

Title: PSI 2015/2016 Explorations in Particle Theory - Burgess - 3

Date: Apr 13, 2016 09:00 AM

URL: <http://pirsa.org/16040022>

Abstract:

$$1\text{eV} \leftrightarrow 10^4\text{K}$$

$$p \approx 0$$

$$\rho \approx 4T^4$$

$$p = \frac{4}{3}T^4$$

$$p = nT$$

$$p = nm + \frac{nT}{\gamma - 1}$$

$$\frac{p}{\rho} = \frac{T}{m} \approx \langle v^2 \rangle$$

$$T \approx \langle mv^2 \rangle$$

$$\left(\frac{\partial p}{\partial \rho}\right)_s = \frac{1}{3} \text{ (radiation)}$$

$$c_p \approx 10^{-9} c \approx 300\text{m/s}$$

$$= \frac{T}{m} \text{ (for non-relativistic matter)}$$

$$c_p^2 \frac{T}{m} = \frac{300\text{K}}{14\text{GeV}} = \frac{10^{-2}\text{eV}}{10\text{GeV}} = \frac{10^{-11}\text{GeV}}{10\text{GeV}} = 10^{-12}$$

$T_{\text{eff}} =$  for DM from cosmology.

structure formation.

$$\rho_r \propto \frac{1}{a^4}$$



$$\rho \approx \# T^4$$

$$P = \frac{\#}{3} T^4$$

$$\frac{P}{\rho} = \frac{T}{\rho} \approx \langle v^2 \rangle$$

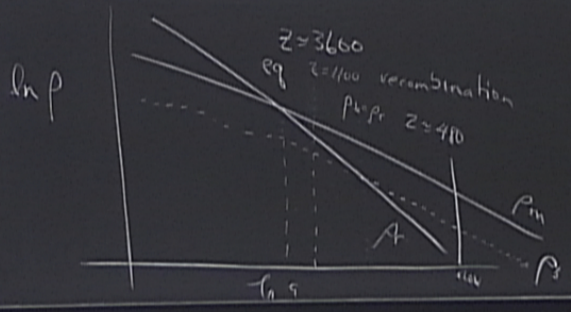
$$T \approx \langle mv^2 \rangle$$

$$= \frac{T}{m} \quad (\text{for NR matter})$$

$$c_{3/2}^2 \frac{T}{m} \approx \frac{300 \text{ K}}{14 \text{ GeV}} = \frac{10^2 \text{ eV}}{10^9 \text{ GeV}} = \frac{10^{-11} \text{ GeV}}{10^9 \text{ GeV}} = 10^{-20}$$

Evidence for DM from cosmology:

- structure formation:



$$\rho_r \propto \frac{1}{a^{4t}}$$

$$\rho_m \propto \frac{1}{a^3}$$

$$A_b \propto \frac{1}{a^2}$$

Class  $\propto a(t)$

$$\propto \frac{\delta \rho}{\rho}$$



$$\rho \approx n T^4$$

$$P = \frac{1}{3} n T^4$$

$$\frac{P}{\rho} = \frac{T}{M} \approx \langle v^2 \rangle$$

$$T \sim \langle mv^2 \rangle$$

$$= \frac{T}{m} \quad (\text{for NR matter})$$

$$c^2 \frac{T}{m} \approx \frac{300 \text{ K}}{14 \text{ GeV}} = \frac{10^2 \text{ eV}}{10^9 \text{ GeV}} = \frac{10^{-11} \text{ GeV}}{10^9 \text{ GeV}} = 10^{-20}$$

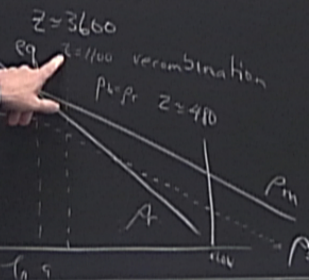
Evidence for DM from cosmology.

