Title: PSI 2015/2016 Explorations in Condensed Matter - Guifre Vidal - 14

Date: Apr 08, 2016 10:15 AM

URL: http://pirsa.org/16040011

Abstract:

Explorations in Condensed Matter – PSI 2016

Lecture 14 (of 14)

Tensor Networks in

(i) D>1 spatial dimensions

(ii) Statistical Mechanics (partition functions)

(iii) Holography

Guifre Vidal, Perimeter Institute

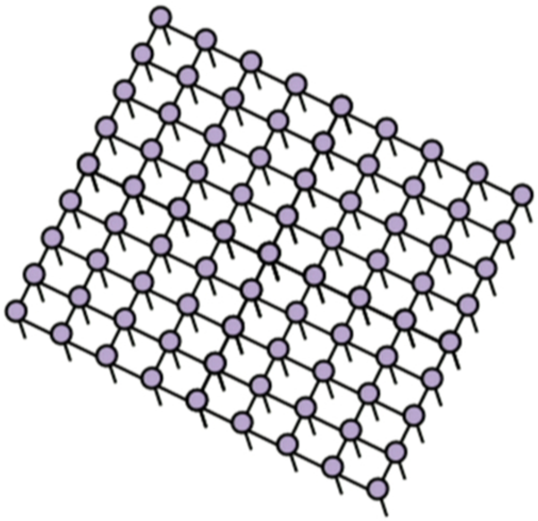
(i) Tensor Networks in D>1 spatial dimensions

area law

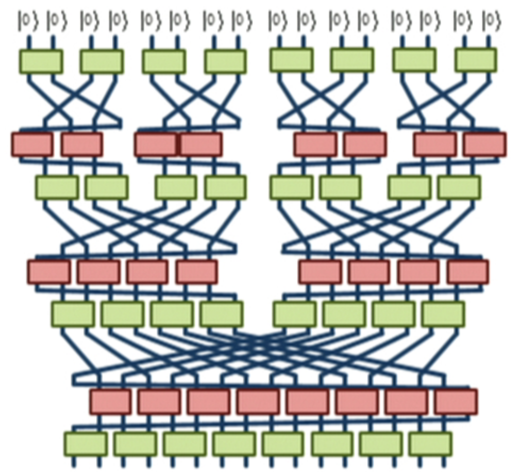
$$S(L) = L^{D-1}$$

logarithmic correction

$$S(L) = L^{D-1}\log(L)$$



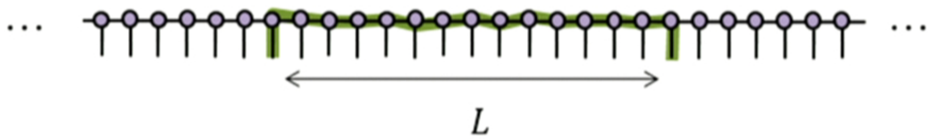
projected entangled pair states
(PEPS)



branching MERA

CORRELATIONS and DISTANCE

matrix product state (MPS)

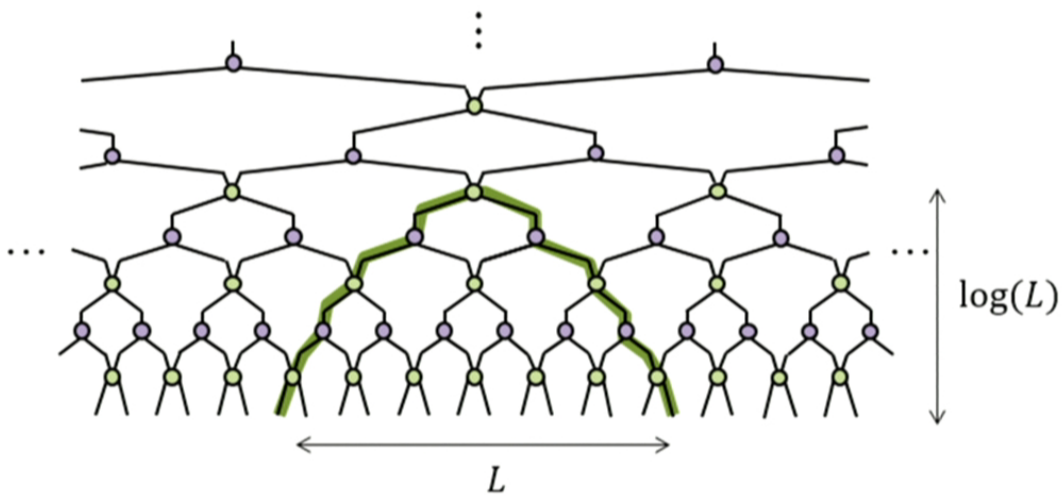


exponential correlators

$C(L) \approx e^{-L/\xi}$

$[C(L) \approx \lambda^L]$

multi-scale entanglement renormalization ansatz (MERA)



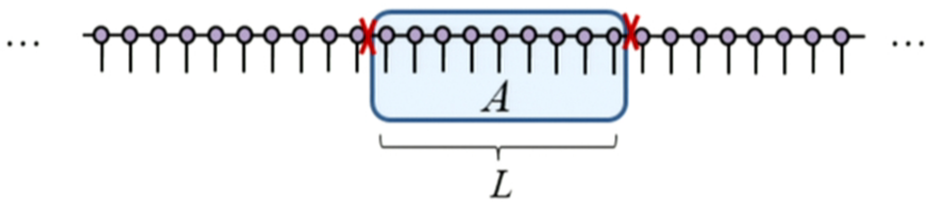
polynomial correlators

$C(L) \approx L^{-p}$

$[C(L) \approx \lambda^{\log(L)}]$

ENTANGLEMENT ENTROPY and CONNECTIVITY

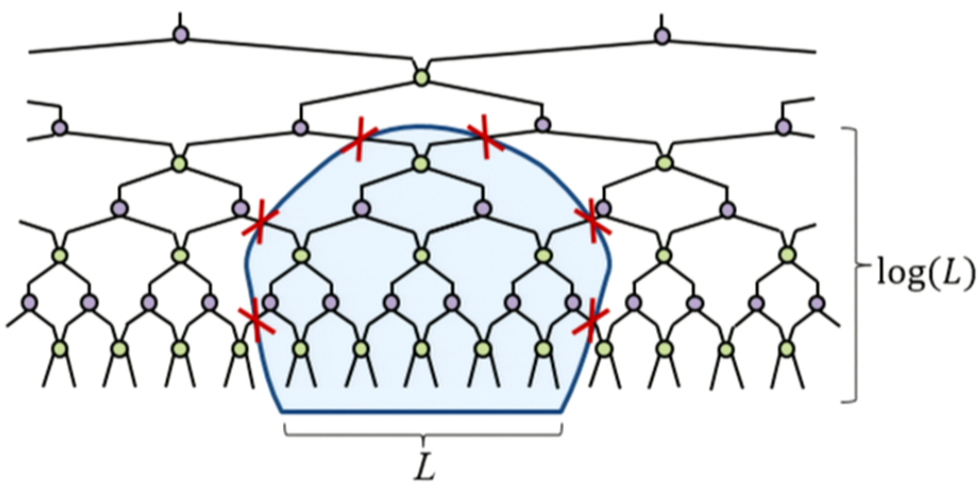
matrix product state (MPS)



constant entropy

$S_L \approx const$

multi-scale entanglement renormalization ansatz (MERA)



logarithmic entropy

$S_L \approx \log(L)$

D=1 spatial dimensions

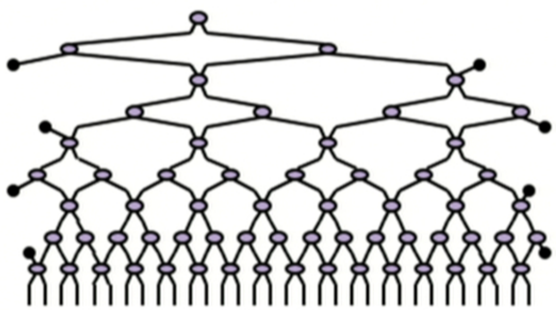
matrix product state
(MPS)

$C(L) \approx e^{-L/\xi}$

$S_L \approx const \ \ (= L^{D-1})$

(gapped systems)

multi-scale entanglement renormalization ansatz
(MERA)



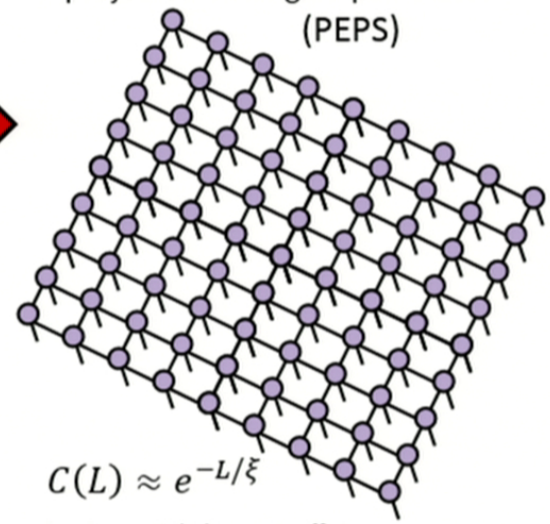
$C(L) \approx L^{-p}$

$S_L \approx \log L \quad (= L^{D-1} \log L)$

(critical systems)

D=2 spatial dimensions

projected entangled pair states
(PEPS)



$C(L) \approx e^{-L/\xi}$

$[\ also\ C(L) \approx L^{-p}\ ]$

$S_L \approx L \quad (= L^{D-1})$

2D MERA

$C(L) \approx L^{-p}$

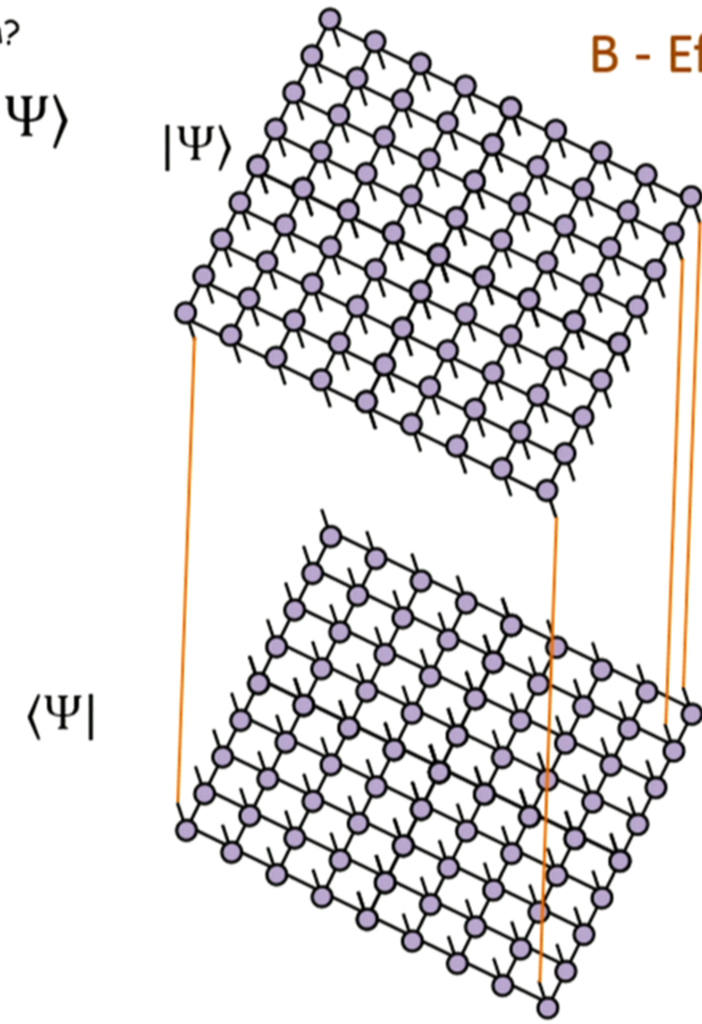
$S_L \approx L \quad (= L^{D-1})$

Projected entangled pair states (PEPS)

Norm?

$\langle\Psi|\Psi\rangle$

B - Efficient manipulation?

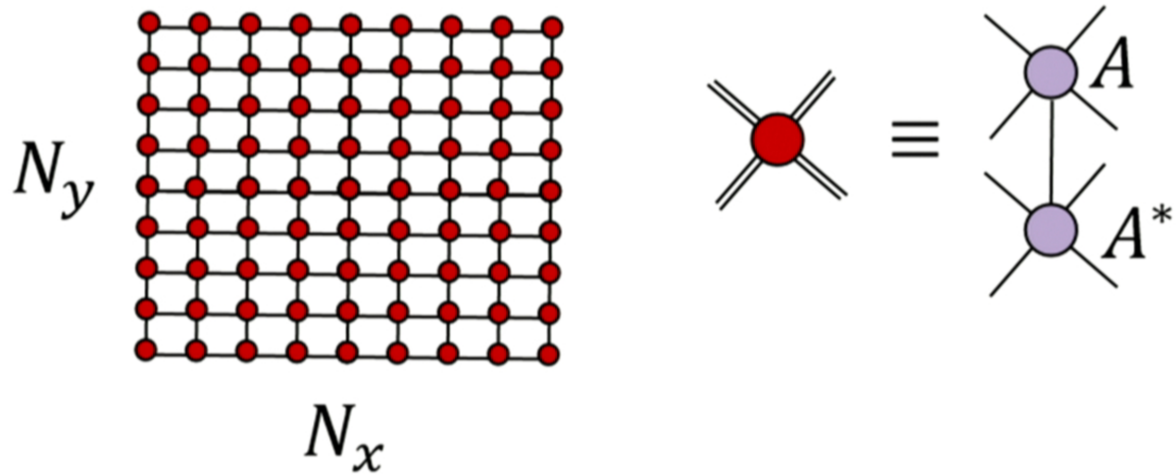


Projected entangled pair states (PEPS)

Norm?

B - Efficient manipulation?

$\langle\Psi|\Psi\rangle$



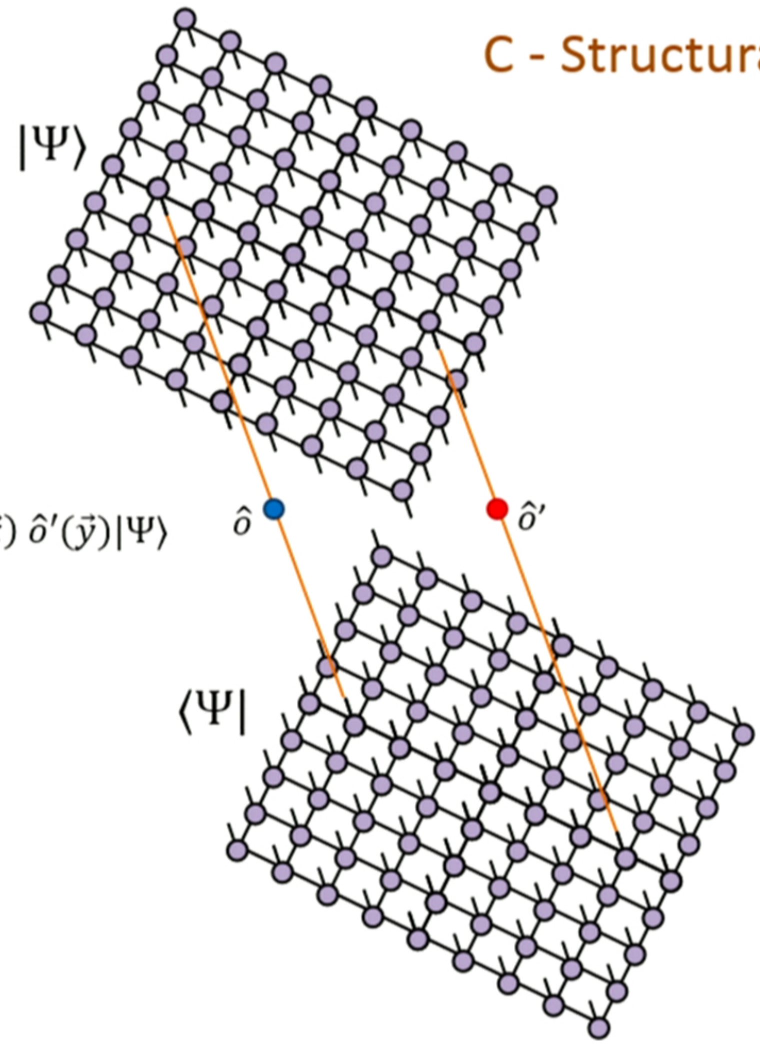
Cost exact contraction $\exp(N_y)$ (if $N_y < N_x$)

Cost approximate contraction $O(N_x N_y)$

Projected entangled pair states (PEPS)

Correlations:

C - Structural properties?



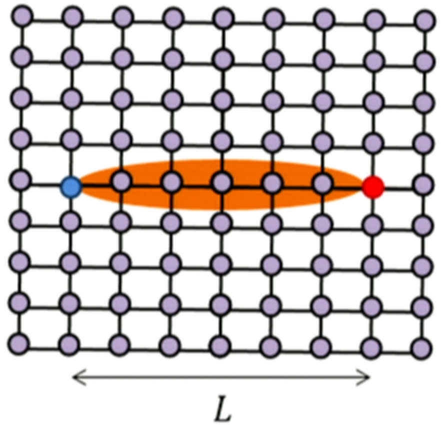
$\langle\Psi|\hat{o}(\vec{x})\ \hat{o}'(\vec{y})|\Psi\rangle$

Projected entangled pair states (PEPS)

Correlations:

$\langle\Psi|\hat{o}(\vec{x})\ \hat{o}'(\vec{y})|\Psi\rangle$

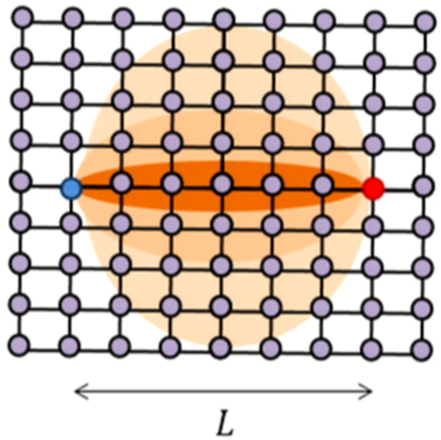
C - Structural properties?



exponential correlations

$C(L) \approx e^{-L/\xi}$

(generic)



polynomial correlations

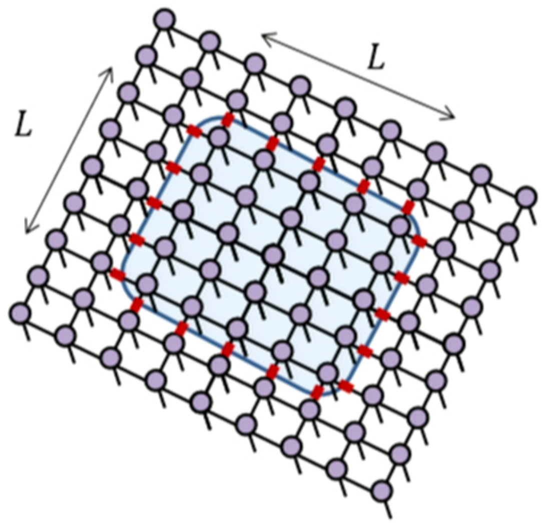
$C(L) \approx L^{-p}$

(fine-tuned)

Projected entangled pair states (PEPS)

Entangled entropy:

C - Structural properties?



boundary law for entanglement entropy

$$S_L \leq 4L \log_2(\chi) \quad (= L^{D-1})$$

D=1 spatial dimensions

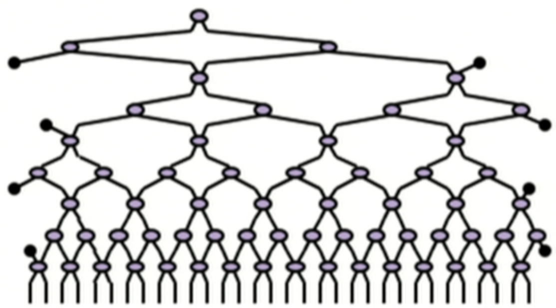
matrix product state
(MPS)

$C(L) \approx e^{-L/\xi}$

$S_L \approx const \ \ (= L^{D-1})$

(gapped systems)

multi-scale entanglement renormalization ansatz
(MERA)



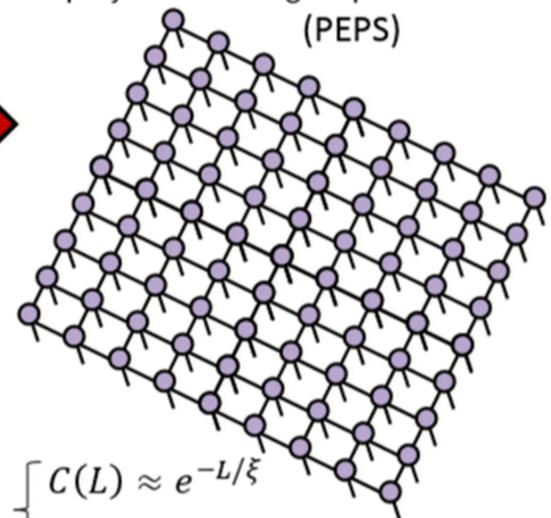
$C(L) \approx L^{-p}$

$S_L \approx \log L \quad (= L^{D-1} \log L)$

(critical systems)

D=2 spatial dimensions

projected entangled pair states
(PEPS)



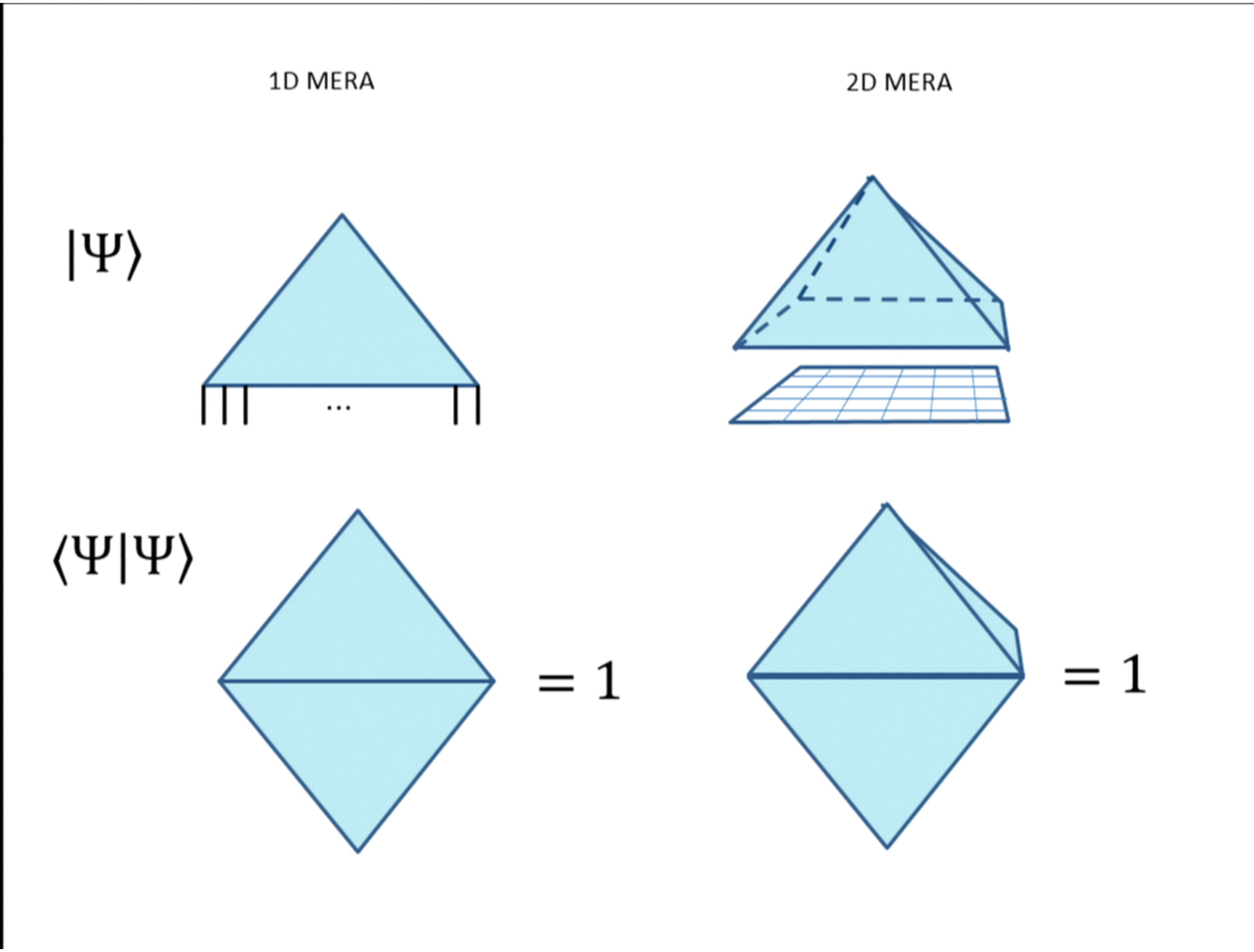
$$\begin{cases} C(L) \approx e^{-L/\xi} \\ C(L) \approx L^{-p} \end{cases}$$

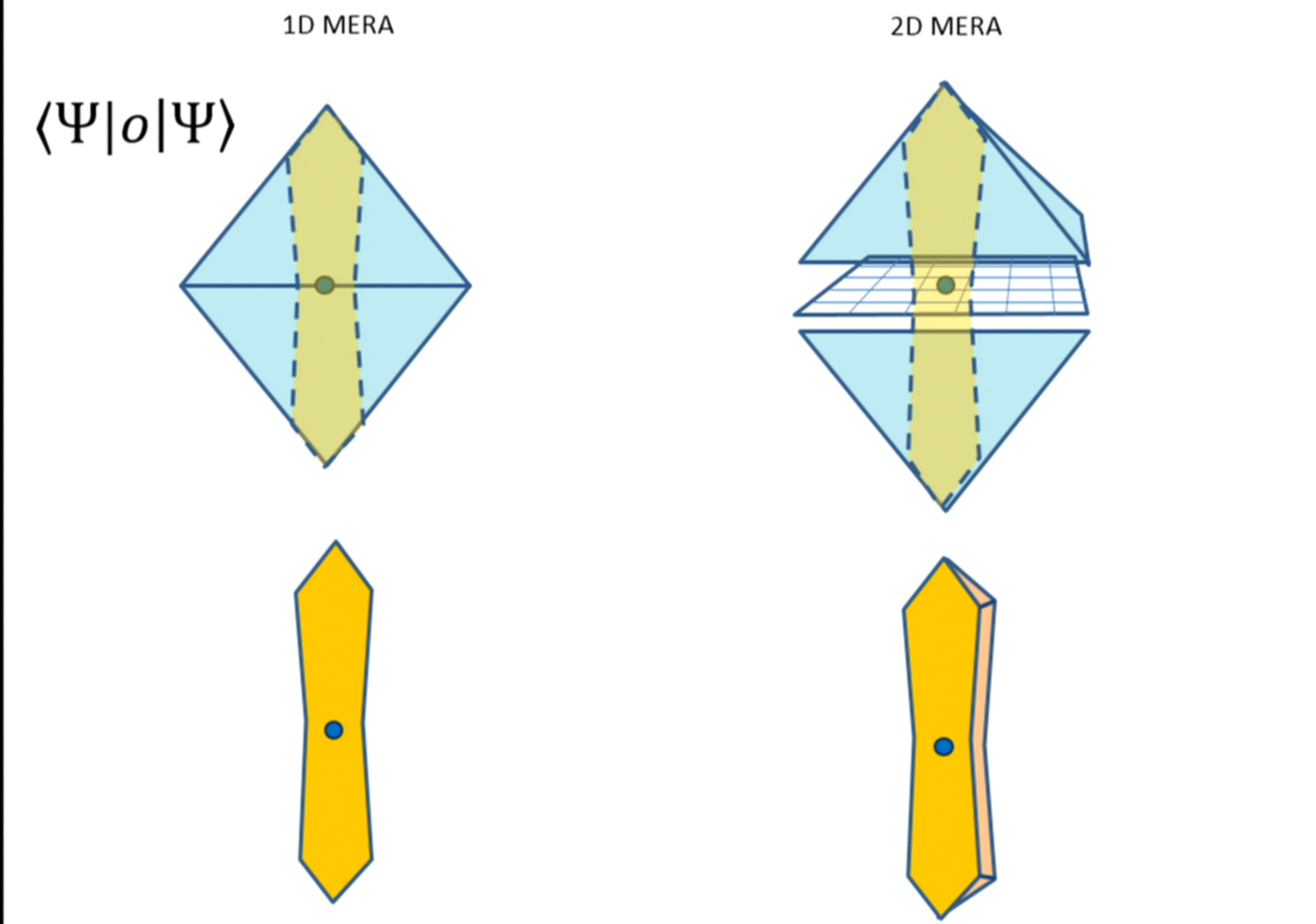
$S_L \approx L \quad (= L^{D-1})$

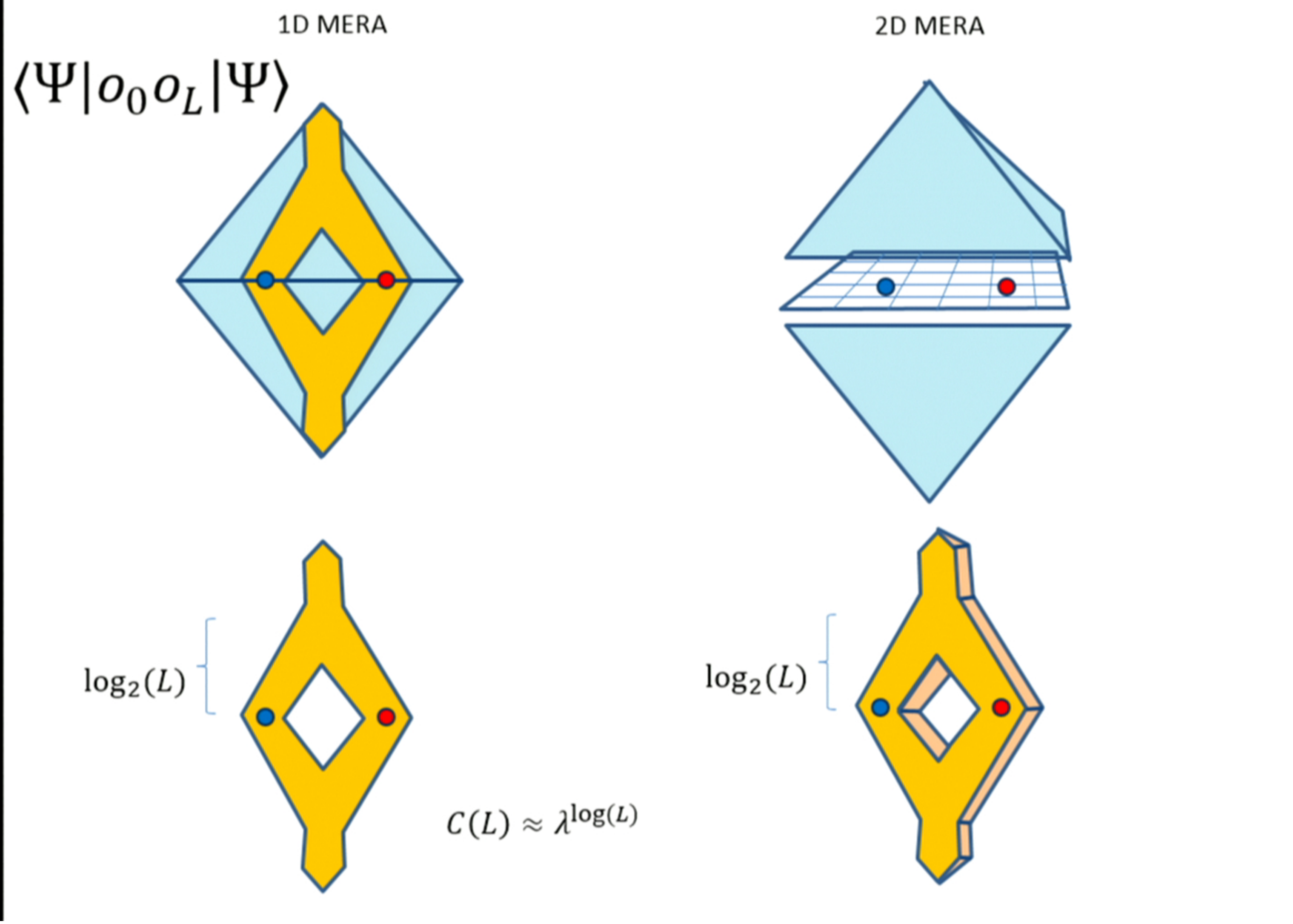
2D MERA

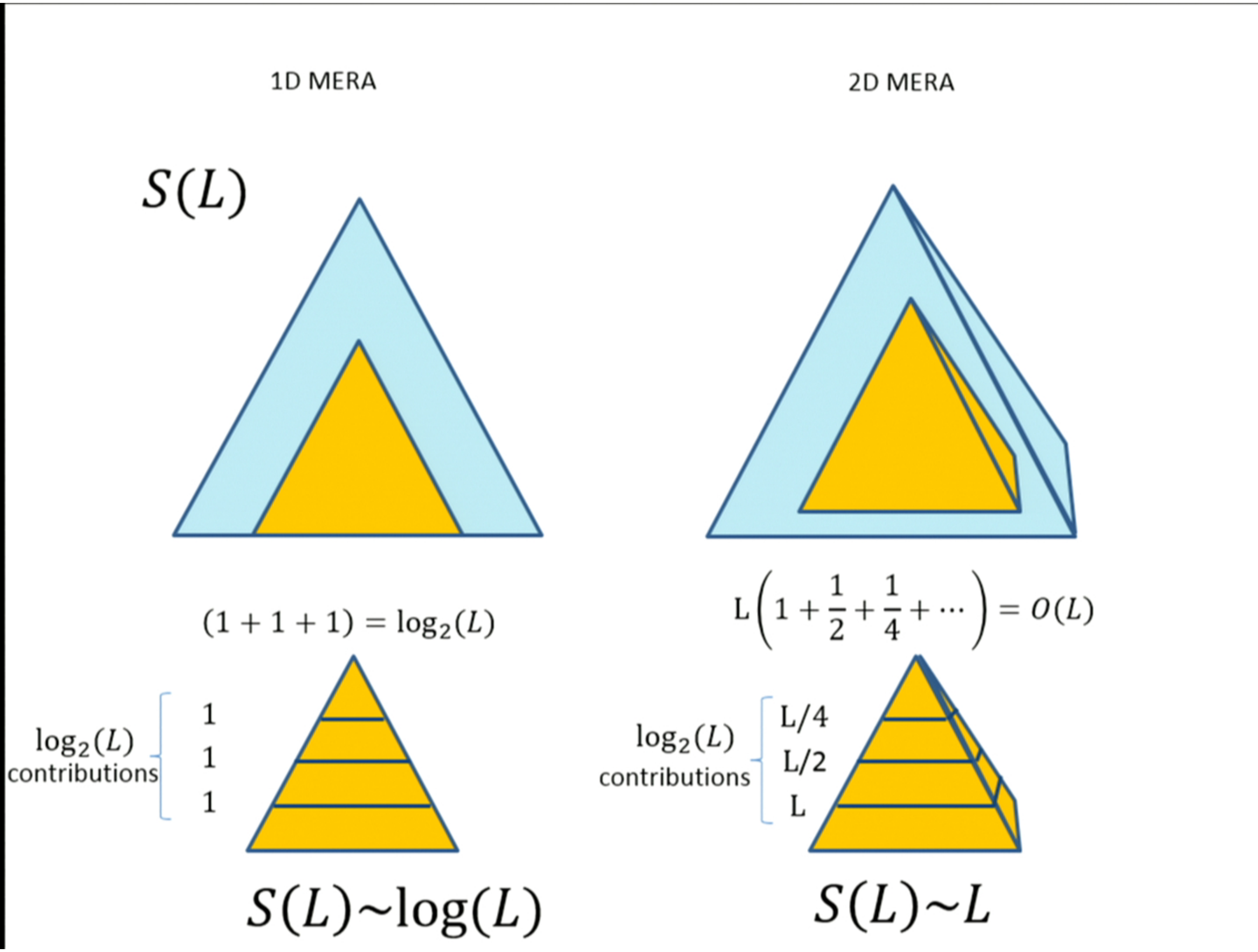
$C(L) \approx L^{-p}$

$S_L \approx L \quad (= L^{D-1})$


1D MERA
2D MERA
$|\Psi\rangle$
$\langle\Psi|\Psi\rangle$
$= 1$
$= 1$

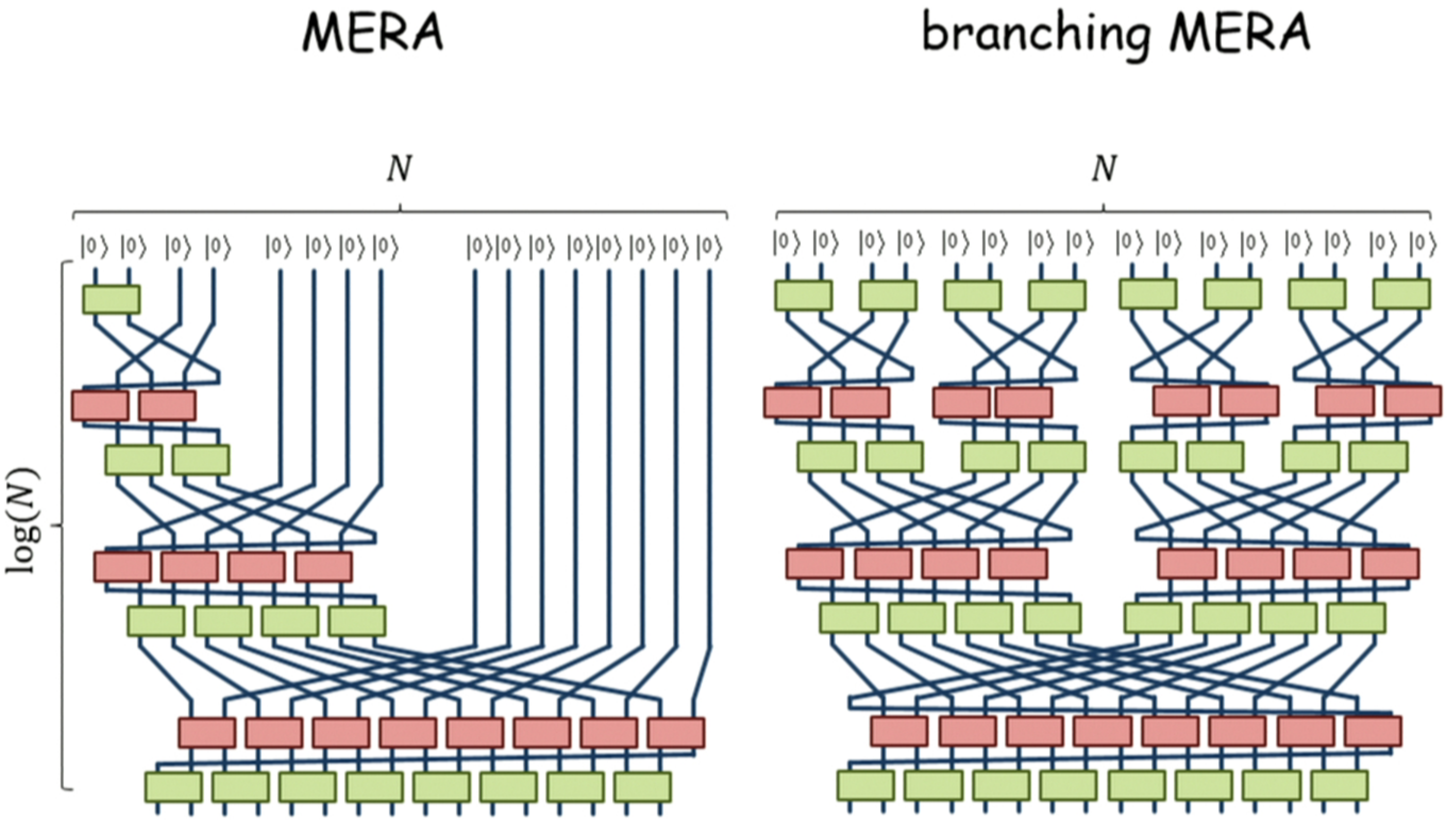

1D MERA
2D MERA
$\langle\Psi|o|\Psi\rangle$

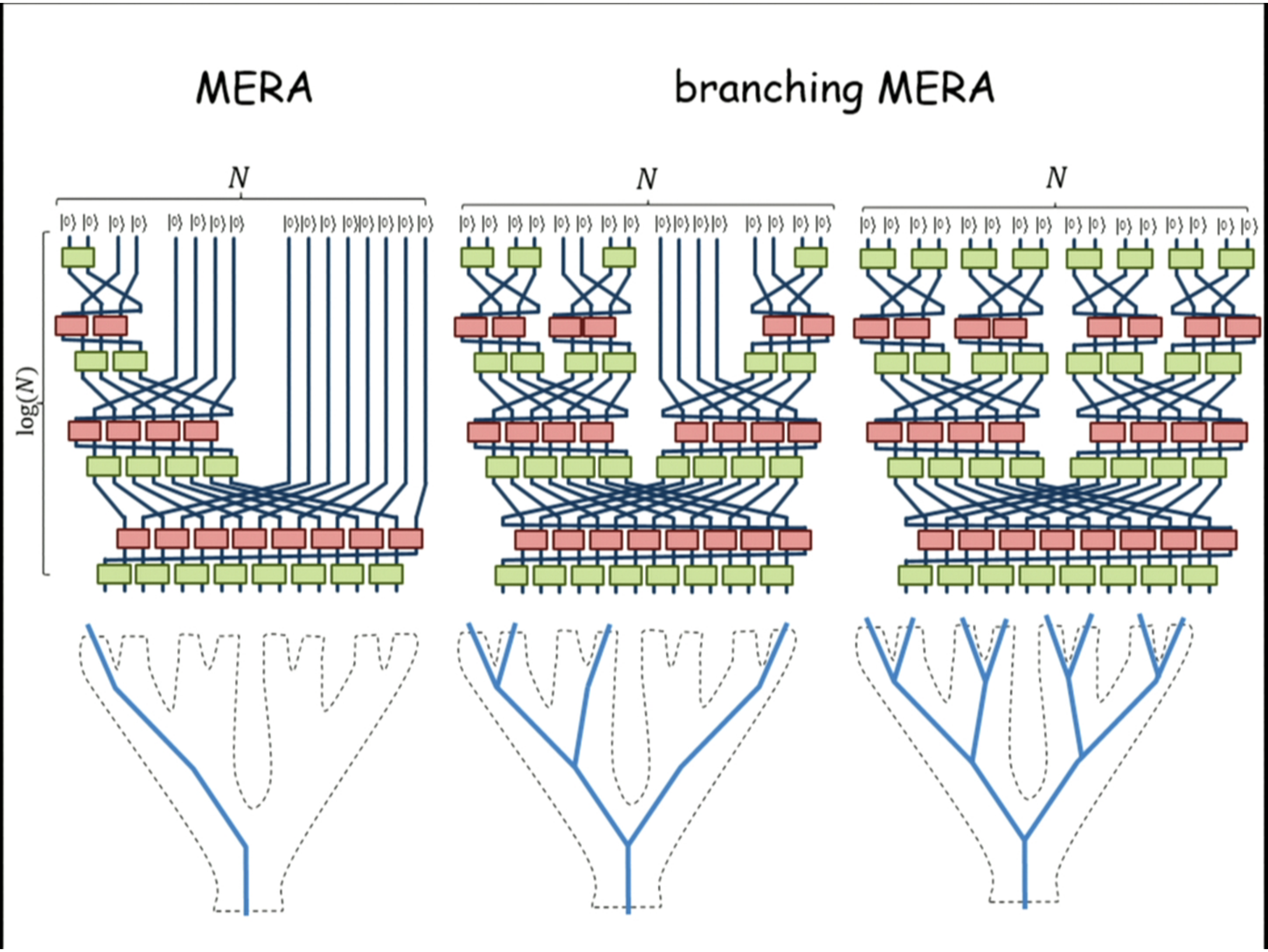

1D MERA
2D MERA
$\langle\Psi|o_0 o_L|\Psi\rangle$
$\log_2(L)$
$\log_2(L)$
$C(L) \approx \lambda^{\log(L)}$



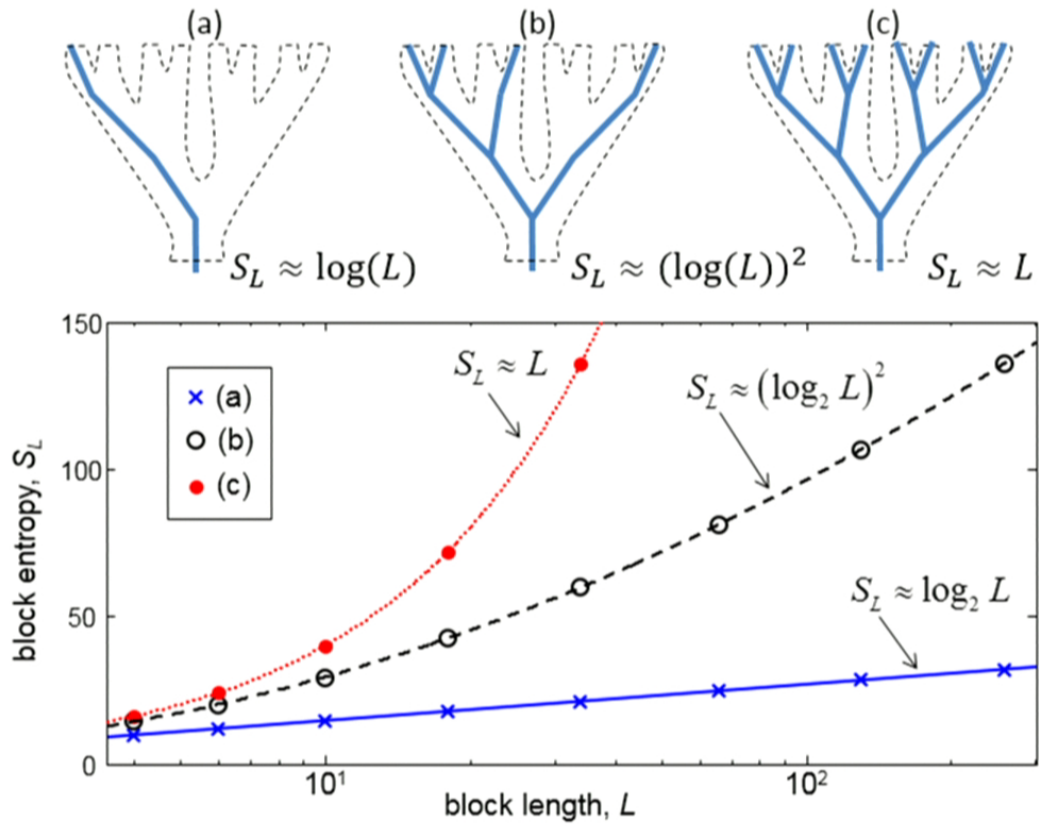
1D MERA
2D MERA
$S(L)$
$(1+1+1) = \log_2(L)$
$L\left(1+\frac{1}{2}+\frac{1}{4}+\cdots\right) = O(L)$
$\log_2(L)$ contributions
1
1
1
$\log_2(L)$ contributions
L/4
L/2
L
$S(L) \sim \log(L)$
$S(L) \sim L$

Branching MERA

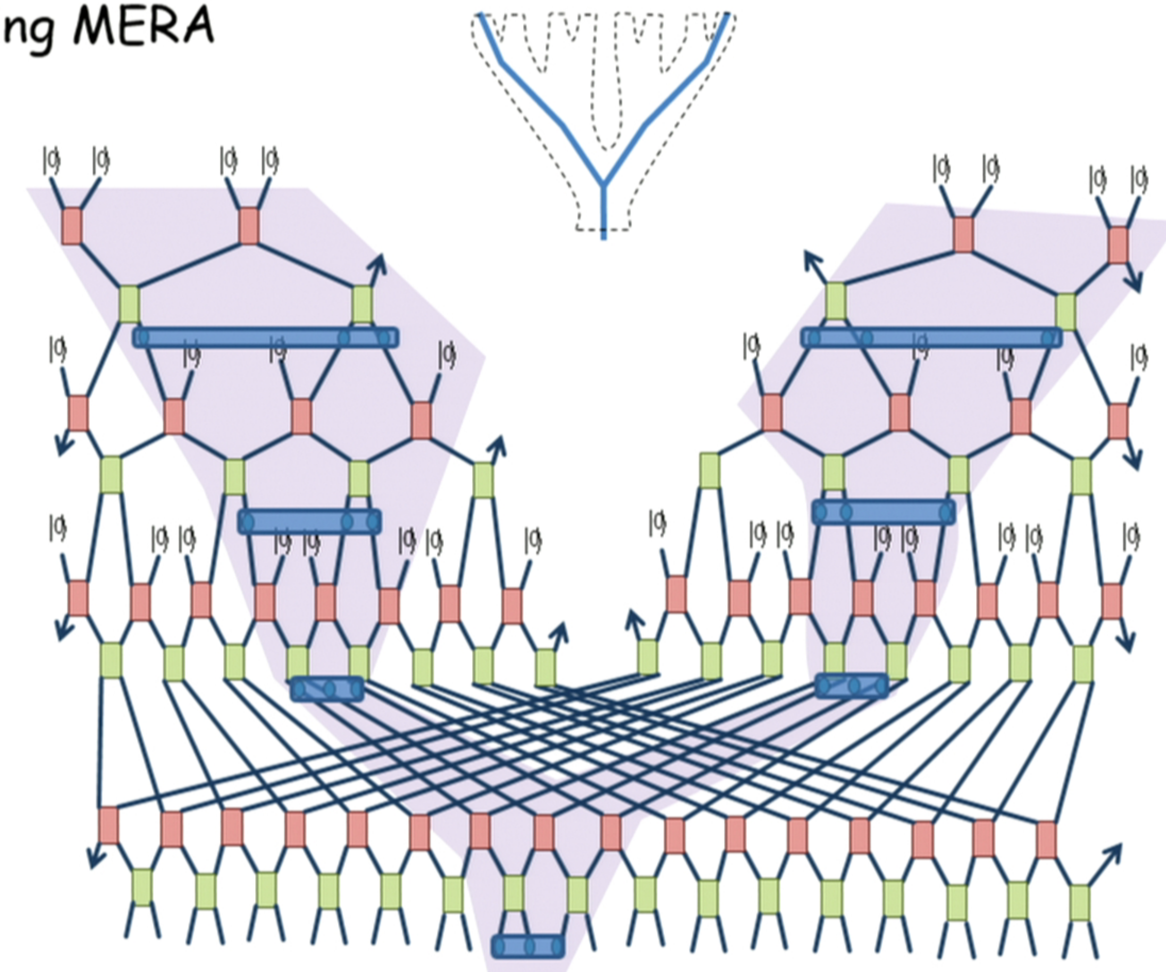



MERA
branching MERA
N
N
N
log(N)

branching MERA



branching MERA

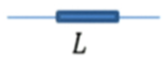
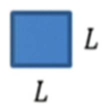
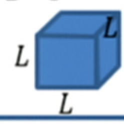


$S_L \approx \log(L) + \log(L)$

branching MERA

D=1 spatial dimensions	$S_L \approx \log(L)$		…		$S_L \approx L$
D>1 spatial dimensions	$S_L \approx L^{D-1}$	…	$S_L \approx L^{D-1}\log(L)$	…	$S_L \approx L^D$

Scaling of entanglement entropy (second week)

Dimension	gapped $\Delta > 0$	gapless $\Delta = 0$ **no** (D-1)- dimensional Fermi surface	gapless $\Delta = 0$ (D-1)- dimensional Fermi surface
D=1 	$S_L \approx const$	N/A	$S_L \approx \log(L)$
D=2 	$S_L \approx L$	$S_L \approx L$	$S_L \approx L \log(L)$
D=3 	$S_L \approx L^2$	$S_L \approx L^2$	$S_L \approx L^2 \log(L)$

Gapped and gapless without a Fermi surface: $S_L \approx L^{D-1}$, boundary law

Gapless with a (D-1)-dimensional Fermi surface: $S_L \approx L^{D-1} \log(L)$, boundary law with logarithmic correction

Tensor networks (third week)

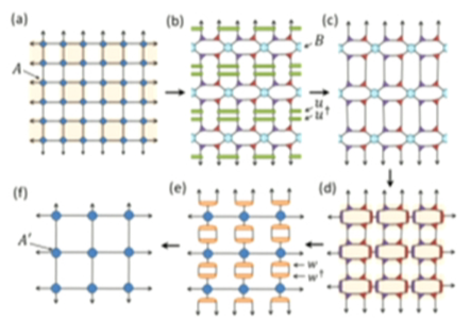
Dimension	gapped $\Delta > 0$	gapless $\Delta = 0$ — **no** (D-1)- dimensional Fermi surface	gapless $\Delta = 0$ — (D-1)- dimensional Fermi surface
D=1  L	$S_L \approx const$ MPS	N/A	$S_L \approx \log(L)$ MERA
D=2  L L	$S_L \approx L$ PEPS	$S_L \approx L$ MERA	$S_L \approx L\log(L)$ branching MERA
D=3  L L L	$S_L \approx L^2$ PEPS	$S_L \approx L^2$ MERA	$S_L \approx L^2\log(L)$ branching MERA

$S_L \approx L^{D-1}$ boundary law (gapped and gapless with no (D-1)-dimensional Fermi surface)

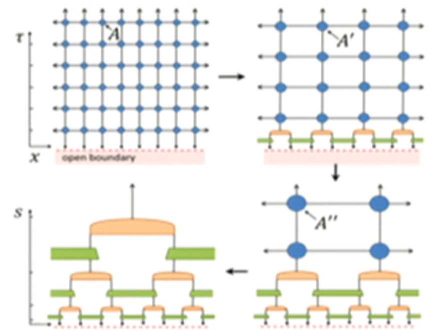
$S_L \approx L^{D-1}\log(L)$ boundary law with logarithmic correction

(ii) Tensor Networks in Statistical Mechanics (partition functions)

RG for statistical partition functions



MERA from Euclidean path integral



Euclidean path integral

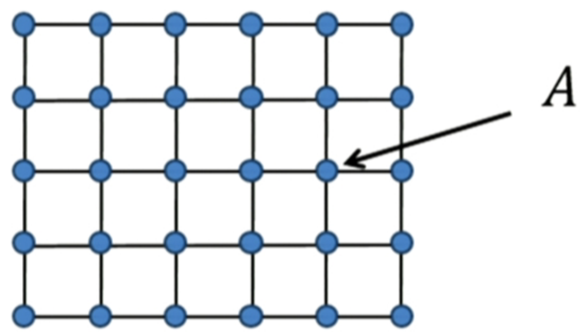
$$Z(\lambda) = tr\; e^{-\beta H_q^{1d}}$$

Statistical partition function

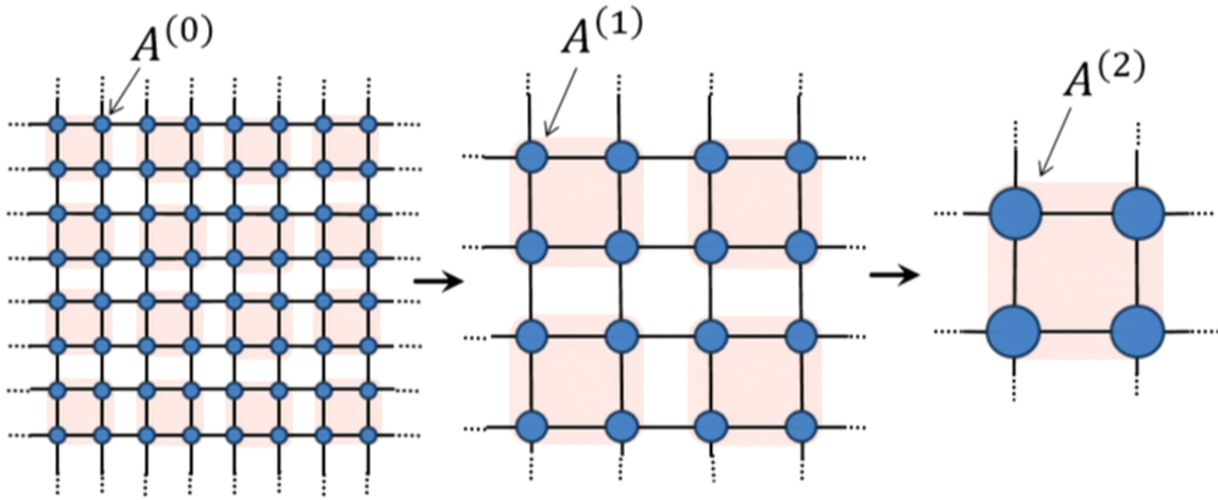
$$Z(T) = \sum_{\{s\}} e^{-\frac{1}{T} H_{cl}^{2d}}$$

as a tensor network

$$Z =$$



Goal: define an RG flow in the space of tensor networks

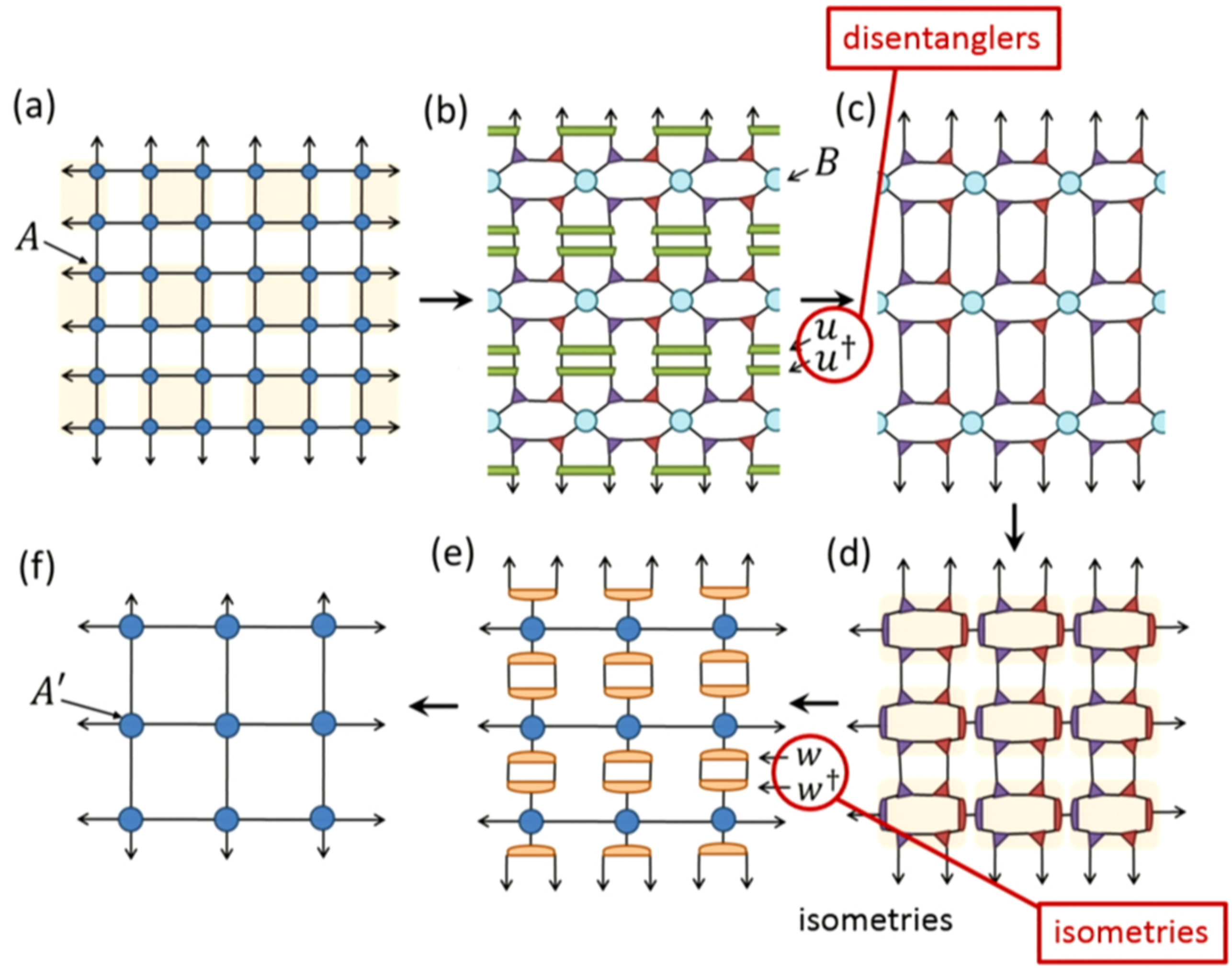


Many proposals, including

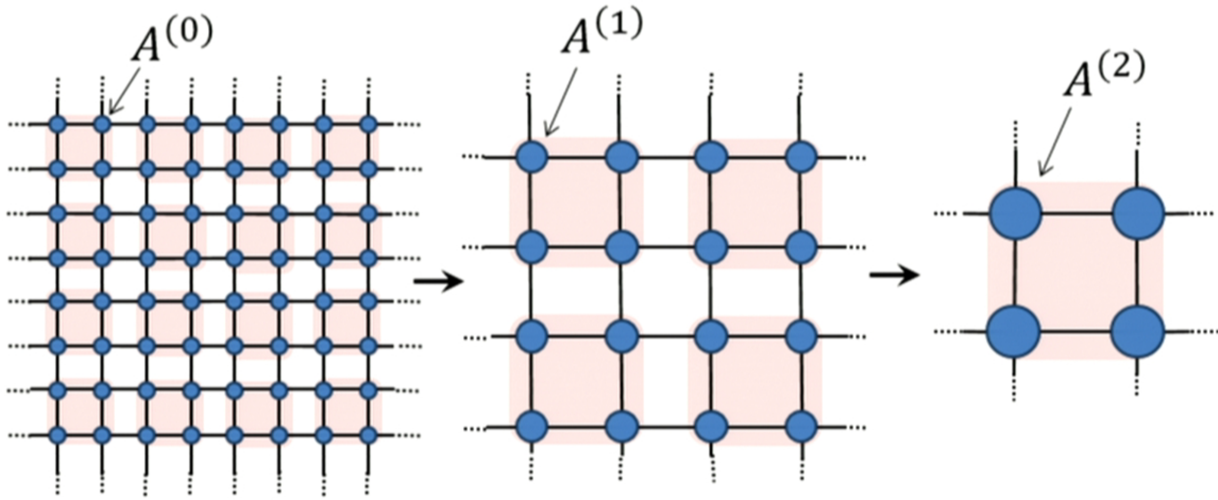
Tensor Renormalization Group (TRG) Levin-Nave, 2006

Tensor Network Renormalization (TNR)

[with Glen Evenbly, 2015 !]



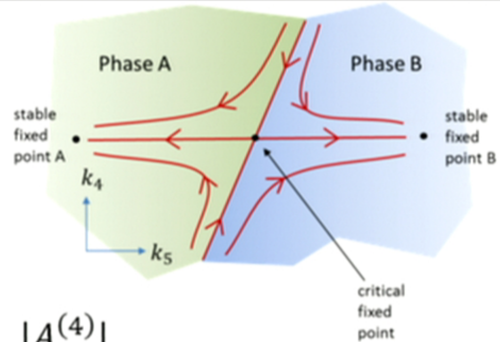
Net result: RG flow in the space of tensor networks



$$A^{(0)} \to A^{(1)} \to A^{(2)} \to \cdots \to A^{fp}$$

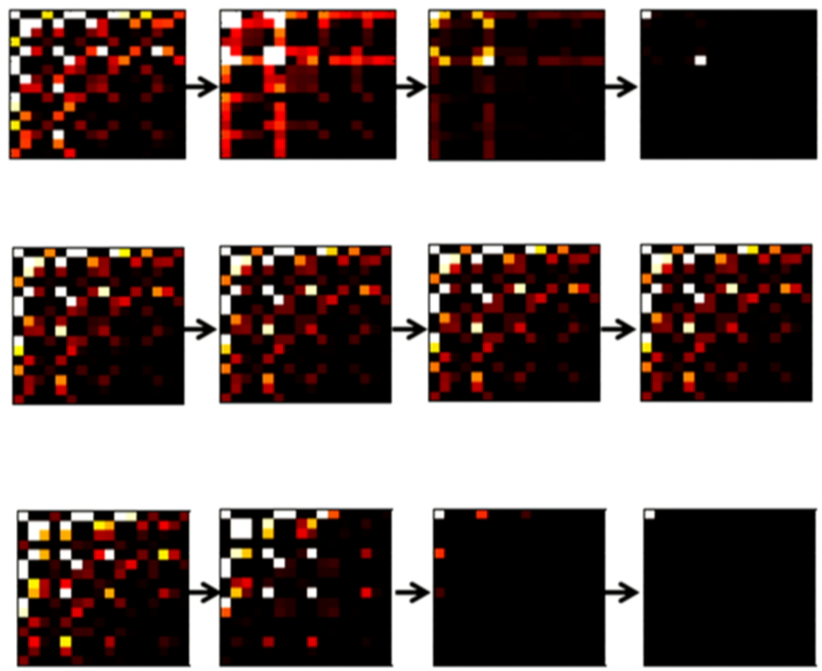
Universal information of the phase or phase transition

TNR -> proper RG flow
Example: 2D classical Ising



$$A^{(0)} \rightarrow A^{(1)} \rightarrow A^{(2)} \rightarrow \cdots \rightarrow A^{fp}$$

$|A^{(1)}|$ $|A^{(2)}|$ $|A^{(3)}|$ $|A^{(4)}|$



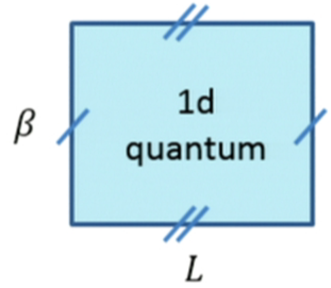
below critical $T = 0.9\,T_c$ → ordered (Z2) fixed point

critical $T = T_c$ → critical fixed point

above critical $T = 1.1\,T_c$ → disordered (trivial) fixed point

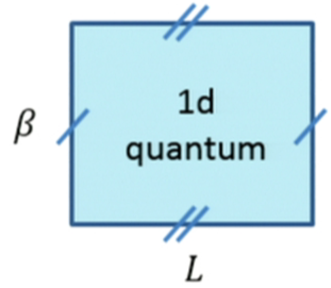
Euclidean path integral

$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$

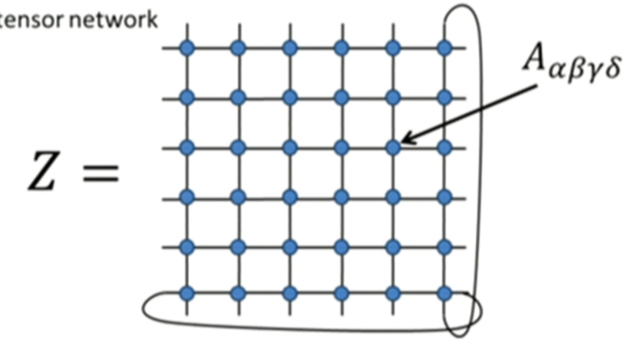


Euclidean path integral

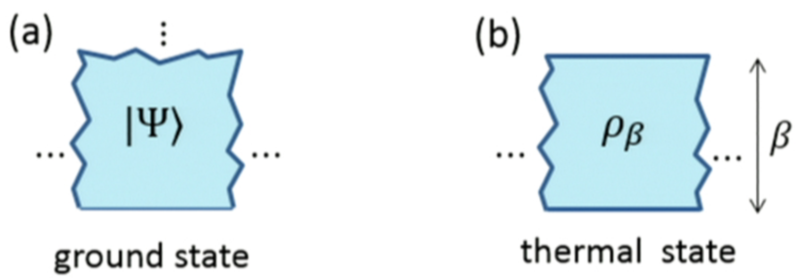
$$Z(\lambda) = tr\, e^{-\beta H_q^{1d}}$$



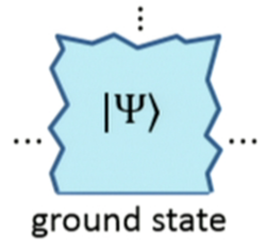
as a tensor network



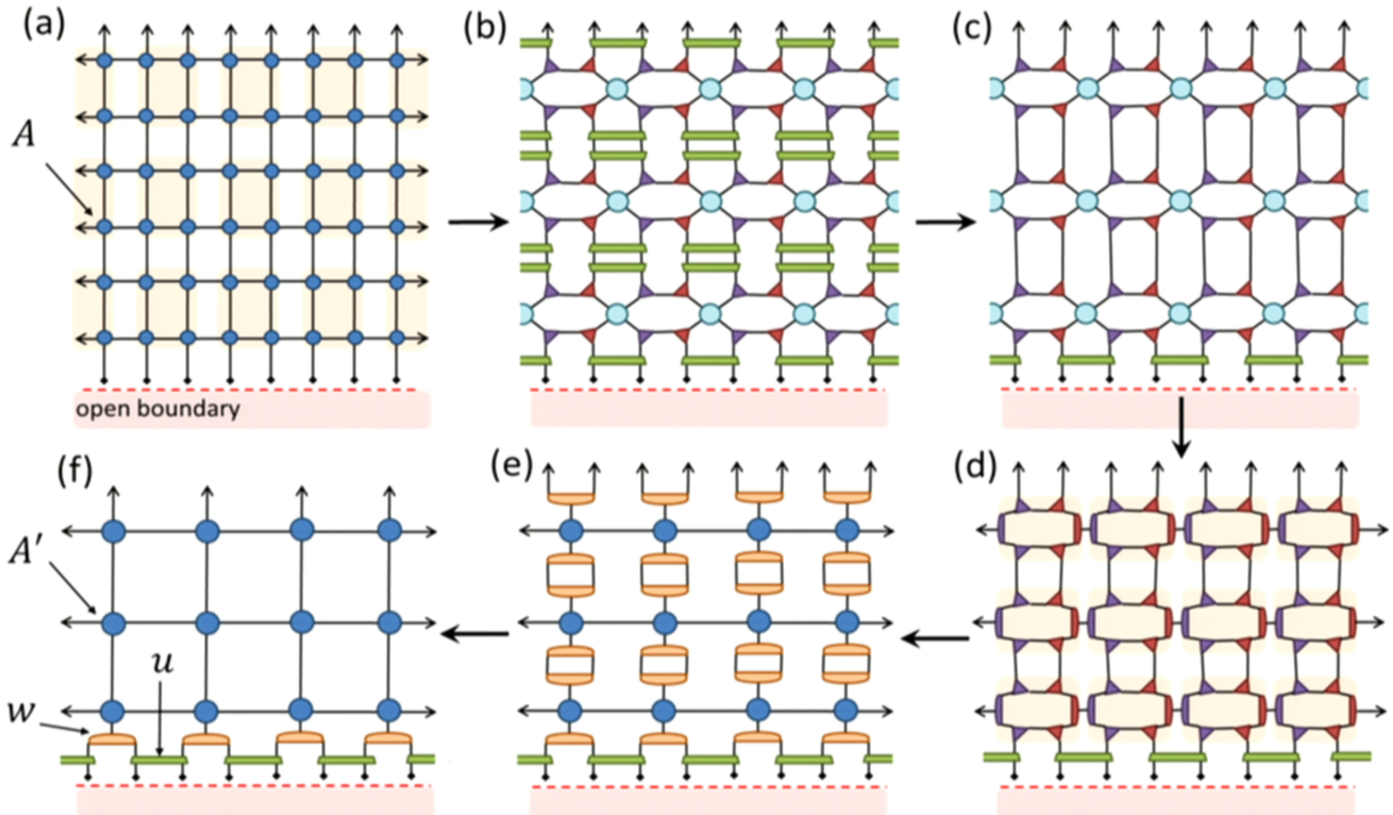
Euclidean time evolution on different geometries



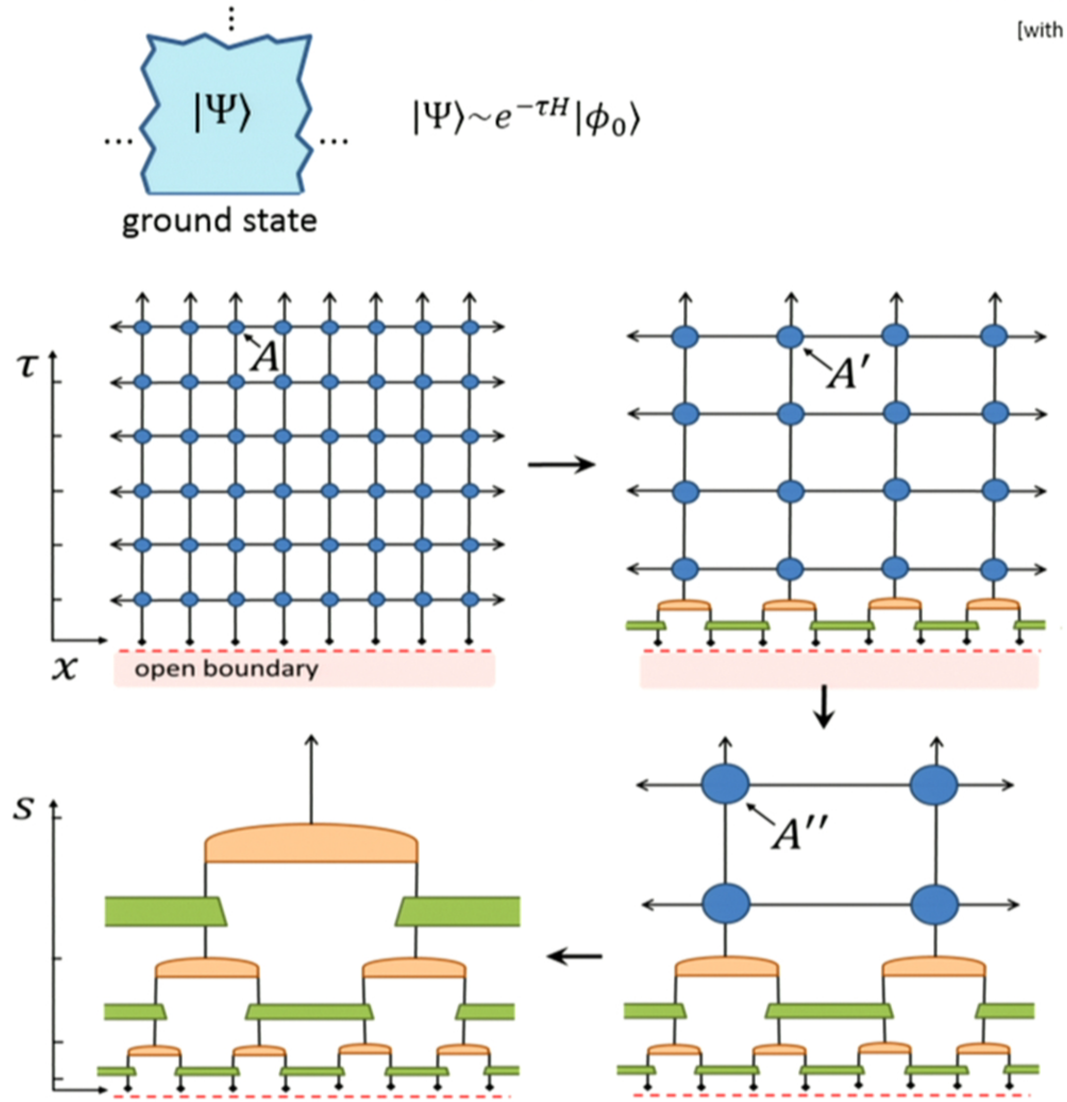
[with Glen Evenbly, 2015]



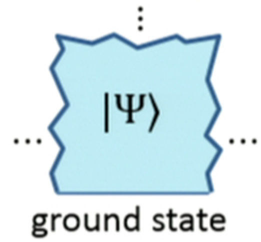
$|\Psi\rangle \sim e^{-\tau H}|\phi_0\rangle$



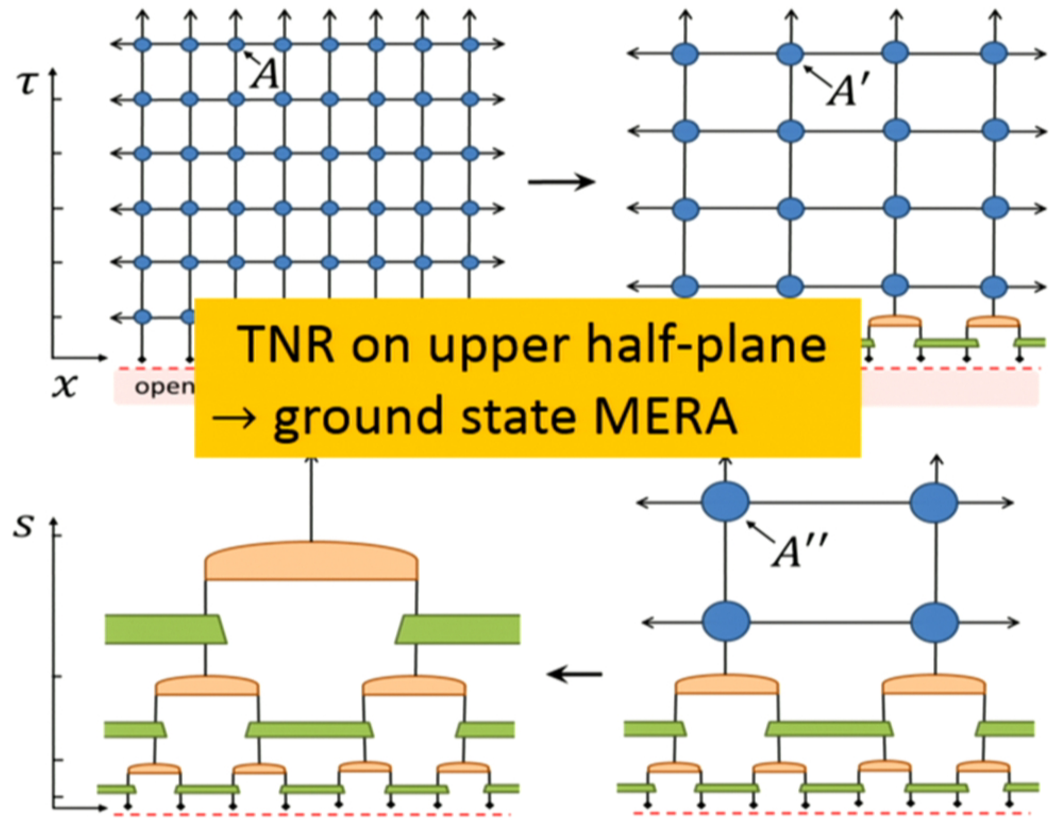
[with Glen Evenbly, 2015]



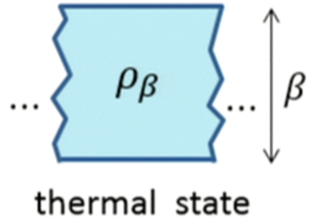
[with Glen Evenbly, 2015]



