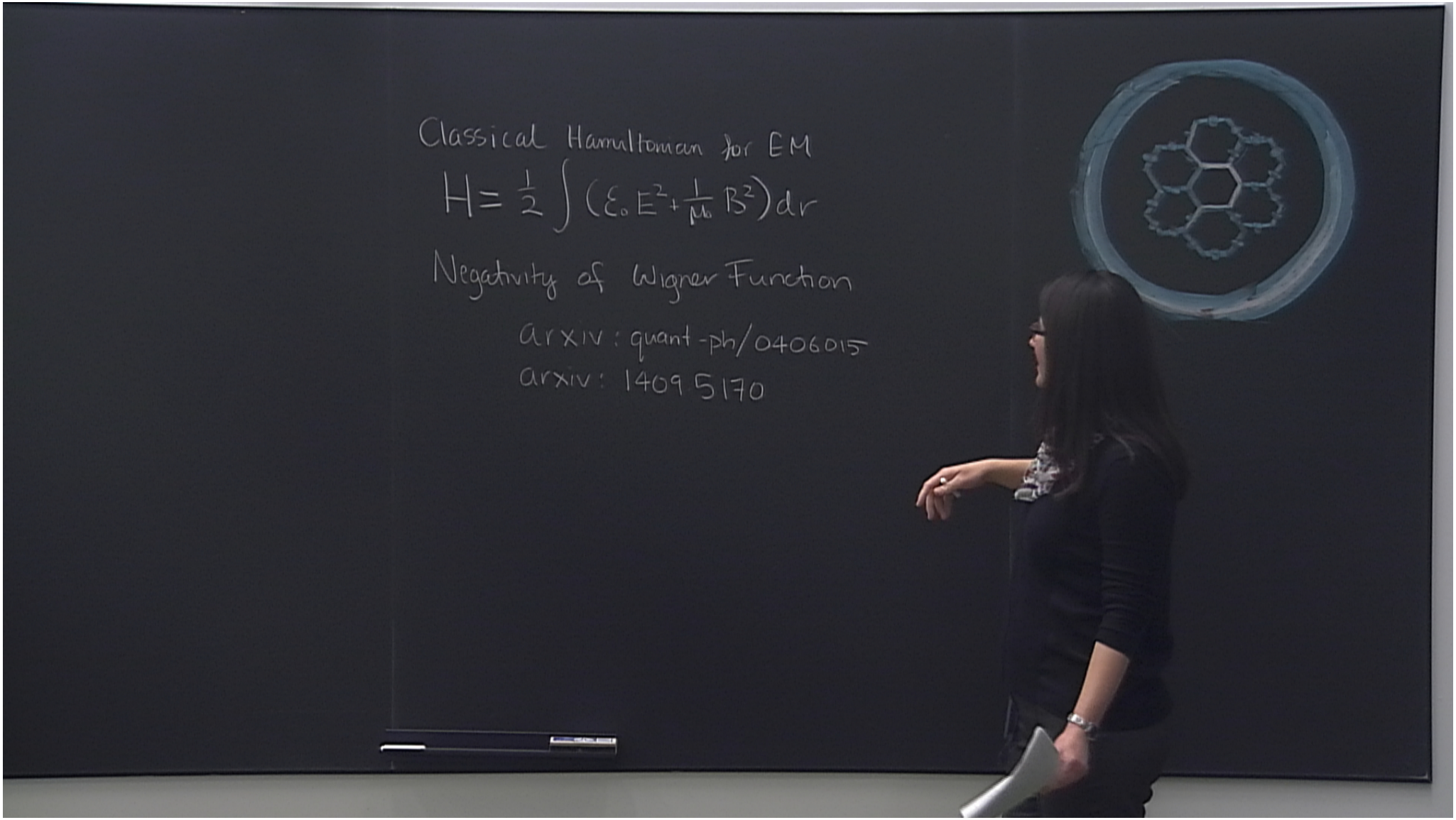
Title: PSI 2015/2016 Explorations in Quantum Information - Agata Branczyk - 11

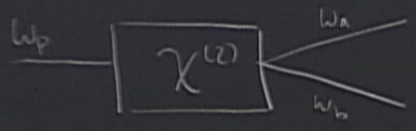
Date: Apr 05, 2016 09:00 AM

URL: http://pirsa.org/16040002

Abstract:


Classical Hamiltonian for EM
$H = \frac{1}{2}\int (\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2) dr$
Negativity of Wigner Function
arxiv: quant-ph/0406015
arxiv: 1409 5170

Spontaneous Parametric Down Conversion



$\omega_p = \omega_a + \omega_b$

$\vec{k}_p = \vec{k}_a + \vec{k}_B$

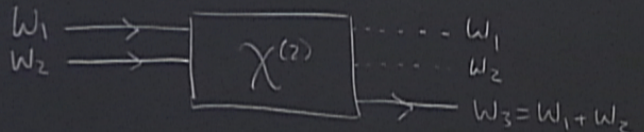
Single mode model

- strong classical pump
- rotating frame
- interaction picture

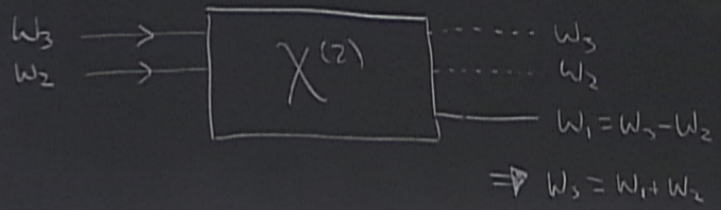
Type I $\quad C \rightarrow O + O$

Classical $\chi^{(2)}$

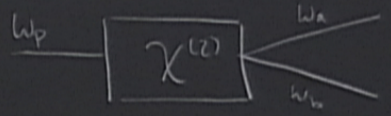
Sum-frequency generation



Difference-frequency generation



Spontaneous Parametric Down Conversi

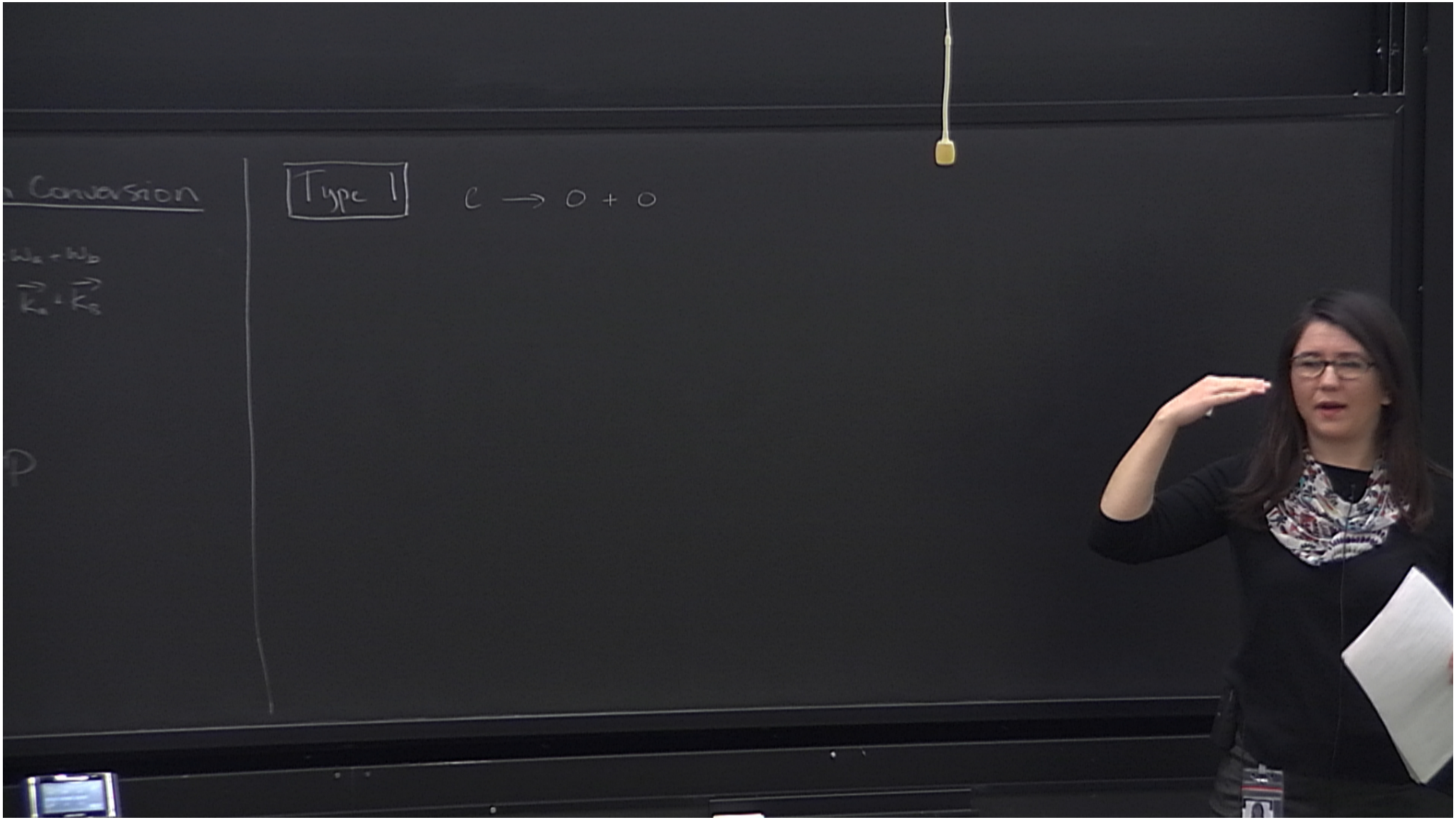


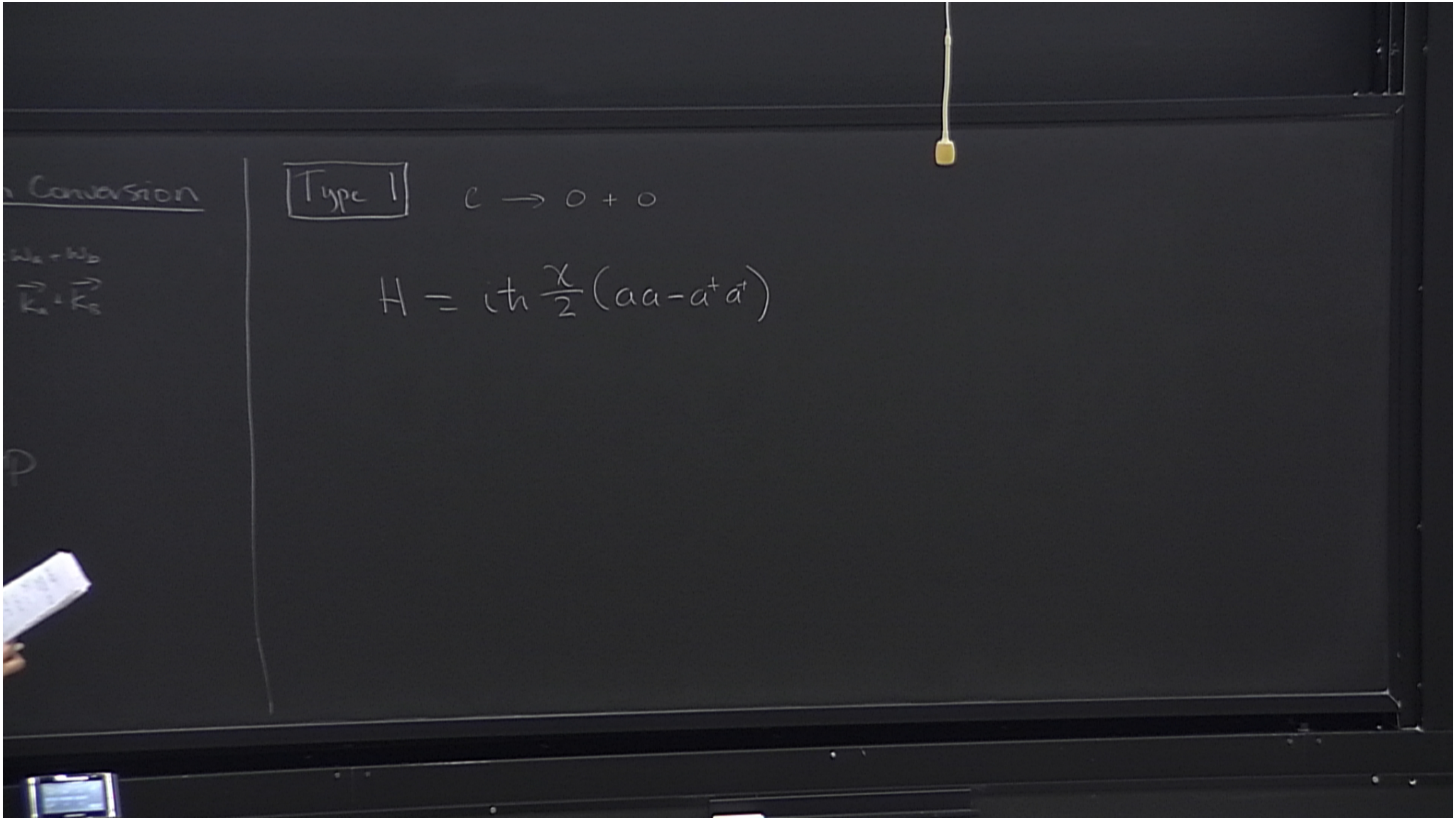
$\omega_p = \omega_a + \omega_b$

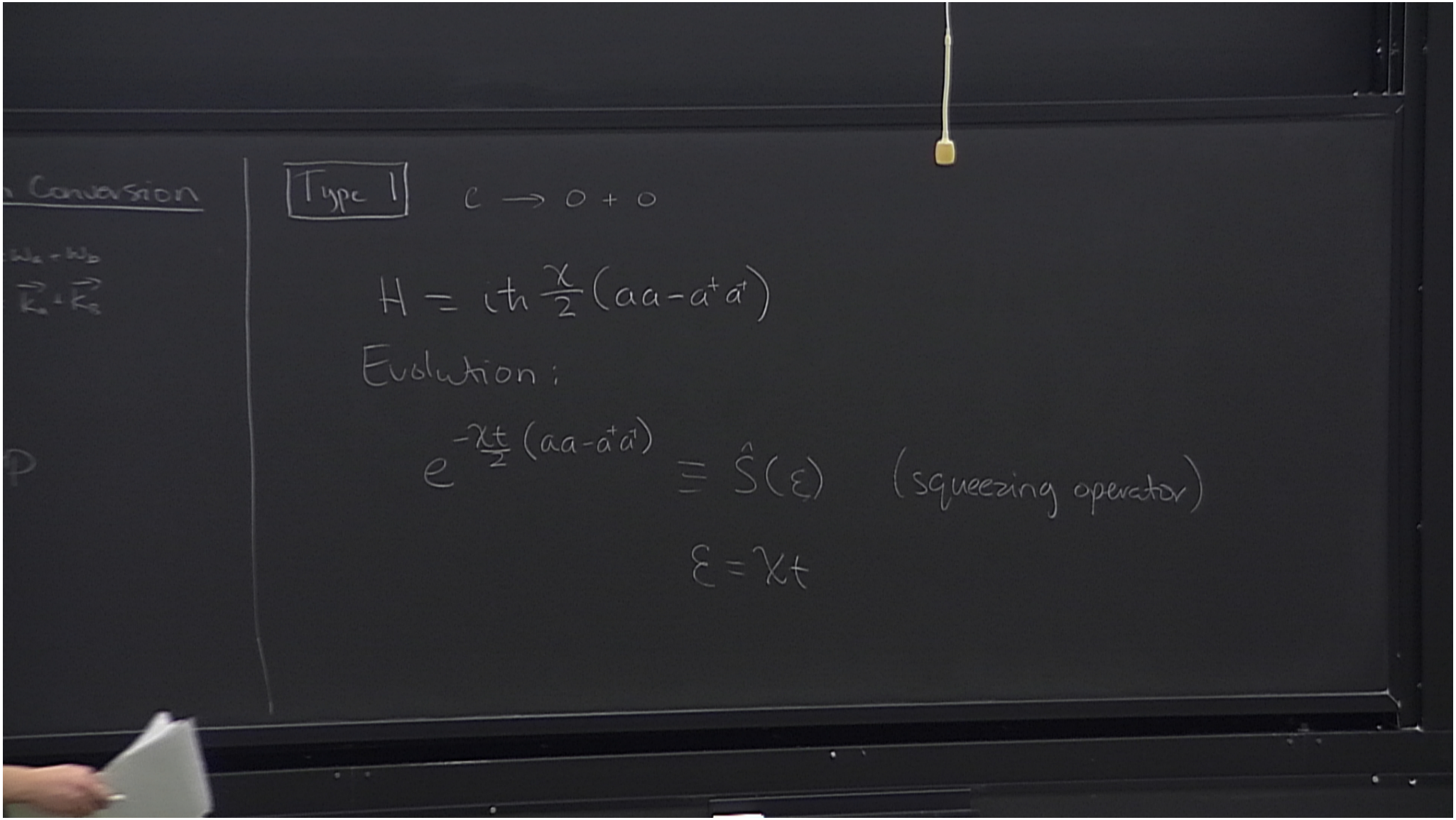
$\vec{k}_p = \vec{k}_a + \vec{k}_b$

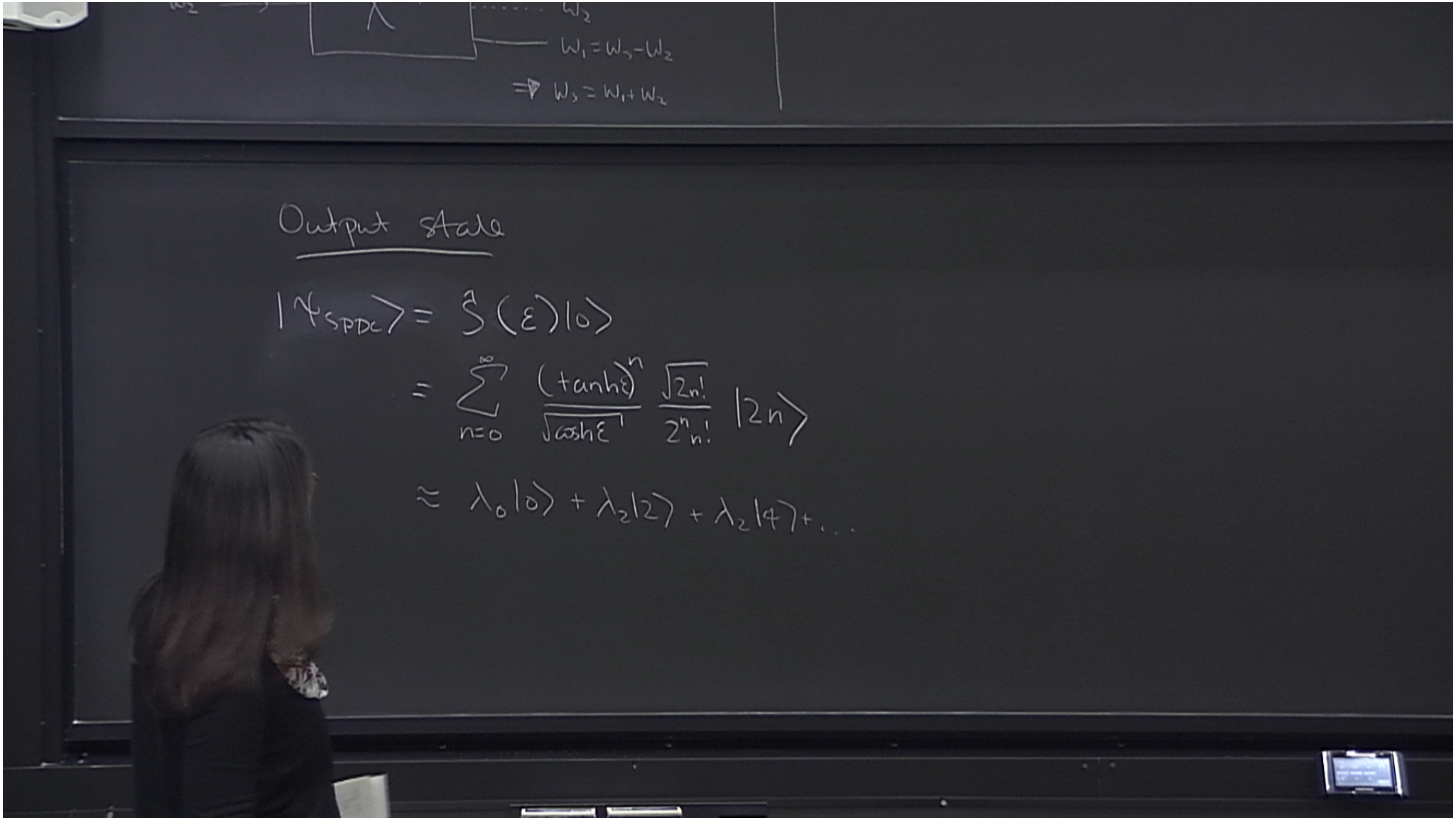
Single mode model

- strong classical pump
- rotating frame
- interaction picture


Conversion
Type I
C → O + O


Conversion
Type I
C → O + O
H = ith χ/2 (aa - a⁺a⁺)


Conversion
Type I
$\chi \rightarrow 0 + 0$
$H = i\hbar \frac{\chi}{2}(aa - a^\dagger a^\dagger)$
Evolution:
$e^{-\frac{\chi t}{2}(aa - a^\dagger a^\dagger)} \equiv \hat{S}(\xi)$ (squeezing operator)
$\xi = \chi t$


W_2
$W_1 = W_3 - W_2$
$\Rightarrow W_3 = W_1 + W_2$
Output State
$|\psi_{SPDC}\rangle = \hat{S}(\xi)|0\rangle$
$= \sum_{n=0}^{\infty} \frac{(\tanh\xi)^n}{\sqrt{\cosh\xi}} \frac{\sqrt{2n!}}{2^n n!} |2n\rangle$
$\approx \lambda_0|0\rangle + \lambda_2|2\rangle + \lambda_2|4\rangle + \ldots$

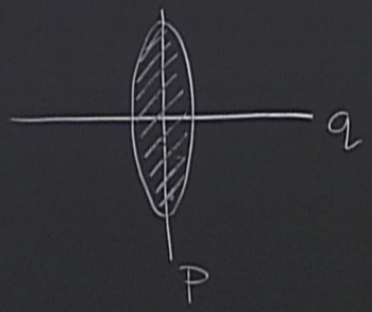
Output state

$$|\psi_{SPDC}\rangle = \hat{S}(\varepsilon)|0\rangle$$

$$= \sum_{n=0}^{\infty} \frac{(\tanh\varepsilon)^n}{\sqrt{\cosh\varepsilon}} \frac{\sqrt{2n!}}{2^n n!} |2n\rangle$$

$$\approx \lambda_0|0\rangle + \lambda_2|2\rangle + \lambda_2|4\rangle + \ldots$$

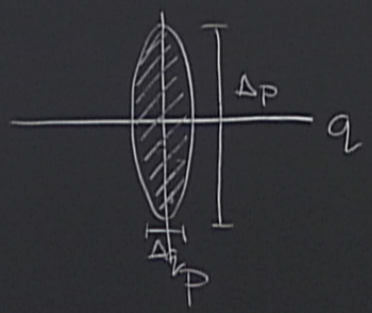
Wigner function



$|2n\rangle$

$\ldots|4\rangle+\ldots$

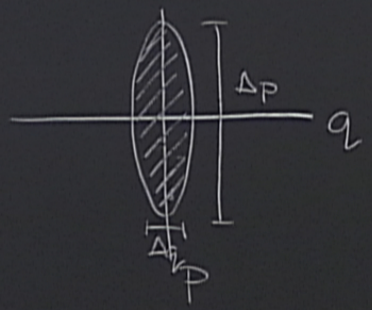
Wigner function



$|2n\rangle$

$c_2|4\rangle + \ldots$

Wigner function

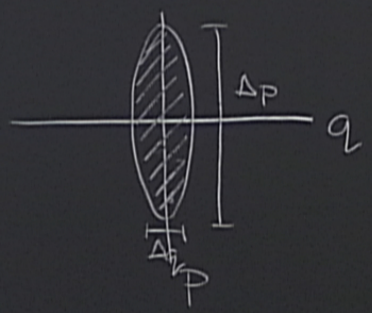


$\Delta q \Delta p = 1$

$|2n\rangle$

$\ldots |4\rangle + \ldots$

Wigner function

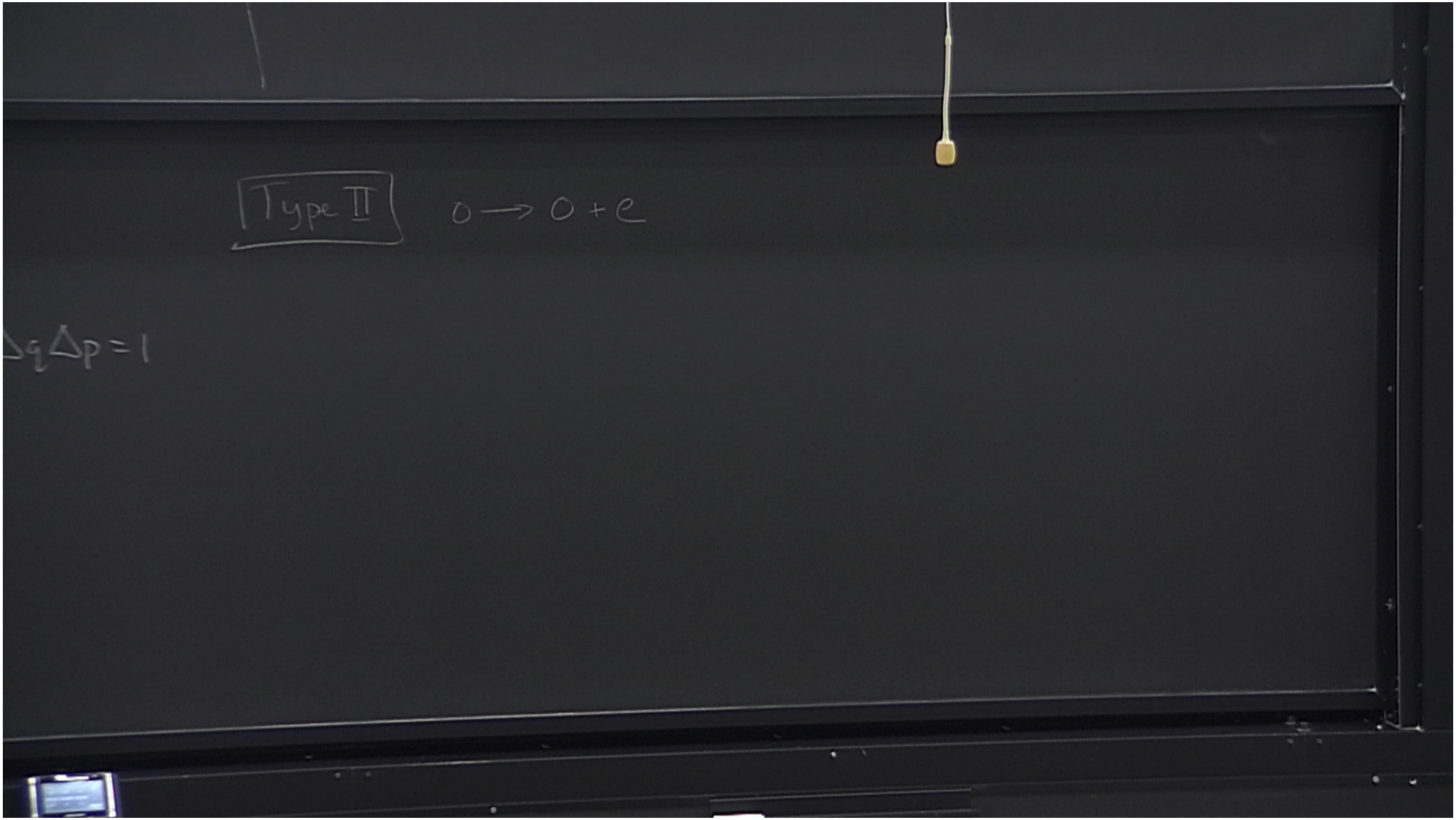


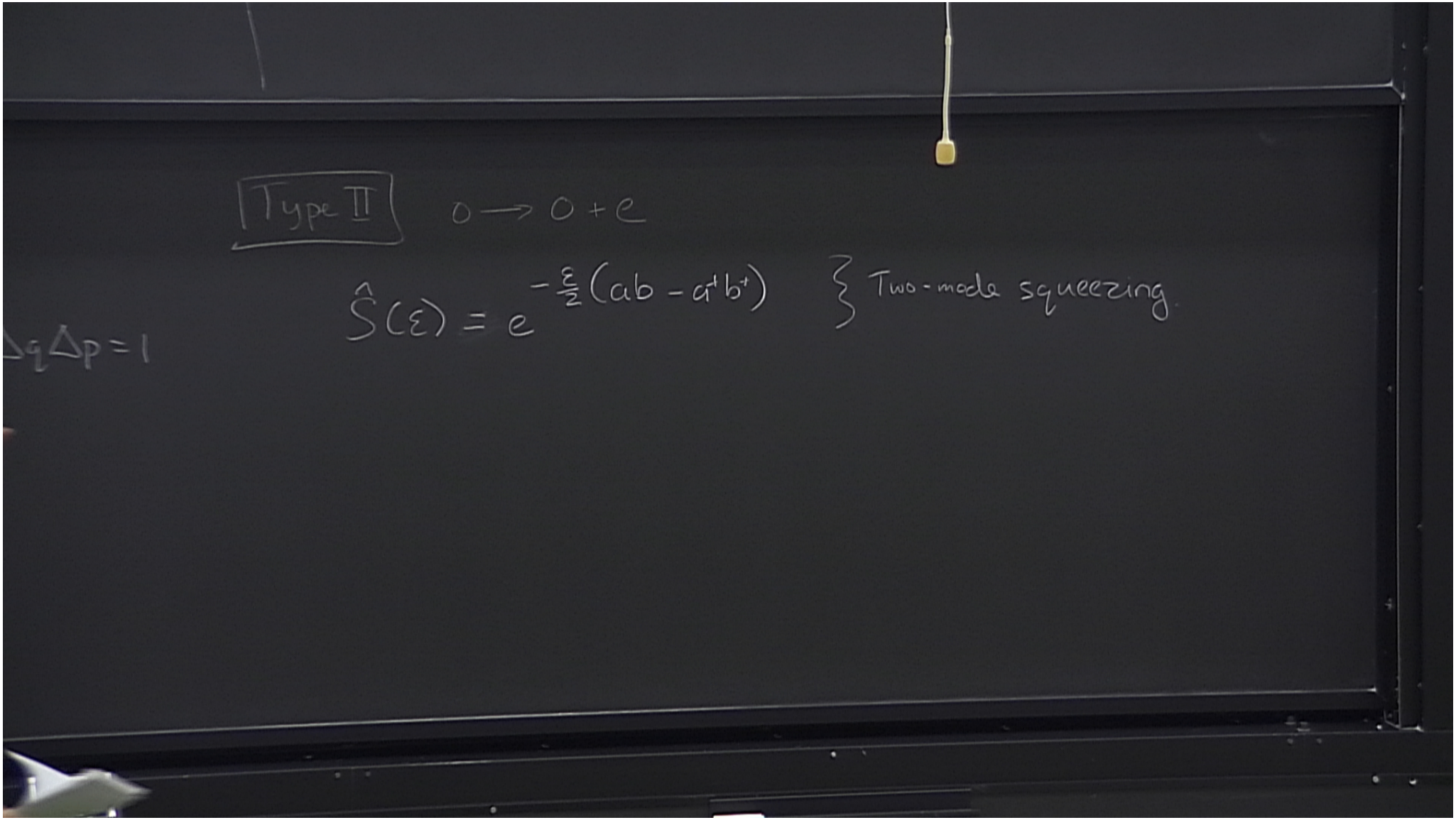
$\Delta q \Delta p = 1$

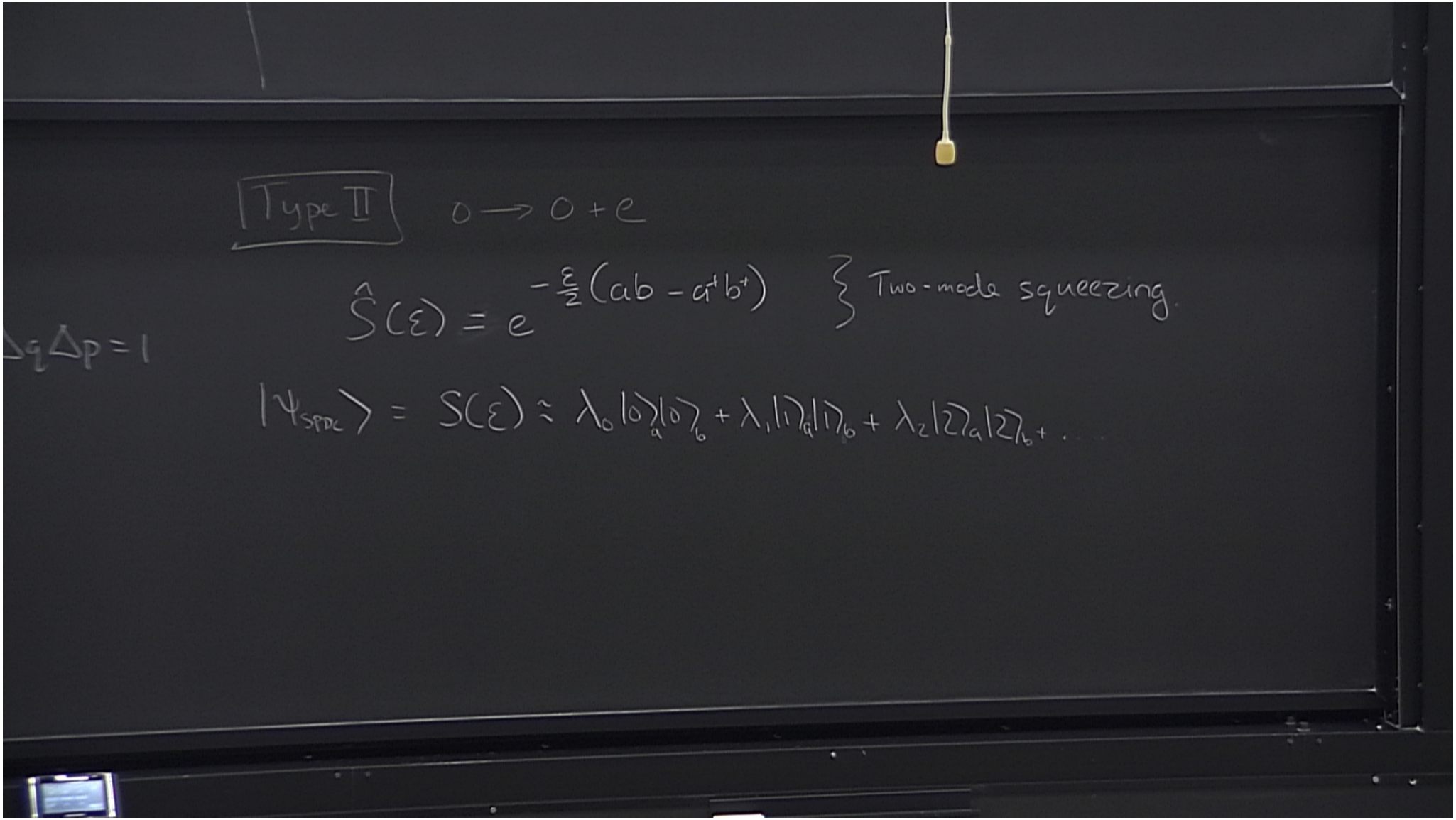
$|2n\rangle$

$|4\rangle + \ldots$

LIGO


Type II
0 → 0 + e
Δq Δp = 1


Type II
o → o + e
$\hat{S}(\xi) = e^{-\frac{\xi}{2}(ab - a^\dagger b^\dagger)}$
Two-mode squeezing
$\Delta q \Delta p = 1$


Type II
0 → 0 + e
$\hat{S}(\xi) = e^{-\frac{\xi}{2}(ab - a^\dagger b^\dagger)}$
Two-mode squeezing
$\Delta q \Delta p = 1$
$|\psi_{SPDC}\rangle = S(\xi) \approx \lambda_0 |0\rangle_a |0\rangle_b + \lambda_1 |1\rangle_a |1\rangle_b + \lambda_2 |2\rangle_a |2\rangle_b + \ldots$

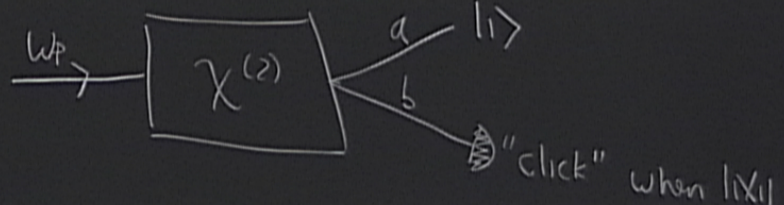
Type II $\quad o \longrightarrow o + e$

$$\hat{S}(\xi) = e^{-\frac{\xi}{2}(ab - a^\dagger b^\dagger)}$$ } Two-mode squeezing.

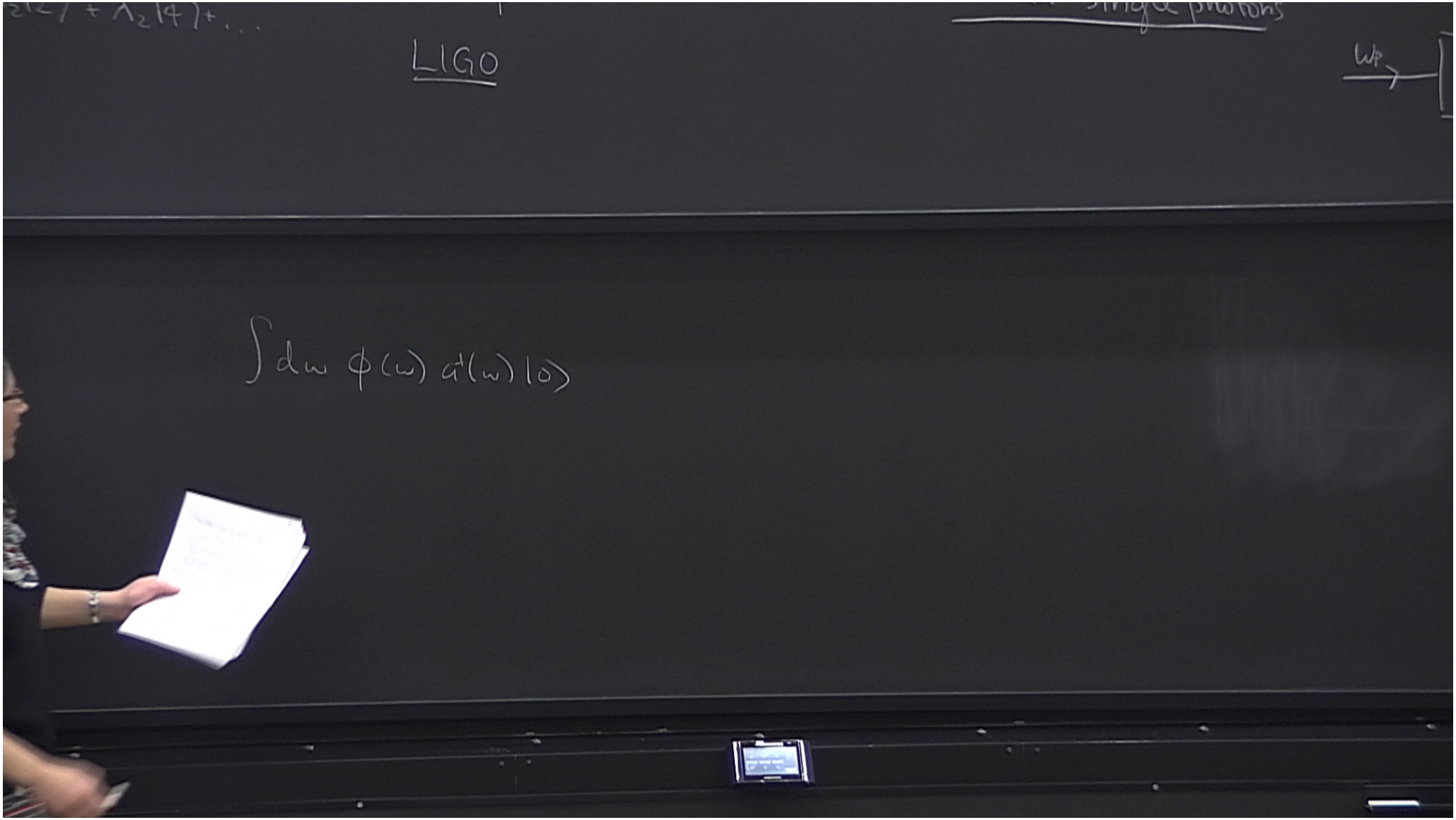
$\Delta q \Delta p = 1$

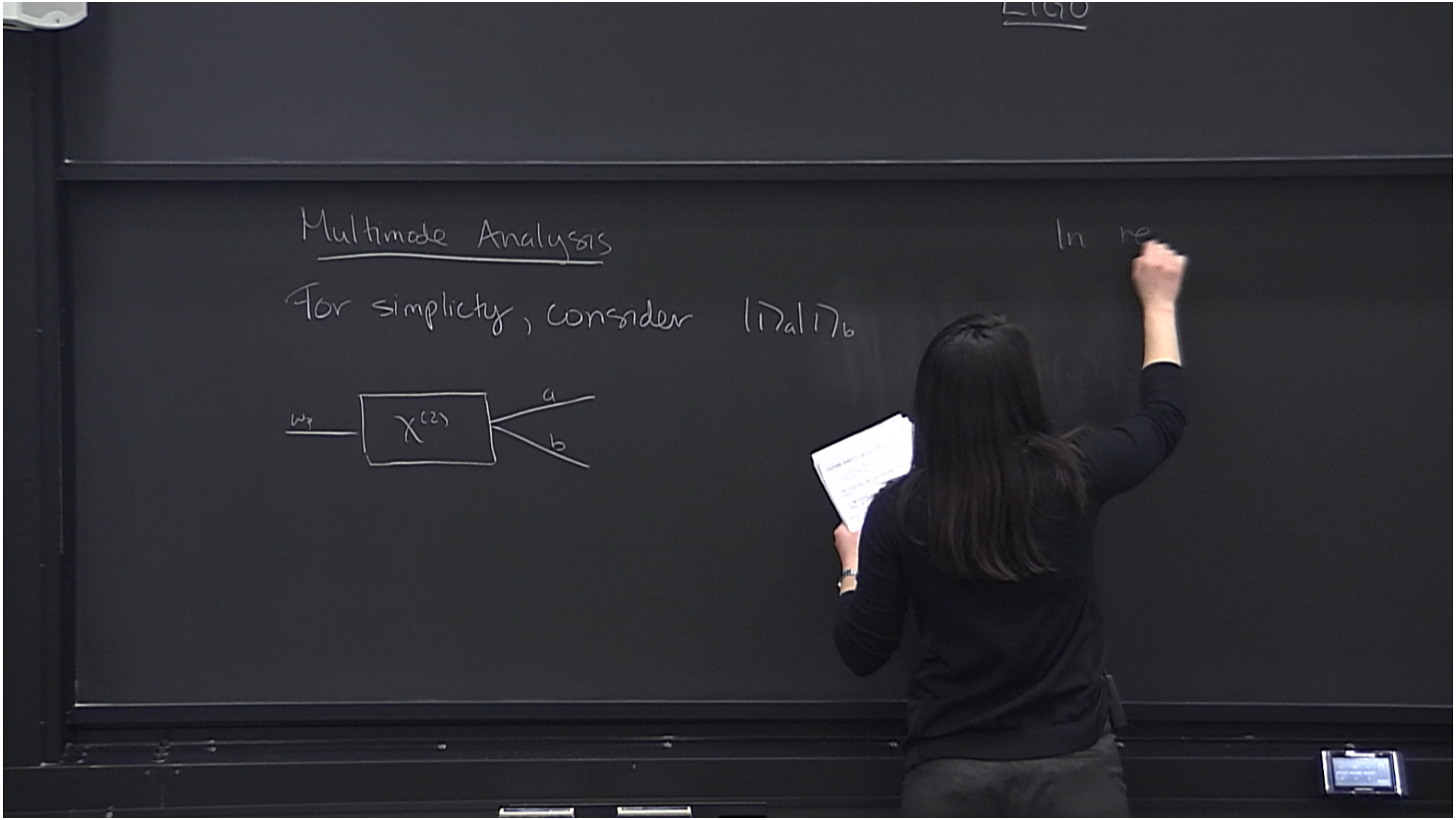
$$|\psi_{SPDC}\rangle = S(\xi) \approx \lambda_0 |0\rangle_a |0\rangle_b + \lambda_1 |1\rangle_a |1\rangle_b + \lambda_2 |2\rangle_a |2\rangle_b + \ldots$$

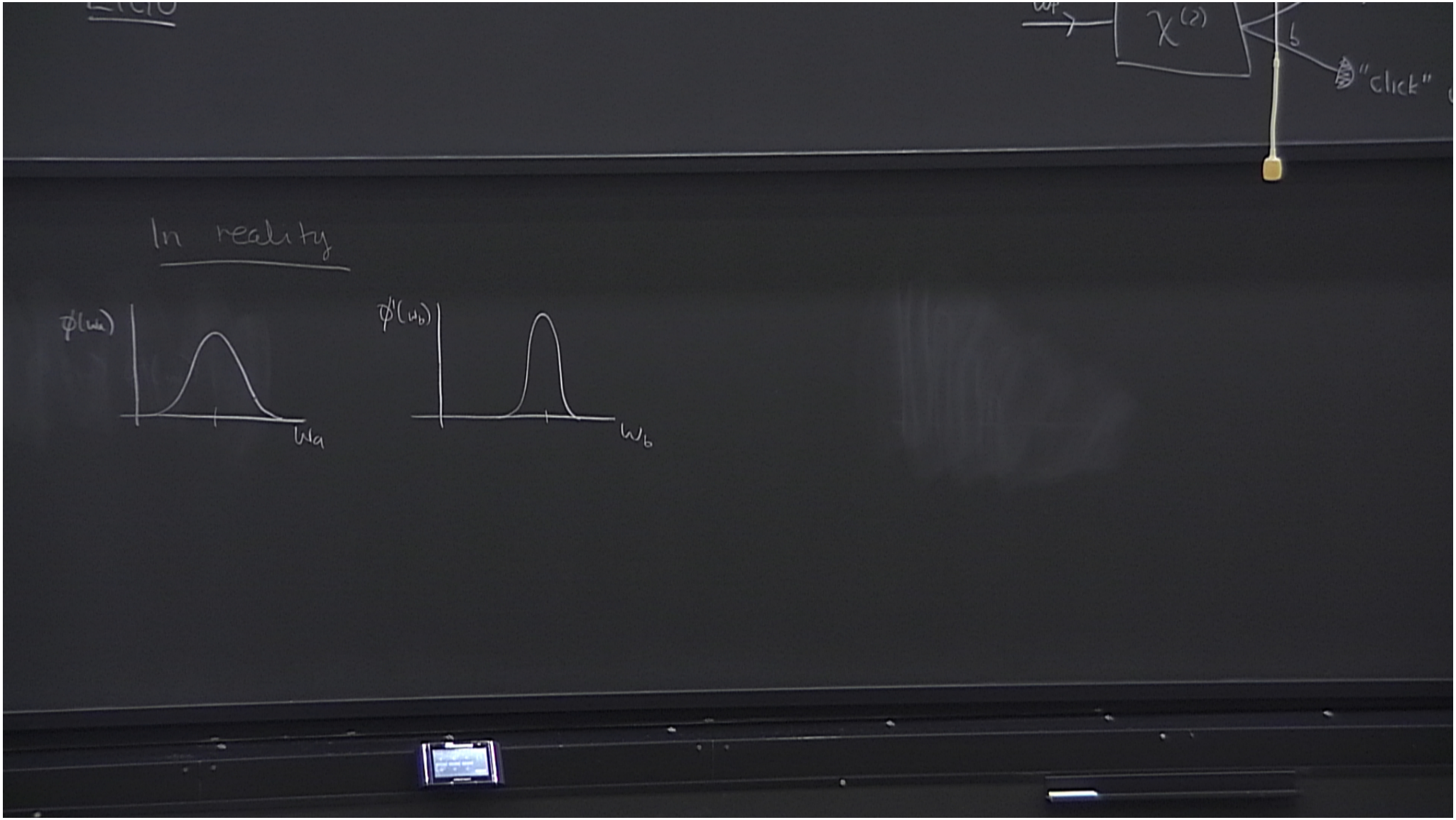
Heralded single photons




ω_p
$\chi^{(2)}$
a
b
$|1\rangle$
"click" when $|\chi_{11}|$
0 1 2 3


LIGO
$\int dw\, \phi(w)\, a^\dagger(w)|0\rangle$

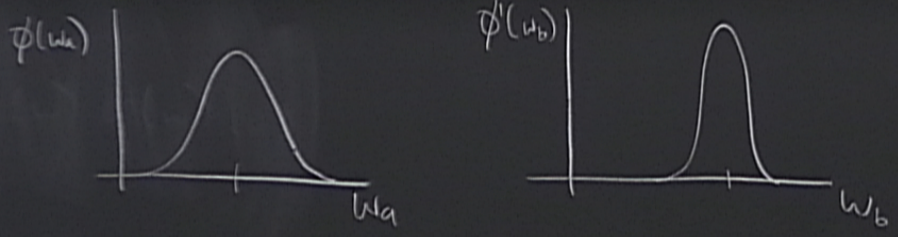

Multimode Analysis
For simplicity, consider
$\chi^{(2)}$
a
b


$\chi^{(2)}$
"click"
In reality
$\phi(\omega_a)$
ω_a
$\phi(\omega_b)$
ω_b

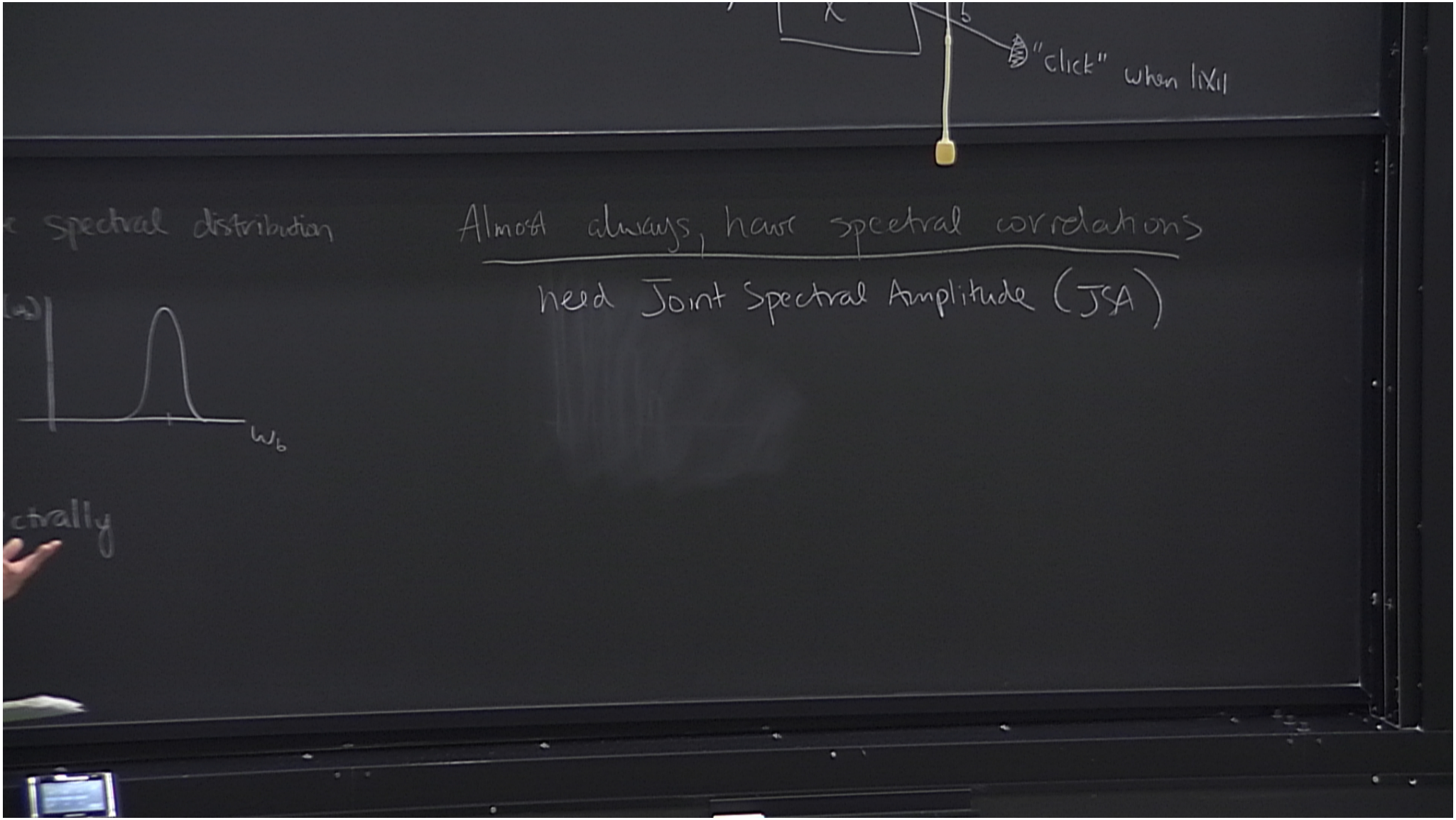
In reality, have spectral distribution

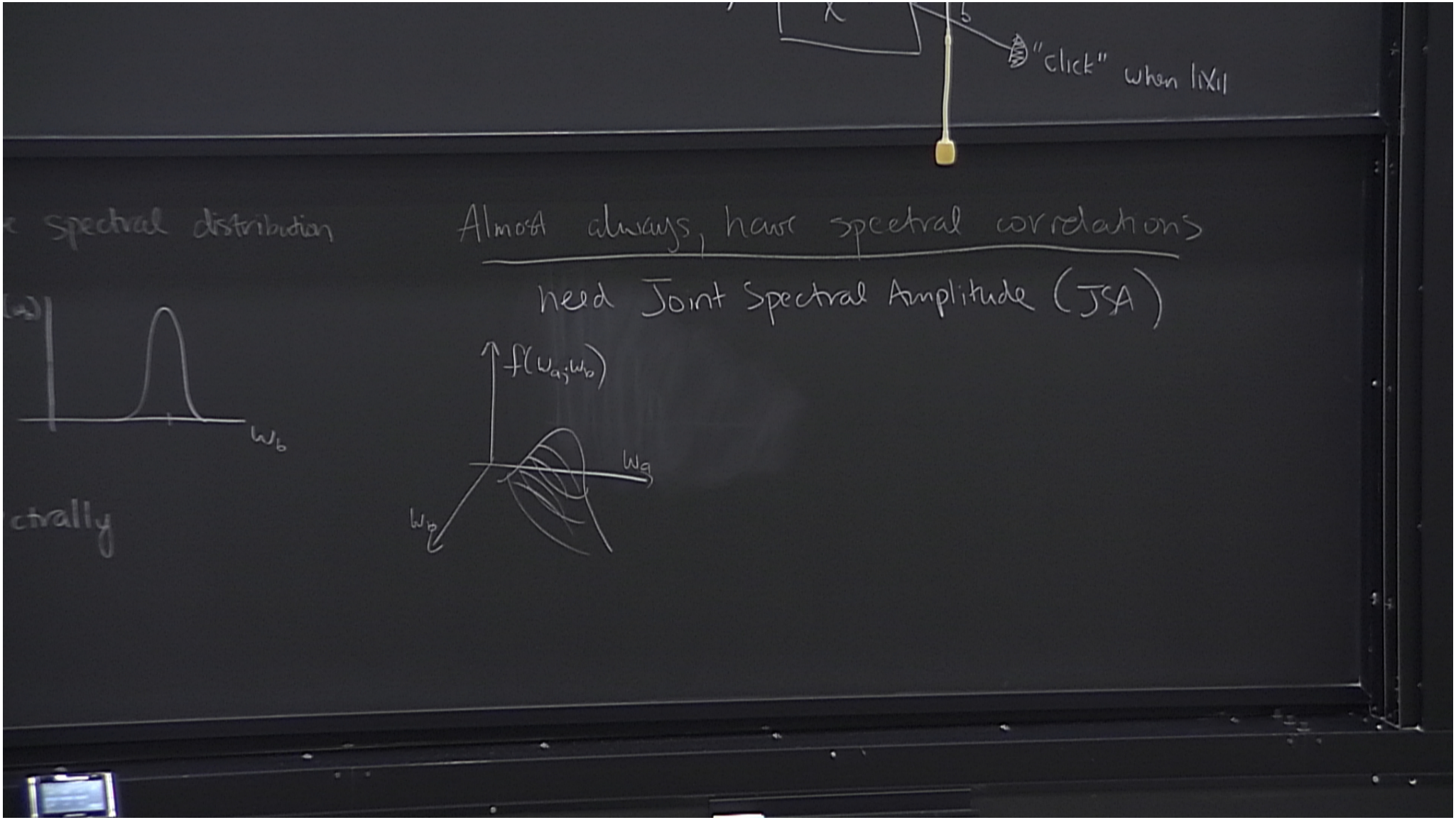
Almost always, h

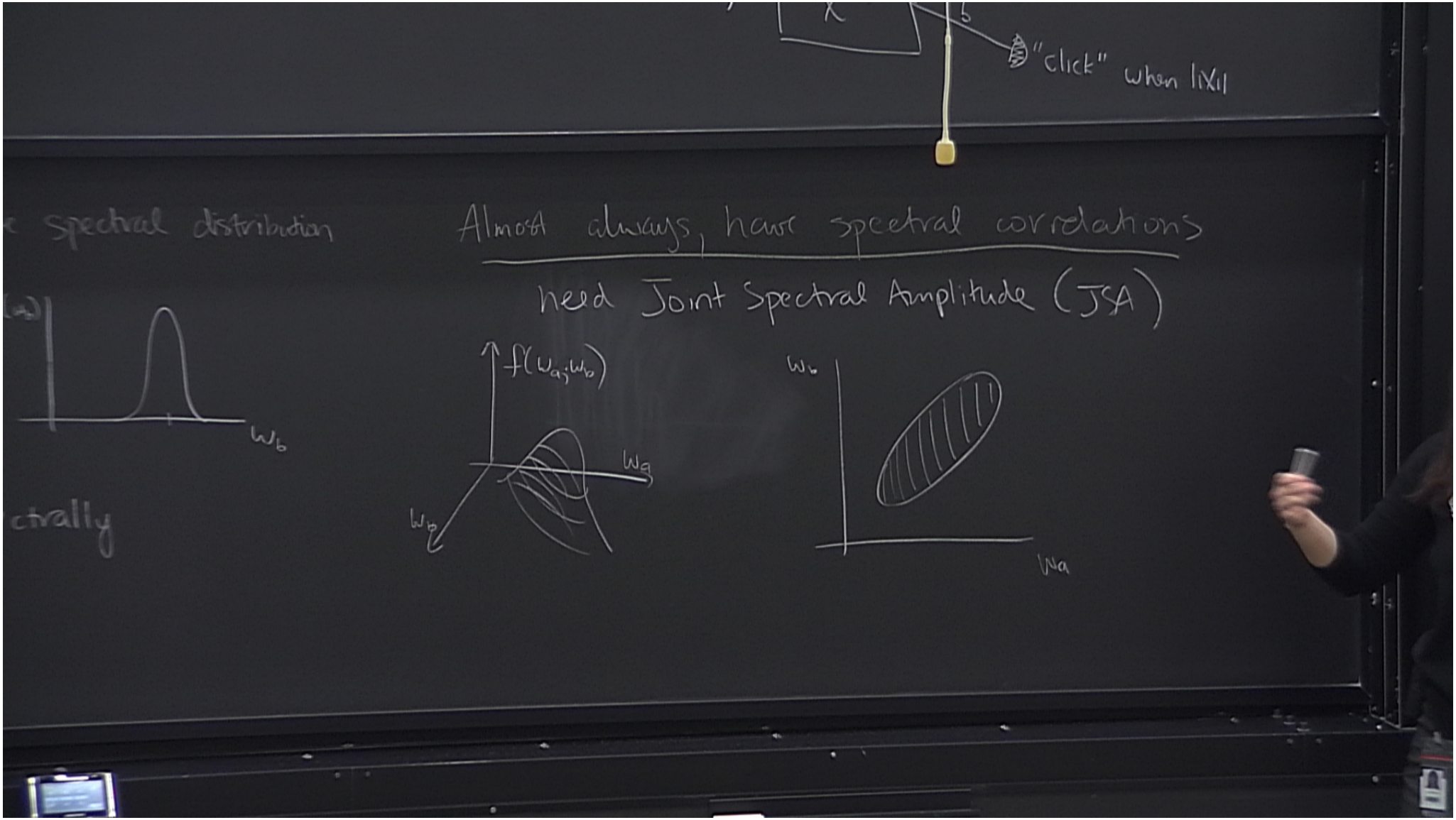
$|\eta_a\rangle|\eta_b\rangle$



This is ok if spectrally uncorrelated


"click" when ||x||
spectral distribution
Almost always, have spectral correlations
need Joint Spectral Amplitude (JSA)
ω_b
ctrally


"click" when ||x||
spectral distribution
Almost always, have spectral correlations
need Joint Spectral Amplitude (JSA)
$f(\omega_a, \omega_b)$
ω_a
ω_b
ω_b
ctrally


"click" when |X|
spectral distribution
Almost always, have spectral correlations
need Joint Spectral Amplitude (JSA)
$f(\omega_a,\omega_b)$
ω_a
ω_b
ω_b
ω_a
ctrally

Two photon state

$$|\psi\rangle = \int d\omega_a \int d\omega_b \, f(\omega_a, \omega_b)\, a^\dagger(\omega_a) b^\dagger(\omega_b) |0\rangle$$

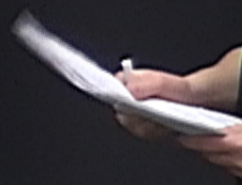
Two photon state

$$|\psi\rangle = \int d\omega_a \int d\omega_b \, f(\omega_a, \omega_b)\, a^\dagger(\omega_a) b^\dagger(\omega_b) |0\rangle$$

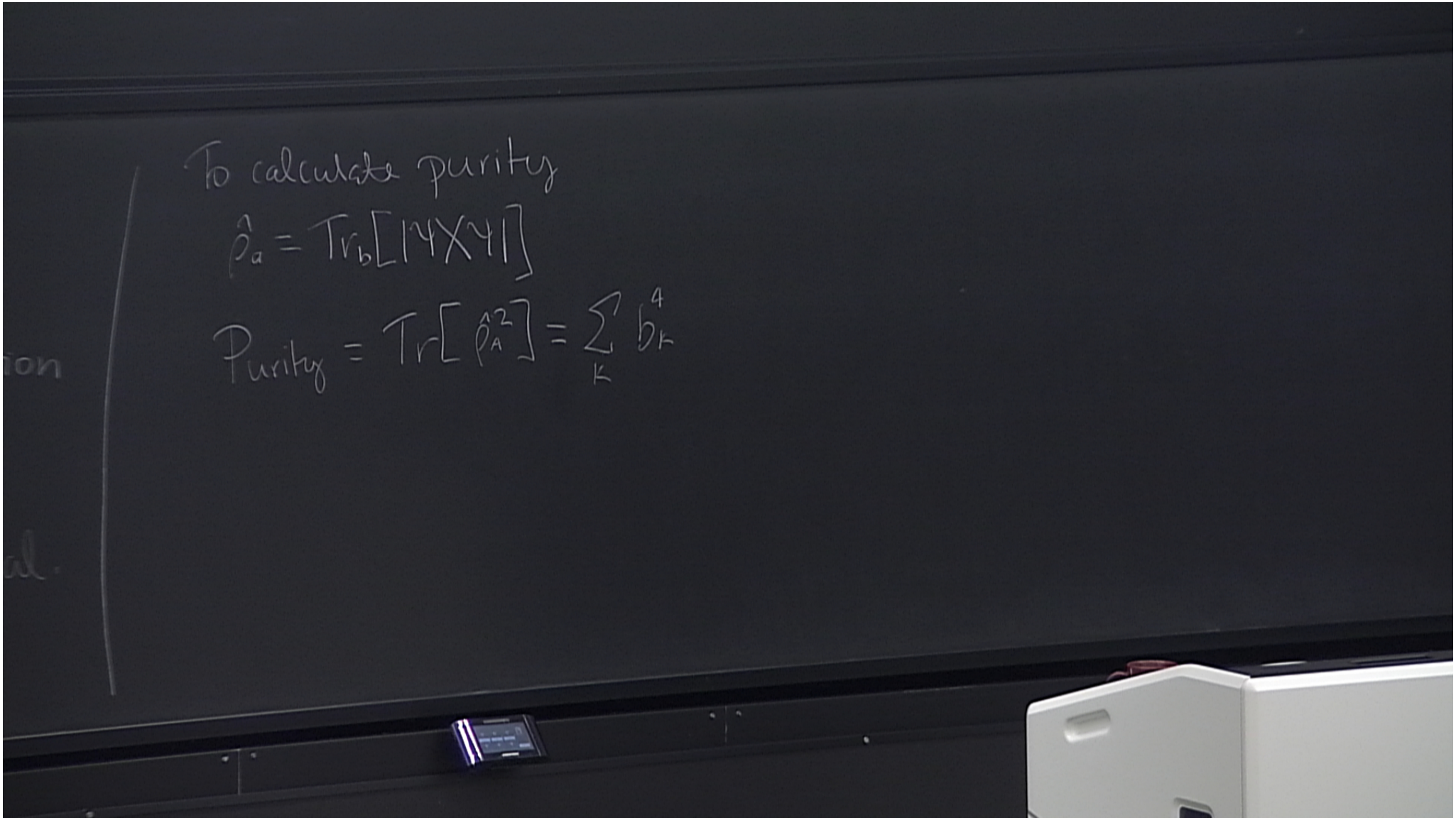
Can always do a Schmidt decomposition

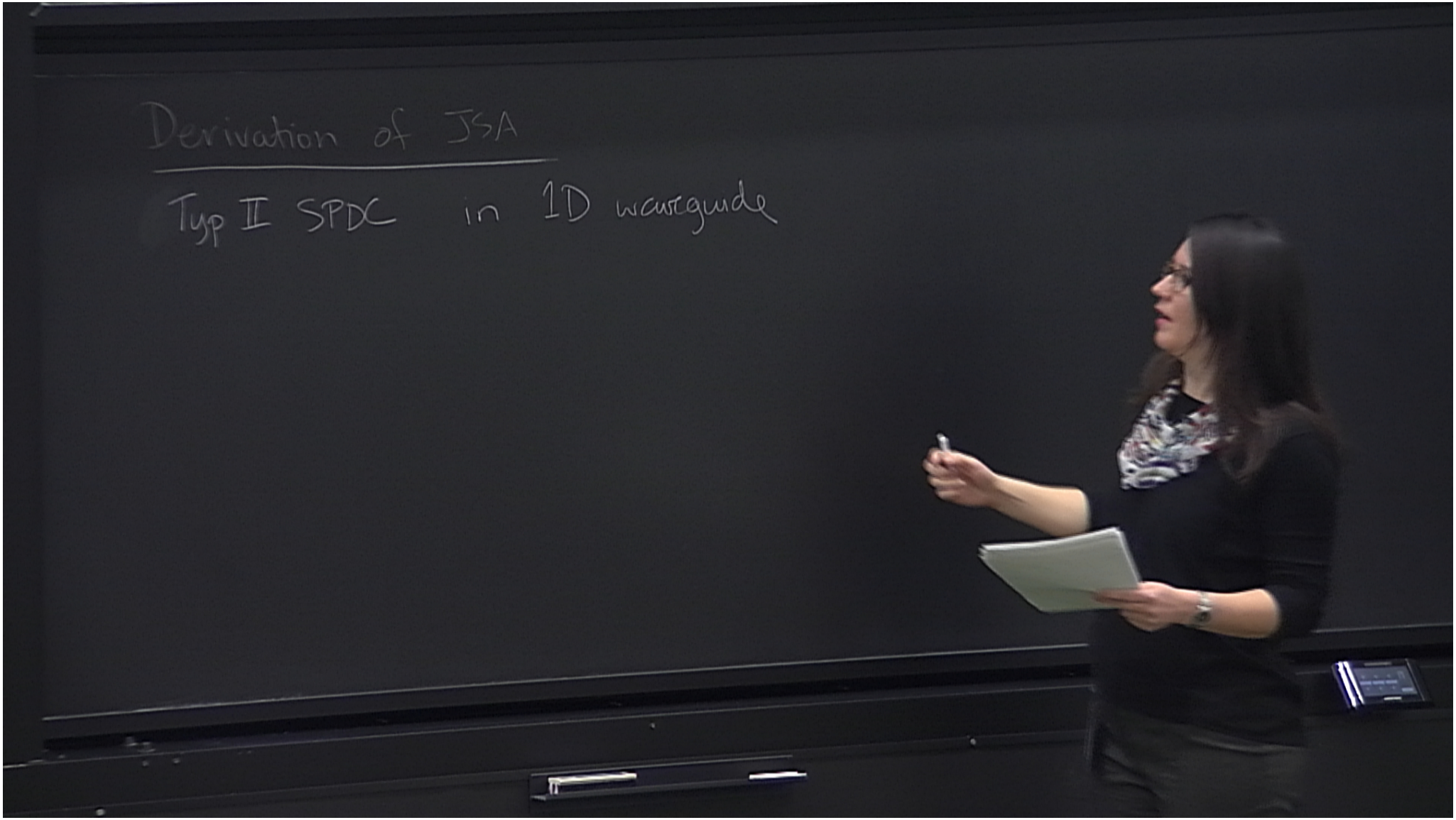
$$f(\omega_a, \omega_b) = \sum_k b_k \, g_k(\omega_a) h_k(\omega_b)$$

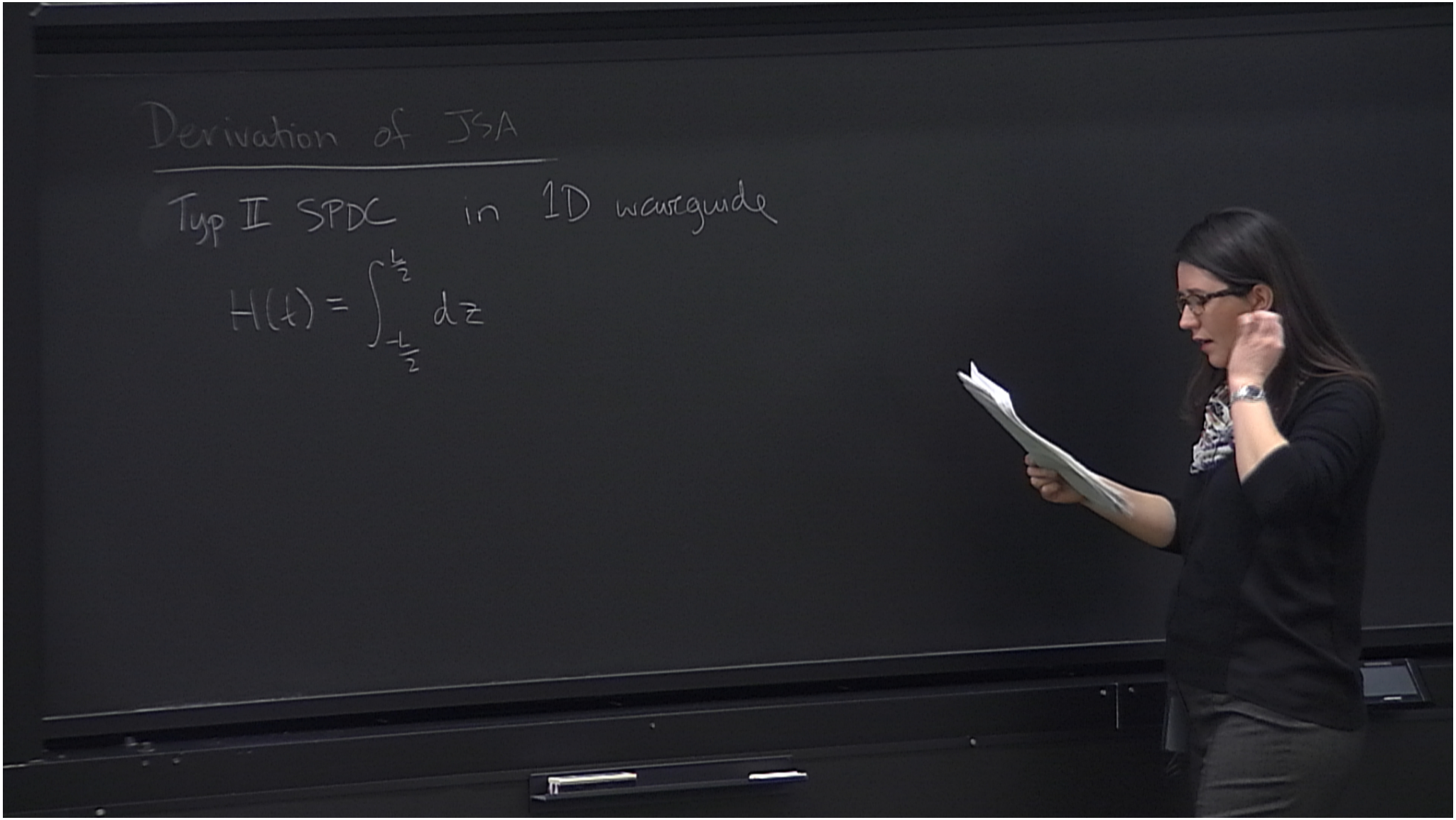
where $h_k(\omega_a)$ and $g_k(\omega)$ are orthonormal.

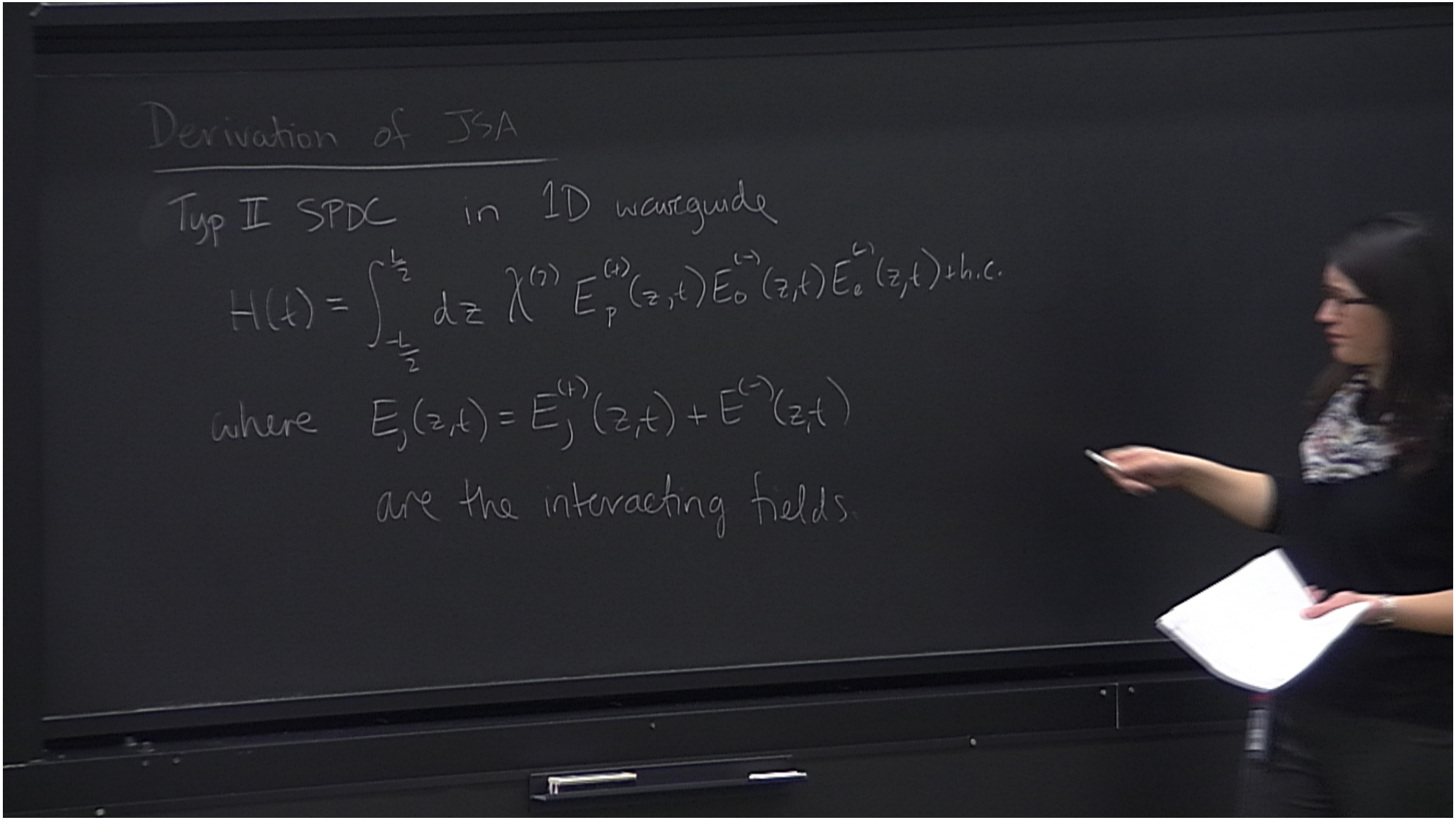


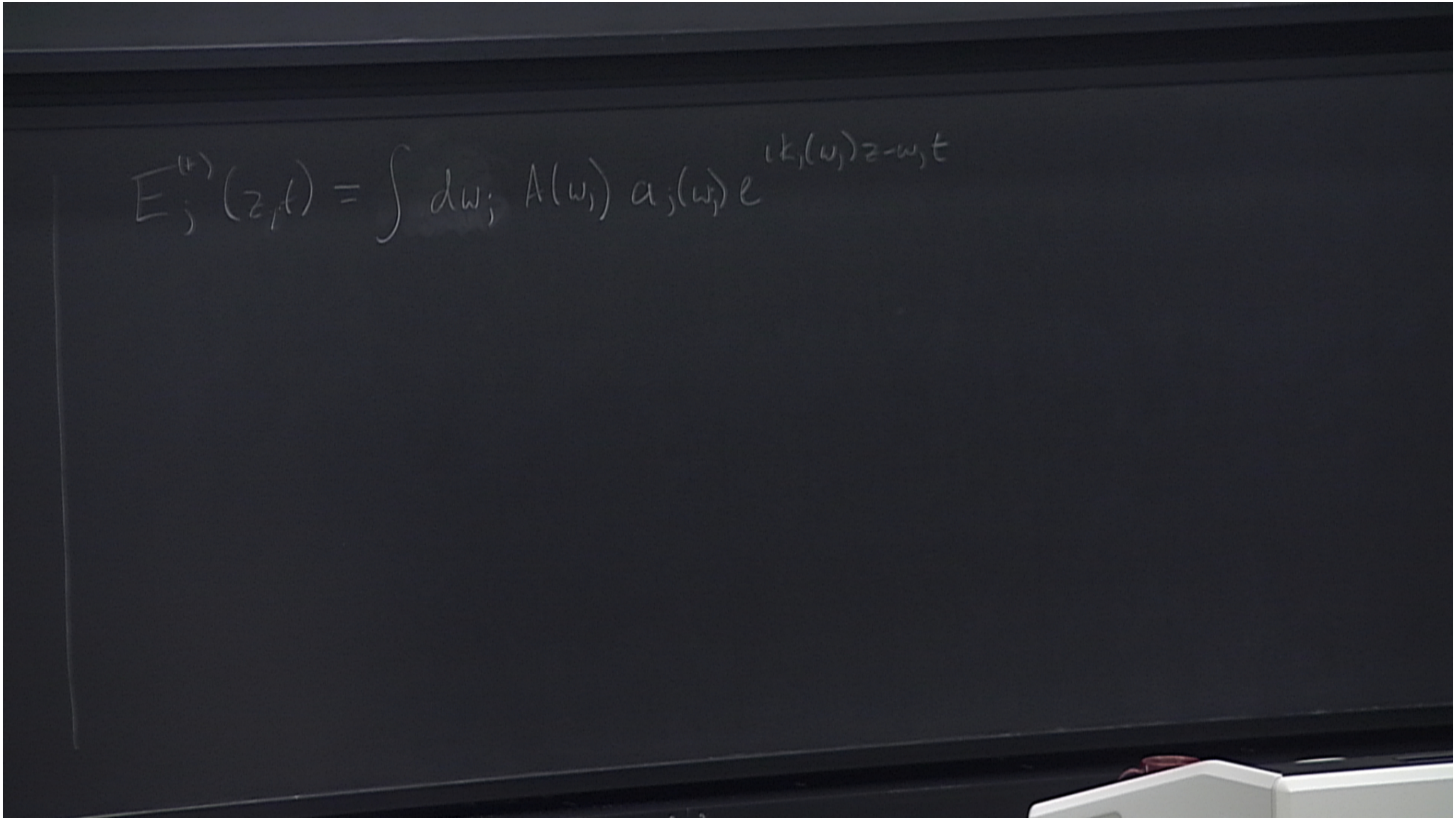

To calculate purity
$\hat{\rho}_a = Tr_b[|\psi\rangle\langle\psi|]$

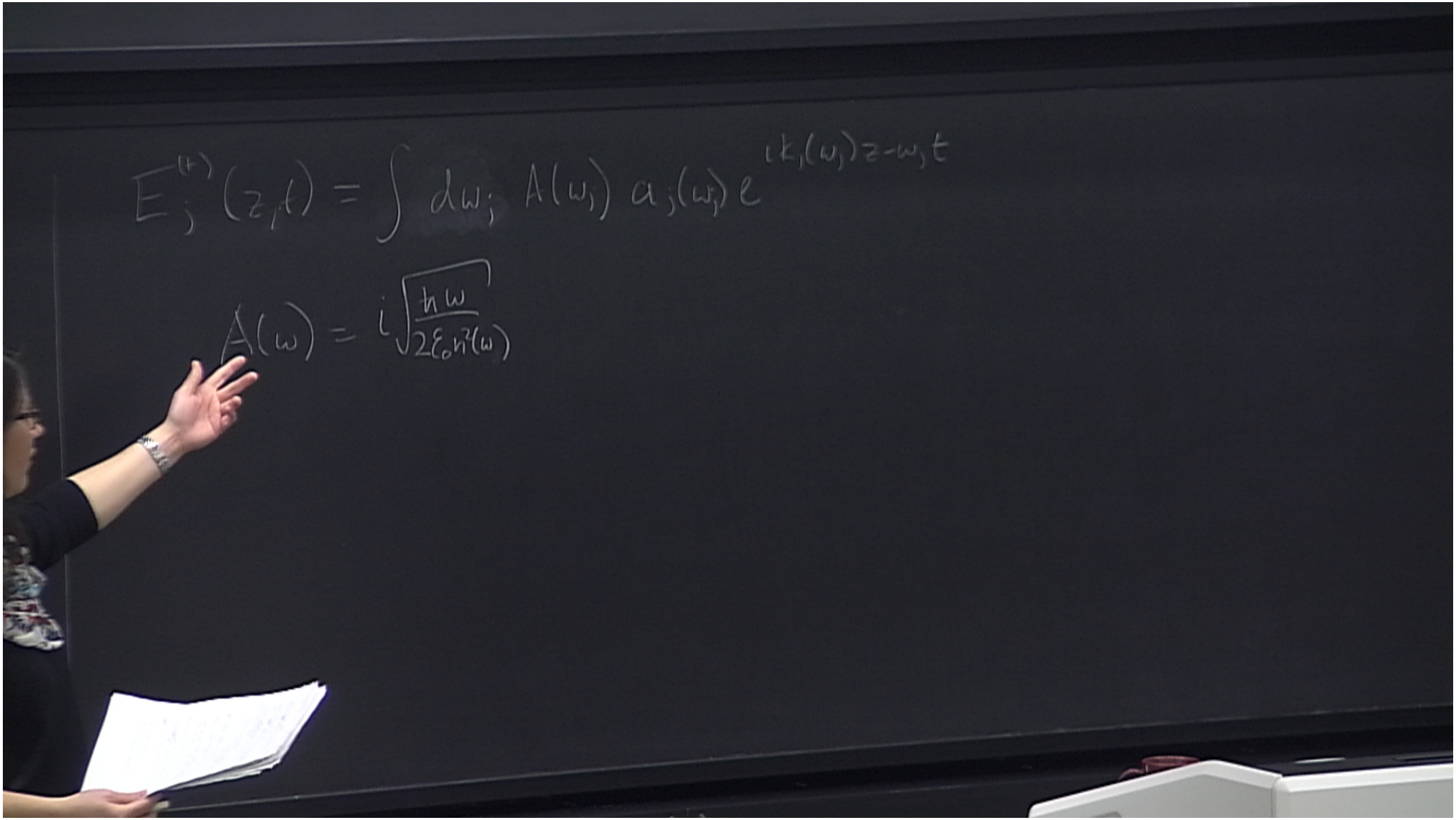

To calculate purity
$\hat{\rho}_a = Tr_b[|\psi\rangle\langle\psi|]$
Purity $= Tr[\hat{\rho}_A^2] = \sum_k b_k^4$

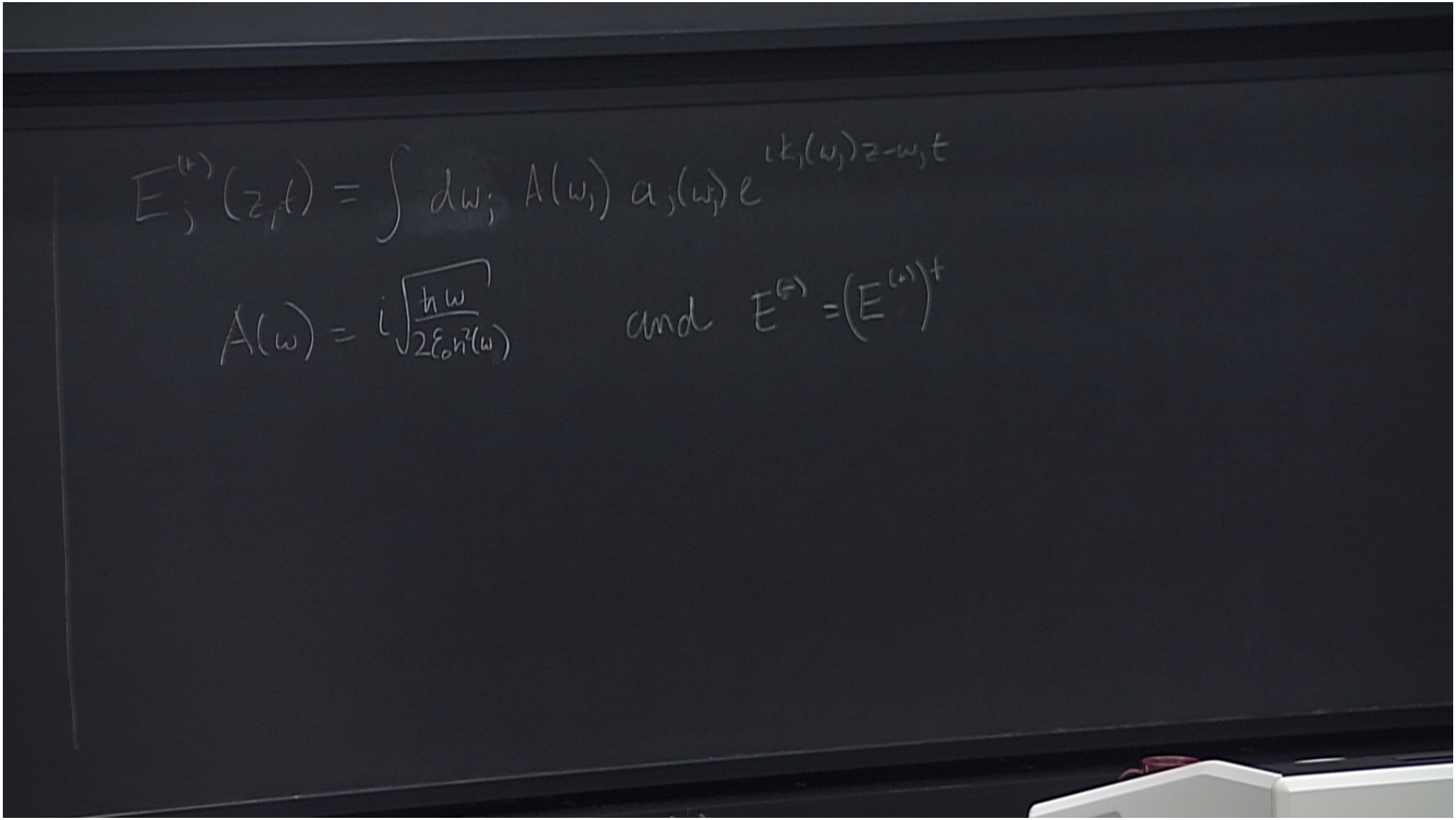

Derivation of JSA
Typ II SPDC in 1D waveguide

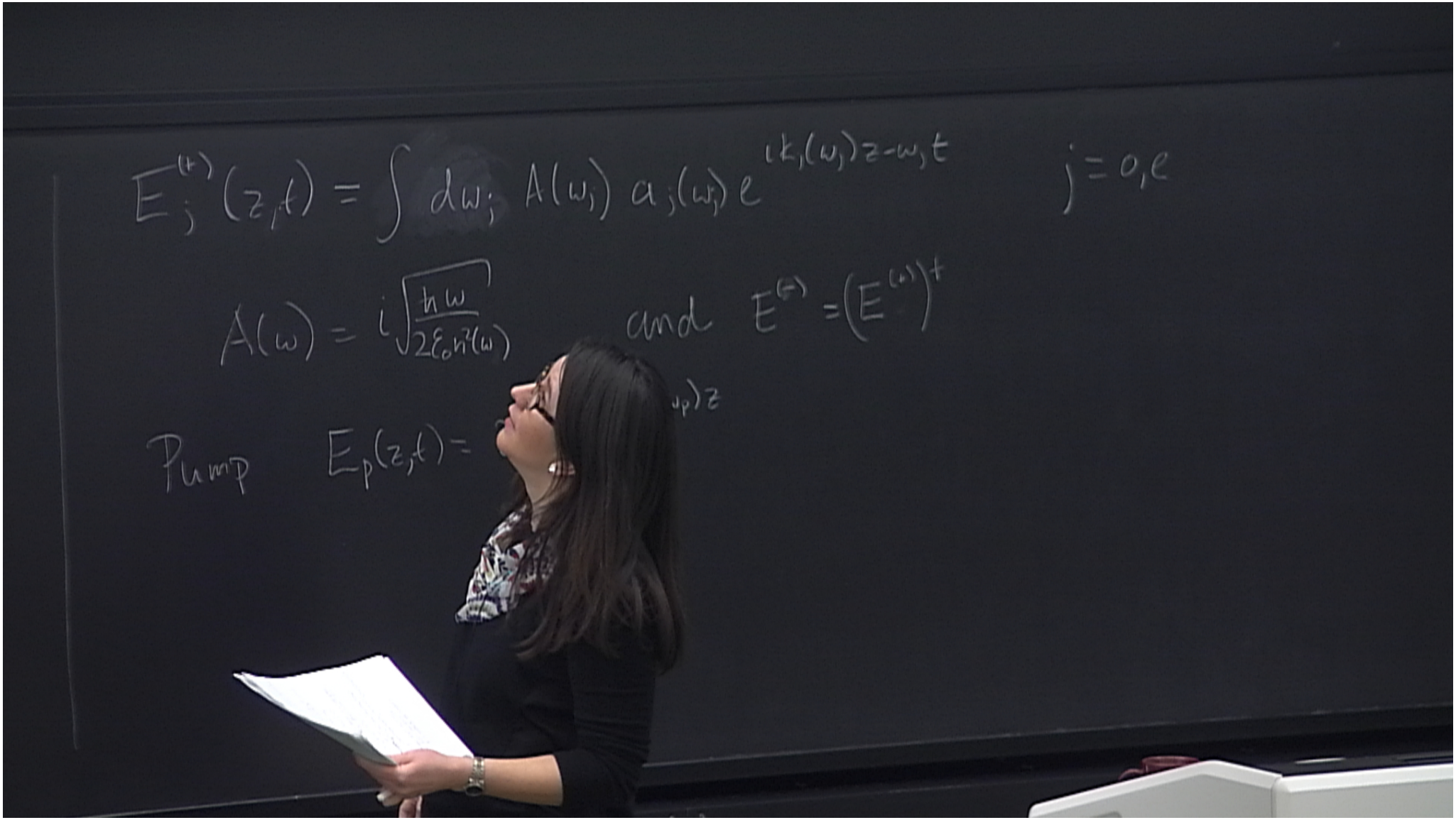

Derivation of JSA
Typ II SPDC in 1D waveguide
$H(t) = \int_{-\frac{L}{2}}^{\frac{L}{2}} dz$


Derivation of JSA
Type II SPDC in 1D waveguide
$H(t) = \int_{-\frac{L}{2}}^{\frac{L}{2}} dz\, \chi^{(2)} E_p^{(+)}(z,t) E_0^{(-)}(z,t) E_e^{(-)}(z,t) + h.c.$
where $E_j(z,t) = E_j^{(+)}(z,t) + E^{(-)}(z,t)$
are the interacting fields.


$E_j^{(+)}(z,t) = \int dw_j\, A(w_j)\, a_j(w_j)\, e^{i k_j(w_j) z - w_j t}$


$E_j^{(+)}(z,t) = \int d\omega_j \, A(\omega_j) \, a_j(\omega_j) \, e^{i k_j(\omega_j) z - \omega_j t}$
$A(\omega) = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 n^2(\omega)}}$

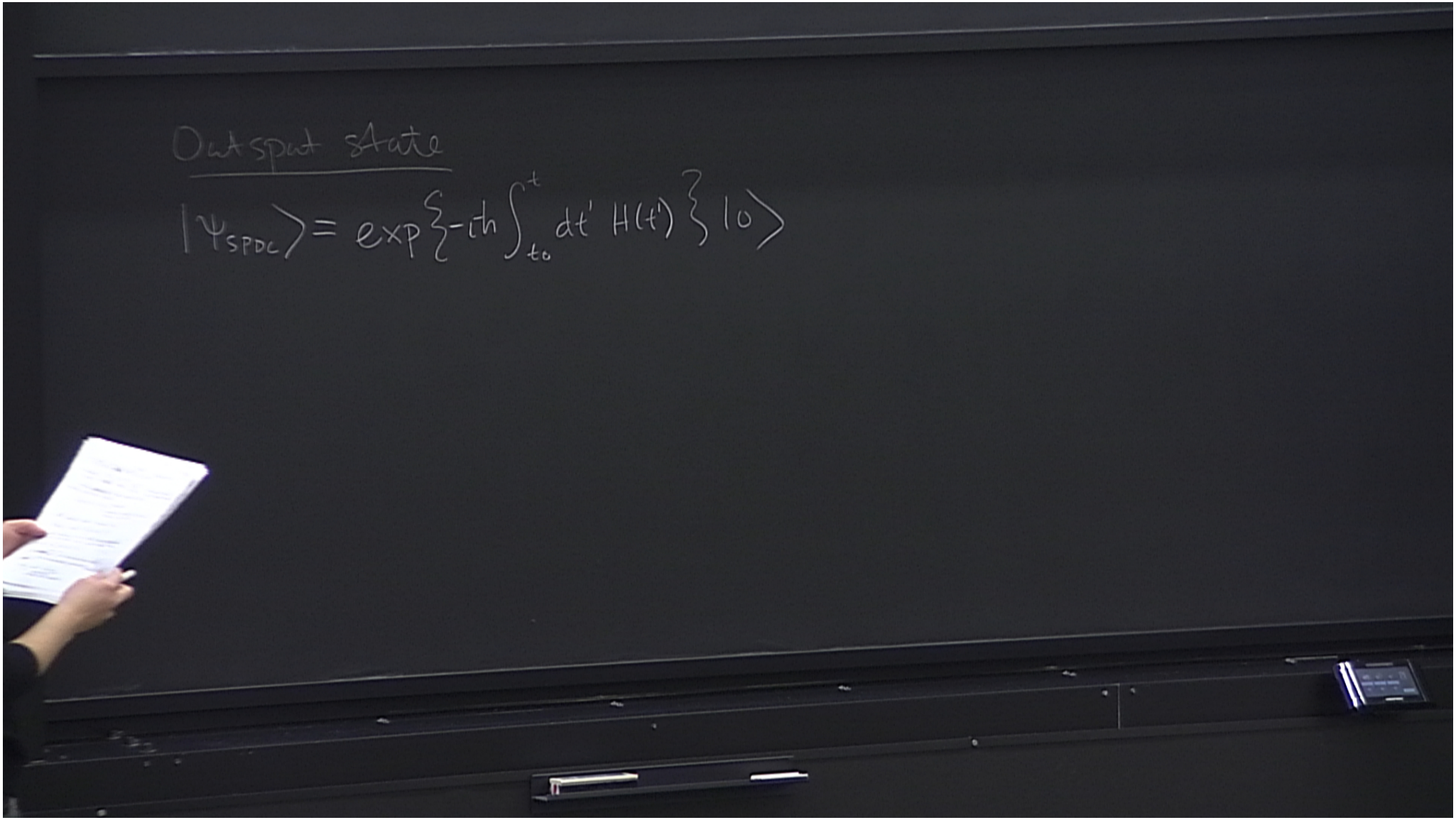

$E_j^{(+)}(z,t) = \int d\omega_j \, A(\omega_j) \, a_j(\omega_j) \, e^{i k_j(\omega_j) z - \omega_j t}$
$A(\omega) = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 n^2(\omega)}}$ and $E^{(-)} = \left(E^{(+)}\right)^\dagger$

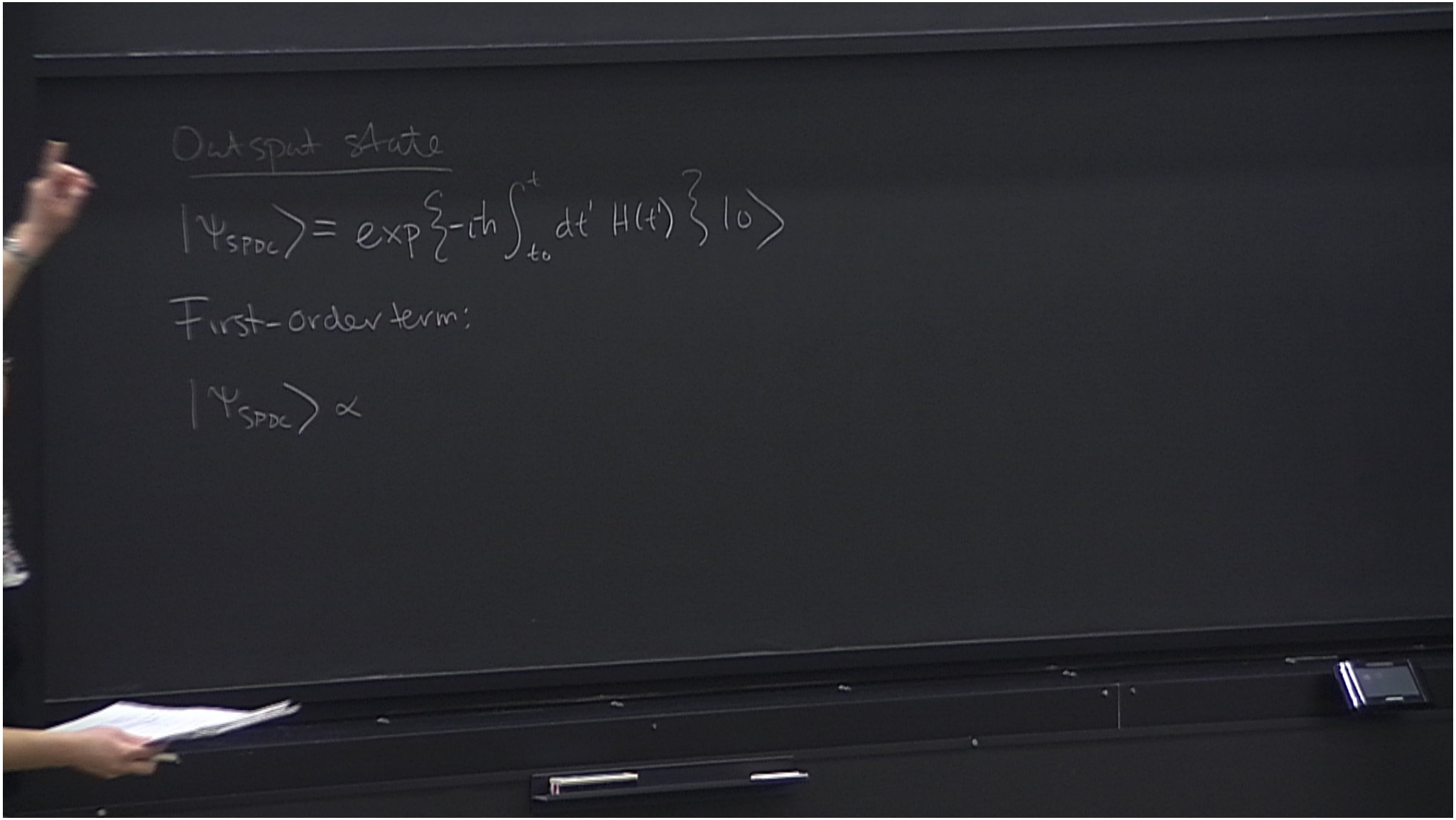

$E_j^{(+)}(z,t) = \int dw_j\, A(w_j)\, a_j(w_j)\, e^{i k_j(w_j) z - w_j t}$
$j = o, e$
$A(w) = i\sqrt{\frac{\hbar w}{2\varepsilon_0 n^2(w)}}$
and $E^{(-)} = (E^{(+)})^+$
Pump $E_p(z,t) =$
$v_p) z$

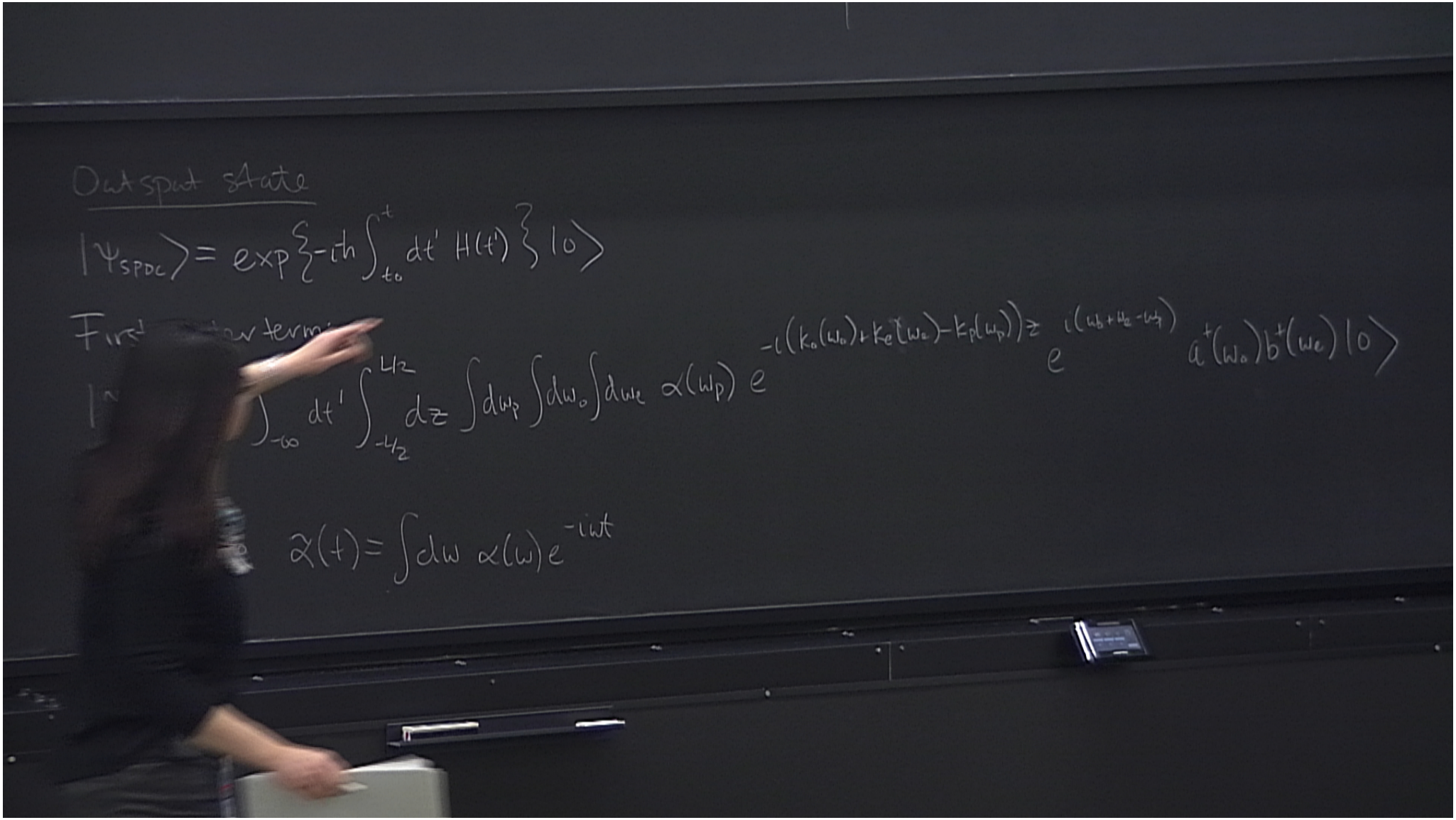
$$E_j^{(+)}(z,t) = \int d\omega_j \; A(\omega_j)\, a_j(\omega_j)\, e^{i k_j(\omega_j) z - \omega_j t} \qquad j = o, e$$

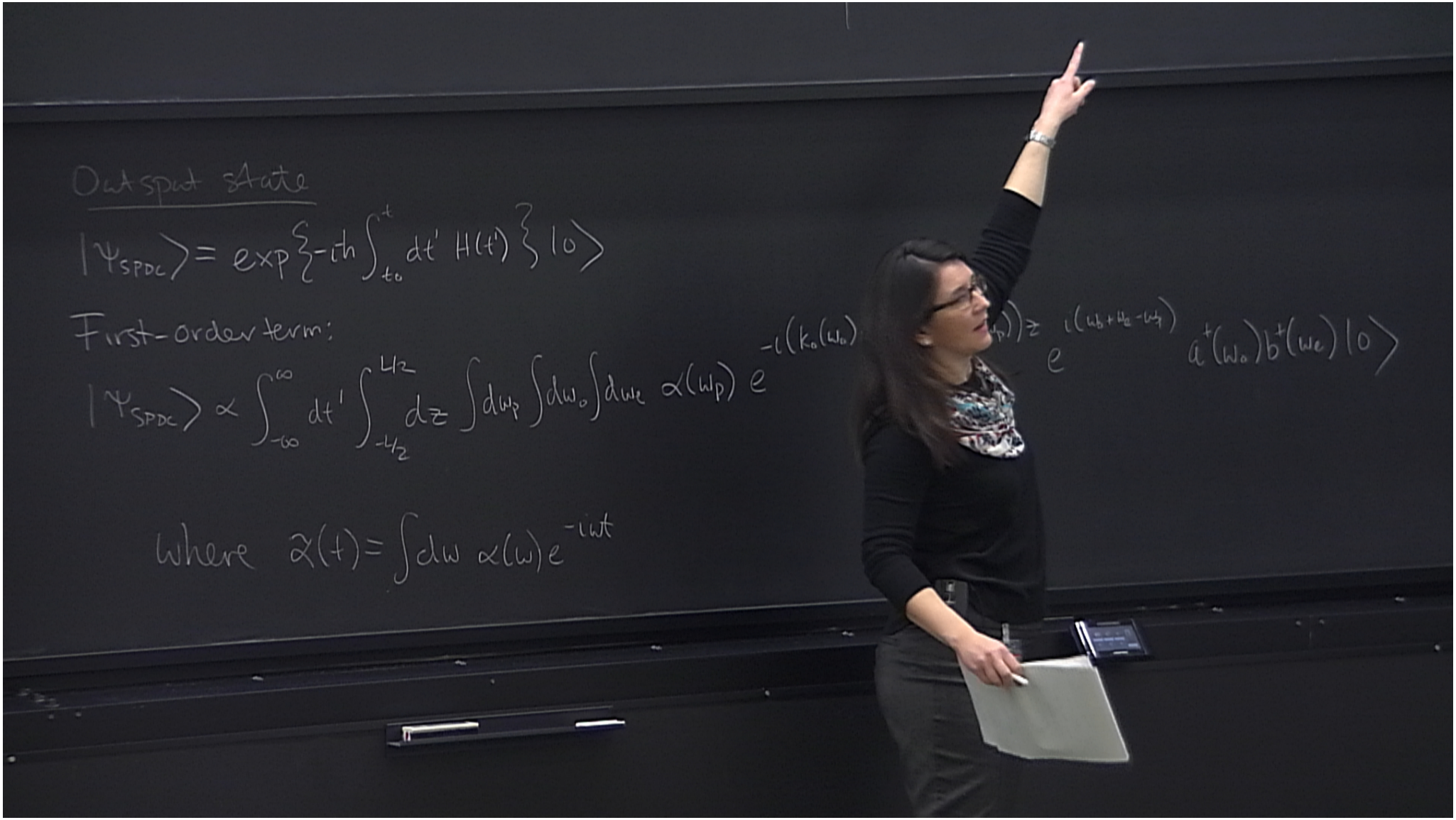
$$A(\omega) = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 n^2(\omega)}} \qquad \text{and} \qquad E^{(-)} = \left(E^{(+)}\right)^{\dagger}$$

Pump $\quad E_p(z,t) = \tilde{\alpha}(t)\, e^{i k_p(\omega_p) z}$


Output state
$|\psi_{SPDC}\rangle = \exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t} dt'\, H(t')\right\}|0\rangle$


Output state
$|\psi_{SPDC}\rangle = \exp\{-\frac{i}{\hbar}\int_{t_0}^{t} dt' H(t')\}|0\rangle$
First-order term:
$|\psi_{SPDC}\rangle \propto$


Output state
$|\psi_{SPDC}\rangle = \exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t} dt'\, H(t')\right\}|0\rangle$
First
$\int_{-\infty} dt' \int_{-L/2}^{L/2} dz \int d\omega_p \int d\omega_o \int d\omega_e\, \alpha(\omega_p)\, e^{-i(k_o(\omega_o)+k_e(\omega_e)-k_p(\omega_p))z}\, e^{i(\omega_o+\omega_e-\omega_p)}\, a^\dagger(\omega_o) b^\dagger(\omega_e)|0\rangle$
$\alpha(t) = \int d\omega\, \alpha(\omega) e^{-i\omega t}$


Output state
$|\Psi_{SPDC}\rangle = \exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t} dt'\, H(t')\right\}|0\rangle$
First-order term:
$|\Psi_{SPDC}\rangle \propto \int_{-\infty}^{\infty} dt' \int_{-L/2}^{L/2} dz \int d\omega_p \int d\omega_o \int d\omega_e\, \alpha(\omega_p)\, e^{-i(k_o(\omega_o)$
$e^{i(\omega_o+\omega_e-\omega_p)z}\, a^\dagger(\omega_o) b^\dagger(\omega_e)|0\rangle$
where $\alpha(t) = \int d\omega\, \alpha(\omega) e^{-i\omega t}$

Output State

$$|\Psi_{SPDC}\rangle = \exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t} dt'\, H(t')\right\}|0\rangle$$

First-order term:

$$|\Psi_{SPDC}\rangle \propto \int_{-\infty}^{\infty} dt' \int_{-L/2}^{L/2} dz \int d\omega_p \int d\omega_o \int d\omega_e\, \alpha(\omega_p)\, e^{-i\left(k_o(\omega_o)+k_e(\omega_e)-k_p(\omega_p)\right)z}\, e^{i(\omega_o+\omega_e-$$

where $\alpha(t) = \int d\omega_p\, \alpha(\omega_p)\, e^{-i\omega_p t}$

$$\int_{t_0}^{t} dt' H(t') \Big\} |0\rangle$$

$$= \int_{-L/2}^{L/2} dz \int dw_p \int dw_o \int dw_e \, \alpha(w_p) \, e^{-i\left(k_o(w_o) + k_e(w_e) - k_p(w_p)\right) z} \, e^{i(w_o + w_e - w_p)t} \, a^\dagger(w_o) b^\dagger(w_e) |0\rangle$$

$$= \int dw_p \, \alpha(w_p) \, e^{-i w_p t}$$

$$\int_{t_0}^{t} dt' H(t') \Big\} |0\rangle$$

$$\int_{-L/2}^{L/2} dz \int dw_p \int dw_o \int dw_e \, \alpha(w_p) \, e^{-i\left(k_o(w_o) + k_e(w_e) - k_p(w_p)\right) z} \, e^{i(w_o + w_e - w_p)t} \, a^\dagger(w_o) b^\dagger(w_e) |0\rangle$$

$$= \int dw_p \, \alpha(w_p) \, e^{-i w_p t}$$

Output state

$$|\Psi_{SPDC}\rangle = \exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t} dt'\, H(t')\right\}|0\rangle$$

First-order term:

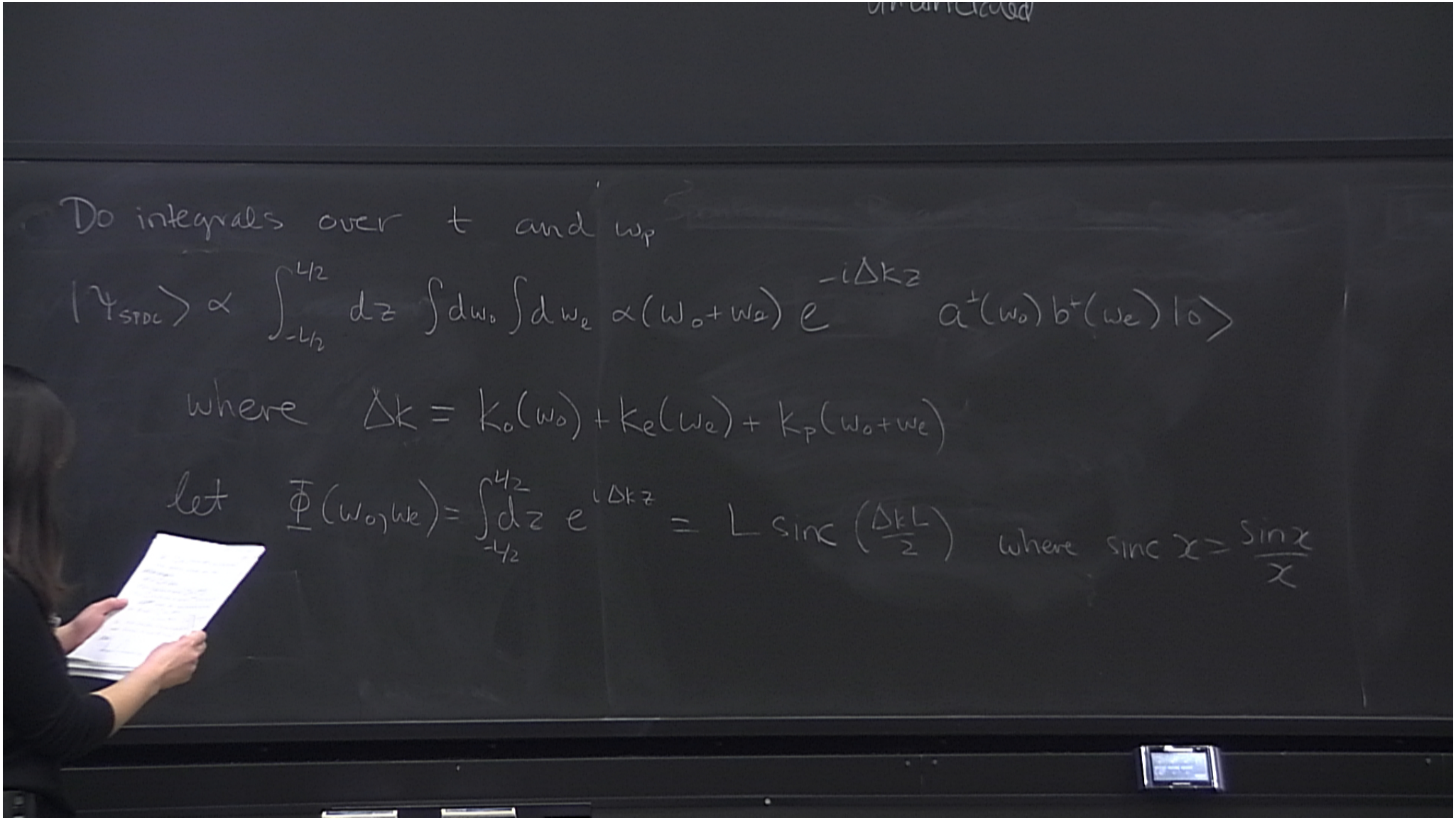
$$|\Psi_{SPDC}\rangle \propto \int_{-\infty}^{\infty} dt' \int_{-L/2}^{L/2} dz \int d\omega_p \int d\omega_o \int d\omega_e\, \alpha(\omega_p)\, e^{-i(k_o(\omega_o)+k_e(\omega_e)-k_p(\omega_p))z}\, e^{i(\omega_o+\omega_e-\omega_p)t}\, a^\dagger(\omega_o) b^\dagger(\omega_e)|0\rangle$$

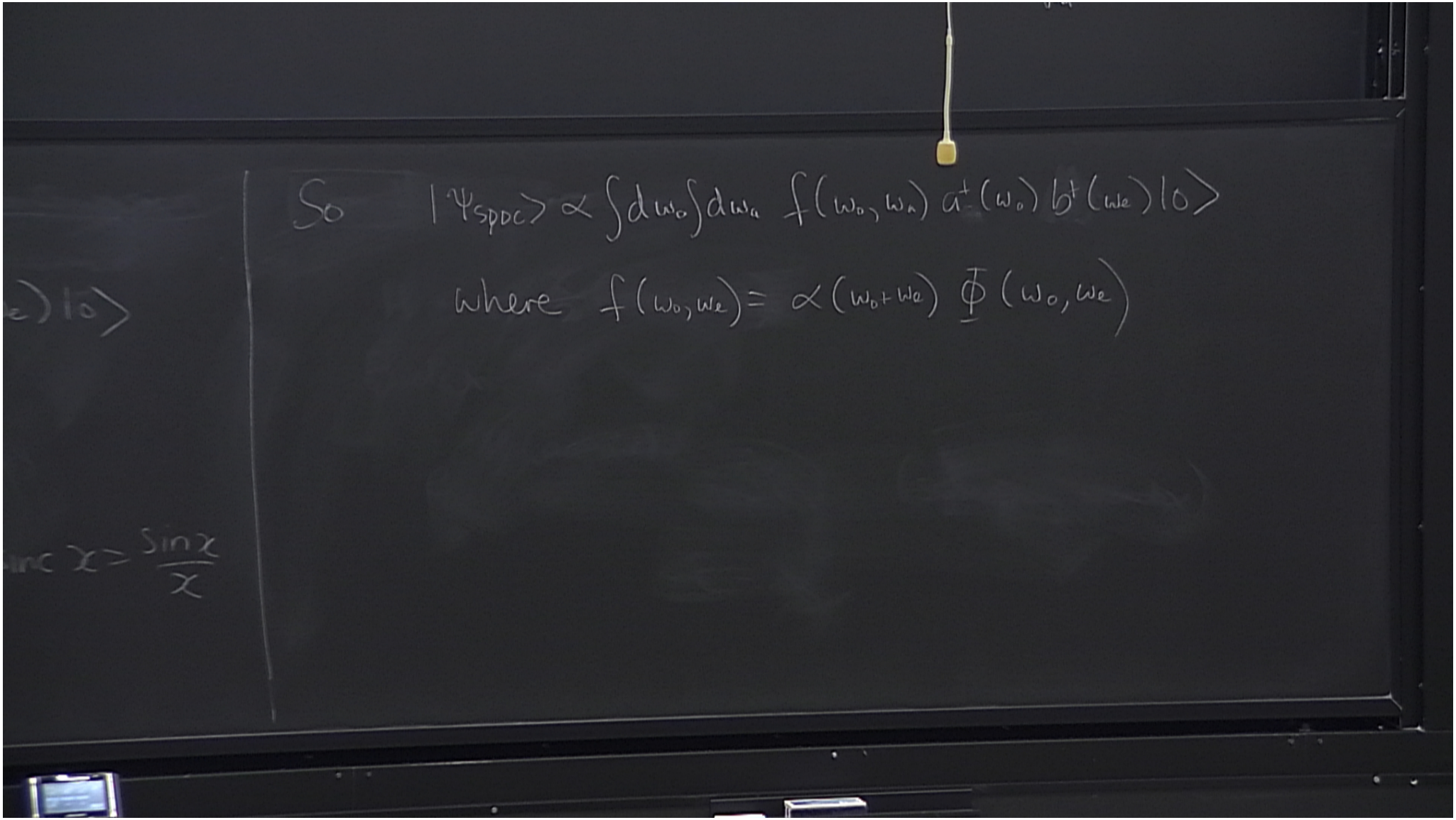
where $\alpha(t) = \int d\omega_p\, \alpha(\omega_p) e^{-i\omega_p t}$ $\qquad 2\pi\delta(\omega_o+\omega_e-\omega_p)$

Do integrals over t and ω_p

$$|\Psi_{SPDC}\rangle \propto \int_{-L/2}^{L/2} dz \int d\omega_o \int d\omega_e \, \alpha(\omega_o+\omega_e)\, e^{-i\Delta k z}\, a^\dagger(\omega_o)\, b^\dagger(\omega_e)\,|0\rangle$$

where $\Delta k = k_o(\omega_o) + k_e(\omega_e) + k_p(\omega_o+\omega_e)$


Do integrals over t and ω_p
$|\Psi_{SPDC}\rangle \propto \int_{-L/2}^{L/2} dz \int d\omega_o \int d\omega_e \, \alpha(\omega_o+\omega_e) \, e^{-i\Delta k z} \, a^\dagger(\omega_o) b^\dagger(\omega_e) |0\rangle$
where $\Delta k = k_o(\omega_o) + k_e(\omega_e) + k_p(\omega_o+\omega_e)$
let $\Phi(\omega_o,\omega_e) = \int_{-L/2}^{L/2} dz \, e^{i\Delta k z} = L \, \mathrm{sinc}\left(\frac{\Delta k L}{2}\right)$ where $\mathrm{sinc}\, x = \frac{\sin x}{x}$


So $|\psi_{SPDC}\rangle \propto \int d\omega_o \int d\omega_e \, f(\omega_o, \omega_e)\, a^\dagger(\omega_o)\, b^\dagger(\omega_e)|0\rangle$
where $f(\omega_o, \omega_e) = \alpha(\omega_o + \omega_e)\, \Phi(\omega_o, \omega_e)$
$\text{sinc}\, x = \frac{\sin x}{x}$

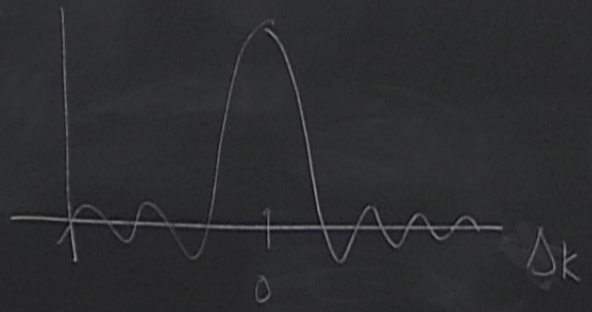
$|0\rangle$

$\text{sinc}\, x = \frac{\sin x}{x}$

So $|\Psi_{SPDC}\rangle \propto \int d\omega_o \int d\omega_e \; f(\omega_o, \omega_e)\, a^\dagger(\omega_o)\, b^\dagger(\omega_e)\, |0\rangle$

where $f(\omega_o, \omega_e) = \alpha(\omega_o + \omega_e)\, \underline{\Phi}(\omega_o, \omega_e)$

$\underline{\Phi}(\Delta k)$



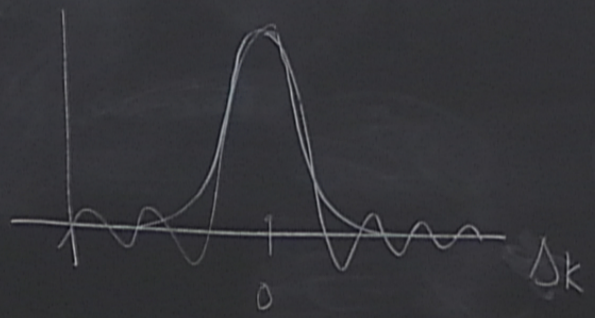
$|\psi\rangle$

$\text{sinc}\, x = \frac{\sin x}{x}$

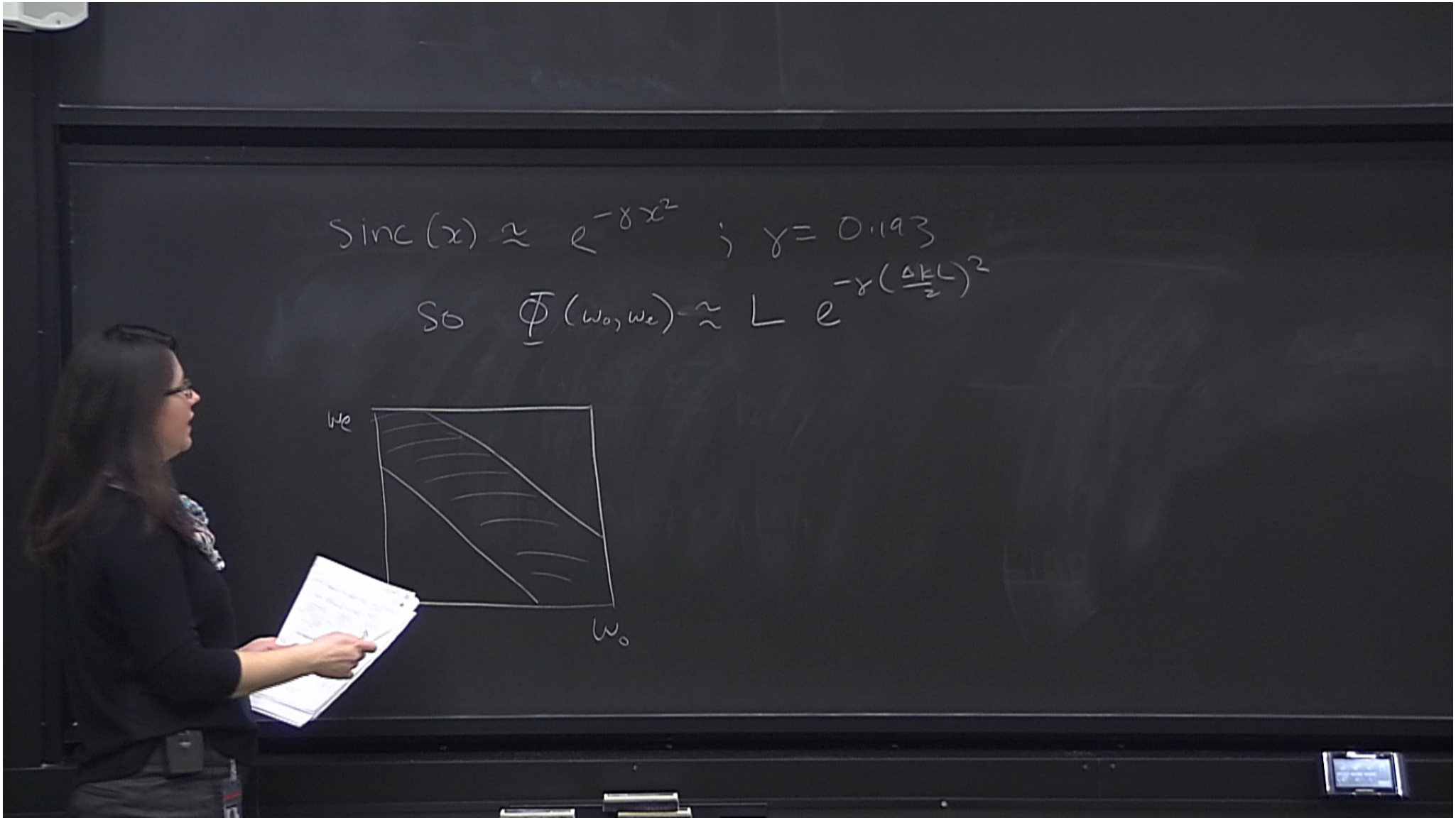
So $|\psi_{SPDC}\rangle \propto \int d\omega_0 \int d\omega_e \; f(\omega_0, \omega_e)\, a^\dagger(\omega_0)\, b^\dagger(\omega_e)|0\rangle$

where $f(\omega_0, \omega_e) = \alpha(\omega_0 + \omega_e)\, \underline{\Phi}(\omega_0, \omega_e)$

$\underline{\Phi}(\Delta k)$

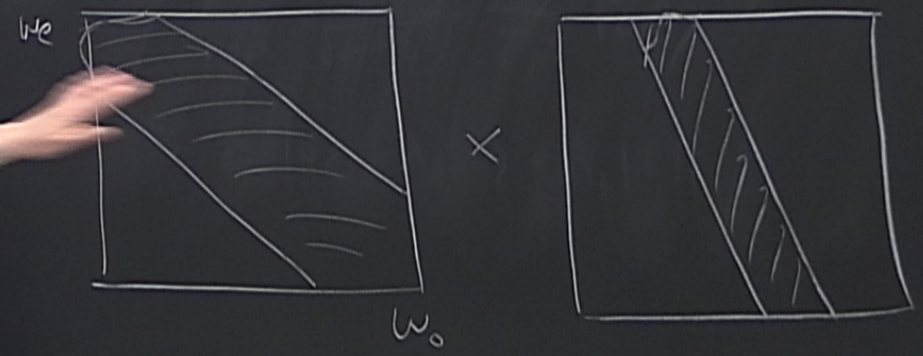


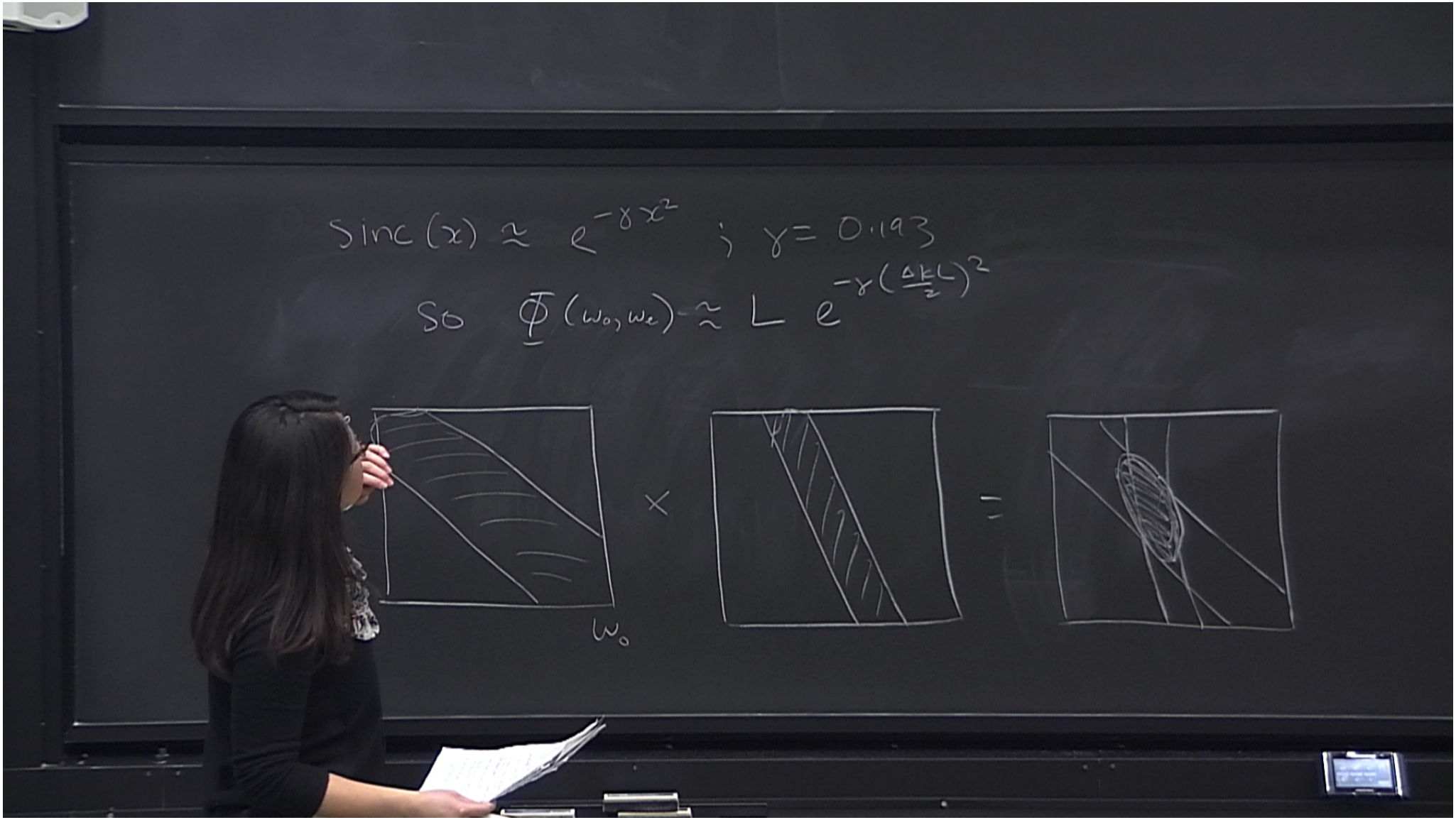
$|\psi_{SPDC}\rangle = S(\xi) = \lambda_0 |0\rangle_1 |0\rangle_2 + \lambda_1 |1\rangle_1 |1\rangle_2 + \lambda_2 |2\rangle_1 |2\rangle_2 + \ldots$

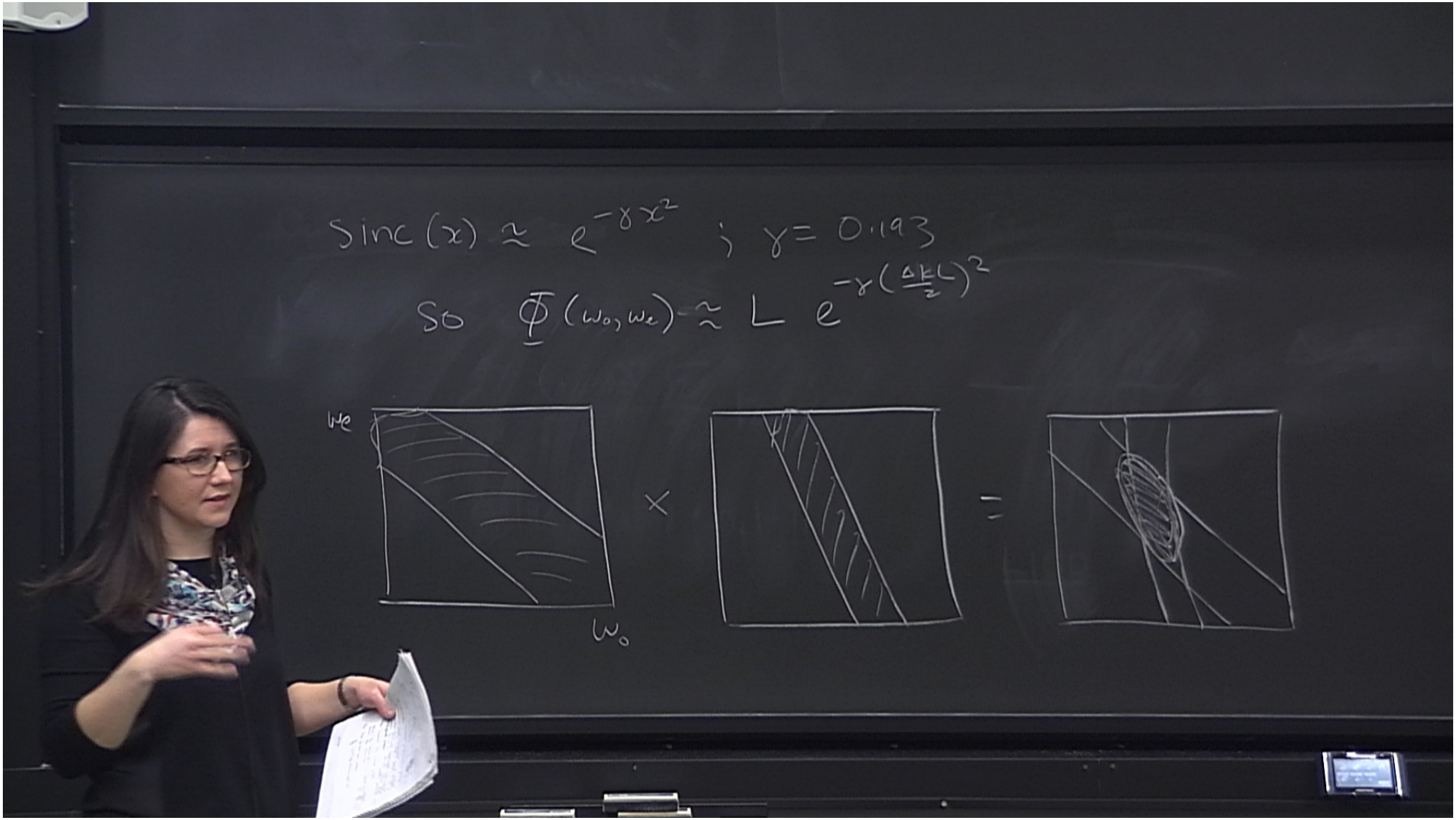

$\mathrm{sinc}(x) \approx e^{-\gamma x^2}$; $\gamma = 0.193$
So $\Phi(\omega_0, \omega_e) \approx L\, e^{-\gamma\left(\frac{\Delta E L}{2}\right)^2}$
ω_e
ω_0

$\mathrm{sinc}(x) \approx e^{-\gamma x^2}$; $\gamma = 0.193$

So $\Phi(\omega_0, \omega_e) \approx L\, e^{-\gamma\left(\frac{\Delta k L}{2}\right)^2}$




$\mathrm{sinc}(x) \approx e^{-\gamma x^2}$; $\gamma = 0.193$
So $\Phi(\omega_o, \omega_e) \approx L\, e^{-\gamma\left(\frac{\Delta k L}{2}\right)^2}$
ω_o


$\mathrm{sinc}(x) \approx e^{-\gamma x^2}$; $\gamma = 0.193$
So $\Phi(\omega_0, \omega_e) \approx L\, e^{-\gamma\left(\frac{\Delta k L}{2}\right)^2}$
ω_e
ω_0
×
=