

Title: PSI 2015/2016 Explorations in Particle Theory - Burgess - 13

Date: Apr 27, 2016 09:00 AM

URL: <http://pirsa.org/16040032>

Abstract:

Effective theory
obtained by integrating out
high-energy sector.

$$e^{iS_{\text{eff}}(L)} = \int \mathcal{D}H e^{iS(L,H)}$$

Claim given $m_h \gg m_\Lambda$

$$S_{\text{eff}} = \int d^4x \mathcal{L}_{\text{eff}} \quad \uparrow \text{local}$$

Locality comes about
because

Effective theory
obtained by integrating out
high-energy sector.

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Claim given $m_h \gg m_\chi$

$$S_h = \int d^4x \mathcal{L}_h \quad \uparrow \text{local}$$

Locality comes about

because $\mathcal{L}_h(x,y) \xrightarrow{m_h \rightarrow \infty} \mathcal{L}_h(x,y)$

Claim

$$m_h \gg m_e$$

$S_h =$

S_h

↑ local

$$S_h(x, y) \xrightarrow{m_h \rightarrow \infty} \delta(x-y)$$

$$G(x, y) = \int d^4 p \frac{e^{ip(x-y)}}{p^2 + m^2} \approx \frac{1}{m^2} \int d^4 p \left(1 + \frac{p^2}{m^2} + \dots\right) e^{ip(x-y)}$$

$$\text{if } |x-y| \gg \frac{1}{m}$$

$$\approx \frac{1}{m^2} \delta(x-y) - \frac{\square}{m^4} \delta(x-y) + \dots$$

Integral dominated by $|p| \ll m$

Locality comes only after expanding in powers of $1/m$

Vacuum energy + the ^{"technical"} naturalness criterion:

$$e^{i\Gamma(g)} = \int \mathcal{D}\phi e^{iS(g, \phi)} \quad \text{for } S = -\int [(\partial\phi)^2 + m^2\phi^2] \sqrt{g}$$
$$= \det^{-1/2}(-\square + m^2)$$

$$\langle T_{\mu\nu} \rangle$$

Vacuum energy + the "technical" naturalness criterion.

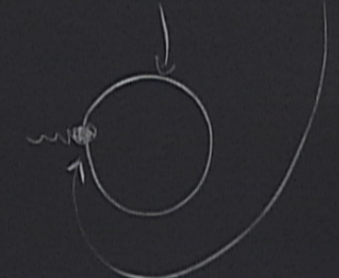
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$$\langle T_{\mu\nu} \rangle = \frac{\delta\Gamma}{\delta g_{\mu\nu}}$$

erion:
 $(\partial\phi)^2 + m^2\phi^2$
 m^2

$$\Gamma = \frac{i}{2} \ln \det (-\mathcal{D} + m^2) = \frac{i}{2} \text{Tr} \ln (-\mathcal{D} + m^2)$$

$$\delta\Gamma = \frac{i}{2} \text{Tr} \left[(-\mathcal{D} + m^2)^{-1} \delta(-\mathcal{D} + m^2) \right]$$

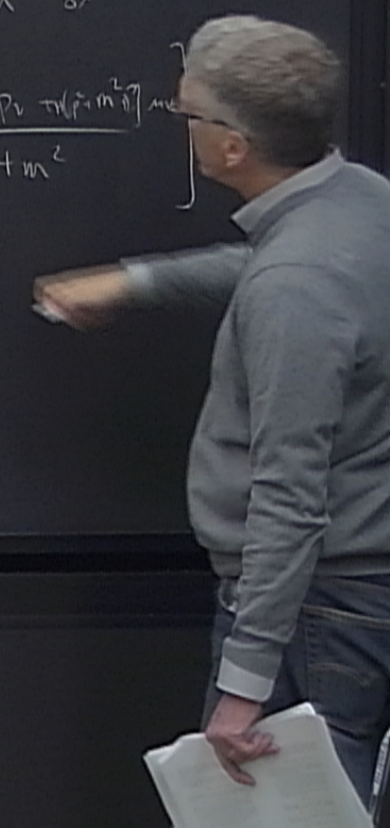


for flat space:

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}((\partial\phi)^2 + m^2\phi^2)$$

$$\delta \ln x = x^{-1} \delta x$$

$$\int \frac{d^4 p}{(2\pi)^4} \frac{p_\mu p_\nu + \eta_{\mu\nu} p^2}{p^2 + m^2}$$



