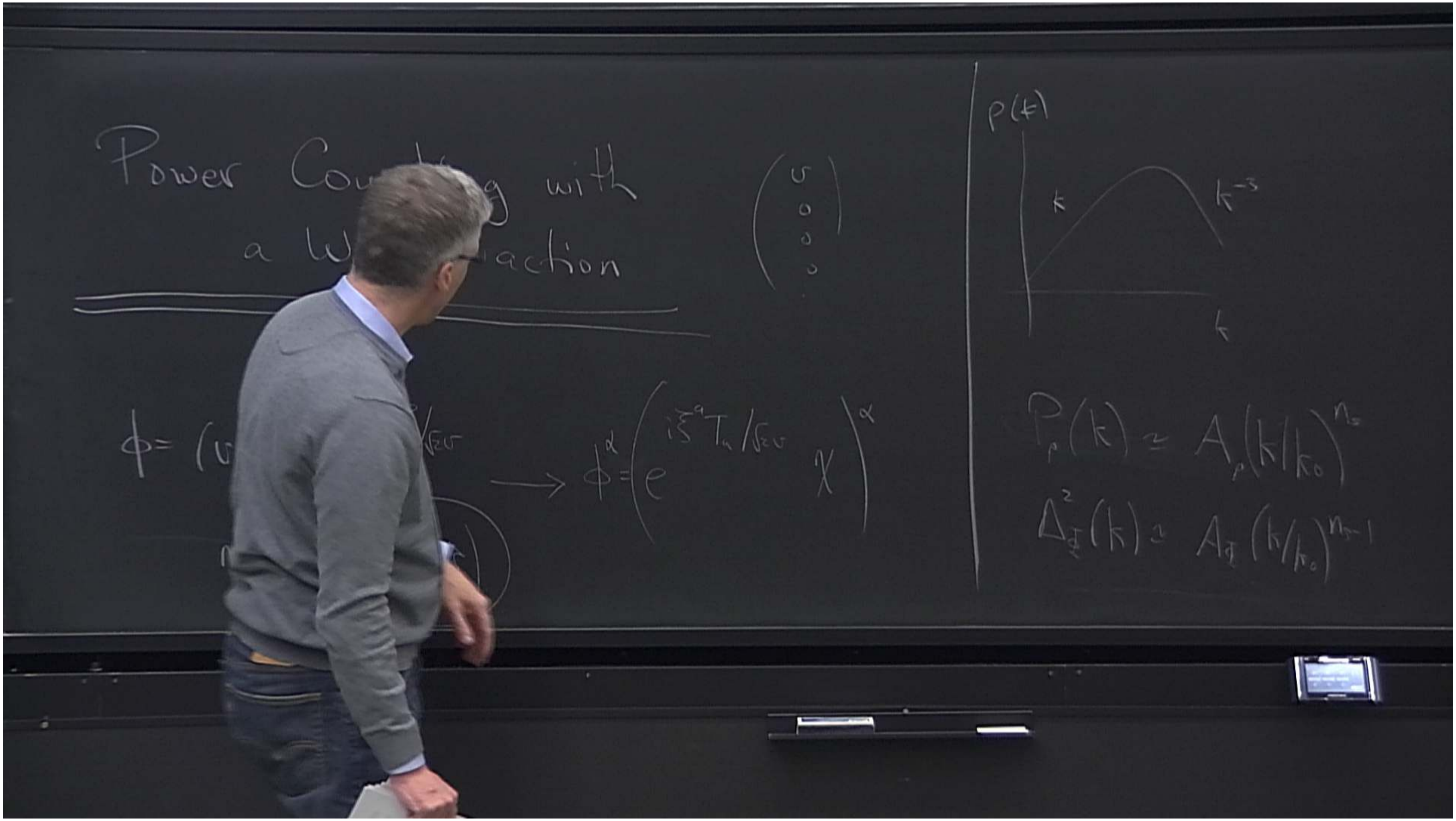
Title: PSI 2015/2016 Explorations in Particle Theory - Burgess - 14

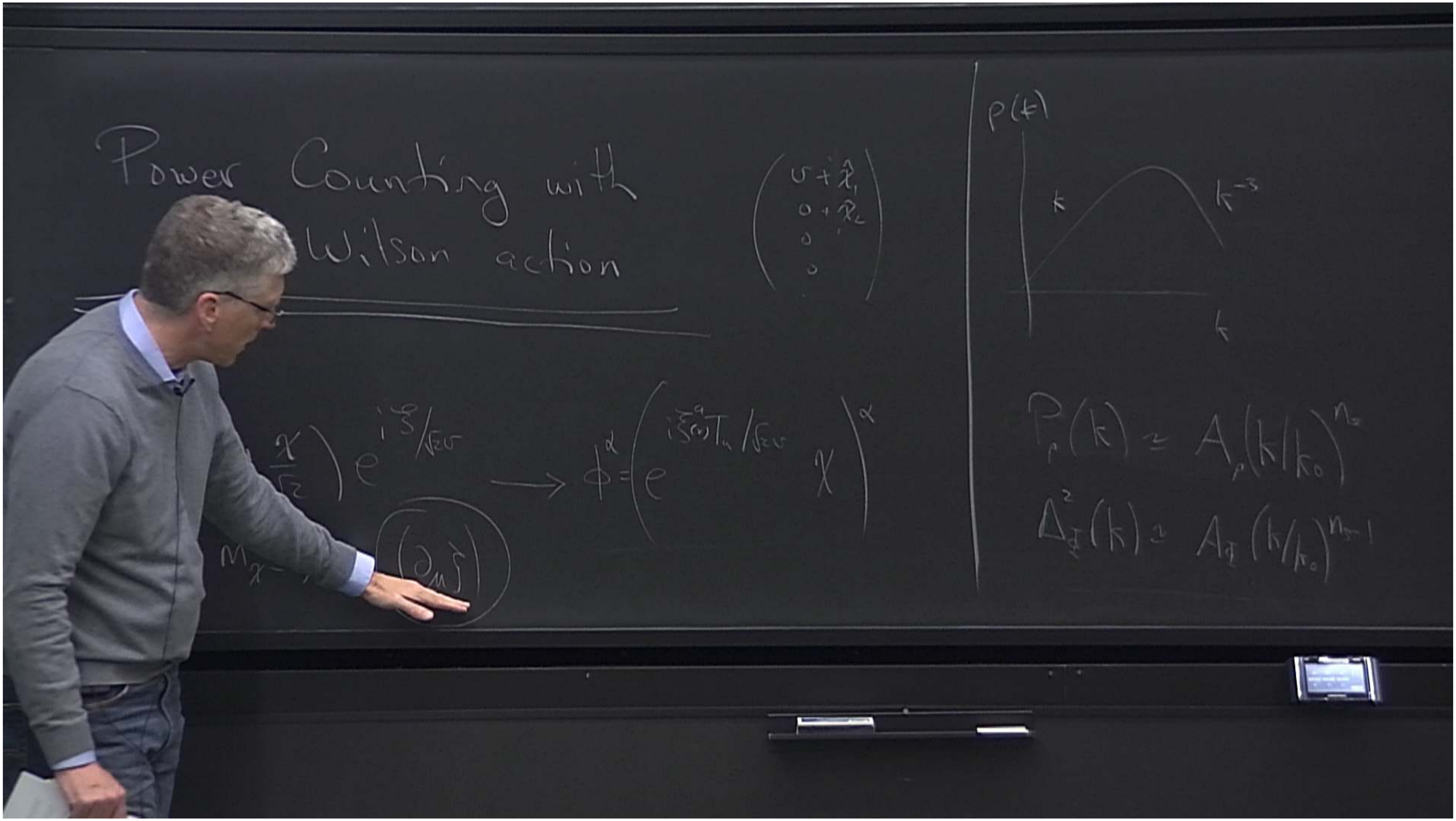
Date: Apr 28, 2016 09:00 AM

URL: http://pirsa.org/16040033

Abstract:


Power Counting with a Wilson action
$\phi = \left(v + \frac{\chi}{\sqrt{2}}\right) e^{i\xi/\sqrt{2}v} \longrightarrow \phi^\alpha = \left(e^{i\xi^a T_a/\sqrt{2}v}\ \chi\right)^\alpha$
$\rho(k)$
k
k^{-3}
k
$P_\rho(k) \simeq A_\rho (k/k_0)^{n_s}$
$\Delta^2_\xi(k) \simeq A_\xi (k/k_0)^{n_s-1}$


