Title: PSI 2015/2016 Explorations in Particle Theory - Burgess - 11

Date: Apr 25, 2016 09:00 AM

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Abstract:

The Story so Far.....

$$\int d^4k \, \ln\left(k^2+m^2-i\epsilon\right)$$

$$\rho_V = \lim_{V\to\infty} \frac{E_0}{V} = \frac{m^4(\varphi)}{64\pi^2}\ln\left(\frac{m^2(\varphi)}{\mu^2}\right) + A\, m^4(\varphi) + B\, m^2(\varphi) + C \qquad m^2(\varphi) = V''(\varphi)$$

regulated with a cutoff $\int_0^\Lambda dk$: $A \simeq \frac{-1}{64\pi^2}\ln \Lambda^2/\mu^2 + a_1$, $B = b_0\Lambda^2 + b_1$ $C = c_0\Lambda^4 +$

" " dimensional reg'n $\int_0^\infty d^n p$ $n = 4-2\epsilon$ $B=C=0$ $A = \frac{a_0}{\epsilon} + a_1$ etc.

$V''(\varphi)$

$= c_0 \Lambda^4 + c_1$

etc.

$$V = V_0 + \frac{1}{2} m^2 \varphi^2 + \frac{1}{4!} \lambda \varphi^4$$

$$\mathcal{L} = -\frac{1}{2} (\partial \varphi)^2 - V$$

$$\uparrow$$

$$\left(\frac{1}{l}\right)^n$$

$$\lambda \longrightarrow \lambda \left(\frac{\mu}{\mu_0}\right)^{4-n}$$

$$= \lambda \left(\mu / \mu_0\right)^{2\epsilon}$$

$$[\varphi] = \frac{n}{2} - 1$$

$$[\varphi^4] = 2n - 4$$

$$[\lambda] = n - [\varphi^4]$$

$$= 4 - n$$

$$V = V_0 + \frac{1}{2} m^2 \varphi^2 + \frac{1}{4!}\hat{\lambda}\varphi^4$$

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$$\hat{\lambda} \to \lambda \mu^{4-n} = \lambda \mu^{2\epsilon}$$

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Want $\langle T_{\mu\nu} \rangle$

If we compute $\langle T_{\mu\nu} \rangle$ we will get p_v, ρ_v

Q: is $p_v = -\rho_v$?

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Q: is $\boxed{p_v = -\rho_v}$?

vacuum is lorentz-inv.

If the regularization scheme breaks Lorentz inv then

in general $p \neq -\rho$

$$\rho_v = \rho_{v_{class}} + \delta\rho_v = V(\varphi) + \frac{m^4}{64\pi^2}\ln\left(\frac{m^2}{\mu^2}\right) + \underbrace{Am^4 + Bm^2 + C}$$

Both the divergences + finite parts of A, B, C are absorbed into unknown couplings V_0, m^2, λ, which are determined by comparison with observables.

If a regulator breaks a desired symmetry then must undo the damage when renormalizing.

Q: Is this always possible?

A: No:

Can
sym
ca

eg if

$=C=0 \quad A = \frac{a_0}{\epsilon} + a_1$ etc.

vacuum is lorentz-inv.

A: No: Cannot fix a failure of symmetry in this way if it cannot be absorbed into a local counterterm.

if $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D}+m)\psi$ then no

but if $\phi \to \phi + \omega$

then $\mathcal{L} = -\frac{\phi}{\omega} F_{\mu\nu}\tilde{F}^{\mu\nu}$ does the job

yes.

eg if $\delta\mathcal{L} = c F_{\mu\nu}\tilde{F}^{\mu\nu}$ must ask does $\exists \mathcal{L}$ s.t. $\delta\mathcal{L} = -c F_{\mu\nu}\tilde{F}^{\mu\nu}$

BPHZ

Effective field theories

if you have a hierarchy of scales

- size of nucleus vs size of atom:

* Nature has "decoupling" property: small distances are <u>largely</u> irrelevant to physics of longer distances.

$l_1 \ll l_2$

…ances,

* math of QFT also does this:

$$f^i(l_1, l_2) \simeq \underbrace{f_0^i(l_2)}_{\text{dominant}} + \frac{l_1}{l_2} f^{i\prime}(l_2) + \dots$$

Pays to expand as early as possible,
(ie in the action itself before
computing observables)

→ BONUS: gives physical understanding of renormalization

lization

Use the "effective action formalism" of the 60's:

Interested in $\langle \phi(x_1) \ldots \phi(x_n) \rangle$

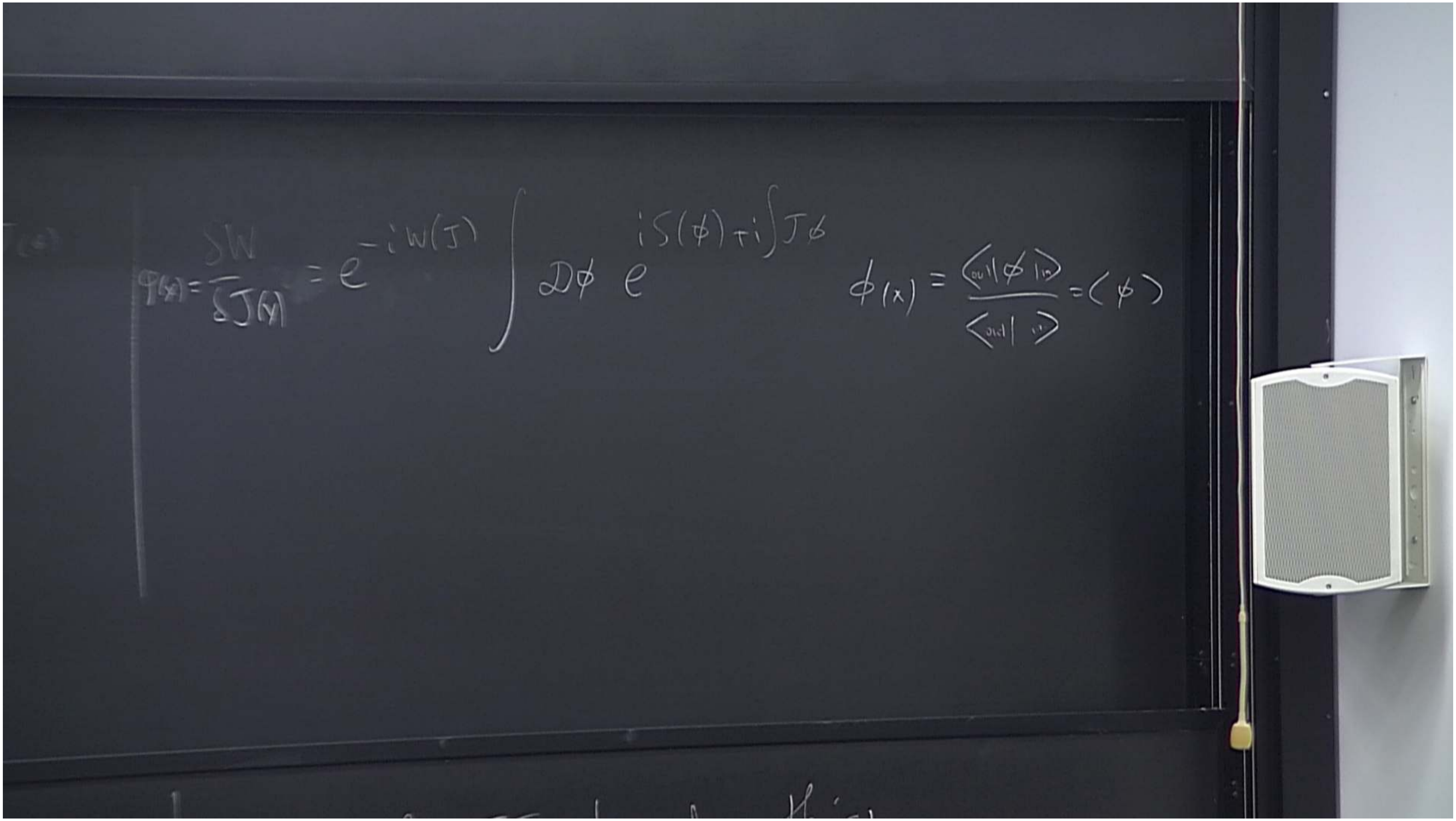
Want: generating functional for these:

$$W[J] = \langle 1 \rangle + \int d^4x \, J(x) \langle \phi(x) \rangle + \frac{1}{2} \int d^4x_1 \, d^4x_2 \, \langle \phi(x_1) \phi(x_2) \rangle J(x_1) J(x_2) + \ldots$$

Define: $W[J]$ by $e^{iW[J]} = \int \mathcal{D}\phi \; e^{iS(\phi) + i\int d^4x\, \phi(x) J(x)}$

if $J=0$ then $e^{iW(0)} = \int \mathcal{D}\phi \, e^{iS(\phi)} = \langle out | in \rangle$

$\frac{\delta W}{\delta J}$


$\phi(x) = \frac{\delta W}{\delta J(x)} = e^{-iW(J)} \int \mathcal{D}\phi \, e^{iS(\phi) + i\int J\phi}$
$\phi(x) = \frac{\langle out|\phi|in\rangle}{\langle out|in\rangle} = \langle \phi \rangle$

Legendre transformation: solve $J = J(\varphi)$

$$\Gamma(\varphi) = W\left[J[\varphi]\right] - \int dy\, \varphi(y)\, J(\varphi)(y)$$

$$\frac{\delta\Gamma}{\delta\varphi(x)} = \cancel{\int d^4y\, \underbrace{\frac{\delta W}{\delta J(y)}}_{\varphi(y)} \frac{\delta J(y)}{\delta\varphi(x)}} - J(x) - \cancel{\int dy\, \varphi(y) \frac{\delta J(y)}{\delta\varphi(x)}} = -J(x)$$

So $\frac{\delta \Gamma}{\delta \varphi} + J = 0$ (quantum corrected eq. of motion for $\langle \varphi \rangle$)

$$e^{i\Gamma(\varphi)} = e^{iW(J) - \int J\varphi} = \int \mathcal{D}\Phi \, e^{iS(\Phi) + i\int J(\Phi - \varphi)}$$

$$\Phi = \varphi + \phi$$

$- J(x)$

$$\boxed{e^{i\Gamma(\varphi)} = \int \mathcal{D}\phi \, e^{iS(\varphi+\phi) + i\int J\phi}}$$