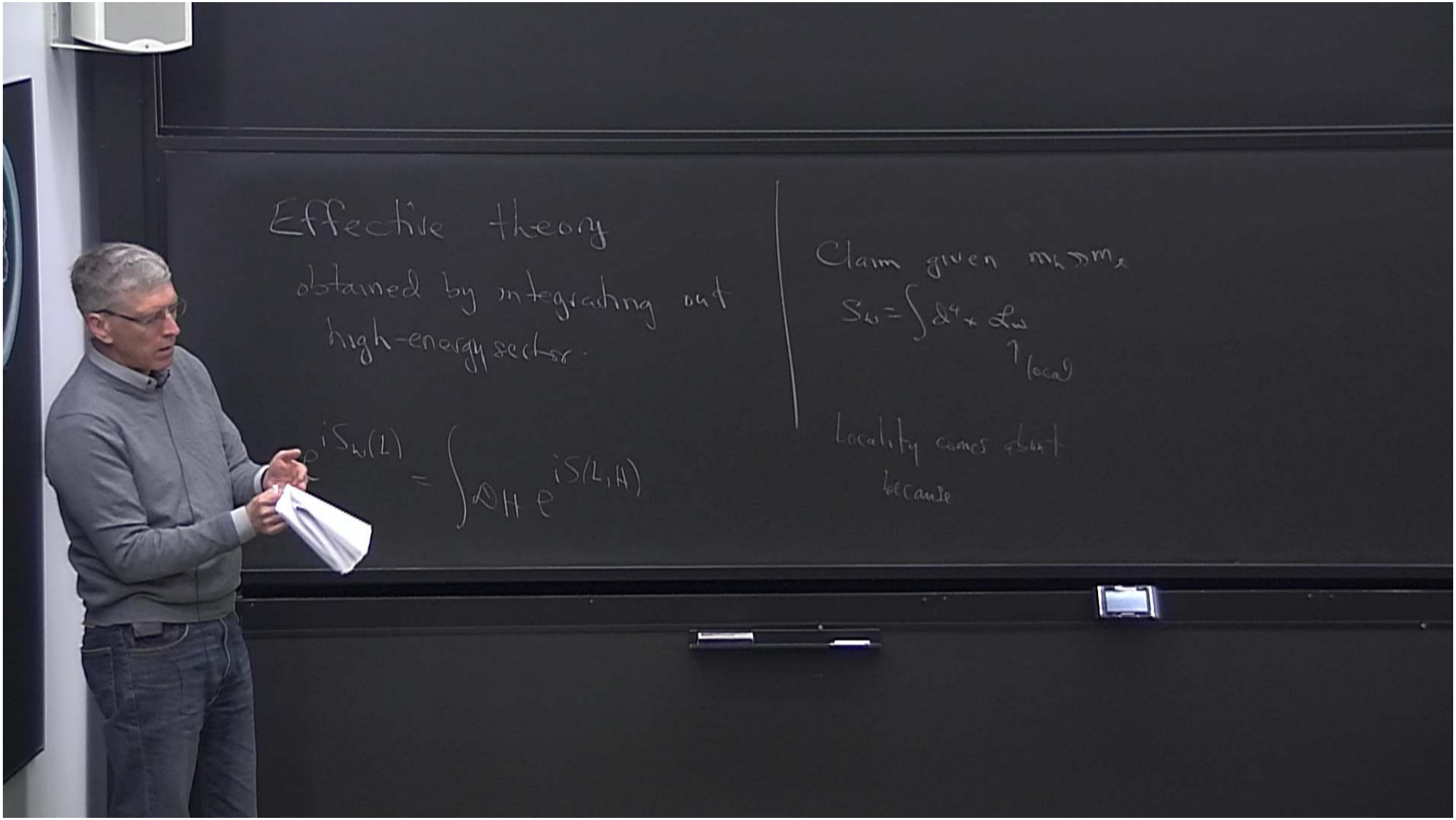
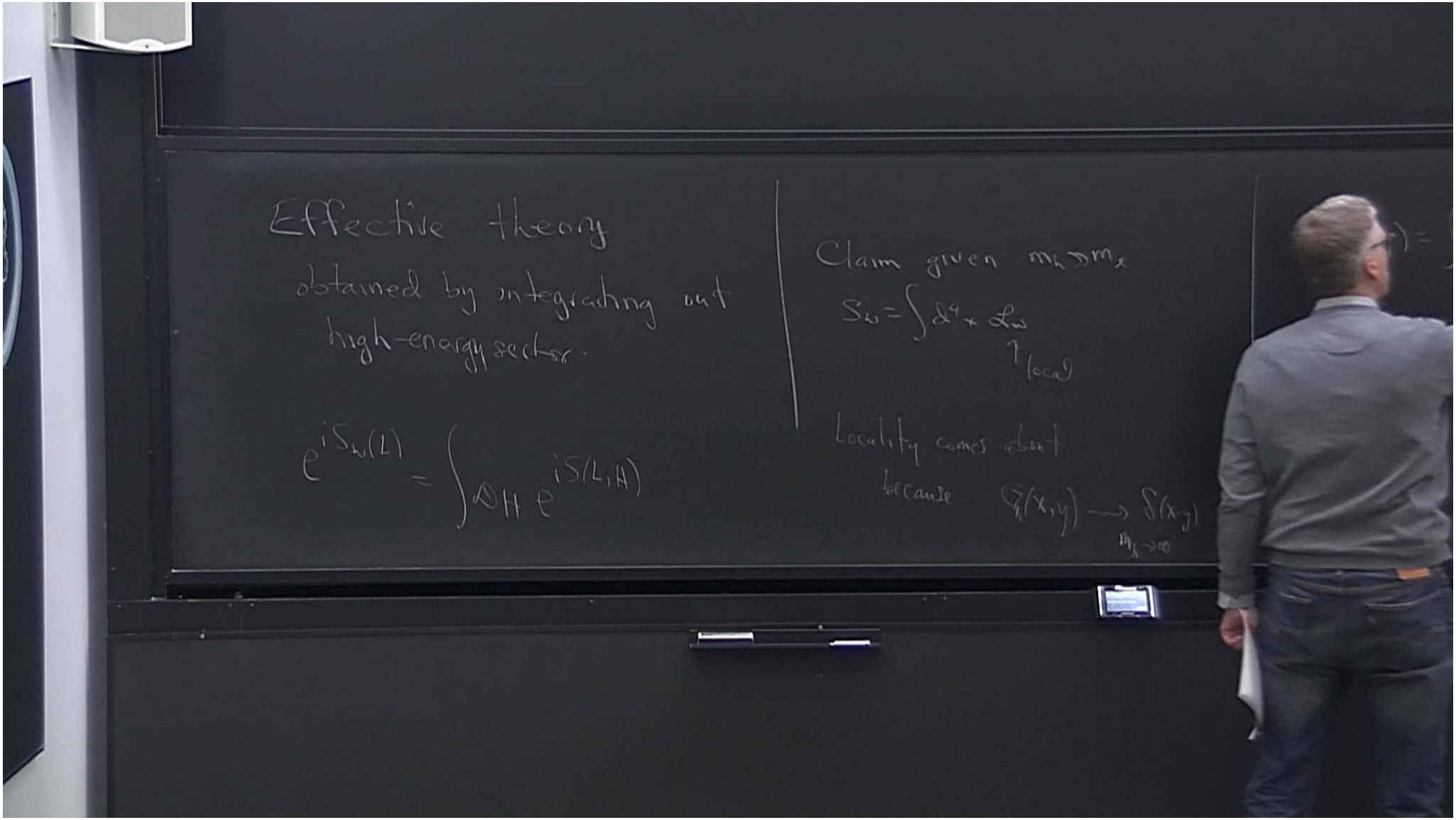
Title: PSI 2015/2016 Explorations in Particle Theory - Burgess - 13