Power
with
action
ρ(k)
k
k^-3
k


Power Counting with
Wilson action
ρ(k)
k
k^-3
k
P_ρ(k) ≃ A_ρ(k/k_0)^{n_s}

k^{-3}

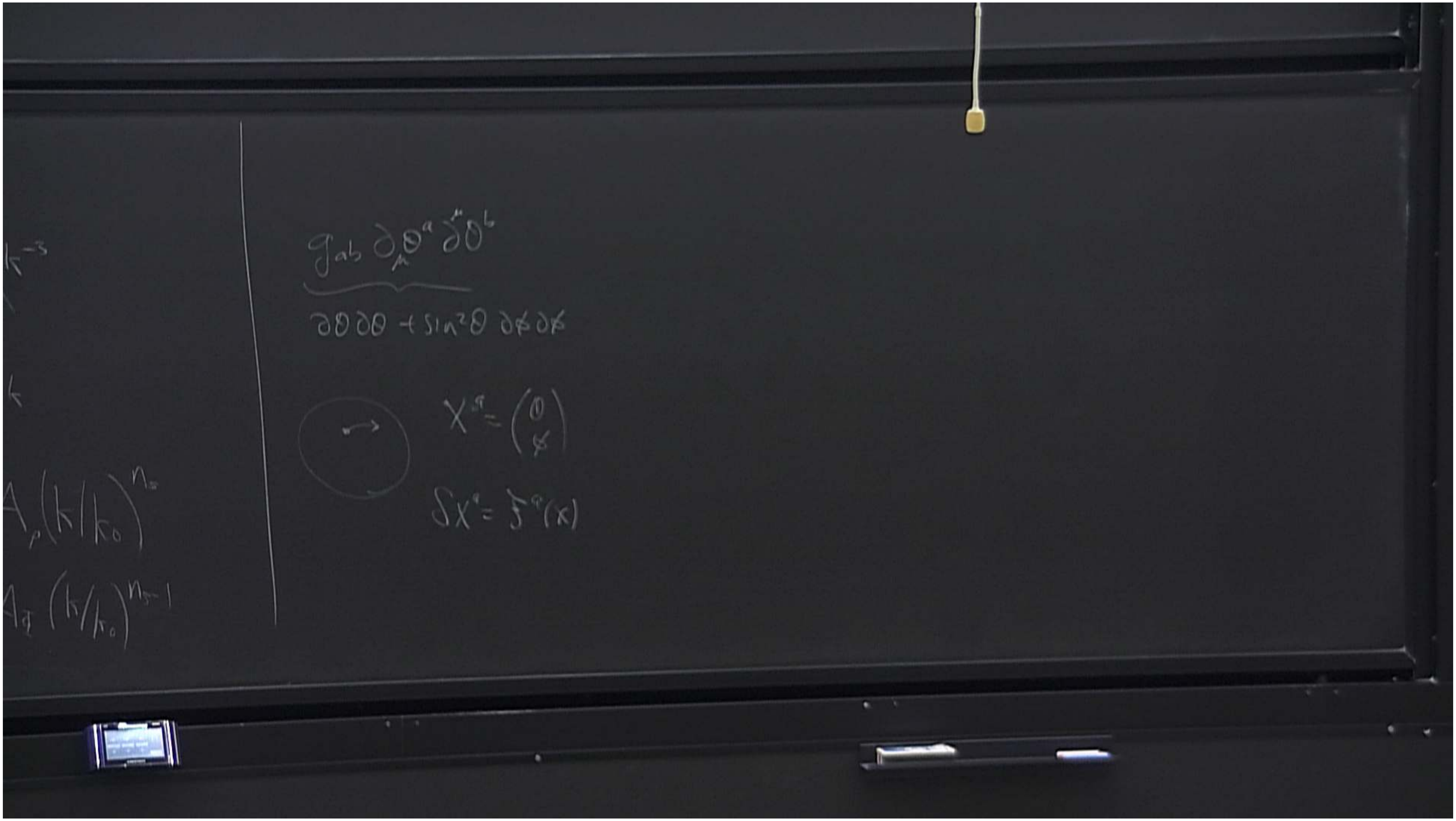
k

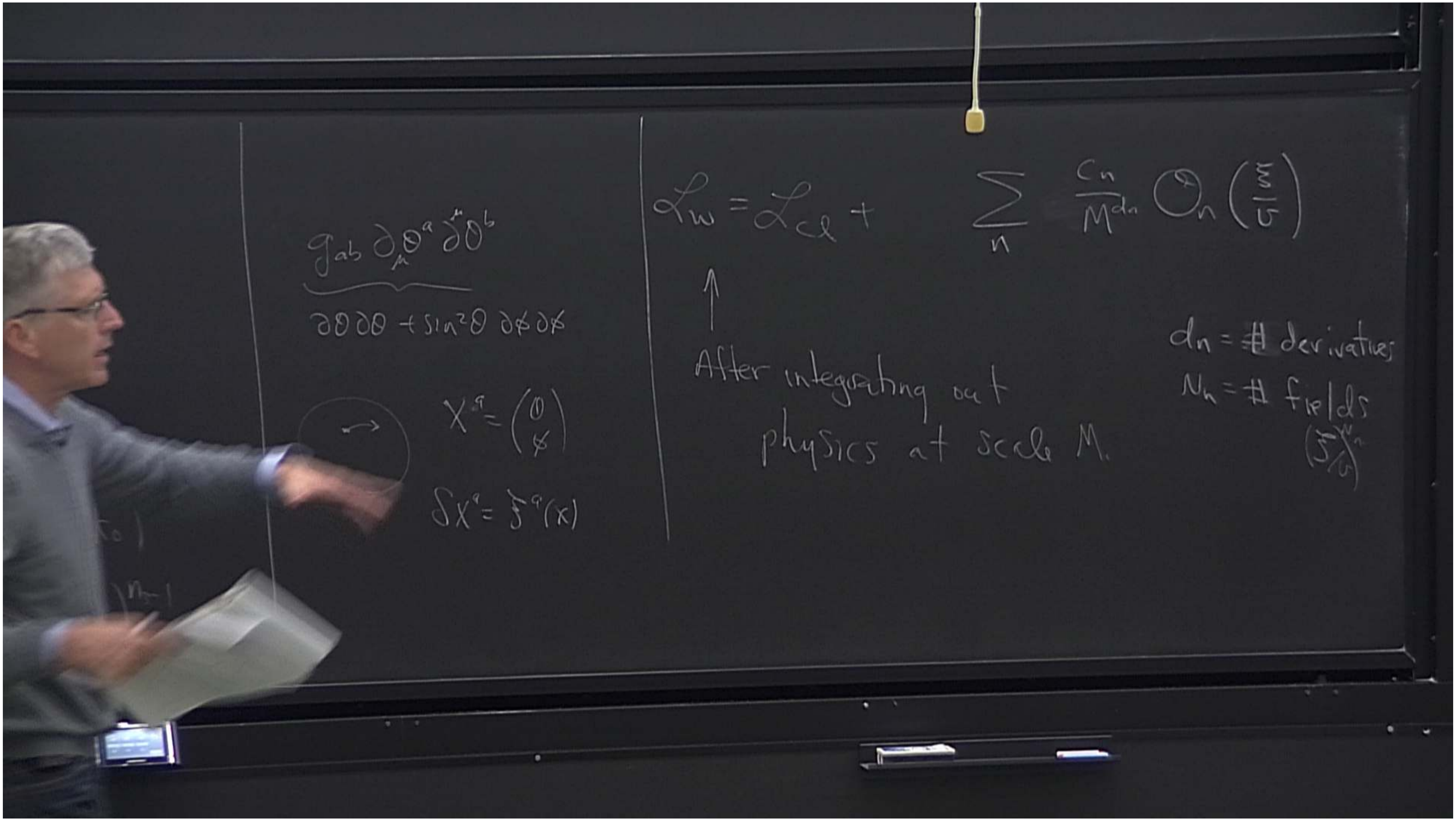
$A_r (k/k_0)^{n_s}$

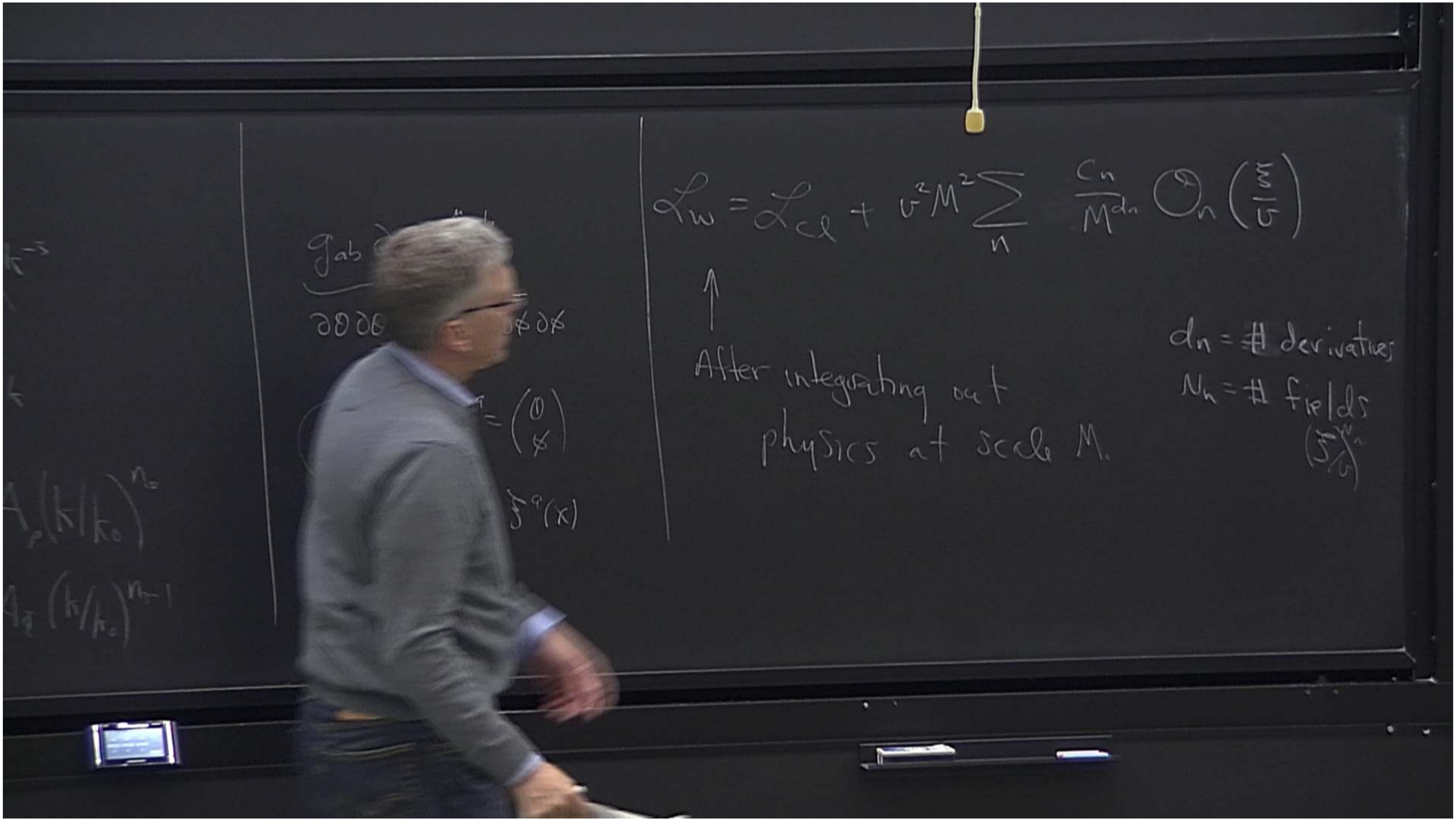
$A_t (k/k_0)^{n_s - 1}$

$$\underbrace{g_{ab}\,\partial_\mu \theta^a\, \partial^\mu \theta^b}$$

$$\partial\theta\,\partial\theta + \sin^2\theta\,\partial\phi\,\partial\phi$$


$g_{ab}\,\partial_\mu\theta^a\,\partial^\mu\theta^b$
$\partial\theta\partial\theta + \sin^2\theta\,\partial\phi\partial\phi$
$X^a = \begin{pmatrix}\theta\\ \phi\end{pmatrix}$
$\delta X^a = \xi^a(x)$
$A_s (k/k_0)^{n_s}$
$A_T (k/k_0)^{n_s-1}$


$g_{ab}\partial_\mu\theta^a\partial^\mu\theta^b$
$\partial\theta\partial\theta + \sin^2\theta\,\partial\phi\partial\phi$
$X^a = \begin{pmatrix}\theta\\ \phi\end{pmatrix}$
$\delta X^a = \xi^a(x)$
$\mathcal{L}_W = \mathcal{L}_{cl} + \sum_n \frac{c_n}{M^{d_n}}\, \mathcal{O}_n\left(\frac{\xi}{v}\right)$
After integrating out
physics at scale M.
d_n = # derivatives
N_n = # fields
$\left(\xi/v\right)^{N_n}$