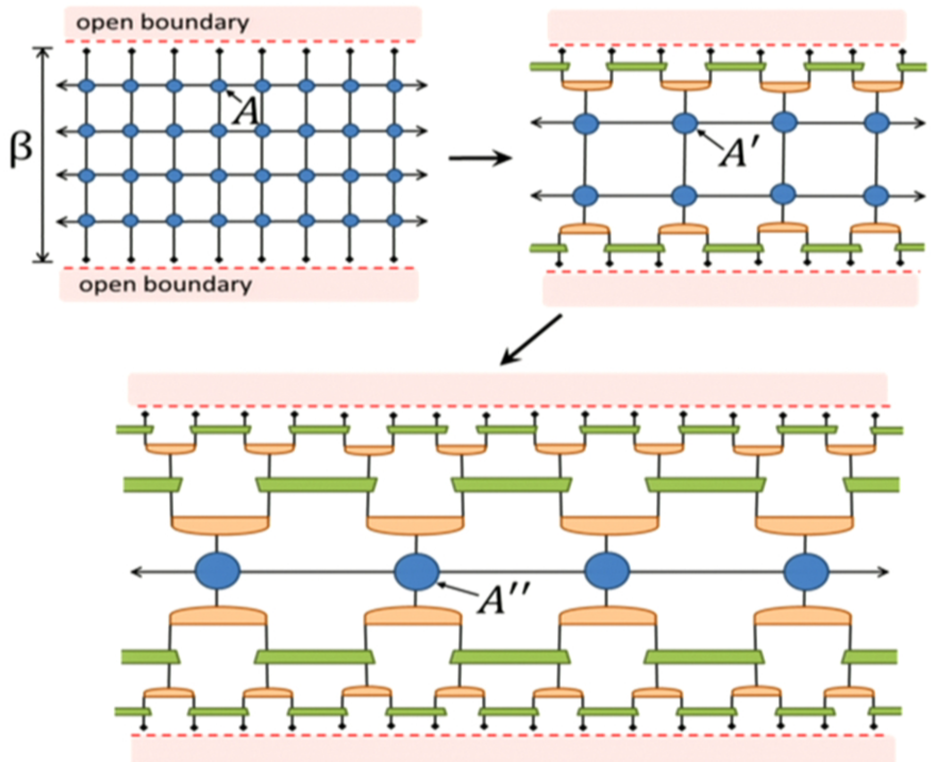
$|\Psi\rangle \sim e^{-\tau H}|\phi_0\rangle$

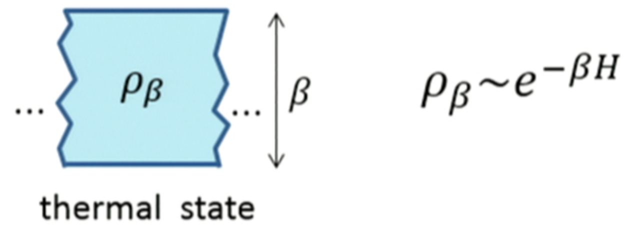


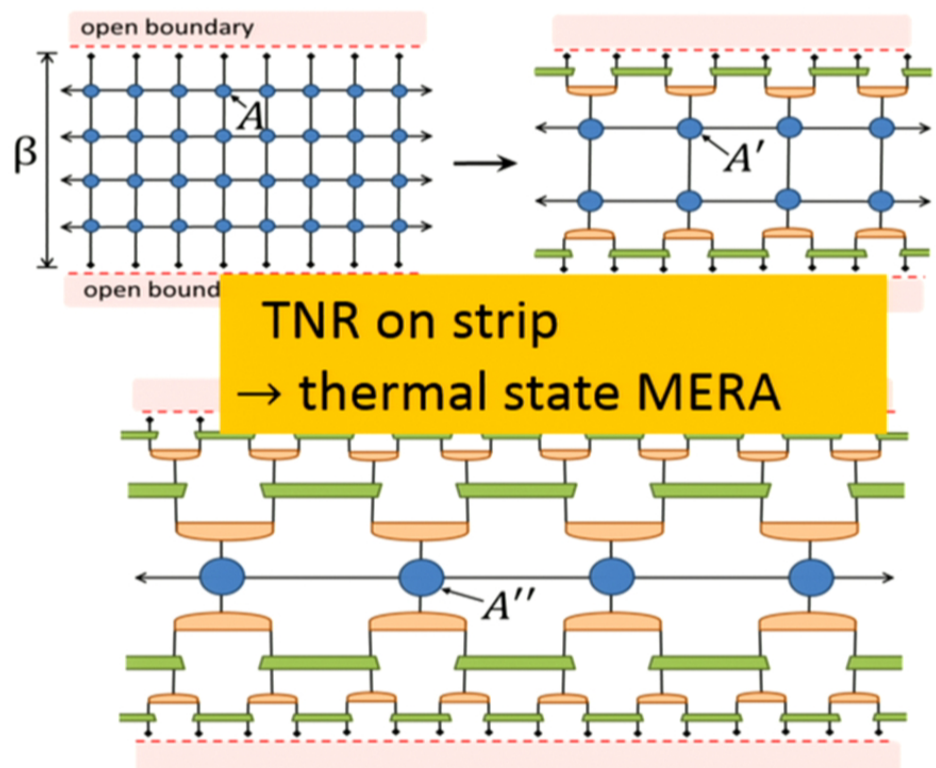
TNR on upper half-plane
→ ground state MERA



$\rho_\beta \sim e^{-\beta H}$



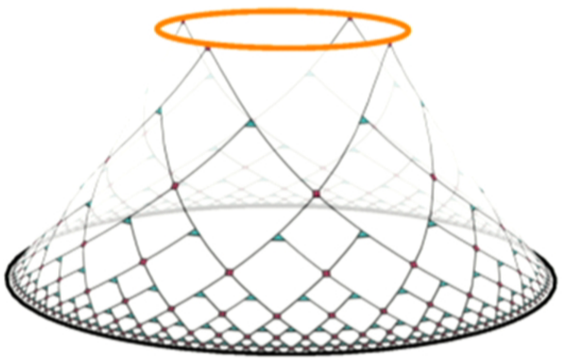




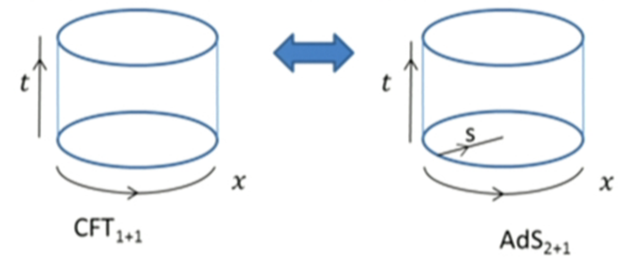
TNR on strip
→ thermal state MERA

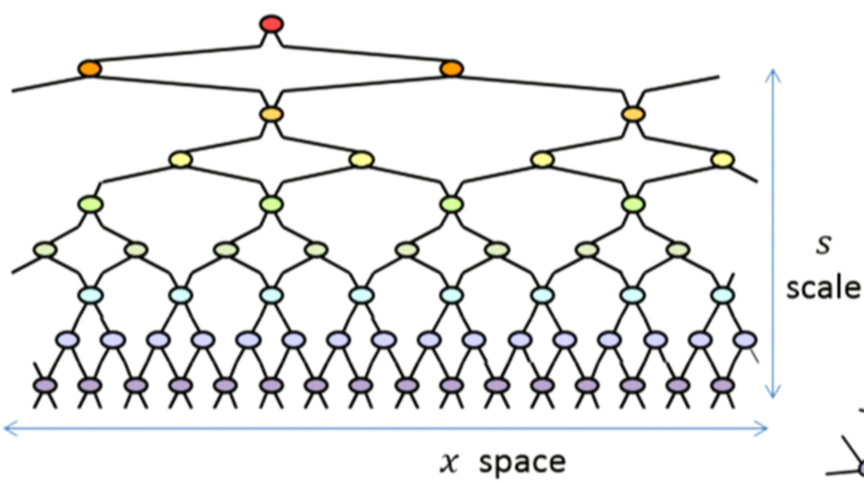
(iii) Tensor Networks in Holography

Toy models for holography/
emergent space-time



MERA and holography?





- entanglement entropy

$$S_L \approx \log(L)$$

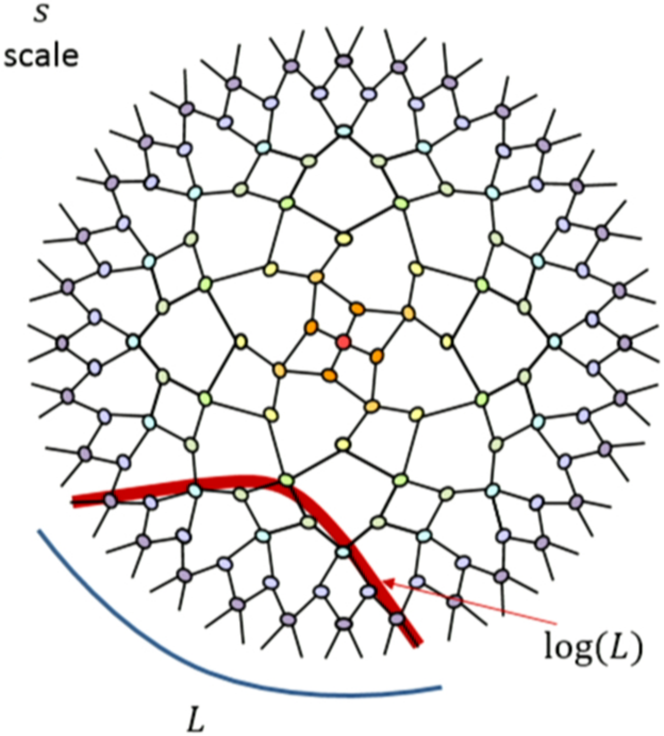
parallel to area of minimal surface in Ryu-Takayanagi

- two-point correlations

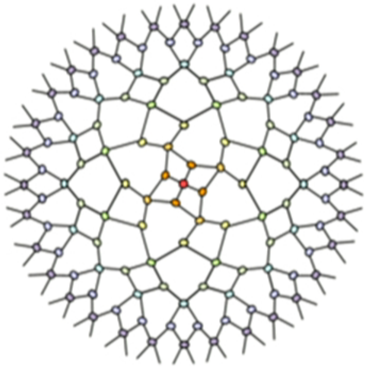
$$C(L) \approx L^{-2\Delta}$$

geodesic distance $D \approx \log(L)$ as in a hyperbolic geometry

$$C(L) \approx e^{-D} = e^{-2\Delta \log(L)} = L^{-2\Delta}$$



MERA and holography?

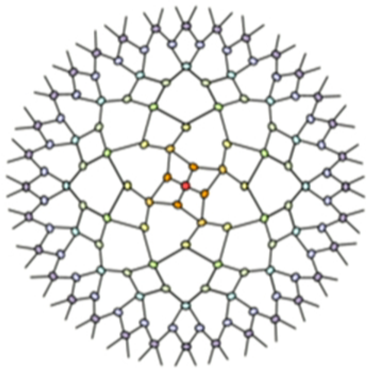


MERA
(2005)

MERA ↔ AdS/CFT

Swingle, 2009

MERA and holography?



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"Entanglement renormalization for quantum fields"
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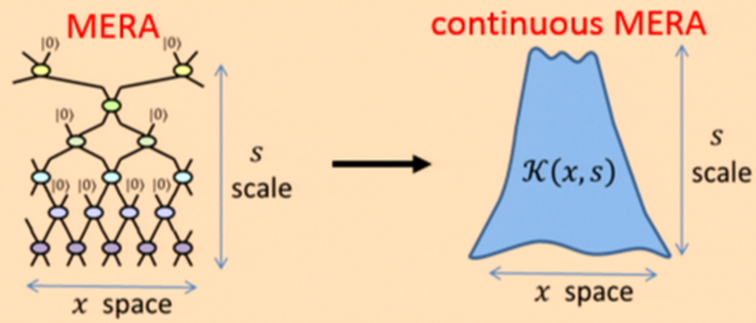

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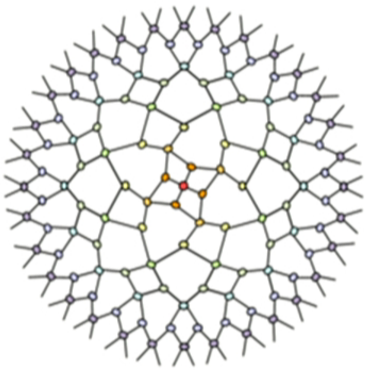

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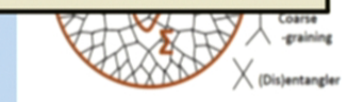
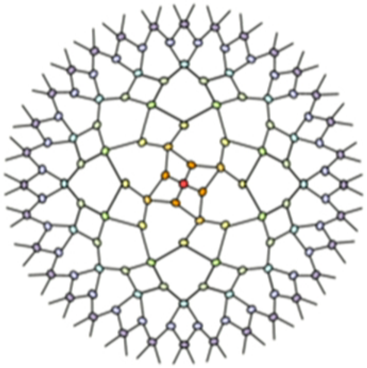
Hartman, Maldacena, 2013

thermal state + time evolution

“Surface/State Correspondence
as a Generalized Holography”
Miyaji, Takayanagi, 2015



MERA and holography?



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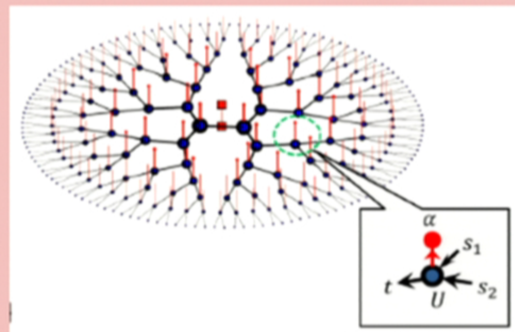
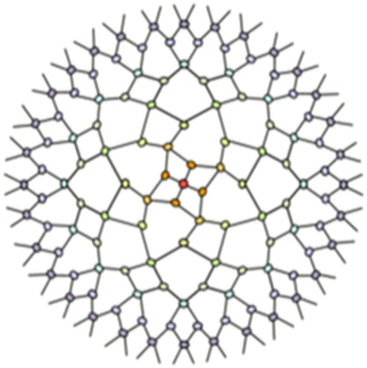
"Exact holographic mapping and emergent space-time geometry"
Xiaoliang Qi, 2013



MERA and holography?



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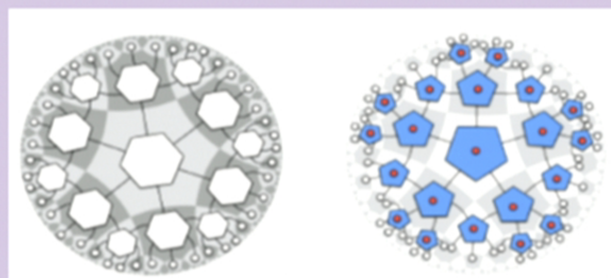
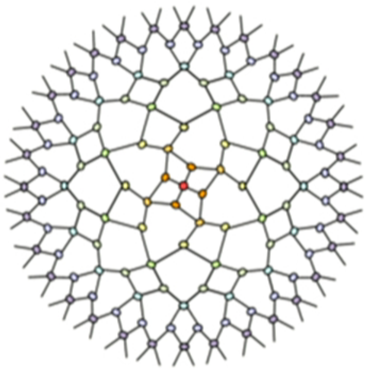


holographic hexagon state holographic pentagon code

MERA and holography?



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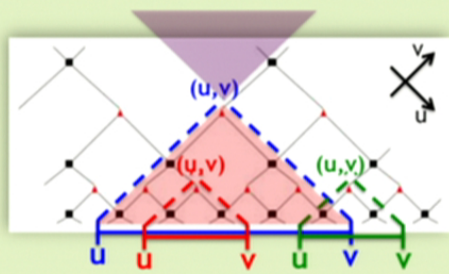
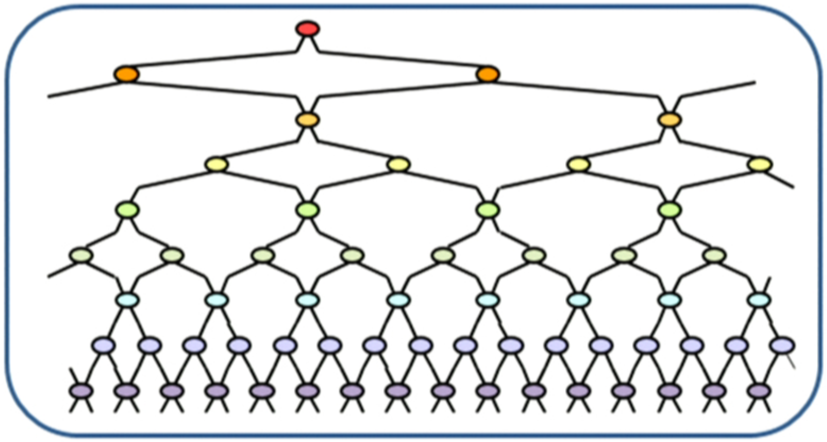
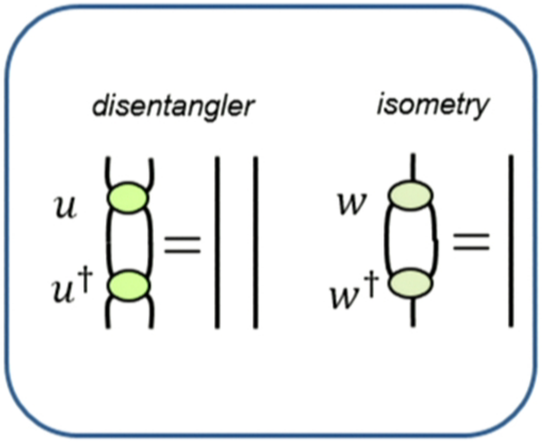
"Integral Geometry and Holography"
Czech, Lamprou, McCandlish, Sully, 2015



MERA = tensor network + isometric/unitary constraints





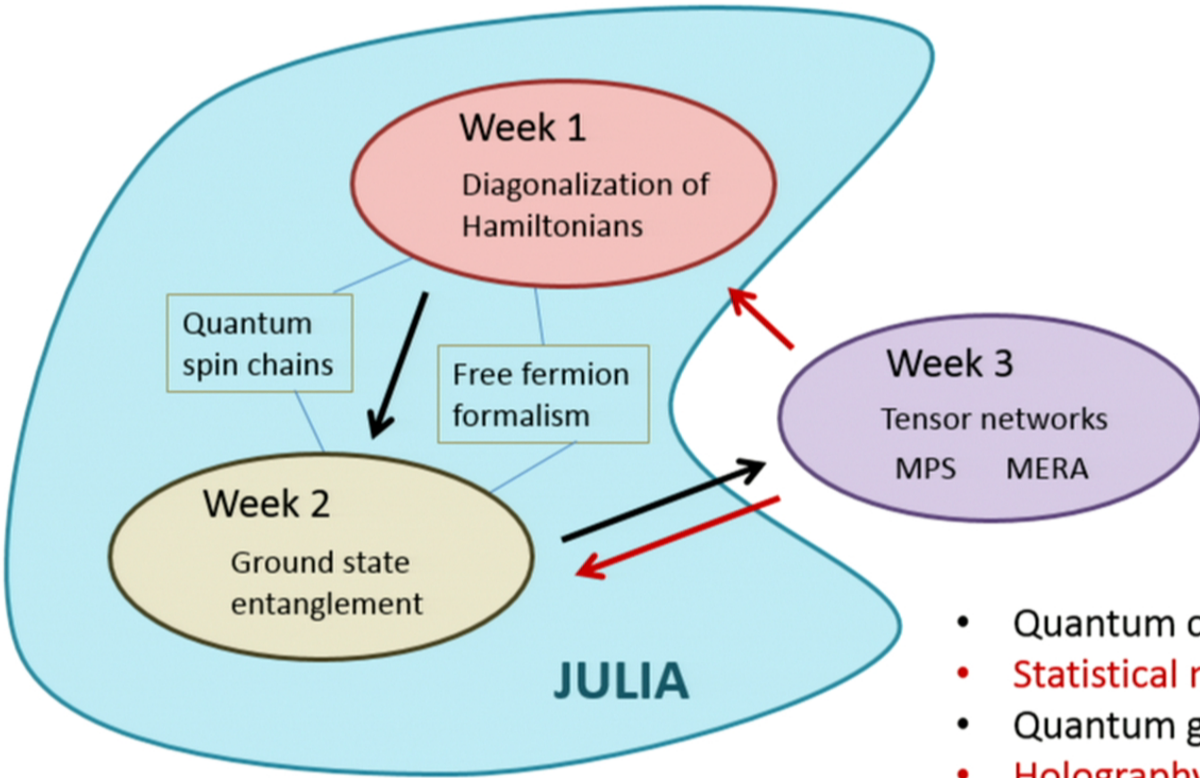
~ hyperbolic plane?
Euclidean metric signature
(Swingle 2009)

~ de Sitter space?
Lorentzian metric signature
(Beny 2011, Czech 2015)

MERA's
causal structure
= Lorentzian signature

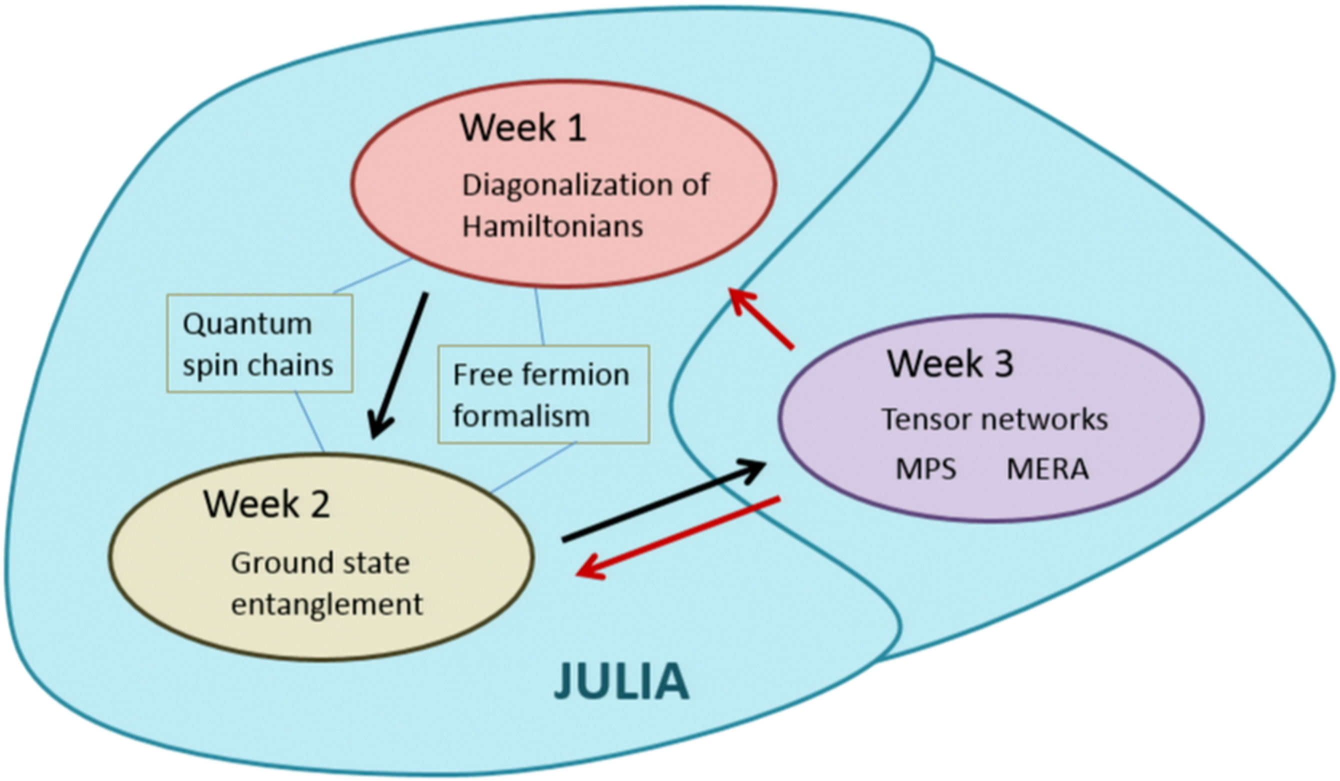
MERA is a discretization
of Kinematic space - Czech
(space of geodesics of the hyperbolic plane,
and NOT the hyperbolic plane)

Summary of course

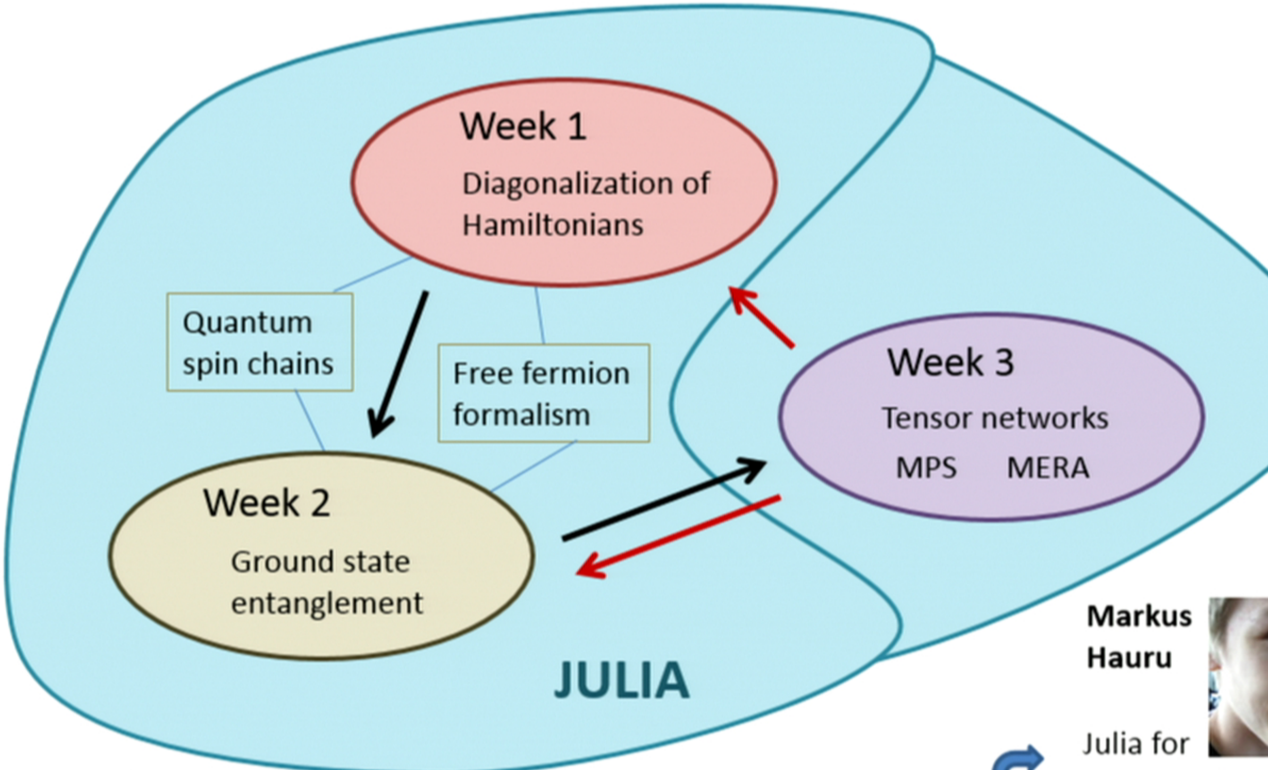


- Quantum chemistry
- Statistical mechanics
- Quantum gravity
- Holography
- Machine learning
- …

What next?


Week 1
Diagonalization of Hamiltonians
Quantum spin chains
Free fermion formalism
Week 2
Ground state entanglement
Week 3
Tensor networks
MPS MERA
JULIA

What next?



Markus Hauru

Julia for tensor networks

Erik Schnetter's talks:

Computational Physics

Tuesday 11:30 am Time room

Julia

Wednesday 11:30 am Time room