Date: Apr 27, 2016 09:00 AM

URL: http://pirsa.org/16040032

Abstract:


Effective theory
obtained by integrating out
high-energy sector.
$e^{iS_W(L)} = \int \mathcal{D}_H e^{iS(L,H)}$
Claim given $m_H \gg m_L$
$S_W = \int d^4x \mathcal{L}_W$
? local
Locality comes about
because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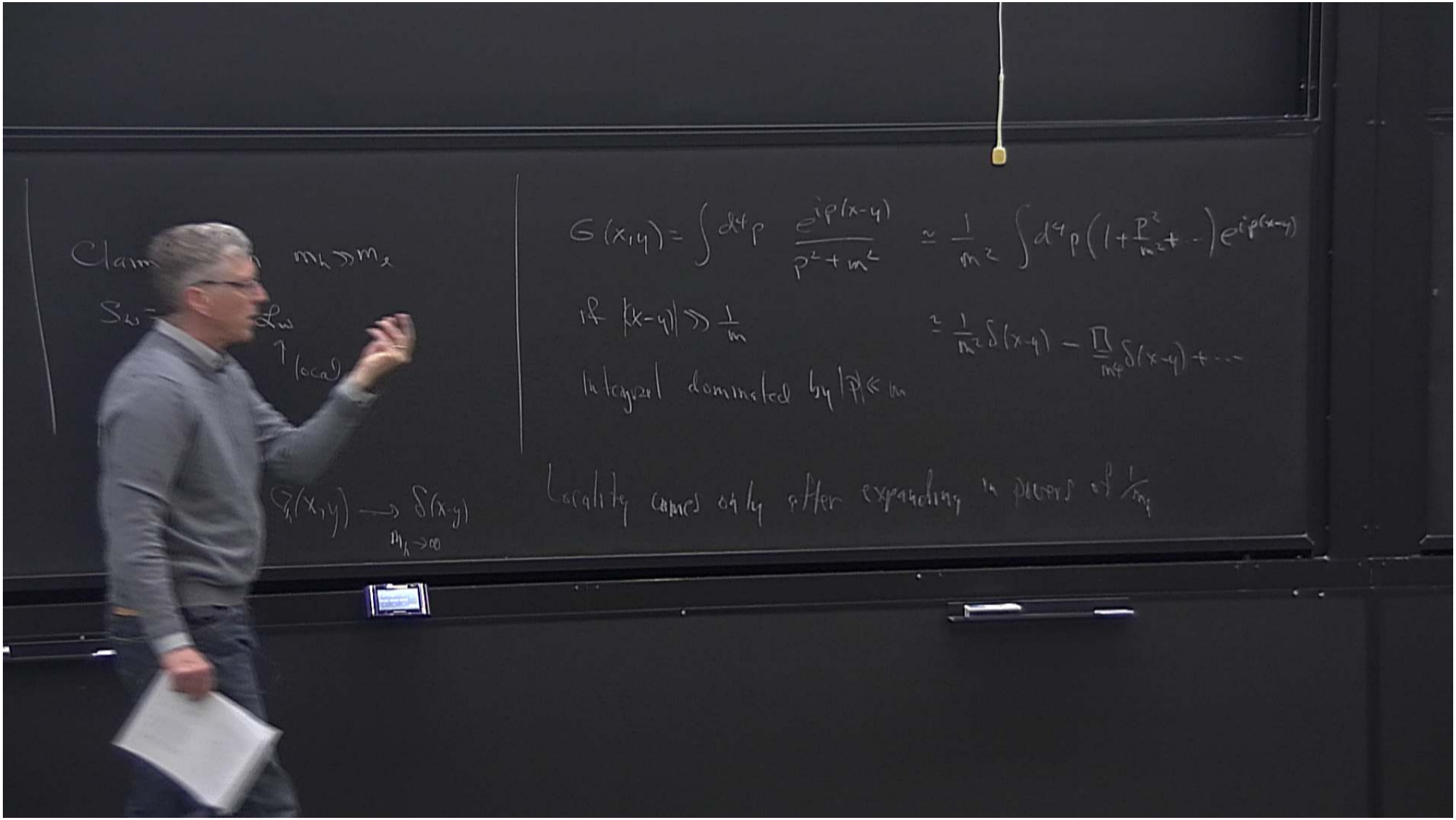
Claim
$m_h \gg m_\ell$
S_w
$\mathcal{L}_w$
local
$G(x,y) = \int d^4p \, \frac{e^{ip(x-y)}}{p^2+m^2} \simeq \frac{1}{m^2} \int d^4p \left(1 + \frac{p^2}{m^2} + \dots\right) e^{ip(x-y)}$
if $|(x-y)| \gg \frac{1}{m}$
$\simeq \frac{1}{m^2}\delta(x-y) - \frac{\Box}{m^4}\delta(x-y) + \dots$
Integral dominated by $|\vec{p}| \ll m$
$G_h(x,y) \longrightarrow \delta(x-y)$
$m_h \to \infty$
Locality comes only after expanding in powers of $1/m_h$

"technical"

Vacuum energy + the naturalness criterion:

$$e^{i\Gamma(g)} = \int \mathcal{D}\phi\, e^{iS(g,\phi)}$$

for $S = -\int \left[(\partial\phi)^2 + m^2\phi^2\right]\sqrt{g}$.