$\mathcal{L}_W = \mathcal{L}_{cl} + v^2 M^2 \sum_n \frac{c_n}{M^{d_n}} \mathcal{O}_n\left(\frac{\xi}{v}\right)$
After integrating out
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d_n = # derivatives
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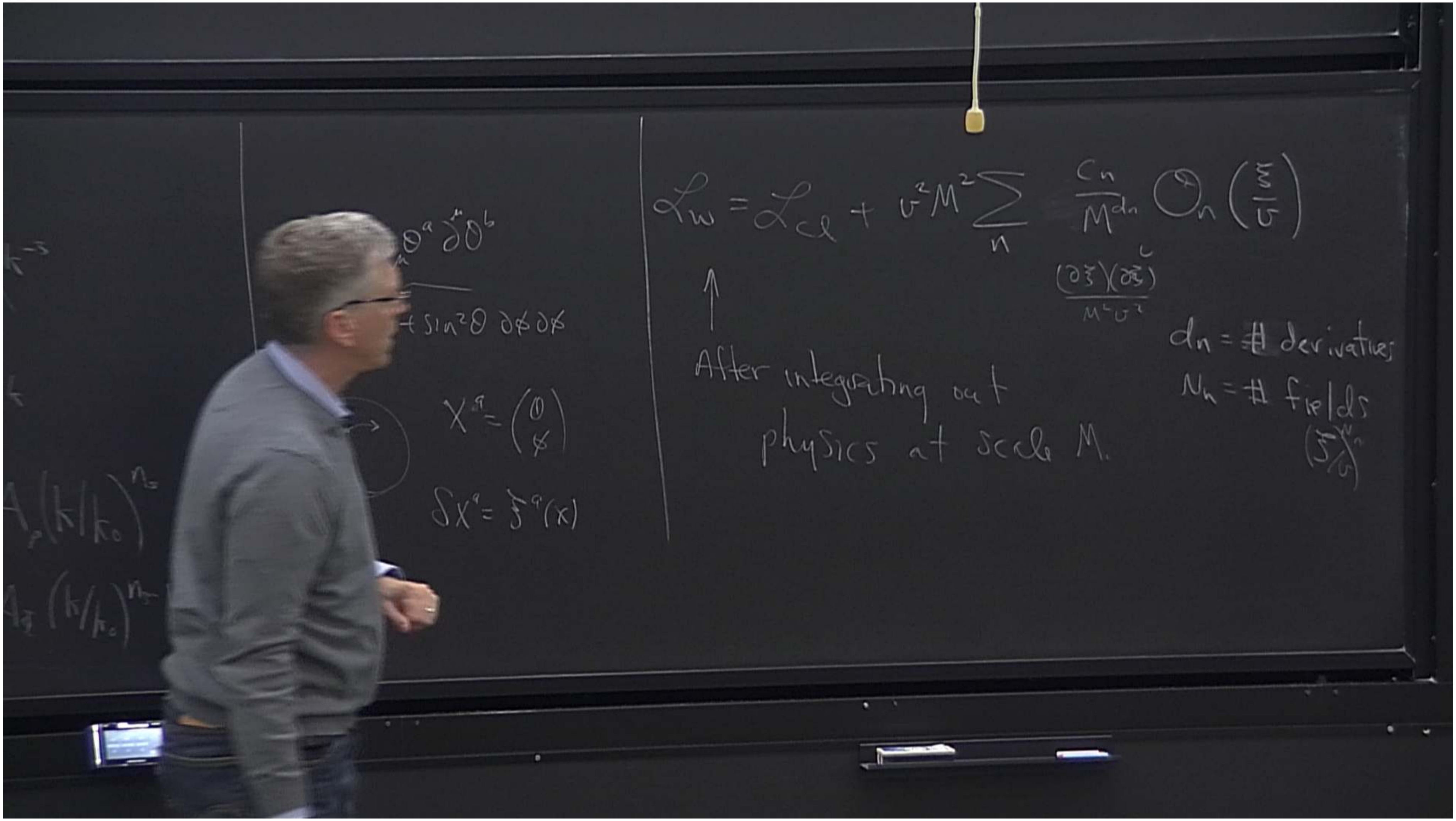
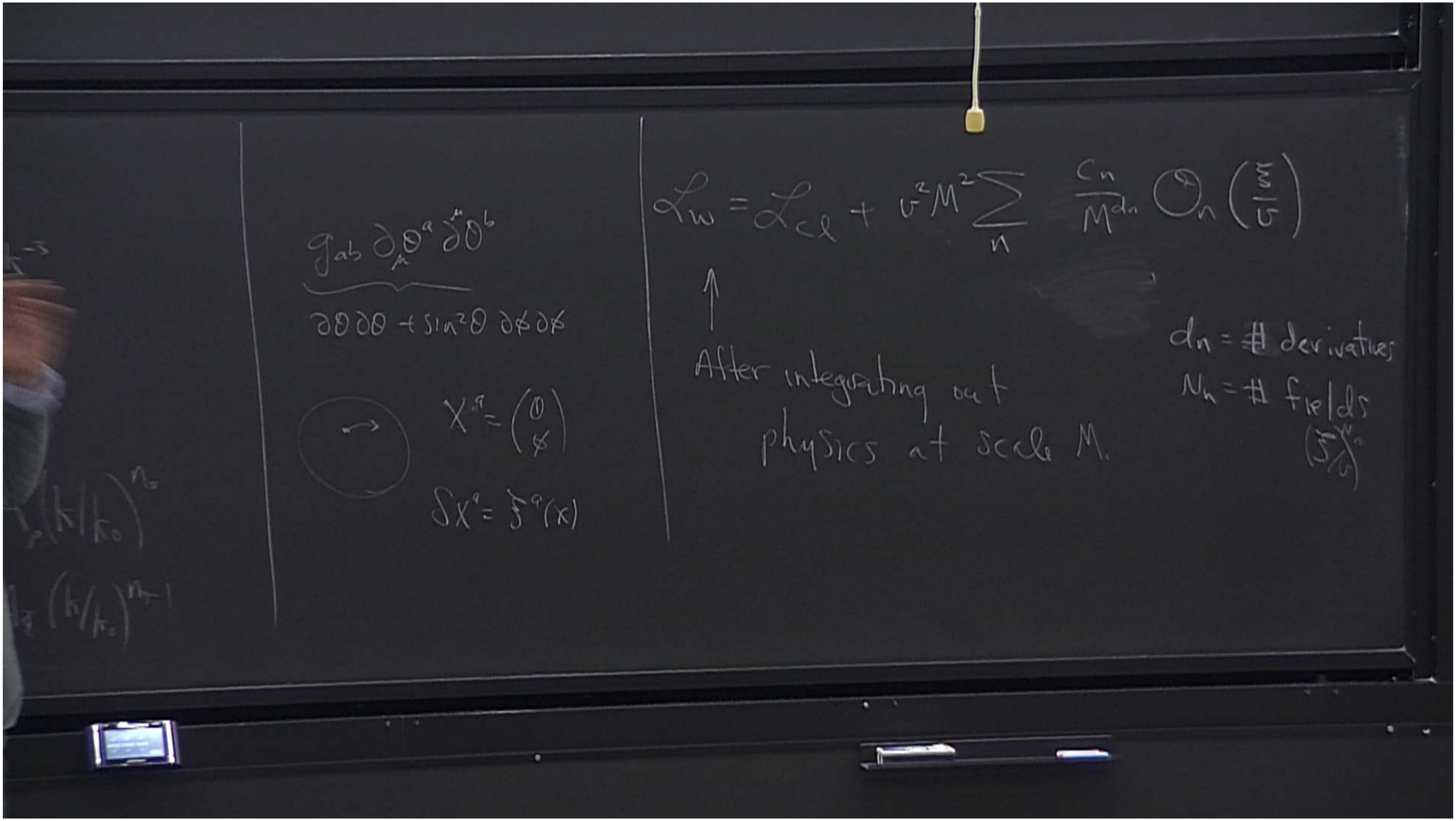
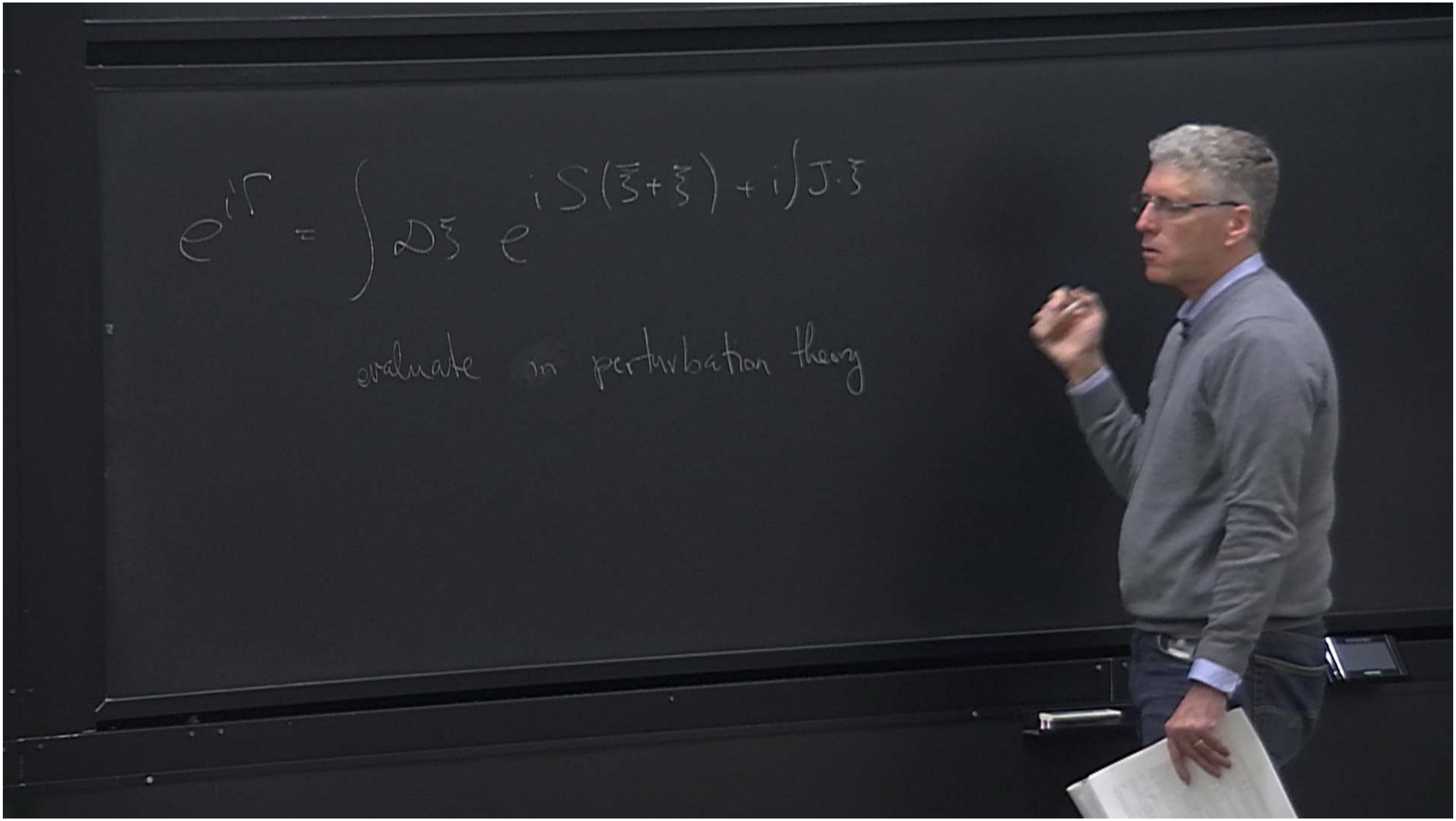
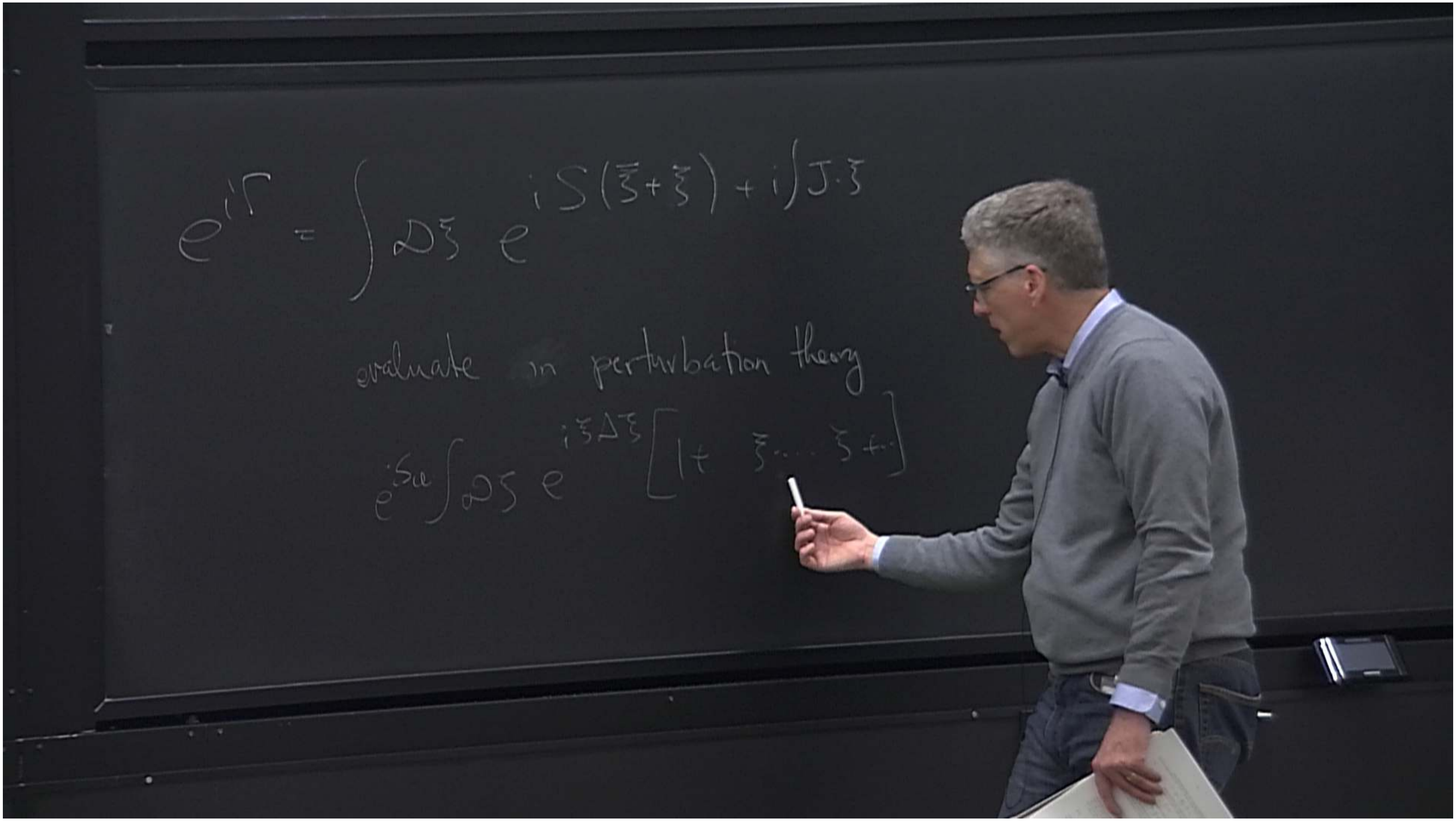
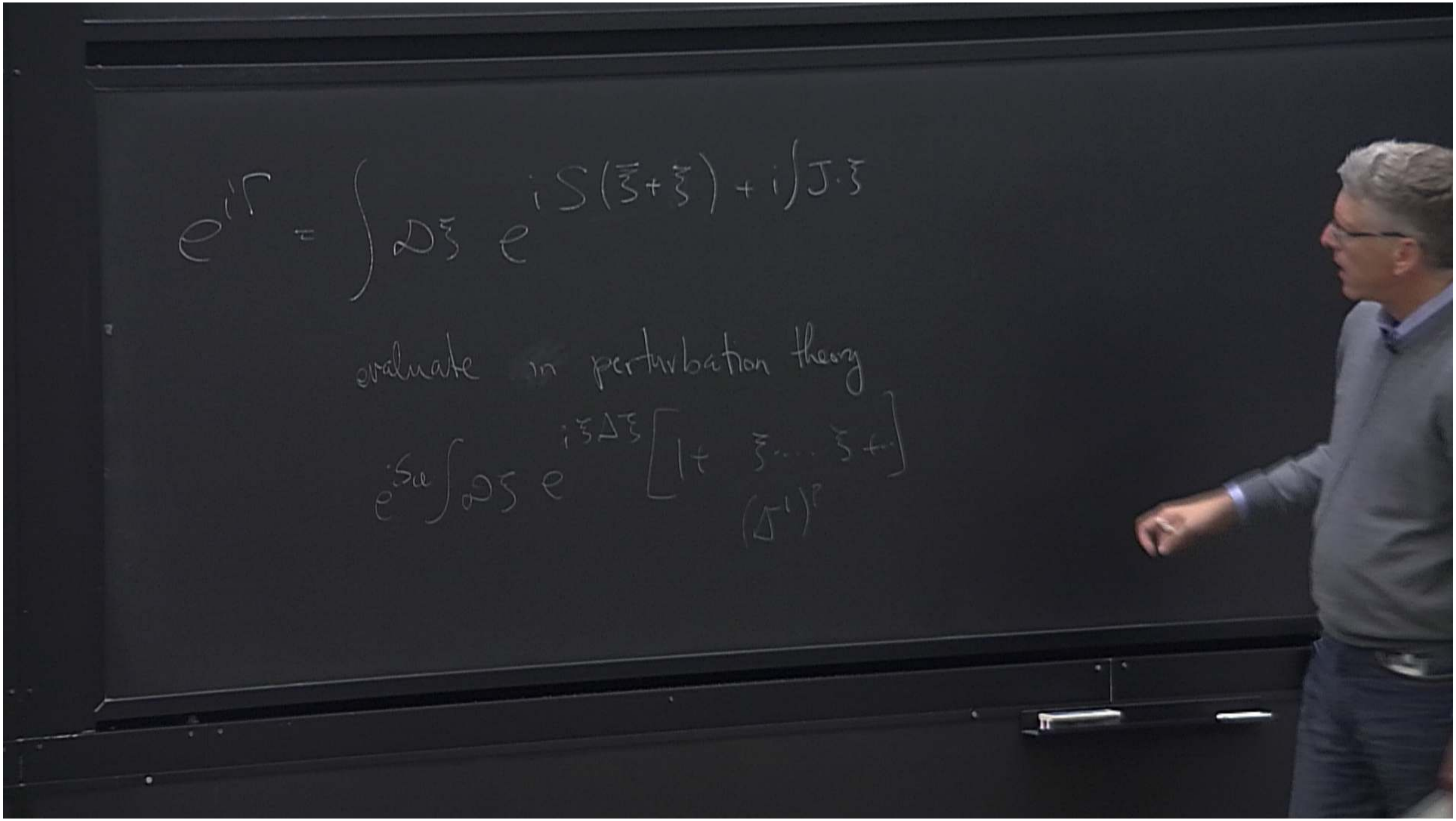