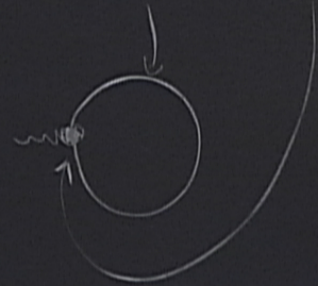
$$\Gamma = \frac{i}{2} \ln \det (-\mathcal{D} + m^2) = \frac{i}{2} \text{Tr} \ln (-\mathcal{D} + m^2)$$

$$\delta \ln X = X^{-1} \delta X$$

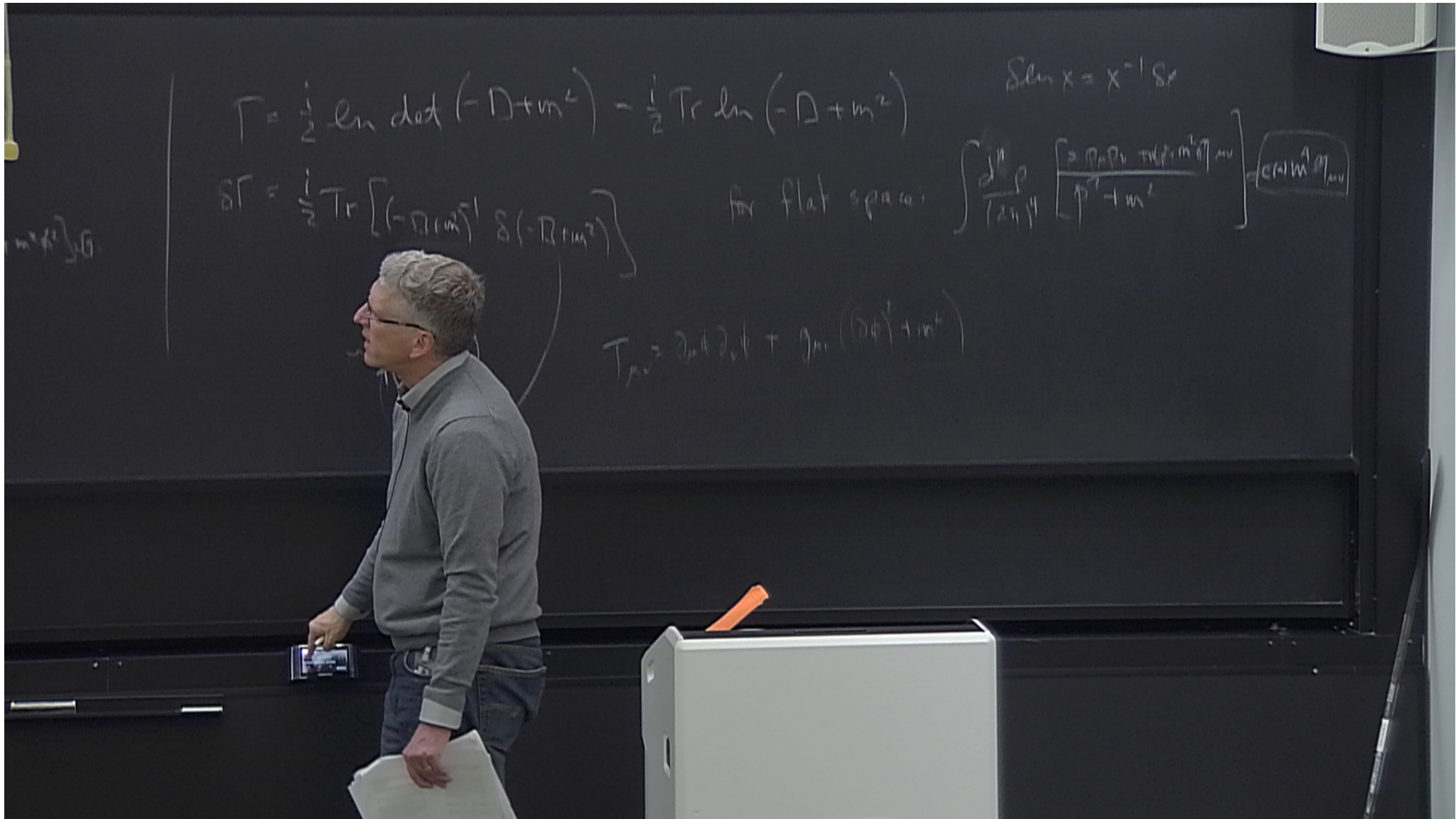
$$\delta \Gamma = \frac{i}{2} \text{Tr} \left[(-\mathcal{D} + m^2)^{-1} \delta (-\mathcal{D} + m^2) \right]$$

for flat space.