$$= \det{}^{-1/2}(-\Box + m^2)$$

$\langle T_{\mu\nu} \rangle$

"technical"

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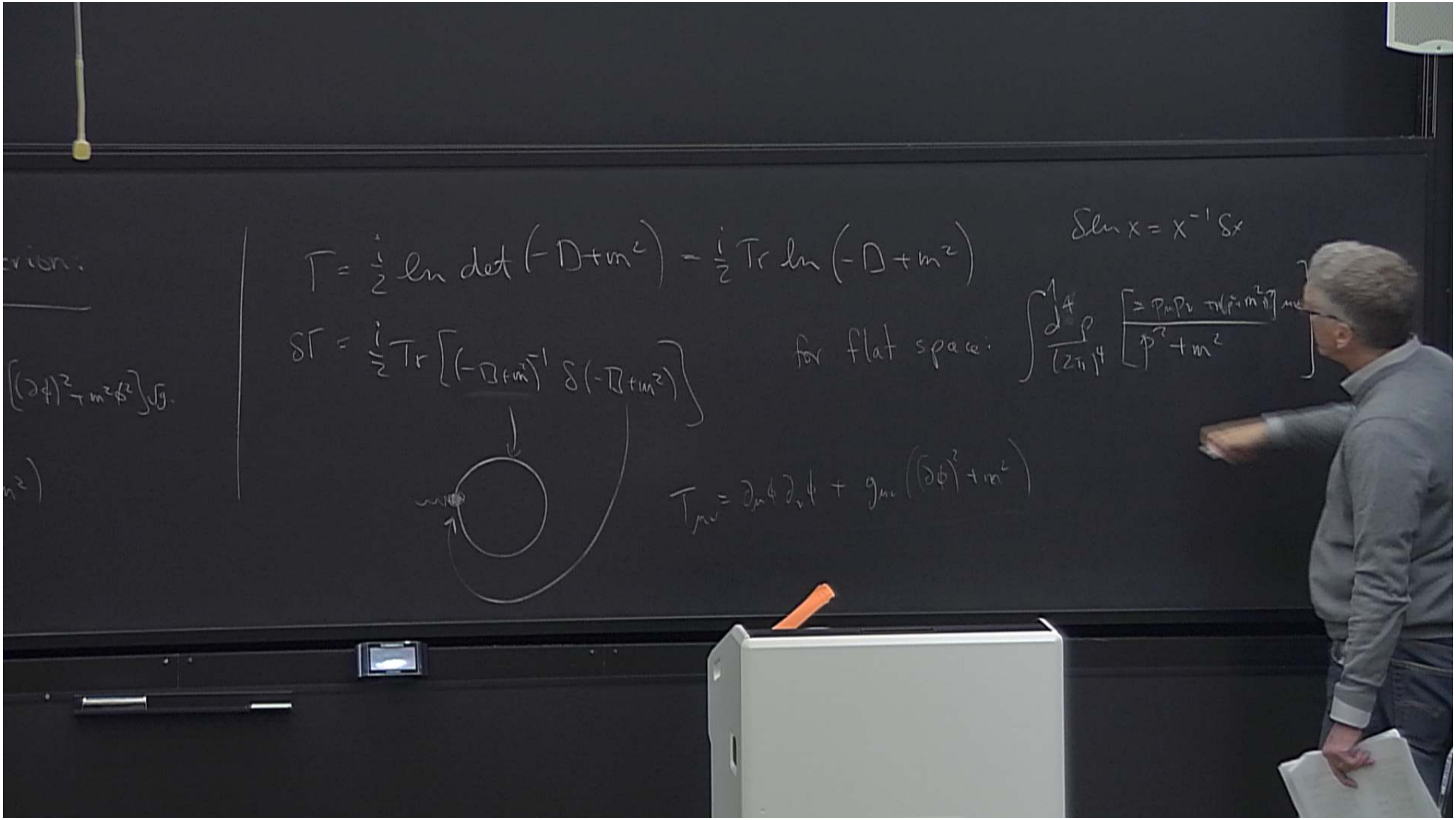
$$\langle T_{\mu\nu} \rangle = \frac{\delta\Gamma}{\delta g_{\mu\nu}}$$

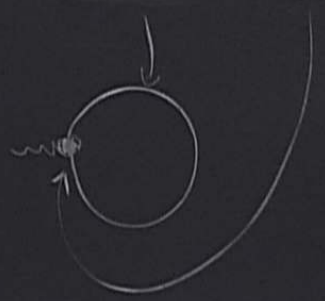

$\Gamma = \frac{i}{2} \ln \det(-\Box + m^2) = \frac{i}{2} \mathrm{Tr} \ln(-\Box + m^2)$
$\delta \ln x = x^{-1} \delta x$
$\delta\Gamma = \frac{i}{2} \mathrm{Tr}\left[(-\Box+m^2)^{-1} \delta(-\Box+m^2)\right]$
for flat space:
$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi + g_{\mu\nu}\left[(\partial\phi)^2 + m^2\right]$

$$\Gamma = \frac{i}{2} \ln \det\left(-\Box + m^2\right) = \frac{i}{2} \mathrm{Tr} \ln\left(-\Box + m^2\right)$$

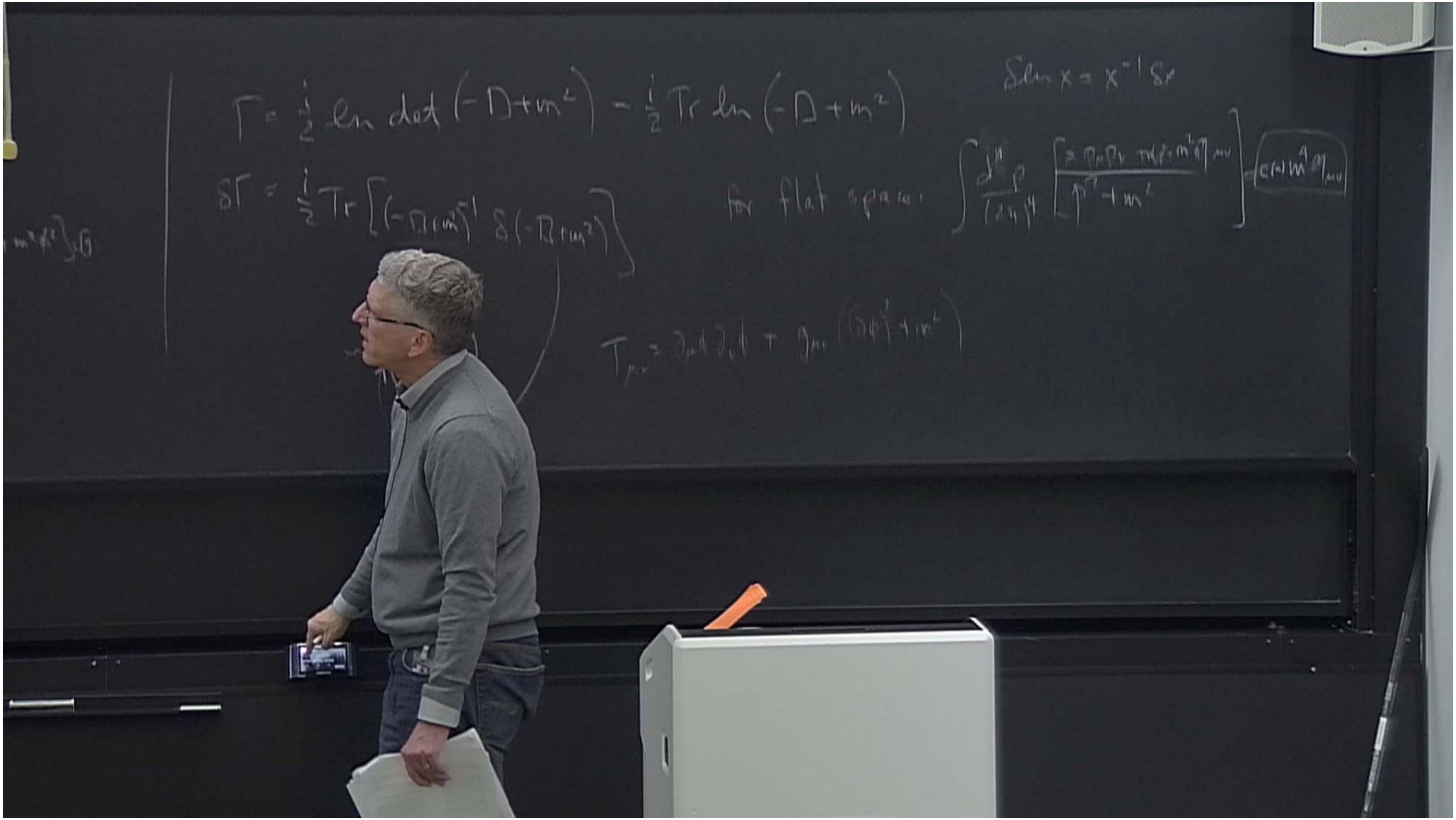
$$\delta \ln x = x^{-1} \delta x$$

$$\delta\Gamma = \frac{i}{2} \mathrm{Tr}\left[\left(-\Box + m^2\right)^{-1} \delta\left(-\Box + m^2\right)\right]$$

for flat space: $\int \frac{d^n p}{(2\pi)^4} \left[\frac{2 p_\mu p_\nu - \eta_{\mu\nu}\left(p^2 + m^2\right)}{p^2 + m^2}\right] = c(n)\, m^4 \eta_{\mu\nu}$



$$T_{\mu\nu} = \partial_\mu \phi\, \partial_\nu \phi + g_{\mu\nu}\left((\partial\phi)^2 + m^2\right)$$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for flat space:
$\delta \ln x = x^{-1} \delta x$
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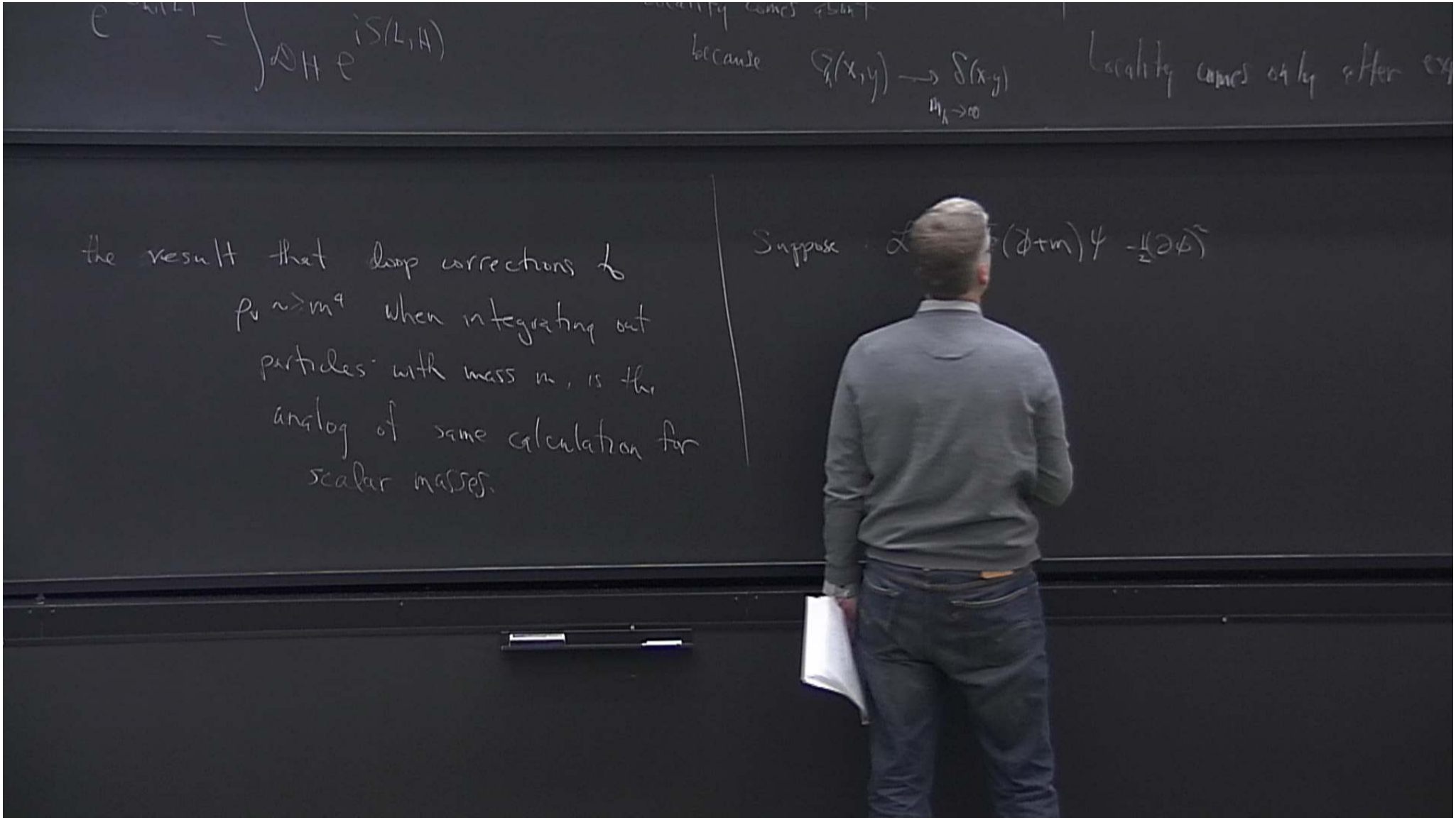

$e^{\quad} = \int \mathcal{D}H\, e^{iS(L,H)}$
because $G_L(x,y) \longrightarrow \delta(x-y)$
$m_H \to \infty$
Locality comes only after
the result that loop corrections to $p_v \sim m^q$ when integrating out particles with mass m, is the analog of same calculation for scalar masses.
Suppose $\mathcal{L} = (\partial\!\!\!/ + m)\psi - \frac{1}{2}(\partial\phi)^2$

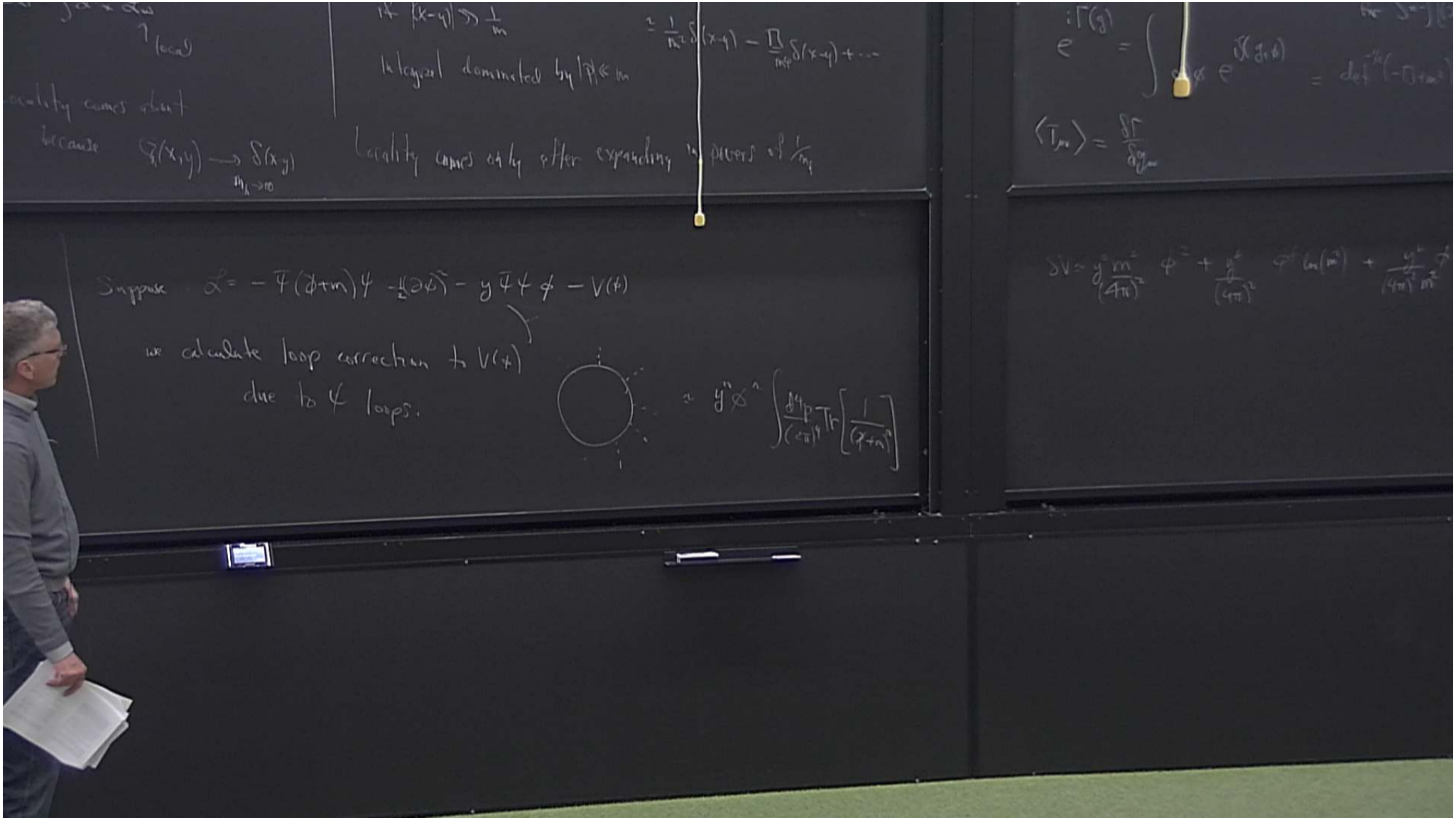
because $G_R(x,y) \longrightarrow \delta(x-y)$ (as $m_\chi \to \infty$)

Locality comes only after expanding in powers of $1/m_\chi$

Suppose $\mathcal{L} = -\bar{\psi}(\partial\!\!\!/ + m)\psi - \frac{1}{2}(\partial\phi)^2 - y\bar{\psi}\psi\phi - V(\phi)$

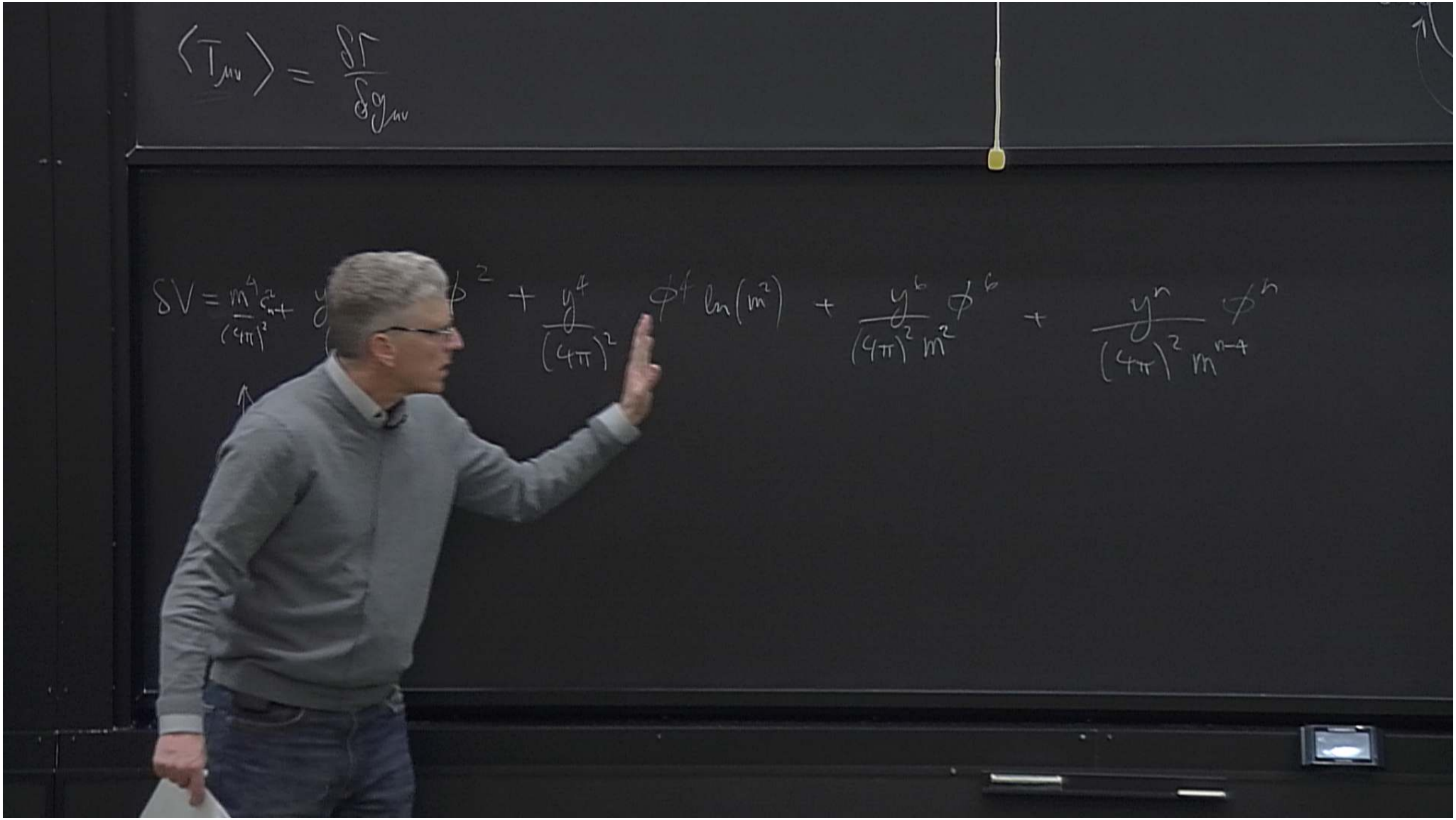
we calculate loop correction to $V(\phi)$ due to ψ loops:

$\sim y^n \phi^n \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{(p\!\!\!/ + m)^n}\right]$


Suppose $\mathcal{L} = -\bar{\psi}(\partial\!\!\!/ + m)\psi - \frac{1}{2}(\partial\phi)^2 - y\bar{\psi}\psi\phi - V(\phi)$
we calculate loop correction to $V(\phi)$
due to ψ loops.
$= y^n \phi^n \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{(p\!\!\!/ + m)^n}\right]$
Locality comes only after expanding in powers of $1/m_\psi$
$\delta V = \frac{y^6 m^2}{(4\pi)^2}\phi^2 + \frac{y^4}{(4\pi)^2}\phi^4 \ln(m^2) + \frac{y^4}{(4\pi)^2 m^2}\phi^6$

$$\langle T_{\mu\nu} \rangle = \frac{\delta \Gamma}{\delta g_{\mu\nu}}$$

$$\delta V \simeq \frac{y^2 m^2}{(4\pi)^2} c_n \phi^2 + \frac{y^4}{(4\pi)^2} \phi^4 \ln(m^2) + \frac{y^6}{(4\pi)^2 m^2} \phi^6 + \frac{y^n}{(4\pi)^2 m^{n-4}} \phi^n$$


$\langle T_{\mu\nu} \rangle = \frac{\delta \Gamma}{\delta g_{\mu\nu}}$
$\delta V = \frac{m^4 c_{n+}}{(4\pi)^2}$
$+ \frac{y^4}{(4\pi)^2} \phi^4 \ln(m^2) + \frac{y^6}{(4\pi)^2 m^2} \phi^6 + \frac{y^n}{(4\pi)^2 m^{n-4}} \phi^n$

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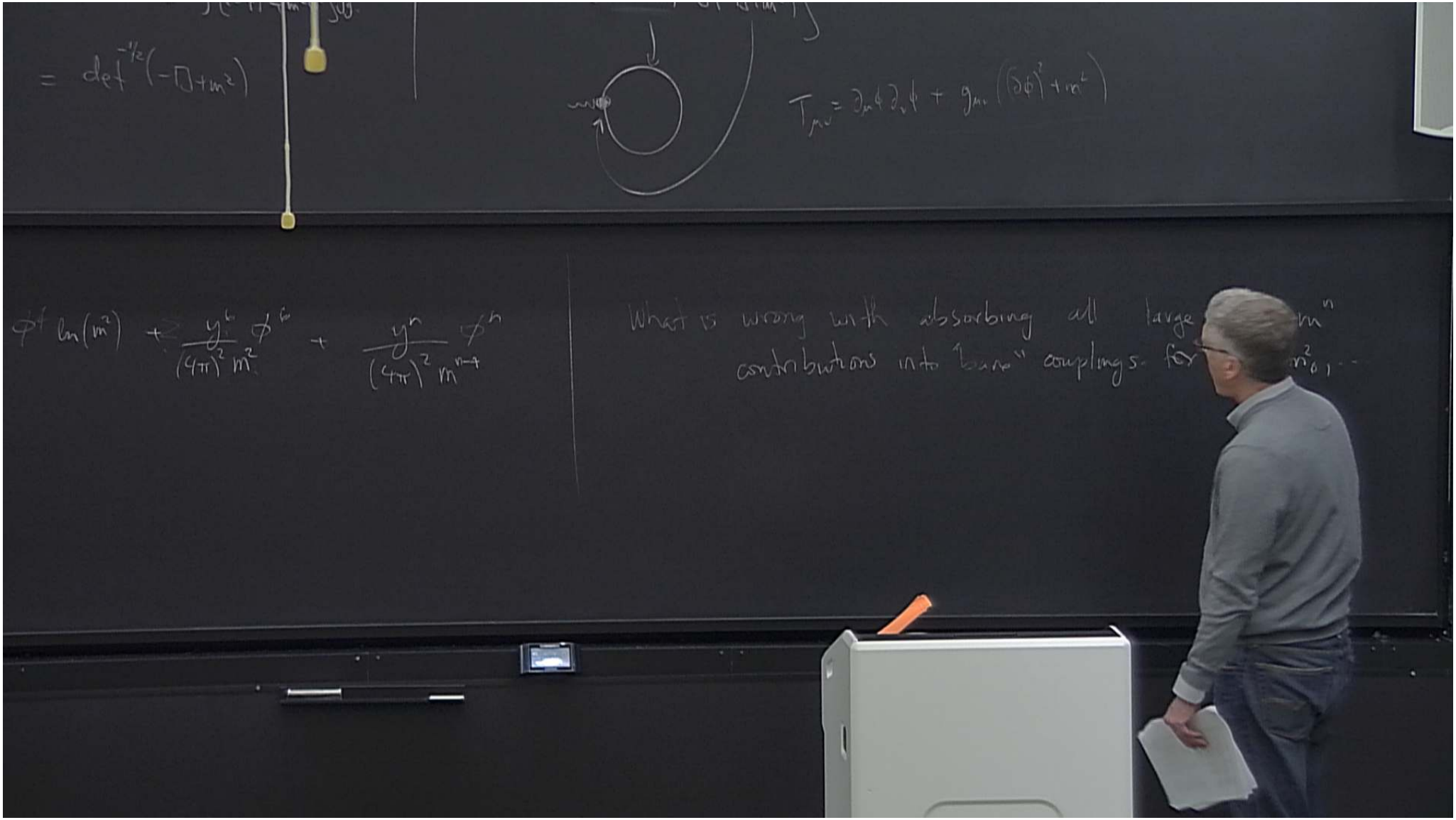
"relevant" interactions