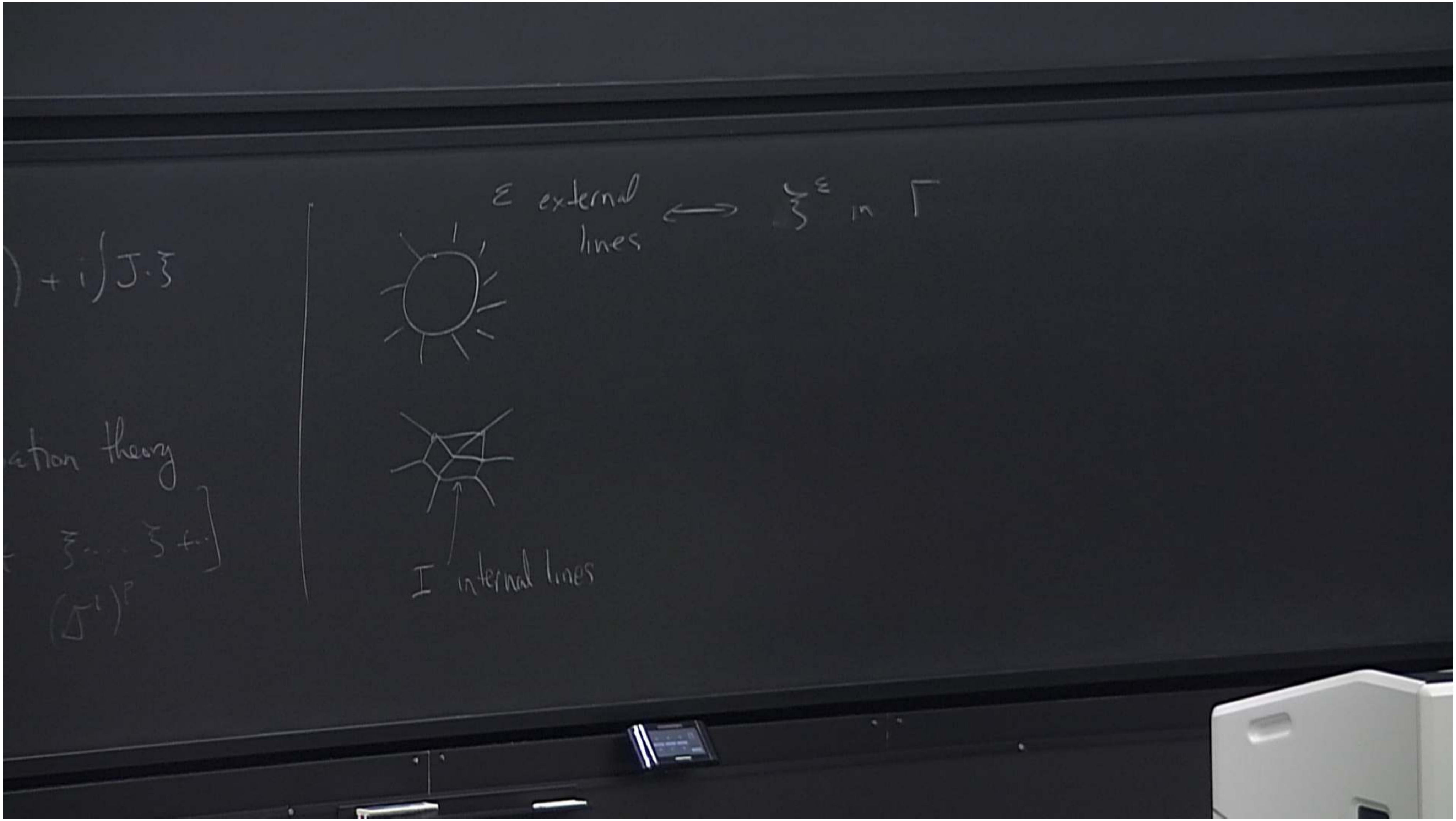
$\mathcal{L}_W = \mathcal{L}_{cl} + v^2 M^2 \sum_n \frac{c_n}{M^{d_n}} \mathcal{O}_n\left(\frac{\xi}{v}\right)$
$\frac{(\partial\xi)(\partial\xi)}{M^2 v^2}$
After integrating out
physics at scale M.
d_n = # derivatives
N_n = # fields
$(\xi/v)^{N_n}$
$+ \sin^2\theta\, \partial\phi\, \partial\phi$
$X^a = \begin{pmatrix}\theta \\ \phi\end{pmatrix}$
$\delta X^a = \xi^a(x)$

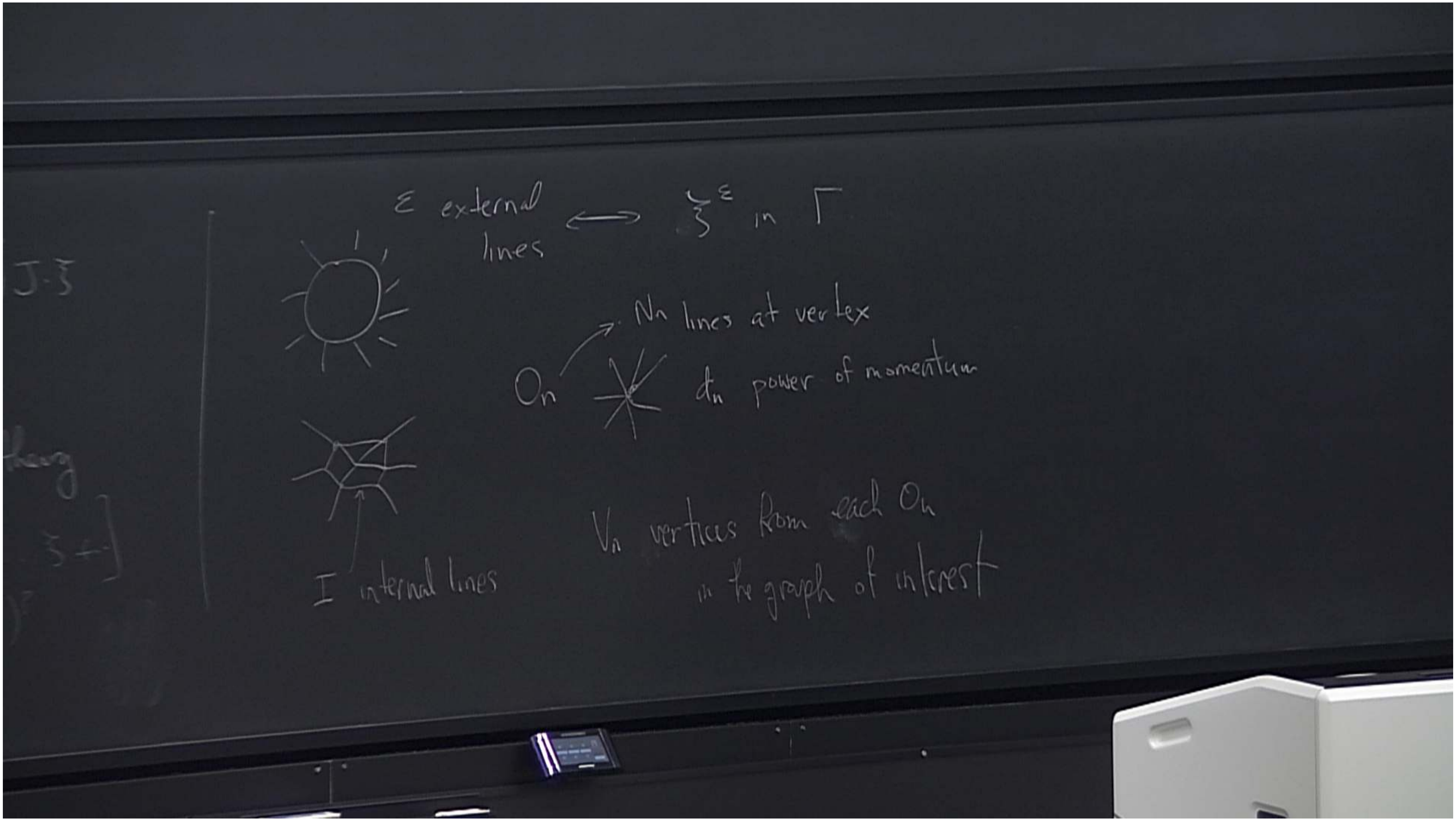

g_ab ∂_μ θ^a ∂^μ θ^b
∂θ∂θ + sin²θ ∂φ∂φ
X^a = (θ, φ)
δX^a = ξ^a(x)
L_W = L_cl + v²M² Σ_n (c_n / M^{d_n}) O_n(ξ/v)
After integrating out
physics at scale M.
d_n = # derivatives
N_n = # fields
(ξ/v)^{N_n}

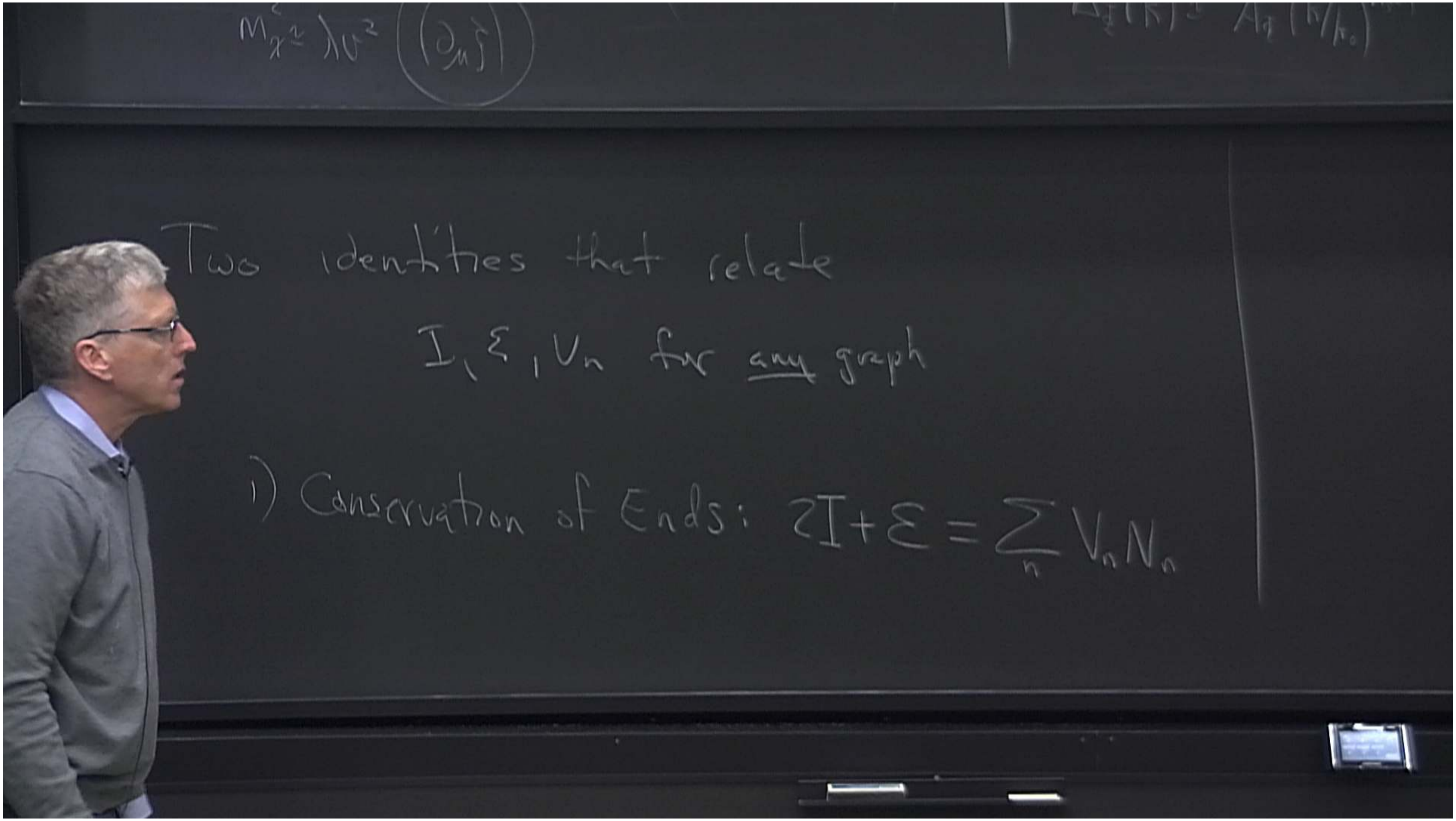

$e^{i\Gamma} = \int \mathcal{D}\xi \, e^{iS(\bar{\xi}+\xi) + i\int J\cdot\xi}$
evaluate in perturbation theory