$$\int \frac{d^n p}{(2\pi)^4} \left[\frac{p_\mu p_\nu + (p^2 + m^2) \eta_{\mu\nu}}{p^2 + m^2} \right] = c \eta_{\mu\nu}$$



$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} (\partial\phi)^2 + m^2$$



$$\Gamma = \frac{i}{2} \ln \det(-D + m^2) - \frac{i}{2} \text{Tr} \ln(-D + m^2)$$

$$\delta\Gamma = \frac{i}{2} \text{Tr} [(-D + m^2)^{-1} \delta(-D + m^2)]$$

for flat space:

$$\int \frac{d^4 p}{(2\pi)^4} \frac{[\gamma_\mu \not{p} + m]}{p^2 + m^2} \quad \left(\frac{1}{(2\pi)^4} \int \frac{d^4 p}{p^2 + m^2} \right)$$

$$T_{\mu\nu} = \partial_\mu \partial_\nu \phi + g_{\mu\nu} (\partial\phi)^2 + m^2 \phi^2$$

$$e^{-iHt} = \int \mathcal{D}\Phi e^{iS(\Phi, A)}$$

locality comes about
because $\delta_k(x, y) \rightarrow \delta(x, y)$
 $m_k \rightarrow \infty$

Locality comes only after ex

the result that loop corrections to $p \sim m^4$ when integrating out particles with mass m , is the analog of same calculation for scalar masses.

Suppose $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$



because $G(x,y) \xrightarrow{m \rightarrow \infty} \delta(x-y)$

Locality comes only after expanding in powers of $1/m$

Suppose $\mathcal{L} = -\bar{\psi}(\not{\partial} + m)\psi - \frac{1}{2}(\partial\phi)^2 - g\bar{\psi}\psi\phi - V(\phi)$

we calculate loop correction to $V(\phi)$
 due to ψ loops:



$$\approx g^2 \phi^n \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left[\frac{1}{(\not{p} + m)^2} \right]$$

$\int dx \rightarrow \int_{\text{local}}$
 locality comes about because $\chi(x,y) \xrightarrow{m \rightarrow \infty} \delta(x,y)$
 if $|x-y| \gg \frac{1}{m}$
 Integral dominated by $|p| \ll m$
 Locality comes only after expanding in powers of $\frac{1}{m}$
 $\frac{1}{m^2} \delta(x-y) = \frac{1}{m^2} \delta(x-y) + \dots$

$$i\Gamma(g) = \int \mathcal{D}\phi e^{iS(\phi)} = \det^{-1/2}(-\partial^2 + m^2)$$

$$\langle T_{\mu\nu} \rangle = \frac{\delta \Gamma}{\delta g^{\mu\nu}}$$

$$\delta V = \frac{g^2 m^2}{(4\pi)^2} \phi^2 + \frac{g^4}{(4\pi)^2} \phi^4 \ln(m^2) + \frac{g^4}{(4\pi)^2 m^2} \phi^4$$

Suppose $\mathcal{L} = -\bar{\psi}(\not{\partial} + m)\psi - \frac{1}{2}(\partial\phi)^2 - g\bar{\psi}\psi\phi - V(\phi)$

we calculate loop correction to $V(\phi)$ due to ψ loops.



$$\approx g^2 \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left[\frac{1}{\not{p} + m} \right]$$

$$\langle T_{\mu\nu} \rangle = \frac{\delta \Gamma}{\delta g_{\mu\nu}}$$

$$\delta V \approx \frac{y^2 m^2 c_n}{(4\pi)^2} \phi^2 + \frac{y^4}{(4\pi)^2} \phi^4 \ln(m^2) + \frac{y^6}{(4\pi)^2 m^2} \phi^6 + \frac{y^n}{(4\pi)^2 m^{n-4}} \phi^n$$

$$\langle T_{\mu\nu} \rangle = \frac{\delta \Gamma}{\delta g_{\mu\nu}}$$

$$\delta V = \frac{m^4}{(4\pi)^2} \phi^2 + \frac{y^4}{(4\pi)^2} \phi^4 \ln(m^2) + \frac{y^6}{(4\pi)^2 m^2} \phi^6 + \frac{y^n}{(4\pi)^2 m^{n-4}} \phi^n$$

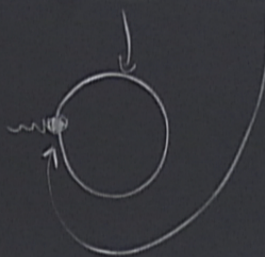
$$\langle T_{\mu\nu} \rangle = \frac{\delta \Gamma}{\delta g_{\mu\nu}}$$

$$\delta V = \frac{m^4 c_n^2}{(4\pi)^2} + y^2 \frac{m^2 c_n}{(4\pi)^2} \phi^2 + \frac{y^4}{(4\pi)^2} \phi^4 \ln(m^2) + \frac{y^6}{(4\pi)^2 m^2} \phi^6 + \frac{y^n}{(4\pi)^2 m^{n-4}} \phi^n$$

