

Title: PSI 2015/2016 Explorations in Quantum Information - Agata Branczyk - 13

Date: Apr 07, 2016 09:00 AM

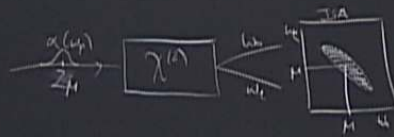
URL: <http://pirsa.org/16040004>

Abstract:

Derived JSA for Type II SPDC

$$|\Psi_{\text{state}}\rangle \propto \int d\omega_s \int d\omega_i f(\omega_s, \omega_i) a^\dagger(\omega_s) b^\dagger(\omega_i) |0\rangle$$

where the JSA is $f(\omega_s, \omega_i) = \alpha(\omega_s, \omega_i) \phi(\Delta k)$



pump spectral distribution

Phase matching (PM)

phase mismatch: $\Delta k = k_s(\omega_s) + k_i(\omega_i) - k_p(\omega_s + \omega_i)$

Taylor expand

$$\begin{aligned} k_s(\omega) &= k_s + k'_s(\omega - \mu) + \dots \\ k_i(\omega) &= k_i + k'_i(\omega - 2\mu) + \dots \end{aligned}$$

where $k'_s = \left. \frac{\partial k_s(\omega)}{\partial \omega} \right|_{\omega=\mu}$
 $k'_i = \left. \frac{\partial k_i(\omega)}{\partial \omega} \right|_{\omega=2\mu}$

$$\Delta k = \underbrace{(k_s + k_i - k_p)}_{\text{phase matching}} + \underbrace{(k'_i - k'_p)(\omega_i - \mu)}_{\text{group velocity matching}} + \underbrace{(k'_s - k'_p)(\omega_s - \mu)}_{\text{group velocity matching}}$$

phase mismatch: $\Delta k = k_o(\omega_o) + k_e(\omega_e) - k_p(\omega_o + \omega_e)$