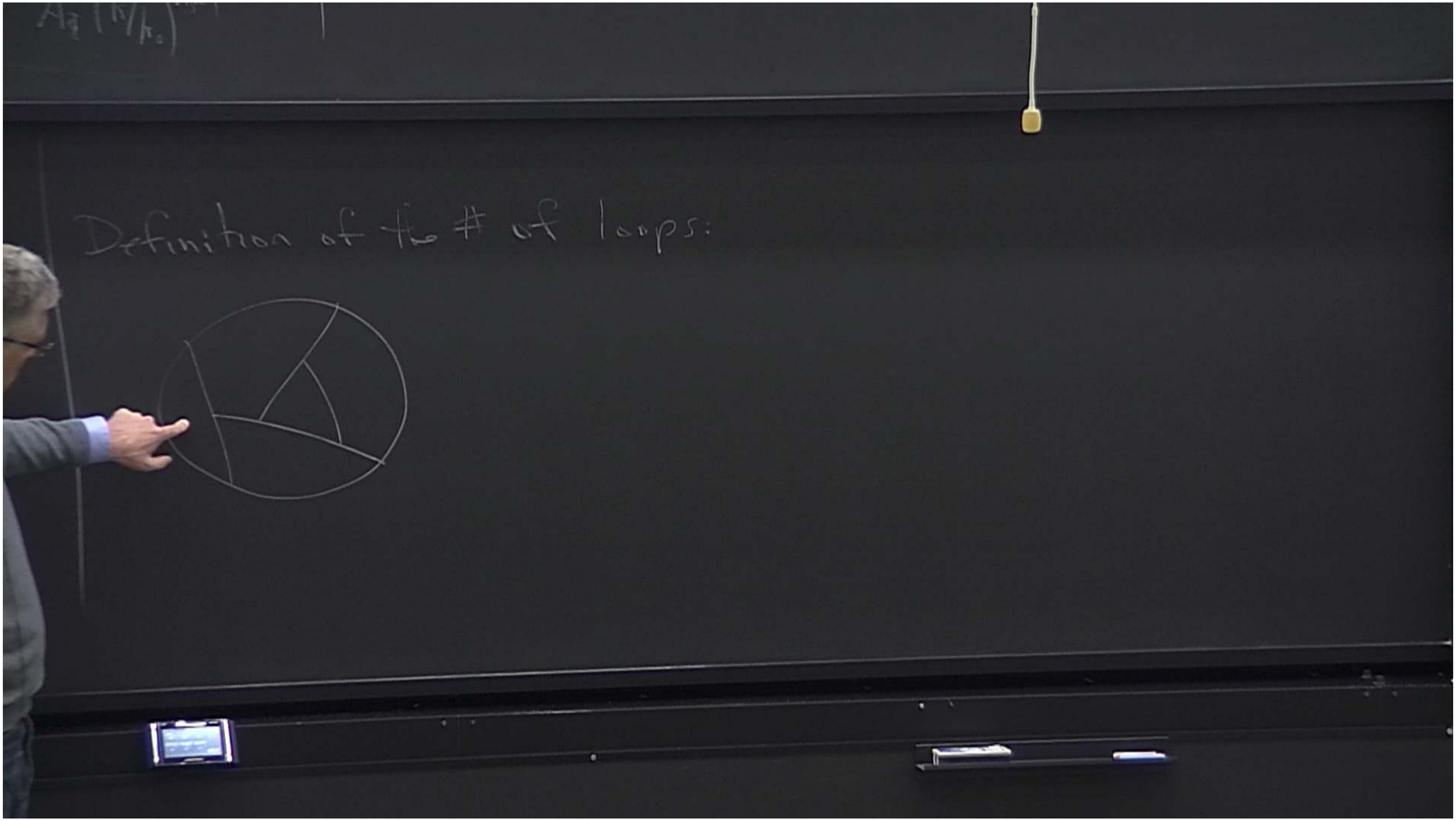

$e^{i\Gamma} = \int \mathcal{D}\xi \; e^{iS(\bar{\xi}+\xi) + i\int J\cdot\xi}$
evaluate in perturbation theory
$e^{iS_{cl}} \int \mathcal{D}\xi \; e^{i\xi\Delta\xi}\left[1 + \xi \dots \xi + \dots\right]$


$e^{i\Gamma} = \int \mathcal{D}\xi \, e^{iS(\bar{\xi}+\xi) + i\int J\cdot\xi}$
evaluate in perturbation theory
$e^{iS_{cl}} \int \mathcal{D}\xi \, e^{i\xi\Delta\xi} \left[1 + \xi \ldots \xi + \ldots \right]$
$(\Delta^{-1})^{p}$


E external lines ⟷ ξ^E in Γ
+ i∫J·ξ
ation theory
I internal lines

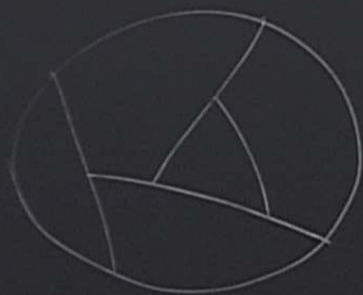

E external lines ⟷ ξ^E in Γ
J·ξ
N_n lines at vertex
O_n
d_n power of momentum
I internal lines
V_n vertices from each O_n
in the graph of interest


Two identities that relate
$I, \mathcal{E}, V_n$ for any graph
1) Conservation of Ends: $2I + \mathcal{E} = \sum_n V_n N_n$


Definition of the # of loops:

$A_q\,(k/k_0)$

Definition of the # of loops:



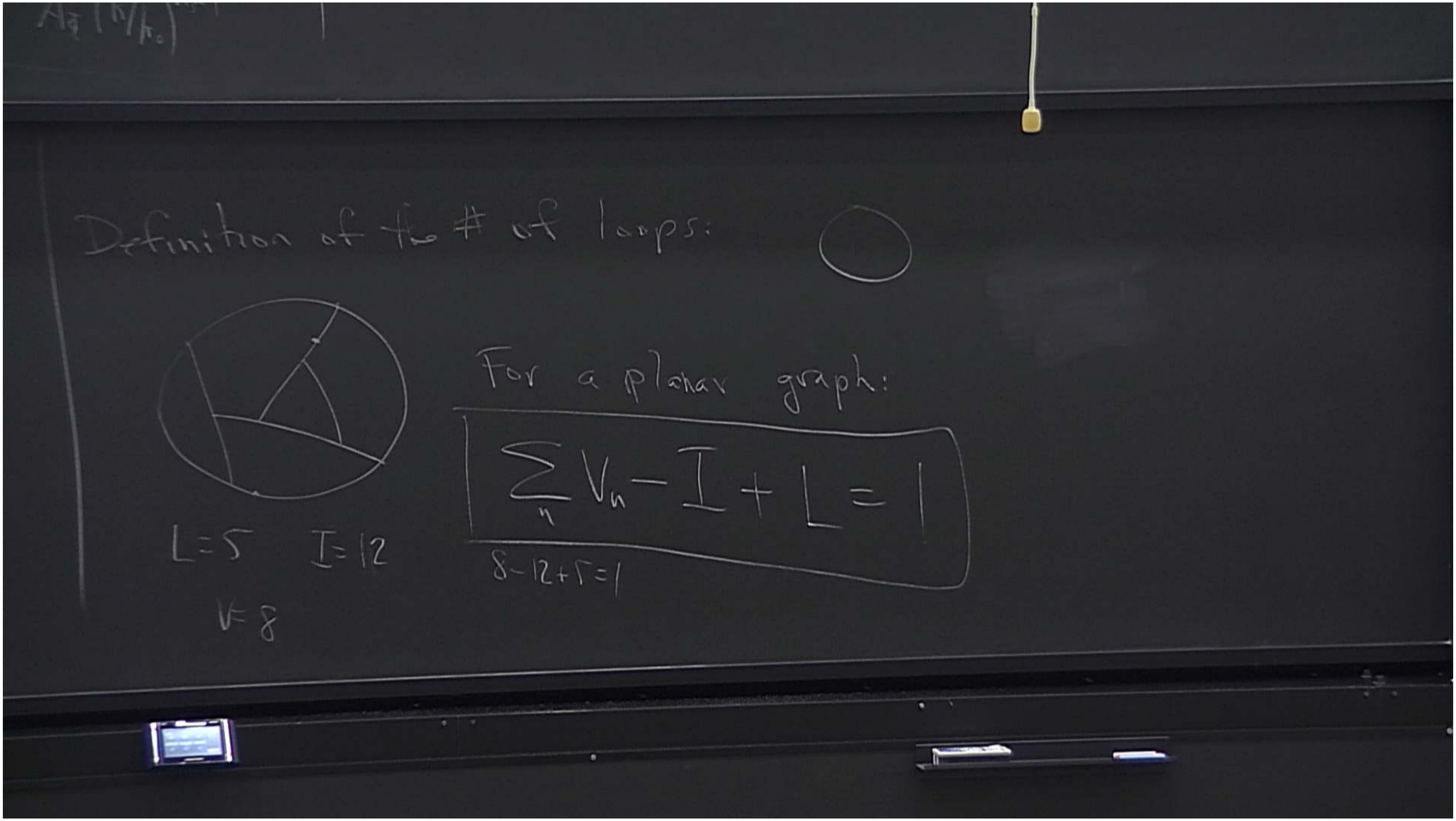
For a planar graph:

$$\sum_n V_n - I + L = 1$$

$L = 5 \quad I = 12$

$V = 8$

$8 - 12 + 5 = 1$


Definition of the # of loops:
For a planar graph:
$\sum_n V_n - I + L = 1$
8-12+5=1
L=5
I=12
V=8

each vertex in the graph
contributes:

$$c_n \left(\frac{1}{M} \right)^{d_n}$$

each vertex in the graph

contributes:

$$c_n \left(\frac{1}{M}\right)^{d_n-2} \left(\frac{1}{\nu}\right)^{N_n-2}$$

each vertex in the graph

contributes:

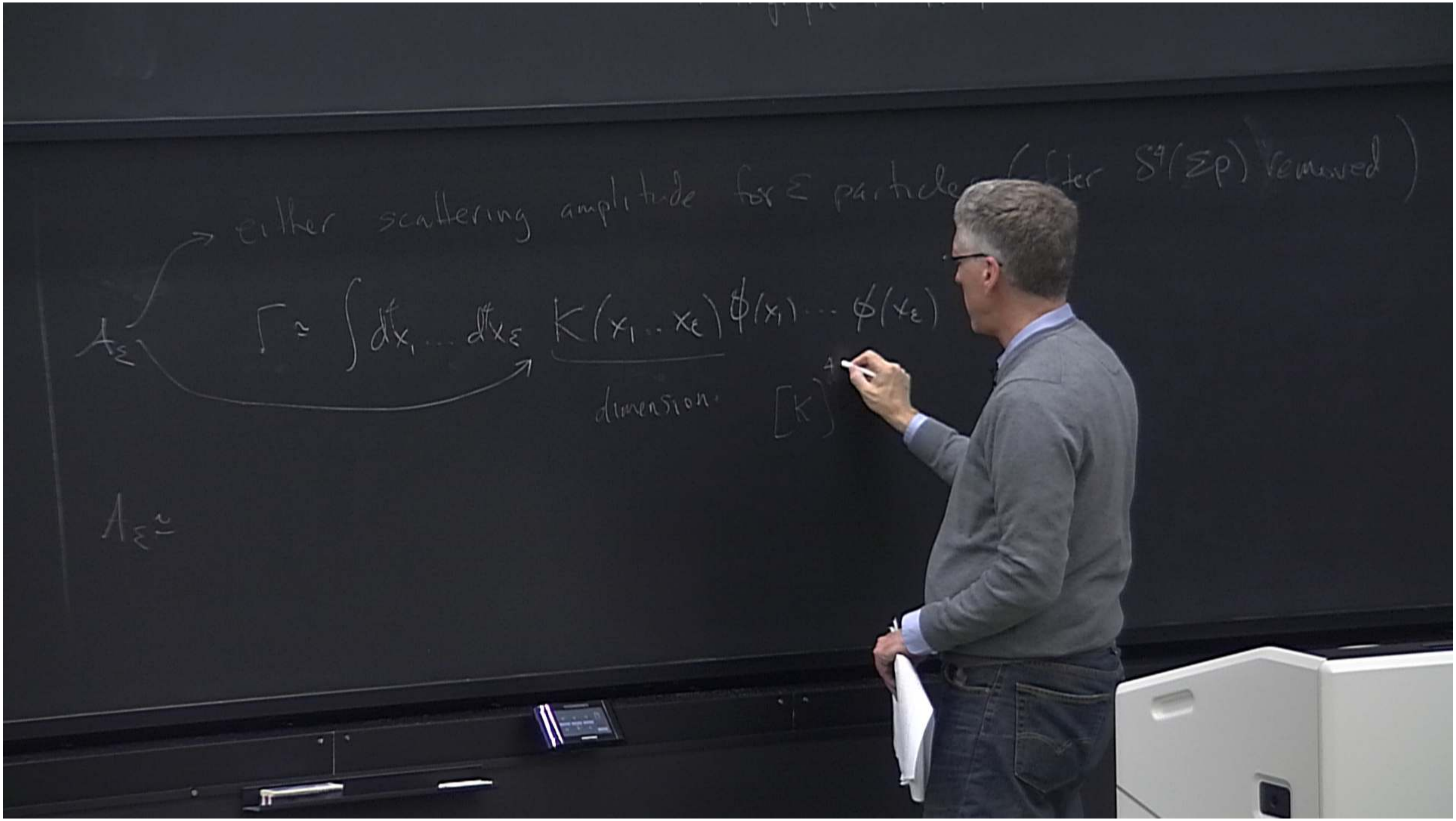
$$\prod_n \left[c_n \left(\frac{1}{M}\right)^{d_n-2} \left(\frac{1}{v}\right)^{N_n-2} \right]^{V_n}$$

each vertex in the graph contributes:

vertex: $$\prod_n \left[c_n \left(\frac{1}{M}\right)^{d_n-2} \left(\frac{1}{v}\right)^{N_n-2} \right]^{V_n}$$

A_E either scattering amplitude for E particles (after $\delta^4(\Sigma p)$ removed)

$$\Gamma = \int dx_1 \ldots dx_E \, \underbrace{K(x_1 \ldots x_E)}_{\text{dimension.}} \, \phi(x_1) \cdots \phi(x_E)$$


either scattering amplitude for E particles (after δ^d(Σp) removed)
A_E
Γ ≃ ∫ d^dx_1 ... d^dx_E K(x_1 .. x_E) φ(x_1) ... φ(x_E)
dimension.
[K]
A_E ≃

either scattering amplitude for E particles (after $\delta^4(\Sigma p)$ removed)

A_E

$$\Gamma \simeq \int d^4x_1 \dots d^4x_E \; \underbrace{K(x_1 .. x_E)}_{\text{dimension.}} \phi(x_1) \cdots \phi(x_E)$$

$[K]^{3E}$

$A_E \simeq$

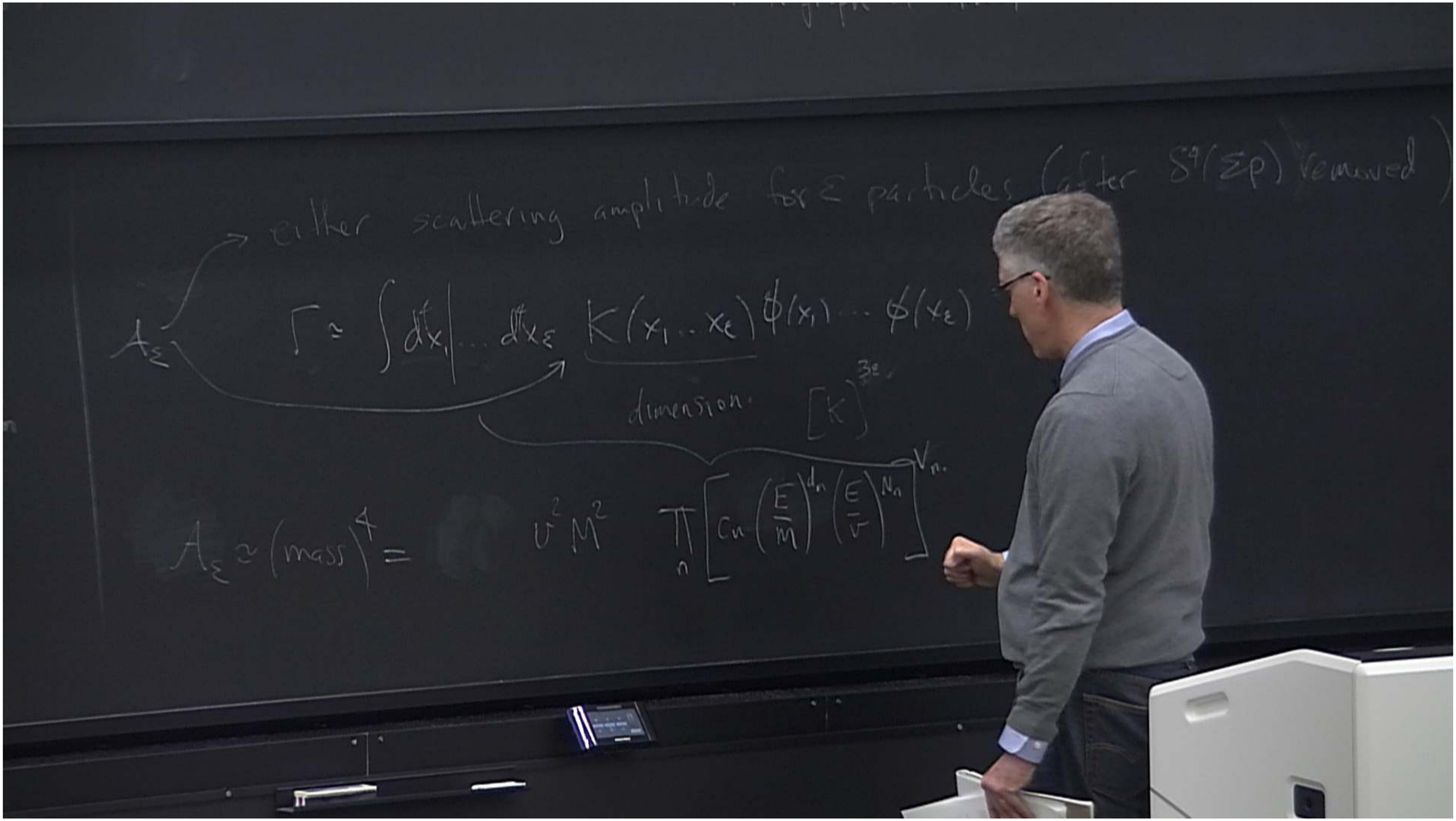
either scattering amplitude for E particles (after $\delta^4(\Sigma p)$ removed)

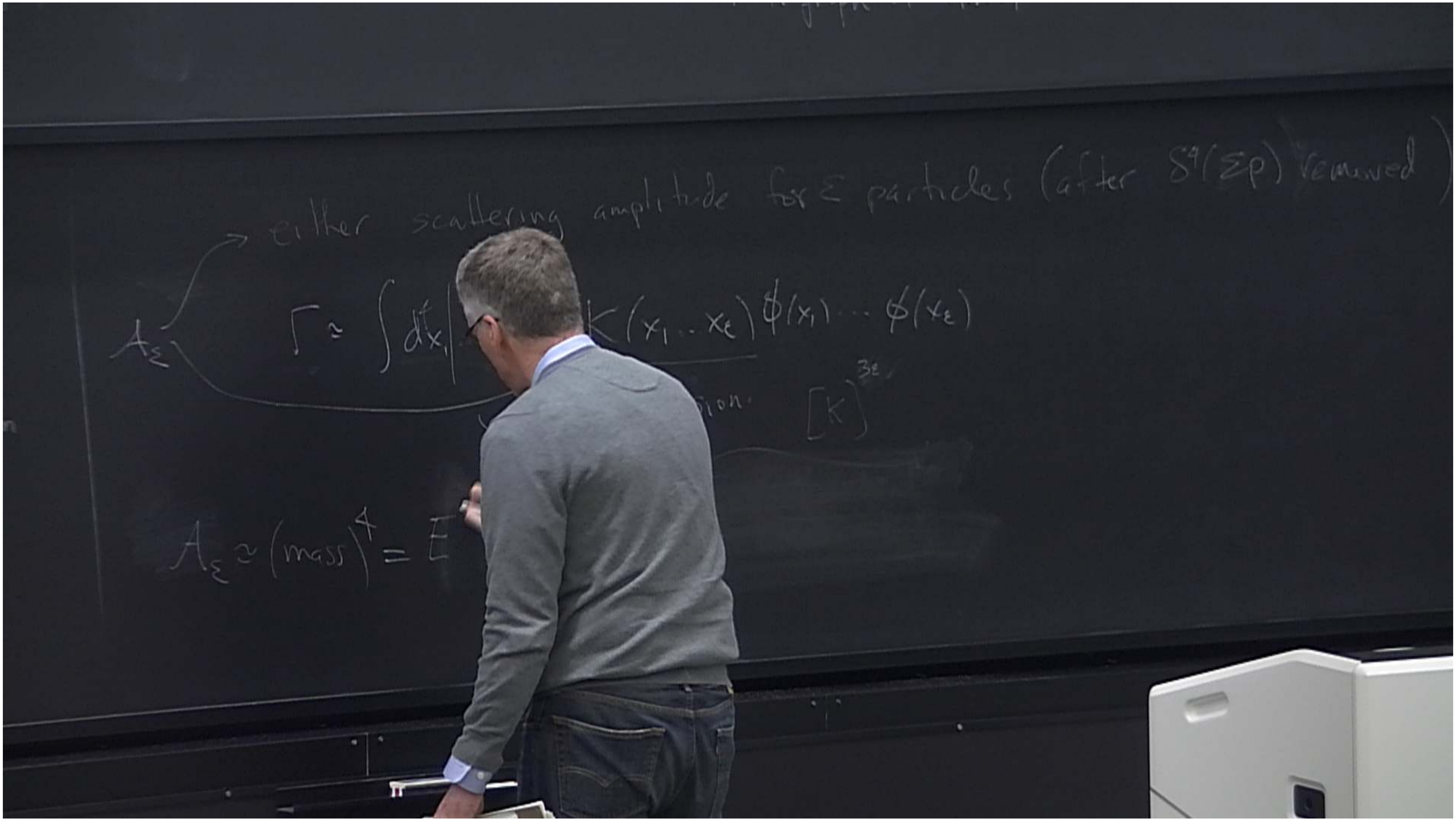
A_E

$$\Gamma \simeq \int d^4x_1 \ldots d^4x_E \, \underbrace{K(x_1 \ldots x_E)}_{\text{dimension: } [K]^{3E}} \phi(x_1) \cdots \phi(x_E)$$