Structure formation.

$$\rho_r \propto \frac{1}{a^{4t}}$$

$$\rho_m \propto \frac{1}{a^3}$$

$$\rho_b \propto \frac{1}{a^3}$$



→ Claim  $\frac{\delta \rho}{\rho} \propto a(t)$

→ Galaxy require  $\frac{\delta \rho}{\rho} \gtrsim 1$

→  $\frac{\delta \rho}{\rho} \approx 10^{-3}$  at recombination



$$\frac{T}{m} \approx \left(\frac{v^2}{c^2}\right)$$

$$= \frac{T}{m} \text{ (for NR matter)}$$

$$G_{\text{eff}} \frac{T}{m} \approx \frac{300 \text{ K}}{14 \text{ GeV}} = \frac{10^{-2} \text{ eV}}{10 \text{ GeV}} \approx \frac{10^{-11} \text{ GeV}}{10 \text{ GeV}} = 10^{-12}$$

cosmology.

→ Claim  $\frac{\delta\rho}{\rho} \propto a(t)$

$$\rho \propto \frac{1}{a^3}$$

→ Galaxy require  $\frac{\delta\rho}{\rho} \gtrsim 1$

$$m \propto \frac{1}{a^3}$$

→  $\frac{\delta\rho}{\rho} \approx 10^{-5}$  at recombination

$$Q \propto \frac{1}{a^2}$$

In order for  $\frac{\delta\rho}{\rho}$  to grow require:

- 1) matter dominated universe
- 2) mode  $|\mathbf{k}| = \frac{2\pi}{\lambda}$  must satisfy  $\frac{k}{a} < H$

$$\frac{T}{m} \approx \left(\frac{v^2}{c^2}\right)$$

$$= \frac{T}{m} \text{ (for NR matter)}$$

$$G^2 \frac{T}{m} \approx \frac{300 \text{ K}}{14 \text{ GeV}} = \frac{10^{-2} \text{ eV}}{10 \text{ GeV}} \approx \frac{10^{-11} \text{ GeV}}{10 \text{ GeV}} = 10^{-12}$$

cosmology.

→ Claim  $\frac{\delta p}{p} \propto a(t)$

$$\rho \propto \frac{1}{a^3}$$

→ Galaxy require  $\frac{\delta p}{p} \gtrsim 1$

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In order for  $\frac{\delta p}{p}$  to grow require:

- 1) matter dominated universe
- 2) mode label  $k = \frac{2\pi}{\lambda}$  must satisfy  $\frac{k}{q} < H$



$$= \frac{1}{m} \text{ (for NR matter)}$$

$$G_{22} \frac{T}{m} = \frac{300 \text{ K}}{10 \text{ GeV}} = \frac{10^{-2} \text{ eV}}{10 \text{ GeV}} \sim \frac{10^{-11} \text{ GeV}}{10 \text{ GeV}} = 10^{-12}$$

→ Claim  $\frac{\delta p}{p} \propto a(t)$

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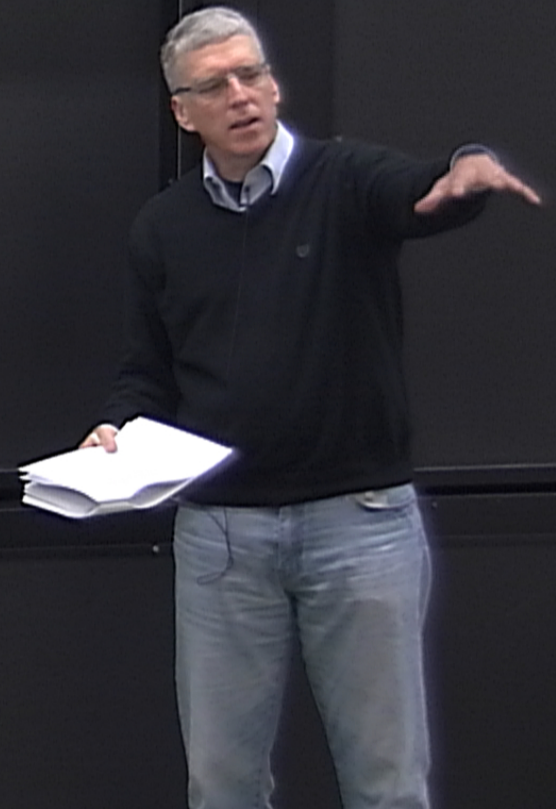
→  $\frac{\delta p}{p} \approx 10^{-5}$  at recombination