$$+ \frac{m^2}{(4\pi)^2} \sqrt{g} R$$


$= \det^{-1/2}(-\Box + m^2)$
$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi + g_{\mu\nu}\left(\partial\phi^2 + m^2\right)$
$\phi^4 \ln(m^2) + \frac{y^6}{(4\pi)^2 m^2}\phi^6 + \frac{y^n}{(4\pi)^2 m^{n-4}}\phi^n$
What is wrong with absorbing all large
contributions into "bare" couplings. for

Given the couplings in $\mathcal{L}_{W_1}$

can compute " " " $\mathcal{L}_{W_n}$ $n>1$

ρ_i = vacuum energy param in $\mathcal{L}_{W_i}$

m_i^2 = scalar mass " " "

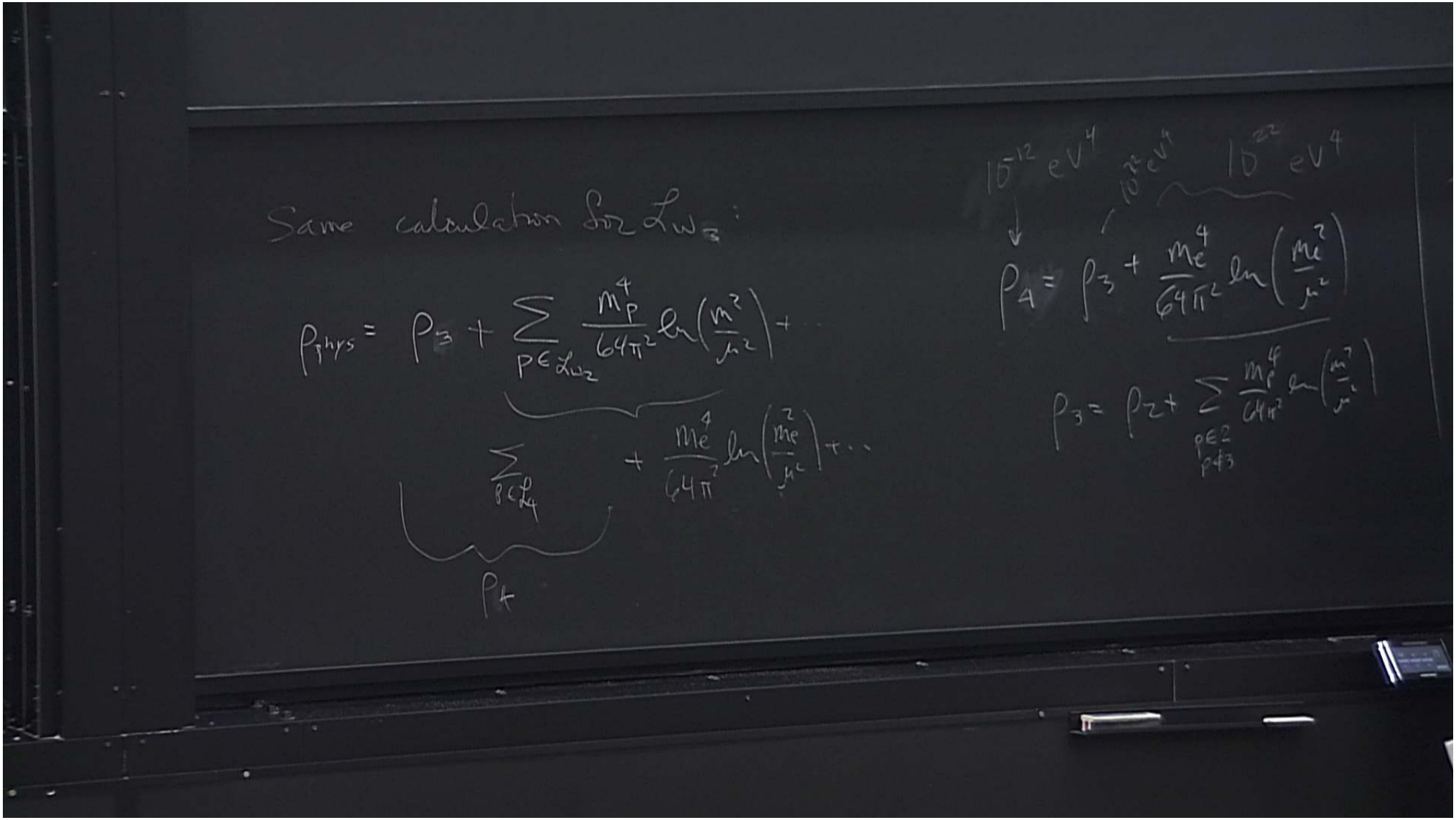
For each $\mathcal{L}_{W_i}$, choose ρ_i to ensure ρ_{phys} has the right value $(10^{-3}\,eV)^4$.

$(10^{-3}\,eV)^4$

$$\rho_{phys} = \rho_s + \sum_{p \in \mathcal{L}_{W_j}} \frac{m_p^4}{64\pi^2} \ln\left(\frac{m_p^2}{\mu^2}\right)$$

learn

$m_p \lesssim 10^{-3}\,eV$ for $\mathcal{L}_{W_j}$


Same calculation for $\mathcal{L}_{W_3}$:
$\rho_{phys} = \rho_3 + \sum_{p \in \mathcal{L}_{W_2}} \frac{m_p^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right) +$
$\sum_{\beta \in \mathcal{L}_4} + \frac{m_e^4}{64\pi^2} \ln\left(\frac{m_e^2}{\mu^2}\right) + \dots$
ρ_+
$10^{-12}\ eV^4$
$10^{-2}\ eV^4$
$10^{-2}\ eV^4$
$\rho_4 = \rho_3 + \frac{m_e^4}{64\pi^2} \ln\left(\frac{m_e^2}{\mu^2}\right)$
$\rho_3 = \rho_2 + \sum_{\substack{p \in \mathcal{L}_2 \\ p \neq 3}} \frac{m_p^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right)$

when integrating out particles with mass m, is the analog of same calculation for scalar masses.

we calculate lo

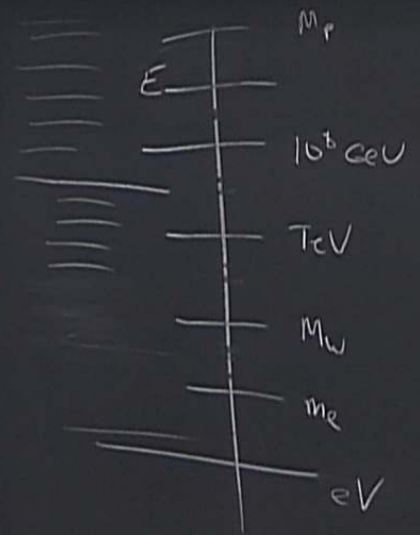
due t

SM

$a_0^{-1} \simeq \alpha_{SM} m_{e_{SM}}$

$r_n \simeq \frac{1}{\Lambda_{QCD}}$

atoms > nuclei? $\quad a_0 \simeq \frac{1}{\alpha m_e} \quad r_n \sim \frac{1}{m_p}$



$m_e = m_{e_{SM}} + \alpha^2 m_e \ln\left(\frac{m_n}{m_e}\right)$

— $\mathcal{L}_{W_1}$

— $\mathcal{L}_{W_2}$ ($\simeq \mathcal{L}_{SM}$)

— $\mathcal{L}_{W_3}$

— $\mathcal{L}_{W_4}$

Given the couplings in $\mathcal{L}_{W_1}$

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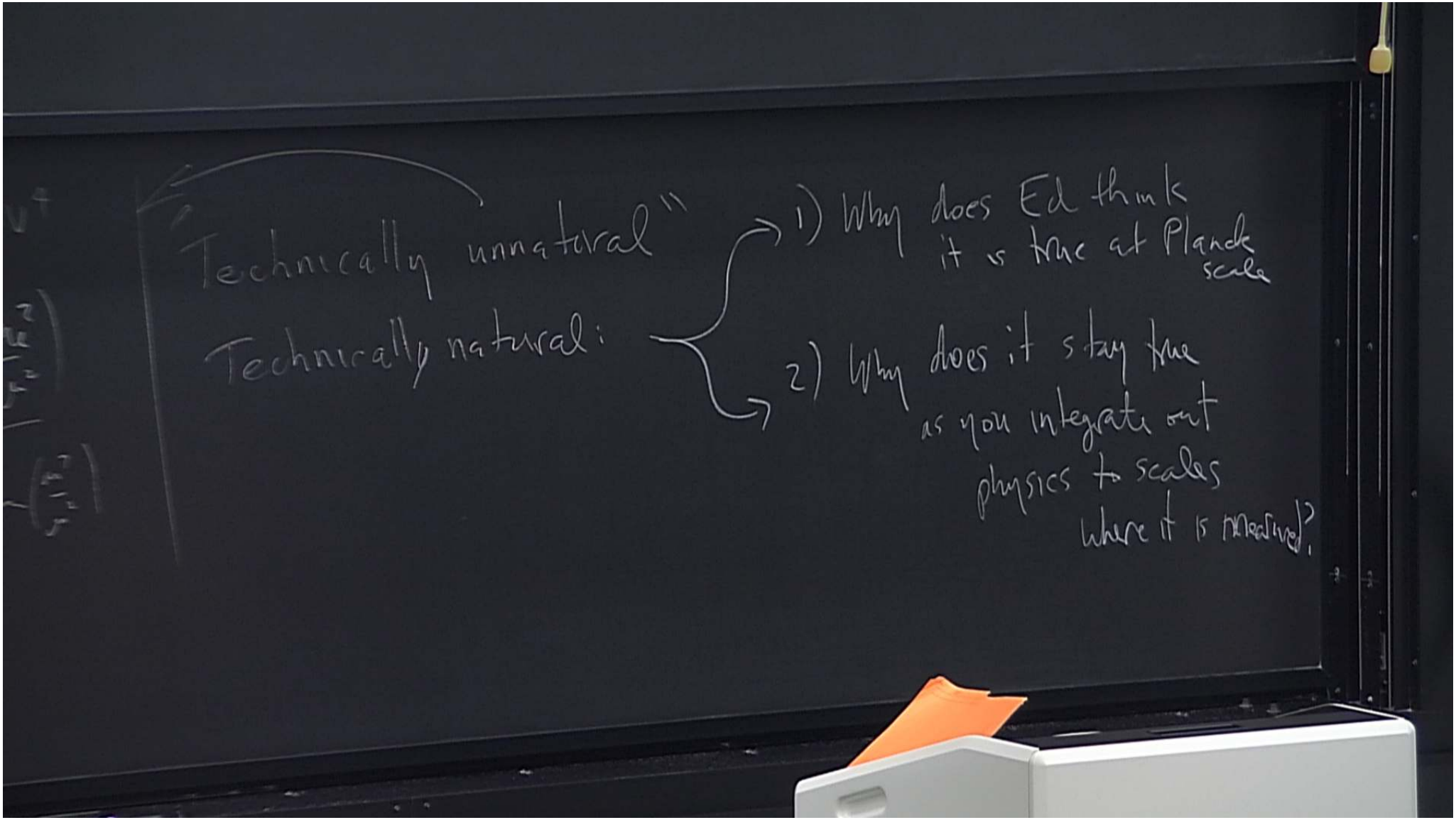
"Technically unnatural"
Technically natural:
1) Why does Ed think it is true at Planck scale
2) Why does it stay true as you integrate out physics to scales where it is measured?