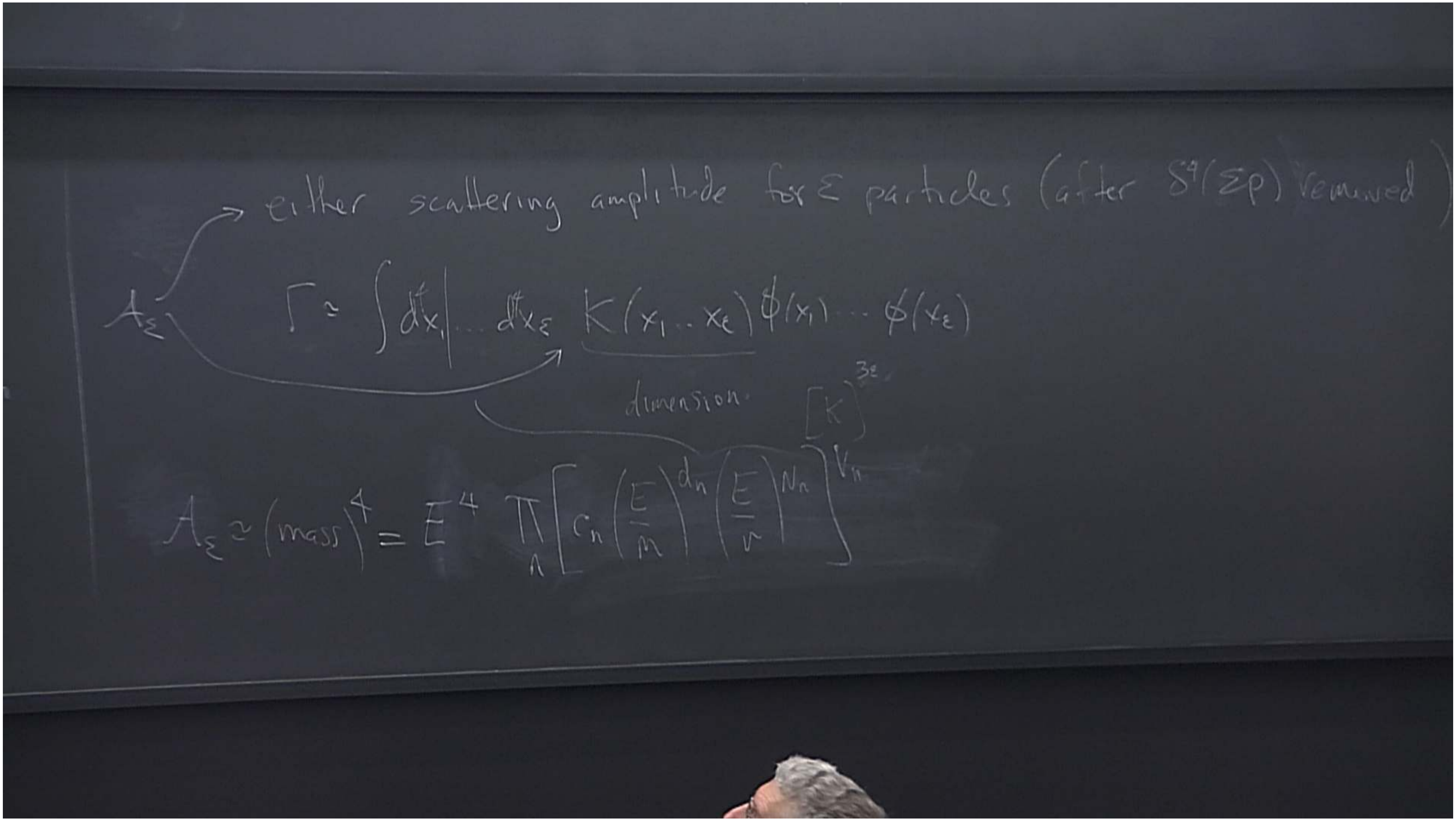
A_E

$A_E \simeq (\text{mass})^4$


either scattering amplitude for E particles (after $\delta^4(\Sigma p)$ removed)
A_E
$\Gamma \simeq \int d^4x_1 \ldots d^4x_E \, K(x_1 \ldots x_E)\, \phi(x_1) \ldots \phi(x_E)$
dimension. $[K]^{3E}$
$A_E \simeq (\text{mass})^4 = v^2 M^2 \prod_n \left[c_n \left(\frac{E}{M}\right)^{d_n} \left(\frac{E}{v}\right)^{N_n} \right]^{V_n}$



either scattering amplitude for E particles (after δ⁴(Σp) removed)
Γ ≃ ∫ d⁴x₁ ... K(x₁ ... x_E) φ(x₁) ... φ(x_E)
A_E
[K]
A_E ≃ (mass)⁴ = E


either scattering amplitude for E particles (after δ⁴(Σp) removed)
A_E
Γ = ∫d⁴x₁ ... d⁴x_E K(x₁ .. x_E) φ(x₁) ... φ(x_E)
dimension
[K]^3E
A_E ≈ (mass)⁴ = E⁴ Π_n [c_n (E/m)^d_n (E/v)^N_n]^V_n

Claim: $$\prod_n \left[c_n \mu^{2-N_n} M^{2-d_n}\right]^{V_n} = \mu^{2-2L-\varepsilon} \prod_n \left[c_n M^{2-d_n}\right]^{V_n}$$

$$2\sum_n V_n - \sum_n N_n V_n$$

$$\prod_n \left[c_n v^{2-N_n} M^{2-d_n} \right]^{V_n} = v^{2-2L-\varepsilon} \prod_n \left[c_n M^{2-d_n} \right]^{V_n}$$

$$2\sum_n V_n - \sum_n N_n V_n$$

$$A_\varepsilon = v^2 E^2 \left(\frac{E}{v}\right)^{\varepsilon} \left(\frac{E}{4\pi v}\right)^{2L} \prod_n \left[c_n \left(\frac{E}{M}\right)^{d_n-2} \right]^{V_n}$$

$$\prod_n \left[c_n v^{2-N_n} M^{2-d_n}\right]^{V_n} = v^{2-2L-\varepsilon} \prod_n \left[c_n M^{2-d_n}\right]^{V_n}$$

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$$A_\varepsilon = v^2 E^2 \left(\frac{E}{v}\right)^{\varepsilon} \left(\frac{E}{4\pi v}\right)^{2L} \prod_n \left[c_n \left(\frac{E}{M}\right)^{d_n-2}\right]^{V_n}$$

$$\prod_n \left[c_n M^{2-d_n} \right]^{V_n}$$

$$\left(\frac{E}{v}\right)^{\varepsilon} \left(\frac{E}{4\pi v}\right)^{2L} \prod_n \left[c_n \left(\frac{E}{M}\right)^{d_n-2} \right]^{V_n}$$

eg 1: Goldstone bosons

$d_n \geqslant 2$ (V is indep of ξ)

Always getting non-neg powers of (E/v) or (E/M).

$$\prod_n \left[c_n v^{2-N_n} M^{2-d_n}\right]^{V_n} = v^{2-2L-\varepsilon} \prod_n \left[c_n M^{2-d_n}\right]^{V_n}$$

$$2\sum_n V_n - \sum_n N_n V_n$$

$$A_\varepsilon = v^2 E^2 \left(\frac{E}{v}\right)^{\varepsilon} \left(\frac{E}{4\pi v}\right)^{2L} \prod_n \left[c_n \left(\frac{E}{M}\right)^{d_n-2}\right]^{V_n}$$

$$\prod_n \left[c_n M^{2-d_n} \right]^{V_n}$$

$$\left(\frac{E}{v}\right)^{\varepsilon} \left(\frac{E}{4\pi v}\right)^{2L} \prod_n \left[c_n \left(\frac{E}{M}\right)^{d_n-2} \right]^{V_n}$$

eg 1: Goldstone bosons

$d_n \geqslant 2$ (V is indep of ξ)

Always getting non-neg powers of (E/v) or (E/M).

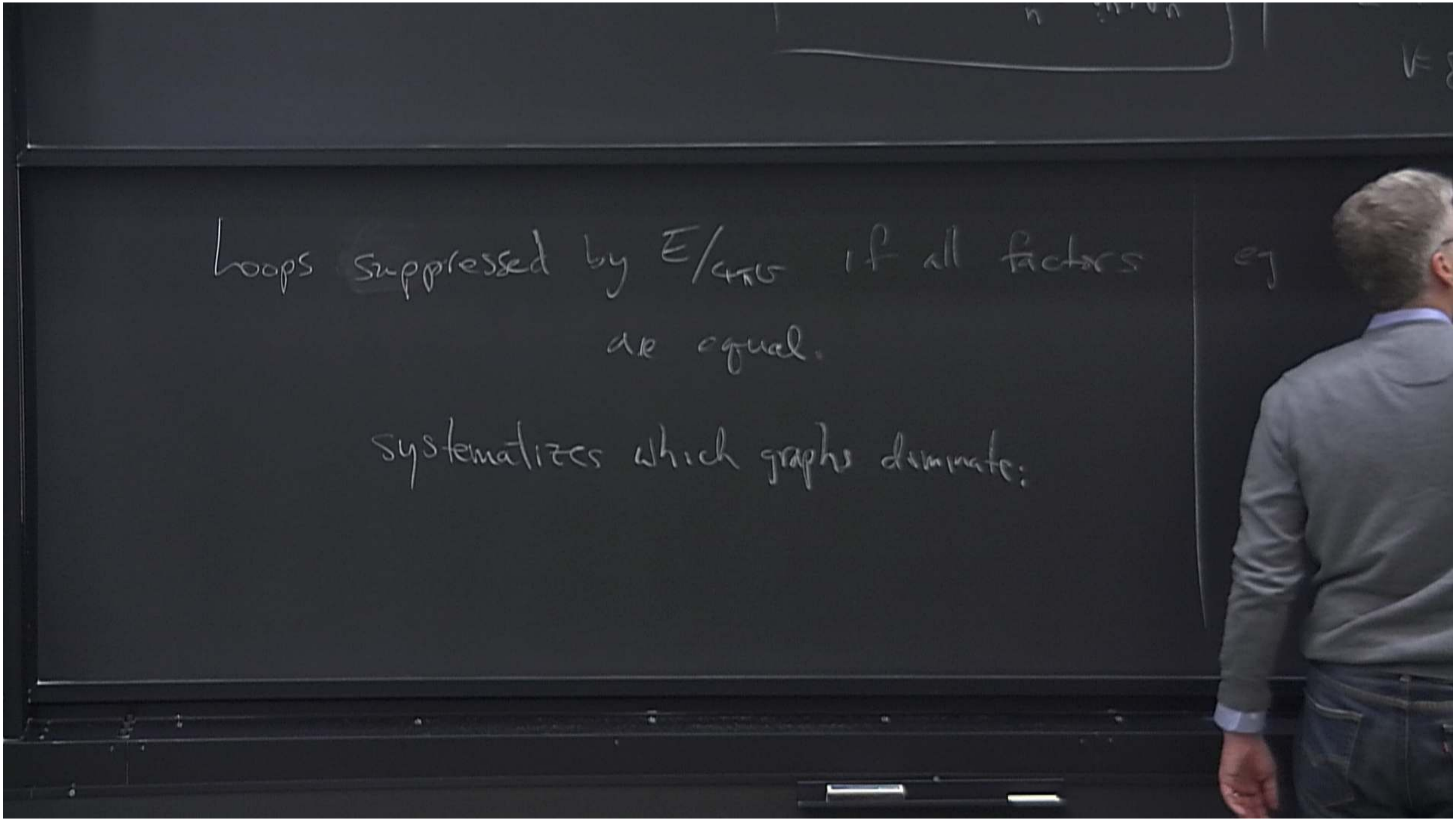
Loops suppressed by

Loops suppressed by $E/4\pi v$ if all factors are equal.

$$\prod_n \left[c_n v^{2-N_n} M^{2-d_n} \right]^{V_n} = v^{2-2L-\varepsilon} \prod_n \left[c_n M^{2-d_n} \right]^{V_n}$$

$$2\sum_n V_n - \sum_n N_n V_n$$

$$A_\varepsilon = v^2 E^2 \left(\frac{[illegible]}{v} \right)^{\varepsilon} \left(\frac{E}{4\pi v} \right)^{2L} \prod_n \left[c_n \left(\frac{E}{M} \right)^{d_n-2} \right]^{V_n}$$


Loops suppressed by E/4πF if all factors
are equal.
systematizes which graphs dominate:
eg

$V=8$ $\quad 8-12+F=1$

eg $2\to 2$ scattering $\xi\xi \to \xi\xi$ at energy E

then dominant contributions have minimal power of E/Λ E/m :

$8-12+\Gamma=1$

$V=8$

$2\to 2$ scattering $\xi\xi\to\xi\xi$ at energy E

then dominant contributions have minimal power of E/σ, E/m:

leading: $L=0$, $V_n\neq 0$ only for $d_n=2$.

$V=8$ $\qquad 8-12+\Gamma=1$

$2\to2$ scattering $\xi\xi\to\xi\xi$ at energy E

then dominant contributions have minimal power of E/G, E/m

leading: $L=0$, $V_n\neq0$ only for $d_n=2$.

(tree level: classical)

$V = 8$ $\quad 8 - 12 + r = 1$

$2 \to 2$ scattering $\xi\xi \to \xi\xi$ at energy E

then dominant contributions have minimal power of E/v

E/m:

leading: $L = 0$, $V_n \neq 0$ only for $d_n = 2$.

(tree level: classical)

subleading $L = 1$, $d_n = 2$ only <u>or</u> $L = 0$, $V_n = 1$ for one $d_n = 4$ term.