In order for  $\frac{\delta p}{p}$  to grow

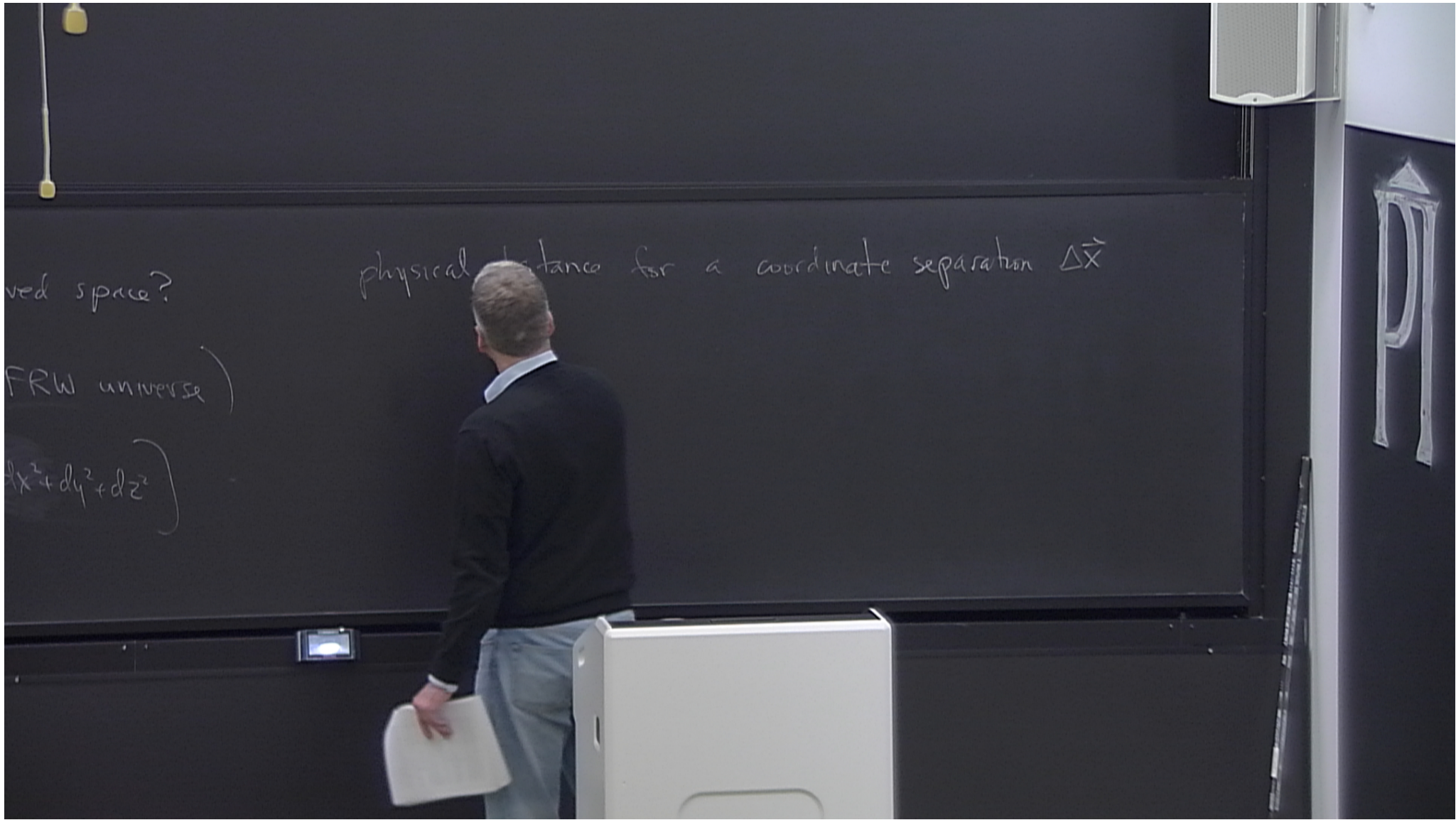
require:

1) matter dominated universe

2) mode label  $k = \frac{2\pi}{\lambda}$  must satisfy  $k/a < H$







ved space?

physical distance for a coordinate separation  $\Delta\vec{x}$

FRW universe

$$dx^2 + dy^2 + dz^2$$



$$ds^2 = -dt^2 + a^2(t) \left( dx^2 + dy^2 + dz^2 \right)$$

$$\phi(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3} \phi_k(t) e^{i\vec{k}\cdot\vec{x}}$$

Consider a relativistic scalar field,  $\phi$ ,  
whose field eq. is  $\square\phi = 0$

$$\begin{aligned} g^{\mu\nu} \nabla_\mu \nabla_\nu \phi &= g^{\mu\nu} (\partial_\mu + \Gamma_{\mu\nu}^\lambda \partial_\lambda) \partial_\nu \phi \\ &= \ddot{\phi} + \underline{3H\dot{\phi}} - \frac{\nabla^2}{a^2} \phi \end{aligned}$$

$$\dot{\phi} = \frac{\partial \phi}{\partial t} \quad H =$$

$$\square\phi_k = 0 = \ddot{\phi}_k - \frac{k^2}{a^2} \phi_k = 0$$

$$\text{if } k \neq 0$$



background.

$$\text{Energy: } \frac{\partial f}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

$$\text{momentum: } \rho \left[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] + \nabla p + \rho \nabla \phi = 0$$

$$\text{Entropy: } \frac{\partial s}{\partial t} + \nabla \cdot (s \vec{v}) = 0$$

$$\text{universal gravitation: } \nabla^2 \phi - 4\pi G \rho = 0$$



background.

Energy:  $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$

$$\frac{dE}{dt} = \int \frac{\partial \rho}{\partial t} d^3x$$

Momentum:  $\rho \left[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] + \nabla p + \rho \nabla \phi = 0$

Entropy:  $\frac{\partial s}{\partial t} + \nabla \cdot (s \vec{v}) = 0$

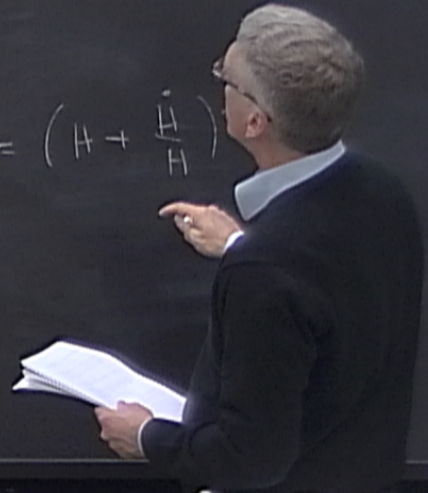
Background:

$$\vec{v}_0 = H(t) \vec{r}$$

$$\dot{\vec{v}}_0 + (\vec{v}_0 \cdot \nabla) \vec{v}_0 = \left( H + \frac{\dot{H}}{H} \right) \vec{v}_0$$

universal gravitation:  $\nabla^2 \phi - 4\pi G \rho = 0$

eq. of state:  $p = p(\rho, s)$





Energy:  $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$

Momentum:  $\rho \left[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] + \nabla p + \rho \nabla \phi = 0$

$$\frac{dE}{dt} = \int_{\Sigma} \frac{\partial p}{\partial t} d^3x = - \int_{\Sigma} \nabla \cdot (\rho \vec{v}) d^3x = - \int_{\partial \Sigma} \vec{n} \cdot \rho \vec{v} d^2x$$

$\rho = 0$   
 $-4\pi G \rho = 0$

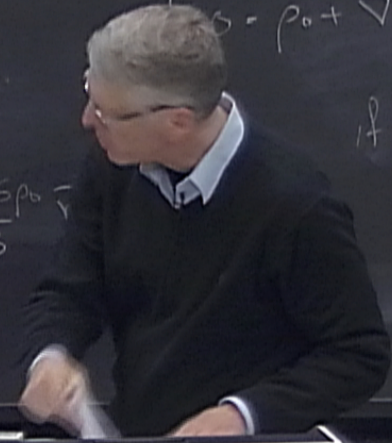
Background:

$$\vec{v}_0 = H(t) \vec{r} \quad \nabla \cdot \vec{v}_0 = 3H$$

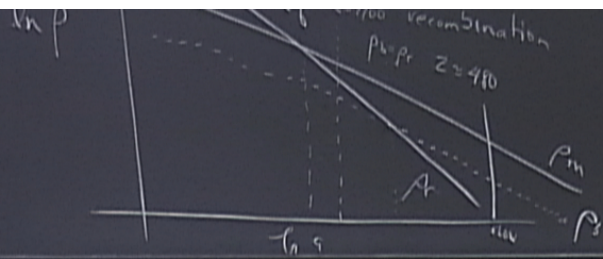
$$\dot{\vec{v}}_0 + (\vec{v}_0 \cdot \nabla) \vec{v}_0 = \left( H + \frac{\dot{H}}{H} \right) \vec{v}_0$$

$$\phi_0 = \frac{2\pi G \rho_0}{3} r^2 \quad -\nabla \phi_0 = -\frac{4\pi G \rho_0}{3} \vec{r}$$

$\rho_0 = \rho_0(t)$   
 $\dot{\rho}_0 + \nabla \cdot (\rho_0 \vec{v}_0) = \dot{\rho}_0 + 3H \rho_0$   
 if  $H = \frac{\dot{a}}{a}$  then  $\rho_0 \propto \frac{1}{a^3}$   
 like for NR matter







$$p_m \propto \frac{1}{a^3}$$

$$p_s \propto \frac{1}{a^3}$$

$$\rightarrow \frac{p_m}{p_s} \approx 10^5 \text{ at recombination}$$

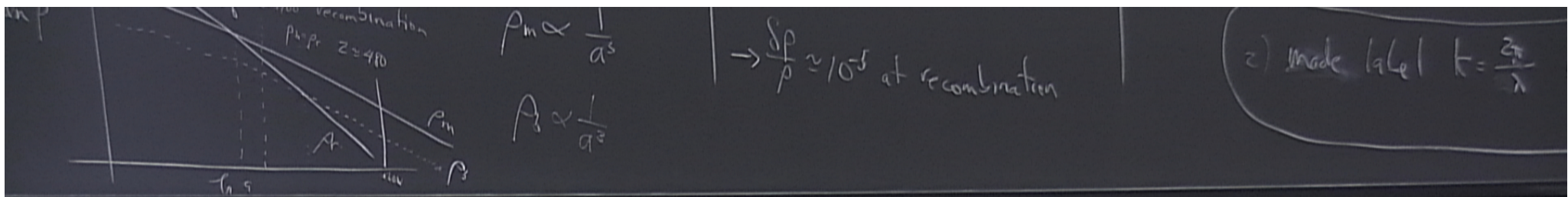
z) mode l

$$\text{if } a = a_0 \left(\frac{t}{t_0}\right)^\alpha$$

$$\text{then } H = \frac{\dot{a}}{a} = \frac{\alpha}{t}$$

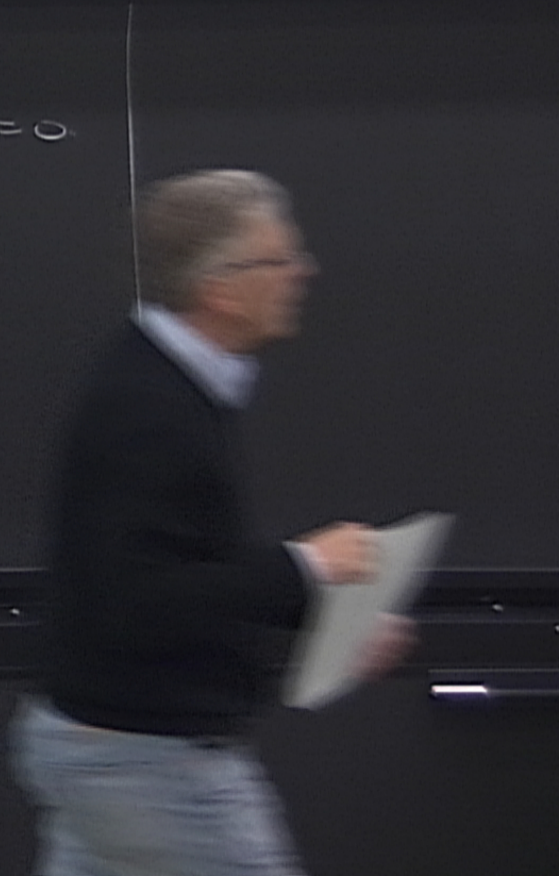
$$p_0 \propto \frac{1}{a^3} \propto t^{-3\alpha}$$





if  $a = a_0 \left(\frac{t}{t_0}\right)^\alpha$   
 then  $H = \frac{\dot{a}}{a} = \frac{\alpha}{t}$   
 $\rho_0 \propto \frac{1}{a^3} \propto t^{-3\alpha}$

then momentum eq:  $\left[ \ddot{H} + H^2 + \frac{4\pi G}{3} \rho_0 \right] \bar{r} = 0$   
 $\rightarrow \alpha = 2/3 \quad a \propto t^{2/3}$   
 matter-dominated background.





$$\rightarrow \frac{\delta p}{p} \approx 10^{-5} \text{ at recombination}$$

2) mode label  $k = \frac{2\pi}{\lambda}$  must satisfy  $k/a < H$

momentum eq:  $\left[ \dot{H} + H^2 + \frac{4\pi G}{3} \rho_0 \right] \vec{r} = 0$

$$\rightarrow \alpha = 2/3 \quad a \propto t^{2/3}$$

matter-dominated background.

$$\vec{V} = \vec{V}_0 + \delta \vec{V}$$

$$\rho = \rho_0 + \delta \rho$$

$$\phi = \phi_0 + \delta \phi$$

$$S = \boxed{S_0} + \delta S$$

$$P = \boxed{P_0} + \delta P$$

(T/m)

$$\delta p = \left( \frac{\partial p}{\partial \phi} \right) \delta \phi + \left( \frac{\partial p}{\partial S} \right) \delta S$$

$\underbrace{\quad}_{c_s^2} \quad (1/3 \text{ for } T)$



matter-dominated background.

$$p = \frac{p_0}{T/m} + \delta p$$

$$\vec{\xi} = \left( \frac{\partial p}{\partial \vec{x}} \right)_p$$

$$\delta_k(t) = \frac{\delta p_k}{\rho_0} = \int \frac{d^3 x}{(2\pi)^3} \frac{\delta p(x,t)}{\rho_0} e^{-i\vec{k} \cdot \vec{x}}$$

claim:  
 $\delta_k \propto t^{2/3}$

$$\nabla^2 \left( \frac{\delta p}{\rho_0} \right) - 4\pi G \rho_0 \left( \frac{\delta p}{\rho_0} \right) = \frac{\ddot{\delta S}}{\rho_0}$$

$\delta S = 0$  choose

$$\ddot{\delta}_k + 2H \dot{\delta}_k + \left( \frac{c_s^2 k^2}{a^2} - 4\pi G \rho_0 \right) \delta_k = 0$$

$$\frac{2}{3} \left( -\frac{1}{3} \right) \delta_k + 2 \left( \frac{2}{3} \right) \frac{\delta_k}{a} - \frac{3}{2} \left( \frac{2}{3} \right) \frac{\delta_k}{a^2}$$

if  $\frac{c_s^2 k^2}{a^2} \ll 4\pi G \rho_0 \approx \frac{3}{2} H^2$

$$\frac{8\pi G \rho_0}{3} = H^2$$

then there is runaway solution