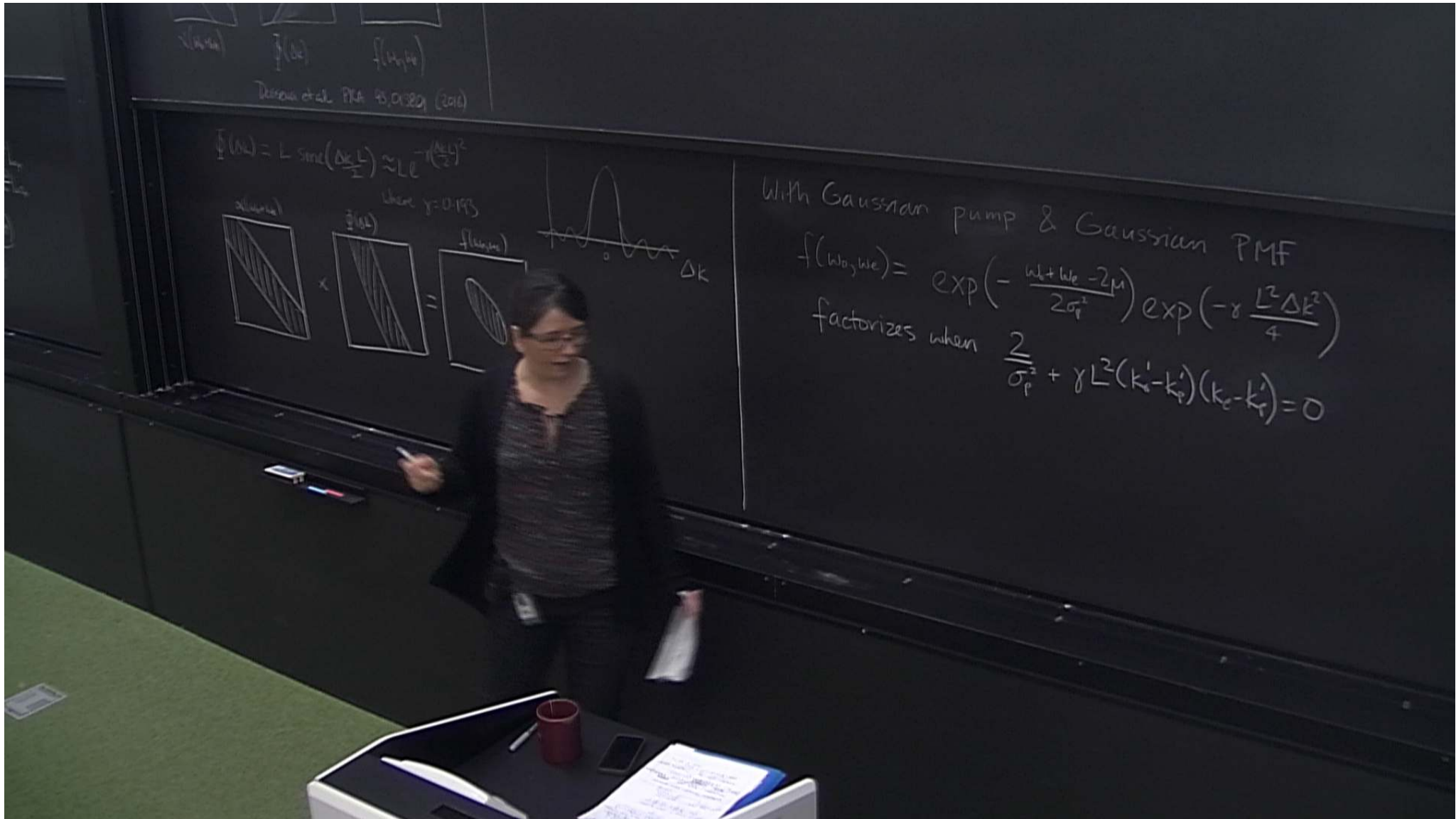
Taylor expand

$$\begin{aligned} k_j(\omega) &= k_j + k'_j(\omega - \mu) + \dots \\ k_p(\omega) &= k_p + k'_p(\omega - 2\mu) + \dots \end{aligned}$$

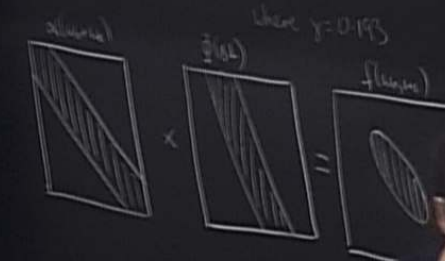
where $k'_j = \left. \frac{\partial k_j(\omega)}{\partial \omega} \right|_{\omega=\mu}$

$k'_p = \left. \frac{\partial k_p(\omega)}{\partial \omega} \right|_{\omega=2\mu}$

$$\Delta k = \underbrace{(k_o + k_e - k_p)}_{\text{phase matching}} + \underbrace{(k'_e - k'_p)(\omega_e - \mu) + (k'_o - k'_p)(\omega_o - \mu)}_{\text{group velocity matching}}$$



$$\Phi(\Delta k) = L \operatorname{sinc}\left(\frac{\Delta k L}{2}\right) \sim L e^{-\frac{(\Delta k L)^2}{4}}$$



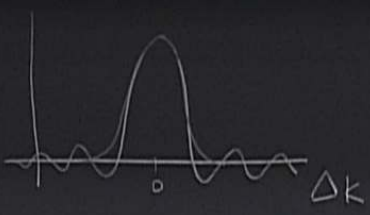
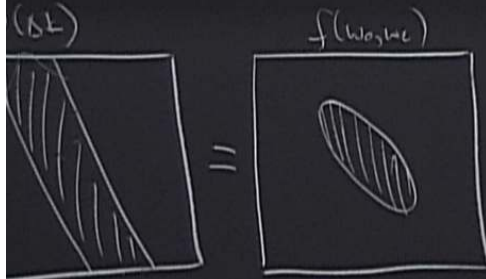
With Gaussian pump & Gaussian PMF

$$f(\omega_b, \omega_c) = \exp\left(-\frac{\omega_b + \omega_c - 2\mu}{2\sigma_p^2}\right) \exp\left(-\gamma \frac{L^2 \Delta k^2}{4}\right)$$

factorizes when $\frac{2}{\sigma_p^2} + \gamma L^2 (k_c' - k_b')(k_c - k_b') = 0$

$f(\omega_s, \omega_e)$
 PRA 93, 013801 (2016)

$\frac{L}{2} \approx L e^{-\gamma \left(\frac{\Delta k L}{2}\right)^2}$
 where $\gamma = 0.143$



With Gaussian pump & Gaussian PMF

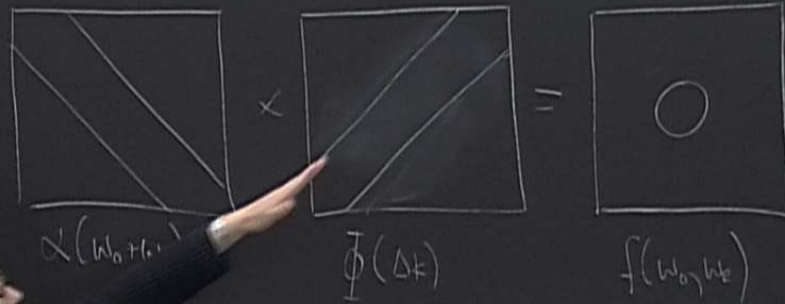
$$f(\omega_s, \omega_e) = \exp\left(-\frac{(\omega_s + \omega_e - 2\mu)^2}{2\sigma_p^2}\right) \exp\left(-\gamma \frac{L^2 \Delta k^2}{4}\right)$$

factorizes when $\frac{2}{\sigma_p^2} + \gamma L^2 (k_s - k_e)(k_e - k_s') = 0$



Satisfied when

$$k_p' = \frac{k_o' + k_e'}{2} \quad \text{and} \quad L = \frac{1}{\sqrt{8\gamma\sigma_p^2(k_o' - k_e')}}}$$

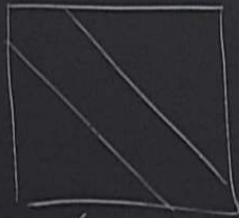


Dusseva et al PRA 93,013801 (2016)

Satisfied when

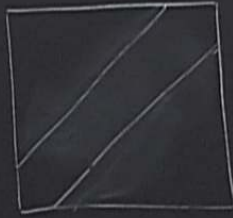
$$k_p' = \frac{k_o' + k_e'}{2}$$

and $L = \frac{1}{\sqrt{8\gamma\sigma_e^2(k_o' + k_e')}}}$



$\chi(w_o + w_e)$

\times



$\Phi(\Delta k)$

$=$

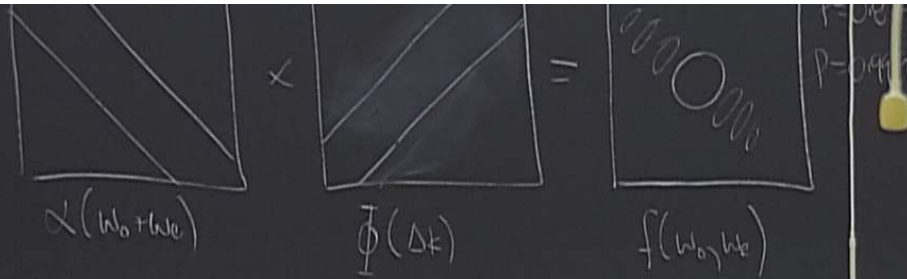


$f(w_o, w_e)$

P-187

Dusseena et al PRA 93, 013801 (2016)

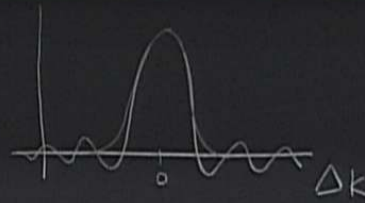




Dusseva et al PRA 93, 013801 (2016)

$$\Phi(\Delta k) = L \operatorname{sinc}\left(\frac{\Delta k L}{2}\right) \approx L e^{-\gamma \left(\frac{\Delta k L}{2}\right)^2}$$

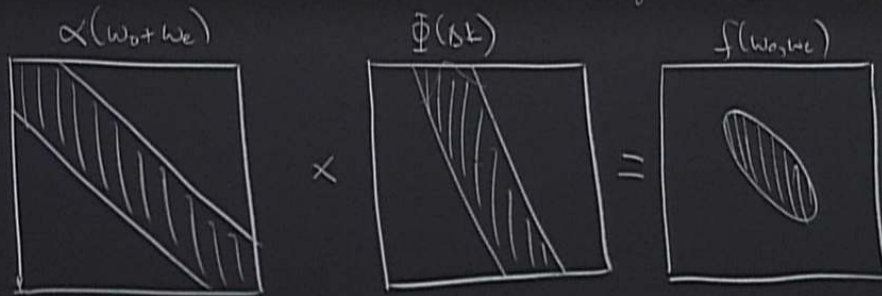
where $\gamma = 0.193$



With Gaussian pump

$$f(\omega_0, \omega_e) = \exp(-)$$

factorizes when



1) Dual rail

$$|0\rangle \equiv \begin{array}{c} a \\ \text{---} \\ b \\ \text{---} \end{array}$$

$$|1\rangle \equiv \begin{array}{c} a \\ \text{---} \\ \text{---} \\ b \\ \text{---} \end{array}$$

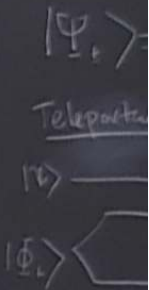


2) Polarization

$$|0\rangle \equiv |H\rangle; |1\rangle \equiv |V\rangle$$

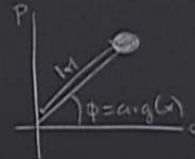


* Bell-basis meas using linear optics & photon counting have success prob $P \leq \frac{1}{2}$
(Calsamiglia & Lütkenhaus, Applied Phys B 72, 67 (2001))

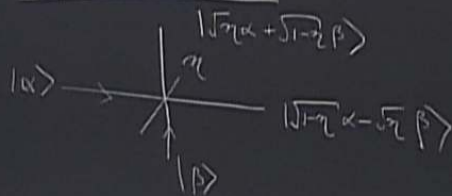


Coherent State Linear Optical Quantum Computing (LOQC)

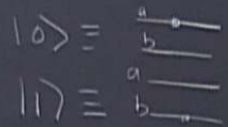
Recall that $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$



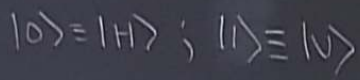
Interference on BS



1) Dual rail



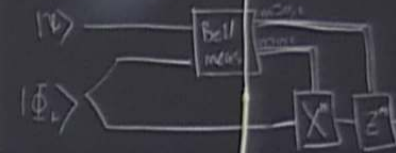
2) Polarization



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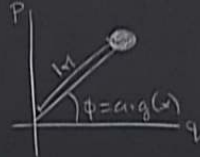
$$|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

Teleportation

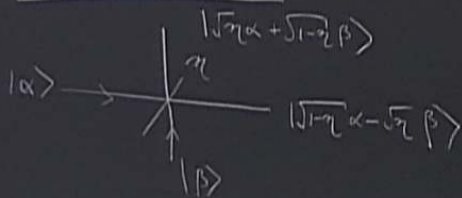


Coherent State Linear Optical Quantum Computing (LOQC)

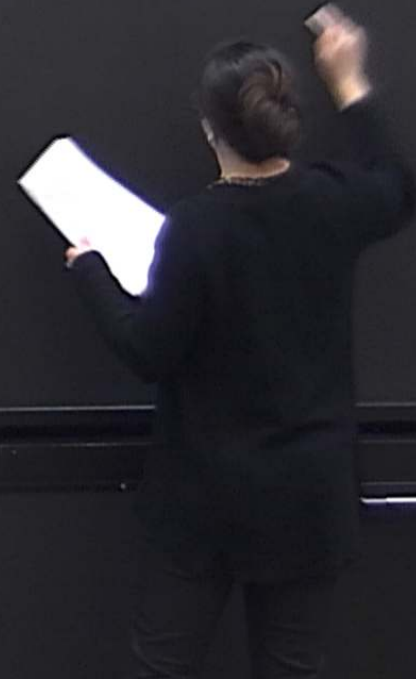
Recall that $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$



Interference on BS

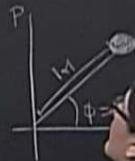


Consider $|\alpha\rangle$ and $|\alpha\rangle$

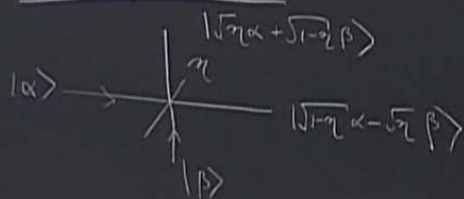


Coherent State Linear Optical Quantum Computing (LOQC)

Recall that $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$



Interference on BS



Consider $|\alpha\rangle$ and $|\alpha\rangle$

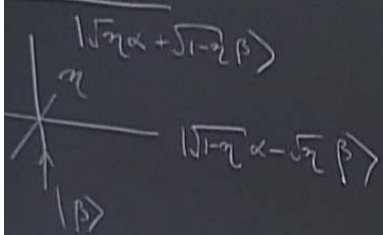
not orthogonal $\langle \alpha | -\alpha \rangle = e^{-4|\alpha|^2}$

e.g. $|\alpha|=2$, overlap $\approx 10^{-7}$

Identify $\pm\alpha$ as computational basis

$$|\alpha\rangle \rightarrow |0_L\rangle$$

$$|-\alpha\rangle \rightarrow |1_L\rangle$$



Identify $\pm\alpha$ as computational basis

$$|\alpha\rangle \rightarrow |0_L\rangle$$

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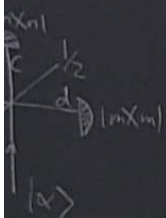
qubit

Non-zero prob of failure

$$P_{\text{fail}} = -|\langle 00|\psi\rangle|^2 = |\mu + \nu|^2 e^{-2|\alpha|^2}$$

$$|\psi\rangle = \mu|\alpha\rangle + \nu|-\alpha\rangle$$

between basis states,



$$|\psi\rangle|\alpha\rangle \rightarrow \mu\sqrt{2}\alpha|0\rangle_d + \nu|0\rangle_c\sqrt{2}\alpha|1\rangle_d$$

Alternatively

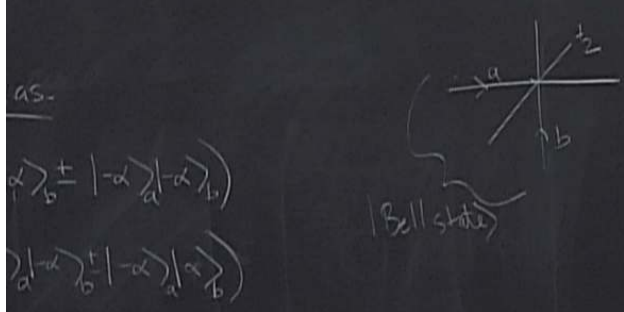
$$\text{"even"} \quad |e\rangle = \mathcal{N}_+ (|d\rangle + |-\alpha\rangle) \rightarrow |0_L\rangle$$

$$\text{"odd"} \quad |d\rangle = \mathcal{N}_- (|\alpha\rangle - |-\alpha\rangle) \rightarrow |1_L\rangle$$

$$\text{where } \mathcal{N}_{\pm} = \frac{1}{\sqrt{2(1 \pm \alpha e^{-2|\alpha|^2})}}$$

$$|e\rangle = 2\mathcal{N}_+ e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{\sqrt{(2n)!}} |2n\rangle$$

$$|d\rangle = 2\mathcal{N}_- e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^{(2n+1)}}{\sqrt{(2n+1)!}} |2n+1\rangle$$



$$|\Phi_+\rangle \rightarrow |E\rangle|0\rangle$$

$$|\Phi_-\rangle \rightarrow |D\rangle|0\rangle$$

$$|\Psi_+\rangle \rightarrow |0\rangle|E\rangle$$

$$|\Psi_-\rangle \rightarrow |0\rangle|D\rangle$$

where $|E/D\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle \pm |-\alpha\rangle)$

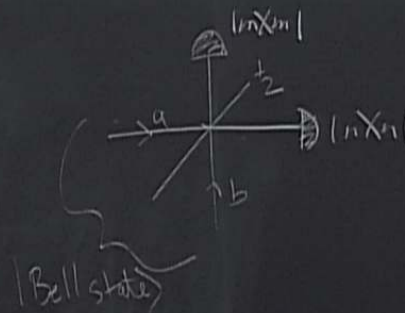


Teleportation

Bell-basis meas.

$$|\Phi_{\pm}\rangle = N_{\pm} (|\alpha\rangle_a |\alpha\rangle_b \pm |-\alpha\rangle_a |-\alpha\rangle_b)$$

$$|\Psi_{\pm}\rangle = N_{\pm} (|\alpha\rangle_a |-\alpha\rangle_b \pm |-\alpha\rangle_a |\alpha\rangle_b)$$



$$|\Phi_{+}\rangle \rightarrow |E\rangle |0\rangle$$

$$|\Phi_{-}\rangle \rightarrow |D\rangle |0\rangle$$

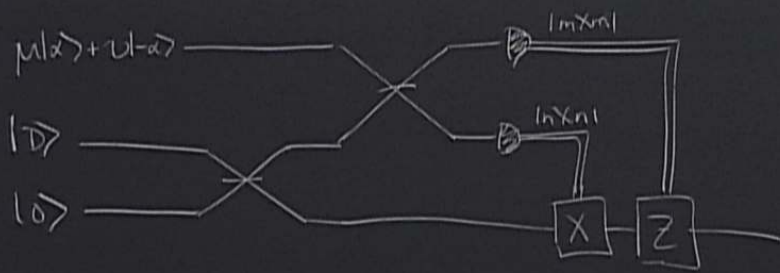
$$|\Psi_{+}\rangle \rightarrow |0\rangle |E\rangle$$

$$|\Psi_{-}\rangle \rightarrow |0\rangle |D\rangle$$

$$|\Psi_{\pm}\rangle = N_{\pm} (|\alpha\rangle_a |\alpha\rangle_b \pm |-\alpha\rangle_a |-\alpha\rangle_b)$$

|Bell state\rangle

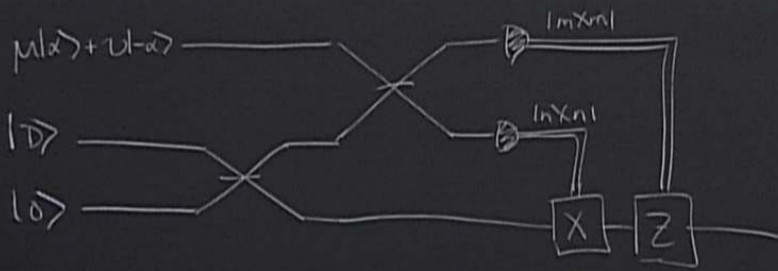
$$|\Psi_{-}\rangle \rightarrow |0\rangle |D\rangle$$



$$|\Psi_{\pm}\rangle = N_{\pm} (|\alpha\rangle_a |\alpha\rangle_b \pm |-\alpha\rangle_a |-\alpha\rangle_b)$$

(Bell state)

$$|\Psi_{-}\rangle \rightarrow |0\rangle |D\rangle$$



X correction

$$\begin{aligned} |\alpha\rangle &\rightarrow |-\alpha\rangle \\ |-\alpha\rangle &\rightarrow |\alpha\rangle \end{aligned}$$

Z correction

$$\begin{aligned} |\alpha\rangle &\rightarrow |\alpha\rangle \\ |-\alpha\rangle &\rightarrow -|-\alpha\rangle \end{aligned}$$

Universal Q.C.

Single qubit



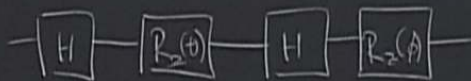
C SIGN



Universal Q.C.

Jeong & Ralph quant-ph/0509137

Single qubit



CSIGN



Resource state

$$|HR\rangle = |\alpha\rangle|\alpha\rangle + |\alpha\rangle|-\alpha\rangle + |-\alpha\rangle|\alpha\rangle - |-\alpha\rangle|-\alpha\rangle$$

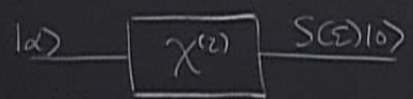
$$|\Psi_{\pm}\rangle = N_{\pm} \left(|\alpha\rangle_a |-\alpha\rangle_b \pm |-\alpha\rangle_a |\alpha\rangle_b \right)$$

(Bell state)

$$|\Psi_{-}\rangle \rightarrow |0\rangle |D\rangle$$

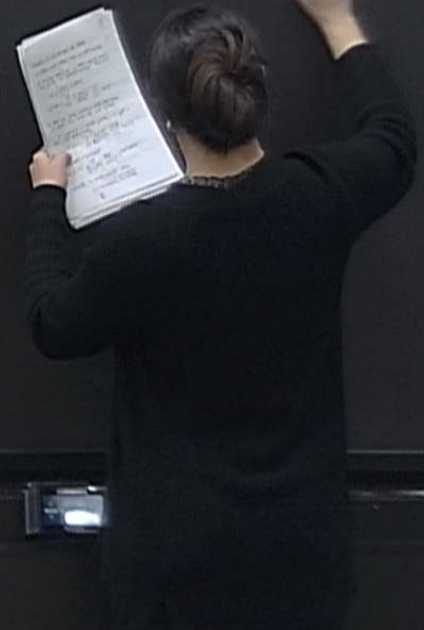
duction of cat states

$S(\epsilon)|0\rangle$ has even photon # terms



abstract single photon

$$a S(\epsilon)|0\rangle = \sum_{n=0}^{\infty} \frac{(-\tanh \epsilon)^n}{(\cosh \epsilon)^{3/2}} \sqrt{(2n+1)!}$$



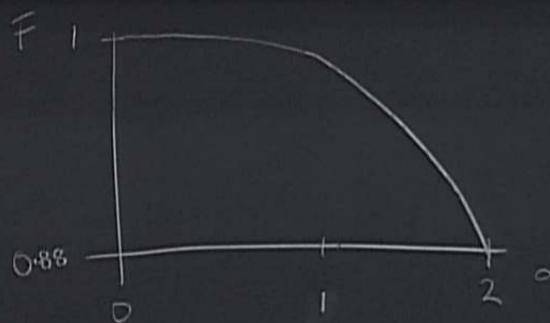
$$|\Psi\rangle \rightarrow |0\rangle |D\rangle$$

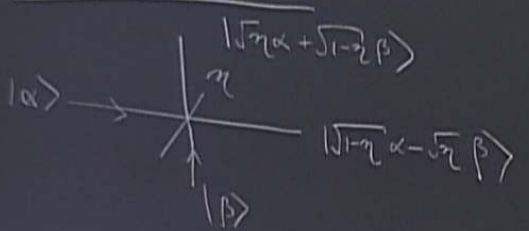
$$a S(\epsilon) |0\rangle = \sum_{n=0}^{\infty} \frac{(-\tanh \epsilon)^n}{(\cosh \epsilon)^{3/2}} \frac{\sqrt{(2n)!}}{2^n n!} |2n+1\rangle$$

$$F(|\Psi\rangle, |\phi\rangle) = |\langle \Psi | \phi \rangle|^2$$

$$F(\alpha, \epsilon) = \frac{e^{-k^2}}{2(1+e^{2k^2})} \frac{4|\alpha|^2}{(\cosh \epsilon)^3} e^{-|\alpha|^2 \tanh \epsilon}$$

$$\text{max when } \epsilon = \text{arcosh} \left(\sqrt{\frac{1}{2} + \frac{1}{6} \sqrt{9+4|\alpha|^2}} \right)$$





Identify $\pm\alpha$ as computational basis

$$|\alpha\rangle \rightarrow |0\rangle$$

$$|-\alpha\rangle \rightarrow |1\rangle$$

Decoherence

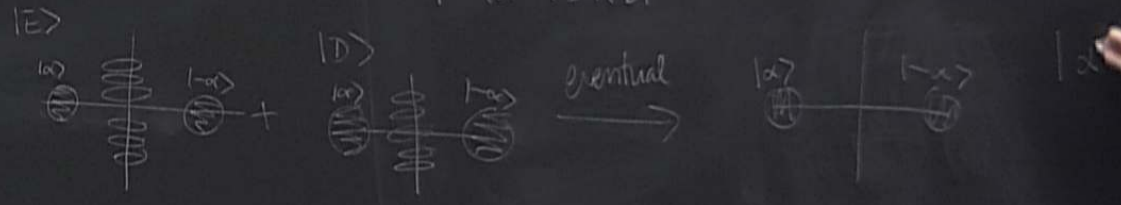
$$H_I = \frac{\hbar\Omega}{2} (a^\dagger \sigma_- + a \sigma_+)$$

$$\sigma_- = |g\rangle\langle e| \quad ; \quad \sigma_+ = |e\rangle\langle g|$$

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} \left(|E_+\rangle |g\rangle_a - |E_-\rangle |e\rangle_a \right)$$

Trace atom

$$|E_+\rangle\langle E_+| - |E_-\rangle\langle E_-|$$



Identify $\pm\alpha$ as computational basis

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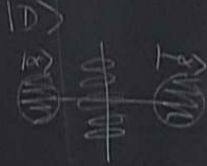
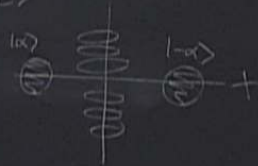
$$|\Psi(t)\rangle = \mathbb{1} - i \frac{\Omega t}{2} (a^\dagger \sigma_- + a \sigma_+) |E\rangle_f |g\rangle_a$$

$$= |E\rangle_f |g\rangle_a - i \frac{\Omega t}{2} |D\rangle_f |e\rangle_a$$

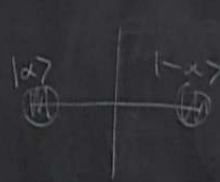
Trace atom

$$|E\rangle\langle E| - \frac{\Omega^2 t^2}{2} |D\rangle\langle D|$$

$|E\rangle$



eventual



$$|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$$