$8-12+\Gamma=1$

$V=8$

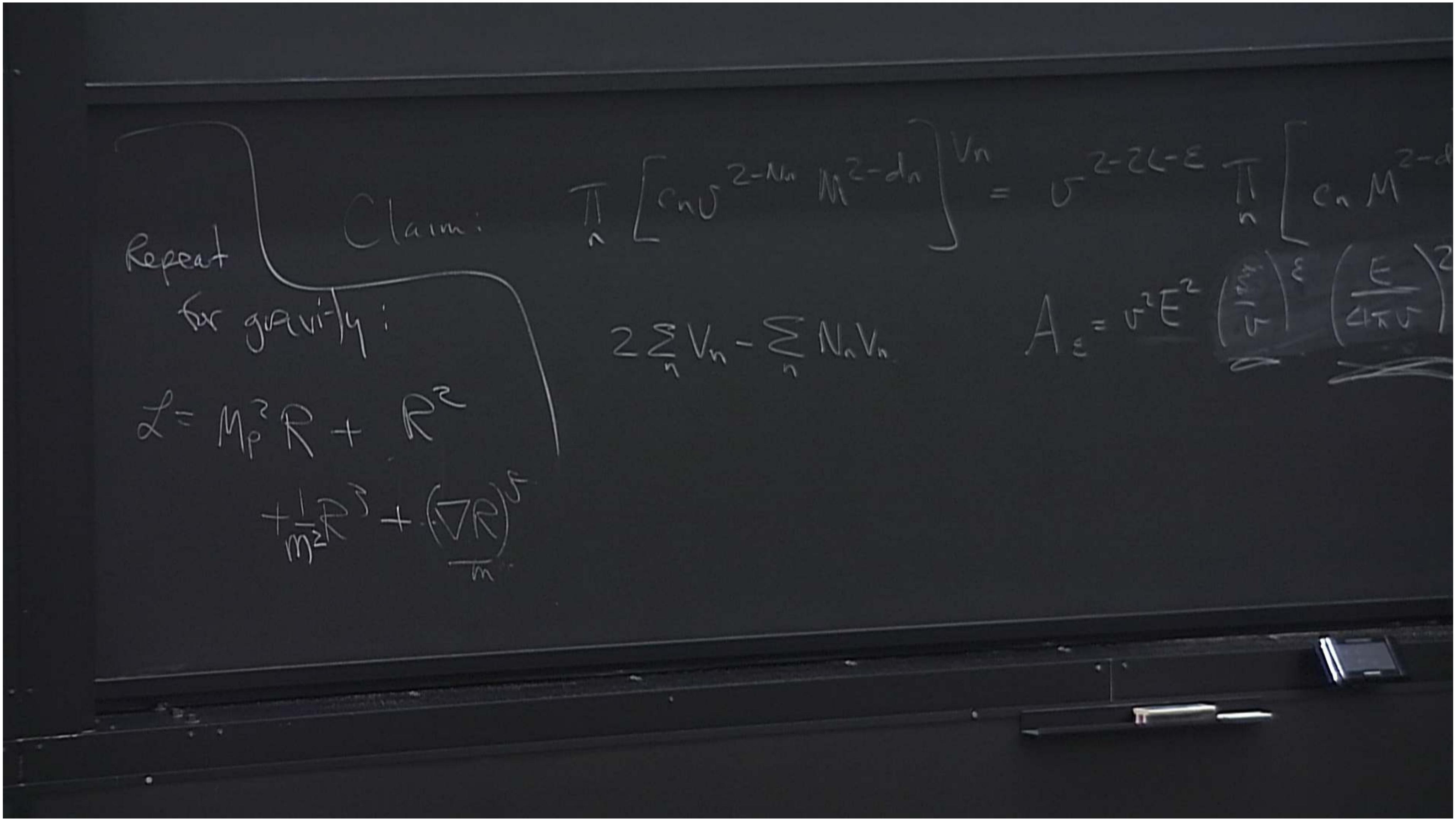
$2\to 2$ scattering $\xi\xi\to\xi\xi$ at energy E

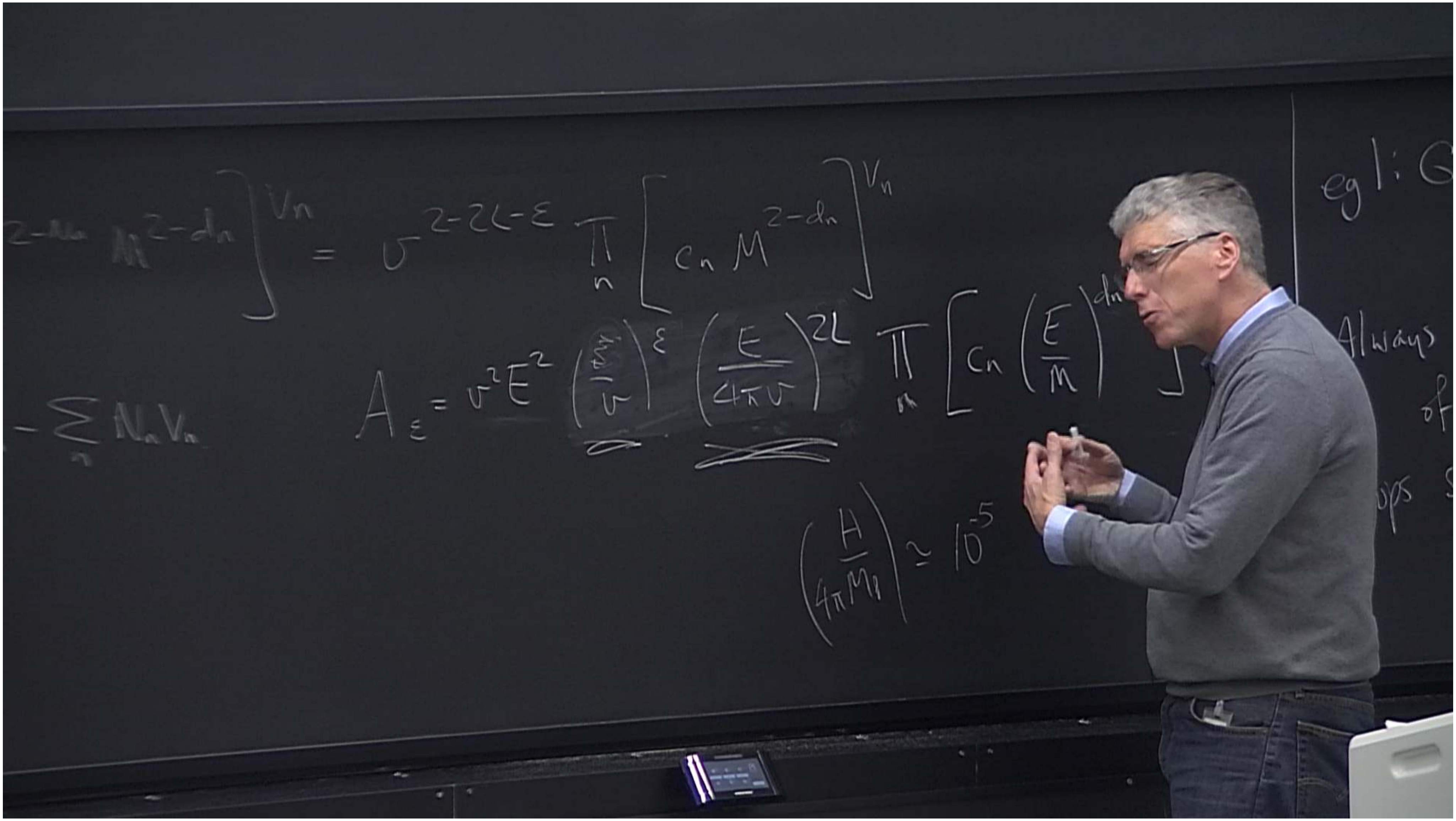
then dominant contributions have minimal power of E/Λ, E/m:

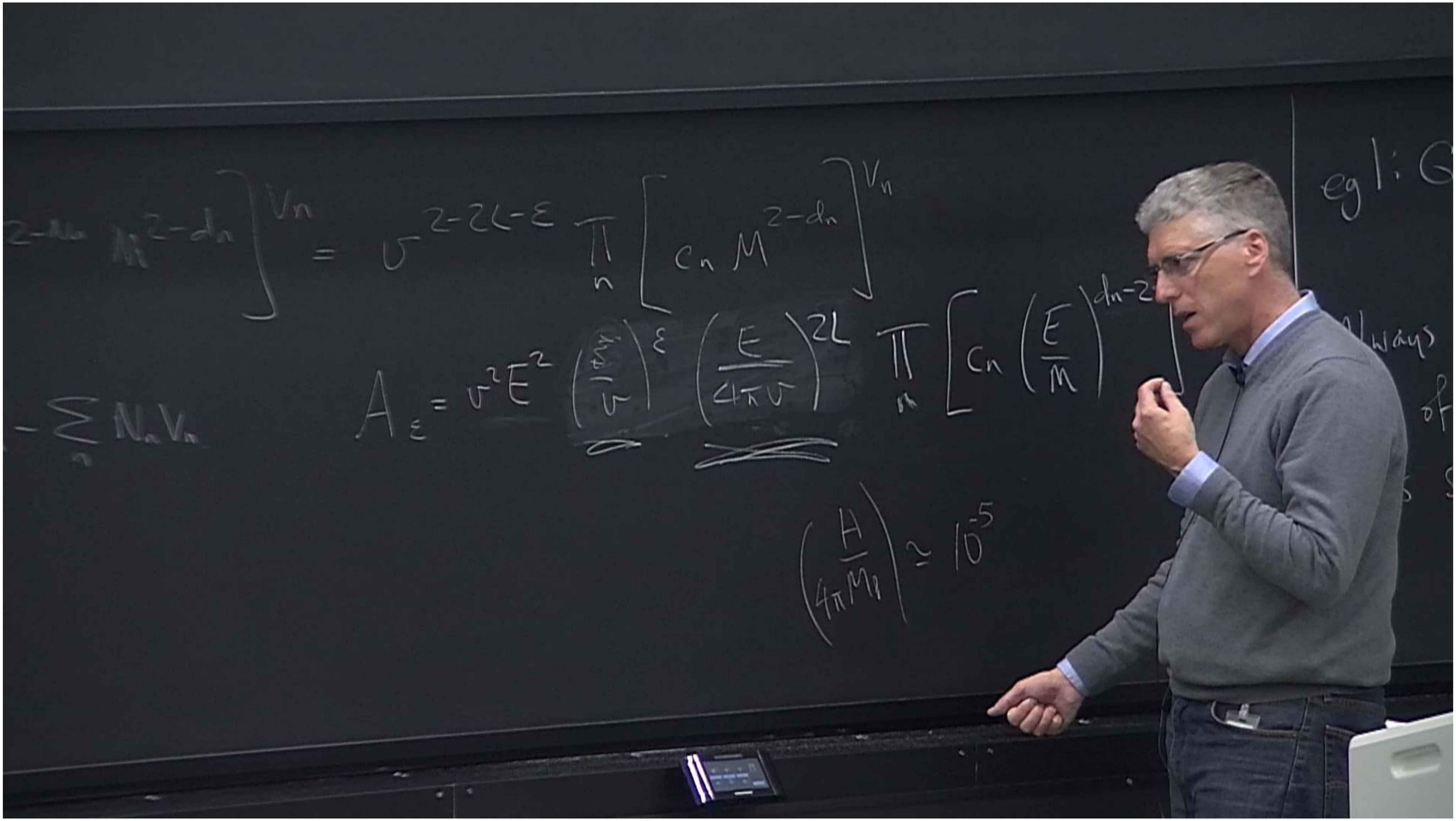
leading: $L=0$, $V_n\neq 0$ only for $d_n=2$

(tree level: classical)

subleading $L=1$, $d_n=2$ only <u>or</u> $L=0$, $V_n=1$ for one $d_n=4$ term.


Repeat for gravity:
$\mathcal{L} = M_p^2 R + R^2$
$+ \frac{1}{m^2} R^3 + \frac{(\nabla R)^2}{m}$
Claim: $\prod_n \left[c_n v^{2-N_n} M^{2-d_n} \right]^{V_n} = v^{2-2L-E} \prod_n \left[c_n M^{2-d} \right.$
$2 \sum_n V_n - \sum_n N_n V_n$
$A_E = v^2 E^2$


$= v^{2-2L-\epsilon} \prod_n \left[c_n M^{2-d_n} \right]^{V_n}$
$A_\epsilon = v^2 E^2 \left(\frac{\epsilon}{v}\right)^\epsilon \left(\frac{E}{4\pi v}\right)^{2L} \prod_n \left[c_n \left(\frac{E}{M}\right)^{d_n} \right]$
$\left(\frac{H}{4\pi M_P}\right) \simeq 10^{-5}$
eg 1:
Always
of
ops


A_ε = v²E² (…/v)^ε (E/4πv)^{2L} Π_n [c_n (E/M)^{d_n−2} …
(H/4πM_p) ≃ 10^{-5}
eg 1: