Title: PSI 2015/2016 Explorations in Particle Theory - Burgess - 7

Date: Apr 19, 2016 09:00 AM

URL: http://pirsa.org/16040026

Abstract:

Detailed balance:

for $I \leftrightarrow F$

$$\prod_{k \in I} \frac{1 \pm \mathcal{N}_k}{\mathcal{N}_k} = \prod_{k \in F} \frac{1 \pm \mathcal{N}_k}{\mathcal{N}_k} \quad \text{for all } I, F$$

$$\Rightarrow \mathcal{N}_k = \left[e^{\beta (u_\mu p^\mu_k - \mu_a q^a_k)} \mp 1 \right]^{-1} \qquad \beta_\mu = \beta u_\mu, \quad u^2 = -1$$

if $u_\mu = \delta_\mu^0$

$$N_k = \left(e^{\beta(E_k - \mu_k)} \mp 1\right)^{-1}$$

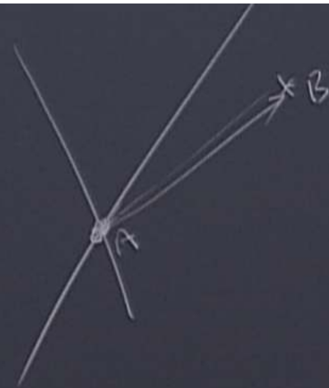
$$\mu_k := \sum_a \mu_a q_k^a$$

(which implies $\bar{\mu}_k = \sum_a \mu_a \bar{q}_k^a = -\mu_k$
since $\bar{q}_k^a = -q_k^a$)

finally for NR $E_k = m_k + \varepsilon_k$ $\left(\varepsilon_k = \frac{p^2}{2m_k} + \cdots\right)$

and $\mu_k = m_k + \delta\mu_k$

so $E_k - \mu_k = \varepsilon_k - \delta\mu_k$.



$a = Q, B, L, \ldots\ldots$ charge

$k = e^-, e^+, \nu, \bar{\nu}, p, \bar{p}, n, \bar{n}, \gamma$ particle.

$l = e, \nu, p, n, \gamma, \ldots$ species

Densities in space (vs phase space)

$$n_k(x) = \int \frac{d^3p}{(2\pi)^3} \mathcal{N}_k.$$

$$\rho(x) = \sum_k \int \frac{d^3p}{(2\pi)^3} E_k(p)\, \mathcal{N}_k$$

$$\vec{P} = \sum_k \int \frac{d^3p}{(2\pi)^3} \vec{p}\, \mathcal{N}_k$$

$$\zeta^a(x) = \sum_{k=p,\bar{p}} q^a_k \int \frac{d^3p}{(2\pi)^3} \mathcal{N}_k = \sum_{k=p,\bar{p}} q^a_k\, n_k.$$

$$= \sum_{\substack{k=p \\ (\text{not } \bar{p})}} q^a_k \left(n_k - \bar{n}_k\right)$$

if $\mu_a = 0 \Rightarrow \mu_k = 0$ for all k.
for all a

$\bar{n}_k = n_k$.

then $\zeta^a = 0$.

so $\mu^a \neq 0$ can be chosen to set ζ^a = given value.

if $\vec{\beta} = 0$ then $\vec{P} = \sum_k \int \frac{d^3p}{(2\pi)^3} \vec{p}_k N_k$

(and if $\varepsilon = \varepsilon(|\vec{p}|)$ then $\vec{P} = 0$.)

$$\Longrightarrow \quad N_k = \left[e^{\beta(u_\mu p^\mu_k - \mu_a q^a_k)} \mp 1 \right] \qquad \beta_\mu = \beta u_\mu, \quad u^2 = -1$$

$$\text{since } \bar{q}^a_k = -q^a_k \;)$$

$$n_k(T, \mu_k) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta(E_k - \mu_k)} \mp 1}$$

$$C_r(y, z) = \frac{1}{2\pi^2} \int_0^\infty dx \, \frac{x^2 (x^2 + y^2)^{r/2}}{e^{\sqrt{x^2+y^2} - z} \mp 1}$$

$$y = \frac{m}{T}, \quad z_k = \frac{\mu_k}{T}$$

"Degenerate" case: $E_k - \mu_k \ll T$

Nondegenerate case $E_k - \mu_k \gg T$

$$n_k = T^3 C_0(y, z_k)$$

$$\rho_k = T^4 C_1(y, z_k)$$

$$\frac{x^2 (x^2+y^2)^{r/2}}{e^{\sqrt{x^2+y^2}-z} \mp 1}$$

$E_k - \mu_k \ll T$

$E_k - \mu_k \gg T$

massless particles with $\mu_k = 0$ $(y = z_k = 0)$

$$C_1^+(0,0) = \frac{7}{8} C_1^-(0,0) = \frac{7\pi^2}{240}$$

$$C_0^+(0,0) = \frac{3}{4} C_0^-(0,0) = \frac{3}{4\pi^2}\zeta(3)$$

$$n_{tot}(T) = \left(N_b + \frac{3}{4} N_f\right)\left(0.1218\, T^3\right)$$

$$\rho_{tot}(T) = \left(N_b + \frac{7}{8} N_F\right)\left(0.3290\, T^4\right)$$

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a_B = Stefan Boltzmann

$$n_{tot}(T) = \left(N_b + \frac{3}{4} N_f\right)\left(0.1218\, T^3\right)$$

$$\rho_{tot}(T) = \left(N_b + \frac{7}{8} N_F\right)\left(0.3290\, T^4\right)$$

For NR particles: $E_k \simeq m_k + \frac{p^2}{2m_k}$.

if $\mu_k = 0$. $(z_k = 0)$

$$C_r^{\pm}(y) = \frac{e^{-y}}{2\pi^2}\int_0^\infty dx\, x^2 \left(y + \frac{x^2}{2y}\right)^r e^{-x^2/2y}.$$

$$= \left(\frac{y}{2\pi}\right)^{3/2} e^{-y} \begin{cases} 1 & \text{if } r=0 \\ y+3/2 & \text{if } r=1 \end{cases}$$

$$\frac{p^2}{2m_K}$$

$$+\frac{x^2}{2y}\Big)^r e^{-x^2/2y}.$$

f $r=0$

f $r=1$

$$n = \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$$\rho = n\left[m + \frac{3T}{2}\right]$$

$$(\gamma = 5/3)$$

m/T

E_h

T

E_ℓ

$e^+ e^- \leftrightarrow \gamma\gamma$

Since p, n, e^- exist $T \ll m$

μ_B (Baryon #) ensures a nonzero $\zeta^B(x)$

μ_Q (electric charge) $\zeta^Q(x)$

$$= \sqrt{m_k^2 + p^2}$$

finally for NR $E_k = m_k + \varepsilon_k \quad \left(\varepsilon_k = \frac{p^2}{2m_k} + \cdots\right)$

and $\mu_k = m_k + \delta\mu_k$

so $E_k - \mu_k = \varepsilon_k - \delta\mu_k$.

$\mu_Q \neq 0$, $\mu_B \neq 0$

$\mu_p = \mu_B + \mu_Q \quad \mu_n = \mu_B \quad \mu_{e^+} = +\mu_Q$

$\mu_{\bar{p}} = -\mu_B - \mu_Q \quad \mu_{e^-} = -\mu_Q$

	B	Q
p	+1	+1
n	+1	0
$\bar{p}$	−1	−1
$\bar{n}$	−1	0
e^+	0	+1
e^-	0	−1

NR case when $\mu_k \neq 0$

$$n_k = \left(\frac{m_k T}{2\pi}\right)^{3/2} e^{\delta\mu_k / T}$$

for $\varepsilon - \delta\mu \gg T$ (non degenerate case of usual interest in cosmol.)

$$n = \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$$\rho = n\left[m + \frac{3T}{2}\right]$$

$$\left(\gamma = 5/3\right)$$

DM (WIMP thermal case)

E_n

T

E_e

$$e^+e^- \leftrightarrow \gamma\gamma$$

Sinc

e

μ_B (B

ensu

μ_Q (ele

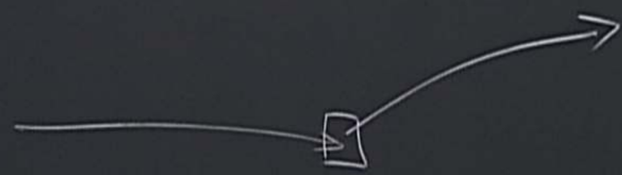
$\uparrow$ DM (WIMP thermal case)

as $n \downarrow$ eventually reaction rates $\downarrow$ so far that equilibrium fails.

Precisely when?

if

If detailed balance fails, how does N_k evolve?



$d^3x \, d^3p$

Boltzmann-equation:

$$D_t \mathcal{N} = C. \quad (\text{collision term})$$

$$\frac{\partial \mathcal{N}}{\partial t} + \dot{x}^i \frac{\partial \mathcal{N}}{\partial x^i} + \left(\dot{p}^i + \Gamma^i_{jk} p^j p^k\right) \frac{\partial \mathcal{N}}{\partial p^i} = \sum_{\substack{\text{initial} \\ \text{final}}} \left(\begin{array}{l}\text{rate to scatter in} \\ - \text{rate to scatter out}\end{array}\right)$$

?

$$D_t N = C. \quad (\text{collision term})$$

$$\frac{\partial N}{\partial t} + \cancel{\dot{x}^i \frac{\partial N}{\partial x^i}} + \cancel{\left(\dot{p}^i + \Gamma^i_{jk} p^j p^k\right) \frac{\partial N}{\partial p^i}} = \sum_{\substack{\text{initial} \\ \text{final}}} \left(\text{rate to scatter in} - \text{rate to scatter out} \right)$$

inhomogeneities + drift

forces pulling you away from geodesic

could vanish if equilibr. or if no interactions

$$a^3 n = \int \frac{d^3p}{(2\pi)^3} \mathcal{N}$$

$$\frac{\partial}{\partial t}(a^3 n) = \int \frac{d^3p}{(2\pi)^3}\left(\text{Collision term}\right)$$

as

if

$$a^3 n = \int \frac{d^3p}{(2\pi)^3} \mathcal{N}$$

$$\frac{\partial}{\partial t}\left(a^3 n\right) = \int \frac{d^3p}{(2\pi)^3}\left(\text{Collision term}\right)$$

$$\dot{n} + 3Hn = \frac{1}{a^3}\int \cdots$$

collision times

H

as

If