↑ ↑
 "relevant"
 interaction

$+ \frac{m^2 \sqrt{g}}{(4\pi)^2} \phi$

$$= \det^{-1/2}(-\square + m^2)$$



$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} (\partial\phi)^2 + m^2$$

$$\phi \ln(m^2) \rightarrow \frac{y^l}{(4\pi)^2 m^2} \phi^l + \frac{y^h}{(4\pi)^2 m^{n-4}} \phi^h$$

What is wrong with absorbing all large m^n contributions into "bare" couplings for $m^2 \gg m_0^2$?

Given the couplings in \mathcal{L}_i
 can compute " " \mathcal{L}_i $n > 1$

ρ_i = vacuum energy param in \mathcal{L}_i
 m_i^c = scalar mass " " "

For each \mathcal{L}_i , choose ρ_i to ensure ρ_{phys} has the right value
 $(10^{-3} \text{eV})^4$

$$\rho_{\text{phys}} = \rho_s + \sum_{p \in \mathcal{L}_i} \frac{m_p^4}{64\pi^2} \ln\left(\frac{m_p^2}{\Lambda^2}\right)$$

learn \uparrow

$m_p \lesssim 10^3 \text{eV}$ for \mathcal{L}_i

Same calculation for L_{w_3} :

$$P_{\text{phys}} = P_3 + \sum_{PE \geq L_{w_2}} \frac{m_p^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right) + \dots$$

$$\underbrace{\sum_{PE \geq L_{w_4}} + \frac{m_e^4}{64\pi^2} \ln\left(\frac{m_e^2}{\mu^2}\right) + \dots}_{P_4}$$

$$10^{12} \text{ eV}^4 \quad 10^{22} \text{ eV}^4$$

$$P_4 = P_3 + \frac{m_e^4}{64\pi^2} \ln\left(\frac{m_e^2}{\mu^2}\right)$$

$$P_3 = P_2 + \sum_{\substack{PE \geq \\ P \geq 3}} \frac{m_p^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right)$$

When integrating out particles with mass m , is the analog of same calculation for scalar masses.

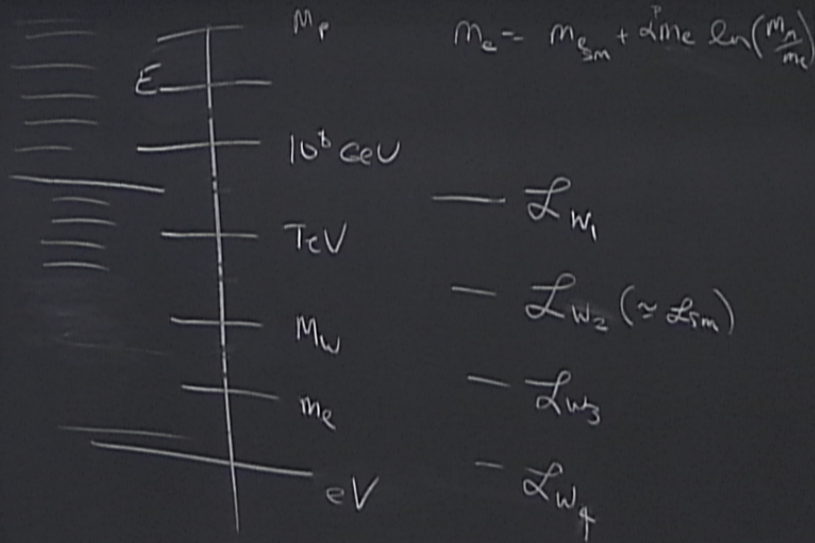
we calculate for due to

$$\frac{5m}{a_0} = \alpha_{SM} M_{Pl} m$$

$$r_n \approx \frac{1}{\Lambda_{GUT}}$$

atoms > nuclei?

$$a_0 \approx \frac{1}{\alpha m_e} \quad r_n \approx \frac{1}{m_p}$$



Given the couplings in L_{W_1} can compute " " " L_{W_n} $n > 1$

ρ_0 = vacuum energy param in L_{W_2}
 M_i = scalar mass " " "

Technically unnatural"

Technically natural:

1) Why does Ed think it is true at Planck scale

2) Why does it stay true as you integrate out physics to scales where it is measured?